

The Ins and Outs of Selling Houses:
Understanding Housing-Market Volatility
Online Appendix

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A Appendices

A.1 Robustness of cyclical patterns in the data

Hodrick-Prescott filtered data (1991Q1–2019Q4) To compare the cyclical properties of the data with the findings of Díaz and Jerez (2013), the seasonally adjusted quarterly time series in natural logarithms are detrended using the Hodrick-Prescott filter (with smoothing parameter 1600). The standard deviations and correlation coefficients are shown in Table A.1.

Table A.1: Cyclical properties of HP-filtered housing-market variables

	Sales	Prices	New listings	Time-to-sell	Houses for sale
			<i>Standard deviations, %</i>		
	6.7	2.5	15.0	11.0	7.3
			<i>Correlation coefficients</i>		
Sales	1				
Prices	0.40	1			
New listings	0.46	0.29	1		
Time-to-sell	−0.76	−0.16	−0.36	1	
Houses for sale	−0.22	0.12	−0.12	0.80	1

Notes: Calculated from HP-filtered (smoothing parameter 1600) natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.

Sources: FHFA and NAR.

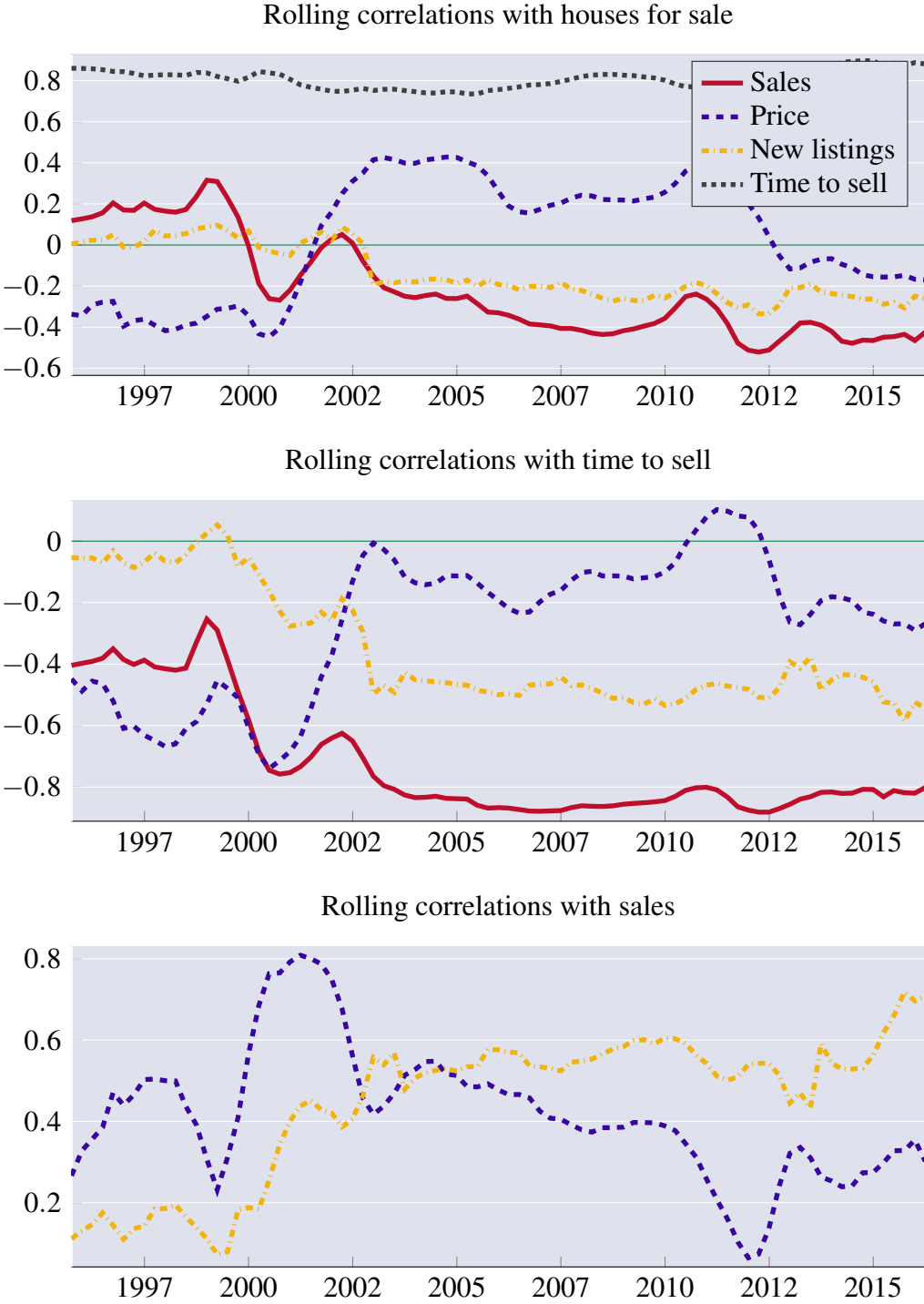
The statistics related to sales, prices, time-to-sell, and houses for sale are similar to those reported in Díaz and Jerez (2013). In addition to the differences in the measurement of time-to-sell and houses for sale discussed in section 2, note also that Table 1 of Díaz and Jerez (2013) uses different time periods for different variables, while the time series here all cover the period 1991Q1–2019Q4. For example, their measure of sales starts from 1968, but the price series starts from either 1975 or 1990.

The overall cyclical patterns are broadly consistent with those presented in Table 2, though the levels of the standard deviations are lower. To highlight a few differences in the correlation coefficients compared to Table 2, the positive correlations between house prices and sales, new listings and sales, and new listings and prices are all weaker. The negative correlation between time-to-sell and new listings is also weaker. Figure A.1 reports rolling correlations in ten-year windows for the HP-filtered data on housing-market variables. This exhibits the same patterns seen in Figure 2.

Data with no detrending (1991Q1–2019Q4) Table A.2 and Figure A.2 report the cyclical properties of the data without any detrending. The standard deviations and correlation coefficients are similar to Table 2 and the patterns of rolling correlations are the same as those in Figure 2.

Redfin data with linear detrending (2012Q2–2019Q4) Table A.3 reports cyclical properties of the Redfin data with linear detrending in comparison with the NAR and FHFA data. The levels of standard deviations and correlation coefficients are the same as those calculated using NAR and FHFA data for the same period. They are both similar to those patterns seen in the raw data from Table 3, except for the mild positive correlation between houses for sale and new listings in the linearly detrended Redfin data.

