On Spacelike Correlations in Algebraic Quantum Field Theory

J. Butterfield: for fourth session of *Philosophical Aspects of QFT on Curved Spacetimes* on 15 February 2022 ...

This handout tries to address two diverse interests: how quantum non-locality plays out in rigorous, in particular algebraic, QFTs; and how to write QFTs on a curved spacetime. The material is needed for the Unruh and Hawking effects, as treated by e.g. Wald 1994. The handout (especially Section 2) will also orient you to the Part III essay on the Hadamard condition on states in AQFT. The closing Section 4 discusses the ever-present (?perennial!) theme of whether there can be "peaceful co-existence" between relativity and quantum theory.

So I will review some conceptual aspects of non-local correlations in algebraic quantum field theory (AQFT) on Minkowski or curved spacetime, especially some aspects of the violation of the Bell inequality in AQFT. This summary will involve:

1. Reviewing a few basic ideas of AQFT, emphasising the three conditions of relativistic causality which have been much studied in Minkowksi spacetime; viz. primitive causality, spacelike commutativity, and the spectrum condition; (Section 1).

2. Discussing how these conditions can be adapted to a curved spacetime. Here the main topic is the spectrum condition, and I shall sketch the developments (since Wald's 1994 book) using microlocal analysis to cope with this condition, and relatedly, the Hadamard condition on states (Section 2).

3. Reviewing the generic violation of Bell inequalities in AQFT. I will confine myself to reporting some results up to 2000 by physicists (especially Landau, Summers and Werner) and philosophers (Clifton and Halvorson). The broad picture is that for most algebraic quantum field theories on Minkowski spacetime, maximal violations of the Bell inequality are endemic (Section 3).

4. Making some remarks about the prospects for "peaceful co-existence" between relativity and quantum theory. In particular, this will include some discussion of: (i) how Section 3's results relate to the assumptions of Bell-type theorems; (ii) recent AQFT-based models of measurement, especially by Fewster and Verch (Section 4).

Some overall references (among many possible) additional to Wald:—
Landsman's review of Haag's book, and Swanson's survey, [1], supplements Section 1. The papers in [2] supplement Section 2. Papers [3,4] by Clifton and Halvorson, which are reprinted as Chapters 6 and 7 of his posthumous *Quantum Entanglements: Selected Papers*, supplement Sections 3 and 4. For Section 4: Landsman's [5], Hofer-Szabo et al. [6], Section 4 of my [7], and the papers in [9] supplement the subsections B, C, D and E respectively.

1 AQFT introduced

The basic idea of AQFT is to associate with each bounded region $\mathcal{O}$ of Minkowski spacetime an algebra $\mathcal{A}(\mathcal{O})$ subject to certain axioms. Observables are self-adjoint elements of these algebras. The basic idea of the association of an observable A with a region $\mathcal{O}$ is that A is a physical quantity pertaining to that part of the field system lying in $\mathcal{O}$, and so is measurable by a procedure confined to $\mathcal{O}$. On the other hand: a state is taken as an expectation functional on all the algebras. (We can recover Hilbert space representations from this abstract setting, primarily by the GNS construction.) Thus in a sense states are global, while observables are local; the global nature of states will be further emphasized in **1.2** below.

The first thing to say about this basic idea is *some anxious warnings!*: as follows.

(1): Note the commitment to the framework applying to arbitrarily small length scales: $\mathcal{O}$ can be arbitrarily small. One might say: this hardly fits with the EFT vision we learnt from e.g. Ken Wilson... In the philosophical literature, this was a focus of the debate between D. Fraser and D. Wallace ca. 2011 about the merits of AQFT; a judicious discussion of the debate is in Section 3 of Swanson's survey paper [1].

(2): $A \in \mathcal{A}(\mathcal{O})$ is to mean that we could measure A by a procedure confined to O. But this association of quantities with regions is rather vague, and operationalist—and Bell would say that it is using the unspeakable word 'measurement' ... In particular:

(3): Note that in almost all AQFT work, the apparatus is not represented explicitly. (Recent exceptions include the papers in [9].) Agreed; that is hardly surprising! We do not have a rigorous Lorentz-invariant interacting quantum field theory in 4 spacetime dimensions: let alone a relativistic account of measurement (cf. A. Kent's proposals discussed in [8]). ... No one knows whether there is a real 'collapse of the wave packet'; and if so, whether it somehow happens 'along the light-cone' ... .

(4): Relatedly: nothing that follows will really address the subtleties of 'improper mixtures': i.e. the troubles about how to interpret the reduced states of component systems whose composite system is in a pure state, troubles first emphasised by Schroedinger in his monumental 'cat' paper of 1935.

(5): We accept the product of arbitrary non-commuting operators: *pace* those authors who restrict products to commuting operators (cf. e.g. the partial Boolean algebra approach of Kochen and Specker (1967)).

