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Endogenous Education and Long-Run Factor Shares†

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We study the determinants of factor shares in a neoclassical environment with capital-skill complementarity and endogenous education. In this environment estimates of the elasticity of substitution between capital and labor that fail to account for human capital levels will be biased upward. We develop a model with overlapping generations, technology-driven neoclassical growth, and ongoing increases in educational attainment. For a class of production functions featuring capital-skill complementarity, a balanced growth path exists and is characterized by an inverse relationship between the rates of capital- and labor-augmenting technological progress and the capital share in national income. (*JEL* D33, E25, J24, O33)

Ever since John Maynard Keynes (1939, p. 48) famously touted the stability of the capital and labor shares in national income as "one of the most surprising, yet best-established, facts in the whole range of economic statistics," growth theorists have been fascinated by the determinants of long-run factor shares and the reasons for their stability. Kaldor (1961) made the constancy of factor shares first of his six "stylized" facts of economic growth, and many economists have observed the continued stability of these shares well beyond the time of his writing. But, in recent years, the labor share declined precipitously, as has been documented and discussed by Elsby, Hobijn, and Şahin (2013); Karabarbounis and Neiman (2014); and many others. Now, the factor shares may well have stabilized again, with workers receiving a new and smaller slice of the economic pie (see, for example, Federal Reserve Bank of St. Louis 2020). These events have revived interest among growth economists in the determinants of the functional distribution of income.

If income shares are stable for long periods in the face of factor accumulation and (biased) technical progress, some equilibrating forces must be at work. A unitary aggregate elasticity of substitution between capital and labor could be one such

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force, because in a Cobb-Douglas world, any persistent divergence between the growth rates of the labor force and the capital stock would be offset by opposing trends in factor returns. But a large body of empirical research suggests that the elasticity of substitution is not equal to one.[1] In Grossman et. al (2017a), we suggested another possible equilibrating force: the endogenous response of education to a rise in the return to schooling could stabilize factor shares in the face of ongoing declines in the prices of investment goods if the aggregate technology exhibits complementarity between capital and skills.[2]

Our previous paper focused on the requirements for balanced growth. We explored a model with fleeting lifespans and derived necessary and sufficient conditions for constant steady-state factor shares in the presence of ongoing capital-augmenting technological progress and a non-unitary aggregate elasticity of substitution between capital and labor. In particular, we identified a class of aggregate production functions characterized by capital-skill complementarity that delivers balanced growth. However, our model could not speak to *changes* in the steady-state factor shares, because our convenient assumption of fleeting lives severed all links between these shares and parameters of the growth process.

In this paper, we allow for longer lives, which renders investment in education a forward-looking decision. In the body of our text, we focus on a model in which all individuals accumulate human capital by spending time in school. But our results are not limited to this model of educational attainment; in the online Appendix, we establish similar results in a model of discrete occupational choice with endogenous fractions of the population opting to become skilled.

We begin in Section I by examining the link between equilibrium factor shares, levels of human capital, and the rental rate for capital in a competitive economy with an aggregate production function characterized by capital-skill complementarity. In such a setting, a greater level of human capital goes hand in hand with a greater capital share, whereas a positive relationship exists between the rental rate and the capital share whenever the elasticity of substitution between capital and raw labor falls short of one. In Section II, we introduce schooling as an intertemporal choice for overlapping generations of the population. In our setting of "perpetual youth" with a constant hazard rate of death, it is optimal for members of each generation to attend school fully until they achieve a (time-varying) target level of human capital, whereupon they enter the work force but continue their education part time to keep pace with the growing human capital threshold. When capital and skills are complementary, there is an inverse equilibrium relationship between the education target and both the rental rate on capital and the difference between the interest rate

[1] See, for example, Chirinko (2008, p. 671), who surveyed many studies that sought to measure this elasticity and concluded that "the weight of the evidence suggests a value of [the elasticity of substitution] in the range of 0.4 to 0.6." In research conducted after that survey was written, Karabarbounis and Neimann (2014) estimate an elasticity of substitution substantially greater than 1, while Herrendorf, Herrington, and Valentinyi (2015) find an elasticity of 0.84 and Oberfield and Raval (forthcoming) estimate it between 0.5 and 0.7 for the US manufacturing sector.

[2] Acemoglu (2003) proposes yet another equilibrating force: when firms choose between capital- and labor-augmenting technological improvements, they may tend toward only the latter in the long run. In his setting, factor shares evolve during a transition phase with capital-augmenting progress but stabilize in the steady state due to the eventual dominance of technical change directed to labor. One difficulty with this story is that quality-adjusted prices of capital goods have declined signficantly over long periods, suggesting an ongoing process of investment-specific technological change.

and growth rate of wages. Taken together, the results in Sections I and II imply that failing to control for variation in human capital will lead to upward bias in estimates of the elasticity of substitution between capital and labor in an economy with capital-skill complementarity.

