Appendix to "The Decision to Move House and Aggregate Housing-Market Dynamics"

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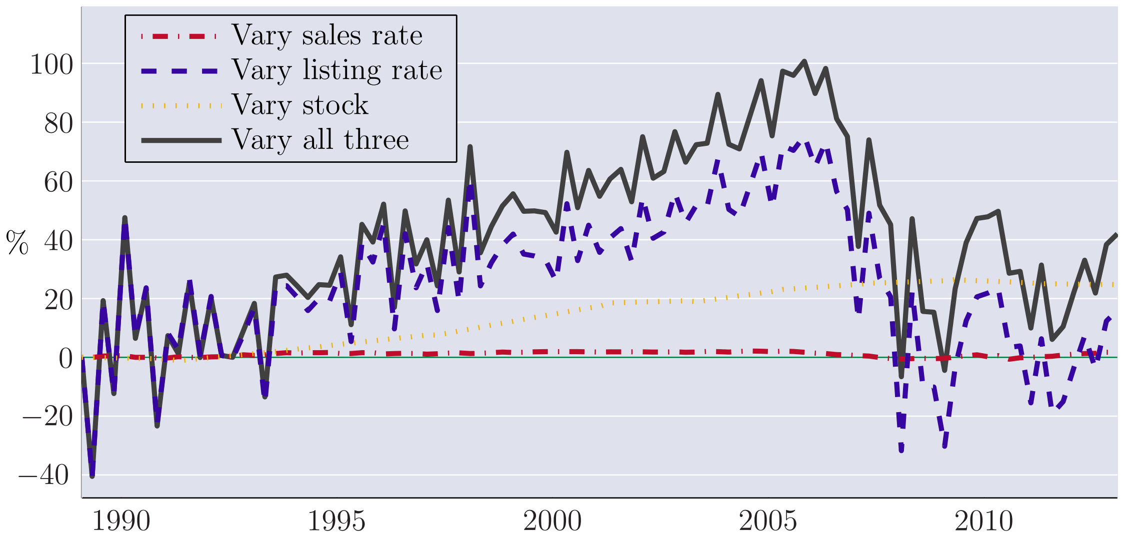
A Appendices

A.1 Measurement of transactions and inventories

The transactions and inventories data from NAR include all existing single-family homes, so homes that are rented out are counted in this data. An estimate of the fraction of single-family homes that are not rented out can be computed from the AHS. This fraction is around 78% on average over the period 1989–2013. If the fraction were constant over time, the counterfactuals presented in section 2.2 would be unaffected and the only change would be to the average level of the listing rate. The effect on the average level of the listing rate would change the implied average time between moves from 15 years to 19.4 years.

However, the data show some changes over time in the fraction of homes that are not rented out. The fraction rises from 77% to 80% during the boom period, and then falls to 75% by 2013. A simple robustness check is to scale the NAR data on transactions and inventories by the fraction of non-rented homes and then recompute the counterfactuals. The results are shown in Figure 11, which are almost identical to those in Figure 2.

Figure 11: *Actual and counterfactual transactions (adjusted for rented homes)*



Notes: The series are reported as log differences relative to their initial values.

A.2 Withdrawals and relistings

Consider the framework introduced in section 2.3. Suppose that the net listing rate $n_t = N_t/(K_t - U_t)$ is equal to a constant n. The sales rate is s, so $S_t = sU_t$, and the total housing stock grows at rate g, that is, $\dot{K}_t/K_t = g$. This implies that equation (1) will hold as in section 2.1 and that the fraction of houses for sale $u_t = U_t/K_t$ will converge to its steady state $u = n/(n + s + g)$.

Let $l_t = L_t/K_t$ denote the stock of houses that have failed to sell but might be relisted relative to the stock of all houses. Using the formula for $\dot{L}_t$, the implied law of motion for l_t is:

$$\dot{l}_t = wu_t - (\rho + \alpha + m + g)l_t.$$

Given that u_t is equal to its steady-state value u, there is a steady state l for l_t:

$$l = \frac{wu}{\rho + \alpha + m + g}, \tag{A.1}$$

and convergence to this steady state takes place at rate $\rho + \alpha + m + g$. Since this rate is strictly greater than the sum of the relisting rate ρ and the rate α at which homeowners give up on attempting a future sale, convergence to the steady state is presumed to be sufficiently fast that l_t is set equal to l in what follows, just as u_t converges quickly enough to u to set $u_t = u$.[1]

Dividing both sides of the accounting identity for net listings by $K_t - U_t$ implies:

$$n = m - w\frac{u_t}{1-u_t} + \rho\frac{l_t}{1-u_t}.$$

If u_t and l_t have reached their steady-state values u and l then the net listing rate n is indeed constant as supposed earlier:

$$n = m - w\frac{u}{1-u} + \rho\frac{l}{1-u},$$

and noting $u/(1-u) = n/(s+g)$ and using the expression for l from (A.1):

$$n = m - \frac{wn}{s+g}\left(1 - \frac{\rho}{\rho+\alpha+m+g}\right) = m - n\frac{w}{s+g}\left(\frac{\alpha+m+g}{\rho+\alpha+m+g}\right).$$

Divide numerator and denominator of the term in parentheses by $\rho+\alpha+m$, and numerator and denominator of its coefficient by s:

$$n = m - \frac{1}{1+\frac{g}{s}}\frac{w}{s}\left(\frac{\frac{\alpha+m}{\rho+\alpha+m} + \frac{g}{\rho+\alpha+m}}{1+\frac{g}{\rho+\alpha+m}}\right)n. \qquad \text{[A.2]}$$

The formulas for the eventual fraction of withdrawals ϕ and the eventual fraction of relistings β imply that:

$$\frac{\phi}{1-\phi} = \frac{w}{s}, \quad \text{and } 1-\beta = \frac{\alpha+m}{\rho+\alpha+m}.$$

These expressions can be substituted into (A.2) to deduce:

$$n = m - \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-\beta+\frac{g}{\rho+\alpha+m}}{1+\frac{g}{\rho+\alpha+m}}\right)\left(\frac{1}{1+\frac{g}{s}}\right)n. \qquad \text{[A.3]}$$

This is the exact link between the moving rate m and the net listing rate n. In addition to ϕ and β, it depends on the sales rate s, the growth rate g, the relisting rate ρ, and the abandonment rate α.

If the growth rate of the total housing stock g is small in relation to the sales rate s, and the sum of the relisting, abandonment, and moving rates $\rho + \alpha + m$ then the terms g/s and $g/(\rho+\alpha+m)$ are negligible. The equation (A.3) linking n and m can then be well approximated by:

$$n \approx m - \frac{\phi(1-\beta)}{(1-\phi)}n.$$

Collecting terms in n on one side, this equation simplifies to:

$$n \approx \frac{m}{1+\frac{\phi(1-\beta)}{1-\phi}},$$

which confirms equation (9) in section 2.3. Given knowledge of the eventual fraction of withdrawals ϕ and the eventual fraction of relistings β, the moving rate m can be calculated from the net listing rate n, or vice versa, the net listing rate can be calculated from the moving rate.

[1] Confirming this assumption would require an empirical measure of the abandonment rate α.

A.3 Value functions and thresholds

Moving and transaction thresholds

The value functions $H_t(\epsilon)$ and J_t and the thresholds x_t and y_t satisfy the equations (14), (15), (17), and (18). No other endogenous variables appear in these equations. Given constant parameters, there is a time-invariant solution $H_t(\epsilon) = H(\epsilon)$, $J_t = J$, $x_t = x$, and $y_t = y$. The time-invariant equations are:

$$rH(\epsilon) = \epsilon\xi - D + a\left(\max\{H(\delta\epsilon), J\} - H(\epsilon)\right); \tag{A.4}$$

$$H(x) = J; \tag{A.5}$$

$$rJ = -F - D + v\int_y (H(\epsilon) - J - C)\mathrm{d}G(\epsilon); \tag{A.6}$$

$$H(y) = J + C. \tag{A.7}$$

Attention is restricted to parameters where the solution will satisfy $\delta y < x$.

Evaluating (A.4) at $\epsilon = x$, noting that $\delta < 1$ and $H(\epsilon)$ is increasing in ϵ:

$$rH(x) = \xi x - D + a(J - H(x)).$$

Since $H(x) = J$ (equation A.5), it follows that:

$$J = H(x) = \frac{\xi x - D}{r}. \tag{A.8}$$

Next, evaluate (A.4) at $\epsilon = y$. With the restriction $\delta y < x$, it follows that $H(\delta y) < H(x) = J$, and hence:

$$rH(y) = \xi y - D + a(J - H(y)).$$

Collecting terms in $H(y)$ on one side and substituting the expression for J from (A.8):

$$(r + a)H(y) = \xi y - D + \frac{a}{r}(\xi x - D) = \xi(y - x) + \left(1 + \frac{a}{r}\right)(\xi x - D),$$

and thus $H(y)$ is given by:

$$H(y) = \frac{\xi x - D}{r} + \frac{\xi(y - x)}{r + a}. \tag{A.9}$$

Combining the equation above with (A.7) and (A.8), it can be seen that the thresholds y and x must be related as follows:

$$y - x = \frac{(r + a)C}{\xi}. \tag{A.10}$$

Using the expression for the Pareto distribution function (10) and using (A.7) to note $H(\epsilon) - J - C = H(\epsilon) - H(y)$, the Bellman equation (A.6) can be written as:

$$rJ = -F - D + vy^{-\lambda}\int_{\epsilon=y}^{\infty} \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)}(H(\epsilon) - H(y))\mathrm{d}\epsilon, \tag{A.11}$$

which assumes $y > 1$. In solving this equation it is helpful to define the following function $\Psi(z)$ for all $z \le y$:

$$\Psi(z) \equiv \int_{\epsilon=z}^{\infty} \frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(H(\epsilon) - H(z))\mathrm{d}\epsilon. \tag{A.12}$$

Since $\delta y < x$ is assumed and $z \le y$, it follows that $\delta z < x$, and thus $H(\delta z) < H(x) = J$. Equation (A.4) evaluated at $\epsilon = z$ therefore implies:

$$rH(z) = \xi z - D + a(J - H(z)).$$

Subtracting this equation from (A.4) evaluated at a general value of ϵ leads to:

$$r(H(\epsilon) - H(z)) = \xi(\epsilon - z) + a\left(\max\{H(\delta\epsilon), J\} - H(\epsilon)\right) - a(J - H(z))$$
$$= \xi(\epsilon - z) - a(H(\epsilon) - H(z)) + a\max\{H(\delta\epsilon) - J, 0\}.$$

Noting that $J = H(x)$ and solving for $H(\epsilon) - H(z)$:

$$H(\epsilon) - H(z) = \frac{\xi}{r+a}(\epsilon - z) + \frac{a}{r+a}\max\{H(\delta\epsilon) - H(x), 0\}. \qquad [\text{A.13}]$$

The equation above can be substituted into (A.12) to deduce:

$$\Psi(z) = \frac{\xi}{r+a}\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(\epsilon - z)\mathrm{d}\epsilon + \frac{a}{r+a}\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}\max\{H(\delta\epsilon) - H(x), 0\}\mathrm{d}\epsilon. \quad [\text{A.14}]$$

First, observe that:

$$\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(\epsilon - z)\mathrm{d}\epsilon = \frac{z}{\lambda - 1}. \qquad [\text{A.15}]$$

Next, make the change of variable $\epsilon' = \delta\epsilon$ in the second integral in (A.14) to deduce:

$$\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}\max\{H(\delta\epsilon) - H(x), 0\}\mathrm{d}\epsilon = \int_{\epsilon'=\delta z}^{\infty}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}\max\{H(\epsilon') - H(x), 0\}\mathrm{d}\epsilon'$$
$$= \int_{\epsilon'=\delta z}^{x}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}0\mathrm{d}\epsilon' + \int_{\epsilon'=x}^{\infty}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}(H(\epsilon') - H(x))\mathrm{d}\epsilon'$$
$$= \left(\frac{\delta z}{x}\right)^{\lambda}\int_{\epsilon=x}^{\infty}\frac{\lambda}{x}\left(\frac{\epsilon}{x}\right)^{-(\lambda+1)}(H(\epsilon) - H(z))\mathrm{d}\epsilon = \left(\frac{\delta z}{x}\right)^{\lambda}\Psi(x),$$

where the second line uses $\delta z < x$ (as $z \leq y$ and $\delta y < x$) and $H(\epsilon') < H(x)$ for $\epsilon' < x$, and the final line uses the definition (A.12). Putting the equation above together with (A.14) and (A.15) yields the following for all $z \leq y$:

$$\Psi(z) = \frac{\xi z}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\left(\frac{\delta z}{x}\right)^{\lambda}\Psi(x). \qquad [\text{A.16}]$$

Evaluating this expression at $z = x$ (with $x < y$):

$$\Psi(x) = \frac{\xi x}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\delta^{\lambda}\Psi(x),$$

and hence $\Psi(x)$ is given by:

$$\Psi(x) = \frac{\xi x}{(r + a(1 - \delta^{\lambda}))(\lambda - 1)}. \qquad [\text{A.17}]$$

Next, evaluating (A.16) at $z = y$ and using (A.17) to substitute for $\Psi(x)$:

$$\Psi(y) = \frac{\xi y}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\left(\frac{\delta z}{x}\right)^{\lambda}\left(\frac{\xi x}{(r + a(1 - \delta^{\lambda}))(\lambda - 1)}\right),$$

and simplifying this equation yields the following expression for $\Psi(y)$:

$$\Psi(y) = \frac{\xi}{(r+a)(\lambda-1)} \left(y + \frac{a\delta^{\lambda} y^{\lambda} x^{1-\lambda}}{r + a(1-\delta^{\lambda})} \right). \quad \text{[A.18]}$$

Using the definition (A.12), equation (A.11) can be written in terms of $\Psi(y)$:

$$rJ = -F - D + vy^{-\lambda}\Psi(y),$$

and substituting from (A.8) and (A.18) yields:

$$\xi x - D = -F - D + vy^{-\lambda} \left(\frac{\xi}{(r+a)(\lambda-1)} \left(y + \frac{a\delta^{\lambda} y^{\lambda} x^{1-\lambda}}{r + a(1-\delta^{\lambda})} \right) \right).$$

This equation can be simplified as follows:

$$x + \frac{F}{\xi} = \frac{v}{(\lambda-1)(r+a)} \left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r + a(1-\delta^{\lambda})} x^{1-\lambda} \right). \quad \text{[A.19]}$$

The two equations (A.10) and (A.19) confirm (26) and (29) given in the main text. These can be solved for the thresholds x and y.

