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(Q4) $\underset{}{y} = \overset{T\times 4}{X}\overset{4\times 1}{\beta} + \varepsilon$; A1, A2, A3 R_{mi}, A4 GM, A5 N. ; Note: $\underset{T\times 4}{X} = [\iota, x_2, x_3, x_4]$; $\beta := (\beta_1, \ldots, \beta_4)'$

$\hat\beta := (X'X)^{-1}X'y$; $\hat\sigma^2 := \frac{(y-\hat y)'(y-\hat y)}{T-4}$

and $\widehat{var}(\hat\beta) := \hat\sigma^2 (X'X)^{-1}$

(i) $\left.\begin{array}{l} H_0 : R\beta = q \\ H_1 : R\beta \neq q \end{array}\right\}$ — (i)

(a) Hypotheses: As in (i) with

$$R = R_a := \begin{bmatrix} 0 & 1 & -3 & 0 \\ 1 & 0 & 0 & -2 \end{bmatrix}, \quad q = q_a := \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Test Statistic: $V := (R\hat\beta - q)'\left(R\,\widehat{Var}(\hat\beta)R'\right)^{-1}(R\hat\beta - q) \div r_a$ where $r_a = 2$

Dist. of V under H_0 : $V \sim F_{2,T-4}$

Sig level : α

Critical values : $V_{crit,1-\alpha} = F_{2,T-4\,;\,1-\alpha}$ → $100(1-\alpha)^{th}$ percentile on the CDF of the $F_{2,T-4}$ distribution

Decision rule : Reject H_0 iff $V > V_{crit,1-\alpha}$

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(b) Hypotheses: As in (i) with

$$R = R_b := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad q = q_b := \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Test Statistic: $V := (R\hat{\beta} - q)' \left(R\, \widehat{Var}(\hat{\beta}) R'\right)^{-1} (R\hat{\beta} - q) \div r_b$ where $r_b = 3$

Dist. of V under H_0: $V \sim F_{3, T-4}$

Sig level: α

Critical values: $V_{crit, 1-\alpha} = F_{3, T-4; 1-\alpha}$

Decision rule: Reject H_0 iff $V > V_{crit, 1-\alpha}$

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(c) Hypotheses: As in (i) with

redundant $\Rightarrow$ exclude this (or any 1 row)

$$R = R_c := \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}, \quad q = q_c := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ie. Row 1 + Row 3 = Row 2

Test Statistic: $V := (R\hat{\beta} - q)' \left(R \widehat{Var}(\hat{\beta}) R' \right)^{-1} (R\hat{\beta} - q) \div r_c$ where $r_c = 2$

Dist. of V under H_0: $V \sim F_{2, T-4}$

Sig level: α

Critical values: $V_{crit, 1-\alpha} = F_{2, T-4; 1-\alpha}$

Decision rule: Reject H_0 iff $V > V_{crit, 1-\alpha}$

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(ii) let $\hat{\varepsilon}_u = y - X\hat{\beta}_u$ where $\underset{T\times 4}{X} := [i\ x_2\ x_3\ x_4]$ and $\hat{\beta}_u = (X'X)^{-1}X'y$

Then define $RSS_u := \hat{\varepsilon}_u'\hat{\varepsilon}_u$.

(a) Consider $y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$.

Imposing the restrictions $\beta_2 - 3\beta_3 = 4$ and $\beta_1 = 2\beta_4$,

$$y_t = 2\beta_4 + (4+3\beta_3)\,x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$$

$$= \beta_4(x_{4t}+2) + 4x_{2t} + \beta_3 3x_{2t} + \beta_3 x_{3t} + \varepsilon_t$$

So, $$(y_t - 4x_{2t}) = \beta_4(x_{4t}+2) + \beta_3(3x_{2t}+x_{3t}) + \varepsilon_t$$

Thus, we can define $y_t^* := y_t - 4x_{2t}$; $z_{1t} := (x_{4t}+2)$; $z_{2t} := (3x_{2t}+x_{3t})$

Then, $\hat{\varepsilon}_{Ra_t} := y_t^* - z_t'\hat{\beta}_{Ra}$ where $z_t := [z_{1t}, z_{2t}]'$ and $\hat{\beta}_{Ra} := \left(\sum_{t=1}^{T} z_t z_t'\right)^{-1} \sum_{t=1}^{T} z_t y_t^*$

Finally, $RSS_{Ra} := \sum_{t=1}^{T} \hat{\varepsilon}^2_{Ra_t}$, and $r_a := 2$

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(b) Consider $y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$.

Imposing the restrictions $\beta_1 = 1$, $\beta_2 = 3\beta_4 - 1$ and $\beta_3 = 0$,

$$y_t = 1 + (3\beta_4 - 1) x_{2t} + \beta_4 x_{4t} + \varepsilon_t$$

$$\Rightarrow y_t - 1 + x_{2t} = \beta_4 (3x_{2t} + x_{4t}) + \varepsilon_t$$

Thus, we can define $\tilde{y}_t := y_t - 1 + x_{2t}$ and $q_t := 3x_{2t} + x_{4t}$

Then, $\hat{\varepsilon}_{Rb_t} := \tilde{y}_t - q_t' \hat{\beta}_{Rb}$ where $\hat{\beta}_{Rb} := \left(\sum_{t=1}^{T} q_t q_t'\right)^{-1} \sum_{t=1}^{T} q_t \tilde{y}_t$

Finally, $RSS_{Rb} := \sum_{t=1}^{T} \hat{\varepsilon}_{Rbt}^2$, and $r_b := 3$

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(c) Consider $y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$.

Imposing the restrictions $\beta_2 = \beta_3 = 2\beta_4$,

$$y_t = \beta_1 + 2\beta_4 x_{2t} + 2\beta_4 x_{3t} + \beta_4 x_{4t} + \varepsilon_t$$
$$= \beta_1 + \beta_4 (2x_{2t} + 2x_{3t} + x_{4t}) + \varepsilon_t$$

Thus, we can define $p_t := [1, 2x_{2t} + 2x_{3t} + x_{4t}]'$

Then, $\hat{\varepsilon}_{RC_t} := y_t - p_t' \hat{\beta}_{RC}$ where $\hat{\beta}_{RC} := \left(\sum_{t=1}^{T} p_t p_t'\right)^{-1} \sum_{t=1}^{T} p_t y_t$

Finally, $RSS_{RC} := \sum_{t=1}^{T} \hat{\varepsilon}^2_{RC_t}$ and, $r_c := 2$

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Now our 3 F-tests are as follows. For $h \in \{a, b, c\}$, we have:

Hypotheses: $H_0^h : R_h \beta = q_h$

$H_1^h : R_h \beta \neq q_h$

$R_a \beta = q_a$

Test Statistic: $V_h := \dfrac{(RSS_{R_h} - RSS_u)/r_h}{RSS_u/(T-4)}$

$$\frac{(\hat{\varepsilon}_{R_a}' \hat{\varepsilon}_{R_a} - \hat{\varepsilon}_u' \hat{\varepsilon}_u)/2}{\hat{\varepsilon}_u' \hat{\varepsilon}_u / T-4}$$

Sig level: α

Critical Value: $V_{crit,1-\alpha}^h = F_{r_h, T-4; 1-\alpha}$

Decision rule: Reject H_0^h iff $V_h > V_{crit,1-\alpha}^h$