On Spacelike Correlations in Algebraic Quantum Field Theory

J. Butterfield: for *Philosophical Aspects of QFT* on 4 Feb 2020 ...

I will review some conceptual aspects of non-local correlations in algebraic quantum field theory (AQFT) on Minkowski or curved spacetime: viz. some aspects of the violation of the Bell inequality in AQFT. This summary will involve:

1. Reviewing a few very basic ideas of AQFT, emphasising the three conditions of relativistic causality which have been much studied in Minkowksi spacetime; viz. primitive causality, spacelike commutativity, and the spectrum condition; (Section 1).

2. Discussing how these conditions can be adapted to a curved spacetime. Here the main topic is the spectrum condition, and I shall report a recent development about it by Hollands and Wald; (Section 2).

3. Reviewing the generic violation of Bell inequalities in AQFT. I will confine myself to reporting some results up to 2000 by physicists (especially Landau, Summers and Werner) and philosophers (Clifton and Halvorson). The broad picture is that for most algebraic quantum field theories on Minkowski spacetime, maximal violations of the Bell inequality are endemic (Section 3).

4. Making some remarks about the prospects for "peaceful co-existence" between relativity and quantum theory. In particular, this will include some discussion of how Section 3's results relate to the assumptions of Bell-type theorems; (Section 4).

Some references (among many possible!):— Landsman's review of Haag's book, [1], supplements Section 1. Section 3.2 of my paper, [2], supplements Section 2. Papers [3,4] by Clifton and Halvorson, which are reprinted as Chapters 6 and 7 of his posthumous *Quantum Entanglements: Selected Papers*, supplement Sections 3 and 4. Landsman's [5], and Section 4 of my paper, [6], also helps to supplement Section 4.

1 AQFT introduced

The basic idea of AQFT is to associate with each bounded region $\mathcal{O}$ of Minkowski spacetime an algebra $\mathcal{A}(\mathcal{O})$ subject to certain axioms. Observables are self-adjoint elements of these algebras. The basic idea of the association of an observable A with a region $\mathcal{O}$ is that A is a physical quantity pertaining to that part of the field system lying in O, and so is measurable by a procedure confined to O. On the other hand:

a state is taken as an expectation functional on all the algebras. (We can recover Hilbert space representations from this abstract setting, primarily by the GNS construction.) Thus in a sense states are global, while observables are local; the global nature of states will be further emphasized in **1.2** below.

The first thing to say about this basic idea is *some anxious warnings!*: as follows.

(1): Not the commitment to the framework applying to arbitrarily small length scales: $\mathcal{O}$ can be arbitrarily small. One might say: this hardly fits with the EFT vision we learnt from e.g. Ken Wilson...

(2): $A \in \mathcal{A}(\mathcal{O})$ is to mean that we could measure A by a procedure confined to O. But: the association of quantities with regions is vague, and operationalist—and Bell would say that it is using the unspeakable word 'measurement' ...

(3): Note that the apparatus is not represented explicitly. Agreed; that is hardly surprising! We do not have a rigorous Lorentz-invariant interacting quantum field theory in 4 spacetime dimensions: let alone a relativistic account of measurement ... No one knows whether there is a real 'collapse of the wave packet'; and if so, whether it somehow happens 'along the light-cone' ... .

(4): Relatedly: nothing that follows will really address the subtleties of 'improper mixtures': i.e. the troubled interpretation if the reduced states of component systems whose composite system is in a pure state.

(5): We accept the product of arbitrary non-commuting operators: *pace* those authors who restrict products to commuting operators (cf. the partial Boolean algebra approach of e.g. Kochen and Specker (1967)).

(6): We impose that expectation functionals are linear on any pair of quantities. That is of course true of quantum expectations in elementary quantum mechanics, and the algebraic approach is to that extent at liberty to retain it in its generalized quantum theory. But recall Bell's (1966), and the pilot-wave theory's, animadversions against it ...