(6): We impose that expectation functionals are linear on any pair of quantities. That is of course true of quantum expectations in elementary quantum mechanics, and the algebraic approach is to that extent at liberty to retain it in its generalized quantum theory. But recall Bell's (1966) stunning critique of it as an assumption of the "no hidden variables" theorems of von Neumann and others; and the pilot-wave theory's escaping it ...

This AQFT framework is far from that of usual 'textbook' QFT; and as both Landsman and Swanson [1] describe, much effort has been devoted to developing the framework, and to linking it to usual QFT. But in this handout, we can specialise somewhat. For the most part, we can assume that the algebras are concrete von Neumann algebras acting on a separable Hilbert space $\mathcal{H}$, and subject to a relatively standard list of axioms. These require, for example:

Isotony: If $\mathcal{O}_1 \subset \mathcal{O}_2$, then $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$;

Poincaré covariance: There is a representation α of the Poincaré group on the algebras, in that: If $g \in \mathcal{P}$, then $\alpha_g(\mathcal{A}(\mathcal{O})) = \mathcal{A}(g(\mathcal{O}))$; thus the structure of the algebras meshes with the Poincare group symmetries of Minkowski spacetime; and

Vacuum: among the states on the net of algebras, there is a unique one Ω, called 'the vacuum', that is invariant under the Poincaré group.

In the rest of this Section, I will, first, briefly state three of these standard axioms, namely three which are expressions of the idea of relativistic causality. (It is of course part of the subtlety of the subject that these are independent of one another ... even in the context of the other axioms.) Then I will briefly report three theorems that bring out the 'global nature' of states in AQFT.

1.1 Three formulations of relativistic causality:

(1): The first of our three formulations expresses the Lorentz invariance of the dynamical evolution of the field system. We begin by observing that in a heuristic quantum field theory, using the Heisenberg picture, operators indexed by spacetime points are subject to Heisenberg equations of motion, while the state is fixed once for all. But these equations are hyperbolic, on analogy with classical field theories using hyperbolic dynamical equations; this means one can show, at least unrigorously, that for any state, all expectation values are determined subluminally, in that the state's restriction to the field operators in a region $\mathcal{O}$ determines all its expectation values for operators in the future *domain of dependence* $D^+(\mathcal{O})$.

In AQFT, this idea is made precise as

(i): *Primitive Causality; the Diamond Axiom*: $\mathcal{A}(D(\mathcal{O})) = \mathcal{A}(\mathcal{O})$.

The idea is: *if* $\mathcal{O}_1 \subset D^+(\mathcal{O}), \mathcal{O}_1 \cap \mathcal{O} = \emptyset$, i.e. $\mathcal{O}_1$ lies in the top half of the "diamond" $D^+(\mathcal{O})$, and $A \in \mathcal{A}(\mathcal{O}_1)$, so that we could measure A by a procedure confined to $\mathcal{O}_1$, *then* we could also instead measure A by a (no doubt different!) procedure confined to $\mathcal{O}$. For thanks to the hyperbolic time-evolution, "the facts in $\mathcal{O}_1$" are already determined by "the facts in $\mathcal{O}$".

(2): The second formulation expresses the physical idea that observables associated with spacelike-related regions should be co-measurable; (especially since in AQFT 'associated with' is meant to imply 'measurable by a procedure confined to'). Elementary quantum theory suggests that co-measurability requires that the observables commute.

This last is of course made a bit more precise in elementary quantum measurement theory: where the no-signalling theorem says that a (NB! *non-selective*) Lüders rule measurement of A cannot affect the measurement probabilities of B, provided $[A, B] = 0$. But we should recall:

(a) warning (4) above, about the subtleties of 'improper mixtures': and

(b) how the violation of the Bell inequality suggests there is indeed case-by-case 'spooky action at a distance'—cf. also Section 4 below ....

Anyway: we have

(ii): *Spacelike commutativity* (also called *micro-causality*): Observables associated with spacelike-related regions commute. In heuristic quantum field theory, treating fermions requires one to also allow anti-commutation; but in AQFT, one distinguishes field algebras and observable algebras, and for the latter imposes only spacelike commutativity. Thus one requires: if $\mathcal{O}_1, \mathcal{O}_2$ are spacelike, then for all $A_1 \in \mathcal{A}(\mathcal{O}_1), A_2 \in \mathcal{A}(\mathcal{O}_2) : [A_1, A_2] = 0$.

(3): The third formulation is perhaps the most direct expression of the prohibition of spacelike processes. It says:

(iii): *Spectrum*: The field system's energy-momentum operator has a spectrum (roughly: set of eigenvalues) confined to the future light-cone.

1.2 Three theorems reflecting the global nature of states:

The broad idea is that the quantum fields degrees of freedom are encoded in the algebras, that are local. But a state assigns expectation values to all elements of all local algebras, and so a state is global.

