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Selection into Trade and Wage Inequality†

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This paper analyzes how intra-industry trade affects the wage distribution when both workers and firms are heterogeneous. Positive assortative matching between worker skill and firm technology generates an employer size-wage premium and an exporter wage premium. Fixed export costs cause the selection of advanced technology, high-skill firms into exporting, and trade shifts the firm technology distribution upwards. Consequently, trade increases skill demand and wage inequality in all countries, both on aggregate and within the upper tail of the wage distribution. This holds when firms receive random technology draws and when technology depends on firm-level R&D. (*JEL* F16, J23, J24, J31)

How does international trade affect the distribution of wages? Traditionally the relationship between trade and inequality has been studied through the lens of the Heckscher-Ohlin model. However, recently this approach has been undermined by a combination of the empirical failings of the Stolper-Samuelson theorem;[1] the fact that changes in the relative demand for skilled labor are better explained by within-industry shifts than between industry reallocation (Berman, Bound, and Griliches 1994); the realization that firm heterogeneity plays a critical role in international trade (Bernard and Jensen 1995; Bernard et al. 2007); and evidence that trade integration leads to higher within-industry wage inequality between workers employed at exporting and nonexporting firms (Verhoogen 2008; Amiti and Davis 2012). Consequently, a new literature has emerged that addresses the links between trade and wages in the context of firm heterogeneity and selection into trade within industries.[2]

This literature has primarily focused on adapting Melitz (2003) to introduce labor market imperfections that generate rent sharing between firms and workers, while maintaining the assumption that workers are homogeneous (Egger and Kreickemeier 2009; Helpman and Itskhoki 2010; Helpman, Itskhoki, and Redding

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[1] Most prominently the fact that many unskilled labor-abundant developing countries have experienced increases in wage inequality following trade liberalization (Goldberg and Pavcnik 2007).

[2] A complementary line of research analyzes how wage inequality is affected by other aspects of globalization, such as offshoring (Feenstra and Hanson 1996) and the formation of cross-border production teams (Antràs, Garicano, and Rossi-Hansberg 2006).

2010; Davis and Harrigan 2011; Helpman et al. 2012).[3] Since these papers abstract from worker heterogeneity, they cannot address the effects of trade on returns to observable worker characteristics. Yet, trade liberalization has been linked to higher returns to observable skills in addition to rising residual wage inequality. For example, Han, Liu, and Zhang (2012) present evidence that the export surge following China's accession to the World Trade Organization (WTO) led to an increase in the college wage premium. Moreover, evidence from matched employer-employee data shows that a large proportion of the exporter wage premium (which lies at the heart of all theories linking intra-industry trade and wage inequality) can be explained by cross-firm differences in workforce composition. In particular, Schank, Schnabel, and Wagner (2007) show using German data that although the average wage paid by exporters is 36.6 percent higher than for nonexporters, controlling for observable employee characteristics reduces the exporter wage premium to just 2.2 percent.

This paper incorporates labor assignment into a trade model based on Melitz (2003) to study how intra-industry trade affects wage inequality when there is observable heterogeneity across both workers and firms. The paper proceeds in two stages. It starts by showing how the labor assignment techniques developed by Costinot and Vogel (2010) can be extended to characterize matching between workers and large, monopolistically competitive firms.[4] Then it uses the matching model to analyze how trade integration affects the equilibrium assignment of workers to firms and, consequently, wage inequality. There are three main conclusions. First, aggregate wage inequality in all countries is higher under trade than in autarky. Second, trade raises wage inequality within high-skill workers employed by exporters, but has ambiguous effects on inequality within workers at the bottom of the wage distribution. Third, while reductions in variable trade costs tend always to increase aggregate wage inequality, inequality is inverse U-shaped in fixed export costs. When there is no fixed cost of exporting inequality is the same as in autarky implying that fixed export costs are necessary for trade to affect wage inequality.

To build the matching model I adapt the production technology used in Melitz (2003) to make each worker's productivity depend on both the technology used by her employer and her skill. Crucially, I show that provided there is perfect substitutability within firms between the output of different workers, then the marginal product of each worker inherits properties of the labor productivity function. Therefore, when labor productivity is log supermodular in firm technology and worker skill and the labor market is competitive, similar reasoning to Costinot and Vogel (2010) implies that there is positive assortative matching between high-skill workers and high technology firms. Although it is well known that log supermodularity implies positive assortative matching when workers sort across tasks or sectors,[5] this is the first paper to solve the matching problem that arises when workers match with large,

[3] Helpman, Itskhoki, and Redding (2010) permit match specific heterogeneity in the productivity of ex ante identical workers and allow for both skilled and unskilled labor, but their model is designed to analyze the effect of trade on wage inequality within, not between, labor types. A notable exception, which allows for labor heterogeneity, is Yeaple (2005). The relationship between this paper and Yeaple (2005) is discussed below.

[4] Costinot and Vogel (2010) analyze how heterogeneous workers sort across tasks in a competitive economy without firms or intra-industry trade.

[5] See, for example, Sattinger (1975); Ohnsorge and Trefler (2007); and Costinot and Vogel (2010).

monopolistically competitive firms.[6] Analyzing sorting across firms instead of tasks has two main benefits. First, it allows us to study the implications of intra-industry trade with heterogeneous firms. By contrast, in Costinot and Vogel (2010) trade has neoclassical roots based on technology or factor endowment differences across countries. Second, it gives concrete empirical content to many of the model's predictions. For example, whenever more technologically advanced firms are larger and select into exporting, positive assortative matching implies both an employer size wage premium (Brown and Medoff 1989) and an exporter wage premium.

A useful property of the assignment equilibrium is that the matching function, which maps worker skill to firm technology, is a sufficient statistic for wage inequality.[7] Log supermodular labor productivity implies that firms with more advanced technologies place greater value on skill. Consequently, whenever the matching function shifts upward on an interval of the skill distribution (meaning that all workers with skill levels in that interval are matched to higher technology firms), wage inequality increases within any group of workers belonging to that interval.[8] It follows that to analyze the effect of trade on wage inequality we must characterize its effect on the equilibrium matching function.

Using the labor market clearing condition, variation in the matching function can be decomposed into two sources: (i) changes in firm-level employment, and (ii) shifts in the firm technology distribution. Trade affects the matching function through both these channels. Fixed export costs imply the existence of a threshold technology above which firms select into exporting. This generates a discontinuity in firm-level employment at the export threshold, which raises the relative demand for high-skill workers and tends to shift the matching function upward for workers employed by exporters. In addition to this firm-level effect, trade leads to shifts in the technology distribution through its impact on firm entry, exit, and R&D. To understand this channel, I consider two alternative general equilibrium frameworks that embody contrasting assumptions on the nature of R&D: a stochastic technology model in which each firm's technology is determined by a random draw from a fixed distribution as in Melitz (2003), and a technology choice model in which technology is determined by the firm's R&D investment. The goal is to identify results that are common to these two approaches.

In the stochastic technology model, the exit cutoff (below which firms choose not to produce) is the only free parameter of the technology distribution. A key prediction of Melitz (2003) is that whenever the fixed export cost is positive, trade raises the exit cutoff, implying the technology distribution shifts upward in the sense of first-order stochastic dominance. With heterogeneous labor, I show that trade integration always increases the exit cutoff when trade costs are sufficiently low and that even in cases where the exit cutoff declines, the matching function shifts upwards for workers

[6] In contemporaneous work, Eeckhout and Kircher (2012) study how workers sort across large firms when the output market is perfectly competitive. In one-to-one matching models, supermodularity guarantees positive assortative matching (Sattinger 1979; Gabaix and Landier 2008; Monte 2011).

[7] The paper assumes the skill distribution is exogenously fixed.

[8] See Proposition 3 for a precise statement of this result. Throughout the paper, wage inequality is measured in terms of second-order stochastic dominance. Therefore, the results hold for any measure of wage inequality that respects scale independence and second-order stochastic dominance.

employed by exporters. Therefore, trade induced variation in the technology distribution tends to reinforce the firm-level effect, meaning that trade raises the demand for skill and wage inequality on aggregate, within high-skill workers employed by exporters and between the upper and lower tails of the skill distribution.

In the technology-choice model, however, the shape and location of the technology distribution are unrestricted ex ante. Free entry requires not simply that firms make zero profits, on average (as in the stochastic technology model), but that the zero-profit condition holds pointwise at all technology levels.[9] Consequently, entry and exit exactly offset variation in firm-level employment, except when changes in fixed costs induce firms to alter their R&D investment. In particular, holding workforce skill constant, entry into exporting causes firms to use more advanced technologies in order to cover their fixed export costs.[10] Thus, trade liberalization shifts the matching function upward at skill levels employed by new exporters, but leaves it unchanged for workers at both nonexporters and continuing exporters. The flexibility of the technology distribution under technology choice means that the impact of trade on the matching function is local to the export sector, not global as in the stochastic technology model. However, the implications for wage inequality are similar. Relative to autarky, trade increases wage inequality, on aggregate, within high-skill workers employed by exporters and between high- and low-skill workers, but leaves wage inequality unchanged within low-skill workers employed by nonexporters.

An important implication of both the stochastic technology and technology choice models is that neither of the channels through which trade affects the matching function operate in the absence of fixed export costs. Thus, the paper implies that when both workers and firms are heterogeneous, intra-industry trade increases aggregate wage inequality in all countries, provided there are fixed costs of exporting. Unlike in neoclassical trade models, the effects of trade on inequality do not depend on a country's endowments or technologies relative to its trading partners. Moreover, the rise in wage inequality is driven by higher inequality between exporting and nonexporting firms and within high-skill workers. The former prediction is consistent with the firm-level findings of Verhoogen (2008) and Amiti and Davis (2012),[11] while the later prediction shows that trade integration can lead to employment and wage polarization. Although the automation of medium-skill intensive routine tasks is the leading explanation for the polarization observed recently in the United States and Europe (Autor, Levy, and Murnane 2003; Autor, Katz, and Kearney 2006; Goos,

[9] In equilibrium, there is cross-firm technology dispersion because technology choice varies with workforce skill. By contrast, if workers were homogeneous then, except in knife-edge cases, all firms would choose the same technology level.

[10] A corollary of this result is that only new exporters invest in technology upgrading following a reduction in variable trade costs. This prediction is consistent with the empirical work of Lileeva and Trefler (2010) and Bustos (2011a) who find that following trade liberalization technology investment increases at firms that are induced to enter exporting, but is unchanged at firms with sufficiently high initial productivity.

[11] Verhoogen (2008) shows that following the Mexican peso devaluation of 1994 relative exports and wages increased at relatively more productive plants, leading to a rise in within industry wage inequality. Amiti and Davis (2012) find, in Indonesia, that cuts in output tariffs increase the average wage paid by exporters, but decrease the average wage at nonexporters. Amiti and Davis (2012) rationalize their findings as resulting from fair wage-based rent sharing between firms and homogeneous workers, but they also note that the data does not allow them to discriminate this story from alternative explanations of their findings.

Manning, and Salomons 2009; Acemoglu and Autor 2011), this result establishes that trade integration is capable of generating similar effects.[12]

This paper builds upon the work of Yeaple (2005), who also predicts that intra-industry trade increases wage inequality when workers are heterogeneous. In particular, the technology-choice model can be interpreted as a generalization of Yeaple's model from two technologies to a continuum of technologies. However, unlike Yeaple (2005), this paper considers the effects of trade in the stochastic technology framework, which, following Melitz (2003), has become the dominant paradigm for studying intra-industry trade with firm heterogeneity. In addition, the paper differs from Yeaple (2005) in several other important ways. First, it solves the general labor assignment problem for matching workers and firms. Second, it shows how trade affects group-level measures of wage inequality, not simply the wage ratio between pairs of workers. Third, the firm-level effect of selection into exporting on the matching function does not operate in Yeaple (2005). Fourth, including more than two technologies allows trade to affect the technology distribution and inequality within the export and nonexport sectors, both of which are constant in Yeaple (2005). Consequently, this paper can ask whether marginal trade liberalizations affect inequality within high- and low-skill workers and lead to wage polarization.

The paper is also related to recent work analyzing the effect of trade on the returns to worker observables by Burstein and Vogel (2010), Bustos (2011b), Harrigan and Reshef (2011), and Monte (2011). Burstein and Vogel (2010) modify Eaton and Kortum (2002) to include skilled and unskilled labor and skill-biased technology, while Bustos (2011b) and Harrigan and Reshef (2011) introduce skilled and unskilled labor into the Melitz (2003) model. Since these papers only include two types of labor, they do not allow for inequality within groups of workers with similar skill levels, or for inequality to evolve differently in the upper and lower tails of the skill distribution. Monte (2011) develops an intra-industry trade model in which both firms and managers are heterogeneous, but workers are homogeneous, and analyzes wage inequality within managers when there is one-to-one matching between managers and firms.

Finally, it is useful to compare this paper with the literature on trade and inequality that assumes homogeneous workers and imperfect labor markets, for example, Helpman, Itskhoki, and Redding (2010). An obvious difference is that this paper focuses on the returns to observable skills rather than residual wage inequality, but even when inequality cannot be decomposed into these two components its predictions differ in interesting and testable ways. To be specific, in Helpman, Itskhoki, and Redding (2010), trade affects wage inequality only because it induces a discontinuity in revenues and wages between exporters and nonexporters. Consequently, wage inequality within workers employed by exporters is constant, inequality when all firms export is the same as in autarky and there is an inverted U-shaped relationship between aggregate wage inequality and either fixed or variable trade costs. By contrast, in this paper, where trade affects wage inequality not only by inducing a size discontinuity between exporters and nonexporters but also by causing shifts in the

[12] In this regard, it is interesting to note that Han, Liu, and Zhang (2012) find that China's WTO accession increased wage inequality primarily within the upper half of the wage distribution.

technology distribution, none of these results hold. In particular, changes in fixed and variable trade costs have qualitatively different effects on inequality. While the relationship between fixed export costs and aggregate wage inequality is in general inverted U-shaped, and inequality is the same when the fixed export cost is zero, as in autarky, a reduction in variable trade costs tends to increase aggregate wage inequality whenever not all firms export.[13] Therefore, unlike Helpman, Itskhoki, and Redding (2010), this paper does not predict that once trade costs are sufficiently low further reductions in variable trade costs will decrease inequality.

The remainder of the paper is organized as follows. Section I develops and solves the matching model in partial equilibrium and characterizes how shifts in the matching function affect wage inequality. Section II introduces trade into the partial equilibrium matching model. Section III considers the impact of trade under stochastic technology, while Section IV presents the technology choice model. Finally, Section V concludes and offers suggestions for future research.

I. Matching Model

A. *Model Setup*

Consider an economy comprising a mass L of workers and a mass M of firms. Both workers and firms are heterogeneous, but for each group there is only a single dimension of heterogeneity, which I call skill for workers and technology for firms.[14]

Let skill be indexed by $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} > 0$ and let $L(\theta)$ be the mass of workers with skill not exceeding θ. Assume that $L(\theta)$ is continuously differentiable and strictly increasing on $[\underline{\theta}, \bar{\theta}]$. Similarly, let technology be indexed by $z \in [\underline{z}, \bar{z}]$ with $\underline{z} > 0$, and let $M(z)$ be the mass of firms with technology no greater than z. Assume that $M(z)$ is continuously differentiable and strictly increasing on $[\underline{z}, \bar{z}]$. Note that the assumptions on $L(\theta)$ and $M(z)$ imply that both the skill distribution and the technology distribution have continuous support and no mass points. These restrictions simplify the presentation of the model, but they are not substantively important for the paper's results. Define $m(z) \equiv \frac{M'(z)}{M}$ to be the density function of the firm technology distribution.

