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A mathematical introduction to general relativity. (English) Zbl 1469.83001

Singapore: World Scientific (ISBN 978-981-12-4377-6/hbk; 978-981-12-4379-0/ebook). xvi, 483 p. (2022).

This book is a genuine mathematical introduction to general relativity theory in that it is thorough, rigorous and mathematically detailed with full proofs given and deals with the physics very generally and elegantly. It is, in my opinion, an excellent text. The first eleven chapters constitute the mathematical part and deal with manifolds, vectors, tensors and their associated fields, metrics, connections (including parallel transport and geodesics) and curvature. (The author, however, only allows a path to be called a geodesic if its parameter is affine which, whilst not uncommon for geometers, is not usually the case for mathematical physicists.) The approach is detailed and carefully explained and many examples are provided both as worked examples in the body of the text and also as exercises for the reader, the latter coming with full solutions at the end of the book. Intuition is appealed to as an aid to the reader, as are clear and instructive diagrams, but at all times the approach is rigorous. All standard aspects of basic differential geometry are dealt with and the author takes much trouble to present them in what seems to be just the right amount of detail. This part of the book also introduces Lorentz manifolds and includes examples on Minkowski space, the Schwarzschild and the Friedmann-Robertson-Walker-Lemaitre (FRWL) metrics and many others. Time orientation, the twin paradox, red shift, spacetime diagrams, etc, are all covered here (and returned to later in the section dealing with physics). That part of the text on connections and curvature deals with light bending, geodesic deviation, tidal forces and Jacobi fields in a novel way. The final five chapters cover the physics. It starts with Minkowski spacetime and includes special relativistic mechanics and electromagnetic field theory in a pleasing and elegant way. Then gravity is introduced first through the ideas of Newtonian theory and secondly, geometrically, as is appropriate in general relativity theory. Einstein's field equations are discussed thoroughly as are the common types of energy momentum tensors. This leads to a study of the Schwarzschild metric, perihelion precession, event horizons, the Kruskal extension and a detailed description of black holes. There are brief mentions of the Kerr, Godel and Reissner-Nordstrom metrics and plane wave spacetimes. The final chapter covers cosmology and gives the FRWL metrics in detail as well as some of their special cases. The mathematical isotropy in the FRWL metrics is dealt with by a brief digression into the sectional curvature function and imposing constant sectional curvature on the cosmic space sections.

The author writes with authority, clarity and care and this book should become a standard reference for general relativity and the associated differential geometry. It is welcomed as a major addition to the literature on this subject.

Reviewer: Graham S. Hall (Aberdeen)

MSC:

83-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to relativity and gravitational theory

Cited in **1** Review

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