Department of Economics, London School of Economics
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A CAPM-Based Model with Feedback Traders, Time-Varying Volatility and Asymmetric News Impact Curve

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Abstract

This work derives an Intertemporal CAPM-based model that incorporates a feedback traders' effect in the form of first-lag serial correlations. In the proposed model, we use a non-standard market equilibrium condition that links traders demand to current market value (wealth distribution over time) and not just to the market portfolio (wealth distribution over assets). The market risk is modeled with a Threshold-GARCH in-mean model in order to account for Time-varying conditional variances of the assets' returns and asymmetric news impact curve. The model is estimated on weekly returns of the S&P index over the years 1980-2008. The empirical analysis reject the Intertemporal CAPM and reveals that relation between expected returns and risk is highly nonlinear. We also find evidence that both returns and conditional variances are serially correlated.

1. Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Linter (1965) and Mossin (1966) became one of the most popular models in the literature of asset pricing. An important insight the CAPM provides is that expected returns should be related to the degree of the asset risk. Particularly, the CAPM imposes a restriction of linearity between expected returns and risk. Because of its simplicity, the CAPM provides a very convenient framework for asset pricing. However, evidence from stock markets behaviour show that the CAPM may be too restrictive in both senses that the relation between expected returns and risk might be nonlinear (Gennotte and Marsh 1993), and in that expected returns seem to be affected by other economic variables (fundamentals), as well as by non-fundamentals (behavioral anomalies). For instance, Cuthbertson and Nitzsche (2004) provides a comprehensive discussion on these issues. Some attempts have been made to directly introduce other economic variables to the model, such as dividend yields, size of firm, volume of the trading, January effect, etc. The present study takes another approach and focuses on several common phenomena of stock markets that can be modeled by only variables that relates directly to the asset prices distributions.

One of the empirical phenomenons this work is concerned with is the existence of noise traders in the markets. By definition, noise traders do not base their asset decisions on fundamental values, but they rather "trade on noise as if it were information" (Black 1986). DeLong, Shleifer, Summers and Waldmann (1990) developed an equilibrium asset pricing model in which noise traders may misperceive the true expected price of the risky assets, corresponding to an independent and identically distributed normal random variable around the true expected price. DeLong, Shleifer, Summers and Waldmann (1991) also showed that noise traders may survive in the market in the long run together with the sophisticated traders.

In this work, we will concentrate on a different type of noise traders, named feedback traders. Feedback traders buy or sell assets according to only the last period trend in the market. Positive feedback traders (also referred to as momentum traders) buy after a price rise, and sell (or short sell) after a price fall. This kind of behaviour is typical, for example, of charities that chase trends, but can also be induced by a mass of stop-loss orders. Some experiments (c.f. Shleifer and Summers 1990, Shiller 1990) show that this kind of momentum behaviour may also stem from pure psychological factors. Feedback trading is closely related to the well-known 'herding behaviour' of investors, and much evidence of its existence can be found in the literature (e.g., Shleifer and Summers 1990, Shiller 1990). Another possibility for traders considered here is to sell (or short sell) after a price rise and to buy after a price fall. This manner of trading is expected from negative feedback traders, who, for instance, follow the rule of 'buy low and sell high'. It is also typical for investors that maintain a constant ratio of each asset within their portfolio.

While empirical research on feedback or noise tends to focus on quantifiable influences induced by the presence of noise traders in the markets, such as positive autocorrelation in the returns and mean-reversion (e.g. Taylor 1985, Fama and French 1988, Poterba and Summers 1988, and Ma, Dare and Donaldson 1990), in this work we follow essentially the line of Sentana and Wadhwani

(1992), and we formulate and fit an explicit CAPM-based model, which accounts for feedback traders as well as for the rational traders that follow a mean-variance efficient strategy. It should be noted that Sentana and Wadhwani (1992) used a univariate 'feedback traders' model that also incorporates the rest of the market characteristics discussed in this essay. However, they resort to somewhat technical and empirical analysis, which lacks economic interpretation. The main goal of this work is to formulate Sentana and Wadhwani's (1992) model as a general multivariate model, and to derive the economic intuition that stands behind the predictions of the model. We also use a non-standard market equilibrium condition that links traders demand to current market value (wealth distribution over time) and not just to the market portfolio (wealth distribution over assets). Crucially, this modified condition seems to correct a problematic result of Sentana and Wadhwani's (1992) model.

A second feature of financial markets discussed in this work is the dynamic and evolving nature of market risk and return. Early evidence of time varying volatilities in stock returns goes back to Mandelbrot (1963) and Fama (1965). With the exception of a relatively few studies (c.f. Gibbons and Ferson 1985, Ferson, Kandel and Stambaugh 1987, and Ferson 1989), a lack of appropriate econometric framework hindered the formulation of an asset pricing model with time-varying risk until the development of the GARCH-in-mean (GARCH-M) models (Engle, Lilien, and Robbins 1987). Indeed, this development led Bollerslev (1987), French, Schwert, and Stambaugh (1987), Chou (1988) and many others to test a CAPM model while taking into account the heteroscedasicity of the market. Overall, the results of these experiments are not supportive of the CAPM, while on the other hand, it was also demonstrated by these studies that the GARCH-M model is sensitive to model specification, so it is possible that these tests' results were influenced by misspecification of the true model. Many other works have been continually attempting to reinvestigate the CAPM model with some more flexible and sophisticated models, the results of which are not yet conclusive.

Finally, this work also deals with the issue of the so-called 'asymmetric news impact curve', which corresponds to the fact that volatility and covariances generally respond in an asymmetric way to positive or negative shocks in share prices. Black (1976) suggested that this asymmetry may be caused by a 'leverage effect', according to which, if the share price of a company falls, the debt-to-equity ratio of that company grows and, thus, the risk of this company to go bankrupt grows. As a result of this increased risk, the company's share becomes more volatile. An alternative view of French, Schwert and Stambaugh (1987) and Campbell and Hentschel (1992) is that the asymmetry effect may be caused if volatilities exhibit positive feedback effect. In other words, the 'volatility feedback' theme is that asymmetric news impact curve may result from a positively autocorrelated volatility, consistent with the GARCH-M approach discussed above. This may happen if large absolute-valued shocks are expected to be followed by further large absolute-valued shocks, which in turn, by a CAPM-type argument, should lower the stock price. For large positive shocks, this mechanism is expected to dampen the initial positive impact of the good news, and therefore acts as a stabilizer of the share price. On the other hand, for initial large negative shock, it intensifies the negative impact of the news and yields higher volatility after market fall. Several empirical studies that investigated the phenomena with GARCH-M models tend to agree on the existence of an asymmetric news impact curve (see, for instance, Nelson 1991, Glosten, Jagannathan and Runkle 1993, and Bekaert and Wu 2000), while the economic interpretation of this effect is still controversial.