This framework is far from that of usual 'textbook' QFT; and as [1] describes, much effort has been devoted to developing the framework, and to linking it to usual QFT. But in this paper, we can specialize somewhat: for the most part, we can assume that the algebras are concrete von Neumann algebras acting on a separable Hilbert space $\mathcal{H}$, and subject to a relatively standard list of axioms. These require, for example: that the structure of the algebras mesh with the Poincare group symmetries of Minkowski spacetime; and that there is a unique state Ω, called the vacuum, that is invariant under all translations.

In the rest of this Section, I will, first, briefly state three of these standard axioms, namely three which are expressions of the idea of relativistic causality. (It is of course part of the subtlety of the subject that these are independent of one another ... even in the context of the other axioms!) Then I will briefly report three theorems that bring out the 'global nature' of states in AQFT.

1.1 Three formulations of relativistic causality:
(1): The first of our three formulations expresses the Lorentz invariance of the dynamical evolution of the field system. We begin by observing that in a heuristic quantum field theory, using the Heisenberg picture, operators indexed by spacetime points are subject to Heisenberg equations of motion, while the state is fixed once for all. But these equations are hyperbolic, on analogy with classical field theories using hyperbolic dynamical equations; this means one can show, at least unrigorously, that for any state, all expectation values are determined subluminally, in that the state's restriction to the field operators in a region O determines all its expectation values for operators in the future domain of dependence $D^+(O)$. In AQFT, this idea is made precise as

(i): *Primitive causality; the Diamond axiom*: We require that $\mathcal{A}(D(O)) = \mathcal{A}(O)$. The idea is: *if* $O_1 \subset D^+(O), O_1 \cap O = \emptyset$, i.e. O_1 lies in the top half of the "diamond" $D^+(O)$, and $A \in \mathcal{A}(O_1)$, so that we could measure A by a procedure confined to O_1, *then* we could also instead measure A by a procedure confined to O. For thanks to the hyperbolic time-evolution, "the facts in O_1" are already determined by "the facts in O".

(2): The second formulation expresses the physical idea that observables associated with spacelike-related regions should be co-measurable; (especially since in AQFT 'associated with' is meant to imply 'measurable by a procedure confined to'). Elementary quantum theory suggests that co-measurability requires that the observables commute.

This last is of course made a bit more precise in elementary quantum measurement theory: where the no-signalling theorem says that a non-selective Lüders rule measurement of A cannot affect the measurement probabilities of B, provided $[A, B] = 0$. But we should recall:

(a) warning (4) above, about the subtleties of 'improper mixtures': and

(b) how the violation of the Bell inequality suggests there is indeed case-by-case 'spooky action at a distance'—cf. also below ....

Anyway: we have

(ii): *Spacelike commutativity* (also called *micro-causality*): Observables associated with spacelike-related regions commute. In heuristic quantum field theory, treating fermions requires one to also allow anti-commutation; but in AQFT, one distinguishes field algebras and observable algebras, and for the latter imposes only spacelike commutativity. Thus one requires: if O_1, O_2 are spacelike, then for all $A_1 \in \mathcal{A}(O_1), A_2 \in \mathcal{A}(O_2) : [A_1, A_2] = 0$.

(3): The third formulation is perhaps the most direct expression of the prohibition of spacelike processes. It says:

(iii): *Spectrum*: The field system's energy-momentum operator has a spectrum (roughly: set of eigenvalues) confined to the future light-cone.

1.2 Three theorems reflecting the global nature of states:

(1): The first, and fundamental, result is:

Theorem (Reeh-Schlieder): Let $\mathcal{O}$ be an open bounded set in spacetime. Then Ω is a cyclic vector for $\mathcal{A}(\mathcal{O})$; *i.e.* the set of vectors $\mathcal{A}(\mathcal{O})\Omega$ is dense in $\mathcal{H}$. Also, any state with bounded energy is cyclic.

Remark: So even with $\mathcal{O}$ a tiny neighbourhood of some point p, we can approximate an arbitrary state of the field by acting on Ω with elements of $\mathcal{A}(\mathcal{O})$: even a state which, far way from (spacelike to) p, is quite unlike Ω. Very surprising! Cf comment (A) in Section 4.