Existence and uniqueness

By using equation (A.10) to replace x with a linear function of y, the equilibrium threshold y is the solution of the equation:

$$\mathcal{I}(y) \equiv \frac{v}{(\lambda-1)(r+a)} \left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r + a(1-\delta^{\lambda})} \left(y - \frac{(r+a)C}{\xi} \right)^{1-\lambda} \right) - y + \frac{(r+a)C}{\xi} - \frac{F}{\xi} = 0. \quad \text{[A.20]}$$

It can be seen immediately (since $\lambda > 1$) that $\mathcal{I}'(y) < 0$, so any solution that exists is unique. A valid solution must satisfy $x > 0$, $y > 1$, and $\delta y < x$. Using equation (A.10), the inequality $\delta y < x$ is equivalent to:

$$\delta y < y - \frac{(r+a)C}{\xi},$$

which is in turn equivalent to:

$$y > \frac{(r+a)C}{(1-\delta)\xi}.$$

Thus, to satisfy $y > 1$ and $\delta y < x$, the equilibrium must feature:

$$y > \max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\}. \quad \text{[A.21]}$$

Observe that $\lim_{y\to\infty} \mathcal{I}(y) = -\infty$ (using A.20 and $\lambda > 1$), so an equilibrium satisfying (A.21) exists if and only if:

$$\mathcal{I}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} \right) > 0. \quad \text{[A.22]}$$

If the condition (A.21) is satisfied then by using (A.10):

$$x > \max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi} > \frac{(r+a)C}{(1-\delta)\xi} - \frac{(r+a)C}{\xi} = \frac{\delta(r+a)C}{(1-\delta)\xi} > 0,$$

confirming that $x > 0$ must hold. Therefore, (A.22) is necessary and sufficient for the existence of a unique equilibrium satisfying all the required conditions. Using equation (A.20), (A.22) is equivalent to:

$$\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\}^{1-\lambda} + \frac{a\delta^\lambda}{r+a(1-\delta^\lambda)}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi}\right)^{1-\lambda} \\ - \frac{(\lambda-1)(r+a)}{v}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi} + \frac{F}{\xi}\right) > 0. \quad \text{[A.23]}$$

Surplus and selling rate

Given x and y, the value functions J, $H(x)$, and $H(y)$ can be obtained from (A.8) and (A.9). The average surplus can be found by combining (A.8) and (A.11) to deduce:

$$\int_{\epsilon=y}^{\infty} \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)} (H(\epsilon) - H(y))\mathrm{d}\epsilon = \frac{y^\lambda}{v}(\xi x + F). \quad \text{[A.24]}$$

Given x and y, the probability π that a viewing leads to a sale, and the expected number of viewings before a sale $V_{\mathrm{s}} = 1/\pi$ are:

$$\pi = y^{-\lambda}, \quad \text{and } V_{\mathrm{s}} = y^\lambda. \quad \text{[A.25]}$$

The selling rate s and the expected time-to-sell $T_{\mathrm{s}} = 1/s$ are given by:

$$s = vy^{-\lambda}, \quad \text{and } T_{\mathrm{s}} = \frac{y^\lambda}{v}. \quad \text{[A.26]}$$

A.4 Prices

Nash bargaining

The price $p_t(\epsilon)$ is determined by combining the Nash bargaining solution $\omega\Sigma_{\mathrm{b},t}(\epsilon) = (1-\omega)\Sigma_{\mathrm{u},t}(\epsilon)$ with the expressions for the buyer and seller surpluses in (11):

$$\omega(H_t(\epsilon) - p_t(\epsilon) - C_{\mathrm{b}} - B_t) = (1-\omega)(p_t(\epsilon) - C_{\mathrm{u}} - U_t),$$

from which it follows that:

$$p_t(\epsilon) = \omega H_t(\epsilon) + (1-\omega)C_{\mathrm{u}} - \omega C_{\mathrm{b}} + ((1-\omega)U_t - \omega B_t). \quad \text{[A.27]}$$

The surplus-splitting condition implies $\Sigma_{\mathrm{b},t}(\epsilon) = (1-\omega)\Sigma_t(\epsilon)$ and $\Sigma_{\mathrm{u},t}(\epsilon) = \omega\Sigma_t(\epsilon)$, with $\Sigma_t(\epsilon) = \Sigma_{\mathrm{b},t}(\epsilon) + \Sigma_{\mathrm{u},t}(\epsilon)$ being the total surplus from (28). The Bellman equations in (12) can thus be written as:

$$rB_t = -F + (1-\omega)v\int_{y_t} \Sigma_t(\epsilon)\mathrm{d}G(\epsilon) + \dot{B}_t, \quad \text{and } rU_t = -D + \omega v\int_{y_t} \Sigma_t(\epsilon)\mathrm{d}G(\epsilon) + \dot{U}_t,$$

and a multiple ω of the first equation can be subtracted from a multiple $1-\omega$ of the second equation to deduce:

$$r((1-\omega)U_t - \omega B_t) = \omega F - (1-\omega)D + ((1-\omega)\dot{U}_t - \omega\dot{B}_t).$$

The stationary solution of this equation is:

$$(1-\omega)U_t - \omega B_t = \frac{\omega F - (1-\omega)D}{r},$$

and by substituting this into (A.27):

$$p_t(\epsilon) = \omega H_t(\epsilon) + (1-\omega)C_{\mathrm{u}} - \omega C_{\mathrm{b}} + \frac{\omega F - (1-\omega)D}{r}. \tag{A.28}$$

Integrating this equation over the distribution of new match quality yields equation (16) for the average transaction price.

Average transactions price

In an equilibrium where the moving and transaction thresholds x_t and y_t are constant over time, the value function $H_t(\epsilon)$ is equal to the time-invariant function $H(\epsilon)$. This means that prices $p_t(\epsilon) = p(\epsilon)$ are also time invariant. Using the Pareto distribution function (10) and equation (16), the average price is:

$$P = \omega \int_y \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)} H(\epsilon)\mathrm{d}\epsilon + (1-\omega)C_{\mathrm{u}} - \omega C_{\mathrm{b}} + \frac{\omega F - (1-\omega)D}{r}.$$

By using equation (14) and (A.8), the above can be written as:

$$P = \omega \int_y \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)} (H(\epsilon) - H(y))\mathrm{d}\epsilon + \omega\left(\frac{\xi x - D}{r} + C\right) + \frac{\omega F - (1-\omega)D}{r} + (1-\omega)C_{\mathrm{u}} - \omega C_{\mathrm{b}},$$

and substituting from (A.24) yields:

$$P = \omega\frac{y^\lambda}{v}(\xi x + F) + \omega C + \frac{\omega \xi x}{r} + (1-\omega)C_{\mathrm{u}} - \omega C_{\mathrm{b}} + \frac{\omega F - D}{r}.$$

Simplifying and using $C = C_{\mathrm{b}} + C_{\mathrm{u}}$, the following expression for the average price is obtained:

$$P = C_{\mathrm{u}} - \frac{D}{r} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)(\xi x + F),$$

which confirms the formula in (34). With the definition of $\kappa = C_{\mathrm{u}}/C$, this equation can also be written as:

$$P = \kappa C - \frac{D}{r} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)(\xi x + F). \tag{A.29}$$

A.5 Stocks and flows

The moving rate

The formula (21) for the moving rate can also be given in terms of inflows $N_t = n_t(1-u_t)$, where u_t is the stock of unsold houses:

$$N_t = a(1-u_t) - a\delta^\lambda x_t^{-\lambda} v \int_{\tau \to -\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau. \tag{A.30}$$

The first term $a(1-u_t)$ is the quantity of existing matches that receive a shock (arrival rate a). The second term is the quantity of existing matches that receive a shock now, but decide not to move. The difference between these two numbers (under the assumption that only those who receive a shock make a moving decision) gives inflows N_t.

Now consider the derivation of the second term in (A.30). The distribution of existing matches (measure $1-u_t$) can be partitioned into vintages τ (when matches formed) and the number k of previous shocks that have been received. At time τ, a quantity u_τ of houses were for sale, and viewings arrived at rate

v. Viewings were draws of match quality ϵ from a Pareto$(1,\lambda)$ distribution, and those draws with $\epsilon \geq y_\tau$ formed new matches, truncating the distribution at y_τ. In the interval between τ and t, those matches that have received k shocks now have match quality $\delta^k\epsilon$. Some of these matches will have been destroyed as a result of these shocks, truncating the distribution of surviving match quality. Because the distribution of initial match quality is a Pareto distribution, these truncations also result in Pareto distributions with the same shape parameter λ.

Consider the matches of vintage τ. All of these were originally from a Pareto distribution truncated at $\epsilon \geq y_\tau$. Subsequently, depending on the arrival of idiosyncratic shocks (both timing and number), this distribution may have been truncated further. Let z denote the last truncation point in terms of the original match quality ϵ (at the time of the viewing). This is $z = y_\tau$ if no shocks have been received, or $z = \delta^{-k}x_T$ if k shocks have been received and the last one occurred at time T when the moving threshold was x_T. Conditional on this last truncation point z, it is shown below that the measure of surviving matches is $z^{-\lambda}vu_\tau$. Furthermore, the original match quality of these surviving matches must be a Pareto(z,λ) distribution.

Now consider the distribution of the number of previous shocks j between τ and t. Given the Poisson arrival rate a, k has a Poisson distribution, so the probability of j is $e^{-a(t-\tau)}(a(t-\tau))^j/j!$. If a shock arrives at time t, matches of current quality greater than x_t survive. If these have received j shocks earlier, this means the truncation threshold in terms of original match quality ϵ is $\epsilon \geq \delta^{-(j+1)}x_t$. Of these matches that have accumulated j earlier shocks, suppose last relevant truncation threshold (in terms of original match quality) was z (this will vary over those matches even with the same number of shocks because the timing might be different), so the distribution of surviving matches in terms of their original match quality is Pareto(z,λ). The probability that these matches then survive the shock at time t is given by $(\delta^{-(j+1)}x_t/z)^{-\lambda}$, and multiplying this by $z^{-\lambda}vu_\tau$ gives the number that survive:

$$(\delta^{-(j+1)}x_t/z)^{-\lambda}z^{-\lambda}vu_\tau = (\delta^\lambda)^{j+1}x_t^{-\lambda}vu_\tau,$$

noting that the terms in z cancel out. This is conditional on z, j, and τ, but since z does not appear above, the distribution of the past truncation thresholds is not needed. Averaging over the distribution of j yields:

$$\sum_{j=0}^{\infty} e^{-a(t-\tau)}\frac{(a(t-\tau))^j}{j!}(\delta^\lambda)^{j+1}x_t^{-\lambda}vu_\tau = \delta^\lambda x_t^{-\lambda}vu_\tau e^{-a(t-\tau)}\sum_{j=0}^{\infty}\frac{(a\delta^\lambda(t-\tau))^j}{j!}$$
$$= \delta^\lambda x_t^{-\lambda}vu_\tau e^{-a(t-\tau)}e^{a\delta^\lambda(t-\tau)} = \delta^\lambda x_t^{-\lambda}vu_\tau e^{-a(1-\delta^\lambda)(t-\tau)},$$

where the penultimate expression uses the Taylor series expansion of the exponential function $e^z = \sum_{j=0}^{\infty} z^j/j!$ (valid for all z). Next, integrating over all vintages τ before the current time t leads to:

$$\int_{\tau\to-\infty}^{t} \delta^\lambda x_t^{-\lambda}e^{-a(1-\delta^\lambda)(t-\tau)}\mathrm{d}\tau = \delta^\lambda x_t^{-\lambda}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}vu_\tau\mathrm{d}\tau.$$

Multiplying this by the arrival rate a of the idiosyncratic shocks confirms the second term of the expression for N_t in (A.30).

This leaves only the claim that the measure of vintage-τ surviving matches with truncation point z (in terms of the original match quality distribution ϵ) is $z^{-\lambda}vu_\tau$. When these matches first form, they have measure $y_\tau^{-\lambda}vu_\tau$ and a Pareto(y_t,λ) distribution, so the formula is correct if no shocks have occurred and $z = y_\tau$. Now suppose the formula is valid for some z and truncation now occurs at a new point $w > z$ (in terms of original match quality). Since matches surviving truncation at z have distribution Pareto(z,λ), the proportion of these that survive the new truncation is $(w/z)^{-\lambda}$, and so the measure becomes $(w/z)^{-\lambda}z^{-\lambda}vu_\tau = w^{-\lambda}vu_\tau$ (with the term in z cancelling out), which confirms the claim.

The distribution of match quality

Now consider the derivation of the law of motion for average match quality Q_t in (22). Let total match quality across all families be denoted by $\mathcal{E}_t$ (those not matched have match quality equal to zero), with

$\mathcal{E}_t = (1-u_t)Q_t$ by definition. Total match quality $\mathcal{E}_t$ changes over time as new matches form, when matches are hit by shocks, and when moving decisions are made. With transaction threshold y_t and new match quality drawn from a Pareto$(1;\lambda)$ distribution, new matches have average quality $(\lambda/(\lambda-1))y_t$. The contribution to the rate of change of total match quality is that average multiplied by $s_t u_t$. Shocks to existing matches arrive randomly at rate a. If no shock is received then there is no change to match quality and no moving decision. For those who receive a shock, let $\underline{\mathcal{E}}_t$ denote the total match quality of those matches that survive (with matches that dissolve counted as having zero match quality). The contribution of the shocks and moving decisions to the rate of change of total match quality is to subtract $a(\mathcal{E}_t - \underline{\mathcal{E}}_t)$. The differential equation for $\mathcal{E}_t$ is therefore:

$$\dot{\mathcal{E}}_t = \frac{\lambda}{\lambda-1} y_t s_t u_t - a(\mathcal{E}_t - \underline{\mathcal{E}}_t). \tag{A.31}$$

Using this formula requires an expression for $\underline{\mathcal{E}}_t$.

Consider the distribution of all matches that formed before time t, survived until time t, and now receive an idiosyncratic shock at time t, but one that is not sufficient to trigger moving. The distribution of surviving matches can be partitioned into vintages τ (when the match formed) and the number of shocks j that have been received previously (not counting the shock at time t). At time τ, a quantity u_τ of houses were for sale, and viewings arrived at rate v. Viewings were draws of match quality ϵ from a Pareto$(1;\lambda)$ distribution, and those draws with $\epsilon \geq y_\tau$ formed new matches, truncating the distribution at y_τ. Subsequently, a number j of idiosyncratic shocks have occurred, with j having a Poisson$(a(t-\tau))$ distribution, and these shocks resulting in the distribution of surviving match quality being truncated. With a shock now occurring at time t after j earlier shocks, match quality is now $\delta^{j+1}\epsilon$, and the distribution is truncated at x_t. In terms of the original match quality ϵ, survival requires $\epsilon \geq \delta^{-(j+1)}x_t$.