In general, the skill and technology distributions and the total mass of workers and firms will be endogenous properties of an economy. From a labor supply perspective, human capital accumulation will be shaped by the returns to skill and the costs of human capital investment and workers may exit the labor force if their wage drops below their outside option. Likewise firms' entry, exit, and R&D decisions will depend on the structure of fixed costs, the set of available technologies, and the nature of output and factor markets. The goal of this paper is to understand how wages are affected by trade integration taking the labor supply as given. Consequently, I assume throughout that $L(\theta)$ is exogenously fixed, meaning that each worker is endowed with a fixed skill level θ and the mass of workers in the

[13] This asymmetry between fixed and variable trade costs is also not found in Yeaple (2005).

[14] Ohnsorge and Trefler (2007) develop a model of labor assignment including two dimensions of worker heterogeneity.

economy is constant. For the first part of the paper I also assume that $M(z)$ is exogenous. Given this partial equilibrium restriction Section IB solves for the equilibrium assignment of workers to firms conditional on $M(z)$. Then in Sections III and IV, I embed the matching model in two alternative general equilibrium frameworks and endogenize $M(z)$.

Firms produce differentiated products and each firm faces a constant elasticity demand function

$$x = Ap^{-\sigma}, \tag{1}$$

where x is the quantity demanded, p is the firm's price, and $A > 0$, $\sigma > 1$ are demand parameters that are common across firms.

Production costs can be split into two components: variable costs and fixed costs.[15] Labor is the only factor used in variable production. I assume that a worker with skill θ employed at a firm using technology z produces $\Psi(\theta, z)$ units of the firm's product, where Ψ is strictly positive, strictly increasing in both its arguments and twice continuously differentiable. I will refer to Ψ as the labor productivity function and the properties of Ψ determine how workers sort across firms. Crucially, I assume that production complementarities between skill and firm technology are sufficiently strong that Ψ is strictly log supermodular. This means that

$$\frac{\Psi(\theta_1, z_1)}{\Psi(\theta_0, z_1)} > \frac{\Psi(\theta_1, z_0)}{\Psi(\theta_0, z_0)}, \qquad \forall\, \theta_1 > \theta_0, z_1 > z_0.$$

Since Ψ is twice continuously differentiable, this is equivalent to assuming $\frac{\partial^2 \log \Psi(\theta, z)}{\partial \theta \partial z} > 0$. To understand what assuming Ψ is log supermodular entails, observe that a constant elasticity of substitution function of two variables is log supermodular if and only if it has an elasticity of substitution less than one. Thus, log supermodularity requires greater complementarity between skill and technology than is found in a Cobb-Douglas production function.

To obtain a firm's total production we simply add up the output produced by each of its employees. Therefore, if $L(\theta; z)$ is the mass of workers with skill not exceeding θ hired by a firm with technology z, then the firm's total output y is given by

$$y = \int_{\underline{\theta}}^{\bar{\theta}} \Psi(\theta, z)\, dL(\theta; z). \tag{2}$$

Firms hire workers in a perfectly competitive labor market, where $w(\theta)$ is the wage paid to a worker with skill θ.

Given the assumption that $M(z)$ is exogenous, the structure of fixed costs can, for now, be left unspecified, but to ensure that the labor market clearing condition is well defined I will assume that any fixed component of production does not use

[15] Fixed costs should be interpreted broadly to include market entry and R&D costs in addition to per period fixed costs.

labor.[16] This completes the specification of the partial equilibrium model. Note that although this paper interprets the model as describing an entire economy, it could also represent a single industry with an industry specific labor force.

B. *Matching Workers and Firms*

This section characterizes the equilibrium assignment of workers to firms assuming that firms maximize profits and the labor market is competitive. Each firm chooses employment to maximize its variable profits $\pi(z)$, taking the wage function $w(\theta)$ as given. Using (1) and (2), and setting supply equal to demand, variable profits are given by

$$(3)\qquad \pi(z) = \max_{\{L(\theta;z)\}} \left\{ A^{\frac{1}{\sigma}} \left[\int_{\underline{\theta}}^{\bar{\theta}} \Psi(\theta, z)\, dL(\theta; z) \right]^{\frac{\sigma-1}{\sigma}} - \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta; z) \right\},$$

and taking the first-order condition for profit maximization implies

$$(4)\qquad \frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}} \Psi(\theta, z) - w(\theta) \leq 0$$

$$\text{with equality } \forall\, \theta \text{ such that } dL(\theta; z) > 0.$$

The profit maximization condition (4) should be interpreted as implying that the marginal revenue product of any worker employed by a firm (the first term on the left-hand side) must equal the firm's marginal cost of employing the worker (the second term on the left-hand side). Since the labor market is competitive, this result is to be expected. More importantly, (4) shows that each worker's marginal revenue product inherits the log supermodularity of Ψ.[17] Consequently, reasoning analogous to that employed by Sattinger (1975) or Costinot and Vogel (2010) can be used to solve the assignment problem. The strong complementarity between worker skill and firm technology embedded in the log supermodularity of Ψ means that firms with better technologies are willing to pay more for the services of high-skill workers, and this leads to positive assortative matching between skill and technology. Hence, I obtain Proposition 1, which is proved in Appendix A.

PROPOSITION 1: *Labor market equilibrium implies positive assortative matching between worker skill and firm technology.*

Since both the skill distribution and the technology distribution have continuous support and no mass points, it follows from Proposition 1 that all workers of a given skill level are hired by firms with the same technology and, conversely, that all firms

[16] It is simple to extend the general equilibrium model in Section IV to allow for the use of labor in fixed costs. See Appendix B for details.

[17] It is straightforward to check that this result does not depend on the firm-level mapping from output to revenue. Instead, the critical assumption required is that equation (2) holds, implying all workers within the firm produce the same product.

with a given technology employ workers with the same skill level.[18] Consequently, we can define a strictly increasing bijection $T : [\underline{\theta}, \bar{\theta}] \to [\underline{z}, \bar{z}]$, such that $l(\theta; z) > 0$ if and only if $z = T(\theta)$, where $l(\theta; z)$ denotes the mass of skill θ workers employed by a technology z firm. T is the matching function and it will play a crucial role in the remainder of the paper. Clearly, $T(\underline{\theta}) = \underline{z}$ and $T(\bar{\theta}) = \bar{z}$. Since T is a strictly increasing bijection its inverse T^{-1} is well defined and gives the skill level of workers employed by a firm with technology z.[19]

It is well known that if heterogeneous workers sort across sectors under perfect competition then log supermodularity of the labor productivity function implies positive assortative matching between workers and sectors.[20] Proposition 1 shows how that result can be extended to characterize the sorting of workers across large firms that produce differentiated products and compete monopolistically.[21] This finding is important because it enables techniques developed in the labor assignment literature to be integrated with the heterogeneous firms models that have dominated recent research in international trade and many areas of macroeconomics. In particular, this paper shows how it can be used to analyze the effects of trade integration on wage inequality.

Empirical support for matching between high-skill workers and high-productivity firms came initially from Brown and Medoff (1989), who find that differences in labor quality account for approximately half of the positive correlation between employer size and wages.[22] Less supportive are the results of Abowd, Creecy, and Kramarz (2002), who find negative correlations between the firm fixed effects and worker fixed effects estimated from wage regressions using matched employer-employee data from France and Washington State. However, Postel-Vinay and Robin (2006) argue that these estimates are likely to suffer from spurious negative correlation, while Lopes de Melo (2008), Lentz (2010), and Eeckhout and Kircher (2011) offer theoretical arguments for why a negative correlation does not imply the absence of positive matching between workers and firms.[23] For example, Eeckhout and Kircher (2011) show that when there are search frictions and each firm hires a single worker, the wage of any given worker is nonmonotonic in firm technology because firms face an opportunity cost of hiring the wrong type of worker. Interestingly, equation (4) shows that the equilibrium assignment in this paper is such that moving any one worker to a firm with a different technology level would lower her marginal revenue product. Although allowing for imperfect matching is beyond the scope of this paper, this observation suggests that wages may be nonmonotonic in firm technology, not because there is an

[18] These claims are established as part of the proof of Proposition 1. See Appendix A for details.

[19] The absence of within-firm skill and wage dispersion implies that the model should be interpreted as a theory of between-firm wage inequality. Empirical work suggests that recent changes in wage inequality have been predominantly driven by between firm variation. See Faggio, Salvanes, and Van Reenen (2010) for the United Kingdom; Barth et al. (2011) for the United States; and Helpman et al. (2012) for Brazil.

[20] This result was first derived by Sattinger (1975) and is at the heart of recent matching models, such as Ohnsorge and Trefler (2007), Costinot (2009), and Costinot and Vogel (2010). Davidson, Matusz, and Shevchenko (2008) develop a model in which the degree of sorting is endogenous to trade integration.

[21] Existing sorting models with both worker and firm heterogeneity assume that all firms produce the same output good and, consequently, are not suitable for analyzing the effects of fixed export costs and selection into exporting. See Sattinger (1979) for the competitive labor market case, Shimer and Smith (2000) for a model with search based labor market frictions, and Antràs, Garicano, and Rossi-Hansberg (2006) for an open economy application.

[22] Larger firms (as measured by either revenue or employment) have higher labor productivity and total factor productivity (TFP). See, for example, Bernard et al. (2007).

[23] See Lentz and Mortensen (2010) for a useful review of this literature.

opportunity cost of hiring, but because the marginal revenue from increasing output is lower at large, technologically advanced firms.

Perhaps the strongest evidence of positive sorting comes from the literature on the exporter wage premium. Bernard and Jensen (1995) first pointed out that firms that export are larger, more productive, and pay higher wages than nonexporters, but the authors were unable to establish whether cross-firm variation in labor quality was responsible for the exporter wage premium. However, as discussed in the introduction, Schank, Schnabel, and Wagner (2007) show, using German data, that although the raw exporter wage premium is 36.6 percent, after controlling for observable worker characteristics, exporters paid only 2.2 percent more than nonexporters. This finding shows that differences in workforce composition explain the vast majority of the exporter wage premium, implying the existence of positive sorting.

Knowing that each firm employs only one type of worker, it is now possible to solve the firm's variable profit maximization problem (3), taking the matching function and wage function as given. If $\theta = T^{-1}(z)$, then profit maximization by a firm with technology z implies

$$l(\theta'; z) = \begin{cases} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} A \Psi(\theta', z)^{\sigma-1} w(\theta')^{-\sigma} & \text{if } \theta' = \theta, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

$$p(z) = \frac{\sigma}{\sigma-1} \frac{w(\theta)}{\Psi(\theta, z)}, \tag{6}$$

$$\pi(z) = \frac{1}{\sigma-1} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} A \left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1}. \tag{7}$$

Note that a firm's marginal production cost depends on both its technology and its employees' skill. However, since there is no within-firm dispersion in worker skill, each firm faces a constant marginal cost of production $\frac{w[T^{-1}(z)]}{\Psi[T^{-1}(z), z]}$, and charges a fixed mark-up over its marginal cost; exactly as occurs in models with heterogeneous firms and homogeneous labor.

Next, the labor market clearing condition can be used to obtain a differential equation for the matching function. Labor market clearing requires that $\forall\ \theta \in [\underline{\theta}, \bar{\theta}]$

$$\int_{\theta}^{\bar{\theta}} dL(\tilde{\theta}) = \int_{T(\theta)}^{\bar{z}} l[T^{-1}(z); z]\, dM(z), \tag{8}$$

where the left-hand side gives the aggregate supply of workers with skill greater than θ and the right-hand side gives total employment at firms that hire workers with skill greater than θ. Differentiating equation (8) with respect to θ gives

$$T'(\theta) = \frac{L'(\theta)}{M'[T(\theta)]} \frac{1}{l[\theta; T(\theta)]}, \tag{9}$$

and using the employment function (5) to substitute for $l[\theta;\, T(\theta)]$, we obtain

$$T'(\theta) = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \frac{1}{A} \frac{L'(\theta)}{M'[T(\theta)]} \frac{w(\theta)^{\sigma}}{\Psi[\theta,\, T(\theta)]^{\sigma-1}}. \tag{10}$$

C. *Wages and Inequality*

The previous section characterized matching between workers and firms taking the wage function as given. However, in equilibrium, the wage function is itself endogenous. This section solves for the wage function and uses it to analyze how shifts in labor demand affect wage inequality.

To obtain the wage function, I first show that firms' optimal choice of workforce skill leads to a simple relationship between the wage function and Ψ. Specifically, the elasticity of the wage function at skill level θ (the returns to skill) equals the elasticity of Ψ with respect to θ at technology $z = T(\theta)$.

PROPOSITION 2: *Labor market equilibrium implies*

$$\frac{\theta}{w(\theta)} w'(\theta) = \frac{\theta}{\Psi[\theta,\, T(\theta)]} \frac{\partial \Psi[\theta,\, T(\theta)]}{\partial \theta}, \tag{11}$$

where $T(\theta)$ is the equilibrium matching function.

The proof of Proposition 2 is given in Appendix A. Intuitively (11) is simply a consequence of cost minimization. The profit function (7) tells us that each firm's profits are decreasing in its marginal production cost $\frac{w(\theta)}{\Psi(\theta,\, z)}$. Consequently, each firm will choose its workforce skill θ to minimize $\frac{w(\theta)}{\Psi(\theta,\, z)}$. Equation (11) follows immediately from the first-order condition of the cost minimization problem.[24]

We can use Proposition 2 to understand how wages vary with firm size. Firm-level employment is given by (5). Observe that there are two countervailing forces affecting how employment varies with z. More productive firms have higher Ψ, which tends to increase employment, but also pay higher wages, which tends to decrease employment. Differentiating (5) with $z = T(\theta)$ and using (11) shows that in order to match the empirical fact that wages are higher at larger firms (Brown and Medoff 1989) we must have

$$(\sigma - 1)\, T'(\theta) \frac{\partial}{\partial z} \Psi[\theta,\, T(\theta)] - \frac{\partial}{\partial \theta} \Psi[\theta,\, T(\theta)] > 0. \tag{12}$$

In Section IVC, I introduce functional form assumptions under which this condition can be expressed in terms of a simple restriction on the parameter space.

[24] This reasoning does not constitute a formal proof of Proposition 2 because it assumes the wage function is differentiable. Appendix A proves the validity of this assumption.

Equations (10) and (11) together form a pair of differential equations that can be solved recursively for the unknown functions $w(\theta)$ and $T(\theta)$.[25] Suppose the wage of workers with skill $\overline{\theta}$ is chosen as the numeraire, meaning $w(\overline{\theta}) = 1$. Given this boundary condition, (11) can be integrated to obtain the equilibrium wage function

$$w(\theta) \;=\; \exp\left[-\int_{\theta}^{\overline{\theta}} \frac{1}{\Psi[s,\, T(s)]}\, \frac{\partial \Psi[s,\, T(s)]}{\partial s}\, ds\right]. \tag{13}$$

This is an important result because it establishes that the matching function is a sufficient statistic for the wage function. Consequently, any movements in wage inequality must result from shifts in the matching function. In particular, differentiating (11) with respect to $z = T(\theta)$ implies

$$\begin{aligned} \frac{\partial}{\partial z}\left[\frac{\theta}{w(\theta)}\, w'(\theta)\right] \;&=\; \theta\, \frac{\partial^2}{\partial \theta\, \partial z} \log\, \Psi(\theta,\, z) \\ &>\; 0, \end{aligned} \tag{14}$$

where the final inequality follows from the log supermodularity of Ψ. Thus, the returns to skill at skill level θ are strictly increasing in $z = T(\theta)$. This result holds because the complementarity between worker skill and firm technology embodied in Ψ implies that the marginal effect of higher skill on log output is greater at more technologically advanced firms. Therefore, when workers are matched to higher technology firms, the returns to skill increase.