Finally, in Section III, we close the model and study neoclassical growth driven by exogenous technological progress. The dynamic equilibrium features ongoing accumulation of physical and human capital. We establish the existence of a unique balanced growth path (BGP) when skills are complementary to capital and human capital enters the aggregate production function in a particular way. Along this path, the downward pressure on the capital share due to accumulation of better and cheaper machinery is offset by upward pressure from investments in skills that are complementary to those machines. In the long run, the human capital target conditional on technology levels is decreasing in the difference between the real interest rate (which makes workers more impatient) and the growth rate of wages conditional on human capital (which makes human capital more valuable). A slowdown in productivity growth—be it capital augmenting or labor augmenting—reduces the interest rate by more than wage growth whenever the intertemporal elasticity of substitution is below one. Consequently, slower growth induces human capital accumulation that leads to a higher capital share and lower labor share.

I. Human Capital and Factor Shares

In this section, we examine the relationship between human capital and the functional distribution of income in a general neoclassical production environment. To this end, we write $Y = F(K, L; h)$, where Y is aggregate output, K and L are physical inputs of capital and labor, and h is some measure of the human capital embodied in that labor. This formulation admits various interpretations for h. For example, h might measure the education achieved by the representative worker, as in Grossman et al. (2017a). Or, h might represent the fraction of the labor force that is "skilled," with the remaining fraction being "unskilled." Then, we could write a three-factor production function $G(K, S, U)$ as in Krusell et al. (2000), with S and U denoting inputs of skilled and unskilled labor, respectively, so that $F(K, L; h) \equiv G(K, hL, (1-h)L)$.

We focus on technologies that exhibit constant returns in the physical inputs, K and L, and that feature *capital-skill complementarity.* We define capital-skill complementarity in terms of the effect of capital accumulation on the marginal product of human capital relative to that of raw labor and invoke the following assumption.

ASSUMPTION 1: $F(K, L; h)$ *is homogeneous of degree one in K and L and exhibits capital-skill complementarity; that is,* $\varphi \equiv d\log(F_h/F_L)/d\log K > 0$ *for all h, L, and K.*

In the most common treatment of human capital, output is taken to be a function of aggregate capital and "efficiency labor," where the latter is defined as the product of raw labor and a productivity term reflecting average human capital per worker; see, for example, Uzawa (1965) and Lucas (1988). In that specification, raw labor and skill are perfect substitutes, and thus capital accumulation impacts their returns similarly. But, following Griliches (1969), Krusell et al. (2000) have emphasized the empirical relevance of capital-skill complementarity and the role it has played in determining the evolution of factor rewards. Using their three-factor production function, $G(K, S, U)$, they associated capital-skill complementarity with a technology in which capital substitutes more closely for unskilled labor than for skilled labor. Our definition coincides with theirs when $G(\cdot)$ takes a nested-CES form (as they assume),[3] while extending the definition to a broader range of production technologies and interpretations of human capital.[4]

Now suppose that the economy is competitive and capital is hired up to the point where its marginal product is equal to the rental rate R, or $F_K(K, L; h) = R$. Define $\theta \equiv RK/Y$ as the capital share in national income (so that $1 - \theta$ is the labor share) and $\sigma \equiv (F_K F_L)/(F F_{KL})$ as the elasticity of substitution between capital and labor for a fixed level of human capital, h. Then, using the definitions of φ, σ, and θ and the first-order condition, $F_K(K, L; h) = R$, it is straightforward to show that[5]

$$d\theta = (1 - \sigma)\theta d\ln R + \sigma\varphi \frac{F_h}{F} dh. \tag{1}$$

Equation (1) relates changes in the capital share to changes in the rental rate and changes in the measure of human capital. In the absence of capital-skill complementarity (i.e., if $\varphi = 0$), the second term drops out and the capital share rises when the rental rate falls if and only if $\sigma > 1$. The positive relationship between changes in the labor share and changes in the rental rate (proxied by changes in the relative price of investment) in cross-country data provides the basis for Karabarbounis and Neiman's (2014) estimation of an elasticity of substitution between capital and labor in excess of one and their attribution of approximately half of the fall in the global labor share in recent years to the fall in the relative price of investment goods since 1975. However, in addition to the usual concerns about the possible endogeneity of R, there is the additional issue that their estimation fails to control for growth in educational attainment, which was widespread in their sample. According to (1), a failure to control for dh will generate an upward bias in estimates of σ in the presence of capital-skill complementarity whenever R and h are negatively correlated. As we shall see, such a negative correlation is a natural outcome in models of optimal human capital accumulation.

II. Determinants of Optimal Education

In Grossman et al. (2017a) we developed a model of growth with endogenous education and capital-skill complementarity. We were interested in the necessary and sufficient conditions for balanced growth, so we invoked a useful shortcut: we assumed that successive generations of workers survive only for an instant, during which they divide their fleeting time between work and education to maximize instantaneous

[3] See the online Appendix for proof of this claim.

[4] With constant returns to scale, we can allow $F(\cdot)$ to represent the output of a "production unit" that employs K units of capital and L units of labor with human capital h. Then, aggregate output is the sum of outputs across all production units. In this manner, we can accomodate nondegenerate distributions of human capital across workers in the labor force.