Consider matches of vintage τ that have previously accumulated k shocks for which the last truncation threshold was z in terms of original match quality (this threshold will depend on when the previous shocks occurred). Since the Pareto distribution is preserved after truncation with the same shape parameter, these matches have a Pareto$(z;\lambda)$ distribution in terms of their original match quality. It was shown above that the measure of surviving vintage-τ matches with truncation point z is $z^{-\lambda}vu_\tau$ (conditional on z, the number of shocks j is irrelevant, though the number of shocks may be related to the value of z). The measure that remain ($\epsilon \geq \delta^{-(j+1)}x_t$) after moving decisions are made at time t is:

$$\left(\delta^{-(j+1)}x_t/z\right)^{-\lambda} z^{-\lambda} v u_\tau = (\delta^\lambda)^{j+1} x_t^{-\lambda} v u_\tau,$$

noting that the terms in z cancel out. The probability of drawing j shocks in the interval between τ and t is $e^{-a(t-\tau)}(a(t-\tau))^j/j!$, and hence averaging over the distribution of j for vintage-τ matches implies that the surviving measure is:

$$\sum_{j=0}^{\infty} e^{-a(t-\tau)} \frac{(a(t-\tau))^j}{j!} (\delta^{j+1})^\lambda x_t^{-\lambda} v u_t = \delta^\lambda x_t^{-\lambda} v u_\tau e^{-a(1-\delta^\lambda)(t-\tau)},$$

which is confirmed by following the same steps as in the derivation of the moving rate above. Integrating these surviving measures over all cohorts:

$$\int_{\tau\to-\infty}^{t} \delta^\lambda x_t^{-\lambda} v u_\tau e^{-a(1-\delta^\lambda)} \mathrm{d}\tau = v\delta^\lambda x_t^{-\lambda} \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau,$$

and since the average match quality among the survivors after the shock at time t is $(\lambda/(\lambda-1))x_t$ for all cohorts, it follows that:

$$\underline{\mathcal{E}}_t = \frac{v\delta^\lambda \lambda}{\lambda-1} x_t^{1-\lambda} \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau.$$

Substituting this and equation (20) into (A.31) implies:

$$\dot{\mathcal{E}}_t = \frac{v\lambda}{\lambda-1} y_t^{1-\lambda} u_t - a\left(\mathcal{E}_t - \frac{v\delta^\lambda \lambda}{\lambda-1} x_t^{1-\lambda} \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau\right). \quad \text{[A.32]}$$

The integral can be eliminated by defining an additional variable Υ_t:

$$\Upsilon_t = \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau, \quad \text{[A.33]}$$

and hence (A.32) can be written as follows:

$$\dot{\mathcal{E}}_t = \frac{v\lambda}{\lambda-1} y_t^{1-\lambda} u_t - a\mathcal{E}_t + \frac{av\delta^\lambda \lambda}{\lambda-1} x_t^{1-\lambda} \Upsilon_t. \quad \text{[A.34]}$$

The evolution of the state variable u_t is determined by combining equations (19), (20), and (21):

$$\dot{u}_t = a(1-u_t) - av\delta^\lambda x_t^{-\lambda} \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau - v y_t^{-\lambda} u_t,$$

where the integral can again be eliminated by writing the equation in terms of the new variable Υ_t from (A.33):

$$\dot{u}_t = a(1-u_t) - av\delta^\lambda x_t^{-\lambda} \Upsilon_t - v y_t^{-\lambda} u_t. \quad \text{[A.35]}$$

Differentiating the integral in (A.33) shows that Υ_t must satisfy the differential equation:

$$\dot{\Upsilon}_t = u_t - a(1-\delta^\lambda)\Upsilon_t. \quad \text{[A.36]}$$

These results can be used to obtain the differential equation for average match quality Q_t in (22). Since the definition implies $Q_t = \mathcal{E}_t/(1-u_t)$, it follows that:

$$\dot{Q}_t = \frac{\dot{\mathcal{E}}_t}{1-u_t} + \frac{\mathcal{E}_t \dot{u}_t}{(1-u_t)^2} = \frac{\dot{\mathcal{E}}_t}{1-u_t} + Q_t \frac{\dot{u}_t}{1-u_t}.$$

Substituting from the differential equations (A.34) and (A.35) leads to:

$$\dot{Q}_t = \left(\frac{v\lambda}{\lambda-1} y_t^{1-\lambda} \frac{u_t}{1-u_t} - aQ_t + \frac{av\delta^\lambda \lambda}{\lambda-1} x_t^{1-\lambda} \frac{\Upsilon_t}{1-u_t}\right) + Q_t\left(a - av\delta^\lambda x_t^{-\lambda} \frac{\Upsilon_t}{1-u_t} - v y_t^{-\lambda} \frac{u_t}{1-u_t}\right),$$

noting that the terms in Q_t on the right-hand side cancel out, so $\dot{Q}_t$ can be written as:

$$\dot{Q}_t = v y_t^{-\lambda}\left(\frac{\lambda}{\lambda-1} y_t - Q_t\right)\frac{u_t}{1-u_t} - \frac{av\delta^\lambda x_t^{-\lambda}\Upsilon_t}{1-u_t}\left(Q_t - \frac{\lambda}{\lambda-1} x_t\right).$$

Comparison with equations (20), (21), and the definition of Υ_t in (A.33) confirms the differential equation for Q_t in (22).

Steady state

Given the moving rate n and the sales rate s, the steady-state stock of houses for sale is:

$$u = \frac{n}{s+n}. \quad \text{[A.37]}$$

The steady-state moving rate n can be derived from the formula (21):

$$n = a - a\delta^{\lambda}x^{-\lambda}v\frac{u}{1-u}\int_{\tau=0}^{\infty} e^{-a(1-\delta^{\lambda})\tau}\mathrm{d}\tau = a - a\delta^{\lambda}\left(\frac{y}{x}\right)^{\lambda} vy^{-\lambda}\frac{n}{s}\frac{1}{a(1-\delta^{\lambda})},$$

where the final equality uses $u/(1-u) = n/s$, as implied by (A.37). Since $s = vy^{-\lambda}$ according to (A.26), the equation above becomes:

$$n = a - \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda} n.$$

Solving this equation for n yields:

$$n = \frac{a}{1+\frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}, \tag{A.38}$$

which confirms the claim in (32).

A.6 Efficiency

The social planner's objective function from (30) can be written in terms of total match quality $\mathcal{E}_t$, the transaction threshold y_t, and houses for sale u_t by substituting $\mathcal{E}_t = (1-u_t)Q_t$ and using equation (20):

$$\varOmega_T = \int_{t=T}^{\infty} e^{-r(t-T)}\left(\xi\mathcal{E}_t - Cvy_t^{-\lambda}u_t - Fu_t - D\right)\mathrm{d}t. \tag{A.39}$$

This is maximized by choosing x_t, y_t, $\mathcal{E}_t$, u_t, and $\varUpsilon_t$ subject to the differential equations for $\mathcal{E}_t$, u_t, and $\varUpsilon_t$ in (A.34), (A.35), and (A.36) (the variable $\varUpsilon_t$ defined in A.33 is introduced because the differential equations A.34 and A.35 are written in terms of $\varUpsilon_t$). The problem is solved by introducing the (current-value) Hamiltonian:

$$\begin{aligned}\mathcal{J}_t = \xi\mathcal{E}_t - Cvy_t^{-\lambda}u_t - Fu_t - D + \varphi_t\left(\frac{v\lambda}{\lambda-1}y_t^{1-\lambda}u_t - a\mathcal{E}_t + \frac{av\delta^{\lambda}\lambda}{\lambda-1}x_t^{1-\lambda}\varUpsilon_t\right)\\ + \vartheta_t\left(a(1-u_t) - av\delta^{\lambda}x_t^{-\lambda}\varUpsilon_t - vy_t^{-\lambda}u_t\right) + \gamma_t\left(u_t - a(1-\delta^{\lambda})\varUpsilon_t\right),\end{aligned} \tag{A.40}$$

where φ_t, ϑ_t, and γ_t are the co-state variables associated with $\mathcal{E}_t$, u_t, and $\varUpsilon_t$. The first-order conditions with respect to x_t and y_t are:

$$\frac{\partial\mathcal{J}_t}{\partial x_t} = av\delta^{\lambda}\lambda x_t^{-\lambda-1}\varUpsilon_t\vartheta_t - av\delta^{\lambda}\lambda x_t^{-\lambda}\varUpsilon_t\varphi_t = 0; \tag{A.41a}$$

$$\frac{\partial\mathcal{J}_t}{\partial y_t} = v\lambda Cy_t^{-\lambda-1}u_t - v\lambda y_t^{-\lambda}u_t\varphi_t + v\lambda y_t^{-\lambda-1}u_t\vartheta_t = 0, \tag{A.41b}$$

and the first-order conditions with respect to the state variables $\mathcal{E}_t$, u_t, and $\varUpsilon_t$ are:

$$\frac{\partial\mathcal{J}_t}{\partial\mathcal{E}_t} = \xi - a\varphi_t = r\varphi_t - \dot{\varphi}_t; \tag{A.41c}$$

$$\frac{\partial\mathcal{J}_t}{\partial u_t} = -Cvy_t^{-\lambda} - F + \frac{v\lambda}{\lambda-1}y_t^{1-\lambda}\varphi_t - (a+vy_t^{-\lambda})\vartheta_t + \gamma_t = r\vartheta_t - \dot{\vartheta}_t; \tag{A.41d}$$

$$\frac{\partial\mathcal{J}_t}{\partial\varUpsilon_t} = \frac{av\delta^{\lambda}\lambda}{\lambda-1}x_t^{1-\lambda}\varphi_t - av\delta^{\lambda}x_t^{-\lambda}\vartheta_t - a(1-\delta^{\lambda})\gamma_t = r\gamma_t - \dot{\gamma}_t. \tag{A.41e}$$

By cancelling common terms from (A.41a), the following link between the moving threshold x_t and the co-states φ_t and ϑ_t can be deduced:

$$\varphi_t = \frac{\vartheta_t}{x_t}. \tag{A.42}$$

Similarly, cancelling common terms from (A.41b) implies a link between the transaction threshold y_t and φ_t and ϑ_t:

$$\frac{C}{y_t} + \frac{\vartheta_t}{y_t} = \varphi_t. \tag{A.43}$$

The differential equation for φ_t in (A.41c) is:

$$\dot{\varphi}_t = (r+a)\varphi_t - \xi,$$

and since $r + a > 0$, the only solution satisfying the transversality condition is the following constant solution:

$$\varphi_t = \frac{\xi}{r+a}. \tag{A.44}$$

With this solution for φ_t, equation (A.43) implies that ϑ_t is proportional to the moving threshold x_t:

$$\vartheta_t = \frac{\xi}{r+a} x_t. \tag{A.45}$$

Eliminating both φ_t and ϑ_t from (A.43) by substituting from (A.44) and (A.45) implies that y_t and x_t must satisfy:

$$y_t - x_t = \frac{(r+a)C}{\xi}. \tag{A.46}$$

Using (A.41d) to write a differential equation for ϑ_t and substituting the solution for φ_t from (A.44):

$$\dot{\vartheta}_t = (r + a + vy_t^{-\lambda})\vartheta_t - \gamma_t + F + Cvy_t^{-\lambda} - \frac{\xi}{r+a}\frac{v\lambda}{\lambda-1}y_t^{1-\lambda}. \tag{A.47}$$

Similarly, (A.41e) implies a differential equation for γ_t, from which φ_t can be eliminated using (A.44):

$$\dot{\gamma}_t = (r + a(1-\delta^\lambda))\gamma_t + av\delta^\lambda x_t^{-\lambda}\left(\frac{\xi}{r+a}x_t\right) - \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\left(\frac{\xi}{r+a}\right),$$

which can be simplified as follows:

$$\dot{\gamma}_t = (r + a(1-\delta^\lambda))\gamma_t - \frac{\xi}{r+a}\frac{av\delta^\lambda}{\lambda-1}x_t^{1-\lambda}. \tag{A.48}$$

It is now shown that there is a solution of the constrained maximization problem where the co-states ϑ_t and γ_t are constant over time. In this case, equations (A.45) and (A.46) require that x_t and y_t are constant over time and related as follows:

$$y - x = \frac{(r+a)C}{\xi}. \tag{A.49}$$

With $\dot{\vartheta}_t = 0$ and $\dot{\gamma}_t = 0$, (A.47) and (A.48) imply the following pair of equations:

$$(r + a + vy^{-\lambda})\vartheta - \gamma + F + Cvy^{-\lambda} - \frac{\xi}{r+a}\frac{v\lambda}{\lambda - 1}y^{1-\lambda} = 0; \tag{A.50}$$

$$(r + a(1-\delta^{\lambda}))\gamma - \frac{\xi}{r+a}\frac{av\delta^{\lambda}}{\lambda-1}x^{1-\lambda} = 0. \tag{A.51}$$

Equation (A.51) yields the following expression for γ in terms of x:

$$\gamma = \frac{\xi a v \delta^{\lambda}}{(\lambda-1)(r+a)(r+a(1-\delta^{\lambda}))}x^{1-\lambda},$$

and substituting this and (A.45) into (A.50) leads to:

$$\frac{\xi(r+a+vy^{-\lambda})}{r+a}x - \frac{\xi a v \delta^{\lambda}}{(\lambda-1)(r+a)(r+a(1-\delta^{\lambda}))}x^{1-\lambda} + F + Cvy^{-\lambda} - \frac{\xi v\lambda}{(\lambda-1)(r+a)}y^{1-\lambda} = 0.$$

Since $(r+a)C = \xi(y-x)$ according to (A.49), multiplying the equation above by $(r+a)$ and substituting for $(r+a)C$ implies:

$$\xi(r+a+vy^{-\lambda})x - \frac{\xi a v\delta^{\lambda}}{(\lambda-1)(r+a(1-\delta^{\lambda}))}x^{1-\lambda} + (r+a)F + \xi v(y-x)y^{-\lambda} - \frac{\xi v \lambda}{(\lambda-1)}y^{1-\lambda} = 0.$$

Dividing both sides by ξ and grouping terms in $(r+a)$ on the left-hand side:

$$(r+a)\left(x + \frac{F}{\xi}\right) = \frac{v\lambda}{\lambda-1}y^{1-\lambda} - vy^{1-\lambda} + \frac{av\delta^{\lambda}}{(\lambda-1)(r+a(1-\delta^{\lambda}))}x^{1-\lambda},$$

and dividing both sides by $r+a$ and simplifying the terms involving $y^{1-\lambda}$ leads to:

$$x + \frac{F}{\xi} = \frac{v}{(\lambda-1)(r+a)}\left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r+a(1-\delta^{\lambda})}x^{1-\lambda}\right). \tag{A.52}$$

The pair of equations (A.49) and (A.52) for x and y are identical to the equations (26) and (29) characterizing the equilibrium values of x and y. The equilibrium is therefore the same as the solution to the social planner's problem, establishing that it is efficient.

A.7 Transitional dynamics and overshooting

Transitional dynamics out of steady state

Equation (21) for the moving rate $n_t = N_t/(1-u_t)$ implies that the quantity of new listings N_t is:

$$N_t = a(1-u_t) - a\delta^{\lambda}x_t^{-\lambda}v\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^{\lambda})(t-\tau)}u_\tau \mathrm{d}\tau, \tag{A.53}$$

and equation (19) implies the differential equation for the stock of houses for sale u_t is:

$$\dot{u}_t = N_t - S_t, \quad \text{where } S_t = s_t u_t \text{ and } s_t = vy_t^{-\lambda}. \tag{A.54}$$

In the equation above, S_t is the number of transactions and s_t is the sales rate, which is taken from (20).