(There is a philosophical literature, mostly inspired by the Everett tradition, of taking the state as the primary physical reality. It goes mostly under the label 'wave-function realism' and is mostly developed just for non-relativistic many particle systems. But in 2010, Wallace and Timpson sketched how to be a "state-space realist" for quantum field theory. They proposed to factorize the universal Hilbert space with factors associated to different spacetime regions, and that the primary physical reality be density matrices on these factors. An assessment, indeed critique, by Swanson is in *British Journal of Philosophy of Science* for 2020: its Section 1 to 3 helpfully expound, including about different Types of von Neumann algebras.)

(1): The first, and fundamental, result is:

Theorem (Reeh-Schlieder): Let $\mathcal{O}$ be an open bounded set in spacetime. Then Ω is a cyclic vector for $\mathcal{A}(\mathcal{O})$; *i.e.* the set of vectors $\mathcal{A}(\mathcal{O})\Omega$ is dense in $\mathcal{H}$. Also, any state with bounded energy is cyclic.

Of the three results, this is the fundamental one. It says: even with $\mathcal{O}$ a tiny neighbourhood of some point p, we can approximate an arbitrary state of the field by acting on Ω with elements of $\mathcal{A}(\mathcal{O})$: even a state which, far way from (spacelike to) p, is quite unlike Ω. Very surprising! Cf. comment (A) in Section 4.

Intermezzo: There is an easy but important relation between a vector being cyclic for an algebra, and it being *separating* for the commutant of the algebra. A vector ψ is called 'separating' for an algebra of operators $\mathcal{A}$ iff for any $A \in \mathcal{A}$: $A\psi = 0$ implies $A = 0$. The label 'separating' alludes to the fact that if ψ is separating, then for any $A_1, A_2 \in \mathcal{A}$: $(A_1 - A_2)\psi = 0$ implies $A_1 = A_2$—so that indeed ψ "can discriminate" between any two $A_1, A_2 \in \mathcal{A}$.The ideas of being cyclic and being separating generalise readlily from a single vector in a Hilbert space $\mathcal{H}$ to a set of vectors $\mathcal{K} \subset \mathcal{H}$. We say that:

(i): $\mathcal{K} \subset \mathcal{H}$ is cyclic for $\mathcal{A}$ if $\mathcal{A}(\mathcal{K}) := \{\psi \in \mathcal{H} : \psi = A(\phi), \text{for some } A \in \mathcal{A} \text{ and some } \phi \in \mathcal{H}\}$ is dense in $\mathcal{H}$; that is: the closure of $\mathcal{A}(\mathcal{K})$ is $\mathcal{H}$.

(ii): $\mathcal{K} \subset \mathcal{H}$ is separating for $\mathcal{A}$ iff for any $A \in \mathcal{A}$: if $A\psi = 0$ for all $\psi \in \mathcal{K}$, then $A = 0$. (So the idea is that $\mathcal{K}$ is "collectively/working as a team" able to discriminate elements of $\mathcal{A}$.)

Then we have: (Bratteli and Robinson, *Operator Algebras and Quantum Statistical Mechanics*, volume 1, Prop. 2.5.3, p. 85):

If $\mathcal{A}$ is a von Neumann algebra on a Hilbert space $\mathcal{H}$, and $\mathcal{K} \subset \mathcal{H}$, then:
(i) $\mathcal{K}$ is cyclic for $\mathcal{A}$ iff (ii) $\mathcal{K}$ is separating for the commutant $\mathcal{A}'$.

Proof: (i) $\Rightarrow$ (ii): Choose $A' \in \mathcal{A}'$ such that $A'(\mathcal{K}) = \{\prime\}$. Then for any $B \in \mathcal{A}$, any $\psi \in \mathcal{K}$, we have $A'B\psi = BA'\psi = 0$. So $A'(\mathcal{A}(\mathcal{K})) = \{0\}$. So $A'(\mathcal{H}) = 0$. So $A' = 0$. (ii) $\Rightarrow$ (i): Let P' be the projector onto the closure of (the span of) $\mathcal{A}(\mathcal{K})$. Then P' is a projector in $\mathcal{A}'$ and $(1 - P')(\mathcal{K}) = \{0\}$. So since $\mathcal{K}$ is separating for $\mathcal{A}$, $(1 - P') = 0$, i.e. $P' = 1$, and so the closure of $\mathcal{A}(\mathcal{K})$ is $\mathcal{H}$.

With this *Intermezzo*, we get ...

(2): A closely related result (Haag, *Local Quantum Physics*, Thm II.5.3.2):

Theorem: If $\mathcal{O}$ has non-empty causal complement, then $\mathcal{A}(\mathcal{O})$ does not contain an operator that annihilates the vacuum: that is, if $A\Omega = 0$ for some $A \in \mathcal{A}(\mathcal{O})$ then $A = 0$. (Nor does it contain an operator that annihilates any state vector with bounded energy.)