[5] See the online Appendix.

income. This shortcut was helpful, because it circumvented thorny aggregation issues; we know of no overlapping generations models in which educational attainment grows in a steady state. Unfortunately, by removing intertemporal considerations from the schooling problem, we severed all links between factor shares and the growth process, because without forward-looking investment, the parameters of the static production function fully determine the functional distribution of income.

To study the determinants of long-run factor shares, we require a setting with meaningful, intertemporal trade-offs. To this end, we wed a model of overlapping generations à la Yaari (1965) and Blanchard (1985) with a model of human capital investment à la Ben-Porath (1967). Cohorts born at every instant exist in a state of "perpetual youth." New generations are born continuously. While alive, individuals divide their time between schooling and work. The cumulation of these choices determines each individual's human capital and thus the supply of skills in the aggregate.[6]

Our economy is populated by a unit mass of identical family dynasties.[7] The representative dynasty comprises a continuum of individuals of mass N_t at time t. Each living individual generates a new member of her dynasty with a constant, instantaneous probability λdt in a period of length dt and faces a constant, instantaneous risk of demise νdt in that same period, with $\lambda > 0, \nu \geq 0$. With these constant hazard rates of birth and death, the size of a dynasty at time t is given by

$$N_t = e^{(\lambda-\nu)(t-t_0)} N_{t_0}.$$

Each newborn enters the world devoid of human capital. An individual is endowed at each instant with a unit of time that she can divide arbitrarily between *working* and *learning*. Work yields a wage at time t that reflects the extant technology and size of the aggregate capital stock as well as the individual's accumulated human capital, h_t. Learning occurs at full-time school or in continuing education. An individual who devotes a fraction ℓ_t of her time to work and the remaining fraction $1 - \ell_t$ to education accumulates human capital according to

$$\dot{h}_t = 1 - \ell_t. \tag{2}$$

The time constraint implies $\ell_t \in [0, 1]$.[8]

[6] The main text focuses soley on educational attainment. But in the online Appendix we show that we can achieve similar results in a model of occupational choice.

[7] We assume that families maximize dynastic utility, including the discounted well-being of unborn generations. Similar qualitative results would be attained in a Yaari (1965) economy with (negative) life insurance and no bequests, as developed in Blanchard and Fischer (1989).

[8] In this formulation, current human capital plays no role in the learning process. However, we could as easily specify

$$\dot{H}_t = H_t^{\varsigma}(1 - \ell_t), \quad \varsigma \in [0, 1],$$

with $H_0 = 1$. This would generate an alternative measure of human capital that is just a monotonic transformation of h_t and that would play the same role as h_t in the analysis that follows. For example, if $\varsigma = 1$,

$$\log H_t = \int_0^t (1 - \ell_z)\, dz = h_t,$$

where the second equality follows from the assumption that $h_0 = 0$.

The representative family maximizes dynastic utility,

$$U_{t_0} = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} N_t \frac{c_t^{1-\eta} - 1}{1-\eta} dt,$$

subject to an intertemporal budget constraint, where c_t is per capita consumption by family members at time t, η is the inverse of the elasticity of intertemporal substitution, and ρ is the subjective discount rate. As usual, the Euler equation implies

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\eta}, \tag{3}$$

where r_t is the real interest rate in terms of consumption goods at time t. To limit the number of cases and conform with widespread empirical evidence, we assume that $\eta > 1$.[9]

Considering that there is a continuum of members in every dynasty and that families maximize dynastic utility, each individual chooses the path of her time allocation $\{\ell_t\}$ to maximize the expected present value of earnings. For an individual born at time τ, the problem is

$$\max \int_{\tau}^{\infty} e^{-\int_{\tau}^{t}(r_z+\nu)dz} \ell_t w_t(h_t)\, dt,$$

subject to $h_\tau = 0$, $\dot{h}_t = 1 - \ell_t$, and $0 \leq \ell_t \leq 1$, where $w_t(h_t)$ is the wage schedule that relates compensation at time t to the worker's human capital. Let μ_t be the costate variable associated with human capital accumulation. Then the first-order conditions imply

$$\left.\begin{array}{l} w_t(h_t) < \mu_t \\ w_t(h_t) = \mu_t \\ w_t(h_t) > \mu_t \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \ell_t = 0 \\ \ell_t \in [0,1] \\ \ell_t = 1 \end{array}\right. \tag{4}$$

and

$$\dot{\mu}_t = (r_t + \nu)\mu_t - \ell_t w_t'(h_t). \tag{5}$$

In this setting, the optimal schooling problem typically has a simple bang-bang solution.[10] Members of each cohort attend school full time beginning at birth until they accumulate human capital equal to a time-varying threshold, h_t^*. Then, the "graduates" enter the labor force, but they continue on with their education to maintain their human capital equal to the (growing) threshold. This education strategy implies that all workers in the labor force share a common level of human capital $h_t = h_t^*$, irrespective of their birth dates.