The rest of this essay is organized as follows. In Section 2 the Sentana and Wadhwani's (1992) version of the Intertemporal CAPM is developed and discussed. The main empirical model proposed in this essay is presented in Section 3. Section 4 estimates the model on the S&P 500 index. Section 5 concludes and summarizes.

2. Background

2.1 Intertemporal CAPM

Let $r_t^1, r_t^2, ..., r_t^N$ be random variables representing the rate of returns on risky assets $i = 1, 2, ..., N$, respectively, between times $t-1$ and t, where $t = 1, 2, ..T$,

$$r_t^i = \frac{\left\{\begin{array}{c}\text{investment value in}\\ \text{asset } i \text{ at time } t\end{array}\right\} - \left\{\begin{array}{c}\text{investment value in}\\ \text{asset } i \text{ at time } t-1\end{array}\right\}}{\left\{\begin{array}{c}\text{investment value in}\\ \text{asset } i \text{ at time } t-1\end{array}\right\}}.$$

We assume that all assets are in strictly positive supply. In addition, the market consists of a risk-free asset (e.g., treasury bills) that yields a safe return of r_t^0 between times $t-1$ and t. We use the notation $r_t^{\mathbf{p}}$ for the corresponding return on an arbitrary risky portfolio, $\mathbf{p}$, such that $\mathbf{p}'\mathbf{1} = 1$, and also by $r_t^{\mathbf{m}}$ the corresponding return on the market portfolio, $\mathbf{m}$, composed of the risky assets aggregately held by the investors in proportion to the total wealth in the economy.

The Intertemporal Capital Asset Pricing Model (ICAPM), developed by Merton (1973, 1980), extends the Sharpe-Linter-Mossin CAPM to captures the successive nature of the market. According to the ICAPM, under some restrictive assumptions (see Bodurtha and Mark 1991, footnote 3, for alternative sets of assumptions) on the economy, the expected returns follow a linear relation in congruence with the standard one-period CAPM, that is to say

$$E_{t-1}\left(r_t^i\right) = r_t^0 + \beta_t^i \cdot \left[E_{t-1}\left(r_t^{\mathbf{m}}\right) - r_t^0\right], \qquad t = 1, 2, ... \tag{1}$$

Here, β_t^i is the (expected) 'beta' of asset i at time t, defined as the conditional correlation between asset i and the market portfolio,

$$\beta_t^i = \frac{Cov_{t-1}\left(r_t^i, r_t^{\mathbf{m}}\right)}{Var_{t-1}\left(r_t^{\mathbf{m}}\right)},$$

and

$$E_{t-1}(\cdot) = E(\cdot|\Psi_{t-1}), Cov_{t-1}(\cdot) = Cov(\cdot|\Psi_{t-1}) \textit{ and } Var_{t-1}(\cdot) = Var(\cdot|\Psi_{t-1})$$

denotes expectation, covariance and variance, respectively, conditional on the past information Ψ_{t-1} available to investors at the end of period $t-1$. It is seen from (1) that the ICAPM allows the expected equilibrium returns to vary across time provided that the conditional correlations, or the beta, are time varying.

Denote the (conditional) 'market price of risk' (or the 'Sharpe ratio') by

$$\lambda_t^{\mathbf{m}} = \frac{\left[E_{t-1}\left(r_t^{\mathbf{m}}\right) - r_t^0\right]}{Var_{t-1}\left(r_t^{\mathbf{m}}\right)}, \tag{2}$$

where subscript **m** highlights the fact that $\lambda_t^{\mathbf{m}}$ represents the aggregated risk aversion of the wide-market (cf. Merton 1980). By linearity properties, the ICAPM equation (1) implies that, for any risky portfolio **p**,

$$E_{t-1}\left(r_t^{\mathbf{p}}\right) = r_t^0 + \lambda_t^{\mathbf{m}} Cov_{t-1}\left(r_t^{\mathbf{p}}, r_t^{\mathbf{m}}\right), \qquad t = 1, 2, ... \tag{3}$$

Moreover, under the assumption of constant relative risk aversion utility functions of the traders, Merton (1980, p. 329) showed that (3) can be approximated by

$$E_{t-1}\left(r_t^{\mathbf{p}}\right) = r_t^0 + \lambda^{\mathbf{m}} Cov_{t-1}\left(r_t^{\mathbf{p}}, r_t^{\mathbf{m}}\right), \qquad t = 1, 2, ..., \tag{4}$$

where now the market price of risk, $\lambda^{\mathbf{m}}$, is independent of time. In particular, the expected risk premium of portfolio **p**, $E_{t-1}\left(r_t^{\mathbf{p}}\right) - r_t^0$, is proportional to the conditional covariance between **p** and the market portfolio. The corresponding equation for the return on the market portfolio is then given by

$$E_{t-1}\left(r_t^{\mathbf{m}}\right) = r_t^0 + Var_{t-1}\left(r_t^{\mathbf{m}}\right), \qquad t = 1, 2, ... \tag{5}$$

To put the ICAPM into an appropriate empirical framework, we define the investor's conditional forecast errors on portfolio p as $\varepsilon_t^p = E_{t-1}\left(r_t^{\mathbf{p}}\right) - r_t^{\mathbf{p}}$. We also assume here, as well as in the rest of the essay, that $E\left(\varepsilon_t^p\right) = 0$, that is to say, the conditional expectation of the investor's is always an unbiased estimate of realized returns. It is straightforward now to see that (4) and (5) can be rewritten as the applicable econometric model,

$$r_t^{\mathbf{p}} = r_t^0 + \lambda^{\mathbf{m}} Cov_{t-1}\left(r_t^{\mathbf{p}}, r_t^{\mathbf{m}}\right) + \varepsilon_t^p, \qquad t = 1, 2, ... \tag{6}$$

and

$$r_t^{\mathbf{m}} = r_t^0 + \lambda^{\mathbf{m}} \left(\sigma_t^{\mathbf{m}}\right)^2 + \varepsilon_t^m, \qquad t = 1, 2, ... \tag{7}$$

with

$$E\left(\varepsilon_t^p\right) = 0 \text{ and } \left(\sigma_t^{\mathbf{m}}\right)^2 = Var_{t-1}\left(\varepsilon_t^m\right).$$

2.2 Sentana-Wadhwani Model

Sentana and Wadhwani (1992) introduced the ICAPM model (7) with a new interaction variable that represents an effect of feedback traders. Sentana and Wadhwani (1992) assumed that the proportional market demand, S_t, of the rational investors (henceforth the 'smart money') in the market follows a simple mean-variance model described by the following equation,

$$S_t = \frac{E_{t-1}\left(r_t^{\mathbf{m}}\right) - \alpha}{\mu\left[\left(\sigma_t^{\mathbf{m}}\right)^2\right]}, \tag{8}$$

with parameter $\alpha > 0$, represents a constant over time rate of return for which the demand by smart money is zero and $\mu\left[\left(\sigma_t^{\mathbf{m}}\right)^2\right]$ is a measure of the perceived riskiness of shares. Note that if the smart money 'takes over' the market, then $S_t = 1$, and relation (8) reduces to a similar relation as the ICAPM relation (7), but with α replacing the time-varying market risk-free rate r_t^0. However, α is not predetermined like r_t^0, but is rather a free parameter estimated by the model.