So if $(A_1 - A_2)\Omega = 0$ then $A_1 = A_2$. So Ω is able to 'discriminate' elements of any local algebra; and so, again: the vacuum is called a 'separating vector'.

Furthermore, it follows immediately (assuming the usual Born rule for qauntum probability) that:–

Any possible outcome of any possible local measurement procedure has non-vanishing probability in the vacuum.

For: with $P \in \mathcal{A}(\mathcal{O})$ representing the outcome of the procedure, $P \neq 0$ implies that $P\Omega \neq 0$, so that $|| P\Omega ||^2 \neq 0$.

(3): Finally, a result about how the vacuum encodes strict correlations, in the same manner as the singlet state in EPR discussions. For the result, and the analogy with the singlet state, cf. Redhead (*Foundations of Physics*, **25**, 1995, pp. 123-137; Theorem 4'):

Theorem For any two spacelike separated bounded open regions $\mathcal{O}_1$and $\mathcal{O}_2$: $\forall \epsilon > 0$, $\forall$ projectors $P_1 \in \mathcal{A}(\mathcal{O}_1)$, $\exists$ a projector $P_2 \in \mathcal{A}(\mathcal{O}_2)$, s.t. $\Omega(P_1 P_2) > (1-\epsilon)\Omega(P_2)$.

Recalling that states are linear functionals on observables and that the projectors P_1 and P_2 commute since their regions are spacelike, we see that this is a statement of strict correlation between the projectors (apart from the 'epsilonics'). Two remarks about this result:

(i): again, the vacuum could be replaced by any state with bounded energy;

(ii): for the analogy between the vacuum—or any state with bounded energy—and the singlet state in *Bell* discussions, i.e. its violation of a Bell inequality, cf. Section 3.

2 Formulating relativistic causality in curved spacetime

Broadly speaking, by the mid 1990s quantum field theory on curved spacetime could be formulated in as satisfactory a manner as heuristic quantum field theory on Minkowski spacetime, subject to three conditions. (Cf Wald, *Quantum field theory on curved spacetime* (1994).) These conditions are:

(a): The curved spacetime is fixed, i.e. there is no back-reaction of the field on the spacetime geometry; (though the curvature can be non-constant).

(b): The field is linear (i.e. not self-interacting).

(c): The spacetime is such that the corresponding classical field theory has a well-posed initial value problem.

For our purposes, condition (c) prompts two comments.

First, 'well-posed initial value problem' refers to the determination of solutions by initial data, as in the classical case. And the most common way to satisfy condition (c) is to restrict attention to globally hyperbolic spacetimes. These are spacetimes with a *Cauchy surface*, i.e. a spacelike slice Σ whose domain of dependence $D(\Sigma)$ is the whole spacetime. In fact, global hyperbolicity is a strong condition of causal "good behaviour"; it implies that spacetime is foliated by Cauchy surfaces, and implies several other causality conditions, including stable causality. The reason for this restriction is that the main theorems securing a well-posed initial value problem for hyperbolic classical equations assume global hyperbolicity (Wald 1984, Theorems 10.1.2-3, p. 250.)

Second, we need to explain 'the corresponding classical theory'. This phrase indicates a standard construction of a heuristic quantum eld theory (with e.g. a Fock space of states built up from a vacuum) from the solution space of a classical theory. (The construction for Minkowski spacetime is in Wald (1994, Sections 3.1-3.2); the adaptation to curved spacetimes is in his Section 4.2.) When this construction is given for Minkowski spacetime, but in a form that is suitable for generalization to curved spacetimes, one sees that:

(i): it involves the choice of a Hilbert space (essentially, of a set of complex solutions of the classical theory), with different Hilbert spaces giving unitarily inequivalent theories; (the Stone-von Neumann theorem asserting unitary equivalence of representations of commutation relations depends on the system being finite-dimensional i.e. not a field); and

(ii): this freedom of choice is characterized by a choice of a bilinear map (inducing an inner product, and so a complex structure); but that

(iii): the Poincare symmetry of Minkowski spacetime gives a preferred bilinear map and so Hilbert space: equivalently, a preferred vacuum with the Hilbert space being the Fock space built up from that vacuum; (Wald 1994, pp. 27-29, 39-42).

In a curved spacetime, property (iii) fails, and so one must either:

(a): seek some other criterion for choosing the bilinear map (or at least a set of them corresponding to a unitary equivalence class of representations); or

(b): treat the different choices (and so unitarily inequivalent representations) on a par.

In some cases, tactic (a) is sensible. For example, for stationary spacetimes, there is a natural unique choice of map; and for a spacetime with a compact Cauchy surface, a condition of physical reasonableness for the state (the *Hadamard condition*, discussed below) constrains the bilinear maps so as to fix a unitary equivalence class. But in general, there is no such choice and one must opt for (b) (Wald 1994, pp. 58-60).