Figure A.1: Rolling correlations of housing-market variables using HP-filtered data



Notes: Correlation coefficients in ten-year windows are calculated using HP-filtered (smoothing parameter 1600) and seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis is the mid-point of each ten-year window.
Sources: FHFA and NAR.

Table A.2: Cyclical properties of housing-market variables without detrending

	Sales	Prices	New listings	Time-to-sell	Houses for sale
	18.7	16.3	25.4	28.6	20.5
	<i>Standard deviations, %</i>				
	<i>Correlation coefficients</i>				
Sales	1				
Prices	0.72	1			
New listings	0.84	0.59	1		
Time-to-sell	-0.70	-0.31	-0.59	1	
Houses for sale	-0.06	0.22	-0.06	0.76	1

Notes: Calculated from natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.

Sources: FHFA and NAR.

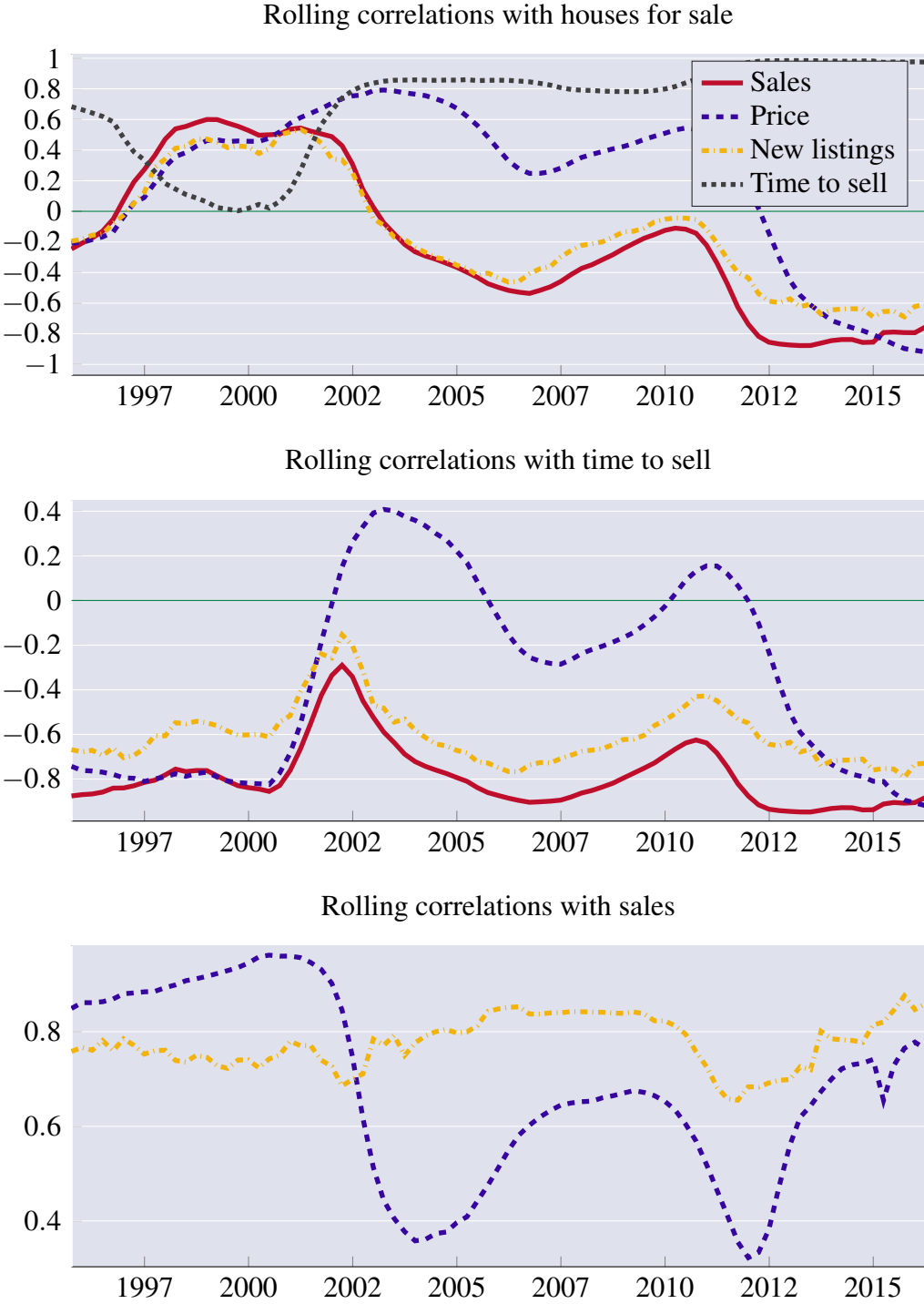
Table A.3: Comparison with linearly detrended Redfin data, 2012Q2–2019Q4

	Sales		Prices		New listings		Time-to-sell		Houses for sale	
	4.1	3.9	2.2	8.0	4.0	8.6	6.4	6.4	4.6	4.7
	<i>Standard deviations, %</i>									
	<i>Correlation coefficients</i>									
Sales	1									
Prices	0.68	0.49	1							
New listings	0.73	0.73	0.68	0.39	1					
Time-to-sell	-0.67	-0.67	-0.65	-0.42	-0.65	-0.51	1			
Houses for sale	-0.14	-0.08	-0.10	-0.16	0.25	-0.08	0.44	0.78	1	

Notes: Calculated from linearly detrended natural logarithms of quarterly time series from 2012Q2 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. Redfin statistics are in bold, adjacent to the equivalent NAR and FHFA statistics.

Sources: Redfin, NAR, and FHFA.

Figure A.2: Rolling correlations of housing-market variables without detrending



Notes: Correlation coefficients in ten-year windows are calculated using the seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis is the mid-point of each ten-year window.
Sources: FHFA and NAR.