On the other hand, proportional market demand for stocks by the feedback traders is assumed to be determined by the rule

$$F_t = \gamma \cdot r_{t-1}^{\mathbf{m}}, \tag{9}$$

where γ represents the feedback value. $\gamma > 0$ indicates positive feedback traders, while $\gamma < 0$ indicates negative feedback traders. Sentana and Wadhwani (1992) now placed the market equilibrium requirement

$$S_t + N_t = 1, \tag{10}$$

reflecting the fact that total supply of assets is finite and strictly positive. Putting (8), (9) and (10) together finally yields the model

$$r_t^{\mathbf{m}} = \alpha + \mu\left[(\sigma_t^{\mathbf{m}})^2\right] - \gamma \cdot \mu\left[(\sigma_t^{\mathbf{m}})^2\right] \cdot r_{t-1}^{\mathbf{m}} + \varepsilon_t, \qquad t = 1, 2, ... \tag{11}$$

Comparing the Sentana-Wadhwani model with the CAPM equation (7), we see that the term $-\gamma\mu\left[(\sigma_t^{\mathbf{m}})^2\right] r_{t-1}^{\mathbf{m}}$ is added to the model and allows for serial correlations in the returns. It is also implied that the serial correlations vary with the volatility. The Sentana-Wadhwani model suggests that stock market price anomalies in the manner of serial correlations between consequent returns are larger when the perceived riskiness of the market is high than when it is low. When expected volatility rises, the demand (or supply) by the smart money drops down, which in turn brings the feedback traders to dominate the market, and causes returns to exhibit stronger serial correlation. Somewhat surprisingly, the model also suggests that returns should exhibit negative serial correlations when the market is influenced by positive feedback traders ($\gamma > 0$), and it should exhibit positive serial correlations if the market is influenced by negative feedback traders ($\gamma < 0$). However, it seems to us that this implication results form a reverse causality arguments on the relative demand of the two type of traders under equilibrium. Consider, for example, the case where the market is influenced by a mass of positive feedback traders. In this case, high positive returns at time $t-1$ will cause high positive demand of the feedback traders at time t. According to Sentana-Wadhwani model, under market equilibrium, this high demand must be met by a corresponding high supply by the smart money. In market equilibrium with positive feedback traders we therefore expect that high positive returns imply not only higher demand by feedback traders, but also higher supply by the smart money traders. However, since smart money buy and sell assets only when it is possible to gain higher expected returns or lower risk, the model 'forces' expected returns to fall after price rise to justify the corresponding supply by the smart money. Therefore the behaviour of the smart money, and also the expected market returns, are affected by the last period returns according to the equilibrium constraint (10). In the following subsection we suggest an alternative equilibrium condition, which, albeit not compatible with the standard ICAPM, seems to correct this seemingly problematic implication of Sentana-Wadhwani model.

Sentana and Wadhwani (1992) implemented a linearised variant of (11) and fitted the model

$$r_t^{\mathbf{m}} = \alpha + \rho\,(\sigma_t^{\mathbf{m}})^2 - \left(\gamma_0 + \gamma_1\,(\sigma_t^{\mathbf{m}})^2\right) r_{t-1}^{\mathbf{m}} + \varepsilon_t, \qquad t = 1, 2, ... \tag{12}$$

to US stock returns daily and hourly data in the years 1855-1988. They used a GARCH-M(1,1), an Exponential GARCH-M[1] (1,1) as well as a semiparametric model to estimate $(\sigma_t^{\mathbf{m}})^2$ together with the parameters α, ρ, γ_0 and γ_1.

[1] The Exponential GARCH-M model (EGARCH-M) was developed by Nelson

They results show that ρ is not statistically different from zero, while γ_1 is statistically significant. This result imply that the influence of the expected conditional volatility on the expected returns works only through the non-linear interaction variable $\left(\sigma_t^{\mathbf{m}}\right)^2 r_{t-1}^{\mathbf{m}}$, and not through $\rho\left(\sigma_t^{\mathbf{m}}\right)^2$ as suggested by the ICAPM.

3. Generalized Asset Pricing Model

3.1 Feedback Traders

Define the vector of risky returns,

$$\mathbf{r}_t = \left(r_t^1, r_t^2, ..., r_t^N\right)', \qquad t = 1, 2, ... \ ,$$

and also let $\mathbf{V}_t$ be the conditional covariance matrix of r_t given the past information Ψ_{t-1}. The demand vector, $\mathbf{S}_t$, of the smart money is chosen to correspond to the mean-variance efficient portfolio,

$$\mathbf{S}_t = \lambda^{-1}\mathbf{V}_t^{-1}\left[E_{t-1}\left(\mathbf{r}_t\right) - r_t^0\mathbf{1}\right], \tag{13}$$

where $\lambda = \lambda^{\mathbf{m}}$ represents the wide-market risk aversion. Similarly to Sentana and Wadhwani (1992), assume that the market consists of feedback traders, whose demand vector for the risky assets, $\mathbf{F}_t = \left(F_t^1, F_t^2, ..., F_t^N\right)'$, depends only on the last period returns, according to the equation

$$\mathbf{F}_t = \lambda^{-1}\mathbf{\Gamma}\mathbf{r}_{t-1}, \tag{14}$$

where $\mathbf{\Gamma}$ is in general a parameter matrix of size $N \times N$.