Now suppose that the moving and transaction thresholds x_t and y_t are constant from some date T onwards, that is, $x_t = x$ and $y_t = y$ for all $t \geq T$. Using (A.54), the number of transactions S_t and the sales rate s_t are given by:

$$S_t = su_t, \quad \text{and } s_t = s = vy^{-\lambda}. \tag{A.55}$$

Equation (A.53) for new listings becomes:

$$N_t = a(1-u_t) - a\delta^\lambda x^{-\lambda} v \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau, \qquad \text{[A.56]}$$

where (A.55) and (A.56) are valid for all $t \geq T$. By taking the derivative of both sides of (A.56) with respect to time t:

$$\dot{N}_t = -a\dot{u}_t - a\delta^\lambda x^{-\lambda} v u_t + a(1-\delta^\lambda)\left(a\delta^\lambda x^{-\lambda} v \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau\right),$$

and using (A.56) to substitute for the integral above an expression involving the current levels of N_t and u_t:

$$\dot{N}_t = -a\dot{u}_t - a\delta^\lambda x^{-\lambda} v u_t + a(1-\delta^\lambda)\left(a(1-u_t) - N_t\right).$$

This differential equation can be simplified as follows:

$$\dot{N}_t = -a\dot{u}_t - a\left((1-\delta^\lambda)N_t + \left(a(1-\delta^\lambda) + \delta^\lambda \left(\frac{y}{x}\right)^\lambda s\right) u_t - a(1-\delta^\lambda)\right), \qquad \text{[A.57]}$$

where s is the constant sales rate from (A.55). Substituting equation (A.55) into (A.54):

$$\dot{u}_t = N_t - s u_t, \qquad \text{[A.58a]}$$

and in turn substituting this equation into (A.57) and simplifying:

$$\dot{N}_t = -a\left(\left(1 + (1-\delta^\lambda)\right) N_t + \left(a(1-\delta^\lambda) + \delta^\lambda \left(\frac{y}{x}\right)^\lambda s - s\right) u_t - a(1-\delta^\lambda)\right). \qquad \text{[A.58b]}$$

Equations (A.58a) and (A.58b) comprise a system of linear differential equations for the stock of houses for sale u_t and new listings N_t.

Now consider a steady state of the system (A.58), that is, a solution $u_t = u$ and $N_t = N$ where $\dot{u}_t = 0$ and $\dot{N}_t = 0$ for all t. Equation (A.58a) implies $N = su$, and substituting this into (A.58b):

$$\left(\left(1 + (1-\delta^\lambda)\right) s + \left(a(1-\delta^\lambda) + \delta^\lambda \left(\frac{y}{x}\right)^\lambda s - s\right)\right) u = a(1-\delta^\lambda),$$

which can be solved for a unique value of u:

$$u = \frac{\frac{a}{1+\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}}{s + \frac{a}{1+\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}} = \frac{n}{s+n}, \quad \text{where } n = \frac{a}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}. \qquad \text{[A.59]}$$

This is of course the steady state found in section 4.3, where s is the steady-state sales rate from (20) and n is the steady-state moving rate (21). Steady-state new listings are $N = su = n(1-u)$.

Now define the percentage deviations of the variables u_t and N_t from their unique steady-state values u and N:

$$\tilde{u}_t = \frac{u_t - u}{u}, \quad \text{and} \quad \tilde{N}_t = \frac{N_t - N}{N}, \quad \text{or equivalently} \quad u_t = u(1+\tilde{u}_t), \quad \text{and} \quad N_t = N(1+\tilde{N}_t), \qquad \text{[A.60]}$$

and the time derivatives of u_t and N_t and $\tilde{u}_t$ and $\tilde{N}_t$ are related as follows:

$$\dot{\tilde{u}}_t = \frac{\dot{u}_t}{u}, \quad \text{and} \quad \dot{\tilde{N}}_t = \frac{\dot{N}_t}{N}. \qquad \text{[A.61]}$$

Using (A.60), (A.61), and $N = su$, the differential equation (A.58a) can be written in terms of $\tilde{u}_t$ and $\tilde{N}_t$ as follows:

$$\dot{\tilde{u}}_t = s\tilde{N}_t - s\tilde{u}_t. \tag{A.62a}$$

Likewise, (A.60), (A.61), and $u/N = 1/s$ imply that the differential equation (A.58b) is equivalent to:

$$\dot{\tilde{N}}_t = -a\left(\left(1 + (1-\delta^\lambda)\right)\tilde{N}_t + \left(\frac{a(1-\delta^\lambda)}{s} + \delta^\lambda\left(\frac{y}{x}\right)^\lambda - 1\right)\tilde{u}_t\right). \tag{A.62b}$$

Make the following definition of a variable $\tilde{n}_t$:

$$\tilde{n}_t = \tilde{N}_t + \frac{n}{s}\tilde{u}_t, \quad \text{and hence } \tilde{N}_t = \tilde{n}_t - \frac{n}{s}\tilde{u}_t \text{ and } \dot{\tilde{n}}_t = \dot{\tilde{N}}_t + \frac{n}{s}\dot{\tilde{u}}_t, \tag{A.63}$$

where the final equation follows from taking the time derivative of the definition of $\tilde{n}_t$. The variable $\tilde{n}_t$ is also approximately the percentage deviation of the moving rate $n_t = N_t/(1-u_t)$ from its steady-state value n, but here, $\tilde{n}_t$ is simply taken as the definition of a new variable. The differential equation (A.62a) can be written exactly in terms of $\tilde{u}_t$ and $\tilde{n}_t$ by using the second equation in (A.63):

$$\dot{\tilde{u}}_t = s\tilde{n}_t - (s+n)\tilde{u}_t. \tag{A.64a}$$

Substituting (A.62b) and (A.64a) into the third equation from (A.63):

$$\begin{aligned}\dot{\tilde{n}}_t &= -a\left(1 + (1-\delta^\lambda)\right)\left(\tilde{n}_t - \frac{n}{s}\tilde{u}_t\right) - a\left(\frac{a(1-\delta^\lambda)}{s} + \delta^\lambda\left(\frac{y}{x}\right)^\lambda - 1\right)\tilde{u}_t + \frac{n}{s}\left(s\tilde{n}_t - (s+n)\tilde{u}_t\right) \\ &= -\left((a-n) + a(1-\delta^\lambda)\right)\tilde{n}_t - \left(\frac{(a-n)}{s}a(1-\delta^\lambda) + a\delta^\lambda\left(\frac{y}{x}\right)^\lambda - (a-n) - (a-n)\frac{n}{s}\right)\tilde{u}_t.\end{aligned}$$

Rearranging the equation for the steady-state moving rate n in (A.59) leads to:

$$a\delta^\lambda\left(\frac{y}{x}\right)^\lambda = \frac{(a-n)}{n}a(1-\delta^\lambda),$$

and substituting this into the equation for $\dot{\tilde{n}}_t$ above implies that the coefficient of $\tilde{u}_t$ can be simplified:

$$\dot{\tilde{n}}_t = -\left((a-n) + a(1-\delta^\lambda)\right)\tilde{n}_t - \frac{(a-n)(s+n)}{s}\left(\frac{a(1-\delta^\lambda)}{n} - 1\right)\tilde{u}_t. \tag{A.64b}$$

Equations (A.64a) and (A.64b) form a system of differential equations in the variables $\tilde{u}_t$ and $\tilde{n}_t$. This system can be written in matrix form as follows:

$$\begin{pmatrix}\dot{\tilde{u}}_t \\ \dot{\tilde{n}}_t\end{pmatrix} = \begin{pmatrix} -(s+n) & s \\ -\frac{n(s+n)\chi_u}{s} & -n\chi_n\end{pmatrix}\begin{pmatrix}\tilde{u}_t \\ \tilde{n}_t\end{pmatrix}, \tag{A.65}$$

where the coefficients χ_u and χ_n are defined by:

$$\chi_u = \frac{a-n}{n}\left(\frac{a(1-\delta^\lambda)}{n} - 1\right), \quad \text{and } \chi_n = \frac{a-n}{n} + \frac{a(1-\delta^\lambda)}{n}. \tag{A.66}$$

The sum of the coefficients χ_u and χ_n is:

$$\chi_u + \chi_n = (1-\delta^\lambda)\left(\frac{a}{n}\right)^2. \tag{A.67}$$

The expression for n in (A.59) implies:

$$\frac{a-n}{n}=\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda, \quad \text{and} \quad \frac{a(1-\delta^\lambda)}{n}=(1-\delta^\lambda)+\delta^\lambda\left(\frac{y}{x}\right)^\lambda,$$

and hence the coefficients χ_u and χ_n from (A.66) are equal to the following:

$$\chi_u=\frac{\delta^\lambda\left(\frac{\delta y}{x}\right)^\lambda\left(\left(\frac{y}{x}\right)^\lambda-1\right)}{1-\delta^\lambda}, \quad \text{and} \quad \chi_n=\frac{(1-\delta^\lambda)^2+(1-\delta^\lambda)\left(\frac{\delta y}{x}\right)^\lambda+\left(\frac{\delta y}{x}\right)^\lambda}{1-\delta^\lambda}. \tag{A.68}$$

The coefficient χ_n is strictly positive, as is χ_u because it is always the case that $y>x$. Since $(\delta y/x)^\lambda<1$ and $(\delta y/x)^\lambda-\delta^\lambda<1-\delta^\lambda$, it follows that $\chi_u<1$. As χ_n is larger than $1-\delta^\lambda+\delta^\lambda(y/x)^\lambda$, which is greater than 1 because $y>x$, it must be the case that $\chi_n>1$.

The set of points where $\dot{\tilde{u}}_t=0$ is given by:

$$-(s+n)\tilde{u}_t+s\tilde{n}_t=0, \quad \text{and hence} \quad \tilde{n}_t=\frac{s+n}{s}\tilde{u}_t,$$

which is an upward-sloping straight line with gradient $(s+n)/s$ in $(\tilde{u}_t,\tilde{n}_t)$ space. To the left and above, $\tilde{u}_t$ is increasing over time, and to the right and below, $\tilde{n}_t$ is decreasing. The set of points where $\dot{\tilde{n}}_t=0$ is given by:

$$-\frac{n(s+n)\chi_u}{s}\tilde{u}_t-n\chi_n\tilde{n}_t=0, \quad \text{and hence} \quad \tilde{n}_t=-\frac{(s+n)}{s}\frac{\chi_u}{\chi_n}\tilde{u}_t.$$

This is a downward-sloping straight line with gradient $-(s+n)\chi_u/s\chi_n$, which is less than the gradient of the $\dot{\tilde{u}}_t=0$ line in absolute value because $\chi_u<\chi_n$. Given that both χ_u and χ_n are positive, $\tilde{n}_t$ is increasing over time to the left and below the line, and decreasing to the right and above.

The characteristic equation for the eigenvalues ζ of the system of differential equations (A.65) is:

$$(\zeta+(s+n))(\zeta+n\chi_n)+n(s+n)\chi_u=0, \tag{A.69}$$

which is a quadratic equation in ζ:

$$\zeta^2+((s+n)+n\chi_n)\zeta+n(s+n)(\chi_u+\chi_n)=0. \tag{A.70}$$

The two eigenvalues ζ_1 and ζ_2 are the roots of this quadratic equation. The eigenvalues are either both real numbers or a conjugate pair of complex numbers. The sum and product of the eigenvalues are:

$$\zeta_1+\zeta_2=-\left((s+n)+n\chi_n\right), \quad \text{and} \quad \zeta_1\zeta_2=n(s+n)(\chi_u+\chi_n).$$

Since χ_u and χ_n are both positive, the sum of the eigenvalues is negative and the product is positive. If both are real numbers then both must be negative numbers. If both are complex numbers then the sum is equal to twice the common real component of the eigenvalues, which must therefore be negative. Hence, in all cases, the real parts of all eigenvalues are negative. This establishes that there is convergence to the steady state in the long run starting from any initial conditions.

The condition for the quadratic equation (A.70) to have two real roots is:

$$((s+n)+n\chi_n)^2\geq 4n(s+n)(\chi_u+\chi_n), \quad \text{or equivalently} \quad \left(\frac{s}{n}+1+\chi_n\right)^2\geq 4(\chi_u+\chi_n)\left(\frac{s}{n}+1\right),$$

where the latter is derived by dividing both sides by the positive number n^2. By expanding the brackets, this condition can be expressed as a quadratic inequality in the ratio of the sales rate to the moving rate:

$$\left(\frac{s}{n}\right)^2+2(1+\chi_n)\frac{s}{n}+(1+\chi_n)^2\geq 4(\chi_u+\chi_n)\frac{s}{n}+4(\chi_u+\chi_n),$$

which can be simplified as follows:

$$\left(\frac{s}{n}\right)^2 + 2\left(1 - (\chi_u + \chi_n) - \chi_u\right)\frac{s}{n} + \left(1 + \chi_n^2 - 2(\chi_u + \chi_n) - 2\chi_u\right) \geq 0.$$

It can be verified directly that the inequality can be factorized:

$$\left(\frac{s}{n} + 1 - \left(\sqrt{\chi_u + \chi_n} - \sqrt{\chi_u}\right)^2\right)\left(\frac{s}{n} + 1 - \left(\sqrt{\chi_u + \chi_n} + \sqrt{\chi_u}\right)^2\right) \geq 0, \qquad \text{[A.71]}$$

which provides a test in terms of the ratio s/n for whether the eigenvalues of the system of differential equations (A.65) are all real (if the test is not satisfied then they are a conjugate pair of complex numbers).

There are three cases. The first case is where the sales rate relative to the moving rate is above the following threshold:

$$\frac{s}{n} > \left(\sqrt{\chi_u + \chi_n} + \sqrt{\chi_u}\right)^2 - 1, \qquad \text{[A.72]}$$

noting that the right-hand side is strictly positive because $\chi_n > 1$. If this holds then the condition (A.71) is satisfied and both eigenvalues ζ_1 and ζ_2 are real numbers. Since $\chi_u > 0$, it follows that $\sqrt{\chi_u + \chi_n} > \sqrt{\chi_n}$ and hence (A.72) implies $s/n > \chi_n - 1$. This means $s + n > n\chi_n$, and as ζ_1 and ζ_2 are roots of the equation (A.69), it must be the case that the negative eigenvalues satisfy $\zeta_1 > -(s+n)$ and $\zeta_2 > -(s+n)$, recalling that χ_u is always positive.

In a model where the moving rate n is exogenous and constant, the only dynamics would come from the differential equation $\dot{\tilde{u}}_t = -(s+n)\tilde{u}_t$ (see A.64a), so $s+n$ would be the rate of convergence to the steady state. In the general model, the speed of convergence is determined by the negative of the real components of the eigenvalues. When (A.72) is satisfied, it follows that $|\zeta_1| < s+n$ and $|\zeta_2| < s+n$, which means the new dynamics coming from match quality dominate the usual dynamics coming from the evolution of the stock of houses for sale. Convergence is therefore slower than it would be in an exogenous moving model with the same moving rate. As (A.72) shows, this case corresponds to the sales rate being sufficiently large, which will make it the empirically relevant one. Using $\sqrt{\chi_u + \chi_n} + \sqrt{\chi_u} < 2\sqrt{\chi_u + \chi_n}$ and the expression for $\chi_u + \chi_n$ in (A.67), a sufficient condition for (A.72) is:

$$\frac{s}{n} > (1 - \delta^\lambda)\left(\frac{a}{n}\right)^2 - 1,$$

which holds for the calibrated version of the model.