A.2 Characterizing aggregate dynamics with a finite number of variables

This section derives a set of equations in a finite number of variables that characterizes the aggregate dynamics of the housing market. Under the assumptions made in section 3.3, the idiosyncratic shock is sufficiently large (δ is sufficiently far below 1) so that $\delta x_t < x_{t'}$ and $\delta y_t < x_{t'}$ for all t and t' . Consequently, there exists a threshold ξ , which lies above y_t and x_t for all t , such that $\delta \varepsilon < x_{t+\tau}$ for any $\varepsilon \leq \xi$. Since $H_{t+\tau}(\varepsilon)$ is increasing in ε , it follows using (17) that $H_{t+\tau}(\delta \varepsilon) - \tau D < J_{t+\tau}$ for all $\varepsilon \leq \xi$ and thus $\max\{H_{t+\tau}(\delta \varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau}$. The Bellman equation (16) for $\varepsilon \leq \xi$ becomes

$$H_t(\varepsilon) = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(\varepsilon) - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}. \quad (\text{A.1})$$

Differentiating with respect to ε gives $H'_t(\varepsilon) = \tau \theta_t + \alpha \beta_t \mathbb{E}_t H'_{t+\tau}(\varepsilon)$, which can be iterated forwards to deduce:

$$H'_t(\varepsilon) = \Theta_t, \quad \text{where } \Theta_t = \tau \mathbb{E}_t [\theta_t + \alpha \beta_t \theta_{t+1} + \alpha^2 \beta_t \beta_{t+1} \theta_{t+2} + \dots].$$

The new variable Θ_t depends only on the exogenous variables θ_t and β_t and satisfies the expectational difference equation

$$\Theta_t = \tau \theta_t + \alpha \beta_t \mathbb{E}_t \Theta_{t+\tau}. \quad (\text{A.2})$$

Since $H'_t(\varepsilon)$ is independent of ε for $\varepsilon \leq \xi$, it follows that $H_t(\varepsilon)$ is linear for $\varepsilon \in [0, \xi]$, that is:

$$H_t(\varepsilon) = \Lambda_t + \Theta_t \varepsilon, \quad (\text{A.3})$$

for some variable Λ_t independent of ε . Substituting back into (A.1) implies $\Lambda_t + \Theta_t \varepsilon = \tau \varepsilon \theta_t + \alpha \beta_t \mathbb{E}_t [\Lambda_{t+\tau} + \Theta_{t+\tau} \varepsilon - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}$, and then replacing Θ_t using (A.2) yields

$$\Lambda_t = \alpha \beta_t \mathbb{E}_t \Lambda_{t+\tau} - \alpha \beta_t \tau D + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}. \quad (\text{A.4})$$

Since $x_t < \xi$, equation (A.3) can be evaluated at $\varepsilon = x_t$, hence $H_t(x_t) = \Lambda_t + \Theta_t x_t$. Using equation (17) that defines the moving threshold x_t , it follows that $\Lambda_t = J_t + \tau D - \Theta_t x_t$. Substituting into (A.4) implies

$$J_t + \tau D - \Theta_t x_t = \beta_t \mathbb{E}_t J_{t+\tau} - \alpha \beta_t \mathbb{E}_t [\Theta_{t+\tau} x_{t+\tau}].$$

Combining this with the Bellman equation (10) to eliminate the joint value function J_t :

$$x_t \Theta_t + \tau F = \alpha \beta_t \mathbb{E}_t [x_{t+\tau} \Theta_{t+\tau}] + \mu \Sigma_t. \quad (\text{A.5})$$

This gives an expectational difference equation for the moving threshold x_t in terms of the surplus Σ_t and the exogenous variable Θ_t .

Using equations (7) and (17) defining the transaction and moving thresholds y_t and x_t , it follows that $H_t(y_t) - H_t(x_t) = \beta_t \mathbb{E}_t J_{t+\tau} + C - J_t - \tau D$. Substituting the Bellman equation (10) implies $H_t(y_t) - H_t(x_t) = \tau F + C - \mu \Sigma_t$. Furthermore, since $y_t < \xi$ and $x_t < \xi$, equation (A.3) yields $H_t(y_t) - H_t(x_t) = \Theta_t (y_t - x_t)$. Putting these equations together leads to the following relationship between the thresholds y_t and x_t :

$$\Theta_t (y_t - x_t) = C + \tau F - \mu \Sigma_t.$$

The term in the surplus Σ_t can be eliminated using (A.5) to leave a simpler relationship between y_t and $x_{t+\tau}$:

$$\Theta_t y_t = C + \alpha \beta_t \mathbb{E}_t [\Theta_{t+\tau} x_{t+\tau}], \quad (\text{A.6})$$

and this equation contains only the thresholds and the exogenous variable Θ_t .

Now consider an arbitrary variable z_t that always satisfies $z_t \leq \xi$. Given z_t , define $\Psi_t(z_t)$ as follows:

$$\Psi_t(z_t) = \int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (H_t(\varepsilon) - H_t(z_t)) d\varepsilon. \quad (\text{A.7})$$

Since $z_t \leq \xi$, equation (A.1) applies and hence $H_t(z_t) = \tau z_t \theta_t + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(z_t) - \tau D] + (1 - \alpha) \beta_t \mathbb{E}_t J_{t+\tau}$. Subtracting this from (16) and using (17) yields

$$\begin{aligned} H_t(\varepsilon) - H_t(z_t) &= \tau \theta_t (\varepsilon - z_t) + (1 - \alpha) \beta_t \max \{H_{t+\tau}(\delta \varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\} \\ &+ \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_t)] = \tau \theta_t (\varepsilon - z_t) + \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t)] \\ &+ \alpha \beta_t \mathbb{E}_t [H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})] + (1 - \alpha) \beta_t \max \{H_{t+\tau}(\delta \varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\}, \end{aligned} \quad (\text{A.8})$$

noting that $\max \{H_{t+\tau}(\delta \varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau} + \max \{H_{t+\tau}(\delta \varepsilon) - \tau D - J_{t+\tau}, 0\} = J_{t+\tau} + \max \{H_{t+\tau}(\delta \varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\}$ because $H_{t+\tau}(x_{t+\tau}) = \tau D + J_{t+\tau}$. Considering the following integral and making the change of variable $\varepsilon' = \delta \varepsilon$, and noting $\delta z_t < x_{t+\tau}$ because $z_t < \xi$:

$$\begin{aligned} &\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \max \{H_{t+\tau}(\delta \varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\} d\varepsilon \\ &= \delta^\lambda \int_{\varepsilon'=\delta z_t}^{\infty} \lambda (\varepsilon')^{-(\lambda+1)} \max \{H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau}), 0\} d\varepsilon' = \delta^\lambda \int_{\varepsilon'=\delta z_t}^{x_{t+\tau}} \lambda (\varepsilon')^{-(\lambda+1)} 0 d\varepsilon' \\ &\quad + \delta^\lambda \int_{\varepsilon'=x_{t+\tau}}^{\infty} \lambda (\varepsilon')^{-(\lambda+1)} (H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau})) d\varepsilon' = \delta^\lambda \Psi_{t+\tau}(x_{t+\tau}), \end{aligned} \quad (\text{A.9})$$

which uses $H_{t+\tau}(\varepsilon') < H_{t+\tau}(x_{t+\tau})$ for $\varepsilon' < x_{t+\tau}$, and the definition of $\Psi_t(z_t)$ from (A.7). Note also:

$$\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} d\varepsilon = z_t^{-\lambda}, \quad \text{and} \quad \int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_t) d\varepsilon = \frac{z_t^{1-\lambda}}{\lambda - 1}. \quad (\text{A.10})$$