In order to resolve the troublesome implication of Sentana-Wadhwani model discussed in subsection 2.2, vectors $\mathbf{S}_t$ and $\mathbf{F}_t$ may sum up to any value and not necessarily to one, as opposed to Sentana-Wadhwani model. These vectors represent now the demand with respect to the level of wealth at the beginning of time t (equivalently, the level of wealth at the end of time $t-1$), and they sum up to a value higher or lower than one in situations of excess demand or excess supply (measured again at the point of time in the beginning of time t, before trading activity starts and leads quickly to a new market equilibrium), respectively. Let $\mathbf{W}_t = \left(W_t^1, W_t^2, ..., W_t^N\right)'$ be a vector of the total wealth invested at time t in each of the risky assets by both two kind of traders, so that the total aggregate wealth invested in the market at time t is equal

(1991) to allow for asymmetric variance effects of the kind discussed in the introduction.

to $W_t^m = \mathbf{W}_t'\mathbf{1}$. In equilibrium, the observed market values must reflect the portfolio desired by investors in the aggregate, i.e. $\mathbf{m}_t = (W_t^m)^{-1}\mathbf{W}_t$. Note that $\mathbf{W}_t$ may be computed directly from the market as the market value of each asset, i.e., the price of asset unit (e.g., a share) times the number of assets units outstanding (see, for instance, Merton 1973). To retain market equilibrium, we require for each asset $i \in \{1, ..., N\}$

$$W_{t-1}^m(\mathbf{S}_t + \mathbf{F}_t) = \mathbf{W}_t.$$

Equivalently, define the vector $\mathbf{W}_{t-1,t} = \left(W_{t-1}^m\right)^{-1}\mathbf{W}_t$. The equilibrium condition then becomes

$$\mathbf{S}_t + \mathbf{F}_t = \mathbf{W}_{t-1,t}. \tag{15}$$

The use of a general vector $\mathbf{W}_{t-1,t}$ enables to regard to relative demand functions among different assets, and not just among smart money and feedback traders in each of the assets. More importantly, unlike in Sentana-Wadhwani model, $\mathbf{W}_{t-1,t}'\mathbf{1} = W_t^m / W_{t-1}^m$ is not necessarily equal to one, but it is rather a random variable that varies over time and may depend on past information Ψ_{t-1}. While a rigorous theory, basically beyond the scope of this paper, would be essential to properly justify the proposed market equilibrium condition, intuitively we assume that changes in the total wealth of the economy are mainly due to variations in asset values, i.e. share price, and not because of reallocation of capital, i.e. the risk-free asset, new issued shares and other alternative assets (cf. Merton 1973, footnote 11). This assumption may be justified, for example, by a constant relative risk aversion of both rational and feedback traders, and by the assumption that the fraction of wealth invested in the market is approximately constant for all investors, so maximizing expected utility does not justify reallocation of capital. Under this setting, we may ascribe changes in market values only to demand and supply forces. Particularly, $\mathbf{W}_t$, the equilibrium market portfolio at time t, is assumed to properly reflect the demand and supply forces evolving over time. Crucially, we do not only take into account the diversification of the market wealth among the different assets but also the diversification among different time periods. The market demand function, based on information given at time $t-1$, determines the market prices at time t, thus acting as a self-fulfilling prophecy. Interesting enough, some similar ideas about self-fulfilling believes also in the context of market that cinsists of technical analysts and momentum traders were suggested by Menkhoff (1997) and Jordan (2006).

To further understand the implication of this modification, consider again the example of the last section where the market is influenced by a mass of positive feedback traders ($\mathbf{\Gamma} > 0$) and returns at time $t-1$ are highly positive. As before, increase in returns at time $t-1$ leads to a high positive demand by feedback traders at time t. This time, however, the feedback traders' demand is met by a corresponding high supply by the smart money only if the latter expect the prices to fall. Alternatively, the feedback traders demand may also lead to an excess demand at the beginning of time t, which in turn raises market prices until the market reaches an equilibrium in which we finally have $\mathbf{W}_{t-1,t} > 1$.

Equations (8), (9) and (10) together yields now

$$E_{t-1}\left(\mathbf{r}_t\right) = r_t^0\mathbf{1} + \lambda\mathbf{V}_t\mathbf{W}_{t-1,t} - \mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1}. \tag{16}$$

It is seen that this model is very similar to the univariate Sentana-Wadhwani model in the sense that the term $-\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1}$, which stands for serial correlations in the returns, is added to standard ICAPM equation. Note that it is possible that returns will exhibit cross-sectional serial correlations if $\mathbf{V}_t\mathbf{\Gamma}$ forms a nondiagonal matrix. As already stressed above, $\mathbf{W}_{t-1,t}$ may depend on $\mathbf{r}_{t-1}$ as well, so serial correlations may also manifest themselves indirectly through the term $\lambda\mathbf{V}_t\mathbf{W}_{t-1,t}$.

To show analogy of the model (16) to the standard ICAPM equation, multiply (16) from left by $\mathbf{p}'$, where $\mathbf{p}$ is an arbitrary risky portfolio. We get

$$\begin{aligned}
E_{t-1}\left(r_t^{\mathbf{p}}\right) &= r_t^0\mathbf{p}'\mathbf{1} + \lambda\mathbf{p}'\mathbf{V}_t\mathbf{W}_{t-1,t} - \mathbf{p}'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1} \\
&= r_t^0\mathbf{p}'\mathbf{1} + \lambda\mathbf{p}'\mathbf{V}_t\mathbf{W}_{t-1,t} - \lambda\mathbf{p}'\mathbf{V}_t\mathbf{F}_t \\
&= r_t^0\mathbf{p}'\mathbf{1} + \lambda\mathbf{p}'\mathbf{V}_t\mathbf{S}_t \\
&= r_t^0\mathbf{1} + \lambda \cdot \mathbf{S}_t'\mathbf{1} \cdot Cov_{t-1}\left(r_t^{\mathbf{p}}, r_t^{\mathbf{S}}\right),
\end{aligned}$$

where $r_t^{\mathbf{p}}, r_t^{\mathbf{S}}$ are the returns at time t of portfolio $\mathbf{p}$ and of portfolio $\mathbf{S}_t / \mathbf{S}_t'\mathbf{1}$ that corresponds the smart money traders, respectively. Since the same relation holds for the portfolio $\mathbf{p} = \mathbf{S}_t / \mathbf{S}_t'\mathbf{1}$, it is straightforward to see that the following version of the standard ICAPM is obtained.

$$E_{t-1}\left(r_t^i\right) = r_t^0 + \beta_t^i \cdot \left[E_{t-1}\left(r_t^{\mathbf{S}}\right) - r_t^0\right], \qquad t = 1, 2, ...,$$

Here, β_t^i is a 'modified beta' of asset i at time t, defined by

$$\beta_t^i = \frac{Cov_{t-1}\left(r_t^{\mathbf{p}}, r_t^{\mathbf{S}}\right)}{Var_{t-1}\left(r_t^{\mathbf{S}}\right)}. \tag{17}$$

It is well known that in the standard ICAPM only the systematic risk, i.e. the risk that is related to the market portfolio $\mathbf{m}_t$, is priced. In the feedback trader model, however, only the systematic risk related to the smart money portfolio $\mathbf{S}_t$, is priced. Stock price anomalies effects caused by feedback traders can be ignored due to the presence of the smart money in the market. In other words, feedback traders' behaviour is already taken into account by the smart money, so that smart money behaviour already reflects all knowledge market prices.