The returns to skill can be interpreted as a local measure of wage inequality. Costinot and Vogel (2010) and Monte (2011) use the wage ratio between pairs of agents with different skill levels to measure wage inequality and show that variation in the returns to skill can be mapped into changes in wage ratios, but they do do not consider wage inequality within groups of more than two workers. A more general result linking the returns to skill and wage inequality can be obtained by characterizing inequality in terms of second-order stochastic dominance. Consider two wage functions w and $\hat{w}$. For the remainder of this paper, I will say that wage inequality over $[\theta_0,\, \theta_1] \subseteq [\underline{\theta},\, \overline{\theta}]$ is higher under $\hat{w}$ than under w if, for any measure of inequality that respects scale independence and second-order stochastic dominance, inequality is strictly higher under $\hat{w}$ than under w within any group of workers satisfying: (i) the skill level of all workers in the group belongs to the interval $[\theta_0,\, \theta_1]$; and (ii) there exists within-group skill dispersion. Sampson (2011) shows that when the returns to skill increase at all skill levels belonging to some interval of the skill distribution, then wage inequality over that interval increases in exactly

[25] Costinot and Vogel (2010) obtain an analogous pair of differential equations that characterize the wage and matching functions in their model of labor assignment across tasks.

this sense. Combining this result with equation (14) gives a simple link between the matching function and wage inequality.

PROPOSITION 3: *Let T and $\hat{T}$ be matching functions such that all workers with skill $\theta \in (\theta_0, \theta_1)$ are matched with higher technology firms under $\hat{T}$ than under T. Then wage inequality over $[\theta_0, \theta_1]$ is higher under the wage function induced by $\hat{T}$ than under T.*

Proposition 3 tells us that any shock that causes all workers belonging to some interval of the skill distribution to match with higher technology firms increases wage inequality within any subset of workers belonging to that interval. This is a powerful result because it implies that monotone shifts in the matching function have unambiguous implications for wage inequality. When workers are employed by higher technology firms then wage inequality increases. Holding the technology distribution $M(z)$ constant, Proposition 3 also implies that wage inequality increases when there is skill downgrading within firms. However, this prediction need not hold in general equilibrium if technology and the matching function are jointly endogenous.

It should also be noted that movements in the matching function can be used to determine changes in between group wage inequality. Suppose inequality between two groups is measured by the ratio of average wages in the two groups. When the matching function shifts upward on an interval (θ_0, θ_1), then the returns to skill increase on (θ_0, θ_1), and this leads to a rise in $\frac{w(\theta_b)}{w(\theta_a)}$ whenever $\theta_a, \theta_b \in [\theta_0, \theta_1]$ with $\theta_b > \theta_a$. Consequently, wage inequality increases between any two groups of workers satisfying: (i) the skill level of all workers in both groups belongs to the interval $[\theta_0, \theta_1]$; and (ii) there is no overlap between the supports of the skill distributions of the two groups. In particular, for any partition of the interval that creates a group of low-skill workers with skill below some threshold and a group of high-skill workers with skill above the threshold, an upward shift in the matching function increases wage inequality between the two groups.

We can now obtain a general result characterizing the effect of employment shifts on wage inequality within high-skill workers. Let $L^D(z) = \int_z^{\bar{z}} M'(\tilde{z}) l[T^{-1}(\tilde{z}); \tilde{z}]\, d\tilde{z}$ denote total employment at firms with technology greater than z. Then $\forall\, z \in [\underline{z}, \bar{z}]$ labor market clearing requires

$$(15) \qquad L - L\left[T^{-1}(z)\right] = L^D(z).$$

Now suppose there is an increase in employment at high technology firms. To be specific, suppose that L^D shifts to $\hat{L}^D$ and that there exists $z_0 \in [\underline{z}, \bar{z})$ such that $\hat{L}^D(z) > L^D(z)\ \forall\, z > z_0$. Let T and $\hat{T}$ be the matching functions corresponding to L^D and $\hat{L}^D$, respectively. Then (15) immediately implies that $\hat{T}(\theta) > T(\theta)\ \forall\, \theta \in \left[\hat{T}^{-1}(z_0), \bar{z}\right)$. Thus, an increase in employment at high technology firms necessitates skill downgrading from the firms' perspective, meaning that workers are matched to higher technology firms. Applying Proposition 3, this means that there is an increase in wage inequality over $\left[\hat{T}^{-1}(z_0), \bar{\theta}\right]$.

PROPOSITION 4: *If total employment at firms with technology greater than z increases for all z above some threshold technology z_0, then there is an increase in wage inequality within workers employed by firms with technology at least z_0.*

Proposition 4 provides a testable prediction linking changes in observed employment to wage inequality. An increase in total employment at firms with technology z has two possible causes: (i) an increase in firm-level labor demand $l\left[T^{-1}(z), z\right]$; or (ii) an increase in the mass of firms $M'(z)$ with technology z. Together these two factors determine the gradient of the matching function given by (9) and, consequently, wage inequality. Proposition 4 is a partial equilibrium result that holds regardless of the cause of employment variation. The remainder of the paper allows employment to be endogenously determined and analyzes how trade affects the matching function through its impact on both firm-level employment and the technology distribution.

II. Trade

This section introduces trade into the model while continuing to assume that $M(z)$ is exogenous. The goal is to show how trade affects the equilibrium conditions of the matching model while maintaining as much generality as possible. In Sections III and IV, I then proceed to analyze the effect of trade on $M(z)$.

Following Melitz (2003), this paper conceptualizes trade as the opportunity for firms that pay a fixed export cost to access new markets. Assume that in the open economy a firm that pays a fixed export cost F_x can enter a foreign market with demand function

$$x^* = A^* p^{*-\sigma}, \tag{16}$$

where an asterisk is used to denote foreign variables. Exports to the foreign market are subject to variable iceberg trade costs $\tau \geq 1$. Equation (16) assumes that the demand elasticity in the foreign market is the same as in the domestic market, but this is the only assumption about the foreign economy required to derive the results obtained in this section. In the general equilibrium models considered in Sections III and IV further restrictions will be placed on the structure of the foreign economy.

Consider a firm with technology z. Given the demand function (16), it is well known that the firm's profit maximizing export choices can be separated from its domestic production decisions.[26] Also, if the firm employs workers with skill θ, its marginal production cost is $\frac{w(\theta)}{\Psi(\theta, z)}$, regardless of whether the output produced is sold at home or abroad. Consequently, the firm employs workers with the same skill level in both domestic and export production implying that the wage function is still given by (13) and Propositions 2 and 3 continue to hold.

[26] Of course, this separability does not apply to the firm's entry, exit, and R&D decisions. The impact of trade on these decisions is considered in Sections III and IV.

Solving for the profit maximizing export price shows that the firm charges the same factory gate price (6) for both domestic and export production. It follows that $p^*(z) = \tau p(z)$. Profit maximization also implies that, conditional on exporting, the quantity of labor used in export production $l_x(\theta; z)$ is

$$l_x(\theta; z) = \frac{A^* \tau^{1-\sigma}}{A} l(\theta; z), \tag{17}$$

where $\theta = T^{-1}(z)$ and $l(\theta; z)$ is given by (5). Similarly, variable export profits are

$$\pi_x(z) = \frac{A^* \tau^{1-\sigma}}{A} \pi(z), \tag{18}$$

where $\pi(z)$ is given by (7).

Obviously, the firm will choose to export if and only if variable export profits exceed the fixed export cost F_x. As profits are strictly increasing in z this implies that there exists a threshold technology z_x satisfying

$$\frac{\Psi\left[T^{-1}(z_x), z_x\right]}{w\left[T^{-1}(z_x)\right]} = \left[(\sigma - 1)\left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \frac{F_x}{A^* \tau^{1-\sigma}}\right]^{\frac{1}{\sigma-1}}, \tag{19}$$

such that only firms with technology $z \geq z_x$ select into exporting. Thus, the model predicts the existence of an exporter wage premium due to matching between high technology firms that select into exporting and high-skill workers who receive higher wages. Note that, holding the matching function constant, z_x is increasing in the export costs τ and F_x and decreasing in foreign demand A^*.

Since only high-technology firms choose to export, the open economy version of the labor market clearing condition (8) is

$$\int_{\theta}^{\bar{\theta}} dL(\tilde{\theta}) = \int_{T(\theta)}^{\bar{z}} \left(l\left[T^{-1}(z); z\right] + I\left[z \geq z_x\right] l_x\left[T^{-1}(z); z\right]\right) dM(z),$$

where I is an indicator function that equals 1 if $z \geq z_x$, and 0 otherwise. Differentiating this labor market clearing condition with respect to θ, and using the employment functions (5) and (17), gives

$$T'(\theta) = \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \frac{1}{A + A^* \tau^{1-\sigma} I\left[T(\theta) \geq z_x\right]} \frac{L'(\theta)}{M'\left[T(\theta)\right]} \frac{w(\theta)^{\sigma}}{\Psi\left[\theta, T(\theta)\right]^{\sigma-1}}. \tag{20}$$

Substituting the wage function (13) into (20) and using (19) shows that equation (20) defines a first-order differential equation for the open economy matching function. Although Costinot and Vogel (2010) analyze the effects of inter-industry trade on wage inequality using a differential equation for the matching function, this paper's focus on firms and intra-industry trade leads to two important differences in the analysis. First, selection into exporting creates a kink in the matching function at the export threshold. Second, while in Costinot and Vogel (2010) technology is exogenous, in Sections III and IV of this paper the technology distribution is endogenous to firms' entry, exit, and R&D choices. As will become apparent, these differences have important consequences for how trade affects wage inequality.

To begin the examination of how trade affects the matching function suppose that some, but not all, firms select into exporting. Thus, $\underline{z} < z_x < \bar{z}$. Then a crucial difference between the open economy and autarky is that exporting induces a discontinuous upward jump in firm-level employment at the export threshold z_x. If we consider two firms on opposite sides of the export threshold, then, for a given matching function, the high-technology firm has higher relative employment in the open economy than in autarky. From (20), the discontinuity tends to make the matching function flatter above the export threshold and steeper below the export threshold, and remembering Proposition 4, the increase in relative employment at high technology firms suggests that selection into exporting tends to increase wage inequality within workers employed by exporters. However, the effect of trade on wage inequality is determined not by its impact on relative employment at the firm level, but by its impact on total employment. Therefore, we must also consider the effects of trade-induced shifts in the demand parameter A and the mass of firms $M(z)$.[27]

A rise in A increases the mass of workers used to produce output for the domestic market by the same proportion at all firms. Similarly, changes in the total mass of firms M shift employment by the same proportion at all technology levels without affecting the relative demand for skill. However, variation in $m(z)$ can generate arbitrary changes in aggregate employment at different technology levels. Therefore, without placing restrictions on how trade liberalization affects $m(z)$ it is not possible to determine its impact on the matching function and, consequently, on wage inequality. The remainder of the paper analyzes how trade affects $m(z)$ under alternative assumptions about the processes that drive firms' entry, exit, and R&D decisions.

III. Stochastic Technology

Theories of trade with heterogeneous firms mostly follow Melitz (2003) in assuming that entering firms receive a random productivity draw from some fixed distribution. After learning its technology, each firm has the option of either producing or exiting immediately, and the presence of a fixed production cost means that there exists an endogenous technology cutoff below which firms choose to exit. The exit cutoff defines the lower bound of the firm technology distribution $\underline{z}$. Moreover, $\underline{z}$ is the only free parameter of the equilibrium technology distribution. The shape of the distribution and the upper bound $\bar{z}$ are invariant characteristics inherited from the sampling distribution.

A key prediction of Melitz (2003) and much of the subsequent literature on trade with heterogenous firms is that trade liberalization increases the exit cutoff $\underline{z}$ leading to efficiency gains as resources are reallocated away from low-productivity firms. When this prediction is combined with the assumption that firms receive stochastic technology draws it imposes strong restrictions on how trade liberalization affects $m(z)$. To

[27] In general, A will depend on domestic income, domestic supply, and the extent of import competition from foreign firms, while variation in $M(z)$ will result from firms' entry, exit, and R&D decisions.

capture these restrictions let us make the following assumption about the relationship between $m(z)$ and $m^a(z)$, where the "a" superscript is used to denote autarky.

ASSUMPTION 1: *Let $m(z)$ be the density function for firm technology in the open economy and $m^a(z)$ be the density function in autarky. Then:* (i) $\underline{z} > \underline{z}^a$; (ii) $\bar{z} = \bar{z}^a$; *and* (iii) $\frac{m(z)}{m(z')} = \frac{m^a(z)}{m^a(z')}$ $\forall\ z, z' \in [\underline{z}, \bar{z}]$.

Assumption 1 means that, compared to the autarky firm technology distribution, the open economy distribution has a greater lower bound, the same upper bound, and the same relative density of firms at any two points in the support of both distributions. It follows that the open economy distribution first-order stochastically dominates the autarky distribution. Assumption 1 will not always hold,[28] but it is of particular interest because it embodies the effect of trade liberalization on the technology distribution in the literature on trade with heterogeneous firms that follows Melitz (2003).

As the technology distribution shifts upwards, the demand for skill increases. Thus, Assumption 1 imposes sufficient restrictions on $m(z)$ and $m^a(z)$ to ensure that shifts in the technology distribution reinforce the increased demand for skill resulting from the firm-level employment effect of selection into exporting. It also implies that the technology of the firms that employ the most skilled workers is unchanged and the technology of the firms that employ the least skilled workers increases. Consequently, when Assumption 1 holds all workers with skill below $\bar{\theta}$ are employed by more technologically advanced firms in the open economy than in autarky: $T(\theta) > T^a(\theta)\ \forall\ \theta \in [\underline{\theta}, \bar{\theta})$. Figure 1 shows the autarky and open economy matching functions given Assumption 1. Observe the kink in the open economy matching function at the export threshold z_x which results from the upward jump in labor demand at the export threshold.[29] Combining the upward shift in the matching function with Proposition 3 implies the following result. A full proof of the proposition is given in Appendix A.

PROPOSITION 5: *Suppose Assumption* 1 *holds. Then in any open economy equilibrium where some, but not all, firms export wage inequality is higher over all workers than in autarky.*

Proposition 5 is a strong result. It shows that if firms receive stochastic technology draws and moving from autarky to the open economy increases the exit cutoff, then wage inequality within all groups of workers is higher under trade than in autarky. An immediate corollary of Proposition 5 is that trade increases wage inequality between any two groups of workers whose skill distributions have nonoverlapping support (for example: workers with above median skill versus workers with below median skill; or workers employed by exporters versus workers employed by nonexporters).

[28] Indeed it does not hold in the general equilibrium model considered in Section IV.

[29] All the functions represented in Figures 1–6 are shown as being piecewise linear, but in each case linearity is used for convenience, not because it is implied by the paper.

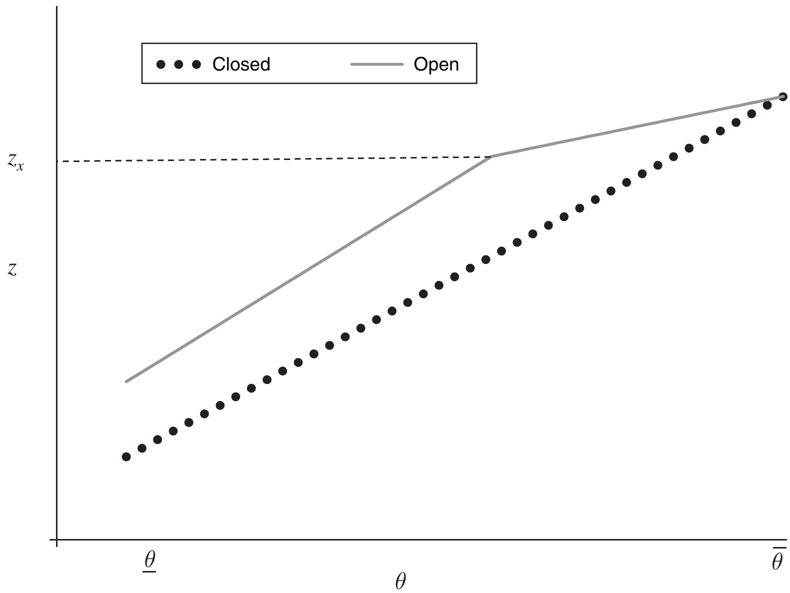

Figure 1. Stochastic Technology—Matching Function

Moreover, Proposition 5 does not require any assumptions on how trade liberalization affects either the domestic demand parameter A or the mass of firms M. In addition, it does not assume symmetry between the domestic and foreign economies and leaves unspecified the exact causes of firms' entry, exit, and R&D decisions. Thus, incorporating labor assignment into Melitz (2003) implies that intra-industry trade increases wage inequality whenever it causes the least productive firms to exit, regardless of the characteristics of an economy's trading partners. However, this degree of generality does come at a cost. Without imposing greater structure on the economy it is not possible to characterize the effects of marginal reductions in trade costs or to show whether Assumption 1 holds. Therefore, the next step is to move beyond partial equilibrium analysis and embed the matching model in a general equilibrium framework with stochastic technology determination.