Since $z_t \leq \xi$ and $z_{t+\tau} \leq \xi$, it follows from (A.3) that $H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) = \Theta_{t+\tau}(\varepsilon - z_{t+\tau})$ for all ε between z_t and $z_{t+\tau}$. Breaking up the range of integration in the following equations and using the definition of $\Psi_t(z_t)$ from (A.7) leads to

$$\begin{aligned} &\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})) d\varepsilon = \int_{\varepsilon=z_t}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} (H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})) d\varepsilon \\ &+ \int_{\varepsilon=z_{t+\tau}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})) d\varepsilon = \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \int_{\varepsilon=z_t}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_{t+\tau}) d\varepsilon \\ &= \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \left(\frac{\lambda}{\lambda - 1} \left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda} \right) \right). \end{aligned} \quad (\text{A.11})$$

Note also that $H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t) = \Theta_{t+\tau}(z_{t+\tau} - z_t)$ using (A.3). By combining equations (A.7), (A.8), (A.9), (A.10), and (A.11), the following result holds for all $z_t \leq \xi$:

$$\begin{aligned} \Psi_t(z_t) &= \tau \theta_t \frac{z_t^{1-\lambda}}{\lambda - 1} + \alpha \beta_t \mathbb{E}_t \Psi_{t+\tau}(z_{t+\tau}) + (1 - \alpha) \delta^\lambda \beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}) \\ &+ \alpha \beta_t \mathbb{E}_t \left[\left((z_{t+\tau} - z_t) z_t^{-\lambda} + \frac{\lambda}{\lambda - 1} \left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda} \right) \right) \Theta_{t+\tau} \right] \\ &= \tau \theta_t \frac{z_t^{1-\lambda}}{\lambda - 1} + \alpha \beta_t \mathbb{E}_t \Psi_{t+\tau}(z_{t+\tau}) + (1 - \alpha) \delta^\lambda \beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}) + \alpha \beta_t \mathbb{E}_t \left[\left(\frac{z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda}}{\lambda - 1} \right) \Theta_{t+\tau} \right]. \end{aligned}$$

Grouping the terms in $z_t^{1-\lambda}$ and using equation (A.2) for Θ_t implies

$$\Psi_t(z_t) - \frac{\Theta_t z_t^{1-\lambda}}{\lambda-1} = \alpha \beta_t \mathbb{E}_t \left[\Psi_{t+\tau}(z_{t+\tau}) - \frac{\Theta_{t+\tau} z_{t+\tau}^{1-\lambda}}{\lambda-1} \right] + (1-\alpha) \delta^\lambda \beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}), \quad (\text{A.12})$$

which holds for any $z_t \leq \xi$ and for all t .

Making the following definition of a variable χ_t , and noting the relationship between the unconditional surplus Σ_t given in (9) and $\Psi_t(z_t)$ from (A.7):

$$\chi_t = \int_{\varepsilon=x_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (H_t(\varepsilon) - H_t(x_t)) d\varepsilon = \Psi_t(x_t), \quad \text{and} \quad \Sigma_t = \Psi_t(y_t). \quad (\text{A.13})$$

With $x_t < \xi$ and $y_t < \xi$, equation (A.12) can be evaluated at $z_t = x_t$ and $z_t = y_t$ and stated in terms of the variables from (A.13):

$$\chi_t - \frac{\Theta_t x_t^{1-\lambda}}{\lambda-1} = \alpha \beta_t \mathbb{E}_t \left[\chi_{t+\tau} - \frac{\Theta_{t+\tau} x_{t+\tau}^{1-\lambda}}{\lambda-1} \right] + (1-\alpha) \delta^\lambda \beta_t \mathbb{E}_t \chi_{t+\tau}, \quad \text{and} \quad (\text{A.14})$$

$$\Sigma_t - \frac{\Theta_t y_t^{1-\lambda}}{\lambda-1} = \alpha \beta_t \mathbb{E}_t \left[\Sigma_{t+\tau} - \frac{\Theta_{t+\tau} y_{t+\tau}^{1-\lambda}}{\lambda-1} \right] + (1-\alpha) \delta^\lambda \beta_t \mathbb{E}_t \chi_{t+\tau}, \quad (\text{A.15})$$

which yields a pair of equations for χ_t and Σ_t in terms of the thresholds x_t and y_t and the exogenous variable Θ_t . The solution for x_t , y_t , χ_t , and Σ_t is determined by (A.5), (A.6), (A.14), and (A.15), with the exogenous variable Θ_t obtained from (A.2).

Given y_t , the value of π_t comes from equation (8), and s_t and T_t from (19). The laws of motion involve equations (20) and (21) for S_t and u_t . Considering equation (22) for new listings N_t , make the following definitions of a new variable Υ_t and a constant ψ :

$$\Upsilon_t = (1-\psi) \sum_{\ell=0}^{\infty} \psi^\ell u_{t-\tau\ell}, \quad \text{where} \quad \psi = \alpha + (1-\alpha) \delta^\lambda. \quad (\text{A.16})$$

Using this new variable, equation (22) for listings becomes

$$N_t = (1-\alpha)(1-u_{t-\tau} + S_{t-\tau}) - \frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi)} x_t^{-\lambda} \Upsilon_{t-\tau}. \quad (\text{A.17})$$

Equation (A.16) defining Υ_t can be stated equivalently as follows:

$$\Upsilon_t = \psi \Upsilon_{t-\tau} + (1-\psi) u_t. \quad (\text{A.18})$$

Given x_t and y_t , the solution for π_t , s_t , T_t , S_t , N_t , u_t , and the auxiliary variable Υ_t is determined by (8), (19), (20), (21), (A.17), and (A.18).