As a result, in equilibrium, the risk of the market is measured relatively to $\mathbf{S}_t$ and not to $\mathbf{m}_t$.

Finally, note that if the smart money 'takes over' the market, then $S_t = \mathbf{m}_t$, and the 'modified beta' (17) reduces to te standard beta. Hence, in this case we get the standard ICAPM equation, even though we used a nonstandard equilibrium equation.

3.2 The Econometric Framework

Ample empirical evidence from stock markets indicates that stock returns share the following common features:

(1) Returns seem to be serially correlated.
(2) Market volatility seems to vary over time.
(3) Market volatility seems to react differently to good news or to bad news.

The proposed econometric model attempts to capture these three features simultaneously. Because of the limitation of time, and lack of available computer programs or code to support applications of multivariate GARCH-M models, only a univariate version of the model is proposed here. A more general multivariate model can be also considered, though the computational burden involved in estimating a multivariate GARCH-M model can be very heavy.

In order to capture feature 1, it is assumed that the market is composed of both smart money and feedback traders consistently with the model developed in the previous subsection. Multiplying equation (16) from left by a market-value weighted portfolio $\mathbf{m}_t = (W_t^m)^{-1}\mathbf{W}_t$, we get

$$\begin{aligned} E_{t-1}\left(r_t^{\mathbf{m}}\right) &= r_t \mathbf{m}_t'\mathbf{1} + \lambda \mathbf{m}_t'\mathbf{V}_t\mathbf{W}_{t-1,t} - \gamma \mathbf{m}_t'\mathbf{V}_t\mathbf{r}_{t-1} \\ &= r_t^0 \mathbf{m}_t'\mathbf{1} + \lambda\left(W_t^m / W_{t-1}^m\right)\mathbf{m}_t'\mathbf{V}_t\mathbf{m}_t - \mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1} \\ &= r_t^0 + \lambda\left(W_t^m / W_{t-1}^m\right)\left(\sigma_t^{\mathbf{m}}\right)^2 - \mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1}. \end{aligned} \tag{18}$$

Note that $W_t^m / W_{t-1}^m = 1 + \left(W_t^m - W_{t-1}^m\right) \Big/ W_{t-1}^m = 1 + r_t^{\mathbf{m}}$. Thus, writing $E_{t-1}\left(r_t^{\mathbf{m}}\right) = r_t^{\mathbf{m}} + \varepsilon_t$, the model (18) becomes

$$r_t^{\mathbf{m}} = \left(1 - \lambda\left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1}\left[r_t^0 + \lambda\left(\sigma_t^{\mathbf{m}}\right)^2 - \mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1} + \varepsilon_t\right]. \tag{19}$$

The term $\left(1 - \lambda\left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1}$ suggests a nonlinear effect of the volatility. By definition of λ (see (2)) we may restrict λ to the range $0 \leq \lambda < \frac{1}{\left(\sigma_t^{\mathbf{m}}\right)^2}$, therefore we expect $\left(1 - \lambda\left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1}$ to be positive and strictly increasing with $\left(\sigma_t^{\mathbf{m}}\right)^2$.

This term stands for higher orders of volatility, since by a Taylor expansion

$$\left(1-\lambda\left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1}=1+\lambda\left(\sigma_t^{\mathbf{m}}\right)^2+\lambda^2\left(\sigma_t^{\mathbf{m}}\right)^4+\ldots .$$

The term $\mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1}$ is difficult to interpret and work with. A convenient simplification is achieved by approximating $\mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\approx\gamma\left(\sigma_t^{\mathbf{m}}\right)^2\mathbf{m}_{t-1}$. In this case we get $\mathbf{m}_t'\mathbf{V}_t\mathbf{\Gamma}\mathbf{r}_{t-1}\approx\gamma\left(\sigma_t^{\mathbf{m}}\right)^2\mathbf{r}_{t-1}^{\mathbf{m}}$, as in Sentana-Wadhwani model. This approximation may be obtained, for instance, if $\mathbf{\Gamma}$ is equal to the outer product $\gamma\mathbf{m}_t\mathbf{m}_{t-1}'$. Alternatively, it can also be obtained if we approximate $\mathbf{V}_t\approx\left(\sigma_t^{\mathbf{m}}\right)\cdot Id$ and $\mathbf{\Gamma}\approx\gamma\cdot Id$, where Id is the identity matrix, and in addition $\mathbf{m}_t\approx\mathbf{m}_{t-1}$. While it is seen that a much more general model is attained with a general matrix $\mathbf{\Gamma}$, we still adopt the proposed simplification for the sake of simplicity, and the model becomes

$$r_t^{\mathbf{m}}\left(1-\lambda\left(\sigma_t^{\mathbf{m}}\right)^2\right)=r_t^0+\lambda\left(\sigma_t^{\mathbf{m}}\right)^2-\gamma\left(\sigma_t^{\mathbf{m}}\right)^2r_{t-1}^{\mathbf{m}}+\varepsilon_t.$$

In a similar manner to Sentana and Wadhwani (1992) model, the following linearised and more flexible variant of the model is considered

$$r_t^{\mathbf{m}}=\left(1-\lambda_0\left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1}\left[\alpha_0r_t^0+\alpha_1+\lambda_1\left(\sigma_t^{\mathbf{m}}\right)^2-\left(\gamma_0+\gamma_1\left(\sigma_t^{\mathbf{m}}\right)^2\right)r_{t-1}^{\mathbf{m}}+\varepsilon_t\right].$$

The parameters $\alpha,\lambda_0,\lambda_1,\gamma_0,\gamma_1$ are restricted to satisfy $0\leq\lambda_0,\lambda_1<\frac{1}{\left(\sigma_t^{\mathbf{m}}\right)^2}$ and $\left|\gamma_0+\gamma_1\left(\sigma_t^{\mathbf{m}}\right)^2\right|<1$ in order to guarantee stability of the process.

Correspondingly to feature 2, we use a GARCH-M (Engle, Lilien, and Robbins 1987) model in order to model the volatility dynamics. The standard GARCH model of Engle (1982) and Bollerslev (1986) uses a representation of the conditional variances and covariances as a weighted average of past squared forecast errors and the past squared returns. The GARCH-M model is suited to estimate the parameters in a CAPM-based equation, while simultaneously fitting a GARCH model for the conditional volatility $\left(\sigma_t^{\mathbf{m}}\right)^2$. We adopt the GARCH-M(1,1) approach, according to which the conditional volatility follows

$$\left(\sigma_t^{\mathbf{m}}\right)^2=a+b\cdot\varepsilon_{t-1}^2+c\cdot\left(\sigma_{t-1}^{\mathbf{m}}\right)^2, \tag{20}$$

where parameters a,b,c satisfy $a>0,\ b,c\geq0$ to ensure positivity of $\left(\sigma_t^{\mathbf{m}}\right)^2$, and $b+c<1$ to ensure stationarity of the volatility process.