Suppose agents consume a final good that is sold at price P and produced under perfect competition as a constant elasticity of substitution aggregate of the available differentiated products with elasticity of substitution σ. Consequently, firms face the demand function given by (1) with

$$A = P^{\sigma-1}E, \tag{21}$$

where E denotes aggregate expenditure on the final good and P is given by

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}\, d\omega\right]^{\frac{1}{1-\sigma}}, \tag{22}$$

where Ω denotes the set of available varieties.

To develop a new product an entering firm must pay a fixed cost of $f_e > 0$ units of the final good. After paying the fixed cost, the entrant receives a technology draw from a known distribution function $G(z)$. Assume that $G(z)$ has continuous support on $[0, \bar{z}]$ and is continuously differentiable with density function $g(z)$. After learning its technology, the firm chooses whether or not to exit. In order to produce it must pay $f > 0$ units of the final good, and in order to export it must pay $f_x > 0$ units of the final good. Thus, the fixed export cost $F_x = f_x P$. Finally, I assume that the domestic and foreign economies are symmetric. These assumptions are sufficient to close the partial equilibrium matching model. All other aspects of the economy are as described in Sections IA and II, except that $M(z)$ is no longer exogenously fixed, but is endogenously determined by firms' entry and exit decisions.

To analyze the general equilibrium stochastic technology model I impose the following assumption:

ASSUMPTION 2: *For all* $z \in [0, \bar{z}]$:

$$(\sigma - 1)\,\frac{1}{\Psi(\underline{\theta}, z)}\,\frac{\partial \Psi(\underline{\theta}, z)}{\partial z} > \frac{g(z)}{\frac{f_e}{f + f_x} + 1 - G(z)}.$$

Assumption 2 is a sufficient, but not necessary, condition to ensure the existence of a unique equilibrium both in autarky and when all firms export. Note that Assumption 2 is guaranteed to hold whenever the entry cost is sufficiently large relative to the fixed costs of production and exporting.

Solving the general equilibrium model shows that the structure of fixed costs plays a key role in determining the exit threshold $\underline{z}$ and the matching function.[30] In autarky, an increase in $\frac{f}{f_e}$ raises $\underline{z}$ and shifts the matching function upward at all skill levels below $\bar{\theta}$. Moving from autarky to an open economy where all firms export is equivalent to replacing f by $f + f_x$ and, consequently, shifts the matching function upward (see Figure 2). The intuition is straightforward. When the fixed costs of production increase, firms must make larger variable profits to break even, which raises $\underline{z}$. Since the resulting technology distribution first-order stochastically dominates the initial distribution, the demand for skill increases and the matching function shifts upwards. By contrast, a higher entry cost reduces product market competition and lowers $\underline{z}$.

When all firms export there is no discontinuity in firm-level employment between exporters and nonexporters to raise the demand for skill. However, since the technology distribution shifts upward, wage inequality is still higher than in autarky.[31] In addition, starting from an equilibrium, where all firms export a marginal decrease in f_x, reduces all firms' fixed costs and shifts the matching function downward at

[30] The details of the solution are included in the proof of Proposition 6 in Appendix A.

[31] By contrast, in Helpman, Itskhoki, and Redding (2010) trade affects wage inequality only through the discontinuity between exporters and nonexporters. Therefore, when all firms export wage inequality is the same as in autarky.

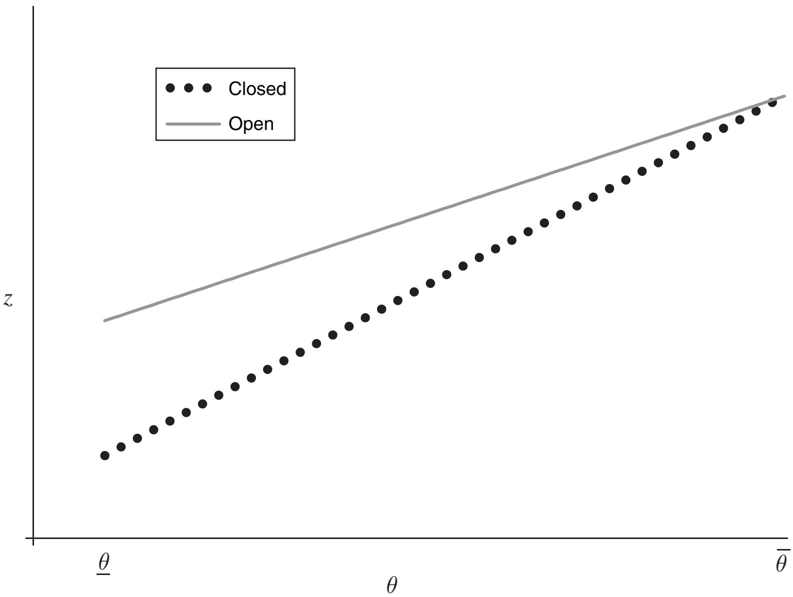

Figure 2. Stochastic Technology All Firms Export—Matching Function

all skill levels below $\overline{\theta}$. Meanwhile, a marginal decrease in the variable trade cost τ leaves the matching function unchanged because it does not affect relative employment at firms with different technologies. Proposition 6 summarizes these results.

PROPOSITION 6 (Stochastic Technology): *Suppose Assumption* 2 *holds and countries are symmetric. Then in any open economy equilibrium, where all firms export wage inequality, is:* (*i*) *higher over all workers than in autarky;* (*ii*) *independent of the variable trade cost*; *and* (*iii*) *higher over all workers when the fixed export cost is larger.*

Proposition 6 has two important corollaries. First, it implies that reductions in fixed and variable trade costs have qualitatively different effects on wage inequality. Inequality is nonmonotonic in the fixed export cost. As f_x declines, aggregate wage inequality peaks at some value of f_x, such that not all firms are exporters, and when f_x is reduced to zero, wage inequality and the matching function are the same as in autarky. However, at arbitrarily low levels of τ wage inequality remains higher than in autarky provided $f_x > 0$. Second, since the exit cutoff $\underline{z}$ is continuous in both f_x and τ, if trade costs are sufficiently low, Assumption 1 must hold even when not all firms are exporters. Thus, applying Proposition 5 implies that at sufficiently high levels of trade integration wage inequality is always greater over all workers than in autarky.

With stochastic technology determination, the model is insufficiently tractable to permit a complete solution of the case where some, but not all, firms export. However, numerical solutions using a range of functional forms and parameter values show that: (i) aggregate wage inequality increases monotonically as τ decreases and is an inverted U-shape as a function of f_x; and (ii) in any open economy equilibrium, wage inequality over workers employed in the export sector is higher than in autarky.[32] In addition, the numerical solutions show that there exist parameterizations of the model in which the fall in labor demand from nonexporters when a small share of firms start to export is sufficient to cause $\underline{z} < \underline{z}^a$ implying that wage inequality within the least skilled workers is lower than in autarky. Therefore, although trade always causes an increase in wage inequality within high-skill workers employed by exporters, it does not necessarily lead to higher inequality in the lower tail of the wage distribution.

IV. Technology Choice

The assumption that technology is randomly determined can be justified by appealing to the uncertainties inherent in R&D, but its ubiquitous presence in the literature probably owes more to the fact that it enables authors to match the empirical observation that there exists productivity dispersion across firms while maintaining tractability. However, a key finding of the R&D literature is that there exists a positive correlation between productivity and R&D at the firm level (Klette and Kortum 2004). To examine whether the effects of trade on wage inequality are robust to alternative assumptions about the determinants of firms' technology levels, this section considers the case where each firm can choose its technology level by paying a technology dependent R&D cost at entry.

This approach would be of little interest in the Melitz (2003) framework because, except in knife-edge cases in which the ratio of the variable profit function to the R&D entry cost was independent of z, all firms would select the same technology.[33] However, the matching model introduced in this paper is different. The existence of heterogeneous workers implies that a firm's optimal technology will, in general, depend on the skill level of its workforce. This enables the model to support equilibria with technology dispersion across firms even when each firm chooses its technology.

The technology choice model uses the same general equilibrium structure introduced in the previous section, except that in order to produce with technology z a firm must pay a fixed cost of $f + \kappa f_e(z)$ units of the final good, where $f \geq 0$, $\kappa > 0$ and $f_e(z)$ is positive, strictly increasing in z, and twice continuously differentiable. Thus, in order to obtain a more advanced technology a firm must invest more in R&D, which increases its fixed production costs.

To distinguish the two components of total fixed costs I will refer to f as the firm's fixed cost and $\kappa f_e(z)$ as its entry or R&D cost. However, when interpreting the comparative statics results below it is important to note that, since firm technology is not

[32] It can be proved that this result always holds if either the parameter restriction $f_e \geq \tau^{\sigma-1}(f + f_x)$ is satisfied, or z_x is sufficiently close to $\bar{z}$, meaning that few firms choose to export.

[33] Costantini and Melitz (2008), Atkeson and Burstein (2010), Lileeva and Trefler (2010), and Bustos (2011a) introduce partial technology choice into the Melitz (2003) model by allowing each firm the option of making a productivity enhancing investment after learning its initial productivity. All of these papers assume homogeneous labor. Yeaple (2005) and Bustos (2011b) develop models in which firms can choose between two technologies and there is observable heterogeneity across workers.

stochastic, the meaningful distinction between the fixed cost and the entry cost lies not in their timings, but in the fact that only the entry cost depends on z.

A. *Closed Economy*

Consider an economy in autarky. To simplify notation I will not use the "a" superscript to denote autarky in this section. Conditional on z, each firm's variable profit maximization problem is exactly as described in Section IB. Consequently, if a firm with technology z employs any workers with skill θ, its variable profits are given by (7).[34] The key general equilibrium restriction is the free entry condition, which requires that variable profits cannot exceed fixed costs. Therefore, letting $\Pi(\theta, z)$ denote total profits, free entry implies

$$\Pi(\theta, z) = \frac{1}{\sigma - 1}\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} A\left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1} - P[f + \kappa f_e(z)] \leq 0, \tag{23}$$

with equality in any equilibrium where both $m(z) > 0$ and a technology z firm chooses to employ workers with skill θ.

Profit maximization implies that firms choose z to maximize $\Pi(\theta, z)$. From (23) the first-order condition of this optimization problem is

$$(\sigma - 1)[f + \kappa f_e(z)] \frac{\partial \Psi(\theta, z)}{\partial z} = \kappa f_e'(z) \Psi(\theta, z), \tag{24}$$

and to ensure that (24) defines a maximum, I assume that whenever (θ, z) satisfies the first-order condition then

$$\kappa f_e''(z) \psi(\theta, z) - (\sigma - 2) \kappa f_e'(z) \frac{\partial \Psi(\theta, z)}{\partial z} - (\sigma - 1)[f + \kappa f_e(z)] \frac{\partial^2 \Psi(\theta, z)}{\partial z^2} > 0. \tag{25}$$

A necessary condition for (25) to hold is that either $f_e(z)$ is convex or Ψ is concave in z. I also assume that for all θ in the support of the skill distribution there exists a solution to (24) with $z > 0$.[35] Given that a solution exists, (25) ensures uniqueness, implying that we can define $z = T(\theta)$ to be the solution of (24) conditional on θ. T defines the equilibrium matching function.

[34] Within-firm dispersion in worker skill cannot be ruled out ex ante, but inspection of (4) shows that (7) holds even if the firm employs more than one type of worker. I show below that in equilibrium a firm's employees all have the same skill level.

[35] This assumption requires that, relative to labor productivity Ψ, the entry cost f_e is sufficiently low that firms choose to enter, but sufficiently high that firms choose a finite technology level. Section IVC gives an example of functional forms that guarantee the existence of a solution.

To obtain the properties of the matching function, we can differentiate (24) with respect to θ to obtain

$$\frac{dz}{d\theta} = \frac{\sigma - 1}{\Psi(\theta, z)} \times \frac{[f + \kappa f_e(z)][\Psi(\theta, z)\Psi_{\theta z}(\theta, z) - \Psi_\theta(\theta, z)\Psi_z(\theta, z)]}{\kappa f_e''(z)\psi(\theta, z) - (\sigma - 2)\kappa f_e'(z)\Psi_z(\theta, z) - (\sigma - 1)[f + \kappa f_e(z)]\Psi_{zz}(\theta, z)} > 0,$$

where the second line follows from the log supermodularity of Ψ and assumption (25). Therefore, $T'(\theta) > 0$, meaning that in equilibrium there is technology dispersion across firms and positive assortative matching between worker skill and firm technology. In addition, as the skill distribution has no mass points and continuous support on $[\underline{\theta}, \bar{\theta}]$, labor market clearing implies that, in equilibrium, the firm technology distribution must also have no mass points and continuous support on $[\underline{z}, \bar{z}]$, where $\underline{z} \equiv T(\underline{\theta})$ and $\bar{z} \equiv T(\bar{\theta})$.[36]

From Proposition 2, the equilibrium wage function must satisfy (11). However, in this case, $w(\theta)$ can also be obtained directly from the free-entry condition (23) giving

$$(26) \qquad w(\theta) = \left[\frac{1}{\sigma - 1}\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \frac{A}{P} \frac{1}{f + \kappa f_e[T(\theta)]}\right]^{\frac{1}{\sigma - 1}} \Psi[\theta, T(\theta)].$$

Imposing that $w(\bar{\theta}) = 1$ by choice of numeraire then implies an explicit relationship between the wage function and the matching function

$$(27) \qquad w(\theta) = \left(\frac{f + \kappa f_e[T(\bar{\theta})]}{f + \kappa f_e[T(\theta)]}\right)^{\frac{1}{\sigma - 1}} \frac{\Psi[\theta, T(\theta)]}{\Psi[\bar{\theta}, T(\bar{\theta})]}.$$

Observe that choosing $z = T(\theta)$ to maximize the right-hand side of (27) gives equation (24), which defines the equilibrium matching function. It follows that in equilibrium each worker is matched with the firm type that can offer her the highest wage subject to the requirement that the firm makes nonnegative profits.

We have now solved for the matching function given by (24) and the wage function (27). Equilibrium also requires that the market for the final good clears, implying that aggregate expenditure on the final good E equals the sum of labor income and firms' expenditure on fixed costs and R&D.

$$(28) \qquad E = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta) + MPf + MP\kappa \int_{\underline{z}}^{\bar{z}} f_e(z) m(z)\, dz.$$

[36] Of course, given that the technology distribution has continuous support and no mass points, Proposition 1 can be invoked to prove positive assortative matching between workers and firms. However, while Section I assumed the technology distribution had these properties, this section shows that free entry with technology choice endogenously generates a technology distribution with continuous support and no mass points.

Using (28) together with the pricing equation (6), the labor market clearing condition (10), the expression for the demand parameter (21), and the price index equation (22), it is now straightforward to solve for the remaining endogenous variables E, A, P, M, and $m(z)$, and show that the closed economy has a unique equilibrium. See the proof of Proposition 7 for details.