(2): A closely related result is (Haag, *Local Quantum Physics*, Thm II.5.3.2):

Theorem: If $\mathcal{O}$ has non-empty causal complement, then $\mathcal{A}(\mathcal{O})$ does not contain an operator that annihilates the vacuum: that is, if $A\Omega = 0$ for some $A \in \mathcal{A}(\mathcal{O})$ then $A = 0$. (Nor does it contain an operator that annihilates any state vector with bounded energy.)

So if $(A_1 - A_2)\Omega = 0$ then $A_1 = A_2$. So Ω is able to 'discriminate' elements of any local algebra (and is therefore called a 'separating vector'). Furthermore, it follows immediately (assuming the usual Born rule for qauntum probability) that:–

Any possible outcome of any possible local measurement procedure has non-vanishing probability in the vacuum.

For: with $P \in \mathcal{A}(\mathcal{O})$ representing the outcome of the procedure, $P \neq 0$ implies that $P\Omega \neq 0$, so that $|| P\Omega ||^2 \neq 0$.

(3): Finally, a result about how the vacuum encodes strict correlations, in the same manner as the singlet state in EPR discussions. For the result, and the analogy with the singlet state, cf. Redhead (*Foundations of Physics*, **25**, 1995, pp. 123-137; Theorem 4'):

Theorem For any two spacelike separated bounded open regions $\mathcal{O}_1$and $\mathcal{O}_2$: $\forall \epsilon > 0$, $\forall$ projectors $P_1 \in \mathcal{A}(\mathcal{O}_1)$, $\exists$ a projector $P_2 \in \mathcal{A}(\mathcal{O}_2)$, s.t. $\Omega(P_1 P_2) > (1 - \epsilon)\Omega(P_2)$.

Recalling that states are linear functionals on observables and that the projectors P_1 and P_2 commute since their regions are spacelike, we see that this is a statement of strict correlation between the projectors (apart from the 'epsilonics'). Two remarks about this result:

(i): again, the vacuum could be replaced by any state with bounded energy;

(ii): for the analogy between the vacuum—or any state with bounded energy—and the singlet state in *Bell* discussions, i.e. its violation of a Bell inequality, cf. Section 3.

2 Formulating relativistic causality in curved spacetime

Broadly speaking, by the mid 1990s quantum field theory on curved spacetime could be formulated in as satisfactory a manner as heuristic quantum field theory on Minkowski spacetime, subject to three conditions. (Cf Wald, *Quantum field theory on curved spacetime* (1994).) These conditions are:

(a): The curved spacetime is fixed, i.e. there is no back-reaction of the field on the spacetime geometry; (though the curvature can be non-constant).

(b): The field is linear (i.e. not self-interacting).

(c): The spacetime is such that the corresponding classical field theory has a well-posed initial value problem. For our purposes, we take this to mean that the spacetime is globally hyperbolic. (This means there is a *Cauchy surface*, i.e. a spacelike slice Σ whose domain of dependence $D(\Sigma)$ is the whole spacetime; this is a strong condition of causal "good behaviour".) Besides, this success was based on adapting the algebraic approach to curved spacetimes (Wald, p. 74f, 84). Thus we naturally hope to carry over directly to such spacetimes **1.1**'s three Minkowski formulations, (i) to (iii), of relativistic causality.

Indeed, there is no problem about (i) and (ii), primitive causality and spacelike commutativity. Global hyperbolicity prevents any "funny business" in the causal structure, such as closed causal curves, so that these conditions can be carried over word for word: 'domain of dependence', 'spacelike' etc. now just refer to the curved spacetime's structure. Besides, the same considerations apply to the case of interacting fields, i.e. to the effort to overcome the limitation (b) above. Thus recent formulations of interacting quantum field theory on curved spacetimes use the algebraic approach, and again there is nothing to prevent carrying over these two conditions intact.