Using the price equation (14), the equations for π_t and Σ_t in (8) and (9), and the Bellman equation (12b) for V_t , the average price paid is given by:

$$P_t = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \frac{\Sigma_t}{\pi_t} = \kappa C + \tau D + V_t + \omega(1-\mu\pi_t) \frac{\Sigma_t}{\pi_t}.$$

By subtracting $\beta_t \mathbb{E}_t P_{t+\tau}$ from P_t , it follows that:

$$P_t - \beta_t \mathbb{E}_t P_{t+\tau} = (1-\beta_t)(\kappa C + \tau D) + V_t - \beta_t \mathbb{E}_t V_{t+\tau} + \omega \left((1-\mu\pi_t) \frac{\Sigma_t}{\pi_t} - \beta_t \mathbb{E}_t \left[(1-\mu\pi_{t+\tau}) \frac{\Sigma_{t+\tau}}{\pi_{t+\tau}} \right] \right),$$

and using the Bellman equation (12b) again to eliminate the value function V_t leads to:

$$P_t = \beta_t \mathbb{E}_t [P_{t+\tau} - \tau D] + (1 - \beta_t) \kappa C + \omega \frac{\Sigma_t}{\pi_t} - \omega \beta_t \mathbb{E}_t \left[(1 - \mu \pi_{t+\tau}) \frac{\Sigma_{t+\tau}}{\pi_{t+\tau}} \right]. \quad (\text{A.19})$$

A.3 A log-linear approximation of the model

Deterministic steady state The deterministic steady state of the model is defined by the absence of aggregate shocks, though individual households still face uncertainty about draws of match quality and the occurrence of idiosyncratic shocks. With $\sigma_\theta = 0$ and $\sigma_r = 0$ in (18), the innovations $\eta_{\theta,t}$ and $\eta_{r,t}$ are always zero, and so $\theta_t = 1$ and $r_t = r$ for all t , the latter implying $\beta_t = \beta$. Using (A.2), this leads to

$$\Theta = \frac{\tau}{1 - \alpha\beta}, \quad (\text{A.20})$$

where a variable without a time subscript denotes the steady-state value of that variable. Equation (A.5) implies the steady-state moving threshold x and surplus Σ are related as follows:

$$x + F = \frac{\mu}{\tau} \Sigma. \quad (\text{A.21})$$

The steady-state thresholds y and x are linked in accordance with equation (A.6):

$$y = \alpha\beta x + \left(\frac{1 - \alpha\beta}{\tau} \right) C. \quad (\text{A.22})$$

The steady-state value of χ can be deduced from equation (A.14):

$$\chi = \frac{x^{1-\lambda}}{(\lambda - 1)} \left(\frac{\tau}{1 - \psi\beta} \right), \quad (\text{A.23})$$

where $\psi = \alpha + (1 - \alpha)\delta^\lambda$ is as defined in (A.16). A relationship between Σ and χ can be derived using equations (A.14) and (A.15):

$$\Sigma - \frac{y^{1-\lambda}}{(\lambda - 1)} \left(\frac{\tau}{1 - \alpha\beta} \right) = \chi - \frac{x^{1-\lambda}}{(\lambda - 1)} \left(\frac{\tau}{1 - \alpha\beta} \right) = \frac{x^{1-\lambda}}{(\lambda - 1)} \left(\frac{\beta(1 - \alpha)\delta^\lambda}{1 - \psi\beta} \right) \left(\frac{\tau}{1 - \alpha\beta} \right),$$

where the second equality follows by substituting from (A.23) and noting $\psi - \alpha = (1 - \alpha)\delta^\lambda$. The steady-state value Σ follows immediately from this:

$$\Sigma = \frac{1}{(\lambda - 1)} \left(\frac{\tau}{1 - \alpha\beta} \right) \left(y^{1-\lambda} + \beta\delta^\lambda \left(\frac{1 - \alpha}{1 - \psi\beta} \right) x^{1-\lambda} \right). \quad (\text{A.24})$$

Eliminating Σ from equations (A.21) and (A.24) implies another equation linking the steady-state thresholds x and y :

$$x + F = \frac{1}{(\lambda - 1)} \left(\frac{\mu}{\tau} \right) \left(\frac{\tau}{1 - \alpha\beta} \right) \left(y^{1-\lambda} + \beta\delta^\lambda \left(\frac{1 - \alpha}{1 - \psi\beta} \right) x^{1-\lambda} \right). \quad (\text{A.25})$$

The steady-state thresholds x and y are the solution of the simultaneous equations (A.22) and (A.25). Equation (A.22) implies a positive relationship between x and y , while equation (A.25) implies a negative relationship between x and y . If a solution exists, it must then be unique. Since (A.22) implies x is positive when $y = 0$, and because (A.25) implies $y \rightarrow 0$ as $x \rightarrow \infty$, while x tends to a positive number when $y \rightarrow \infty$, it follows that a unique solution $x > 0$ and $y > 0$ exists. However, the equations are only meaningful if $y > 1$ and

$\delta y < x$. The solution features $y > 1$ if and only if

$$\left(\frac{1 - \left(\frac{1 - \alpha\beta}{\tau} \right) C}{\alpha\beta} \right) + F < \frac{1}{(\lambda - 1)} \left(\frac{\mu}{\tau} \right) \left(\frac{\tau}{1 - \alpha\beta} \right) \left(1 + \beta\delta^\lambda \left(\frac{1 - \alpha}{1 - \psi\beta} \right) \left(\frac{1 - \left(\frac{1 - \alpha\beta}{\tau} \right) C}{\alpha\beta} \right)^{1 - \lambda} \right),$$

and it can also be verified whether δ is sufficiently far below 1 so that $\delta y < x$.

The steady-state acceptance probability is $\pi = y^{-\lambda}$ from (8), the steady-state selling probability $s = \mu\pi$ and time-to-sell $T = (1/\pi)(\tau/\mu)$ from (19). Equations (20) and (21) imply $S = su$ and $N = S$, hence $S = N = \mu y^{-\lambda} u$. Noting that $Y = u$ from (A.18), equation (A.17) in steady state implies

$$N = (1 - \alpha)(1 - u + \mu y^{-\lambda} u) - \frac{\mu(1 - \alpha)\delta^\lambda}{(1 - \psi)} x^{-\lambda} u.$$

Combined with $N = \mu y^{-\lambda} u$, this can be solved for the steady-state u :

$$u = \frac{(1 - \alpha)}{(1 - \alpha) + \mu \left(\alpha y^{-\lambda} + \delta^\lambda x^{-\lambda} \frac{(1 - \alpha)}{(1 - \psi)} \right)} = \frac{1}{1 + \mu \left(\frac{\alpha}{1 - \alpha} y^{-\lambda} + \frac{\delta^\lambda}{1 - \psi} x^{-\lambda} \right)}. \quad (\text{A.26})$$

The steady state implied by the price equation (A.19) is:

$$P = \kappa C - \beta \left(\frac{\tau}{1 - \beta} \right) D + \omega \left(\frac{1 - \beta(1 - \mu\pi)}{1 - \beta} \right) \left(\frac{\tau}{\mu} \right) \left(\frac{x + F}{\pi} \right), \quad (\text{A.27})$$

which uses (A.21) to substitute for Σ .