PROPOSITION 7 (Technology Choice): *There exists a unique closed economy equilibrium. In equilibrium there is technology dispersion across firms and positive assortative matching between worker skill and firm technology.*

Before considering the effects of trade it is worth making some observations about the equilibrium. With technology choice the free entry condition must hold pointwise for all z. This implies that there is effectively a separate market for workers of each skill level and, consequently, the matching function only depends on the available production technologies and is independent of both the size of the economy and the skill distribution. It follows that variation in $L(\theta)$ caused by births, deaths, immigration, emigration, or human capital accumulation leaves both the matching function and wage inequality unchanged. Instead, variation in the labor supply is absorbed by changes in the mass of firms at each technology level. Given this observation, it makes sense to interpret the technology choice model as capturing the long-run equilibrium that obtains after firms' R&D investment and the technology distribution have fully adjusted to any shocks to the economy. In addition, note that an upward shift in the skill distribution causes an upward shift in the support of the technology distribution. Therefore, an economy with higher skill workers will have more productive firms not only because more skilled workers have higher labor productivity as in Manasse and Turrini (2001), but also because firms that employ higher skill workers invest in more advanced technologies as in Yeaple (2005). However, whereas there are only two technologies in Yeaple (2005), in the technology choice model firms can select from a continuum of technologies. This distinction will be important when analyzing how trade cost variation affects wage inequality.

To understand how the matching function depends on the fixed costs of production and entry, we can differentiate (24) with respect to f and κ while holding θ constant. This gives

$$(29)\quad dz = \frac{1}{\kappa}\,\frac{(\sigma-1)\,\Psi_z(\theta,z)(\kappa df - f d\kappa)}{\kappa f_e''(z)\,\psi(\theta,z) - (\sigma-2)\,\kappa f_e'(z)\,\Psi_z(\theta,z) - (\sigma-1)[f+\kappa f_e(z)]\,\Psi_{zz}(\theta,z)}.$$

Remembering from assumption (25) that the denominator of this expression is positive, it follows that an increase in the technology independent fixed cost f shifts the matching function upward and generates an increase in wage inequality over all workers, while a proportional increase in the technology dependent entry cost $\kappa f_e(z)$ has the opposite effect provided $f > 0$. For any given technology z, an increase in f reduces the share of total fixed costs due to the entry cost. This reduces the elasticity of total fixed costs with respect to z and, consequently, firms opt to invest in more advanced technologies. An increase in κ has the opposite effect. These comparative statics are the same as those obtained in the stochastic technology model implying that the impact on wage inequality of variation in the fixed costs of production and R&D is not sensitive to how firm technology is determined.

B. *Open Economy*

Suppose that in addition to serving the domestic market, firms can also choose to become exporters. The export opportunity is as described in Section II with foreign demand given by (16). The fixed cost of exporting is the same as in Section III—in order to export a firm must pay $f_x > 0$ units of the final good. Since each firm's export profit maximization problem is unchanged from Section II, it immediately follows that only firms whose technology exceeds the export threshold z_x defined by (19) select into exporting; firms that export use workers with the same skill level in both domestic and export production; employment in export production is given by (17) and variable export profits are given by (18); and open economy labor market clearing implies (20).

In the open economy, the free entry condition is

$$\text{(30)} \qquad \begin{aligned} \Pi(\theta, z) \;=\; & \frac{1}{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(A + A^{*}\tau^{1-\sigma} I[z \geq z_x]\right)\left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma-1} \\ & - P\left[f + f_x I[z \geq z_x] + \kappa f_e(z)\right] \\ \leq\; & 0, \end{aligned}$$

with equality whenever both $m(z) > 0$ and a technology z firm chooses to employ workers with skill θ. Note that at technology levels below the export threshold, this expression is identical to the autarky free entry condition (23).[37] As in autarky, firms choose z to maximize $\Pi(\theta, z)$. Therefore, when $z < z_x$ the first-order condition (24) continues to hold. This means that for workers employed by firms that do not export, the matching function in the open economy is the same as the autarky matching function $T^a(\theta)$. When $z > z_x$, maximization of $\Pi(\theta, z)$ gives

$$\text{(31)} \qquad (\sigma-1)\left[f + f_x + \kappa f_e(z)\right] \frac{\partial \Psi(\theta, z)}{\partial z} \;=\; \kappa f_e'(z)\, \Psi(\theta, z).$$

The inclusion of f_x on the left-hand side of this expression is the only difference from (24). Consequently, assumptions analogous to those made in the closed economy guarantee that (31) has a unique solution $z = T_x(\theta)$, where $T_x(\theta)$ is differentiable and strictly increasing in θ.[38]

[37] Since firms must always pay both the fixed cost f and the R&D cost $\kappa f_e(z)$, it is never optimal for a firm to export, but not sell domestically.

[38] To be specific, I assume that a solution to (31) exists and that $\kappa f_e''(z)\psi(\theta, z) - (\sigma-2)\kappa f_e'(z)\frac{\partial \Psi(\theta, z)}{\partial z} - (\sigma-1)\left[f + f_x + \kappa f_e(z)\right]\frac{\partial^2 \Psi(\theta, z)}{\partial z^2} > 0$ whenever (θ, z) satisfies (31) and $z > 0$, which is the open economy equivalent of (25).

Comparing (24) and (31) shows that the inclusion of f_x in (31) is equivalent to an increase in the fixed cost f. Remembering from (29) that an increase in f shifts the matching function upward, it follows that $T_x(\theta) > T^a(\theta)\ \forall\ \theta$. Therefore, all workers employed by exporters are matched with higher technology firms in the open economy than in autarky. This shift in the matching function could be driven by firm entry and exit or by existing firms upgrading their technology or downgrading the skill level of their workforce. Since it is static, the model does not speak to the relative contribution of these possible factors but, I hypothesize that in a dynamic model in which labor market frictions make reallocating labor costly and firms face sunk costs, technology upgrading by existing firms would drive the adjustment from autarky to the open economy equilibrium. However, regardless of the relative importance of different margins of adjustment, note that changes in the technology distribution, not differences in labor demand between exporters and nonexporters, drive variation in the matching function between autarky and the open economy.

The next step in solving for the open economy matching function is to characterize which workers are employed by exporters. The open economy free entry condition (30) implies that the wage function $w_x(\theta)$ for workers employed by exporters is

$$(32)\quad w_x(\theta) = \left[\frac{1}{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\frac{1}{P}\frac{A + A^*\tau^{1-\sigma}}{f + f_x + \kappa f_e[T_x(\theta)]}\right]^{\frac{1}{\sigma-1}}\Psi[\theta,\, T_x(\theta)],$$

while the wage function $w_d(\theta)$ for workers employed by nonexporters is given by (26). Substituting (32) into equation (19), which defines the export threshold, implies that z_x satisfies

$$(33)\qquad \frac{A}{A^*\tau^{1-\sigma}} f_x = f + \kappa f_e(z_x).$$

It is easy to check that $\frac{w_x(\theta)}{w_d(\theta)}$ is strictly increasing in θ and that $w_x(\theta)$ and $w_d(\theta)$ intersect at skill level θ_x, satisfying[39]

$$(34)\qquad \left(1 + \frac{A^*\tau^{1-\sigma}}{A}\right)\frac{f + \kappa f_e[T^a(\theta_x)]}{f + f_x + \kappa f_e[T_x(\theta_x)]}\left(\frac{\Psi[\theta_x,\, T_x(\theta_x)]}{\Psi[\theta_x,\, T^a(\theta_x)]}\right)^{\sigma-1} = 1.$$

Since each worker matches with the firm that makes her the highest wage offer, θ_x is a skill threshold, such that workers with $\theta \leq \theta_x$ are employed by nonexporters

[39] See the proof of Proposition 8 for details.

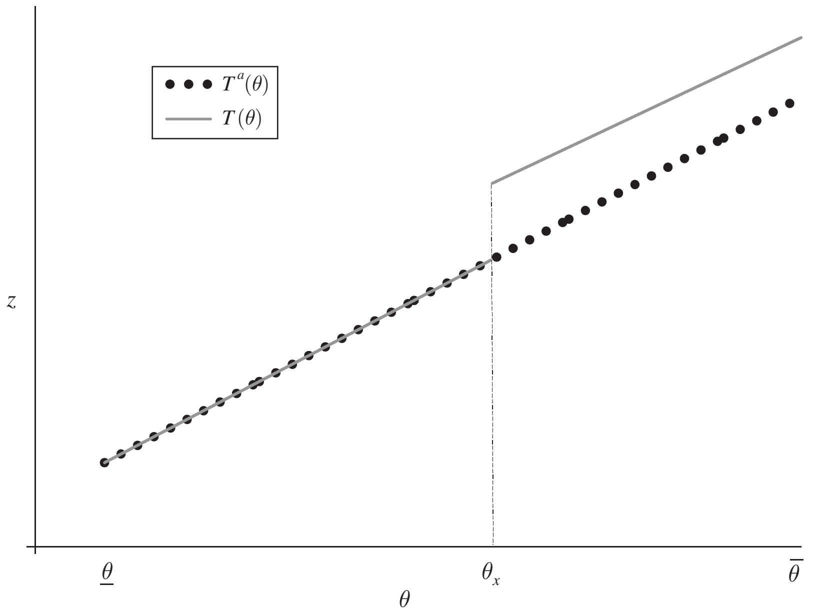

Figure 3. Technology Choice—Matching Function

and workers with $\theta \geq \theta_x$ are employed by exporters.[40] Therefore, the open economy matching function $T(\theta)$ is[41]

$$T(\theta) = \begin{cases} T^a(\theta) & \text{if} \quad \theta \leq \theta_x, \\ T_x(\theta) & \text{if} \quad \theta \geq \theta_x. \end{cases}$$

As $T^a(\theta_x) < T_x(\theta_x)$, the matching function implies that there is positive assortative matching between worker skill and firm technology, and that whenever $\theta_x \in [\underline{\theta}, \overline{\theta}]$, the support of the firm technology distribution is discontinuous. There exists a set of low-technology firms with $z \in [T^a(\underline{\theta}), T^a(\theta_x)]$ that do not export and a set of high-technology firms with $z \in [T_x(\theta_x), T_x(\overline{\theta})]$ that are exporters. Both the upper bound and the range of the firm technology distribution are greater in the open economy than in autarky. Figure 3 shows the autarky and open economy matching functions.

Remembering Proposition 3, comparing $T(\theta)$ and $T^a(\theta)$ implies that wage inequality over workers employed by exporters is higher in the open economy than in autarky, while inequality over workers employed by nonexporters is unchanged. In addition, if $\theta_1 > \theta_x \geq \theta_0$, then $\frac{w(\theta_1)}{w(\theta_0)} > \frac{w^a(\theta_1)}{w^a(\theta_0)}$, implying that inequality between

[40] Without placing further restrictions on the model's functional forms and parameters, there is no guarantee that $\theta_x \in [\underline{\theta}, \overline{\theta}]$. If $\theta_x < \underline{\theta}$, all firms export, while if $\theta_x > \overline{\theta}$, there is no trade. I focus primarily on the situation where some, but not all, firms export since this appears to be the empirically relevant case.

[41] Given that $T^a(\theta_x) \neq T_x(\theta_x)$, the matching function is technically a correspondence, but since the skill distribution does not have any mass points, this observation is unimportant.

high- and low-skill workers is higher in the open economy. Proposition 8 summarizes these results. The proof is in Appendix A.

PROPOSITION 8 (Technology Choice): *In any open economy, equilibrium wage inequality compared to autarky is:* (*i*) *higher over all workers employed by exporters;* (*ii*) *unchanged over all workers employed by nonexporters*; *and* (*iii*) *higher between workers employed by exporters and workers employed by nonexporters.*

Since it does not require any restrictions on the foreign demand parameter A^*, or on how imports affect the domestic demand parameter A, Proposition 8 holds regardless of the size, skill endowment, or production structure of the foreign economy. It follows simply from free entry, and the assumption that firms that select into exporting face an additional technology independent fixed cost f_x. The fixed export cost shifts downward both the total profit function $\Pi(\theta, z)$ and the elasticity of total fixed costs with respect to z. Consequently, exporters require higher variable profits to break even and, because the lower elasticity means total fixed costs are less sensitive to z, it is optimal for exporters to increase their variable profits by raising their R&D investment.

To understand the relationship between the technology choice approach and the stochastic technology framework, it is interesting to compare Proposition 8 to Proposition 5 above. Both propositions imply that due to its asymmetric impact on high- and low-technology firms, trade raises wage inequality at the aggregate level and within workers employed by exporters. However, the mechanisms that cause higher inequality are different. In Proposition 5, the assumptions that firms receive random technology draws and trade raises the exit cutoff are sufficient to ensure that trade induced variation in the firm technology distribution cannot overturn the increased demand for skill caused by the discontinuous upward jump in firm-level employment at the export threshold. By contrast, in the technology choice model the technology distribution is unrestricted ex ante. Among nonexporters, changes in the mass of firms using each technology exactly offset the variation in firm-level employment caused by moving from autarky to the open economy, and the matching function is unchanged. However, among exporters, the technology distribution shifts upward, which increases the demand for skill and raises inequality. In summary, Proposition 5 holds because trade increases relative employment at high technology firms, but Proposition 8 holds because trade causes exporters to use more advanced technologies.

To complete the solution of the open economy model more information about the foreign economy is required. I assume the domestic and foreign economies are identical, implying $A = A^*$. Given this restriction, the open economy has a unique equilibrium. The proof is in Appendix A.

PROPOSITION 9 (Technology Choice): *With symmetric countries there exists a unique open economy equilibrium.*

Now we can analyze how changes in trade costs affect wage inequality in an open economy. First, consider a decrease in the variable trade cost τ. From (24) and (31), the matching functions $T^a(\theta)$ and $T_x(\theta)$ are independent of τ. Although a fall in τ raises export profits and exporters' employment ceteris paribus, free entry

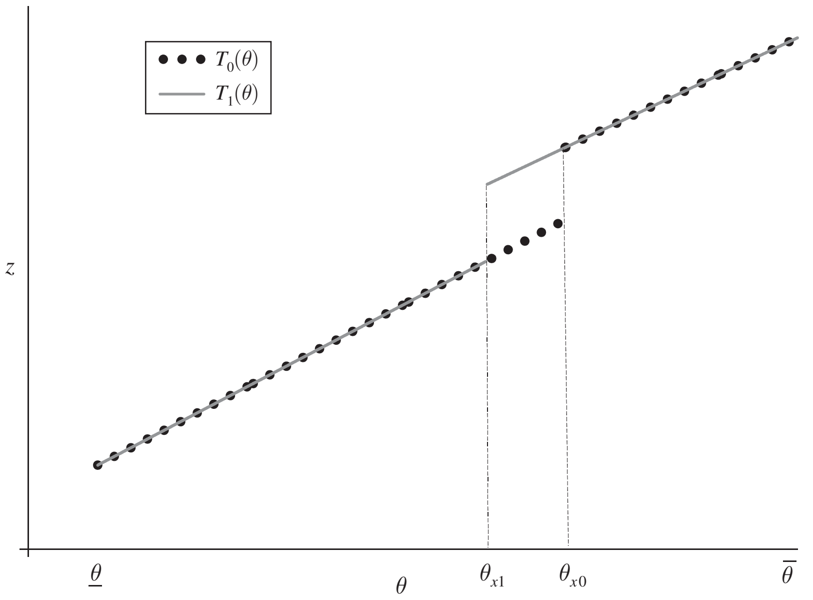

FIGURE 4. REDUCTION IN VARIABLE TRADE COSTS—MATCHING FUNCTION

exactly offsets this effect leaving the matching function conditional on exporting unchanged. Therefore, using $A = A^*$ and $\sigma > 1$, it follows from (26) and (32) that a fall in τ shifts $w_x(\theta)$ upward relative to $w_d(\theta)$, implying that the skill threshold θ_x declines. Similarly, from (33), the export threshold z_x falls, implying that firms start exporting at lower technology levels. These new exporters hire workers who were previously employed by nonexporters using less advanced technologies. For all other workers the matching function is unchanged. Figure 4 shows the matching functions T_0, before, and T_1, after, a reduction in τ. The implications of these changes for wage inequality are summarized in Proposition 10.[42]

PROPOSITION 10 (Technology Choice): *A fall in the variable trade cost causes the skill threshold above which workers are matched with exporters to decline. In the new equilibrium, wage inequality is:* (*i*) *higher over all workers that switch from nonexporters to exporters;* (*ii*) *unchanged both within workers employed by nonexporters in the new equilibrium and within workers employed by exporters in the initial equilibrium*; *and* (*iii*) *higher between workers employed by exporters and workers employed by nonexporters.*

Figure 5 depicts how lower variable trade costs affect the wage function. It shows log wages plotted against log skill, meaning that the gradient equals the wage elasticity. Consequently, at skill levels for which the matching function is unaffected by the reduction in τ, the gradient of the log wage function is also unchanged. Observe that if $\theta_b \geq \theta_{x0}$ and $\theta_{x1} \geq \theta_a$ where θ_{x0}, θ_{x1} denote the skill thresholds for

[42] With nonsymmetric countries, an increase in $\frac{A^*}{A}$ has the same effect on the matching function as a fall in τ. Therefore, an increase in foreign's relative demand will increase domestic wage inequality in the manner described in Proposition 10.