[9] See, for example, Hall (1988), Campbell (2003), and Yogo (2004) for estimates using macro data and Attanasio and Weber (1993) and Vissing-Jørgensen (2002) for estimates using micro data.

[10] In the online Appendix, we show that the bang-bang solution is optimal under the technical conditions detailed in Assumption A.1.

The human capital threshold h_t^* equals the education level at which an individual is indifferent between school and work. The benefit of additional schooling is the present value of human capital, μ_t, while the instantaneous cost is the foregone wage, w_t. Substituting $\mu_t = w_t(h_t^*)$ in (5) and rearranging terms gives

$$r_t + \nu - g_{w|h_t^*,t} = \frac{w_t'(h_t^*)}{w_t(h_t^*)}, \tag{6}$$

where $g_{w|h,t}$ is the growth rate of wages (for a given level of human capital, h) at time t. Then, as we show formally in the online Appendix, for any aggregate production function $F(\cdot)$ that satisfies Assumption 1 and that generates an interior choice of h_t^*, (6) gives an inverse relationship between human capital and both the rental rate on capital and the difference between the interest rate and the growth rate of wages. The former observation underlies our claim at the end of Section II that optimal human capital accumulation implies a negative correlation between h and R when capital and skill are complementary. Intuitively, when a rise in the rental rate reduces demand for capital, it also reduces the marginal returns to skill, $w_t'(h_t^*)/w_t(h_t^*)$. So, the demand for education also falls. Meanwhile, the latter observation—which does not require capital-skill complementarity—shows that the growth process also influences human capital accumulation inasmuch as a high interest rate discourages investment while a high rate of wage growth makes additional schooling more attractive.

III. Optimal Education and Balanced Growth

To study the determinants of long-run factor shares, we need to close the model. We prefer to do so in a way that preserves balanced growth, both for reasons of tractability and because factor shares were stable for decades after WWII and, after a substantial realignment over some 20 years, seem to have stabilized again.

The task of generating a BGP might seem daunting. First, the presence of ongoing capital-augmenting technical progress is inconsistent with constant factor shares in a standard neoclassical setting with a non-unitary elasticity of substitution between capital and labor; see Uzawa (1961). Yet, Gordon (1990); Greenwood, Hercowitz, and Krusell (1997); and others have documented a significant decline in the relative price of capital, which is suggestive of capital-augmenting progress. Second, a falling rate of return on capital goes hand in hand with ongoing human capital accumulation, which means that different cohorts will target different levels of education before entering the labor force. Aggregation becomes an immediate technical concern. Third, growing educational attainment means falling labor force participation, and so the growth rate of labor supply need not be constant. Yet, capital accumulates at a constant rate along a BGP. Despite these hurdles, we are able to close our model in a way that admits balanced growth by building upon the insights in Grossman et. al (2017a). By combining the technology introduced in that paper with the Yaari-Blanchard model of overlapping generations and the Ben-Porath (1967) model of educational investment, we are able to solve for a BGP and to study its properties.

To generate long-run growth, we introduce capital- and labor-augmenting technology into the model of Section I. A firm that hires K units of capital and L units of labor with human capital h produces

$$Y_t = F(A_t K, B_t L, h) \tag{7}$$

units of output at time t, where A_t now represents the state of disembodied, capital-augmenting technology and B_t the state of labor-augmenting technology.[11] We retain Assumption 1 from Section I, which imposes capital-skill complementarity and constant returns to scale; the latter allows us to use (7) also for the aggregate production function. Next we borrow from Grossman et al. (2017a) the assumption that $F(\cdot)$ falls within a particular class of production functions.

ASSUMPTION 2: *The production function can be written as* $F(A_t K, B_t L, h) = \tilde{F}(e^{-ah} A_t K, e^{bh} B_t L)$, *with* $a > 0, b > \lambda \geq 0$, *where*

(*i*) $f(k) \equiv \tilde{F}(k,1)$ *is strictly increasing, twice differentiable, and strictly concave for all* k;

(*ii*) $\lim_{k\to 0} kf'(k)/f(k) < b/(a+b)$.

As we discussed in our earlier paper, this class of production functions makes schooling akin to capital-using (or labor-saving) technical progress; that is, an increase in human capital raises the demand for capital relative to that for raw labor at the initial factor prices. While it may be tempting to interpret Assumption 2 as positing that human capital reduces the efficiency of physical capital, the fact that h enters $\tilde{F}(\cdot)$ in two places renders this interpretation specious. To see this, note that Assumption 2 is formally equivalent to assuming that the production function can be written as

$$F(A_t K, B_t L, h) = (B_t L)^{1-\beta} \mathcal{F}\left(A_t K, e^{bh/\beta} B_t L\right)^{\beta},$$

with $\beta = b/(a+b)$. This alternative formulation expresses output as a Cobb-Douglas function of raw labor and a composite input produced by capital and a measure of worker skills. Then it is clear that h raises the marginal productivity of physical capital for any K; that is, human capital accumulation shifts the $K-L$ isoquants inward while at the same time rotating them to induce greater demand for capital. Together with Assumption 1, which stipulates capital-skill complementarity, our restriction on the technology ensures $\sigma < 1$, which is in keeping with the findings of Oberfield and Raval (forthcoming), who estimate σ from a factor-share equation after controlling for workers' human capital. Assumption 2.ii ensures that the marginal product of human capital is positive for all K, L, and h.