This suggests that we should adopt the framework of algebraic quantum field theory, which takes abstract algebras of observables as the primary notion, with states being linear functionals on them. And indeed, adapting the standard construction mentioned above to the algebraic approach, we find that the (Weyl) algebra of observables that naturally arises is independent of the choice of bilinear map (Wald 1994, p. 74f.). Furthermore, this approach is at first sight very promising for the topic of precise formulations of relativistic causality, since this approach again associates abstract algebras of observables with open bounded regions of any globally hyperbolic spacetime (Wald, 1994 p. 84). Thus we naturally hope to carry over directly to such spacetimes the three Minkowski formulations, (i) to (iii), of Section **1.2**.

Indeed, there is no problem about the first two conditions, primitive causality and spacelike commutativity. The global hyperbolicity assumption prevents any "funny business" in the causal structure, such as closed causal curves, so that these conditions can be carried over word for word: 'domain of dependence', 'spacelike' etc. now just refer to the curved spacetime's structure.

Besides, though I have so far confined my summary to the easier, and better understood, case of non-interacting fields, the same considerations apply to the interacting case. As we shall see shortly, recent formulations of interacting quantum eld theory on curved spacetimes use the algebraic approach, and again there is nothing to prevent carrying over these two conditions intact.

But there is a problem about the third condition, the spectrum condition: though it is a problem that in the twenty-five years since Wald's book has been solved. In effect, the problem was that no one knew how to define the spectrum condition's topic, i.e. the energy-momentum operator, in a curved spacetime: all one knew was how to define a class of physically reasonable states that gave a well-defined expectation value. But in the last twenty-five years years, the problem has been solved by exploiting a mathematical theory, *microlocal analysis*.

The details are worth summarising since the problem is much more general, and thus its solution much more impressive, than my mentioning just one observable can suggest. Indeed, the solution secures a perturbative formulation of interacting heuristic quantum field theory on a globally hyperbolic spacetime that is about as precise as those we have for Minkowski spacetime. So this achievement is worth reporting.

The problem of definability arises from the fact that the energy and momentum of the field are encoded in the stress-energy tensor $\hat{T}$, which involves the square of the quantum field $\hat{\phi}$. But $\hat{\phi}$ is a distribution, and the product of distributions at a single spacetime point is mathematically undefined; so that some prescription is needed in order that $\hat{T}$, make sense.

Until about 1995, it was not known how to do this "directly", i.e. by enlarging the algebra of observables to include some suitably smeared version of $\hat{T}$. So one aimed only to characterize a class of physically reasonable states ω for which the expectation value $\langle\hat{T}\rangle_\omega$ was well-defined. This was work enough since, in particular, the standard prescription for Minkowski spacetime (normal ordering, which corresponds to subtracting off the infinite sum of the zero-point energies of the oscillators comprising the field) depends on a preferred vacuum—which is generally unavailable in a globally hyperbolic spacetime.

In fact, there is a compelling characterization of such states. Since it builds on Hadamard's work on distributional solutions to hyperbolic equations, they are called *Hadamard states*. A bit more precisely: one requires the short-distance singularity structure of the two-point function $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle$ to be "as close as possible" to the corresponding structure of the two-point function of the Minkowski vacuum (Wald 1994, p. 94). This definition turns out to be very successful, as shown by both existence and uniqueness results. That is: (i) any globally hyperbolic spacetime has a large class of Hadamard states, and (ii) Hadamard states satisfy some natural axioms, and do so uniquely up to a local curvature term; (Wald, 1994: pp. 89-95 for (ii), and pp. 95-97 for (i)).

But since 1994, various authors have exploited microlocal analysis so as to achieve the original goal ("direct" in the preceding paragraph). Indeed, they have defined, not just the energy-momentum, and stress-energy operators; but also the other products of field operators and their derivatives, and polynomials of such products, and time-ordered products, that are crucial in order to formulate the perturbation theory of an interacting quantum field theory. I will just gesture at what is involved, with an eye on our interest in the spectrum condition. For more detail, cf. Hollands and Wald (2001, 2002), who build on previous work, especially by Brunetti, Fredenhagen and collaborators.

The fundamental idea is to use microlocal analysis' definition of the set of directions (at each point of the spacetime) in which a distribution is singular: it is called the 'wave front set' of the distribution. Using this and related ideas, one can define the above operators so as to satisfy appropriate properties, in particular locality and covariance. To give the flavour, we need only note the following characterization of Hadamard states in any globally hyperbolic spacetime (and thereby of the spectrum condition in Minkowski spacetime): the two point function of a Hadamard state has a wave front set consisting of pairs comprising:

(i) at any given spacetime point x, a future-pointing null vector k; and

(ii) at any other point x' on the null geodesic through x generated by parallel-transporting k, the corresponding tangent vector, i.e. the parallel-transport of k from x to x'.