Log linearizations Log deviations of variables from their deterministic steady-state values are denoted using sans serif letters, for example, $x_t = \log x_t - \log x$. The log linearization of equation (A.2) for Θ_t is

$$\Theta_t = (1 - \alpha\beta)\theta_t + \alpha\beta\beta_t + \alpha\beta\mathbb{E}_t\Theta_{t+\tau},$$

which uses the steady-state values $\theta = 1$ and Θ from (A.20). The discount factor is $\beta_t = e^{-\tau r_t}$ in terms of the discount rate r_t , and $\beta = e^{-\tau r}$ is its steady-state value. It follows that $\beta_t = \log \beta_t - \log \beta = -\tau(r_t - r) = -\tau r_t$, where $r_t = r_t - r$ is the deviation of the discount rate from its steady-state level. The log-linearized equation for Θ_t can then be written as

$$\Theta_t = (1 - \alpha\beta)\theta_t - \alpha\beta\tau r_t + \alpha\beta\mathbb{E}_t\Theta_{t+\tau}. \quad (\text{A.28})$$

Noting (A.20) and (A.21), the log linearization of the moving-threshold equation (A.5) is

$$x_t = \alpha\beta\mathbb{E}_t x_{t+\tau} + (1 - \alpha\beta) \frac{(x + F)}{x} \Sigma_t - (1 - \alpha\beta)\theta_t. \quad (\text{A.29})$$

The transaction threshold equation (A.6) can be log linearized as follows:

$$y_t = \frac{x}{y} \alpha\beta (\mathbb{E}_t \Theta_{t+\tau} + \mathbb{E}_t x_{t+\tau} - \tau r_t) - \Theta_t, \quad (\text{A.30})$$

and this can be used to deduce that

$$\begin{aligned}
y_t - \alpha\beta\mathbb{E}_t y_{t+\tau} &= \frac{x}{y}\alpha\beta\left(\mathbb{E}_t[\Theta_{t+\tau} - \alpha\beta\mathbb{E}_{t+\tau}\Theta_{t+2\tau}] + \mathbb{E}_t[x_{t+\tau} - \alpha\beta\mathbb{E}_{t+\tau}x_{t+2\tau}] - \tau(r_t - \alpha\beta\mathbb{E}_t r_{t+\tau})\right) \\
- (\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) &= \frac{x}{y}\alpha\beta\mathbb{E}_t\left[\left((1-\alpha\beta)\theta_{t+\tau} - \alpha\beta\tau r_{t+\tau}\right) + (1-\alpha\beta)\left(\frac{(x+F)}{x}\Sigma_{t+\tau} - \theta_{t+\tau}\right)\right] \\
+ \frac{x}{y}\alpha\beta\tau(r_t - \alpha\beta\mathbb{E}_t r_{t+\tau}) &- \left((1-\alpha\beta)\theta_t - \alpha\beta\tau r_t\right) \\
&= \frac{(x+F)}{y}(1-\alpha\beta)\alpha\beta\mathbb{E}_t\Sigma_{t+\tau} - (1-\alpha\beta)\theta_t + \frac{(y-x)}{y}\alpha\beta\tau r_t, \quad (\text{A.31})
\end{aligned}$$

where the subsequent expressions follow from substituting (A.28) and (A.29).

For equation (A.14) for χ_t , by using (A.20) and (A.23), the log linearization is

$$\begin{aligned}
\chi_t &= \left(\alpha + (1-\alpha)\delta^\lambda\right)\beta\mathbb{E}_t\chi_{t+\tau} + \left(\frac{1-\psi\beta}{1-\alpha\beta}\right)\left((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(x_t - \alpha\beta\mathbb{E}_t x_{t+\tau})\right) \\
&\quad - \left(\left(\alpha + (1-\alpha)\delta^\lambda\right) - \alpha\left(\frac{1-\psi\beta}{1-\alpha\beta}\right)\right)\beta\tau r_t,
\end{aligned}$$

and with the definition of $\psi = \alpha + (1-\alpha)\delta^\lambda$ from (A.16):

$$\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + \frac{(1-\psi\beta)}{(1-\alpha\beta)}\left((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(x_t - \alpha\beta\mathbb{E}_t x_{t+\tau})\right) - \frac{(1-\alpha)\delta^\lambda}{(1-\alpha\beta)}\beta\tau r_t.$$

Substituting from (A.28) and (A.29):

$$\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + (1-\psi\beta)\left(\theta_t - \frac{\alpha}{1-\alpha\beta}\beta\tau r_t + (1-\lambda)\left(\frac{(x+F)}{x}\Sigma_t - \theta_t\right)\right) - \frac{(1-\alpha)\delta^\lambda}{(1-\alpha\beta)}\beta\tau r_t,$$

and by collecting terms and simplifying:

$$\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + (1-\lambda)\frac{(x+F)}{x}(1-\psi\beta)\Sigma_t + \lambda(1-\psi\beta)\theta_t - \psi\beta\tau r_t, \quad (\text{A.32})$$

which again uses the definition of $\psi = \alpha + (1-\alpha)\delta^\lambda$.