[11] Recall from Section II that all workers in the labor force have the same human capital, so we do not need to specify the output by heterogeneous labor. If workers were to differ in skills, we could subdivide each firm into units with homogeneous labor and sum the output across these units; see footnote 4.

Output can be used for consumption or investment. A unit of output produces one unit of the consumption good or q_t units of the investment good at time t, where growth in q_t captures investment-specific technological change, as in Greenwood, Hercowitz, and Krusell (1997). Thus,

$$Y_t = C_t + I_t/q_t$$

and

$$\dot{K}_t = I_t - \delta K_t,$$

where C_t and K_t are aggregate consumption and the aggregate capital stock, respectively; I_t is gross investment; and δ is the constant rate of capital depreciation.

Technology evolves exogenously in our model. Let $\gamma_L = \dot{B}/B$ be the constant rate of labor-augmenting technological progress, $g_A = \dot{A}/A$ the constant rate of *disembodied* capital-augmenting progress, and $g_q = \dot{q}/q$ the constant rate of *embodied* (or investment-specific) technological progress. Define $\gamma_K \equiv g_A + g_q$ as the *total* rate of capital-augmenting technological progress. We are interested in the relationship between these parameters that describe the growth process and the long-run factor shares.

A. *Characterizing a BGP*

In order to solve for a BGP, we impose some further parameter restrictions.

ASSUMPTION 3: *The parameters of the economy satisfy*

(*i*) $a > \gamma_K$;

(*ii*) $\lim_{k \to 0} \frac{kf'(k)}{f(k)} > \frac{\Omega}{1+\Omega} > \lim_{k \to \infty} \frac{kf'(k)}{f(k)}$, *where*

$$\Omega \equiv \frac{b-\lambda}{a} - \frac{(\eta - 1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right) + \rho - (\lambda - \nu)}{a - \gamma_K};$$

(*iii*) $(\eta - 1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right) + \rho - (\lambda - \nu) > 0.$

Assumption 3 ensures the existence of an equilibrium with finite dynastic utility. It also generates interior choices for continuing education among those that have already joined the labor force.

A competitive firm takes the rental rate as given. A firm that hires a unit of labor bearing human capital h at time t will combine that labor with $\kappa_t(h)$ units of physical capital, where $\kappa_t(h)$ is given implicitly by

$$e^{-ah} A_t \tilde{F}_K\left[e^{-ah} A_t \kappa_t(h), e^{bh} B_t\right] = R_t. \tag{8}$$

The worker is paid her marginal product, which, with constant returns, is the difference between revenue and capital costs, or

$$w_t(h) = \tilde{F}(\cdot) - e^{-ah} A_t \kappa_t(h) \tilde{F}_K(\cdot). \tag{9}$$

Individuals use the wage schedule $w_t(h)$ together with their rational expectations of the evolution of wages and the interest rate to make their optimal schooling decisions, summarized in (6).

Let us define a BGP as a dynamic equilibrium with constant growth rates of output, consumption, and capital, and with factor income shares that are constant and strictly positive. A constant growth rate of consumption implies a constant interest rate, by the Euler equation (3). We conjecture a constant division of time between work and education, ℓ, for those that have completed full-time school. We prove in the online Appendix the following lemma that describes important features of the BGP.

LEMMA 1: *Suppose* g_q, g_A, *and* γ_L *are constants and Assumptions 1, 2, and 3 are satisfied. Then there exists a unique BGP characterized by*

$$\ell = 1 - \frac{\gamma_K}{a} \tag{10}$$

and

$$z_t \equiv \frac{e^{-ah_t^*} A_t K_t}{e^{bh_t^*} B_t L_t} = z^* \quad \textit{for all } t. \tag{11}$$

Here, z_t adjusts the effective capital-labor ratio at time t (i.e., $A_t K_t / B_t L_t$) for the prevailing level of human capital of those in the workforce, taking into account the different complementarity between human capital and each of the primary factors of production. We henceforth refer to z_t as the schooling-adjusted effective capital-to-labor ratio.

Equation (10) implies that the human capital threshold increases linearly with time,

$$\dot{h}_t^* = \frac{\gamma_K}{a}. \tag{12}$$

The optimal schooling strategies are depicted in Figure 1. Here, the lines with unit slope represent the human capital accumulation by each cohort while its members remain full-time students. Once a cohort's human capital reaches h_t^*, the members devote a fraction γ_K / a of their time to continuing education, just like all others that have completed their full-time schooling.