But there is a problem about the spectrum condition, (iii): though it is a problem that has recently been largely solved. Since the solution is important, and bears on the project of formulating *interacting* quantum field theory in curved spacetime, I will give some details. (Section 3.2.2 of my [2] gives some references—and thanks to Wald for teaching!)

In effect, the problem was that no one knew how to define the spectrum condition's topic, i.e. the energy-momentum operator, in a curved spacetime: all one knew was how to define a class of physically reasonable states that gave a well-defined expectation value. But in recent years, the problem has been solved by exploiting a mathematical theory, microlocal analysis. The problem of definability arises from the fact that the energy and momentum of the field are encoded in the stress-energy tensor $\hat{T}$, which involves the square of the quantum field $\hat{\phi}$. But $\hat{\phi}$ is a distribution, and the product of distributions at a single spacetime point is mathematically undefined; so that some prescription is needed in order that $\hat{T}$ make sense.

Until about 2000, it was not known how to do this "directly", i.e. by enlarging the algebra of observables to include some suitably smeared version of $\hat{T}$; so one aimed only to characterize a class of physically reasonable states ω for which the expectation value $< \hat{T} >_\omega$ was well-defined. (This was work enough since, in particular, the standard prescription for Minkowski spacetime (normal ordering, which corresponds to subtracting off the infinite sum of the zero point energies of the oscillators comprising the field) depends on a preferred vacuum—which is generally unavailable in a globally hyperbolic spacetime.) In fact, there is a compelling characterization of such states. Since it builds on Hadamard's work on distributional solutions to hyperbolic equations, they are called 'Hadamard states'.

But in recent years, various authors have exploited microlocal analysis so as to achieve the original goal ("direct" in the above paragraph). Indeed, they have defined, not just the energy-momentum, and stress-energy operators, and so the spectrum condition, our (iii); but also the other products of field operators and their derivatives, and polynomials of such products, and time-ordered products, that are crucial in order to formulate the perturbation theory of an interacting quantum field theory.

3 AQFT violates the Bell Inequality

Let us return to Minkowski spacetime. The non-local correlations encoded in the vacuum (and many other states) of AQFT have been shown, by authors such as Landau, Summers and Werner, to support a violation of Bell-type inequalities. This violation is endemic in the sense that it occurs for generic obervables (with the right spectrum) on generic (sorts of) regions for generic states, in most rigorous AQFTs. This Section gives a few details about this.

First, there is a sense in which it is endemic that the violation is *maximal*. To explain this, we first recall that it is convenient to consider a local 'classical' or 'hidden variable' model of a correlation experiment that uses 'left observables' A_1, A_2 and 'right observables' B_1, B_2 that are, not projectors, but rather self-adjoint contractions. Thus given a projector E, we define $A := 2E - 1$, so that $-1 \leq A = A^* \leq 1$. Then the Bell inequality, for a state ϕ taken (as in algebraic quantum theory) as an expectation functional, and for left and right algebras of observables $\mathcal{A}, \mathcal{B}$, says:— For any self-adjoint contractions $A_i \in \mathcal{A}, B_j \in \mathcal{B}, i, j = 1, 2$: $\mid \phi(A_1(B_1 + B_2) + A_2(B_1 - B_2)) \mid \leq 2$. Then the maximal correlation of $\mathcal{A}$ and $\mathcal{B}$ in the state ϕ is defined to be:

$$\beta(\phi, \mathcal{A}, \mathcal{B}) := \sup \ \frac{1}{2}\phi(A_1(B_1 + B_2) + A_2(B_1 - B_2))$$

where the supremum is taken over all self-adjoint contractions $A_i \in \mathcal{A}, B_j \in \mathcal{B}$. So the Bell inequality is: $\beta(\phi, \mathcal{A}, \mathcal{B}) \leq 1$. In fact, for any state ϕ on any C^*-algebra with commuting subalgebras $\mathcal{A}$ and $\mathcal{B}$, there is a more permissive bound (Cirel'son 1980): $\beta(\phi, \mathcal{A}, \mathcal{B}) \leq \sqrt{2}$.