Finally, feature 3 is modeled by elaborating the standard GARCH model to get a Threshold GARCH (TGARCH). Introduced by Zakoian (1991) and Glosten, Jagannathan and Runkle (1993), the TGARCH models divide the distribution of the innovations, which represents the unanticipated news effecting the share prices, into two disjoint intervals depending on the sign of the last innovation, and then approximate a piecewise linear function for the conditional volatility.

Applying the TGARCH(1,1) model to our case, equation (20) is changed to

$$(\sigma_t^{\mathbf{m}})^2 = a + b_0 \cdot \varepsilon_{t-1}^2 + b_s \cdot \text{sign}\{\varepsilon_{t-1}\} \cdot \varepsilon_{t-1}^2 + c \cdot \left(\sigma_{t-1}^{\mathbf{m}}\right)^2,$$

where b_0, b_s are two parameters and $\text{sign}\{\cdot\}$ is the standard sign operator with values ± 1. The absolute value of the parameter b_s may be thought of as a measure of the asymmetry effect in the volatility. $b_s = 0$ suggests that volatility is symmetric with respect to unanticipated news, while large absolute values of b_s clearly indicates that news impact curve is asymmetric.

4. Evidence from the S&P 500

4.1 Empirical Setup

We consider 1475 successive observations of weekly realized returns for the market-value weighted S&P 500 index over the period January 1st 1980 to March 31st March 2008. This time period covers several of the contemporary financial turmoils took place in the stock markets, among which are the market crash of October 1987, the 1990 Gulf war, the 1998 Russian government default on its debt payments, the 2000 dotcom bubble pop, the 9/11/2001 attack on the US world trade center and Pentagon, the 2003 Iraq war and the recent 2007 subprime mortgage crisis. The risk-free rate r_t^0 are determined as the weekly returns on a 10 years Treasury bill. All data were taken from the Yahoo Finance historical prices listings available in the internet. Figures 1 and 2 contain graphs of the S&P 500 index value and rate of returns over the test period.

The market model developed in the previous section is implemented on the data. Assuming that the forecast errors are normally distributed,

$$\varepsilon_t | \Psi_{t-1} \sim N\left(0, \sigma_t^{\mathbf{m}}\right), \tag{21}$$

it is possible to apply a standard Gaussian MLE procedure to estimate the TGARCH-M(1,1) system

$$\begin{aligned} r_t^{\mathbf{m}} &= \left(1 - \lambda_0 \left(\sigma_t^{\mathbf{m}}\right)^2\right)^{-1} \left[\alpha_0 r_t^0 + \alpha_1 + \lambda_1 \left(\sigma_t^{\mathbf{m}}\right)^2 - \left(\gamma_0 + \gamma_1 \left(\sigma_t^{\mathbf{m}}\right)^2\right) r_{t-1}^{\mathbf{m}} + \varepsilon_t\right], \\ (\sigma_t^{\mathbf{m}})^2 &= a + b_0 \cdot \varepsilon_{t-1}^2 + b_s \cdot \text{sign}\{\varepsilon_{t-1}\} \cdot \varepsilon_{t-1}^2 + c \cdot \left(\sigma_{t-1}^{\mathbf{m}}\right)^2, \end{aligned} \tag{22}$$

with respect to the ten parameters $\alpha_0, \alpha_1, \lambda_0, \lambda_1, \gamma_0, \gamma_1, a, b_0, b_s, c$.

The log-likelihood function of the model with time horizon $t = 1, ..., T$ is given

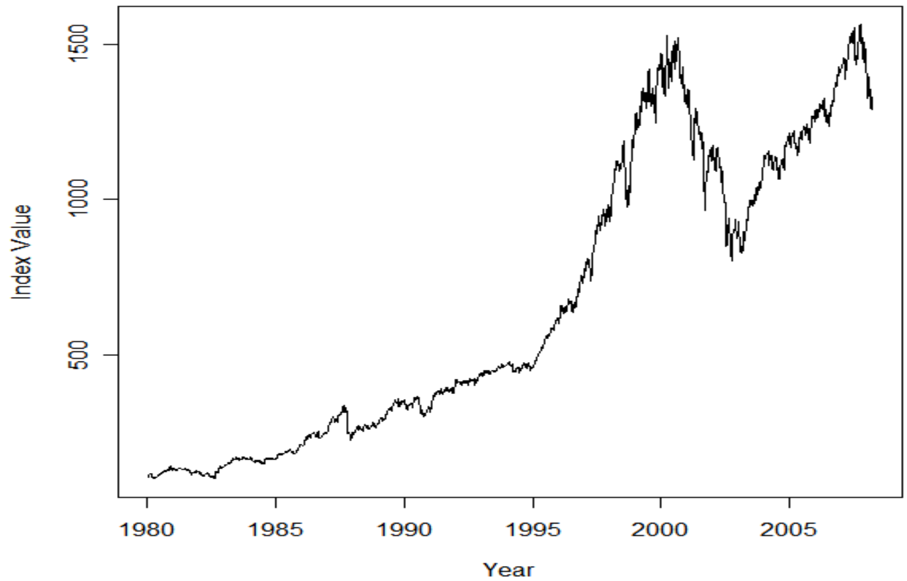

Fig. 1. S&P 500 index value

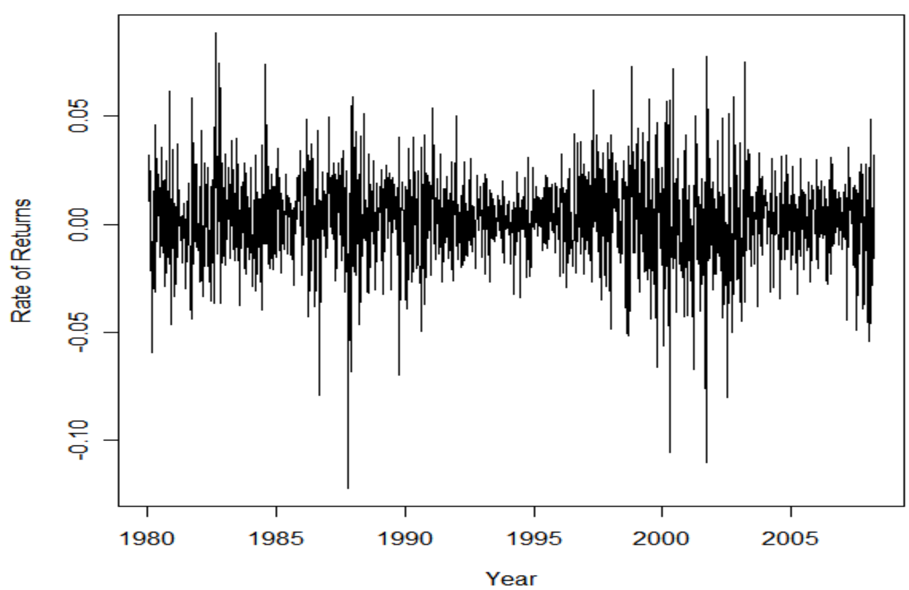

Fig. 2. S&P 500 index returns

(up to a constant) by

$$\log \mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} L_t(\boldsymbol{\theta}),$$