The second case is where the sales rate is relatively low, specifically $s/n < (\sqrt{\chi_u + \chi_n} - \sqrt{\chi_u})^2 - 1$. The condition in (A.71) will be satisfied, so both eigenvalues would be negative real numbers. In this case, $s/n < \chi_n - 1$ since $\sqrt{\chi_u} + \sqrt{\chi_n} > \sqrt{\chi_u + \chi_n}$, which means that $s + n < n\chi_n$. Given that the eigenvalues are roots of equation (A.69), it follows that $\zeta_1 < -(s+n)$ and $\zeta_2 < -(s+n)$ and hence $|\zeta_1| > s+n$ and $|\zeta_2| > s+n$. Convergence to the steady state is actually faster than an exogenous moving model, and the new dynamics of match quality do not play an important role compared to the usual dynamics coming from the evolution of the stock of houses for sale. This case is not empirically relevant because the required sales rate would need to be too low compared to the moving rate.

The third case is where the dynamics of match quality and the dynamics of the stock of houses for sale are of similar importance, which occurs when s/n lies between $(\sqrt{\chi_u + \chi_n} - \sqrt{\chi_u})^2 - 1$ and $(\sqrt{\chi_u + \chi_n} + \sqrt{\chi_u})^2 - 1$. In this case, condition (A.71) does not hold and both eigenvalues ζ_1 and ζ_2 are complex numbers. This case features damped oscillations around the steady state, but is not empirically relevant because the required sales rate is too low compared to the moving rate. In what follows, attention is restricted to the empirically plausible case where the sales rate is sufficiently high that condition (A.72) holds.

There are two eigenvectors of the form $(1, \nu)$ associated with the two eigenvalues (as will be seen, the first element can be normalized to 1). The values of ν in the eigenvectors are solutions of the following equations:

$$(-(s+n) - \zeta) + s\nu = 1, \quad \text{and hence } \nu = \frac{(s+n)+\zeta}{s}. \qquad \text{[A.73]}$$

This equation holds for ν_1 when $\zeta = \zeta_1$ and for ν_2 when $\zeta = \zeta_2$. Without loss of generality, let ζ_1 denote the eigenvalue with the greater absolute value. Consequently, as both are negative, $\zeta_1 < \zeta_2$. Since it has been shown above that $\zeta_1 > -(s+n)$ and $\zeta_2 > -(s+n)$, equation (A.73) implies that both ν_1 and ν_2 are positive, with $\nu_1 < \nu_2$. Geometrically in $(\tilde{u}_t, \tilde{n}_t)$ space, both eigenvectors are upward-sloping straight lines, and since both ζ_1 and ζ_2 are negative, their gradient is less than the $\dot{\tilde{u}}_t = 0$ line. The eigenvector associated with the dominant eigenvalue ζ_2 has a steeper gradient than the eigenvector associated with ζ_1.

Having found the eigenvalues and eigenvectors of the system of differential equations (A.65), the solution can be stated as follows:

$$\tilde{u}_t = k_1 e^{\zeta_1(t-T)} + k_2 e^{\zeta_2 t - T}, \quad \text{and} \quad \tilde{n}_t = k_1\nu_1 e^{\zeta_1(t-T)} + k_2\nu_2 e^{\zeta_2(t-T)}, \qquad \text{[A.74]}$$

where T is the date from which the moving and transaction thresholds will be constant at x and y respectively, and k_1 and k_2 are coefficients to be determined. Since there is convergence to the steady state for any initial conditions, the coefficients k_1 and k_2 are pinned down by knowing the values of $\tilde{u}_T$ and $\tilde{n}_T$:

$$\tilde{u}_T = k_1 + k_2, \quad \text{and} \quad \tilde{n}_T = k_1\nu_1 + k_2\nu_2,$$

and these equations can be solved for k_1 and k_2:

$$k_1 = \frac{\nu_2\tilde{u}_T - \tilde{n}_T}{\nu_2 - \nu_1}, \quad \text{and} \quad k_2 = \frac{\tilde{n}_T - \nu_1\tilde{u}_T}{\nu_2 - \nu_1}.$$

Substituting these expressions into (A.74) yields the solution conditional on given initial conditions at date T:

$$\tilde{u}_t = (\nu_2\tilde{u}_T - \tilde{n}_T)\left(\frac{1}{\nu_2 - \nu_1}\right) e^{\zeta_1(t-T)} + (\tilde{n}_T - \nu_1\tilde{u}_T)\left(\frac{1}{\nu_2 - \nu_1}\right) e^{\zeta_2 t - T}; \qquad \text{[A.75a]}$$

$$\tilde{n}_t = (\nu_2\tilde{u}_T - \tilde{n}_T)\left(\frac{\nu_1}{\nu_2 - \nu_1}\right) e^{\zeta_1(t-T)} + (\tilde{n}_T - \nu_1\tilde{u}_T)\left(\frac{\nu_2}{\nu_2 - \nu_1}\right) e^{\zeta_2(t-T)}. \qquad \text{[A.75b]}$$

As T tends to infinity, the vector $(\tilde{u}_t, \tilde{n}_t)$ must approach the origin approximately along the eigenvector associated with the dominant eigenvalue ζ_2, that is, $(1, \nu_2)$.

Given the initial values of $\tilde{u}_T$ and $\tilde{n}_t$, (A.75) gives an exact solution of the system of differential equations (A.65) for variables $\tilde{u}_t$ and $\tilde{n}_t$ defined in (A.60) and (A.63). Using the definition in (A.63), this implies an exact solution for $\tilde{N}_t$ as well. The exact solution for the variables u_t and N_t can then be recovered using the definitions in (A.60). Once u_t and N_t are known, the exact solution for the moving rate can be computed using the definition $n_t = N_t/(1-u_t)$. Finally, the sales rate s_t is simply equal to the constant s, and transactions $S_t = s_t u_t$ can be found given the solution for u_t.

Overshooting

Next, consider how the initial values of $\tilde{u}_T$ and $\tilde{n}_T$ are determined following a change to the moving or transaction thresholds x_t and y_t. Suppose that x_t and y_t were previously constant at x_0 and y_0, and then move permanently to x and y from date T onwards. The previous sales rate was s_0, and the steady-state values of the moving rate and houses for sale were n_0 and u_0. After the change to x and y, there is a new steady-state sales rate s that is reached immediately at date T. Houses for sale is a stock that cannot instantaneously jump, so this variable remains equal to its old steady-state value initially, that is, $u_T = u_0$. The new steady state for u_t is u, and this can be used to compute $\tilde{u}_T = (u_0 - u)/u$.

Using (32) and (33), an increase in the moving threshold x implies a higher moving rate n and a higher u compared to u_0, and thus a negative value of $\tilde{u}_T$. Using (31), (32), and (33), an increase in y implies a lower ratio s/n, which means a higher value of u compared to u_0, and thus a negative value of $\tilde{u}_T$. Therefore, either an increase in x or an increase in y implies $\tilde{u}_T < 0$.

Now consider the moving rate at date T when the moving and transaction thresholds change. With houses for sale u_t at its old steady-state value of u_0 for $t < T$, equation (21) implies:

$$n_T = a - \frac{a\delta^\lambda x_T^{-\lambda} v_0}{1-u_T}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_0 \mathrm{d}\tau = a - \frac{a\delta^\lambda x^{-\lambda} v_0}{1-u_0}\frac{u_0}{a(1-\delta^\lambda)} = a - \frac{\delta^\lambda}{1-\delta^\lambda}\frac{u_0}{1-u_0} v_0 x^{-\lambda},$$

which uses $x_T = x$ and $u_T = u_0$, and where v_0 denotes the old value of the parameter v (which may change at date T). Noting that $u_0 = n_0/(s_0+n_0)$, where s_0 and n_0 are the old sales and moving rates, it can be seen that $u_0/(1-u_0) = n_0/s_0$, where $s_0 = v_0 y_0^{-\lambda}$. Substituting this into the equation above:

$$n_T = a - \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y_0}{x}\right)^\lambda n_0, \qquad \text{[A.76]}$$

and substituting the expression for the old steady state n_0 from (32):

$$n_T = a - \frac{a\left(\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y_0}{x}\right)^\lambda\right)}{1+\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y_0}{x_0}\right)^\lambda} = \left(1 + \frac{\delta^\lambda}{1-\delta^\lambda} y_0^\lambda\left(\frac{1}{x_0^\lambda} - \frac{1}{x^\lambda}\right)\right) n_0. \qquad \text{[A.77]}$$

This implies that an increase in y has no impact on the initial value of n_T, while an increase in x raises n_T above n_0.

Next, the value of n_T is compared to the new steady-state value n. When y increases, equation (32) implies the steady-state value of n is lower. Therefore, $n < n_T = n_0$ following an increase in y, and thus $\tilde{n}_T > 0$. Now suppose x increases with no change in y. As explained above, (A.77) implies $n_T > n_0$. Combined with (A.76):

$$n_T > a - \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y_0}{x}\right)^\lambda n_T, \quad \text{and therefore } n_T > \frac{a}{1+\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y_0}{x}\right)^\lambda}.$$

After the increase in x, the right-hand side is equal to the new steady-state moving rate n, so the inequality above implies $n_T > n$. This means the moving rate overshoots its new steady state in the short run. Therefore, following either an increase in x or an increase in y, the initial deviation of the moving rate from its new steady state is such that $\tilde{n}_T > 0$.

In summary, an increase in either x or y leads to $\tilde{u}_T < 0$ and $\tilde{n}_T > 0$, and a decrease in x or y leads to $\tilde{u}_T > 0$ and $\tilde{n}_T < 0$. The transitional dynamics therefore follow the example paths illustrated in Figure 7.

A.8 Model with heterogeneous distributions of idiosyncratic shocks

The search process is the same as in the basic model. u_t denotes the measure of houses for sale, b_t denotes the measure of buyers, and $\mathcal{V}(u_t, b_t)$ denotes the meeting function. The viewing rate for both buyers and sellers is $v = \mathcal{V}(u_t, b_t)/u_t$ given that $u_t = b_t$ in equilibrium. Following a viewing, the buyer draws a match-specific quality ϵ from the Pareto distribution $G(\epsilon)$ in (10). If the match quality is sufficiently high then a transaction takes place with the price determined by Nash bargaining.

Value functions, thresholds, and prices

The new aspect of the model with heterogeneity emerges after a transaction occurs. After the buyer has moved in, a type $i \in \{1, \ldots, q\}$ is drawn from a distribution with probabilities θ_i, where $\sum_{i=1}^q \theta_i = 1$. The number of types is $q \geq 1$, and the special case $q = 1$ corresponds to the basic version of the model. A type-i homeowner faces idiosyncratic shocks that scale down match quality ϵ by a factor δ_i, with these shocks arriving at rate a_i. The value of occupying a house with match quality ϵ for a type-i homeowner is $H_{i,t}(\epsilon)$. The Bellman equations (the equivalent of 17) are:

$$rH_{i,t}(\epsilon) = \epsilon\xi - D + a_i\left(\max\{H_{i,t}(\delta_i\epsilon), J_t\} - H_{i,t}(\epsilon)\right) + \dot{H}_{i,t}(\epsilon), \quad \text{for all } i \in \{1, \ldots, q\}. \qquad \text{[A.78]}$$

The moving threshold $x_{i,t}$ for type-i households is the solution of the equation:

$$H_{i,t}(x_{i,t}) = J_t = B_t + U_t, \qquad \text{[A.79]}$$

for each $i = 1, \ldots, q$, which is the equivalent of (18), but now each threshold $x_{i,t}$ depends on a type-specific value function $H_{i,t}(\epsilon)$. The value J_t is the sum of values from being a buyer B_t and a seller V_t as in the basic version of the model. The Bellman equations for B_t and V_t are the same as the basic version of the model (the equations in 12). The surpluses $\Sigma_{\mathrm{b},t}(\epsilon)$ and $\Sigma_{\mathrm{u},t}(\epsilon)$ are also as given in (11), but in the model with heterogeneity, the value function $H_t(\epsilon)$ is a weighted average of the type-specific value functions $H_{i,t}(\epsilon)$:

$$H_t(\epsilon) = \sum_{i=1}^{q} \theta_i H_{i,t}(\epsilon). \qquad \text{[A.80]}$$

The total surplus $\Sigma_t(\epsilon)$ is also as given before in (13), with $H_t(\epsilon)$ specified by (A.80) here. Using (13), the equation $\Sigma_t(y_t) = 0$ for the transaction threshold is the same as the basic version of the model, namely equation (14), which depends on the $H_t(\epsilon)$ given in (A.80). The Bellman equation (15) for J_t is also unchanged. Assuming Nash bargaining over transaction prices, the same method in appendix A.4 can be used to show the expression in (16) for the average price is unchanged.

Stocks and flows

The stock-flow accounting identity (19) for houses for sales u_t is the same as in the basic version of the model. The process by which transactions occur is also the same as in the basic model, so equation (20) for the sales rate s_t holds as before. Let $\sigma_{i,t}$ denote the measure of homeowners of type-i at time t. The stock-flow accounting identity for $\sigma_{i,t}$ and the link with houses for sale u_t are:

$$\dot{\sigma}_{i,t} = \theta_i S_t - N_{i,t}, \quad \text{and} \quad \sum_{i=1}^{q} \sigma_{i,t} = 1 - u_t, \qquad \text{[A.81]}$$

where $N_{i,t}$ denotes the number of type-i homeowners who put their houses up for sale at date t. The same steps used in deriving equation (A.30) in appendix A.5 can be applied to show that the $N_{i,t}$ are given by:

$$N_{i,t} = a_i \sigma_{i,t} - \theta_i a_i \delta_i^{\lambda} x_{i,t}^{-\lambda} v \int_{\tau \to -\infty}^{t} e^{-a_i(1-\delta_i^{\lambda})(t-\tau)} u_\tau \mathrm{d}\tau, \qquad \text{[A.82]}$$

which holds for $i = 1, \ldots, q$. The aggregate number of houses newly put up for sale is $N_t = \sum_{i=1}^{q} N_{i,t}$, and by using equation (A.82), the moving rate $n_t = N_t/(1-u_t)$ is:

$$n_t = \frac{\sum_{i=1}^{q} a_i \sigma_{i,t}}{1-u_t} - \frac{v \sum_{i=1}^{q} \theta_i a_i \delta_i^{\lambda} x_{i,t}^{-\lambda} \int_{\tau \to -\infty}^{t} e^{-a_i(1-\delta_i^{\lambda})(t-\tau)} u_\tau \mathrm{d}\tau}{1-u_t}. \qquad \text{[A.83]}$$

Equilibrium

As in section 4, if the parameters are constant over time, the equilibrium of the model has constant moving and transaction thresholds, so time subscripts are dropped in what follows.