So in the context of AQFT, we say that a state ϕ and two algebras $\mathcal{A}(\mathcal{O}_1)$, $\mathcal{A}(\mathcal{O}_2)$ maximally violate the Bell inequality if $\beta(\phi, \mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)) = \sqrt{2}$. We can now state how this maximal violation is endemic in AQFT. Namely: Summers and Werner show that for most rigorous AQFTs, for all pairs of regions, $\mathcal{O}_1, \mathcal{O}_2$ that have a certain shape (e.g. each is a double cone) and a certain spatiotemporal relationship (e.g. they are tangent to each other), there is maximal violation of the Bell inequality for *all* normal states. (Roughly speaking: a normal state is a density operator.)

Furthermore: If we do not require a maximal violation, then violation is endemic in two other senses. The first relates to *which quantities* give the violation; the second to *which states*, i.e. how generic a state, gives a violation.

(1): Roughly: Landau (1987) shows that we can be 'as choosy as we please' about which quantities give the violation. That is: it follows from his results (Prop 3 and 5 of *Physics Letters* **120A**,pp. 54-56) that for $\mathcal{O}_1, \mathcal{O}_2$ strictly spacelike, and any quantities $A_i \in \mathcal{A}(\mathcal{O}_1)$ and $B_i \in \mathcal{A}(\mathcal{O}_2)$ (with $[A_1, A_2] \neq 0 \neq [B_1, B_2]$, and each with spectrum ± 1), there is a state violating the Bell inequality.

(2): Roughly: Clifton and Halvorson (2000) show that once we pick a Hilbert space $\mathcal{H}$ carrying a representation of a AQFT, either on Minkowski spacetime or on any globally hyperbolic spacetime, and any two open spacelike-related regions $\mathcal{O}_1, \mathcal{O}_2$: there is an open dense subset of the unit ball of $\mathcal{H}$, i.e. a set of unit vectors of $\mathcal{H}$, each of which, considered as a state, violates the Bell inequality. That is, writing ϕ for the state: $\beta(\phi, \mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)) > 1$. (This summarizes Propositions 1, 3, 4 of their [3].)

4 Peaceful co-existence: some themes

I confine myself to making four points, labelled (A) to (D). All are supportive of the broad idea that there is "peaceful co-existence" between AQFT and the idea of relativistic causality—but I admit, there are plenty of issues yet to explore! The first three points are important. The fourth is my personal perspective on a line of work, principally by Redei, about the fate, within AQFT, of Reichenbach's Principle of the Common Cause.

The first two points bear more on "separability" than "locality". The first (due to Clifton and Halvorson [4]) is about the Reeh-Schleider theorem, and the impossibility of destroying entanglement. The second point, due to Landsman [5], is rather different: it uses the apparatus of algebraic quantum theory to argue for "peaceful co-existence" between Bohr and Einstein!

The third and fourth are each about the relations between AQFT's Bell inequality violation, and the assumptions of Bell-type theorems; and so bear more on "locality" than "separability".

(A): *Reeh-Schleider and the indestructibility of entanglement*:— Part (1) of Section 1.2 stated the Reeh-Schleider theorem. At first sight, it suggests action-at-a-distance, as follows: for any given state ϕ—perhaps specified in terms of its expectation values for observables in a region spacelike to a point p—there is an arbitrarily close state that one can "produce" by acting on the vacuum Ω with some element A of the algebra $\mathcal{A}(\mathcal{O})$ associated with a tiny neighbourhood $\mathcal{O}$ of p. That is: A can be chosen to make $A\Omega$ arbitrarily close to ϕ.

Clifton and Halvorson (Section 3 of [4]) reply to this threat, essentially by emphasising that in general A will represent a selective operation. I take it that this reply is orthodox. In any case, it assimilates the situation to the familiar one, whereby the no-signalling theorem is considered compatible with the change in statistics arising from a *selective* projective measurement. (And so it leads to the themes in (C) and (D) below.)