Taking equation (A.15) for Σ_t and log linearizing, making use of the steady-state equations (A.21), (A.23), and (A.24):

$$\begin{aligned}
\Sigma_t &= \alpha\beta\mathbb{E}_t\Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{x^{1-\lambda}}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} \\
&+ \frac{1}{(\lambda-1)}\frac{y^{1-\lambda}}{(x+F)}\frac{\mu}{(1-\alpha\beta)}\left((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(y_t - \alpha\beta\mathbb{E}_t y_{t+\tau})\right) \\
&\quad - \left(\alpha - \frac{1}{(\lambda-1)}\left(\frac{y^{1-\lambda}}{(x+F)}\frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)}\frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\right)\beta\tau r_t.
\end{aligned}$$

Substituting (A.28) and (A.31) into this equation yields

$$\begin{aligned} \Sigma_t = & \alpha\beta\mathbb{E}_t\Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{x^{1-\lambda}}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{y^{1-\lambda}}{(x+F)}\left(\theta_t - \frac{\alpha}{1-\alpha\beta}\beta\tau r_t\right) \\ & - \mu\frac{y^{1-\lambda}}{(x+F)}\left(\frac{(x+F)}{y}\alpha\beta\mathbb{E}_t\Sigma_{t+\tau} - \theta_t + \frac{(y-x)}{y}\frac{\alpha}{1-\alpha\beta}\beta\tau r_t\right) \\ & - \left(\alpha - \frac{1}{(\lambda-1)}\left(\frac{y^{1-\lambda}}{(x+F)}\frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)}\frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\right)\beta\tau r_t, \end{aligned}$$

and cancelling terms, simplifying, and writing the equation in terms of $\pi = y^{-\lambda}$:

$$\begin{aligned} \Sigma_t = & \alpha\beta(1-\mu\pi)\mathbb{E}_t\Sigma_{t+\tau} + \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} + \mu\pi\frac{\lambda}{(\lambda-1)}\frac{y}{(x+F)}\theta_t \\ & - \left(\alpha\left(1 + \frac{(y-x)}{(x+F)}\frac{\mu\pi}{1-\alpha\beta}\right) + \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\beta\tau r_t. \quad (\text{A.33}) \end{aligned}$$

Log linearizations of the transaction probability, sales rate, and time-to-sell equations from (8) and (19) are

$$\pi_t = -\lambda y_t, \quad s_t = \pi_t, \quad \text{and} \quad \Upsilon_t = -\pi_t. \quad (\text{A.34})$$

Using (A.21) and (A.27), the price equation (A.19) is log linearized as follows:

$$\begin{aligned} \left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega\frac{(1-\beta(1-\mu\pi))}{(1-\beta)}\frac{\tau}{\mu}\frac{(x+F)}{\pi}\right)(P_t - \beta\mathbb{E}_tP_{t+\tau}) = & \omega\frac{\tau}{\mu}\frac{(x+F)}{\pi}(\Sigma_t - \pi_t) \\ - \left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega\frac{(1-\beta(1-\mu\pi))}{(1-\beta)}\frac{\tau}{\mu}\frac{(x+F)}{\pi} - \tau D - \kappa C - \omega(1-\mu\pi)\frac{\tau}{\mu}\frac{(x+F)}{\pi}\right)\beta\tau r_t \\ & - \beta\omega\frac{\tau}{\mu}\frac{(x+F)}{\pi}\mathbb{E}_t[(1-\mu\pi)(\Sigma_{t+\tau} - \pi_{t+\tau}) - \mu\pi\pi_{t+\tau}], \end{aligned}$$

and simplifying the coefficients in this equation leads to:

$$P_t = \beta\mathbb{E}_tP_{t+\tau} + \frac{\frac{\omega\tau(x+F)}{\mu\pi}(\Sigma_t - \beta(1-\mu\pi)\mathbb{E}_t\Sigma_{t+\tau} - \pi_t + \beta\mathbb{E}_t\pi_{t+\tau}) - \frac{\tau(\omega(x+F)-D)\beta\tau}{(1-\beta)}r_t}{\kappa C - \frac{\beta\tau D}{(1-\beta)} + \frac{\omega\tau(1-\beta(1-\mu\pi))(x+F)}{(1-\beta)\mu\pi}}. \quad (\text{A.35})$$

Log-linearizations of the sales (20) and houses for sale (21) equations are

$$S_t = s_t + u_t, \quad \text{and} \quad u_t - u_{t-\tau} = \mu\pi(N_t - S_{t-\tau}), \quad (\text{A.36})$$

where $\pi = y^{-\lambda}$ and the steady-state equation $N = S = su$ have been used. Equation (A.17) has the following log-linearization:

$$N_t = \lambda\delta^\lambda\left(\frac{y}{x}\right)^\lambda\left(\frac{1-\alpha}{1-\psi}\right)x_t + (1-\alpha)S_{t-\tau} - \left(\frac{1-\alpha}{\mu\pi}\right)u_{t-\tau} - \delta^\lambda\left(\frac{y}{x}\right)^\lambda\left(\frac{1-\alpha}{1-\psi}\right)\Upsilon_{t-\tau}, \quad (\text{A.37})$$

which uses $N = S = su$, $s = \mu\pi$, and $\pi = y^{-\lambda}$. Finally, log-linearizing equation (A.18) for the auxiliary state variable Υ_t from (A.16):

$$\Upsilon_t = \psi\Upsilon_{t-\tau} + (1-\psi)u_t, \quad (\text{A.38})$$

which makes use of $\Upsilon = u$.

In summary, the system of equations (A.29), (A.30), (A.32), (A.33), (A.34), (A.35), (A.36), (A.37), and (A.38) can be solved for x_t , y_t , χ_t , Σ_t , π_t , s_t , Υ_t , P_t , S_t , N_t , u_t , and Υ_t . These equations are in recursive form with only contemporaneous (t), one-period lagged ($t - \tau$), and expected one-period ahead ($t + \tau$) values of the variables appearing.

The auxiliary variable χ_t can be eliminated as follows. Note that (A.33) implies

$$\begin{aligned} \Sigma_t - \psi\beta\mathbb{E}_t\Sigma_{t+\tau} &= \alpha\beta(1 - \mu\pi)\mathbb{E}_t[\Sigma_{t+\tau} - \psi\beta\Sigma_{t+2\tau}] + \mu\pi\frac{\lambda}{(\lambda - 1)}\frac{y}{(x + F)}(\theta_t - \psi\beta\mathbb{E}_t\theta_{t+\tau}) \\ &- \left(\alpha\left(1 + \frac{(y-x)}{(x+F)}\frac{\mu\pi}{1 - \alpha\beta}\right) + \mu\pi\frac{(y/x)^\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\frac{(1 - \alpha)\delta^\lambda}{(1 - \psi\beta)} \right) \beta\tau(r_t - \psi\beta\mathbb{E}_tr_{t+\tau}) \\ &+ \mu\pi\frac{(y/x)^\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\frac{(1 - \alpha)\delta^\lambda\beta}{(1 - \psi\beta)}\mathbb{E}_t[\chi_{t+\tau} - \psi\beta\chi_{t+2\tau}], \end{aligned}$$

which makes use of the law of iterated expectations, and then by using equation (A.32):

$$\begin{aligned} \mu\pi\frac{(y/x)^\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\frac{(1 - \alpha)\delta^\lambda\beta}{(1 - \psi\beta)}\mathbb{E}_t[\chi_{t+\tau} - \psi\beta\chi_{t+2\tau}] &= \mu\pi\frac{\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\left(\frac{y}{x}\right)^\lambda(1 - \alpha)\delta^\lambda\beta\mathbb{E}_t\theta_{t+\tau} \\ &- \mu\pi(1 - \alpha)\delta^\lambda\left(\frac{y}{x}\right)^\lambda\beta\mathbb{E}_t\Sigma_{t+\tau} - \mu\pi\frac{(y/x)^\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\frac{(1 - \alpha)\delta^\lambda\psi\beta}{(1 - \psi\beta)}\beta\tau\mathbb{E}_tr_{t+\tau}. \end{aligned}$$