Using the definition of the moving threshold x_i in (A.79) and evaluating the type-i homeowner's value function $H_i(\epsilon)$ at $\epsilon = x_i$:

$$(r + a_i) H_i(x_i) = \xi x_i - D + a_i J, \qquad \text{[A.84]}$$

which holds for all $i = 1, \ldots, q$. Together with (A.79) this implies:

$$x_i = \frac{rJ + D}{\xi} = x, \qquad \text{[A.85]}$$

which means that all types of homeowners share a common moving threshold $x_i = x$ in equilibrium. The value J can be written in terms of parameters and the common threshold x:

$$J = \frac{\xi x - D}{r}. \tag{A.86}$$

As in the basic model, idiosyncratic shocks are assumed large enough so that all marginal homebuyers would move upon receiving a shock. This must be true for all types of homeowner, that is, $\delta_i y < x$. It then follows by evaluating the type-i homeowner's value function (A.78) at $\epsilon = y$ that:

$$(r + a_i)H_i(y) = \xi y - D + a_i J.$$

Substituting J from (A.86) and rearranging yields:

$$(r + a_i)H_i(y) = \xi y - D + a_i \frac{\xi x - D}{r} = \xi(y - x) + \left(\frac{a_i + r}{r}\right)(\xi x - D),$$

and thus the type-i homeowner value function evaluated at the transaction threshold is:

$$H_i(y) = \frac{\xi(y - x)}{r + a_i} + \frac{\xi x - D}{r}.$$

Averaging across all types of homeowners:

$$H(y) = \xi(y - x)\sum_{i=1}^{q} \frac{\theta_i}{r + a_i} + \frac{\xi x - D}{r}. \tag{A.87}$$

An equation linking the moving and transaction thresholds x and y can be derived from (14), (A.86), and (A.87):

$$\xi(y - x)\sum_{i=1}^{q} \frac{\theta_i}{r + a_i} + \frac{\xi x - D}{r} = \frac{\xi x - D}{r} + C,$$

which can be rearranged as follows:

$$y - x = \frac{\left(\frac{1}{\sum_{i=1}^{q} \frac{\theta_i}{r + a_i}}\right) C}{\xi}. \tag{A.88}$$

This reduces to the equilibrium condition (26) of the basic model if there is only one type of homeowner ($q = 1$). Another equilibrium condition linking x and y can be obtained by combining equation (14) for the transaction threshold with the Bellman equation (15) for the value J:

$$rJ = -F - D + vy^{-\lambda} \int_{\epsilon = y}^{\infty} \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)} (H(\epsilon) - H(y))\,\mathrm{d}\epsilon, \tag{A.89}$$

which uses the Pareto distribution of new match quality ϵ from (10). Given the expression for $H(\epsilon)$ in (A.80), this equation can be rewritten as follows:

$$rJ = -F - D + vy^{-\lambda}\sum_{i=1}^{q} \theta_i \Psi_i(y), \quad \text{where } \Psi_i(z) = \int_{\epsilon = z}^{\infty} \frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)} (H_i(\epsilon) - H_i(z))\mathrm{d}\epsilon. \tag{A.90}$$

Expressions for the functions $\Psi_i(z)$ defined above when evaluated at $z = y$ can be found explicitly using the same steps used to derive equation (A.18) in appendix A.3:

$$\varPsi_i(y) = \frac{\xi}{(\lambda-1)(r+a_i)}\left(y + \frac{a_i\delta_i^\lambda y^\lambda x^{1-\lambda}}{r+a_i(1-\delta_i^\lambda)}\right). \tag{A.91}$$

By substituting (A.86) and (A.91) into the Bellman equation from (A.90):

$$\xi x - D = -F - D + vy^{-\lambda}\sum_{i=1}^{q}\frac{\xi\theta_i}{(\lambda-1)(r+a_i)}\left(y + \frac{a_i\delta_i^\lambda y^\lambda x^{1-\lambda}}{r+a_i(1-\delta_i^\lambda)}\right),$$

which simplifies to:

$$x = \frac{v\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}\left(y^{1-\lambda} + \frac{a_i\delta_i^\lambda}{r+a_i(1-\delta_i^\lambda)}x^{1-\lambda}\right)}{\lambda-1} - \frac{F}{\xi}. \tag{A.92}$$

This is the second equilibrium condition involving x and y. It reduces to the equivalent condition (29) from the basic model when $q=1$.

The equilibrium thresholds x and y are solutions of the equations (A.88) and (A.92). By substituting for x in (A.92) using (A.88), any equilibrium value of y is a solution of the equation $\mathcal{I}(y)=0$, where the function $\mathcal{I}(y)$ is given below:

$$\mathcal{I}(y) = \frac{v}{\lambda-1}\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}\left(y^{1-\lambda} + \frac{a_i\delta_i^\lambda}{r+a_i(1-\delta_i^\lambda)}\left(y - \left(\frac{1}{\sum_{j=1}^{q}\frac{\theta_j}{r+a_j}}\right)\frac{C}{\xi}\right)^{1-\lambda}\right) - y + \left(\frac{1}{\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}}\right)\frac{C}{\xi} - \frac{F}{\xi},$$

which is the equivalent of (A.20) when $q=1$. This function is such that $\mathcal{I}'(y)<0$ and $\lim_{y\to\infty}\mathcal{I}(y)=-\infty$ because $\lambda>1$. Since $\mathcal{I}(y)$ is strictly decreasing in y, any solution (if it exists) must be unique. A solution must satisfy $x>0$, $y>1$, and $\delta_i y < x$ for all $i=1,\ldots,q$. Generalizing the argument used to derive (A.21), the inequalities involving y are equivalent to:

$$y > \max\left\{1, \frac{C}{\left(1-\max_{i\in\{1,\ldots,q\}}\delta_i\right)\xi}\left(\frac{1}{\sum_{j=1}^{q}\frac{\theta_j}{r+a_j}}\right)\right\}.$$

Thus, there exists a solution of the equation $\mathcal{I}(y)=0$ (which is unique) if and only if the function $\mathcal{I}(y)$ is positive when evaluated at the right-hand side of the inequality above. The same argument following (A.22) can be used to show that the inequality for y implies $x>0$, so all the requirements for an equilibrium are satisfied.

Average transaction price

Using the Pareto distribution from (10) and the definitions of $C=C_\text{b}+C_\text{u}$ and $\kappa=C_\text{u}/C$, the average transaction price in (16) can be written as:

$$P = \omega\int_{\epsilon=y}^{\infty}\frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)}(H(\epsilon)-H(y))\mathrm{d}\epsilon + \omega H(y) + (\kappa-\omega)C + \frac{\omega F-(1-\omega)D}{r}.$$

Using the definition of the transaction threshold from (14) and equations (A.86) and (A.89):

$$P = \omega\frac{(\xi x - D)+F+D}{vy^{-\lambda}} + \omega\left(C + \frac{\xi x - D}{r}\right) + (\kappa-\omega)C + \frac{\omega F-(1-\omega)D}{r},$$

which can be simplified to:

$$P = \kappa C - \frac{D}{r} + \omega \left(\frac{1}{r} + \frac{y^\lambda}{v} \right) (\xi x + F). \qquad \text{[A.93]}$$

Given a moving threshold x, this is the same equation for P as (34) in the basic model.

Average moving rate

Consider a steady state where $u_t = u$, $\sigma_{i,t} = \sigma_i$, and $N_{i,t} = N_i$ for all t. Using the result $x_{i,t} = x_i = x$ from (A.85), equation (A.82) for the number of houses put up for sale by type-i homeowners becomes:

$$N_i = a_i \sigma_i - \theta_i a_i \delta_i^\lambda x^{-\lambda} v \left(\frac{u}{a_i(1-\delta_i^\lambda)} \right).$$

In steady state, equation (A.81) implies $N_i = \theta_i S = \theta_i s u$, where $S = su$ is the number of transactions. Substituting into the equation above and dividing both sides by a_i:

$$su \frac{\theta_i}{a_i} = \sigma_i - x^{-\lambda} v u \theta_i \frac{\delta_i^\lambda}{a_i(1-\delta_i^\lambda)}.$$

Sum over all $i = 1, \ldots, q$ and make use of the link between σ_i and u from (A.81):

$$su \sum_{i=1}^q \frac{\theta_i}{a_i} = 1 - u - x^{-\lambda} v u \sum_{i=1}^q \theta_i \frac{\delta_i^\lambda}{a_i(1-\delta_i^\lambda)}.$$

Dividing both sides by $1-u$ and noting that (33) implies $u/(1-u) = n/s$:

$$s \frac{n}{s} \sum_{i=1}^q \frac{\theta_i}{a_i} = 1 - \frac{n x^{-\lambda} v}{s} \sum_{i=1}^q \theta_i \frac{\delta_i^\lambda}{a_i(1-\delta_i^\lambda)},$$

and substituting the expression for the sales rate s from (31):

$$n \sum_{i=1}^q \frac{\theta_i}{a_i} = 1 - n \left(\frac{y}{x} \right)^\lambda \sum_{i=1}^q \theta_i \frac{\delta_i^\lambda}{a_i(1-\delta_i^\lambda)}.$$

This can be rearranged to give a formula for the steady-state moving rate n:

$$n = \frac{1}{\sum_{i=1}^q \frac{\theta_i}{a_i} + \left(\frac{y}{x}\right)^\lambda \sum_{i=1}^q \theta_i \frac{\delta_i^\lambda}{a_i(1-\delta_i^\lambda)}}. \qquad \text{[A.94]}$$

The basic model is a special case of this when $q = 1$.

A.9 The hazard function and the elasticity of the moving rate

The analysis here considers the general model with heterogeneity in the distributions of idiosyncratic shocks across q types of homeowners. The results are applicable to the basic model by considering the special case $q = 1$.

The distribution of time spent in a house

Consider an equilibrium where parameters are expected to remain constant. In this case, the moving and transaction thresholds x and y are constant over time. Let $\psi_i(T)$ denote the survival function for new matches of type-i homeowners, in the sense of the fraction of matches forming at time t that survive until at least $t+T$. Each cohort starts with a match quality distribution $\epsilon \sim \text{Pareto}(y; \lambda)$ at $T = 0$. Now consider some $T > 0$. Moving occurs only if the value of ϵ after shocks have occurred (ϵ') is such that $\epsilon' < x$. Shocks arrive at a Poisson rate a_i, so the number k of shocks that would occur to a match over an interval of time T has a Poisson($a_i T$) distribution, which means the probability that j shocks occur is $e^{-a_i T}(a_i T)^j/j!$. If no shocks occur, $\epsilon' = \epsilon$, so no moving occurs. If $j \geq 1$ shocks have occurred then $\epsilon' = \delta_i^j \epsilon$, where ϵ is the initial draw of match quality. These matches survive only if $\epsilon' \geq x$, that is, $\epsilon \geq x/\delta_i^j$. Since the original values of ϵ are drawn from a Pareto distribution truncated at $\epsilon = y$ with shape parameter λ, this probability is $((x/\delta_i^j)/y)^{-\lambda}$ (this expression is valid for all $j \geq 1$ since $\delta y < x$). Therefore, the survival function $\psi_i(T)$ is given by:

$$\psi_i(T) = e^{-a_i T} + \sum_{j=1}^{\infty} e^{-a_i T} \frac{(a_i T)^j}{j!} \left(\frac{x/\delta_i^j}{y} \right)^{-\lambda}$$
$$= e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} e^{-a_i T} \sum_{j=1}^{\infty} \frac{(a_i \delta_i^{\lambda} T)^j}{j!} = e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} e^{-a_i T} e^{a_i \delta_i^{\lambda} T},$$

where the final equality uses the (globally convergent) series expansion of the exponential function. Conditional on each type $i = 1, \ldots, q$, the survival function $\psi_i(T)$ is thus:

$$\psi_i(T) = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} e^{-a_i(1-\delta_i^{\lambda})T}.$$

Given the random assignment of types with probabilities $\theta_1, \ldots, \theta_q$, the survival function $\psi(T)$ for all members of a cohort of new homeowners is:

$$\psi(T) = \sum_{i=1}^{q} \theta_i \psi_i(T) = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \sum_{i=1}^{q} \theta_i e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \theta_i e^{-a_i(1-\delta_i^{\lambda})T}, \qquad \text{[A.95]}$$

observing that $\psi(0) = 1$.

For new matches, the distribution $\mu(T)$ of the time T until the next move can also be obtained from the survival function $\psi(T)$ using $\mu(T) = -\psi'(T)$. Hence, by taking the derivative of the survival function $\psi(T)$ from (A.95) with respect to the duration T:

$$\mu(T) = -\psi'(T) = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \sum_{i=1}^{q} \theta_i a_i e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \theta_i a_i (1 - \delta_i^{\lambda}) e^{-a_i(1-\delta_i^{\lambda})T}. \qquad \text{[A.96]}$$

The definition of the hazard function $h(T)$ is the proportional decrease in the survival function for a small change in duration, that is, $h(T) = -\psi'(T)/\psi(T)$. Using (A.95) and (A.96):

$$h(T) = \frac{\left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \sum_{i=1}^{q} \theta_i a_i e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \theta_i a_i (1 - \delta_i^{\lambda}) e^{-a_i(1-\delta_i^{\lambda})T}}{\left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \sum_{i=1}^{q} \theta_i e^{-a_i T} + \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \theta_i e^{-a_i(1-\delta_i^{\lambda})T}}, \qquad \text{[A.97]}$$

and by simplifying this expression, equation (35) is confirmed.

The expected time T_{n} between moves is the expected value of the probability distribution $\mu(T)$:

$$T_{\mathrm{n}} = \int_{T=0}^{\infty} T\mu(T)\mathrm{d}T = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \sum_{i=1}^{q} \frac{\theta_i}{a_i} + \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \frac{\theta_i}{a_i(1-\delta_i^{\lambda})}, \qquad \text{[A.98]}$$

which can be simplified to derive the expression for T_{n} in (38). It is also the case that T_{n} is equal to the reciprocal of the average moving rate n, as can be seen by comparing equations (A.94) and (A.98).