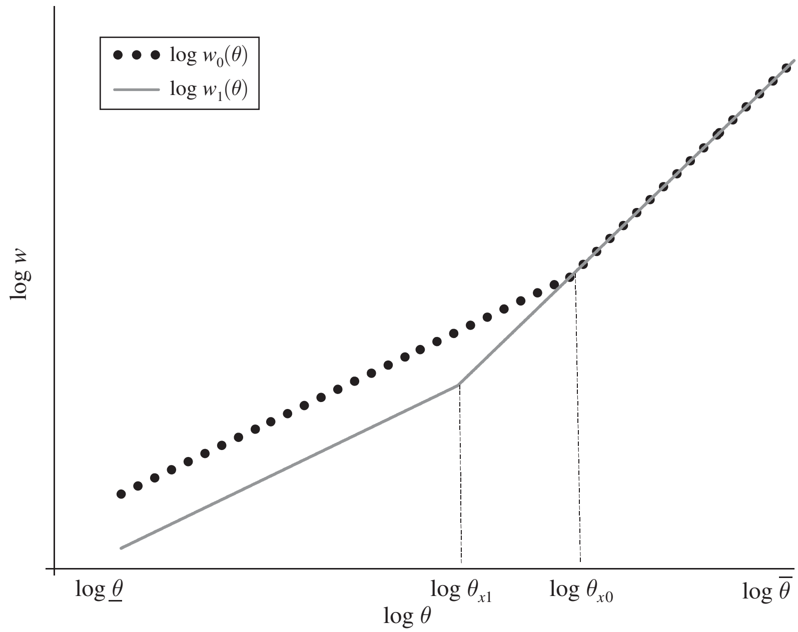

Figure 5. Reduction in Variable Trade Costs—Wage Function

employment in the export sector before and after, respectively, the reduction in τ, then the wage ratio $\frac{w(\theta_b)}{w(\theta_a)}$ is higher in the new equilibrium, even though the matching function is unchanged at both θ_b and θ_a.

The predictions of Proposition 10 are supported by the findings of recent empirical work that uses firm-level data to analyze the consequences of trade liberalization. Verhoogen (2008) finds that following the Mexican peso devaluation of 1994, wages and exports increased at more productive Mexican plants leading to a rise in within-industry wage inequality. Verhoogen argues that exported goods are better quality than output sold domestically and are produced by higher quality workers. Consequently, wage increases at high-productivity plants resulted from plants upgrading the quality of their workforce in order to increase export production. Amiti and Davis (2012) show, using firm-level data from Indonesia, that cuts in output tariffs increased the average wage paid by exporters, but decreased the average wage at non-exporters. On the technology side, Lileeva and Trefler (2010) and Bustos (2011a) present evidence that trade liberalization increases technology investment at firms that are induced to start exporting, but does not affect the technology used by firms with very high initial productivity. To the extent that the upward shift in the matching function for workers employed by new exporters shown in Figure 4 results from within firm technology upgrading, these findings are consistent with Proposition 10. Less evidence is available on whether trade affects wage inequality within high-skill workers employed in the export sector. The results of Han, Liu, and Zhang (2012) suggest that China's WTO accession increased wage inequality within workers in the upper tail of the skill distribution, but more work using matched employer-employee data and considering alternative trade liberalization episodes is clearly required.

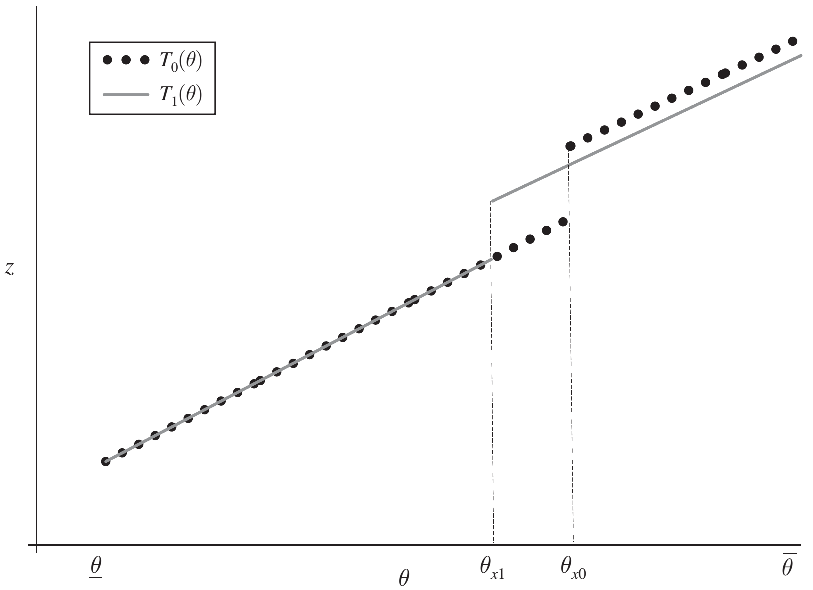

Figure 6. Reduction in Fixed Export Cost—Matching Function

Now, let us consider a decrease in the fixed export cost f_x. As discussed above, a fall in f_x is equivalent to a reduction in f in the closed economy, and shifts the matching function for exporters $T_x(\theta)$ downward. The matching function for nonexporters $T^a(\theta)$ is unaffected. Differentiating $\frac{w_x(\theta)}{w_d(\theta)}$ with respect to f_x and using (31) implies that the wage ratio shifts upward when f_x declines, implying a fall in θ_x. Likewise, (33) shows that z_x decreases. Therefore, a lower fixed cost of exporting induces firms to start exporting at lower technology levels and leads to some workers switching from nonexporters to exporters. In addition, it causes existing exporters to use less advanced technologies, which reduces their demand for skill, leading to a fall in wage inequality over workers initially employed by exporters. The effect on aggregate wage inequality and on wage inequality between workers employed by exporters and workers employed by nonexporters is, in general, ambiguous. Figure 6 shows the matching functions T_0, before, and T_1, after, a fall in f_x, and Proposition 11 summarizes the effects of a decrease in f_x.

PROPOSITION 11 (Technology Choice): *A fall in the fixed export cost causes the skill threshold above which workers are matched with exporters to decline, and shifts the matching function for workers employed by exporters downward. In the new equilibrium wage, inequality is:* (*i*) *higher over all workers that switch from nonexporters to exporters;* (*ii*) *unchanged over workers employed by nonexporters in the new equilibrium*; *and* (*iii*) *lower over all workers employed by exporters in the initial equilibrium.*

I am not aware of any empirical work that studies how changes in fixed export costs impact technology investment or wage inequality. This would be an interesting avenue for future research.

Propositions 10 and 11 show that the effects of reductions in trade costs on aggregate wage inequality in the technology choice model are similar to those found with stochastic technology determination. Lower variable trade costs increase aggregate wage inequality whenever not all firms are exporters, but the effect of lower fixed export costs is nonmonotonic. When few firms export technology, upgrading by new exporters dominates technology downgrading by existing exporters, and a reduction in f_x increases aggregate wage inequality. However, if most or all firms export the technology, downgrading effect dominates, and reducing fixed export costs lowers aggregate wage inequality. When $f_x = 0$, the matching function and wage inequality are the same as in autarky.

Although the technology choice and stochastic technology models lead to similar predictions for how trade integration affects aggregate wage inequality and inequality within the export sector, they have different implications for inequality within workers employed by nonexporters. In the stochastic technology model, shocks to the economy lead to global shifts in the matching function that affect all workers. However, in the technology choice model, since the free entry condition must hold pointwise for all z, trade does not affect the matching function for nonexporters. Consequently, the effects of trade on wage inequality are local to the export sector.

C. *Functional Forms*

Beyond assuming log supermodularity, the analysis above does not impose any restrictions on the labor productivity function. This section analyzes the technology choice model after assuming specific functional forms for $\Psi(\theta, z)$ and $f_e(z)$. The purpose of the analysis is to show that there exist functional form restrictions under which: (i) there exists a unique matching function (i.e., there exists a solution to equation (24), and assumption (25) is satisfied); (ii) the model is highly tractable; and (iii) the equilibrium wage, employment, and revenue distributions provide a good approximation to their empirical counterparts.

Assume the following functional forms for labor productivity Ψ and the entry cost f_e:

ASSUMPTION 3: $\Psi(\theta, z) = \left(\theta^{\frac{\gamma-1}{\gamma}} + z^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$, $f_e(z) = z^{\alpha}$, *where* $0 < \gamma < 1$ *and* $\frac{\sigma-1}{\sigma} < \alpha < \sigma - 1$.

The requirement that $\gamma < 1$ implies Ψ is log supermodular, while $\alpha > \frac{\sigma-1}{\sigma}$ guarantees (12) holds meaning that employment is strictly increasing in z and $\alpha < \sigma - 1$ ensures the second-order condition (25) holds. For simplicity, I also assume throughout this section that $f = 0$ and $\kappa = 1$.

Under these functional form assumptions the model is highly tractable and, in autarky, it gives closed form expressions for the main outcomes of interest. For example, the autarky matching and wage functions are

$$T^a(\theta) = \left(\frac{\alpha}{\sigma - 1 - \alpha}\right)^{\frac{\gamma}{\gamma-1}} \theta, \qquad w^a(\theta) = \left(\frac{\theta}{\bar{\theta}}\right)^{\frac{\sigma-1-\alpha}{\sigma-1}}.$$

In addition, if skill has a truncated Pareto distribution on $[\underline{\theta}, \overline{\theta}]$, then worker wages, firm employment, firm revenue, and firm variable profits all have truncated Pareto distributions. In the open economy, the existence of the fixed export cost f_x means these variables do not have exact truncated Pareto distributions, but it can be shown that their distribution functions converge to power functions as θ or z become large. Thus, the upper tail of each of these distributions is well approximated by a Pareto distribution. Proposition 12 summarizes these results. The proof is in Appendix A.[43]

PROPOSITION 12 (Technology Choice): *Suppose the functional form restrictions in Assumption* 3 *hold and worker skill has a truncated Pareto distribution. Then, in equilibrium, the distributions of worker wages*, *firm employment*, *firm revenue, and firm variable profits are truncated Pareto in autarky and converge to Pareto distributions in their upper tails in the open economy.*

It has frequently been observed that the Pareto distribution provides a good fit to the distribution of firm employment and sales (Axtell 2001; Luttmer 2007), while the upper tail of the wage distribution is also well approximated by a Pareto distribution (Neal and Rosen 2000). Proposition 12 offers a unified explanation for these observations. Technology choice together with matching between workers and firms implies that dispersion in firm technologies is inherited from dispersion in worker skill. Under Assumption 3, the matching, wage, employment, revenue, and variable profit functions are all (asymptotically) power functions. Consequently, if worker skill has a truncated Pareto distribution, then the wage, employment, revenue, and variable profit distributions all behave like Pareto distributions in their upper tails. This result demonstrates how an integrated treatment of worker and firm heterogeneity can generate insights not obtained when the two topics are addressed in isolation. In addition, it suggests that under appropriate functional form restrictions the modeling framework developed in this paper may have sufficient empirical realism to justify its future use in calibration exercises that seek to quantitatively evaluate the effects of trade on wage inequality.

V. Conclusion

To understand the channels through which trade integration impacts wage inequality we must account for the role played by firm and worker heterogeneity. This paper analyzes the effects of trade on wage inequality in a framework that highlights the importance of: (i) intra-industry trade with fixed export costs; and (ii) positive assortative matching between high-skill workers and advanced technology firms. By creating new opportunities for firms with better technologies, trade liberalization increases the demand for skill and raises wage inequality. Unlike neoclassical trade theories based on differences between countries, in which trade moves inequality in opposite directions in different countries, intra-industry trade raises wage inequality

[43] As noted previously, the fixed cost f and the export cost f_x have equivalent effects on the matching function. It follows that if $f > 0$, then both the autarky and open economy distributions of worker wages, firm employment, firm revenue, and firm variable profits have approximate Pareto upper tails.

in all countries. In addition, because only the most productive firms that employ the best workers select into exporting, the impact of trade is strongest in the upper tail of the wage distribution, where it increases wage inequality within high-skill workers employed by exporters. The paper shows that these results hold regardless of whether firms receive stochastic technology draws, as in Melitz (2003), or technology is endogenous to firm-level R&D.

By demonstrating how labor assignment theory can be incorporated into a canonical heterogeneous firms model to endogenize both the quantity and the quality of labor hired, the paper develops a framework that enables an integrated treatment of worker and firm heterogeneity. It is hoped that this methodology will prove useful for analyzing labor markets across a wide range of issues where firm heterogeneity is acknowledged to play an important role. For example, the matching model could be used to quantify the effects of skill-biased technical change, to analyze how human capital accumulation and firm-level R&D interact to shape the long-run growth rate, or to study how business cycle fluctuations affect wages and employment for different types of workers.

Although the paper's predictions are broadly consistent with existing empirical findings, further work is required that tests the precise mechanisms identified in the paper. In particular, matched employer-employee datasets offer the opportunity to identify the channels through which trade affects firm-level wages. The functional form restrictions introduced in Section IVC could also be used as a starting point to calibrate the model to firm-level data and evaluate the effects of trade on wage inequality quantitatively. Moreover, it would be interesting to develop a dynamic version of the model for analyzing the short-term impact of trade by studying the transition between steady states. For example, if shifts in the matching function are brought about by firm entry and exit or by the reallocation of labor across firms, then trade may temporarily raise unemployment during the transition period. However, if changes in the matching function are driven by variation in firm-level R&D, then the principal short-term impact of trade will be on technology investment. Which of these channels is more important will depend on the costs of adjustment along each margin. Therefore, a multi-period model could be used both to shed light on the dynamics of trade integration and to quantify the empirical relevance of different adjustment costs. Finally, skill and wage dispersion within firms could be incorporated into the model by allowing for multi-product firms with technology variation across products. If trade liberalization reduces the number of products produced (Bernard, Redding, and Schott 2011; Mayer, Melitz, and Ottaviano 2012), then it may cause within- and between-firm wage inequality to move in opposite directions.

Appendix A: Proofs

PROOF OF PROPOSITION 1:

Define the following correspondence:

$$T : [\underline{\theta}, \bar{\theta}] \rightarrow [\underline{z}, \bar{z}],$$

$$T(\theta) = \{z : dL(\theta, z) > 0\}.$$

For each θ, T gives the set of technologies used by firms that employ workers with skill θ.

