The Quantization of Linear Dynamical Systems II: Infinite Systems

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This document, and its predecessor on the Quantization of Linear Dynamical Systems with *Finitely* many degrees fo freedom, expound a rigorous quantization procedure developed by Irving Segal and others in the 1960s. This means we postpone to the second half of term, coverage of algebraic quantum theory; which will include e.g. inequivalent representations, 'getting out of Fock space', Haag's theorem etc. (cf. eg Emch 1972). But the present material:

(i) gives a strong grip on the first (forbiddingly concise!) third of Wald 1994, which is the basis for the rest of that book on QFT in curved spacetime and thus e.g. the Unruh effect (an essay!);

(ii) is of intrinsic interest... though please be warned that here you will find: no Lagrangian, no path integrals, no renormalization, no gauge theory, no curved spacetime, no gravitation; indeed, no interactions, and overall, not much physics ... we will focus on the harmonic oscillator (!),the free KG field and spin-chains (and without putting a Hamiltonian on the chain...). Nor will you find much straight-up philosophy ... but perhaps the light here shed on field/wave vs.particle counts as philosophy, since wave vs.particle is, like continuum vs. discrete, a perennial dichotomy of *natural philosophy...*

The 'bottom-line' for Parts I and II together is that we have a procedure for constructing a representation of the Weyl algebra for any of a special class of classical systems. The simple harmonic oscillator and the free real bosonic field both belong to this class, but only in the case of the simple harmonic oscillator does this construction pick out a unique representation.

We begin in Section 1 by recalling from Part I:

(i) quantization as the construction of a representation of the *Weyl algebra* associated with some classical system's phase space; and as "unitarizing" a Hamiltonian evolution in a symplectic space so as to give an evolution in a complex Hilbert space;

(ii) the idea of a *one particle structure*;

(iii) the Stone-von Neumann Theorem, which essentially guarantees that the quantization of the paradigm *finite* system, viz. point particles in $\mathbb{R}^n$, is unique (up to unitary equivalence).

Then we work up slowly to the free real bosonic field. We first look at ways the premises of the Stone-von Neumann Theorem can fail: viz. with

(a) failure of weak continuity (Section 2);

(b) a classical configuration space other than $\mathbb{R}^n$, e.g. the circle S_1 (Section 3).

Besides, while we saw in Part I that if we wish to represent the CARs, not the CCRs, on a *finite* system, for example on a finite spin chain, then there is uniqueness (up to unitary equivalence): for an *infinite* system, e.g. an infinite spin chain, one can easily show by construction that uniqueness fails (Section 4).

In the last two Sections we describe the free real bosonic field. Section 5 describes the free boson field on any one particle structure. In effect, this is an exposition of symmetric Fock space without regard to the details of dynamics. Finally, section 6 focusses exclusively on the free real bosonic field, subject to the Klein-Gordon equation, and various interpretative issues, including particle localization and the interpretation of the local field operators $\Phi(\mathbf{x})$.

Mottoes:

There is thus a complete harmony between the wave and light-quantum descriptions of the interaction. (Dirac, 1927, p. 245).

First quantization is a mystery, but second quantization is a functor. (E.Nelson).

The life of a theoretical physicist consists of solving the harmonic oscillator at ever higher levels of abstraction. (S. Coleman)

Contents

Sections 1 to 4 owe much to Chapters 2 and 3 of Ruetsche (2011). Sections 5 and 6 are based on Baez *et al* (1992, Section 1) and Halvorson (2001).