The elasticity of the moving rate

Imposing a common moving threshold $x_t = x_{i,t}$ for all $i = 1, \ldots, q$ (as shown in appendix A.8) and differentiating the moving rate n_t from (A.83) with respect to x_t:

$$\frac{\partial n_t}{\partial x_t} = \frac{\lambda x_t^{-\lambda-1} v \sum_{i=1}^{q} \theta_i a_i \delta_i^{\lambda} \int_{\tau\to-\infty}^{t} e^{-a_i(1-\delta_i^{\lambda})(t-\tau)} u_\tau \mathrm{d}\tau}{1-u_t}.$$

Let η denote the elasticity of the moving rate n_t with respect to x_t evaluated at the steady state. The partial derivative above implies:

$$\eta = \left.\frac{\partial \log n_t}{\partial \log x_t}\right|_{u_t=u, n_t=n} = \frac{\lambda x^{-\lambda} v u \sum_{i=1}^{q} \theta_i a_i \delta_i^{\lambda} \int_{\tau\to-\infty}^{t} e^{-a_i(1-\delta_i^{\lambda})(t-\tau)} \mathrm{d}\tau}{n(1-u)}. \qquad \text{[A.99]}$$

Note that the integrals appearing in the expression above are:

$$\int_{\tau\to-\infty}^{t} e^{-a_i(1-\delta_i^{\lambda})(t-\tau)} \mathrm{d}\tau = \frac{1}{a_i(1-\delta_i^{\lambda})},$$

and by substituting these into (A.99):

$$\eta = \frac{\lambda x^{-\lambda} v \sum_{i=1}^{q} \theta_i \frac{a_i \delta_i^{\lambda}}{a_i(1-\delta_i^{\lambda})}}{n(1-u)} = \lambda x^{-\lambda} v \frac{u}{n(1-u)} \sum_{i=1}^{q} \theta_i \frac{\delta_i^{\lambda}}{1-\delta_i^{\lambda}}.$$

Equation (33) implies $u/(1-u) = n/s$, and by substituting this and the expression for s from (31) into the above:

$$\eta = \lambda \frac{v x^{-\lambda} n}{n v y^{-\lambda}} \sum_{i=1}^{q} \theta_i \frac{\delta_i^{\lambda}}{1-\delta_i^{\lambda}} = \lambda \left(\frac{y}{x}\right)^{\lambda} \sum_{i=1}^{q} \theta_i \frac{\delta_i^{\lambda}}{1-\delta_i^{\lambda}}. \qquad \text{[A.100]}$$

This confirms the formula for η given in (37).

A.10 Estimates of time-to-sell

This section provides further discussion of alternative estimates of time-to-sell. Using the 'Profile of Buyers and Sellers' survey collected by NAR, Genesove and Han (2012) report that for the time period 1987–2008, the average time-to-sell is 7.6 weeks, the average time-to-buy is 8.1 weeks, and the average number of homes visited by buyers is 9.9. They also discuss other surveys that have reported similar findings.

These numbers are significantly smaller than the 6 months estimate of time-to-sell derived from the NAR data on sales and inventories. However, the estimates of time-to-sell and time-to-buy derived from survey data are likely to be an underestimate of the actual time a new buyer or seller would expect to spend in the housing market. The reason is that the survey data include only those buyers and sellers who have successfully completed a house purchase or sale, while the proportion of buyers or sellers who withdraw from the market (at least for some time) without a completed transaction is substantial.

To understand the impact withdrawals can have on estimates of time-to-sell, suppose houses on the market have sales rate s and withdrawal rate w as in section 2.3. Let $\tilde{T}_{\mathrm{s}}$ denote the average time taken to sell among those houses that are successfully sold, measuring the time on the market from the start of the most recent listing. Let T_{s} denote the average of the total time spent on the market by houses that are successfully sold, ignoring the times between listings when houses are off the market. The two measures of time-to-sell are:

$$\tilde{T}_{\mathrm{s}} = \frac{1}{s+w}, \quad T_{\mathrm{s}} = \frac{1}{s}, \quad \text{and hence } T_{\mathrm{s}} = \frac{\tilde{T}_{\mathrm{s}}}{1-\phi}.$$

The final equation gives the relationship between the two measures in terms of the fraction ϕ of houses eventually withdrawn from sale. Estimates of time-to-sell based on survey data are typically measuring $\tilde{T}_{\mathrm{s}}$. On the other hand, the NAR data provide an estimate of the sales rate s, and taking the reciprocal yields a measure of T_s.

The studies by Anenberg and Laufer (2017) and Carrillo and Williams (2015) discussed in section 2.3 suggest that the fraction of properties eventually withdrawn from sale lies between 50% and 60%. Using these numbers, the formula above suggests that estimates of T_s should be around 2 to 2.5 times higher than estimates of $\tilde{T}_s$. This simple observation goes a long way in reconciling the magnitudes of the different estimates. Carrillo and Williams (2015) also show directly that controlling for withdrawals substantially increases the estimated value of average time-to-sell. Similarly, in comparing the efficiency of different platforms for selling properties, Hendel, Nevo and Ortalo-Magné (2009) explicitly control for withdrawals and report a time-to-sell of 15 weeks (using the Multiple Listing Service for the city of Madison).

An alternative approach to estimating time-to-sell unaffected by withdrawals is to look at the average duration for which a home is vacant using data from the American Housing Survey. In the years 2001–2005, the mean duration of a vacancy was 7–8 months. However, that number is likely to be an overestimate of the expected time-to-sell because it is based on houses that are 'vacant for sale'. Houses that are for sale but currently occupied would not be counted in this calculation of average duration. Another approach that avoids the problem of withdrawals is to look at the average time taken to sell newly built houses. Díaz and Jerez (2013) use the Census Bureau 'New Residential Sales' report to find that the median number of months taken to sell a newly built house is 5.9 (for the period 1991–2012). This is only slightly shorter than the average of the time-to-sell number constructed using NAR data on existing single-family homes, but there is reason to believe that newly built homes should sell faster than existing homes owing to greater advertising expenditure and differences in the target groups of buyers.

A.11 Calibration of the model with heterogeneity

This section shows how the parameters of the model with heterogeneity can be set to give the best fit to the empirical aggregate hazard function for moving house, as well as matching other empirical targets. There are parameters q, $\{\theta_i\}$, $\{a_i\}$, and $\{\delta_i\}$ that describe the distributions of idiosyncratic shocks faced by homeowners, and parameters λ, v, C, F, D, κ, ω, and r that are related to other aspects of the model.

The number of types q determines the dimension of the parameter space, and as discussed in section 5.2, this can be chosen to be large enough to give a sufficiently good fit to the aggregate hazard function. Here, q is taken as given. Three of the other parameters (κ, ω, and r) are also set directly. The remaining parameters are chosen to minimize a weighted sum of squared deviations between the empirical hazard function and the hazard function $h(T)$ implied by the model, and to match five empirical targets exactly: time-to-sell T_s, viewings per sale V_s, the transaction cost to price ratio c, the flow search cost to price ratio f, and the flow maintenance cost to price ratio d.

The calibration procedure has two stages. First, a numerical search over parameters θ_i, a_i, and δ_i^λ to find the solution to:[2]

$$\min_{\substack{\{\theta_i\}_{i=1}^q, \{a_i\}_{i=1}^q, \{\delta_i^\lambda\}_{i=1}^q \\ \text{s.t. } \sum_{i=1}^q \theta_i = 1}} \sum_T \varpi(T)(\hat{h}(T) - h(T))^2, \tag{A.101}$$

where T denotes a duration for which data on the hazard function is available, $\hat{h}(T)$ is the estimated hazard rate described in section 5.2.1, $h(T)$ is the model-implied hazard function given in (35), and $\varpi(T)$ is the weight assigned to duration T. The weights $\varpi(T)$ are assumed to be proportional to the number of data points available to calculate the empirical hazard rate $\hat{h}(T)$. An alternative weighting scheme makes the weights proportional to the model-implied survival function $\psi(T)$ from (A.95) (initialized with the parameters obtained from the first weighting scheme, and then iterating until convergence). There are $3q - 1$ independent parameters given that $\sum_{i=1}^q \theta_i = 1$, and the parameters must satisfy the restrictions $0 < \theta_i \leq 1$, $a_i > 0$, and $0 \leq \delta_i^\lambda < 1$. The procedure specifies δ_i^λ rather than δ_i because this turns out to be more convenient, and the admissible range of δ_i^λ is the same as that of δ_i.

[2]The numerical method used is to draw many initial conditions at random from the parameter space, and then perform a search for the minimum using a simplex algorithm starting from each initial condition, before finally choosing the parameter vector with the smallest value of the objective function from among all the searches.

To compute the model-implied hazard function $h(T)$ using (35) it is necessary to know the value of the parameter λ and the values of the endogenous variables x and y in addition to $\{\theta_i\}$, $\{a_i\}$, and $\{\delta_i^\lambda\}$. Conditional on the parameters given at the first stage, the values of λ, x, and y are pinned down by the requirement of matching the other five empirical targets: T_{s}, V_{s}, c, f, and d. This is the second stage of the calibration procedure.

By dividing the cost parameters C, F, and D (search, transactions, and maintenance) by the average transaction price P from equation (A.93), the model's predictions for the targets c, f, and d are:

$$c = \frac{\frac{C}{\xi}}{\kappa\frac{C}{\xi} - \frac{D}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}; \tag{A.102a}$$

$$f = \frac{\frac{F}{\xi}}{\kappa\frac{C}{\xi} - \frac{D}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}; \tag{A.102b}$$

$$d = \frac{\frac{D}{\xi}}{\kappa\frac{C}{\xi} - \frac{D}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}. \tag{A.102c}$$

Note that the model contains one other parameter ξ in addition to those listed earlier, but in all equations determining observables, ξ enters only as a ratio to other parameters. The parameter ξ is therefore normalized to $\xi = 1$. Now take equation (A.93) for the average price and divide both sides by P:

$$\kappa c - \frac{d}{r} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(\frac{x}{P} + f\right) = 1.$$

Using the expression for T_{s} in (A.26), the equation above can be solved for x/P as follows:

$$\frac{x}{P} = \frac{1 - \kappa c + \frac{d}{r}}{\omega\left(\frac{1}{r} + T_{\mathrm{s}}\right)} - f. \tag{A.103}$$

Now take the linear equation (A.88) involving the thresholds x and y and divide both sides by P (recalling that $\xi = 1$):

$$\frac{y}{P} = \frac{x}{P} + \frac{c}{\sum_{i=1}^{q} \frac{\theta_i}{r+a_i}} = \frac{1 - \kappa c + \frac{d}{r}}{\omega\left(\frac{1}{r} + T_{\mathrm{s}}\right)} + \frac{c}{\sum_{i=1}^{q} \frac{\theta_i}{r+a_i}} - f, \tag{A.104}$$

and then dividing both sides by x/P and using (A.103):

$$\frac{y}{x} = \frac{y/P}{x/P} = 1 + \frac{\frac{c}{\sum_{i=1}^{q} \frac{\theta_i}{r+a_i}}}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)} - f}. \tag{A.105}$$

Dividing both sides of the second equation (A.92) for the thresholds x and y by $P\sum_{i=1}\theta_i/(r+a_i)$ and rearranging leads to:

$$\frac{\sum_{i=1}^{q} \frac{\theta_i}{r+a_i}\left(1 + \frac{a_i\delta_i^\lambda}{r+a_i(1-\delta_i^\lambda)}\left(\frac{y}{x}\right)^{\lambda-1}\right)}{(\lambda-1)\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}} = \frac{y^\lambda\left(\frac{x}{P} + f\right)}{v\frac{y}{P}\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}}.$$

Using $T_{\mathrm{s}} = y^\lambda/v$ from (A.26) together with (A.103), (A.104), and (A.105), this equation can be written as follows:

$$\varPhi(\lambda) = \frac{1 + \aleph_\delta \aleph_{yx}^{\lambda-1}}{\lambda - 1} - \aleph = \frac{1 + \aleph_\delta e^{(\log\aleph_{yx})(\lambda-1)}}{\lambda-1} - \aleph = 0, \tag{A.106}$$

where the coefficients $\aleph$, $\aleph_\delta$, and $\aleph_{yx}$ can be derived from the calibration targets:

$$\aleph = \frac{\frac{\left(1-\kappa c+\frac{d}{r}\right)T_\mathrm{s}}{\omega\left(\frac{1}{r}+T_\mathrm{s}\right)}}{c+\left(\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_\mathrm{s}\right)}-f\right)\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}}, \quad \aleph_\delta = \frac{\sum_{i=1}^{q}\frac{\theta_i a_i \delta_i^\lambda}{r+a_i(1-\delta_i^\lambda)}}{\sum_{i=1}^{q}\frac{\theta}{r+a_i}}, \quad \text{and} \quad \aleph_{yx} = 1+\frac{\frac{c}{\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}}}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_\mathrm{s}\right)}-f}. \quad \text{[A.107]}$$

Observe that the function $\Phi(\lambda)$ becomes an arbitrarily large positive number as λ tends to 1, and since $\aleph_{yx} > 1$, it is also the case that $\Phi(\lambda)$ eventually becomes arbitrarily large as λ increases. Note that the derivative of $\Phi(\lambda)$ is:

$$\Phi'(\lambda) = \frac{\left((\lambda-1)^2-1\right)\aleph_\delta e^{(\log\aleph_{yx})(\lambda-1)}-1}{(\lambda-1)^2}.$$

The denominator of this expression is always positive given that $\lambda > 1$. The sign of the numerator depends only on $\aleph_\delta((\lambda-1)^2-1)-e^{-(\log\aleph_{yx})(\lambda-1)}$, which is strictly increasing in λ for all $\lambda > 1$. Since $\Phi'(1) < 0$, it follows that the function $\Phi(\lambda)$ is initially decreasing in λ and subsequently increasing in λ after passing a threshold value of λ. For any $\lambda > 1$, it must be the case that $e^{(\log\aleph_{yx})(\lambda-1)} > 1+(\log\aleph_{yx})(\lambda-1)$ because $\log\aleph_{yx} > 0$. This inequality implies the function $\Phi(\lambda)$ from (A.106) has the following lower bound:

$$\Phi(\lambda) > \frac{1+\aleph_\delta}{\lambda-1}-(\aleph-\aleph_\delta\log\aleph_{yx}). \quad \text{[A.108]}$$

As a solution for λ requires $\Phi(\lambda) = 0$, a necessary condition for a solution to exist is $\aleph > \aleph_\delta\log\aleph_{yx}$. When this condition is satisfied, the inequality above implies a lower bound $\underline{\lambda}$ for a solution (if one exists):

$$\lambda > \underline{\lambda}, \quad \text{where} \quad \underline{\lambda} = 1+\frac{1+\aleph_\delta}{\aleph-\aleph_\delta\log\aleph_{yx}},$$

which follows because the bound (A.108) on $\Phi(\lambda)$ is decreasing in λ. The parameter λ must also satisfy an upper bound. It is required that $\delta_i y < x$ for all $i = 1,\ldots,q$. This is equivalent to $(y/x)^\lambda\delta_i^\lambda < 1$ for all i, and hence $\aleph_{yx}^\lambda \max\delta_i^\lambda < 1$ since $y/x = \aleph_{yx}$. By taking logarithms of both sides, this implies an upper bound for λ:

$$\lambda < \overline{\lambda}, \quad \text{where} \quad \overline{\lambda} = \frac{-\log\max\delta_i^\lambda}{\log\aleph_{yx}}, \quad \text{[A.109]}$$

with $\aleph_{yx} > 1$ taken from (A.107). Given the properties of the $\Phi(\lambda)$ function established above, the necessary and sufficient conditions for the existence of a unique solution $\lambda > 1$ to the equation $\Phi(\lambda) = 0$ with $1 < \lambda < \overline{\lambda}$ are that $\underline{\lambda} > 1$ and $\Phi(\overline{\lambda}) < 0$. When these conditions are met, the solution for λ can be found by searching the interval $(\underline{\lambda}, \overline{\lambda})$ because $\Phi(\underline{\lambda}) > 0$ and $\Phi(\overline{\lambda}) < 0$.