Let s_τ denote years in full-time school (or "educational attainment") for the cohort born at time τ. This is the time it takes for them to catch up with the human capital threshold, that is, $s_\tau = h^*_{\tau+s_\tau}$. With the threshold rising according to (12), $h^*_{\tau+s_\tau} = h^*_\tau + s_\tau \gamma_K / a$. Thus, educational attainment also increases linearly,

$$\dot{s}_\tau = \frac{\gamma_K}{a - \gamma_K}. \tag{13}$$

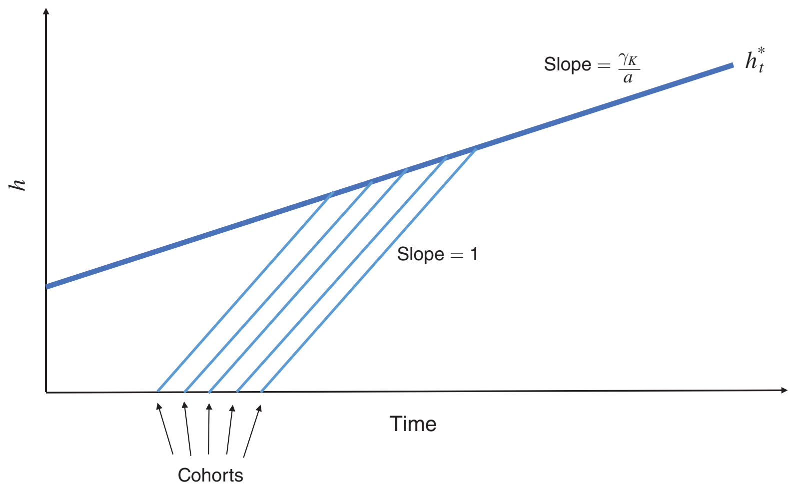

Figure 1. Human Capital Accumulation by Birth Cohort

Recalling that $a > \gamma_K$ by Assumption 3.i, educational attainment rises in the steady state if and only if the rate of capital-augmenting technical progress is strictly positive.

Lemma 1 states that the schooling-adjusted effective capital-labor ratio converges to a constant value, z^*, in the long run.[12] This is the key to balanced growth in the presence of capital-augmenting technological progress and an elasticity of substitution between capital and labor less than one. As capital accumulates and becomes more productive, the capital share in national income would tend to fall when $\sigma < 1$. However, the capital-skill complementarity implies an increased return to schooling. The extra schooling is capital using, which puts upward pressure on the capital share. For the class of production functions described in Assumption 2, the offsetting forces just balance, and the capital share remains constant.[13]

Why then is it optimal for active workers to upgrade their human capital continuously so as to keep z_t constant? For an interior choice of $\ell \in (0,1)$, the indifference condition (6) must be satisfied in the steady state, when r_t and $g_{w|h_t^*,t}$ are constants. Meanwhile, Assumption 2 implies

$$(14) \qquad \frac{w_t'(h_t)}{w_t(h_t)} = b - a\frac{\theta[z_t(h_t)]}{1-\theta[z_t(h_t)]},$$

where $\theta(z_t) \equiv z_t f'(z_t)/f(z_t)$ is the capital share. Notice that the capital share depends only on the schooling-adjusted effective capital-to-labor ratio. So, a choice of h_t^* that

[12] In our working paper with a different title and focus, Grossman et al. (2017b), we used numerical methods to suggest the presumed stability of the BGP.

[13] Put differently, (12) implies that $e^{-ah_t^*}A_t q_t$ is constant along a BGP. So, the induced investment in human capital is just what is needed to offset the exogenous improvement in capital productivity.

keeps z_t constant also keeps $w_t'(h_t^*)/w_t(h_t^*)$ constant, which is consistent with the steady-state requirements of (6).[14]

Using the optimal allocation of time, we can now calculate the (constant) growth rates of the labor force, wages, and output per capita, along with the constant interest rate and capital share. The aggregate labor force at time t is the product of the fraction of time that the typical worker devotes to gainful employment and the mass of the surviving population that has completed its phase of full-time schooling. The measure of individuals that were born at τ and that are still alive at t is $\lambda N_\tau e^{-\nu(t-\tau)} = \lambda N_t e^{-(\lambda-\nu)(t-\tau)} e^{-\nu(t-\tau)} = \lambda N_t e^{-\lambda(t-\tau)}$. All those who were born at or before $t - h_t^*$ have already entered the labor force. Therefore,

$$L_t = \left(1 - \frac{\gamma_K}{a}\right)\int_{-\infty}^{t-h_t^*} \lambda N_t e^{-\lambda(t-\tau)} d\tau = \left(1 - \frac{\gamma_K}{a}\right) N_t e^{-\lambda h_t^*}. \tag{15}$$

It follows from (15) that labor force participation, L_t/N_t, shrinks at the rate $g_L - g_N = -\lambda\gamma_K/a < 0$. Declining labor force participation mirrors increasing educational attainment, which requires longer stays in school for successive cohorts.