The first step in proving Proposition 1 is to show that T is a surjection. A sufficient condition for this to hold is that all firms have nonzero labor demand. Suppose that for some $z \in [\underline{z}, \bar{z}]$, $z \notin T(\theta') \ \forall \ \theta' \neq \theta$. Then, from (3), a technology z firm chooses employment $l(\theta; z)$ of skill θ workers to maximize variable profits

$$\pi(z) = A^{\frac{1}{\sigma}}\left[\Psi(\theta, z)\, l(\theta; z)\right]^{\frac{\sigma-1}{\sigma}} - w(\theta)\, l(\theta; z).$$

Since $\frac{\sigma-1}{\sigma} < 1$, optimal labor demand is positive and $z \in T(\theta)$. Therefore, T is surjective.

The second, and most important, step in the proof is to show that T is nondecreasing. Assume $z \in T(\theta)$. Suppose there exists $z_1 > z$ and $\theta_0 < \theta$ with $z_1 \in T(\theta_0)$. Then we must have

$$\begin{aligned}
\frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} y(z_1)^{-\frac{1}{\sigma}} \Psi(\theta, z_1) - w(\theta) &= w(\theta_0)\frac{\Psi(\theta, z_1)}{\Psi(\theta_0, z_1)} - w(\theta), \\
&> w(\theta_0)\frac{\Psi(\theta, z)}{\Psi(\theta_0, z)} - w(\theta), \\
&= \Psi(\theta, z)\left[\frac{w(\theta_0)}{\Psi(\theta_0, z)} - \frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} y(z)^{-\frac{1}{\sigma}}\right], \\
&\geq 0,
\end{aligned}$$

where the second line follows from the log supermodularity of Ψ and all other lines are implied by equation (4). This result contradicts profit maximization (4) implying that $z_1 \in T(\theta_0)$ is not possible. Similarly, we cannot have $z_0 < z$ and $\theta_1 > \theta$ with $z_0 \in T(\theta_1)$. Therefore, it follows that T is nondecreasing.

The third step is to show that T is single valued. Suppose T is multi-valued, implying that for some θ there exist $z, z_1 \in T(\theta)$ with $z_1 > z$. Since the technology distribution has support $[\underline{z}, \bar{z}]$, we must have that $\forall \ z' \in (z, z_1)$ there exist firms with technology z'. Then, because T is nondecreasing, it immediately follows that $\forall \ z' \in (z, z_1)$, $z' \in T(\theta')$ if and only if $\theta' = \theta$. However, because all firms have positive labor demand and there are no mass points in the skill distribution, this contradicts labor market clearing—we cannot have that a positive mass of firms all demand workers with the same skill. It follows that T is single valued and is a well-defined function.

Finally, similar reasoning can be combined with the assumption that there are no mass points in the technology distribution to show that T is an injection. Combining this result with the fact that T is nondecreasing implies that T is strictly increasing. This completes the proof.

PROOF OF PROPOSITION 2:

The profit maximization condition (4) evaluated at $z = T(\theta)$ implies

$$\frac{\sigma - 1}{\sigma} A^{\frac{1}{\sigma}} y^{-\frac{1}{\sigma}} = \frac{w(\theta)}{\Psi[\theta, T(\theta)]},$$

and substituting this expression back into (4) gives

$$\frac{\Psi[\theta + d\theta, T(\theta)]}{\Psi[\theta, T(\theta)]} \leq \frac{w(\theta + d\theta)}{w(\theta)}.$$

Similarly, we must have

$$\frac{w(\theta + d\theta)}{w(\theta)} \leq \frac{\Psi[\theta + d\theta, T(\theta + d\theta)]}{\Psi[\theta, T(\theta + d\theta)]},$$

and rearranging the two expressions above, we obtain

$$\begin{aligned}\frac{\Psi[\theta + d\theta, T(\theta)] - \Psi[\theta, T(\theta)]}{d\theta} &\leq \frac{\Psi[\theta, T(\theta)]}{w(\theta)} \frac{w(\theta + d\theta) - w(\theta)}{d\theta}, \\ &\leq \frac{\Psi[\theta, T(\theta)]}{\Psi[\theta, T(\theta + d\theta)]} \\ &\quad \times \frac{\Psi[\theta + d\theta, T(\theta + d\theta)] - \Psi[\theta, T(\theta + d\theta)]}{d\theta}.\end{aligned}$$

Now taking the limit as $d\theta \to 0$, and using the continuity of T and Ψ, implies that the wage function is differentiable and satisfies equation (11). This completes the proof.

PROOF OF PROPOSITION 3:

The result follows immediately from the discussion in the main text and Lemma 2 in Sampson (2011).

PROOF OF PROPOSITION 5:

To prove the proposition, I will show that $T(\theta) > T^a(\theta)\ \forall\ \theta \in (\underline{\theta}, \overline{\theta})$. The result then follows from Proposition 3.

Before beginning the proof, it is useful to make some observations and introduce some definitions. To simplify notation, define ϵ_C^D to be the elasticity of function D with respect to variable C. Thus, for example, the returns to skill $\frac{\theta w'(\theta)}{w(\theta)}$ can be written as $\epsilon_\theta^w(\theta)$.

Since L and M are continuously differentiable and Ψ and w are continuous, equation (10) implies that $T^{a\prime}$ is continuous on $[\underline{\theta}, \bar{\theta}]$, and equation (20) implies that T' is continuous at all points except $\hat{\theta} \equiv T^{-1}(z_x)$, where it has a discontinuous downward jump. The assumption that some, but not all, firms export implies $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$. For all $\theta \in [\underline{\theta}, \bar{\theta}]$, define

$$\tilde{T}'(\theta) = \frac{A + A^* \tau^{1-\sigma} I[T(\theta) \geq z_x]}{A + A^* \tau^{1-\sigma} I[T(\theta) > z_x]} T'(\theta).$$

$\tilde{T}'$ and T' are identical except at $\hat{\theta}$, and $\tilde{T}'(\hat{\theta})$ is equal to the limit of $T'(\theta)$ as $\theta \to \hat{\theta}$ from below. Note $\tilde{T}'(\hat{\theta}) > T'(\hat{\theta})$. We are now ready to start the proof. There are three steps.

Step 1: suppose there exists $\theta \in (\underline{\theta}, \bar{\theta})$ with $T^a(\theta) > T(\theta)$. Assumption 1 implies $T(\bar{\theta}) = T^a(\bar{\theta})$ and $T(\underline{\theta}) > T^a(\underline{\theta})$. As both T and T^a are continuous, it follows that there exist $\theta_0, \theta_1 \in [\underline{\theta}, \bar{\theta}]$, such that $\theta_0 < \theta_1$; $T(\theta_0) = T^a(\theta_0) = z_0$; $T(\theta_1) = T^a(\theta_1) = z_1$; and $T^a(\theta) > T(\theta)\ \forall\ \theta \in (\theta_0, \theta_1)$. In addition, we must have $T^{a\prime}(\theta_0) \geq T'(\theta_0)$ and $T^{a\prime}(\theta_1) \leq \tilde{T}'(\theta_1)$. Therefore, $\tilde{T}'(\theta_1) T^{a\prime}(\theta_0) \geq T^{a\prime}(\theta_1) T'(\theta_0)$. However, (10) and (20) together imply

$$\frac{\tilde{T}'(\theta_1)}{T^{a\prime}(\theta_1)} \frac{T^{a\prime}(\theta_0)}{T'(\theta_0)} = \frac{m(z_0) m^a(z_1)}{m(z_1) m^a(z_0)} \frac{A + A^* \tau^{1-\sigma} I[z_0 \geq z_x]}{A + A^* \tau^{1-\sigma} I[z_1 > z_x]} \left[\frac{w(\theta_1) w^a(\theta_0)}{w^a(\theta_1) w(\theta_0)}\right]^\sigma.$$

The first fraction on the right-hand side of this expression equals one by Assumption 1. The second fraction cannot exceed 1 because $z_0 < z_1$. Finally, the third fraction must be less than 1 because $T^a(\theta) > T(\theta)\ \forall\ \theta \in (\theta_0, \theta_1) \Rightarrow \epsilon_\theta^{w^a}(\theta) > \epsilon_\theta^w(\theta)\ \forall\ \theta \in (\theta_0, \theta_1) \Rightarrow \frac{w(\theta_1)}{w^a(\theta_1)} < \frac{w(\theta_0)}{w^a(\theta_0)}$. Therefore, we have $\tilde{T}'(\theta_1) T^{a\prime}(\theta_0) < T^{a\prime}(\theta_1) T'(\theta_0)$, which contradicts the result derived above. Consequently, it must be that $T(\theta) \geq T^a(\theta)\ \forall\ \theta \in (\underline{\theta}, \bar{\theta})$.

Step 2: suppose $T(\hat{\theta}) = T^a(\hat{\theta})$. Since $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ and $T(\theta) \geq T^a(\theta)\ \forall\ \theta \in (\underline{\theta}, \bar{\theta})$, the fact that T and T^a cannot cross at $\hat{\theta}$ implies $\tilde{T}'(\hat{\theta}) \leq T^{a\prime}(\hat{\theta}) \leq T'(\hat{\theta})$. However, the definition of $\tilde{T}'$ shows that $\tilde{T}'(\hat{\theta}) > T'(\hat{\theta})$, giving a contradiction. It follows that $T(\hat{\theta}) > T^a(\hat{\theta})$.

Step 3: suppose there exists $\theta_0 \in (\hat{\theta}, \bar{\theta})$ with $T(\theta_0) = T^a(\theta_0)$. Then, since T and T^a cannot cross at θ_0, we must have $T'(\theta_0) = T^{a\prime}(\theta_0)$. Equations (10) and (20) then imply

$$\text{(A1)} \qquad \frac{w^a(\theta_0)^\sigma}{A^a M^a m^a[T^a(\theta_0)]} = \frac{w(\theta_0)^\sigma}{(A + A^* \tau^{1-\sigma}) M m[T(\theta_0)]}.$$

Using (13) to substitute for $w^a(\theta)$ in (10) gives an integro-differential equation for $T^a(\theta)$. Integrating this equation between θ and θ_0 shows that $\forall\ \theta \in (\hat{\theta}, \theta_0]$, the autarky matching function $T^a(\theta)$ satisfies the following integral equation

$$T^a(\theta) = T^a(\theta_0) - \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \frac{w^a(\theta_0)^{\sigma}}{A^a M^a m^a\left[T^a(\theta_0)\right]}$$

$$\times \int_{\theta}^{\theta_0} \frac{m^a\left[T^a(\theta_0)\right]}{m^a\left[T^a(\tilde{\theta})\right]} \frac{L'(\tilde{\theta})}{\Psi\left[\tilde{\theta}, T^a(\tilde{\theta})\right]^{\sigma-1}} \exp\left[-\sigma \int_{\tilde{\theta}}^{\theta_0} \frac{1}{\Psi\left[s, T^a(s)\right]} \frac{\partial \Psi\left[s, T^a(s)\right]}{\partial s}\, ds\right] d\tilde{\theta}.$$

Similarly, integrating (20) gives an analogous integral equation for the open economy matching function

$$T(\theta) = T(\theta_0) - \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \frac{w(\theta_0)^{\sigma}}{(A + A^* \tau^{1-\sigma}) M m\left[T(\theta_0)\right]}$$

$$\times \int_{\theta}^{\theta_0} \frac{m\left[T(\theta_0)\right]}{m\left[T(\tilde{\theta})\right]} \frac{L'(\tilde{\theta})}{\Psi\left[\tilde{\theta}, T(\tilde{\theta})\right]^{\sigma-1}} \exp\left[-\sigma \int_{\tilde{\theta}}^{\theta_0} \frac{1}{\Psi\left[s, T(s)\right]} \frac{\partial \Psi\left[s, T(s)\right]}{\partial s}\, ds\right] d\tilde{\theta}.$$

Now observe that equation (A1), Assumption 1 and $T(\theta_0) = T^a(\theta_0)$ together imply that $T(\theta)$ and $T^a(\theta)$ satisfy the same integral equation. A standard, if somewhat lengthy, application of Banach's fixed point theorem can be used to show that this integral equation has a unique solution. It follows that $T(\theta) = T^a(\theta)\ \forall\ \theta \in (\hat{\theta}, \theta_0]$. However, since T and T^a are both continuous this contradicts $T(\hat{\theta}) > T^a(\hat{\theta})$. Therefore, we must have $T(\theta) > T^a(\theta)\ \forall\ \theta \in (\hat{\theta}, \bar{\theta})$.

Finally, a similar argument can be combined with the fact that $T(\underline{\theta}) > T^a(\underline{\theta})$ to show that $T(\theta) > T^a(\theta)\ \forall\ \theta \in (\underline{\theta}, \hat{\theta})$. This completes the proof.

PROOF OF PROPOSITION 6:

The proof consists of three steps. First, demonstrating that there is a unique autarky equilibrium. Second, characterizing how the autarky equilibrium depends on the fixed costs of entry and production. Third, showing that moving from autarky to an open economy where all firms export has the same effect on the matching function as increasing the fixed cost of production.

Step 1: To solve for the autarky equilibrium start by noting that the exit threshold $\underline{z}$ must satisfy $\pi(\underline{z}) = fP$, where $\pi(z)$ is given by (7). In addition, market clearing for the final good requires that aggregate expenditure equals the sum of labor income and firms' fixed cost payments

$$E = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta) + MP\left[f + \frac{f_e}{1 - G(\underline{z})}\right],$$

while free entry implies

$$f_e P = \int_{\underline{z}}^{\bar{z}} \pi(z)\, dG(z) + [1 - G(\underline{z})] P.$$

Combining these conditions with (6), (7), (21), and (22), substituting into (10) and using $M'[T(\theta)] = \frac{Mg[T(\theta)]}{1 - G(\underline{z})}$ gives

$$\text{(A2)} \qquad T'(\theta) = \frac{L'(\theta)}{g[T(\theta)]} \frac{w(\theta)^{\sigma}}{\Psi[\theta, T(\theta)]^{\sigma-1}} x,$$

where $x = X[\underline{z}, E, w(\theta)]$ is defined by

$$\text{(A3)} \qquad X[\underline{z}, E, w(\theta)] \equiv \frac{\sigma}{\sigma - 1} \frac{1}{E} \left[\frac{\Psi(\underline{\theta}, \underline{z})}{w(\underline{\theta})}\right]^{\sigma-1} \left[\frac{f_e}{f} + 1 - G(\underline{z})\right],$$

and

$$\text{(A4)} \qquad E = \frac{\sigma}{\sigma - 1} \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta).$$

Substituting (13) into (A2) gives an integro-differential equation for $T(\theta)$, which, by the same reasoning used in the proof of Proposition 5, has a unique solution for any given value of $x \geq 0$. From the solution $T(\theta)$, we know $\underline{z} = T(\underline{\theta})$, and we can use equations (13) and (A4) to solve for $w(\theta)$ and E, respectively. Thus, we can write $X[\underline{z}, E, w(\theta)]$ as a function of x. Equilibrium occurs when $x = X(x)$.