(This is, *modulo* some technicalities, Radzikowski 1996, Theorem 5.1.) Building on this sort of characterization, one requires of the local and covariant operators, a generalized msicrolocal spectrum condition, which is a microlocal analogue of the translation invariance of Minkowski spacetime; (cf. e.g. Hollands and Wald (2001, Definitions 3.3, 4.1)). Thus the spectrum condition, (iii) of Section **1.2**, is carried over to curved spacetimes.

For more up-to-date details about this, indeed about AQFT in general (but not the topics of Sections 3 and 4), cf. e.g. the October 2020 course of lectures for the "AQFT in the UK" network, by Siemsen and Capoferri: sponsored by the London Mathematical Society. So go to:

https://www.lms.ac.uk/grants/joint-research-groups-uk-scheme-3/online-lectures

and click low down on: AQFT in the UK, to get to:

https://www.youtube.com/playlist?list=PLsDn5JyJXoYIEeyBx0F2Apee0O4*underline*TRkGt

To sum up this Section:— In Section **1.2**, we saw three precise formulations of relativistic causality for quantum theories on Minkowski spacetime, which prescind from interpretative controversies about such notions as causation, or signal, or quantum measurement, and which are logically independent. In this Section, these three formulations were adapted to globally hyperbolic spacetimes.

3 AQFT violates the Bell Inequality

Let us return to Minkowski spacetime. The non-local correlations encoded in the vacuum (and many other states) of AQFT have been shown, by authors such as Landau, Summers and Werner, to support a violation of Bell-type inequalities. This violation is endemic in the sense that it occurs for generic obervables (with the right spectrum) on generic (sorts of) regions for generic states, in most rigorous AQFTs. This Section gives a few details about this.

First, there is a sense in which it is endemic that the violation is *maximal*. To explain this, we first recall that it is convenient to consider a local 'classical' or 'hidden variable' model of a correlation experiment that uses 'left observables' A_1, A_2 and 'right observables' B_1, B_2 that are, not projectors, but rather self-adjoint contractions. Thus given a projector E, we define $A := 2E - 1$, so that $-1 \leq A = A^* \leq 1$. Then the Bell inequality, for a state ϕ taken (as in algebraic quantum theory) as an expectation functional, and for left and right algebras of observables $\mathcal{A}, \mathcal{B}$, says:— For any self-adjoint contractions $A_i \in \mathcal{A}, B_j \in \mathcal{B}, i, j = 1, 2$: $\mid \phi(A_1(B_1 + B_2) + A_2(B_1 - B_2)) \mid \leq 2$. Then the maximal correlation of $\mathcal{A}$ and $\mathcal{B}$ in the state ϕ is defined to be:

$$\beta(\phi, \mathcal{A}, \mathcal{B}) := \sup \ \frac{1}{2}\phi(A_1(B_1 + B_2) + A_2(B_1 - B_2))$$

where the supremum is taken over all self-adjoint contractions $A_i \in \mathcal{A}, B_j \in \mathcal{B}$. So the Bell inequality is: $\beta(\phi, \mathcal{A}, \mathcal{B}) \leq 1$. In fact, for any state ϕ on any C^*-algebra with commuting subalgebras $\mathcal{A}$ and $\mathcal{B}$, there is a more permissive bound (Cirel'son 1980): $\beta(\phi, \mathcal{A}, \mathcal{B}) \leq \sqrt{2}$.

So in the context of AQFT, we say that a state ϕ and two algebras $\mathcal{A}(\mathcal{O}_1)$, $\mathcal{A}(\mathcal{O}_2)$ maximally violate the Bell inequality if $\beta(\phi, \mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)) = \sqrt{2}$. We can now state how this maximal violation is endemic in AQFT. Namely: Summers and Werner show that for most rigorous AQFTs, for all pairs of regions, $\mathcal{O}_1, \mathcal{O}_2$ that have a certain shape (e.g. each is a double cone) and a certain spatiotemporal relationship (e.g. they are tangent to each other), there is maximal violation of the Bell inequality for *all* normal states. (Roughly speaking: a normal state is a density operator.)

Furthermore: If we do not require a maximal violation, then violation is endemic in two other senses. The first relates to *which quantities* give the violation; the second to *which states*, i.e. how generic a state, gives a violation.

(1): Roughly: Landau (1987) shows that we can be 'as choosy as we please' about which quantities give the violation. That is: it follows from his results (Prop 3 and 5 of *Physics Letters* **120A**,pp. 54-56) that for $\mathcal{O}_1, \mathcal{O}_2$ strictly spacelike, and any quantities $A_i \in \mathcal{A}(\mathcal{O}_1)$ and $B_i \in \mathcal{A}(\mathcal{O}_2)$ (with $[A_1, A_2] \neq 0 \neq [B_1, B_2]$, and each with spectrum ± 1), there is a state violating the Bell inequality.