where

$$L_t(\boldsymbol{\theta}) = -\frac{1}{2}\left[\log\left(\sigma_t^{\mathbf{m}}\right)^2 + \left(\frac{\varepsilon_t}{\sigma_t^{\mathbf{m}}}\right)^2\right].$$

θ is a 10-dimensional vector that contains all unknown parameters for ε_t and $(\sigma_t^{\mathbf{m}})^2$. The maximum likelihood estimator $\widehat{\boldsymbol{\theta}}$ is then obtain as the parameter value that maximizes $\log \mathcal{L}(\boldsymbol{\theta})$.

Under sufficient regularity conditions and particularly under the assumption that the model is correctly specified, the parameter estimators are expected to be consistent and asymptotically normal with asymptotic variance corresponding to the Cramér-Rao theorem,

$$\sqrt{T}\left(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right) \sim_d N\left(0, \boldsymbol{\Gamma}(\boldsymbol{\theta}_0)^{-1}\right),$$

where $\boldsymbol{\Gamma}(\boldsymbol{\theta})$ is the Fisher information matrix of $L_t(\boldsymbol{\theta})$. Furthermore, under slightly stronger conditions (cf. Engle, Lilien, and Robbins 1987, p. 397), $\boldsymbol{\Gamma}(\boldsymbol{\theta}_0)$ may be well approximated by $\mathbf{S}'\mathbf{S}/T$ where $\mathbf{S}$ is the $T \times p$ matrix of the first derivatives of the conditional likelihood function of each single observation,

$$[\mathbf{S}]_{t,i} = \frac{\partial L_t\left(\widehat{\boldsymbol{\theta}}\right)}{\partial \boldsymbol{\theta}_i}.$$

Therefore we get the property

$$\left(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right) \sim_d N\left(0, (\mathbf{S}'\mathbf{S})^{-1}\right),$$

$\mathbf{S}_{t,i}$ may be conveniently estimated numerically, thus asymptotic variances can be used to assess the significance levels of the parameters by performing t-tests.

The estimation was mades on an R 2.6.1 software. In order to locate the maximum likelihood estimate, the likelihood functions was numerically maximized with the Broyden-Fletcher-Goldfard-Shanno variant of Davidon-Fletcher-Powell algorithm [2]. The model is nonlinear and high dimensional, therefore the maximization algorithm may easily yield estimators corresponding to local maximum of the likelihood function. In order to enhance the chances of the algorithm to find the global maximum, a relatively extensive search procedure for the likelihood maximum was employed using numerous starting parameters values (above 500), chosen randomly within the parameter space. For each of the parameters, a two-tailed t-test with 1465 (={#{observations}- #{parameters}) degrees of freedom was performed corresponding to the hypotheses that the parameters is insignificant.

[2] A quasi-Newton maximization method. The code for the algorithm was taken from the website of Daniel F. Heitjan at http://www.cceb.upenn.edu/pages/heitjan/optimize/

Parameter	Estimate	t statistic	p-value
α_0	0.010	0.443	0.65
α_1	0.812	30.830	0.00
λ_0	263.124	4.147	0.00
λ_1	84.459	1.386	0.16
γ_0	-0.169	18.500	0.00
γ_1	156.253	8.598	0.00
a	$6.2 \cdot 10^{-5}$	3.446	0.00
b_0	0.079	4.120	0.00
b_s	0.020	1.590	0.11
c	0.801	17.333	0.00

Table 1. TGARCH-M(1,1) model: parameter estimates

4.2 Summary of Results

The estimation values results are reported in Table 1. The results do not reject that the parameters α_0 is zero at the 5% significance level. The consequence $\alpha_0 = 0$ implies that the risk-free rate does not influence the index returns in line with standard pricing models. On the other hand the t-test rejects the hypothesis that α_1 or λ_0 are zero, maybe indicating that a more practical measure of excess returns is of the form $f\left(\sigma_t^{\mathbf{m}}\right) \cdot r_t^{\mathbf{m}} - \alpha_1$, rather than $r_t^{\mathbf{m}} - r_t^0$, where $f\left(\sigma_t^{\mathbf{m}}\right)$ is a possibly nonlinear function of the market volatility. The t-test also does not reject the hypothesis $\lambda_1 = 0$ at the 5% level. This result contradicts the ICAPM predictions, but is consistent with similar conclusions of Sentana and Wadhwani (1992) and other empirical works. The test statistic for the γ_0, γ_1 parameters reject that they are zero. The values $\gamma_0 = -0.169$, $\gamma_1 = 156.253$, indicate that weekly stock returns are likely to exhibit positive serial correlation most of the time, but when volatility is higher than a critical value corresponding to $\left(\sigma_t^{\mathbf{m}}\right)^2 = -\frac{\gamma_0}{\gamma_1} = 1 \cdot 10^{-3}$, then returns are more likely to exhibit negative serial correlation. A similar result was also reported by Sentana and Wadhwani (1992) for their daily data. The t-test do not reject the hypothesis that the threshold parameter b_s, representing the asymmetric news impact curve, is zero, thus providing an evidence against the asymmetric effect, albeit with a relatively small p-value (0.11). The rest of the GARCH parameters estimates a, b_0 and c are significant at the required level and their values $b_0 + c = 0.88$ suggest a high degree of persistence in volatility. Figure 3 presents the conditional market variance $\left(\sigma_t^{\mathbf{m}}\right)^2$ estimated by the model.

