Institutional Specialization
Online Appendix

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A Derivations of the theoretical results

A.1 Proof of Proposition 1

The first-order conditions for maximizing C_p subject to (9) and (12) equate marginal rates of substitution between investment and endowment goods across all individuals. Given preferences in (4), this means all individuals i should have the same ratio $C_I(i)/C_E(i)$ of consumption of the two goods. These are the same conditions for Pareto efficiency in respect of the allocation of consumption goods.

Market economy

Consider a market economy with free exchange of goods domestically at relative price π and incomes subject to taxes and transfers. Workers have incomes $Y_w = Q - T_Q$ in units of the endowment good after a tax T_Q is levied on their endowment Q. Investors have incomes $Y_k = (Q - T_Q) + (\pi - T_k)$, where T_k is a tax on producing capital. Incumbents have incomes $Y_p = (Q - T_Q) + V$, where V is a transfer representing the private benefit of being in power.

The market economy has international trade conducted by competitive import-export firms that choose X_E and X_I to maximize their profits subject to the trade budget constraint (2). There can be a proportional tariff τ (if positive, or subsidy, if negative) on imports of the investment good ($X_I < 0$), which raises revenue $-\tau\pi^* X_I$. The profits of a representative import-export firm are $-X_E - \pi X_I + \tau\pi^* X_I$, which is $((1+\tau)\pi^* - \pi)X_I$ after imposing (2). There is no competitive equilibrium unless

$$\pi = (1+\tau)\pi^*, \tag{A.1}$$

as otherwise profits would be unbounded. The tariff drives a wedge between the domestic market-clearing price π and the price π^* in world markets.[1]

The definition of GDP Y in units of endowment goods, and the fiscal budget constraint are

$$Y = Q + \pi K + (\pi^* - \pi)X_{\mathrm{I}}, \quad \text{and} \;\; pV = T_Q + T_K K - \tau\pi^* X_I, \tag{A.2}$$

where the final term in GDP accounts for some production X_{I} being exported and sold at world price π^* rather than domestic price π. Using post-tax-and-transfer incomes Y_w, Y_k, and Y_p, the domestic-foreign price relationship (A.1), and firm and government budget constraints in (2) and (A.2):

$$pY_p + KY_k + (1-p-K)Y_w = (Q - X_E) + \pi(K - X_I) = Y, \tag{A.3}$$

which says that GDP Y is also the sum of incomes (noting firms' profits are zero) and the sum of the value of domestic sales of the two goods $Q - X_E$ and $K - X_I$ (the trade balance is zero).

With individual incomes $Y(i)$, each person maximizes $C(i)$ from (4) subject to a budget constraint $C_E(i) + \pi C_I(i) = Y(i)$. Combining the first-order conditions $C_I(i)/C_E(i) = \alpha\pi^{-\varepsilon}/(1-\alpha)$ with the budget constraints implies the demand functions in (15). Marginal rates of substitution, and hence

[1] Since π and π^* are relative prices in terms of the endowment good, the effects of a tariff on the endowment good are equivalent here to subsidizing the investment good, and vice versa.

the ratios $C_I(i)/C_E(i)$, are aligned across all individuals i because everyone faces the same relative price. The resource constraints (9) are also the market-clearing conditions for endowment and investment goods. Using the demand functions (15) and the expressions for GDP in (A.3), the two market-clearing conditions are equivalent to $(1-\alpha+\alpha\pi^{1-\varepsilon})(Q-X_E)=(1-\alpha)((Q-X_E)+\pi(K-X_I))$ and $(1-\alpha+\alpha\pi^{1-\varepsilon})(K-X_I)=\alpha\pi^{-\varepsilon}((Q-X_E)+\pi(K-X_I))$. Both equations hold at the relative price π given in (16), confirming this is the market clearing price.

Hence, the consumption allocation for the equilibrium institutions with $C_w=C/F(p)$ and $C_k=(1+\theta)C/F(p)$ and equal marginal rates of substitution can be implemented by a market economy with taxes $T_Q=Q-(1-\alpha+\alpha\pi^{1-\varepsilon})^{1/(1-\varepsilon)}C/F(p)$ and $T_k=\pi-\theta(1-\alpha+\alpha\pi^{1-\varepsilon})^{1/(1-\varepsilon)}C/F(p)$. These formulas follow from the binding constraints (12), the definitions of Y_w and Y_k, and the utility-maximizing value of the consumption basket $C(i)$ from (15). In a market economy, (15) and (A.3) imply the consumption baskets C_p, C_k, and C_w satisfy

$$pC_p+KC_k+(1-p-K)C_w=\frac{Y}{(1-\alpha+\alpha\pi^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}}=\frac{(Q-X_E)+\pi(K-X_I)}{(1-\alpha+\alpha\pi^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}}, \tag{A.4}$$

where the left- and right-hand sides are aggregate consumption and the economy's real GDP.