Let $T_i(\theta)$ be the solution when $x = x_i$, $i = 1, 2$. Then, if $x_2 > x_1$ it must be that $T_2(\theta) < T_1(\theta)\ \forall \theta \in [\underline{\theta}, \bar{\theta})$. To prove this claim, first note that $T_2(\bar{\theta}) = T_1(\bar{\theta}) = \bar{z}$, but $T_2'(\bar{\theta}) > T_1'(\bar{\theta})$. By differentiability of the matching function, it follows that $\exists \epsilon > 0$, such that $T_2(\theta) < T_1(\theta)\ \forall\ \theta \in (\bar{\theta} - \epsilon, \bar{\theta})$. Suppose the claim does not hold. Then $\tilde{\theta} \equiv \sup\{\theta \in [\underline{\theta}, \bar{\theta}) \mid T_2(\theta) \geq T_1(\theta)\}$ is well-defined and differentiability of the matching function implies $T_2(\tilde{\theta}) = T_1(\tilde{\theta})$ and $T_2'(\tilde{\theta}) \leq T_1(\tilde{\theta})$. In addition, because T_2 lies below T_1 on $(\tilde{\theta}, \bar{\theta})$ and $w_2(\bar{\theta}) = w_1(\bar{\theta}) = 1$, we have $w_2(\tilde{\theta}) > w_1(\tilde{\theta})$. Therefore, equation (A2) implies

$$\frac{T_2'(\tilde{\theta})}{T_1'(\tilde{\theta})} = \left(\frac{w_2(\tilde{\theta})}{w_1(\tilde{\theta})}\right)^{\sigma} \frac{x_2}{x_1} > 1,$$

which contradicts $T_2'(\tilde{\theta}) \leq T_1(\tilde{\theta})$. Hence, we must have $T_2(\theta) < T_1(\theta)\ \forall\ \theta \in [\underline{\theta}, \bar{\theta})$. It follows that raising x reduces $\underline{z}$, while increasing both $w(\theta)\ \forall\ \theta \in [\underline{\theta}, \bar{\theta})$ and E. Invoking Assumption 2 then implies that $X'(x) < 0$. Since $X(0) > 0$, this means that $x = X(x)$ has a unique solution. Given the equilibrium value of x, it is straightforward to check that there exist unique solutions for all other endogenous variables. Thus, there exists a unique autarky equilibrium.

Step 2: From (13) and (A2), observe that both the matching function and the wage function are independent of f and f_e for a given value of x. Therefore, (A3) implies that a decrease in $\frac{f_e}{f}$ shifts $X(x)$ downward and reduces the equilibrium value of x. Moreover, from step 1 we know that a lower x leads to an upward shift in the matching function for all $\theta \in [\underline{\theta}, \bar{\theta})$. Using Proposition 3, it follows that either an increase in the fixed production cost f or a decrease in the entry cost f_e leads to a rise in wage inequality over all workers.

Step 3: Consider an open economy where all firms export. Since both countries are symmetric, $A = A^*$, and using (18), it follows that the exit threshold must satisfy $\pi(\underline{z}) + \pi_x(\underline{z}) = (1 + \tau^{1-\sigma})\pi(\underline{z}) = (f + f_x)P$. Combining the open economy versions of the equilibrium conditions used in step 1 with the differential equation for the open economy matching function given in (20) shows that in the open economy, where all firms export, the equilibrium matching function continues to be the solution to equations (13), (A2), (A3), and (A4) provided that f is replaced by $f + f_x$ in (A3). It follows that moving from autarky to an open economy, where all firms export, is equivalent to an increase in f. From step 2, an increase in f leads to a rise in wage inequality over all workers. Moreover, provided all firms export both before and after the shock, an increase in f_x shifts the matching function upward on $[\underline{\theta}, \bar{\theta})$ and generates a rise in wage inequality over all workers, while variation in τ leaves the matching function unchanged. This completes the proof.

PROOF OF PROPOSITION 7:

Equation (24) defines the equilibrium matching function. Conditional on the matching function, equation (27) gives the wage function. It remains to solve for aggregate expenditure E, the price index P, the demand parameter A, the total mass of firms M, and the firm productivity distribution $m(z)$.

Substituting (6), (21), and (26) into the expression for the price index (22) gives

$$E = \sigma MP \int_{\underline{z}}^{\bar{z}} [f + \kappa f_e(z)]\, m(z)\, dz,$$

and combining this result with the market clearing condition (28) implies that aggregate expenditure E is given by (A4).

Taking the ratio of the two expressions for the wage function (26) and (27), then using (21) and the solution for E, gives

$$A = \left((\sigma - 1)\left(\frac{\sigma}{\sigma - 1}\right)^{\frac{\sigma(\sigma-1)-1}{\sigma-1}} \frac{f + \kappa f_e(\bar{z})}{\Psi(\bar{\theta}, \bar{z})^{\sigma-1}} \left[\int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta)\right]^{\frac{-1}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma-2}},$$

and we can then solve for P by substituting this expression and equation (A4) into (21). Finally, the labor market clearing condition (10) can be used to solve for M and $m(z)$. Thus, there is a unique closed economy equilibrium.

PROOF OF PROPOSITION 8:

Remember that we have defined ϵ_C^D to be the elasticity of function D with respect to variable C. Differentiating the ratio $\frac{w_x(\theta)}{w_d(\theta)}$ and using (24) and (31) gives

$$\begin{aligned}\frac{d}{d\theta}\left[\frac{w_x(\theta)}{w_d(\theta)}\right] &\propto \epsilon_\theta^\Psi[\theta, T_x(\theta)] - \epsilon_\theta^\Psi[\theta, T^a(\theta)],\\ &> 0,\end{aligned}$$

where the second line is implied by (14) and $T_x(\theta) > T^a(\theta)$. Therefore, $\frac{w_x(\theta)}{w_d(\theta)}$ is strictly increasing in θ. The proposition then follows immediately from the discussion in the main text and the fact that

$$\frac{d}{d\theta}\left[\frac{w(\theta)}{w^a(\theta)}\right] \propto \epsilon_\theta^w(\theta) - \epsilon_\theta^{w^a}(\theta).$$

PROOF OF PROPOSITION 9:

The main text solves for the equilibrium matching function $T(\theta)$, export threshold z_x and skill threshold θ_x. It remains to solve for the wage function $w(\theta)$, aggregate expenditure E, the price index P, the demand parameter A, the total mass of firms M, and the firm productivity distribution $m(z)$.

Given the skill threshold θ_x, equations (26) and (32), together with the numeraire condition $w(\bar{\theta}) = 1$, can be used to solve for the open economy wage function $w(\theta)$.

$$w(\theta) = \begin{cases}\left(\frac{f+\kappa f_e[T^a(\theta_x)]}{f+\kappa f_e[T^a(\theta)]}\,\frac{f+f_x+\kappa f_e[T(\bar{\theta})]}{f+f_x+\kappa f_e[T_x(\theta_x)]}\right)^{\frac{1}{\sigma-1}}\frac{\Psi[\theta, T^a(\theta)]}{\Psi[\theta_x, T^a(\theta_x)]}\,\frac{\Psi[\theta_x, T_x(\theta_x)]}{\Psi[\bar{\theta}, T(\bar{\theta})]} & \text{if } \theta \le \theta_x,\\ \left(\frac{f+f_x+\kappa f_e[T(\bar{\theta})]}{f+f_x+\kappa f_e[T_x(\theta)]}\right)^{\frac{1}{\sigma-1}}\frac{\Psi[\theta, T_x(\theta)]}{\Psi[\bar{\theta}, T(\bar{\theta})]} & \text{if } \theta \ge \theta_x.\end{cases}$$

The open economy version of the final good market clearing condition (28) is

$$E = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta)\, dL(\theta) + MPf + MP\int_{T(\underline{\theta})}^{T(\bar{\theta})}\left(\kappa f_e(z) + f_x I[z \ge T_x(\theta_x)]\right) m(z)\, dz,$$

and, since $p^*(z) = \tau p(z)$, and the two economies are symmetric, the price index is given by

$$P = \left[\int_{T(\underline{\theta})}^{T(\bar{\theta})}\left(1 + \tau^{1-\sigma} I[z \ge T_x(\theta_x)]\right) p(z)^{1-\sigma}\, dM(z)\right]^{\frac{1}{1-\sigma}}.$$

Combining these expressions for E and P with (6), (21), (26), and (32) shows that as in the closed economy, aggregate expenditure is given by (A4). Substituting the numeraire condition $w(\bar{\theta}) = 1$ into (32) gives one equation in the two unknowns

A and P. Combining (21) and (A4) gives a second equation, allowing us to solve for A and P. Finally, labor market clearing in the open economy implies (20) holds, which can be used to obtain M and $m(z)$. This completes the proof.

PROOF OF PROPOSITION 12:

Consider the closed economy. Solving (24) implies the matching function satisfies $T^a(\theta) \propto \theta$. Using (27), we then have $w^a(\theta) \propto \theta^{\frac{\sigma-1-\alpha}{\sigma-1}}$. Given these solutions for the matching function and the wage function, equations (5) and (7) imply

$$l^a\left[(T^a)^{-1}(z);\, z\right] \;\propto\; z^{\frac{\alpha\sigma+1-\sigma}{\sigma-1}}, \qquad R^a(z) \;\propto\; \pi^a(z) \;\propto\; z^{\alpha}, \tag{A5}$$

where $R^a(z)$ denotes the autarky revenue function. Thus, the matching, wage, employment, revenue, and variable profits functions are all power functions.

The next step is to characterize the firm technology distribution. From the closed economy labor market clearing condition (10), we have

$$\begin{aligned} m(z) \;&=\; \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\frac{1}{A}\,\frac{L'(\theta)}{M}\,\frac{1}{T'(\theta)}\,\frac{w(\theta)^{\sigma}}{\Psi(\theta, z)^{\sigma-1}}, \\ &\propto\; L'(\theta)\; z^{\frac{\sigma-1-\alpha\sigma}{\sigma-1}}, \end{aligned} \tag{A6}$$

where $\theta = (T^a)^{-1}(z)$, and the second line follows from using Assumption 3 and the expressions for the matching function and the wage function given above. Now suppose worker skill has a truncated Pareto distribution on $[\underline{\theta}, \overline{\theta}\,]$ with shape parameter $\beta > 0$. This implies

$$L(\theta) \;=\; \frac{\underline{\theta}^{-\beta} - \theta^{-\beta}}{\underline{\theta}^{-\beta} - \overline{\theta}^{-\beta}}\, L.$$

Differentiating this expression with respect to θ and substituting the result into (A6), and then using the matching function to eliminate θ, shows that firm technology has a truncated Pareto distribution with shape parameter $\beta + \frac{\alpha\sigma}{\sigma-1} - 1$.

Since any power function of a truncated Pareto random variable also has a truncated Pareto distribution, it follows that wages, employment, revenue, and variable profits all have truncated Pareto distributions. By transforming the skill distribution and technology distribution, it is easy to check that the wage distribution has shape parameter $\frac{\beta(\sigma-1)}{\sigma-1-\alpha}$, the employment distribution has shape parameter $\frac{\beta(\sigma-1)}{\alpha\sigma+1-\sigma} + 1$, and both the revenue and variable profit distributions have shape parameter $\frac{\beta}{\alpha} + \frac{\alpha\sigma+1-\sigma}{\alpha(\sigma-1)}$.

Solving the open-economy model is slightly less straightforward. From (31) the open economy matching function satisfies

$$T(\theta) \;=\; \left[\frac{\alpha}{\sigma-1-\alpha+(\sigma-1)f_x I[\theta \geq \theta_x]\; T(\theta)^{-\alpha}}\right]^{\frac{\gamma}{\gamma-1}}\theta.$$

This equation does not give a closed form expression for the matching function. However, suppose the upper bound of the skill distribution $\bar{\theta}$ is arbitrarily large. Then, as $\theta \to \infty$, we must have $T(\theta)^{-\alpha} \to 0$. Consequently, the closed economy result $T(\theta) \propto \theta$ holds asymptotically as $\theta \to \infty$ in the open economy. Similarly, it can be shown that in the open economy $w(\theta) \propto \theta^{\frac{\sigma-1-\alpha}{\sigma-1}}$ holds asymptotically as $\theta \to \infty$, and that the expressions for employment, revenue, and variable profits in (A5) and the technology distribution in (A6) hold asymptotically as $z \to \infty$. Consequently, if worker skill has a truncated Pareto distribution then in the open economy the distribution function of worker wages converges asymptotically to a power function as θ becomes large and the distribution functions of firm employment, revenue, and variable profits converge to power functions as z becomes large. This completes the proof.

Appendix B: Labor-Based Fixed Costs

In the main body of the paper I assume that all fixed costs are denominated in units of the final good, implying that labor is not used in fixed production. Suppose instead that fixed costs are denominated in units of firm output.[44] Under this assumption firms must employ workers to meet their fixed costs in addition to the workers used in variable production. However, it is relatively straightforward to show that this modification does not affect either the main results or the tractability of the technology choice model. In particular, Propositions 7–12 continue to hold. In the interest of brevity, I will not present complete solutions of the closed and open economy models under labor-based fixed costs. Instead, I will make four observations that demonstrate why including labor-based fixed costs does not substantively change the model.

First, and most important, each firm chooses to employ workers with the same skill level in both fixed and variable production. To obtain this result note that if a firm with technology z employs workers with skill θ in fixed production, then its unit cost function for fixed production is $\frac{w(\theta)}{\Psi(\theta, z)}$. Section IC shows that, when hiring workers for variable production, firms choose θ to minimize this unit cost function. It immediately follows that each firm will choose the same skill level for its fixed production workforce as for its variable production workforce.[45] Since there is no within-firm heterogeneity in worker skill, the log supermodularity of Ψ continues to imply positive assortative matching between worker skill and firm technology, and unit cost minimization ensures that equation (11) still defines the relationship between the wage function and the matching function.

Second, equations (24) and (31), which define the matching functions for non-exporters and exporters, respectively, are unchanged, except that σ is replaced by

[44] This is equivalent to assuming that fixed costs are denominated in terms of a firm-specific investment good, where labor productivity in investment good production is given by Ψ.

[45] Under stochastic technology, if firms face fixed costs of production and exporting that are denominated in units of firm output, then cost minimization again implies that firms will employ workers with the same skill level in both fixed and variable production. However, requiring that the entry cost is paid using labor is problematic because the firm's technology is unknown at the time of entry.

$\sigma + 1$. To show that this is true for nonexporters, start by observing that with labor-based fixed costs, the total profit function is

$$\Pi(\theta, z) = \frac{1}{\sigma - 1}\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} A\left[\frac{\Psi(\theta, z)}{w(\theta)}\right]^{\sigma - 1} - [f + \kappa f_e(z)]\frac{w(\theta)}{\Psi(\theta, z)}.$$

Combining the first-order condition from choosing z to maximize profits with the free entry condition $\Pi(\theta, z) = 0$ gives an equation identical to (24), except that σ is replaced by $\sigma + 1$ on the left-hand side.[46] Identical logic applies for exporters. Given these results, the properties of T^a and T are unaffected by the introduction of labor-based fixed costs.

Third, free entry implies that the wage function is given by

$$w(\theta) = \frac{\sigma - 1}{\sigma}\left[\frac{A}{\sigma - 1}\frac{1 + \tau^{1-\sigma} I[\theta \geq \theta_x]}{f + f_x I[\theta \geq \theta_x] + \kappa f_e[T(\theta)]}\right]^{\frac{1}{\sigma}} \Psi[\theta, T(\theta)].$$

Consequently, a fall in either τ or f_x shifts $w_x(\theta)$ upward relative to $w_d(\theta)$, implying that the skill threshold θ_x declines. In addition, equation (33), which defines the export threshold z_x, continues to hold. Therefore, declines in trade costs still lead to reductions in θ_x and z_x when there are labor-based fixed costs.

Fourth, when labor is required for fixed production firm-level labor demand is

$$\begin{aligned} l(\theta, z) + l_x(\theta, z) &= \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} A(1 + \tau^{1-\sigma} I[z \geq z_x])\frac{\Psi(\theta, z)^{\sigma - 1}}{w(\theta)^{\sigma}} \\ &\quad + \frac{f + f_x I[z \geq z_x] + \kappa f_e(z)}{\Psi(\theta, z)}, \\ &= \sigma\frac{f + f_x I[z \geq z_x] + \kappa f_e(z)}{\Psi(\theta, z)}, \end{aligned}$$

where $\theta = T^{-1}(z)$. The importance of this observation is that employment in fixed production and employment in variable production are both proportional to $\frac{f + f_x I[z \geq z_x] + \kappa f_e(z)}{\Psi(\theta, z)}$. This property of the model with labor-based fixed costs ensures that the labor market clearing condition remains tractable.

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[46] Similarly, the second-order condition analogous to (25) can be obtained by replacing σ with $\sigma + 1$ in (25).

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