The Quantization of Linear Dynamical Systems I: Finite Systems

Spoken by JNB, but all due to Adam Caulton (adam.caulton@balliol.ox.ac.uk);
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This document, and its succcessor on the Quantization of Linear Dynamical Systems with Infinitely many degrees fo freedom, expound a rigorous quantization procedure developed by Irving Segal and others in the 1960s. This means we postpone to the second half of term, coverage of algebraic quantum theory; which will include e.g. inequivalent representations, 'getting out of Fock space', Haag's theorem etc. (cf. eg Emch 1972). But the present material:

(i) gives a strong grip on the first (forbiddingly concise!) third of Wald 1994, which is the basis for the rest of that book on QFT in curved spacetime and thus e.g. the Unruh effect (an essay!);

(ii) is of intrinsic interest... though please be warned that here you will find: no Lagrangian, no path integrals, no renormalization, no gauge theory, no curved spacetime, no gravitation; indeed, no interactions, and overall, not much physics ... we will focus on the harmonic oscillator (!),the free KG field and spin-chains (and without putting a Hamiltonian on the chain...). Nor will you find much straight-up philosophy ... but perhaps the light here shed on field/wave vs.particle counts as philosophy, since wave vs.particle is, like continuum vs. discrete, a perennial dichotomy of *natural philosophy...*

In this document, we consider only finitely many degrees fo freedom, and lead up to the Stone-von Neumann Theorem, which essentially guarantees that the quantization of point particles in $\mathbb{R}^n$ is unique. We begin by introducing the Weyl form of the CCRs; and posing the quest for its representations (Section 1). Then we present the complexification and realification of vector spaces, complex structures etc. (Section 2); and symplectic vector spaces and manifolds (Section 3). Then we present linear systems, both classical and quantum; and thus the harmonic oscillator (Section ??). With all this in hand, we can then see the task of quantization as "unitarizing" a Hamiltonian evolution in a symplectic space so as to give an evolution in a complex Hilbert space. This gives the idea of a *one particle structure*, both in general and for the harmonic oscillator as an example (Section 5). The key to successful quantization, which see at work in the harmonic oscillator example, turns out to be the *two out of three property* of the unitary group: which concerns its relation to certain orthogonal and symplectic groups (Section 6)). Then we treat the case of finitely many harmonic oscillators, and so the occupation number representation: which can be described in a "Fock-space way" (Section 7). Finally, we state (i) the Stone-von Neumann Theorem; and (ii) an analogous theorem (the Jordan-Wigner theorem) about the uniqueness of the representation of the CARs (as against CCRs) of a *finite* system, such as a spin chain (Section 8).

Mottoes:

Let us try to introduce a quantum Poisson Bracket which shall be the analogue of the classical one....we are thus led to the following definition for the quantum Poisson Bracket of any two variables u and v: $uv - vu = i\hbar[u, v]$. Dirac (1930/1958, Section 21)

There is thus a complete harmony between the wave and light-quantum descriptions of the interaction. (Dirac, 1927, p. 245).

First quantization is a mystery, but second quantization is a functor. (E.Nelson).

Probably all these connections would have been clarified long ago, if quantum physicists had not been hampered by a prejudice in favor of complex and against real numbers. (Freeman Dyson)

The life of a theoretical physicist consists of solving harmonic oscillator at ever higher levels of abstraction. (S. Coleman)

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