Combining these two equations yields

$$\begin{aligned} \Sigma_t &= \left(\psi + (1 - \mu\pi)\alpha - \mu\pi(1 - \alpha)\delta^\lambda\left(\frac{y}{x}\right)^\lambda \right) \beta\mathbb{E}_t\Sigma_{t+\tau} - (1 - \mu\pi)\alpha\psi\beta^2\mathbb{E}_t\Sigma_{t+2\tau} \\ &+ \mu\pi\frac{\lambda}{\lambda - 1}\left(\frac{y}{(x+F)}(\theta_t - \psi\beta\mathbb{E}_t\theta_{t+\tau}) + \frac{x}{(x+F)}\left(\frac{y}{x}\right)^\lambda(1 - \alpha)\delta^\lambda\beta\mathbb{E}_t\theta_{t+\tau} \right) \\ &- \alpha\left(1 + \frac{(y-x)}{(x+F)}\frac{\mu\pi}{1 - \alpha\beta}\right)\beta\tau(r_t - \psi\beta\mathbb{E}_tr_{t+\tau}) - \mu\pi\frac{(y/x)^\lambda}{(\lambda - 1)}\frac{x}{(x+F)}\frac{(1 - \alpha)\delta^\lambda}{(1 - \psi\beta)}\beta\tau r_t. \end{aligned}$$

The auxiliary variable Υ_t can also be eliminated by using (A.37) to obtain an equation for $N_t - \psi N_{t-\tau}$ and then substituting (A.38):

$$\begin{aligned} N_t = \psi N_{t-\tau} + \frac{\lambda\delta^\lambda(y/x)^\lambda(1 - \alpha)}{(1 - \psi)}(x_t - \psi x_{t-\tau}) &+ (1 - \alpha)(S_{t-\tau} - \psi S_{t-2\tau}) \\ &- (1 - \alpha)\left(\frac{1}{\mu\pi} + \delta^\lambda\left(\frac{y}{x}\right)^\lambda\right)u_{t-\tau} + \frac{(1 - \alpha)\psi}{\mu\pi}u_{t-2\tau}. \end{aligned}$$

A.4 A single housing-demand shock

Table A.4 reports the standard deviations and correlation coefficients predicted by the model with only a single housing-demand shock through changes in expenditure θ_t .

Table A.4: *Model-predicted correlations with only expenditure shocks*

	Sales	Prices	New listings	Time-to-sell	Houses for sale
<i>Correlation coefficients among housing-market variables</i>					
Sales	1				
Prices	0.99	1			
New listings	1.00	0.99	1		
Time-to-sell	-0.99	-1.00	-0.99	1	
Houses for sale	0.95	0.95	0.95	-0.90	1
<i>Correlations between housing variables and shocks</i>					
Expenditure (data)	0.78	0.93	0.68	-0.34	0.19
Expenditure (model)	1.0	1.0	1.0	-1.0	0.95

Notes: Simulated moments of the theoretical model with $\phi_\theta = 0.9873^{1/13}$, $\sigma_\theta = \sqrt{1 - \phi_\theta^2} \times 0.0965$, and $\sigma_r = 0$ so that only the expenditure shock occurs.

A.5 The special case of exogenous moving decisions

A model with exogenous moving is a special case of the parameters of the model in section 3 for which the moving decision effectively becomes exogenous. If the size of the idiosyncratic shock to match quality becomes very large, that is, $\delta = 0$, then moving occurs if and only if an exogenous idiosyncratic shock is received. Adjusting the parameter α so that the average length of time between moving house remains the same provides an otherwise identical model with exogeneity of the moving decision as the only difference. The model-implied standard deviations and correlation coefficients subject to the same aggregate shocks are displayed in Table A.5.

The model with exogenous moving predicts that new listings are perfectly negatively correlated with houses for sale. This is because, irrespective of market conditions, listings are proportional to the previous number of homeowners not trying to sell. Furthermore, given that houses for sale are small on average as a fraction of all houses, the predicted volatility of new listings is tiny. Empirically, new listings are highly volatile and have a correlation with houses for sale that changes between positive and negative over time (see Table 2 and Figure 2). More generally, the exogenous-moving model predicts that correlations of houses for sale with other variables are always the negative of correlations of new listings with those variables. Hence, the model can only predict a change in the sign of the correlation between houses for sale and sales or prices if the sign of the new listings correlation with prices or sales changes. According to Figure 2, the correlations among sales, price, and new listings are stable.

Table A.5: Predictions of the exogenous-moving special case of the model

Expenditure	Interest rate	Sales	Prices	New listings	Time-to-sell	Houses for sale
<i>Standard deviations, %</i>						
9.7	0.86	0.41	8.20	0.06	2.32	2.23
<i>Correlation coefficients</i>						
	Sales	1				
	Prices	0.25	1			
	New listings	0.15	0.96	1		
	Time-to-sell	-0.32	-0.97	-0.98	1	
	Houses for sale	-0.15	-0.96	-1.00	0.98	1
<i>Correlations with shocks</i>						
	Expenditure (data)	0.78	0.93	0.68	-0.34	0.19
	Expenditure (model)	0.29	0.99	0.98	-0.99	-0.98
	Interest rate (data)	-0.03	-0.10	0.04	-0.13	-0.21
	Interest rate (model)	0.25	-0.12	0.10	-0.14	-0.10

Notes: Simulated moments of the $\delta = 0$ special case of the theoretical model with $\phi_\theta = 0.9873^{1/13}$, $\phi_r = 0.8033^{1/13}$, $\sigma_\theta = \sqrt{1 - \phi_\theta^2} \times 0.0965$, and $\sigma_r = \sqrt{1 - \phi_r^2} \times 0.0086$.