(2): Roughly: Clifton and Halvorson (2000) show that once we pick a Hilbert space $\mathcal{H}$ carrying a representation of a AQFT, either on Minkowski spacetime or on any globally hyperbolic spacetime, and any two open spacelike-related regions $\mathcal{O}_1, \mathcal{O}_2$: there is an open dense subset of the unit ball of $\mathcal{H}$, i.e. a set of unit vectors of $\mathcal{H}$, each of which, considered as a state, violates the Bell inequality. That is, writing ϕ for the state: $\beta(\phi, \mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)) > 1$. (This summarizes Propositions 1, 3, 4 of their [3].)

4 Peaceful co-existence: five themes

I confine myself to making five points, labelled (A) to (E). All are supportive of the broad idea that there is "peaceful co-existence" between AQFT and the idea of relativistic causality—but I admit, there are plenty of issues yet to explore!

The first three points are central to the issues. The fourth is my personal perspective on a line of work, principally by Redei, about the fate, within AQFT, of Reichenbach's Principle of the Common Cause. The fifth reports recent work that addresses my *anxious warnings*, (2) and (3) at the start of Section 1: that AQFT's association of quantities with regions is vague and operationalist—and Bell would say that it uses the unspeakable word 'measurement' ...

Another orienting remark:— The first two points bear more on "separability" than "locality". The first (due to Clifton and Halvorson [4]) is about the Reeh-Schlieder theorem, and the impossibility of destroying entanglement. The second point, due to Landsman [5], is rather different: it uses the apparatus of algebraic quantum theory to argue for "peaceful co-existence" between Bohr and Einstein!

On the other hand: the third and fourth points are each about the relations between AQFT's Bell inequality violation, and the assumptions of Bell-type theorems; and so bear more on "locality" than "separability".

(A): *Reeh-Schlieder and the indestructibility of entanglement*:—
Part (1) of Section **1.2** stated the Reeh-Schlieder theorem. At first sight, it suggests action-at-a-distance, as follows: for any given state ϕ—perhaps specified in terms of its expectation values for observables in a region spacelike to a point p—there is an arbitrarily close state that one can "produce" by acting on the vacuum Ω with some element A of the algebra $\mathcal{A}(\mathcal{O})$ associated with a tiny neighbourhood $\mathcal{O}$ of p. That is: A can be chosen to make $A\Omega$ arbitrarily close to ϕ.

Clifton and Halvorson (Section 3 of [4]) reply to this threat, essentially by emphasising that in general A will represent a *selective* operation. I take it that this reply is orthodox. In any case, it assimilates the situation to the familiar one, whereby the no-signalling theorem is considered compatible with the change in statistics arising from a *selective* projective measurement. (And so it leads to the themes in (C) and (D) below.)

Clifton and Halvorson go on (Section 4 of [4]) to discuss:

(i) how local operations in a region $\mathcal{O}$ cannot disentangle the field system's state in $\mathcal{O}$ from that in $\mathcal{O}$'s spacelike complement; (this follows from the type III_1 structure of local algebras, especially in the light of a characterization of type III_1 by Connes and Størmer); but also

(ii) how the indestructibility of entanglement in (i) is not a practical problem.

(B): *Bohr vs Einstein Revisited*:—
Following [5], I will: (i) state a theorem (due to Raggio and Bacciagaluppi), and (ii) urge the sense in which it makes peace between Bohr and Einstein.

Beware, on both counts!

(i): The statement below is rough. I will not here explain even the idea of C^*-algebras ... let alone specify that a state being separable requires only that it be in the w^* closure of the convex hull of the product states; or that the tensor product used should be the projective one.

(ii): As Landsman discusses (citing Spinoza and Maimonides!), the peace is not complete ...

(i): *Theorem*: The following three conditions on two C^*-algebras $\mathcal{A}, \mathcal{B}$ are equivalent:

(a): each state on $\mathcal{A} \otimes \mathcal{B}$ is separable i.e. a mixture of product states;
(b): $\mathcal{A}$ or $\mathcal{B}$ (or both) is commutative;
(c): each state on $\mathcal{A} \otimes \mathcal{B}$ satisfies the Bell inequality.

(ii): Now recall Einstein's belief that (roughly speaking!) physics requires each subsystem to have its own real state (*Trennungsprinzip*), and Bohr's belief that (roughly speaking!) physics requires a classical description of the measurement apparatus. It is natural to translate these beliefs, respectively, into:

(1): For each pure state of a joint system, its restriction to a subsystem is pure;

(2): The algebra of observables of a measurement apparatus is commutative; while a quantum system has a non-commutative algebra.

Punchline:— Now let us apply the Theorem to a joint system comprising a measured quantum object $\mathcal{A}$, and a measurement spparatus $\mathcal{B}$. Imposing (2), we infer that condition (b) holds; and therefore, so do (a) and (c). In particular: (a) implies (1). Thus in this framework, "Bohr implies Einstein".

