EC402 (2023/24)

RAGVIR'S SPEAKING NOTES FOR CLASS

Vassilis PS3 (WK 4, AT)

PLAN

(Q1)

(Q2)

(Q3) (a) in detail

(Q1) MATH

$y = X\beta + \varepsilon$, where X is a $T \times K$ matrix. Suppose (WLOG) the last regressor is scaled by λ.

i.e. $Q := XA$ where $A := \begin{bmatrix} I_{K-1} & 0_{K-1} \\ 0'_{K-1} & \lambda \end{bmatrix}$

Say we use OLS: $y = X\hat{\beta} + \hat{\varepsilon}$. Then we scale X by A. That is, we fit $y = Q\hat{\alpha} + \hat{\eta}$.

If we force $\hat{\varepsilon} = \hat{\eta}$ (which is another way of saying we use OLS again), then clearly:

$$X\hat{\beta} = Q\hat{\alpha} = XA\hat{\alpha} \Rightarrow \hat{\beta} = A\hat{\alpha}$$

So that $\hat{\alpha} = A^{-1}\hat{\beta} = \begin{bmatrix} I_{K-1} & 0_{K-1} \\ 0'_{K-1} & 1/\lambda \end{bmatrix} \hat{\beta}$

$$\therefore \hat{\alpha}_j = \begin{cases} \hat{\beta}_j, & j = 1, \ldots, K-1 \\ \hat{\beta}_j/\lambda, & j = K. \end{cases}$$

INTUITION

x_K is measured originally in £ million, and now in £000s, i.e. $\lambda = 1000$.

If it's confusing, see this:

	Salary (£m)	Salary (£000)
Elon Musk	0.3	300
Ragvir	92	92,000

i.e. $\lambda = 1000$ here

So the regression coefficients will be scaled by a factor of 0.001 relative to the original results.

The estimated marginal effect on y of "1 unit" change in x_K will be

think big |effect| ← £ $\hat{\beta}_K$ when the "1 unit" is "1 million", but it will be £ $\hat{\alpha}_K = \hat{\beta}_K / 1000$ when the "1 unit" is just "1 thousand".

think smaller |effect|

Q2) FIRST GLIMPSE OF PANEL DATA

A3Rmi : $E[\varepsilon | X] = E[\varepsilon]$ – "STRONG"

A3Rsm : $E[\varepsilon_{it} x_{h,it}] = 0$ for every $h = 1, \dots, k$.

"WEAK"

Assume: $\underset{NT\times 1}{y} = \overset{NT\times K}{X} \underset{K\times 1}{\beta} + \varepsilon^{NT\times 1}$, $E(\varepsilon) = 0$

$$\begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1T} \\ \hline \vdots \\ \hline y_{N1} \\ y_{N2} \\ \vdots \\ y_{NT} \end{bmatrix} = X\beta + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1T} \\ \hline \vdots \\ \hline \varepsilon_{N1} \\ \varepsilon_{N2} \\ \vdots \\ \varepsilon_{NT} \end{bmatrix};$$

Also assume:

(i) $\varepsilon_{it} \perp\!\!\!\perp \varepsilon_{js}$ for $i \neq j$ (for any t, s)

(ii) $E(\varepsilon_{it}\varepsilon_{is}) \neq 0$ for all i (")

eg. unobserved heterogeneity

(a) A3F $\Rightarrow$ A3Rmi, A3Rsru.

(c) $y_{it} = \beta x_{it} + \varepsilon_{it}$

$$E[x_{it}\,\varepsilon_{it-s}] \begin{cases} = 0, & \text{for } s \leq 0 \\ \neq 0, & \text{for } s > 0 \end{cases}$$

$\Rightarrow$ A3Rmi, A3Rsru.