With the solution of (A.106) for λ, the transaction threshold y can be obtained from viewings per sale V_s:

$$y = V_\mathrm{s}^{\frac{1}{\lambda}}, \quad \text{[A.110]}$$

and the moving threshold x can be derived from the above along with equation (A.105):

$$x = \frac{V_\mathrm{s}^{\frac{1}{\lambda}}}{1+\frac{\frac{c}{\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}}}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_\mathrm{s}\right)}-f}}. \quad \text{[A.111]}$$

With λ, x, and y, the hazard function $h(T)$ can be computed using the formula in (35) given the values of $\{\theta_i\}$, $\{a_i\}$, and $\{\delta_i^\lambda\}$. This allows the weighted sum of squared deviations (A.101) to be computed, and hence the calibration procedure can be implemented as described above.

Once θ_i, a_i, and δ_i^λ have been chosen to minimize (A.101), the parameters $\{\delta_i\}$ can be obtained from $\{\delta_i^\lambda\}$ using the value of λ that solves (A.106) and $\delta_i = (\delta_i^\lambda)^{1/\lambda}$. The remaining parameters v, C, F, and D can be obtained as follows. Using (A.25) and (A.26), the ratio of viewings per sale V_s and time to sell T_s determines the meeting rate v:

$$v = \frac{V_\mathrm{s}}{T_\mathrm{s}}. \tag{A.112}$$

Combining equations (A.104) and (A.110) leads to the following expression for P:

$$P = \frac{V_\mathrm{s}^{\frac{1}{\lambda}}}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_\mathrm{s}\right)} + \frac{c}{\sum_{i=1}^{q}\frac{\theta_i}{r+a_i}} - f}, \tag{A.113}$$

and this can be used to obtain the parameters C, F, and D using $C = cP$, $F = fP$, and $D = dP$.

A.12 Calibration of the basic model without heterogeneity

This section shows how the 10 parameters a, δ, λ, v, C, F, D, κ, ω, and r can be determined in the basic version of the model (with no heterogeneity in idiosyncratic shock distributions, that is, $q = 1$). When $q = 1$, the general expressions for the elasticity of the moving rate η and time-to-move T_n from (37) and (38) reduce to:

$$\eta = \lambda\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda; \tag{A.114}$$

$$T_\mathrm{n} = \frac{1+\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}{a};. \tag{A.115}$$

Three of the parameters (κ, ω, and r) are set directly. The other seven are obtained indirectly from five calibration targets: time-to-sell T_s, viewings per sale V_s, the transaction cost to price ratio c, the flow search cost to price ratio f, and the flow maintenance cost to price ratio d, together with the two targets derived from information contained in the hazard function, namely the steady-state elasticity η of the moving rate with respect to the moving threshold and time-to-move T_n.

The price equation (A.29) is identical to (A.93) in the model with heterogeneity, so the expressions in (A.102a)–(A.102c) for the ratios of costs (search, transactions, and maintenance) to the average price are also valid here. The model contains one other parameter ξ, but as in appendix A.11, in all equations determining observables, ξ enters only as a ratio to other parameters, hence it can be normalized to $\xi = 1$.

The calibration method begins by setting κ, ω, and r directly. Next, consider a guess for T_δ, the expected time until an idiosyncratic shock occurs. This conjecture determines the parameter a using:

$$a = \frac{1}{T_\delta}. \tag{A.116}$$

The admissible range for T_δ is $0 < T_\delta < T_\mathrm{n}$.

Using equation (A.115) for time-to-move T_n and the expressions for η and T_δ from equations (A.114) and (A.116):

$$T_\mathrm{n} = \left(1+\frac{\eta}{\lambda}\right)T_\delta. \tag{A.117}$$

Rearranging equation (A.117) and using the calibration targets T_n and η and the conjecture for T_δ:

$$\lambda = \frac{\eta T_\delta}{T_\mathrm{n}-T_\delta}. \tag{A.118}$$

This yields the value of the parameter λ.

Since the equation for the average price is the same as the model with heterogeneity, the same steps used to derive the expression for x/P in (A.103) are valid here. Now take the linear equation (A.10) involving the thresholds x and y and divide both sides by P, recalling that $\xi = 1$:

$$\frac{y}{P} = \frac{x}{P} + (r+a)c = \frac{1 - \kappa c + \frac{d}{r}}{\omega\left(\frac{1}{r} + T_{\mathrm{s}}\right)} + (r+a)c - f, \qquad \text{[A.119]}$$

which uses the formula for x/P from (A.103). Dividing both sides by x/P and using (A.103) again:

$$\frac{y}{x} = \frac{y/P}{x/P} = 1 + \frac{(r+a)c}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)} - f}. \qquad \text{[A.120]}$$

This gives the ratio y/x implied by the calibration targets. Equation (A.114) for η can be rearranged to obtain an expression for δ^λ:

$$\delta^\lambda = \frac{\eta}{\eta + \lambda\left(\frac{y}{x}\right)^\lambda},$$

and hence the value of the parameter δ is:

$$\delta = \left(\frac{\eta}{\eta + \lambda\left(\frac{y}{x}\right)^\lambda}\right)^{\frac{1}{\lambda}}. \qquad \text{[A.121]}$$

Given λ, the transaction threshold y must satisfy equation (A.110) in terms of viewings per sale V_{s} as (A.25) holds for the model with and without heterogeneity. An expression for the moving threshold x can be derived using (A.110) and (A.120):

$$x = \frac{V_{\mathrm{s}}^{\frac{1}{\lambda}}}{1 + \frac{(r+a)c}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)} - f}}. \qquad \text{[A.122]}$$

The parameter υ must satisfy (A.112) in terms of viewings per sale V_{s} and time to sell T_{s} given that equations (A.25) and (A.26) are the same with and without heterogeneity. Combining equations (A.119) and (A.110) leads to the following expression for P:

$$P = \frac{V_{\mathrm{s}}^{\frac{1}{\lambda}}}{\frac{1-\kappa c+\frac{d}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)} + (r+a)c - f},$$

and this can be used to obtain the parameters C, F, and D using $C = cP$, $F = fP$, and $D = dP$. Finally, equation (A.19) must also hold, which requires:

$$x + \frac{F}{\xi} = \frac{\upsilon}{(\lambda-1)(r+a)}\left(y^{1-\lambda} + \frac{a\delta^\lambda}{r + a(1-\delta^\lambda)}x^{1-\lambda}\right),$$

and this is used to verify the initial conjecture for T_δ.

A.13 Productivity and interest rates

Suppose that a family's flow utility is $\mathcal{C}_t^{1-\upsilon}\mathcal{H}_t^{\upsilon}$, where $\mathcal{C}_t$ denotes consumption and $\mathcal{H}_t$ denotes housing, and where υ indicates the importance of housing in the utility function ($0 < \upsilon < 1$). This adds non-housing goods to the model and replaces the flow utility $\xi\epsilon$ assumed earlier. The form of the flow utility function assumes complementarity between consumption and housing services. The housing variable $\mathcal{H}_t$ that enters the utility function is equal to the match quality ϵ of a family with its house, and the evolution of this variable in response to idiosyncratic shocks and moving and transaction decisions is the same as before. The discount rate for future utility flows is the rate of pure time preference ϱ. The lifetime utility function from time T onwards is therefore:

$$\mathcal{U}_T = \int_{t=T}^{\infty} e^{-\varrho(t-T)} \mathcal{C}_t^{1-\upsilon} \mathcal{H}_t^{\upsilon} \mathrm{d}t. \quad \text{[A.123]}$$

Suppose there are complete financial markets for securities with consumption payoffs contingent on any state of the world, and suppose all families receive the same real income (with no aggregate risk) and initially all have equal financial wealth. Note that only state-contingent consumption, not housing services, can be traded in these markets. With complete financial markets there is full consumption insurance of idiosyncratic risk coming from shocks to match quality and the uncertainties in the search process, implying that the marginal utility of consumption must be equalized across all families. The marginal utility of consumption is $X_t^{-\upsilon}$, where $X_t = \mathcal{C}_t/\mathcal{H}_t$ is the ratio of consumption to housing match quality. If r is the real interest rate (in terms of consumption goods) then maximization of utility (A.123) subject to the lifetime budget constraint requires that the following consumption Euler equation holds:

$$\upsilon \frac{\dot{X}_t}{X_t} = r - \varrho. \quad \text{[A.124]}$$

In equilibrium, the sum of consumption $\mathcal{C}_t$ across all families must be equal to aggregate real income Y_t, which is assumed to be an exogenous endowment growing at rate g over time. Given equalization of $X_t = \mathcal{C}_t/\mathcal{H}_t$ across all families at a point in time and given a stationary distribution of match quality $\mathcal{H}_t = \epsilon$ across all families, it follows that all families have a value of X_t proportional to aggregate real income Y_t at all times:

$$X_t = \varkappa Y_t, \quad \text{where } \varkappa = \frac{1}{(1-u)Q}. \quad \text{[A.125]}$$

The constant $\varkappa$ is the reciprocal of total match quality $(1-u)Q$ in steady state (noting that unsatisfied owners receive no housing utility flows). Substituting this into the consumption Euler equation (A.124) implies that the equilibrium real interest rate is:

$$r = \varrho + \upsilon g. \quad \text{[A.126]}$$

Since (A.125) implies $\mathcal{C}_t = \varkappa Y_t \mathcal{H}_t$, it follows that lifetime utility (A.123) can be expressed as follows:

$$\mathcal{U}_T = \varkappa^{1-\upsilon} \int_{t=T}^{\infty} e^{-\varrho(t-T)} Y_t^{1-\upsilon} \mathcal{H}_t \mathrm{d}t.$$

With Y_t growing at rate g, income at time t can be written as $Y_t = e^{g(t-T)} Y_T$. By substituting this into the lifetime utility function and using the expression for the real interest rate r in (A.126):

$$\mathcal{U}_T = \varkappa^{1-\upsilon} Y_T^{1-\upsilon} \int_{t=T}^{\infty} e^{-(r-g)(t-T)} \mathcal{H}_t \mathrm{d}t. \quad \text{[A.127]}$$

Lifetime utility is therefore a discounted sum of match quality $\mathcal{H}_t = \epsilon$. The coefficient of match quality (this is the parameter ξ in the main text) is increasing in the current level of real income, and the discount rate (denoted r in the main text) is the difference between the market interest rate and the growth rate of real income. This provides a justification for interpreting a rise in real incomes as an increase in ξ and a fall in the market interest rate as a lower discount rate.

A.14 The effects of credit availability

Table 8 reports the results for a one-quarter reduction in buyers' transaction costs C_b. The effects are shown in isolation and in combination with the other factors.

Table 8: *Improvement in credit conditions*

Factor	Transactions	Listings	Sales rate	Moving rate	Houses for sale	Prices
C_b	15%	15%	12%	15%	3%	−2%
C_b, productivity, & search	28%	28%	19%	28%	9%	32%
All factors	32%	32%	9%	33%	24%	69%
Data (1995–2003)	27%	34%	14%	34%	13%	31%

To study the role of credit availability during the boom-and-bust cycle considered in section 5.5, Table 9 also has buyers' transaction costs C_b go down by one quarter during the boom and up by one quarter during the bust in addition to the factors considered in Table 7.

Table 9: *Boom and bust predictions with credit availability*

	Boom (1995–2006)		Bust (2007-2009)	
	Model	Data	Model	Data
Transactions	29%	30%	−13%	−13%
Price	38%	47%	−6%	−17%

A.15 Sensitivity analysis

Table 10 below conducts a sensitivity analysis in respect of some of the calibration targets to check the robustness of the results and to identify the key mechanisms at work in the model.

The first exercise is to explore the relative importance of the two search frictions discussed in section 3.2 that are found in the model. The first friction relates to the time taken to find suitable houses to view. The second friction relates to houses having a range of possible match qualities with different buyers that only become known to a buyer once a house is viewed. The sensitivity analysis considers separately a reduction in the first friction and a reduction in the second friction.

Lowering the first friction is equivalent to increasing the viewing rate v, while lowering the second friction is equivalent to increasing the shape parameter λ of the Pareto distribution of match quality. To increase v while keeping λ constant requires holding the time-to-sell T_s constant. This can only be done by increasing viewings per sale V_s since $v = T_s/V_s$. To increase λ while keeping v constant essentially means decreasing average viewing per sale V_s and lowering time-to-sell T_s in proportion to the reduction in V_s. The effects of reducing the two frictions by half are reported in the 'low first friction' and 'low second friction' rows. The second row clearly demonstrates the importance of the housing mismatch and the need to inspect houses before purchasing.

The next exercise is to vary the size of transaction costs C. This has a large effect on the results, with stronger effects of endogenous moving found when transaction costs are high relative to house prices. To understand this, note that in the special case of zero transactions costs, the model has the surprising feature that its steady-state equilibrium is isomorphic to an exogenous moving model with the parameter

a redefined as $a(1-\delta^{\lambda})$. The logic behind this is that equation (26) implies $y = x$ when $C = 0$. From (32), this means that $n = a(1-\delta^{\lambda})$, so the moving rate is independent of the equilibrium moving and transaction thresholds. Hence, only those parameters directly related to the shocks received by homeowners affect the moving rate. The equilibrium value of y is determined by replacing x with y in equation (29) and simplifying to:

$$\frac{vy^{1-\lambda}}{(r + a(1-\delta^{\lambda}))(\lambda - 1)} = y + \frac{F}{\xi}.$$

This has the same form as (29) when $\delta = 0$, that is, when moving is exogenous, so all steady-state predictions of the two models would be the same if $C = 0$.

As can be seen in Table 10, the size of the flow cost of search F has a much smaller impact on the results than transaction costs C. Finally, the extent of the seller's bargaining power ω does make a difference to the results, with higher seller's bargaining power increasing the strength of the results. At first glance, this is surprising because in the model, bargaining power should affect only prices, not quantities. However, changes in bargaining power require changes in the other parameters to continue to match the calibration targets. As can be seen from equation (34), an increase in ω raises average transaction prices, which requires an increase in transaction costs C to match the calibration target for $c = C/P$. Following the discussion above, it is the required increase in C after raising ω that has a large impact on the results for quantities.

Table 10: *Sensitivity analysis*

Target to vary	Transactions	Listings	Sales rate	Moving rate	Houses for sale	Prices
Frictions in the search process						
Low 1st fric.: $T_s = 6.5/12$, $V_s = 20$	17%	17%	11%	18%	6%	33%
Low 2nd fric.: $T_s = 6.5/24$, $V_s = 5$	2%	2%	2%	2%	0%	33%
Transaction costs						
Low: $c = 0.05$	11%	11%	6%	11%	5%	33%
High: $c = 0.15$	23%	23%	15%	24%	8%	34%
Flow costs of search						
Low: $f = 0.0125$	17%	17%	12%	17%	5%	35%
High: $f = 0.05$	18%	18%	9%	18%	9%	30%
Bargaining power of the seller						
Low: $\omega = 0.25$	11%	11%	8%	11%	3%	34%
High: $\omega = 0.75$	23%	23%	14%	24%	9%	32%
Baseline	17%	17%	11%	18%	6%	33%
Data (1995–2003)	27%	34%	14%	34%	13%	31%

Notes: The table shows the long-run steady-state effects of the changes to productivity and internet search in the basic version of the model.