Next we derive the growth rate of wages. Compensation rises thanks to ongoing technological progress as well as ongoing investments in physical and human capital. Using (8) and (9), we calculate that, along a BGP, the wage paid to each worker in the labor force (who has growing human capital of h_t^*) increases at rate[15]

$$g_w = \gamma_L + \frac{b}{a}\gamma_K.$$

Since factor shares are constant along the BGP, aggregate output is proportional to labor income, so the growth rate of output per capita can be expressed as

$$g_y = g_w + g_L - g_N = \gamma_L + \frac{b-\lambda}{a}\gamma_K.$$

Combining this expression with Assumption 3.iii implies that the present value of utility is finite. Also, per capita consumption is proportional to per capita output, so (3) gives the long-run interest rate,

$$r = \rho + \eta g_y = \rho + \eta\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right). \tag{16}$$

[14] Note that for (14) to be satisfied with a constant value of z_t, we need a sufficiently large range for $zf'(z)/f(z)$. We show in the online Appendix that Assumption 3.ii guarantees the existence of a solution to (14).

[15] We substitute for the arguments of $\tilde{F}(\cdot)$ and $\tilde{F}_K(\cdot)$ using $z_t = e^{-(a+b)h_t^*} A_t \kappa_t(h_t^*)/B_t$ and note that z_t is constant along a BGP. The no-arbitrage condition for capital accumulation implies that $R_t q_t - \dot{q}_t/q_t - \delta = \iota_t$, and thus, when the interest rate and the rate of investment-specific technical progress are constant, $\dot{R}_t/R_t = -g_q$. Totally differentiating (8) and (9) with z_t constant implies

$$-g_q = g_A - a\dot{h}_t^*$$

and

$$\frac{\dot{w}_t}{w_t} = \gamma_L + b\dot{h}_t^*,$$

from which it follows that

$$\frac{\dot{w}_t}{w_t} = \gamma_L + \frac{b}{a}\gamma_K.$$

Finally, we come to the steady-state factor shares. In the steady state, (6) and (14) imply

$$\gamma_L + \frac{b}{a}\gamma_K = r + \nu - \left(1 - \frac{\gamma_K}{a}\right)\left(b - a\frac{\theta}{1-\theta}\right)$$

or

$$\frac{\theta}{1-\theta} = \frac{b + \gamma_L - (r + \nu)}{a - \gamma_K}. \tag{17}$$

Next we substitute for the long-run interest rate, using (16), which gives us a relationship between the long-run capital share and the primitive parameters of the economy, namely

$$\frac{\theta}{1-\theta} = \frac{b-\lambda}{a} - \frac{(\eta - 1)\left(\gamma_L + \frac{b-\lambda}{a}\gamma_K\right) - \lambda + \nu + \rho}{a - \gamma_K}. \tag{18}$$

We summarize our characterization of the BGP as follows.

PROPOSITION 1: *Suppose the aggregate production function obeys Assumptions 1 and 2; the parameters satisfy Assumption 3; and g_q, g_A, and γ_L are constant. Then there exists a unique BGP along which new cohorts are full-time students until their human capital reaches a threshold h_t^* that grows linearly with time. Once a cohort enters the labor force, its members devote a constant fraction $\ell = 1 - \gamma_K/a$ of their time to work and the remainder to continuing education. Wages grow at constant rate $\gamma_L + (b/a)\gamma_K$ and per capita income grows at constant rate $\gamma_L + (b - \lambda)\gamma_K/a$. The long-run real interest rate is given by (16), and the long-run factor shares are given by (18).*

B. *Determinants of Long-Run Factor Shares*

We are ready to discuss the determinants of the long-run distribution of national income. We begin with (17), which expresses θ as a function of γ_K and γ_L, taking the real interest rate as given. If, for example, the aggregate economy comprises a continuum of small regional economies or similar industries that face a common interest rate due to nationwide asset trade, then (17) would describe the cross-sectional relationship between growth rates of output and factor shares. From this equation, it is clear that θ would be *positively* correlated with both γ_K and γ_L *in the cross section*; regions and industries with faster rates of capital- or labor-augmenting technological progress would have higher shares of their income paid to capital in an economy with a uniform interest rate.

But in a closed economy (or a global economy), the interest rate is endogenous and responds to changes in the growth process. Equation (18) informs us about the long-run relationship between factor shares and rates of technological progress. Recall our assumption that $\eta > 1$, that is, that the elasticity of intertemporal substitution is less than 1. By differentiating the expression on the right-hand side of (18) and making use of Assumption 3.iii, we establish our key result.

PROPOSITION 2: *When* $\eta > 1$, *an increase in* γ_K *or* γ_L *raises the long-run labor share*, $1 - \theta$.

Proposition 2 states than an acceleration of technological progress of any sort will shift the distribution of national income from capital to labor. Of course, a productivity slowdown does just the opposite. Our model thus predicts a *negative* correlation between the growth rate and the capital share across steady states.

