EC402 (20|21) - MT WK 6 -

ASYMPTOTICS TRILOGY : PART 1
"GENTLE INTRO. TO ASYMPTOTICS"

RAGUIR'S SPEAKING NOTES (UNOFFICIAL CONTENT)

(#1) "Ragvir, why are you so cruel? I have enough to do!"

- In previous years, my students really struggled when Vassilis started talking about asymptotics.

- To help, I thought it would be useful to introduce terminology now rather than in a few weeks. This is basically just "Item 6" of my "Stats check" slides (and effectively a stats "review" type thing).

- As usual, there is no pressure to read any of this material. No problem if you want to skip it.

(#2) "Ok, fine, I'm curious, Ragvir. What's this trilogy of yours?"

- Vassilis will soon start saying things like "Best CAN".
- In Part 1, I explain intuition for "C", consistency.
- In Part 2, —— " —— "AN", asymp. Normality.
- In Part 3, —— " —— "Best", asymp. Efficiency.

(#3) "Let me hear about Part 1 first... but keep it simple, ok?"

Consider $x_i \overset{IID}{\sim} N(\mu, \sigma^2)$ for $i = 1, \ldots, N$

Assume $0 < \sigma^2 < \infty$. Object of inference: μ, for $|\mu| < \infty$.

Step 1: let $\bar{x}_N := \frac{1}{N} \sum_{i=1}^{N} x_i$

Then, $E(\bar{x}_N) = \mu$ and $Var(\bar{x}_N) = \sigma^2/N$

Step 2: But then $MSE(\bar{X}_N) := E\left[(\bar{X}_N - \mu)^2\right]$

$$= E\left[\bar{X}_N^2 - 2\bar{X}_N\mu + \mu^2\right]$$

$$= E\left[\bar{X}_N^2\right] - \mu^2$$

$$= \frac{\sigma^2}{N} + \mu^2 - \mu^2$$

$$= \frac{\sigma^2}{N}.$$

(No surprise since I've just replicated the proof that $MSE(\cdot) = Var(\cdot) + Bias^2(\cdot)$ for an unbiased estimator!)

Step 3: So, $\lim_{N\to\infty} E\left[(\bar{X}_N - \mu)^2\right]$

$$= \lim_{N\to\infty} \frac{\sigma^2}{N}$$

$$= 0$$

since $\sigma^2 < \infty$.

i.e. $\bar{X}_N \xrightarrow{\text{M.S.}} \mu$.

(if you've never truly had convergence concepts explained to you, do talk to me, eg. what is { conv in m.s.? / conv in p.? / conv in d.? }

Step 4: Chebyshev's inequality $\Rightarrow$

For any $\varepsilon > 0$,

$$\lim_{N \to \infty} P\left(|\bar{X}_N - \mu| > \varepsilon \right) \leq \frac{\sigma^2}{N\varepsilon^2} = 0.$$

i.e. $\bar{X}_N \xrightarrow{p} \mu$, or "$\operatorname{plim}_{N \to \infty} \bar{X}_N = \mu$".

or "$\bar{X}_N$ is a consistent estimator for μ".

(#4) "Ragvir, enough. I knew you'd make it complicated. Can you get to the point before Christmas?!"

Yes, my friends, what we have just proved is indeed

Chebyshev's WLLN.

This is exactly Theorem 6 on page 48 of Vassilis' "Supplementary Technical Notes"

And along the way we've recapped ~~to~~ many concepts:

- Item 1 from "Status Check"
- Bias | Variance | MSE
- Convergence in m.s. and p
- Chebyshev's inequality
- What is an LLN
- Why MME makes sense
- Consistency & "plim" | root-N consistency
- Next term, we will compare with ergodicity

All this in just 3-4 slides!

And that's why I love this little proof. Any questions, just find me!

R (29/10/20)