(b) Say $R=1$ and $x_{it} := y_{it-1}$

· To verify A3Rmi, we need to condition on leads and lags as well as the current value of regressors. That is,

check $E[\varepsilon_{it} \mid (\ldots, y_{it-2}, y_{it-1}, y_{it} \ldots)]$

but this is impossible since realisation of ε_{it} affects y_{it} for $t \geq 1$.

If still confused, here's some extra intuition

"We can't have it both ways" - either we treat ε_{it} as random in which case we cannot claim all regressor values/leads are given.

OR we treat ε_{it} as given so you can sensibly condition, but then why are we asking about $E[\cdot]$ of a non-random object?

Next consider A3R sm.

Say $y_{it} = \beta y_{it-1} + \varepsilon_{it}$ and of course $E[\varepsilon_{it}\varepsilon_{it-1}] \neq 0$.

Then, $E[y_{it-1}\varepsilon_{it}] = E[(\beta y_{it-2} + \varepsilon_{it-1})\varepsilon_{it}] \neq 0$.

$\Rightarrow$ ~~A3R mi~~, ~~A3R sm~~.

Q3(a)

(1) $y_t = \beta_1 x_{2t}^{\beta_2} x_{3t}^{\beta_3} \varepsilon_t$, for $t = 1, \dots, T$

Step 1: PROBLEM WITH MODEL (1)

Model (1) is non-linear in parameters. We cannot directly use linear regression methods to estimate this specification.

Step 2: PROPOSED SOLUTION

Model (1) may be linearised using a log transformation:

$\log y_t = \log \beta_1 + \beta_2 \log x_{2t} + \beta_3 \log x_{3t} + \log \varepsilon_t$, or

$\tilde{y}_t = \gamma_1 + \beta_2 \tilde{x}_{2t} + \beta_3 \tilde{x}_{3t} + \eta_t$, where

$$\begin{cases} \tilde{y}_t := \log y_t \,;\ \tilde{x}_{ht} := \log \tilde{x}_{ht} \text{ for } h = 2, 3; \\ \eta_t := \log \varepsilon_t \,;\ \text{and } \gamma_1 := \log \beta_1. \end{cases}$$

(2)

Step 3: Brief Analysis of Model (2)

Model (2) is linear in parameters as far as γ_1, β_2 and β_3 are concerned. It is a standard multiple linear regression model where $\tilde{y}_t$ is regressed on a constant, $\tilde{x}_{2t}$ and $\tilde{x}_{3t}$.

Of course the model remains non-linear with respect to β_1.

(a) the economists among us can maybe help explain why we may not care about β_1 in a Cobb-Douglas context?

(b) as Vassilis' solution explains, we retain consistency for $\hat{\beta}_1 := \exp[\hat{\gamma}_1]$.

↳ i.e. non-linearity wrt β_1 may not be a big deal.

We also will need to ensure that a log transformation is feasible.

Assumption 1: $y_t > 0$, $x_{2t} > 0$, $x_{3t} > 0$, $\varepsilon_t > 0$ for all t.

Step 4: FURTHER ANALYSIS OF MODEL (2)

PROPOSITION 1: Say we had Vassilis' "A1" under Model (1) and x_{2t} and x_{3t} were not constant over t.

Then $\tilde{X} := [\,i \;\; \tilde{x}_2 \;\; \tilde{x}_3\,]$ is a full rank matrix (provided $\log(\cdot)$ does not introduce any linear dependence)

PROPOSITION 2: Say we had Vassilis' "A3Rfi" and "A4GMiid"

Then, $\tilde{X} \perp\!\!\!\perp \eta$, and $E[\eta\eta'] = v I_T$ for a positive constant scalar v.

Under Model 2, ...

by Propositions 1 and 2 and Assumption 1, the OLS estimator for $\theta := (\gamma_1, \beta_2, \beta_3)'$ is the best linear unbiased estimator due to the Gauss-Markov theorem. If, additionally, we assume log-normality for ε_t, then the OLS estimator of θ is the best unbiased estimator.