What accounts for this shift in factor shares? Note first from (16) that, in response to an exogenous shock to the growth process, the interest rate moves in the same direction as the growth rate of per capita income. Moreover, with $\eta > 1$, the response of the former is greater than that of the latter. Thus, an acceleration of technological progress that causes g_y to rise will cause $r - g_y$ to rise as well. On a BGP, wages grow at a rate similar to per capita income, so $r - g_{w|h}$ also rises. This term appears in the expression for the optimal human capital threshold (6); whereas an increase in the growth rate of wages makes staying in school more desirable, a rise in the interest rate makes extended schooling less palatable. In the long run, the latter effect dominates, so by a combination of (6) and (14), z^* eventually rises. In other words, we find that the long-run schooling-adjusted effective capital-to-labor ratio rises in response to an acceleration of technological progress once proper adjustment is made for the optimal response of targeted human capital and the greater complementarity of schooling with physical capital than with raw labor. Finally, with an elasticity of substitution between capital and labor less than one, a rise in the schooling-adjusted effective capital-labor ratio spells a reallocation of income from capital to labor. To avoid possible confusion, note that although faster productivity growth reduces the steady-state human capital target conditional on technology levels, equation (12) shows that an increase in the rate of capital-augmenting technological progress γ_K also raises the rate at which h_t^* increases as technology improves. Conversely, a productivity slowdown that raises the human capital target can also reduce the long-run growth of schooling.

Recent history has, however, witnessed not an acceleration in technological progress but rather a slowdown in productivity growth; see, for example, Gordon (2010, 2016) and Fernald (2014). Our analysis suggests that a productivity slowdown will contribute to a redistribution of income from labor to capital in a world of capital-skill complementarity with ongoing gains in educational attainment. This could be a partial explanation for the recent fall in the global labor share.[16] Indeed, in their study of the functional distribution of income in the United States, the United Kingdom, and France from the late 1800s until recently, Charpe, Bridji, and McAdam (2020) find long cycles in the labor share that are positively correlated with growth in per capita income.

[16] In the online Appendix, we discuss how to calibrate the model and explore its quantitative properties. We find that, for plausible calibrations, a productivity slowdown that reduces trend labor productivity growth by 1 percentage point increases capital's income share by several percentage points. The parameter restrictions imposed in Assumption 3 are satisfied in all our calibrations.

IV. Concluding Remarks

We see three main contributions in this paper.

First, we have shown that education affects the division of national income between capital and labor in the presence of capital-skill complementarity. When skills and capital are complementary, the accumulation of embodied human capital raises the marginal return to physical capital and thus the share of income that accrues to any given stock of machinery and equipment. Moreover, optimal investment in education induces a negative correlation between the level of human capital and the return to physical capital in the presence of capital-skill complementarity. In such circumstances, using time series correlation between capital returns and capital shares will produce upwardly biased estimates of the elasticity of substitution between capital and labor.

Second, features of the growth process will affect long-run factor shares in the presence of capital-skill complementarity, even if those shares are stable in a steady state. We have shown that an increase in rates of technological progress will redistribute income from capital to labor, and conversely, a productivity slowdown will boost the capital share. The effects work through the endogenous response of investments in schooling. We have made these points in a neoclassical model of growth with competitive goods and factor markets and exogenous technological progress. But similar mechanisms exist in models with imperfect competition and endogenous growth. Many models of automation and robotization feature capital-skill complementarity, as automated equipment and robots are operated by more-skilled workers while substituting closely for less-skilled workers. Therefore, the spread of robots in the production process is bound to affect the distribution of income across skill groups.

Third, we have developed a growth model that admits balanced growth and stable factor shares despite ongoing capital-augmenting technical progress, ongoing growth in educational attainment, ongoing changes in labor force participation, and elasticities of substitution between factors that differ from one. Moreover, we have done so in a setting with overlapping generations, where the arrival of new cohorts introduces heterogeneity in schooling choices and labor force participation that makes aggregation potentially complex. The combination of perpetual youth à la Yaari (1965) and Blanchard (1985), human capital accumulation à la Ben-Porath (1967), and capital-skill complementarity à la Grossman et. al (2017a) solves the aggregation problem. This purely technical contribution may prove useful in other contexts.

Our paper suggests several directions for future research. On the theoretical side, one might wish to move away from the assumption of "perpetual youth" to a more realistic model with finite lifetimes. However, such a modification would likely threaten the existence of a BGP and would surely complicate dynamics, as is evident from Cass and Yaari (1967), who show that multiple steady states and complex transition dynamics can emerge even in a simple neoclassical setting that neglects human capital accumulation. On the empirical side, perhaps the most pressing need is for estimates of the degree of capital-skill complementarity in aggregate production. Not only is such complementarity necessary for our theoretical results, but the degree of complementarity determines the quantitative importance of the mechanism we highlight as well as the relationship between human capital accumulation and output that would underpin a growth-accounting exercise using our production function. We note that, given exogenous variation in the capital rental rate R and human capital h, equation (1) could be used to simultaneously estimate capital-skill complementarity and the elasticity of substitution between capital and raw labor.

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