Clifton and Halvorson go on (Section 4 of [4]) to discuss:

(i) how local operations in a region $\mathcal{O}$ cannot disentangle the field system's state in $\mathcal{O}$ from that in $\mathcal{O}$'s spacelike complement; (this follows from the type III_1 structure of local algebras, especially in the light of a characterization of type III_1 by Connes and Størmer); but also

(ii) how the indestructibility of entanglement in (i) is not a practical problem.

(B): *Bohr vs Einstein Revisited*:— Following [5], I will: (i) state a theorem (due to Raggio and Bacciagaluppi), and (ii) urge the sense in which it makes peace between Bohr and Einstein.

Beware, on both counts! (i): The statement is rough: I will not specify that a state being separable requires only that it be in the w^* closure of the convex hull of the product states; nor that the tensor product used should be the projective one. (ii): As Landsman discusses, the peace is not complete ...

(i): *Theorem*: The following three conditions on two C^*-algebras $\mathcal{A}, \mathcal{B}$ are equivalent:

(a): each state on $\mathcal{A} \otimes \mathcal{B}$ is separable i.e. a mixture of product states;

(b): $\mathcal{A}$ or $\mathcal{B}$ (or both) is commutative;

(c): each state on $\mathcal{A} \otimes \mathcal{B}$ satisfies the Bell inequality.

(ii): Now recall Einstein's belief that (roughly speaking!) physics requires each subsystem to have its own real state (*Trennungsprinzip*), and Bohr's belief that (roughly speaking!) physics requires a classical description of the measurement apparatus. It is natural to translate these beliefs, respectively, into:

(1): For each pure state of a joint system, its restriction to a subsystem is pure;

(2): The algebra of observables of a measurement apparatus is commutative; while a quantum system has a non-commutative algebra.

Punchline:— Now let us apply the Theorem to a joint system comprising a measured quantum object $\mathcal{A}$, and a measurement spparatus $\mathcal{B}$. Imposing (2), we infer that condition (b) holds; and therefore, so do (a) and (c). In particular: (a) implies (1). Thus in this framework, "Bohr implies Einstein".

Similarly conversely, "from Einstein to Bohr". that is: Imposing (1), and so (a), we infer (b); and so, assuming $\mathcal{A}$ is non-commutative, we infer (2).

(C): *Outcome dependence in AQFT*:— In discussions of quantum non-locality, it is usual to say that the assumption of a Bell theorem that is shown false by the violation of the Bell inequalities is (in Shimony's jargon) outcome independence, rather than parameter independence. ('Usual' in the sense of 'orthodox'! That is: We here set aside the pilot-wave (causal) interpretation of quantum theory: for which the "culprit" assumption would instead be parameter independence.)

As I see matters, AQFT does not change this situation. That is: this usual/orthodox verdict against outcome independence can be maintained, also in AQFT. (In fact, AQFT encodes parameter independence in the fact that different local algebras have a common unit.)

(D): *SEL in AQFT*:— Finally: my own wee contribution (cf. [6], Section 4) relates to (C). More specifically, it relates to a line of work (principally by Redei) about the fate, within AQFT, of Reichenbach's Principle of the Common Cause (which is close to the outcome independence of the Bell theorem).

(1) I consider a precise formulation suitable for AQFT of a physical locality condition, called 'stochastic Einstein locality' (SEL), which was introduced in the Bell theorem literature as suiting Minkowski spacetime. The intuitive idea of SEL is that for an event E occurring in a spacetime region R, the probability at an earler time (spacelike hypersurface) t that E occurs should be determined by history (*i.e.* the events that occurred) within that part of the backward light cone of R that lies before t; i.e. by history within $C^-(R) \cap C^-(t)$. More precisely, it turns out that this intuitive idea has two inequivalent formulations; both generally and in the versions suitable for AQFT.

(2) I argue that one of these formulations follows from the AQFT axioms (especially Isotony and Diamond); that the second is endemically violated by AQFT, as a result of the strong non-local correlations coded in the vacuum state (or any vector state of bounded energy); and that this violation corresponds to the endemic outcome dependence discussed in (C) above.

5 References

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