Free trade

Given a tariff τ, the net exports of profit-maximizing firms are finite only if (A.1) holds, which in equilibrium means X_E and X_I adjust until the domestic market-clearing price π in (16) satisfies (A.1). Combining equations (2), (16), and (A.1) gives the implied levels of net exports

$$X_E=\frac{\alpha\pi^{*1-\varepsilon}Q-(1-\alpha)(1+\tau)^{\varepsilon}\pi^*K}{(1-\alpha)(1+\tau)^{\varepsilon}+\alpha\pi^{*1-\varepsilon}}, \quad \text{and } X_I=\frac{(1-\alpha)(1+\tau)^{\varepsilon}K-\alpha\pi^{*-\varepsilon}Q}{(1-\alpha)(1+\tau)^{\varepsilon}+\alpha\pi^{*1-\varepsilon}}. \tag{A.5}$$

Varying τ over its maximum range $-1<\tau<\infty$ is equivalent to X_E moving between Q and $-\pi^*K$, and X_I moving between $-Q/\pi^*$ and K, which are the full ranges of values of net exports consistent with the budget and resource constraints (2) and (9) for non-negative levels of consumption.

Substituting the price relationship (A.1) and net exports (A.5) into GDP (A.3) demonstrates that real GDP $Y/(1-\alpha+\alpha\pi^{1-\varepsilon})^{1/(1-\varepsilon)}$ is $D(\tau)(Q+\pi^*K)/(1-\alpha+\alpha\pi^{*1-\varepsilon})^{1/(1-\varepsilon)}$, where $D(\tau)$ is the impact of the tariff τ on real GDP $(Q+\pi^*K)/(1-\alpha+\alpha\pi^{*1-\varepsilon})^{1/(1-\varepsilon)}$ at world prices:

$$D(\tau)=\frac{\left(1-\alpha+\alpha(1+\tau)^{1-\varepsilon}\pi^{*1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{\left(1-\alpha+\alpha(1+\tau)^{-\varepsilon}\pi^{*1-\varepsilon}\right)\left(1-\alpha+\alpha\pi^{*1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}}}. \tag{A.6}$$

The function $D(\tau)$ is strictly positive, satisfies $D(0)=1$, and has derivative

$$D'(\tau)=-\frac{\alpha(1-\alpha)\pi^{*1-\varepsilon}D(\tau)(1+\tau)^{-\varepsilon-1}\tau}{\left(1-\alpha+\alpha(1+\tau)^{-\varepsilon}\pi^{*1-\varepsilon}\right)\left(1-\alpha+\alpha(1+\tau)^{1-\varepsilon}\pi^{*1-\varepsilon}\right)}.$$

This shows the first-order condition $D'(\tau)=0$ holds only for $\tau=0$, and also that $D''(0)<0$, demonstrating that $D(\tau)$ is a strictly quasi-concave function maximized at $D(0)=1$ by $\tau=0$.

Since equilibrium institutions have consumption payoffs where $pC_p + KC_k + (1-p-K)C_w$ equals real GDP from (A.4), net exports X_E and X_I must maximize real GDP subject to the budget constraint (2). This requires the domestic market-clearing price is $\pi = \pi^*$ (noting the partial effect of π on real GDP is zero at the market-clearing price 16), and hence the tariff τ in (A.1) is zero. Equivalently, τ maximizes $D(\tau)$, the impact of trade on real GDP, which requires $\tau = 0$. International trade under the equilibrium institutions thus can be implemented in a market economy with no tariffs or subsidies driving a wedge between domestic and foreign prices. Given constraints (2) and (9), Pareto efficiency in respect of international trade requires the common marginal rate of substitution across individuals is equated to π^*, which is the same as $\pi = \pi^*$ because marginal rates of substitution are equal to π in a market economy.

With $K = \lambda\chi$ from (12), the economy's level of real GDP C under the equilibrium institutions follows from (A.4) with $\pi = \pi^*$ and using (2), or by noting $D(0) = 1$ at $\tau = 0$.

A.2 Proof of Proposition 2

The partial derivative of the incumbent payoff (19) with respect to power sharing p is given in the main text and the first-order condition is equivalent to equation (21) following the steps there.

(i) Since $a(p) > 1$ and $m(p) > 0$ under Assumption 1, it follows immediately for each p that (21) implies a value of s between 0 and 1.

(ii) The second partial derivative of the incumbent payoff (19) with respect to p is

$$\frac{\partial^2 C_p}{\partial p^2} = -\frac{CF'(p)}{pF(p)^2}\left(2 + 2(1-p+\chi\theta\lambda)\frac{F'(p)}{F(p)} - (1-p+\chi\theta\lambda)\frac{F''(p)}{F'(p)}\right) - \frac{2}{p}\frac{\partial C_p}{\partial p}.$$

Evaluating the second derivative at a level of power sharing p where the first partial derivative is zero ($\partial C_p/\partial p = 0$) and writing it in terms of $m(p) = F'(p)$ and $a(p) = F(p)/p$:

$$\left.\frac{\partial^2 C_p}{\partial p^2}\right|_{\frac{\partial C_p}{\partial p}=0} = -\frac{m(p)C}{p^3 a(p)^2}\left(2 + 2(1-p+\chi\theta\lambda)\frac{m(p)}{pa(p)} - (1-p+\chi\theta\lambda)\frac{m'(p)}{m(p)}\right). \tag{A.7}$$

With $m'(p) \leq 0$ under Assumption 2, the above is necessarily negative because C, $m(p)$, $a(p)$, and $1-p+\chi\theta\lambda$ are all positive. Hence, the second derivative (A.7) is strictly negative whenever the first derivative is zero. This implies C_p in (19) is a strictly quasi-concave function of p, which means the first-order condition (21) is necessary and sufficient for an interior value of p that maximizes C_p.