Similarly conversely, "from Einstein to Bohr". that is: Imposing (1), and so (a), we infer (b); and so, assuming $\mathcal{A}$ is non-commutative, we infer (2).

(C): *Outcome dependence in AQFT*:—
Recall that interpretative discussions of quantum non-locality revolve around which assumption of a proof of a Bell inequality one should deny. A broad motivation of the

locality assumptions is that correlations between spacelike events should be explained by a common cause occurring in the intersection of their past light cones. This is made precise in Reichenbach's Principle of the Common Cause, with 'explained by' taken as 'probabilistically screened-off by', i.e. 'rendered probabilistically independent by conditioning on'. Hofer-Szabo et al [6] is a full analysis of the Principle in general: for the application to quantum non-locality, including AQFT, cf. Chapters 8.1 and 9.1.

In these discussions, it is usual to say that the assumption of a Bell theorem that is shown false by the violation of the Bell inequalities is (in the now-prevalent jargon proposed by Shimony) *outcome independence*, rather than *parameter independence*. Here, outcome independence means: probabilistic independence of the two outcomes, conditional on a specification of which two quantities are measured; and parameter independence means: probabilistic independence of one wing's outcomes from a specification of which quantity is measured in the distant wing. (So as Shimony agreed: a better name for parameter independence would be 'setting independence'.) Quantum theory obeys parameter independence in the sense that it has the no-signalling theorem mentioned in (2) of Section **1.1**: a non-selective Lüders rule measurement of A cannot affect the measurement probabilities of a quantity B that commutes with A.[1]

Broadly speaking, AQFT does not change this situation. That is: this usual/orthodox verdict against outcome independence can be maintained, also in AQFT. (In fact, AQFT encodes parameter independence in the fact that different local algebras have a common unit.)

(D): *Stochastic Einstein locality (SEL) in AQFT*:—
My own contribution to the discussion in (C) (cf. [7], Section 4) relates to the fate, within AQFT, of appropriate formulations of outcome independence and Reichenbach's Principle of the Common Cause.

(1) I consider a precise formulation suitable for AQFT of a physical locality condition, called *stochastic Einstein locality* (SEL), which was introduced in the Bell theorem literature as suiting Minkowski spacetime.

The intuitive idea of SEL is to combine Minkowski spacetime structure with the idea of objective time-dependent chances for events, as follows:—

For an event E occurring in a spacetime region $\mathcal{O}$, the probability at an earlier time (spacelike hypersurface) t that E occurs should be determined by history (i.e. the events that occurred) within that part of the backward light cone of $\mathcal{O}$ that lies before t; i.e. by history within $J^-(\mathcal{O}) \cap J^-(t)$:—

[1] I write 'usual to say' in the sense of 'orthodox to say'! That is: We here set aside the pilot-wave (causal) interpretation of quantum theory: for which the "culprit" assumption would instead be parameter independence. For a glimpse of how, despite the orthodoxy, the conceptual situation is very open, cf. my [8], which analyses a proposal of Kent's (especially its Section 2).

More precisely, it turns out that this intuitive idea has two *inequivalent* formulations; both generally and in the versions suitable for AQFT.

(2) I argue that:

one of these formulations follows from the AQFT axioms (especially Isotony and Diamond);

the second is endemically violated by AQFT, as a result of the strong nonlocal correlations coded in the vacuum state (or any vector state of bounded energy); and

this violation corresponds to the endemic outcome dependence discussed in (C) above.

(E): *Models of relativistic measurement—and their limits?*:—
As stated in my anxious warnings (2) and (3) of Section **1.1**: the algebraic approach to quantum theory has an operational flavour, with description of measurement being mostly confined to the words, and not expressed in the formalism. In particular, AQFT traditionally associates to each bounded region of Minkowski spacetime an algebra of operators, whose self-adjoint elements are interpreted as the physical quantities that can be measured by an operation confined to the region. There was some, but not much, formal analysis of this operation, e.g. in the work of Hellwig and Kraus, and of Davies and Lewis.

But recently, Fewster and Verch (cf. [9]) have given a detailed analysis along these lines, which includes a theorem expressing no-signalling, i.e. that spacelike correlations involve no superluminal causation. On the other hand: in 1993, Sorkin argued that assuming that arbitrary field-theoretic quantities could be measured would imply superluminal signalling.

More recently, two papers have addressed anew the topic of measurements that are "impossible because they imply superluminal signalling—their results being apparently at loggerheads, even contradictory. Borsten and co-authors give a criterion for a measurement to be no-signalling, that implies strong limitations on what can be measured. Bostelmann and co-authors argue that Sorkins argument, i.e. his protocol for signalling, does not hold good in the Fewster-Verch framework.

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