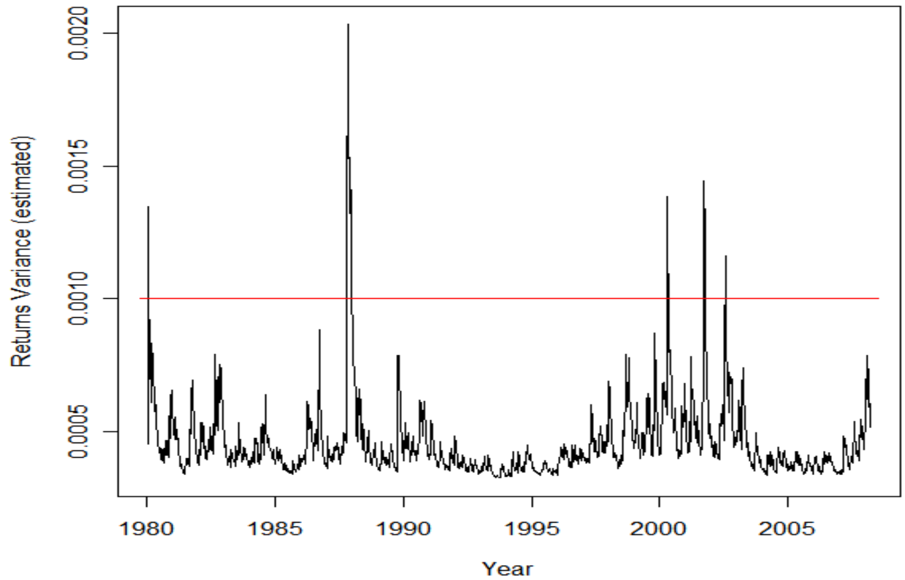

Fig. 3. S&P 500 index returns estimated conditional variance. The red line indicates the critical conditional varaince, above which the serial correlations are expected to be negative

This figure shows that besides in very few extremely volatile weeks, indeed the variance is usually lower than the critical value $1 \cdot 10^{-3}$, marked by the horizontal red line in the figure.

Finally, the model's standardized residuals $\varepsilon_t / \sigma_t^{\mathbf{m}}$ are analyzed graphically in Figure 4 and Figure 5.These figures show that the residuals come from a distribution that appear to be close to a normal distribution, consistent with the basic model assumption, though the left tail of the distribution that corresponds to extreme negative returns is somewhat longer than what is expected from standard normal distribution.

5. Conclusion

This essay develops a general asset pricing model that consists of feedback traders along with rational traders that follow a mean-variance efficient strategy. The proposed equilibrium condition accounts not only for the market portfolio but also for the market value, both are obtained as a 'self-fulfilling prophecy' induced by the traders demand functions. Based on this model, an empirical univariate model that allows also for time-varying conditional variances, serial correlations and asymmetric news impact curve, is estimated using weekly data from the S&P 500 index.

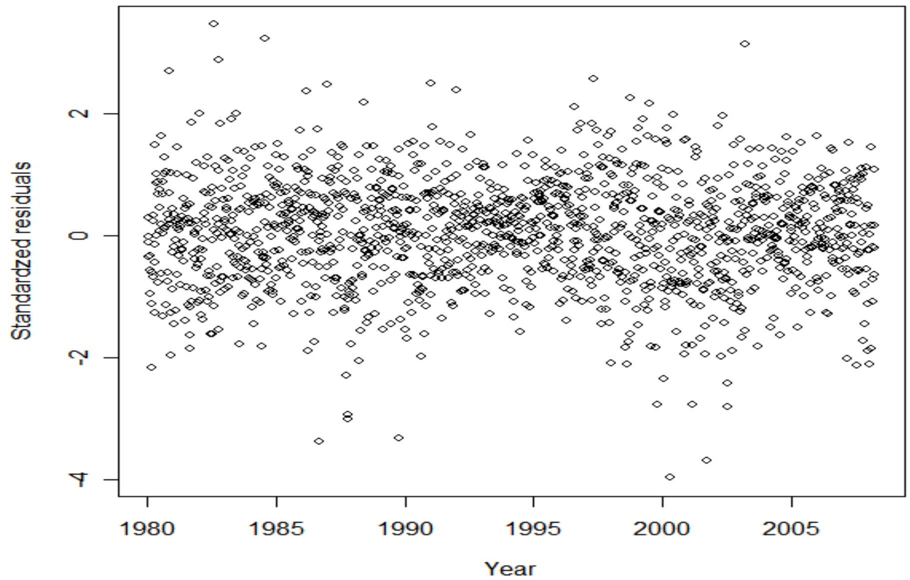

Fig. 4. The estimated model standardized residuals $\varepsilon_t / \sigma_t^{\mathbf{m}}$

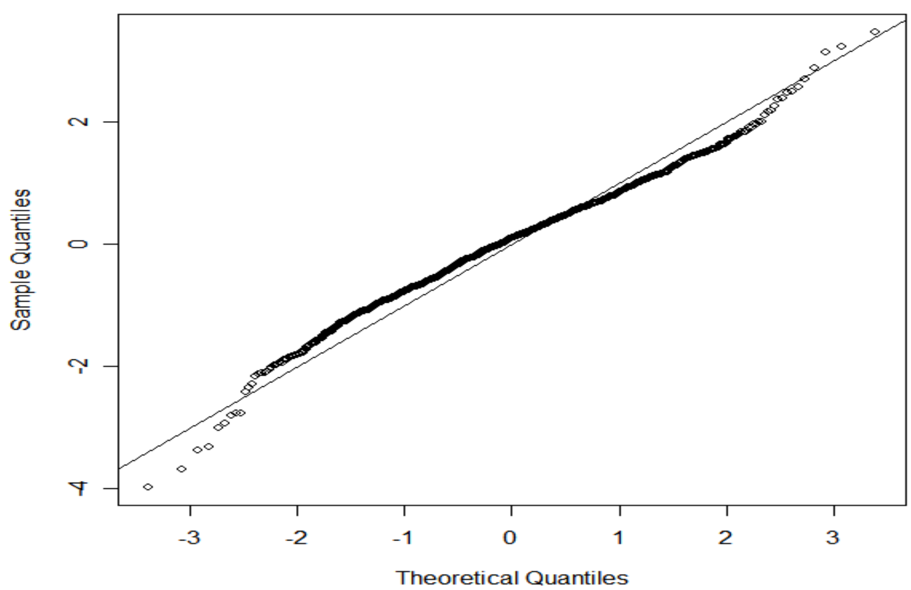

Fig. 5. Normal QQ-plot of the standardized residuals $\varepsilon_t / \sigma_t^{\mathbf{m}}$

The results indicate that the returns are serially correlated, where the serial correlations depends also on the expected risk. In addition, we find clear evidence that conditional variances change through time and that they are highly persistent. The results are not supportive of the ICAPM, and they suggest that relation between expected returns and risk is highly nonlinear.

We also do not find support for a ‘leverage effect’ in the volatility. Hence our rsults are consistent with the ‘volatility feedback’ theory, which claims that asymmetric news impact curve may result from a positively autocorrelated volatility.

Overall, our results conform with some previous works, and in particular with the conclusions of Sentana and Wadhwani (1992).

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