(iii) Differentiating the incumbent income share s from (21) with respect to power sharing p:

$$\frac{\partial s}{\partial p} = \frac{(a(p)-1)}{(a(p)+m(p))^2}m'(p) - \frac{(1+m(p))}{(a(p)+m(p))^2}a'(p). \tag{A.8}$$

With $a(p) - 1 > 0$ given Assumption 1, the lower bound on $m'(p)$ in Assumption 3 implies

$$\frac{\partial s}{\partial p} \geq \frac{(1+m(p))}{(a(p)+m(p))^2}\left(-a'(p) - \frac{(a(p)-m(p))^2}{2pa(p)}\right) = \frac{(1+m(p))(-a'(p))}{2a(p)(a(p)+m(p))},$$

where the second expression follows by noting $(a(p)-m(p))/p=-a'(p)$ and simplifying. Since $a'(p)<0$ under Assumption 3, the right-hand side is positive, so the s given by (21) rises with p.

A.3 Proof of Proposition 3

(i) The partial derivative of the incumbent payoff (19) with respect to power sharing is $\partial C_p/\partial p = ((1-s)F'(p)C_w-(C_p-C_w))/p$. Using the political constraint $C_w=C/F(p)$ from (12) and the definitions $s=pC_p/C$, $a(p)=F(p)/p$, and $m(p)=F'(p)$, this derivative can be written as

$$\frac{\partial C_p}{\partial p}=\frac{(a(p)+m(p))C}{pa(p)}\left(\frac{1+m(p)}{a(p)+m(p)}-s\right),$$

Comparison with the first-order condition (21) shows that C_p is increasing in p subject to the constraint (22) if the (p,s) satisfying (22) lies below the first-order condition (21) in Figure 1.

Given Assumption 1, at $p=0$, the first-order condition (21) yields a value of s satisfying $s\geq 0$. Under Assumption 4, the constraint (22) implies $s=1-(1+\chi\theta\lambda)/F(0)$, thus $s<1-(1+\chi\theta\lambda)=-\chi\theta\lambda\leq 0$ for any $\lambda\in[0,1]$ because $F(0)<1$. Geometrically, this means the first-order condition is initially above the combined constraint in Figure 1 at $p=0$, and as C_p is increasing in p in this region, there cannot be a corner equilibrium at $p=0$.

To rule out a corner equilibrium with $p=1-\chi$ for all $\lambda\in[0,1]$ (if $p=1-\chi$ and $\lambda=1$, the number of workers is zero), it suffices that the first-order condition (21) lies below the constraint (22) at $p=1-\chi$ for all $\lambda\in[0,1]$. This would imply C_p is decreasing in p in a neighbourhood of $p=1-\chi$, so there cannot be an equilibrium at $p=1-\chi$. Since the value of s implied by (22) is decreasing in λ, it is sufficient to confirm the first-order condition is below the constraint at this point when $\lambda=1$. At $p=1-\chi$ and $\lambda=1$, (22) implies $1-s=(\chi/(1-\chi))((1+\theta)/a(1-\chi))$ using $a(p)=F(p)/p$, and (21) yields $1-s=(a(1-\chi)-1)/(a(1-\chi)+m(1-\chi))$. Hence, the first-order condition (21) is below the constraint (22) if $(\chi/(1-\chi))((1+\theta)/a(1-\chi))<(a(1-\chi)-1)/(a(1-\chi)+m(1-\chi))$, which holds because it is a rearrangement of the second condition stated in Assumption 4. That condition is satisfied for sufficiently small χ because the left-hand side approaches zero as χ does, while the right-hand side approaches a positive number given Assumption 1.

Hence, under Assumption 1 and Assumption 4, for any $\lambda\in[0,1]$, the value of p that maximizes C_p is an interior equilibrium with $0<p<1-\chi$. The first-order condition (21) is necessary for an interior maximum, so the equilibrium conditional on λ is found at an intersection point of (21) and (22) in Figure 1. The partial derivative of the constraint $s=1-(1-p+\chi\theta\lambda)/F(p)$ in (22) is

$$\left.\frac{\partial s}{\partial p}\right|_{\text{Constraint}}=\frac{1}{F(p)}+\frac{(1-p+\chi\theta\lambda)F'(p)}{F(p)^2}=\frac{1+(1-s)m(p)}{pa(p)}, \tag{A.9}$$

where the second expression substitutes back the constraint itself and uses the definitions of $a(p)$ and $m(p)$. This derivative is positive given Assumption 1, so (22) is upward sloping in Figure 1.

At a point of intersection, (21) implies $1-s=(a(p)-1)/(a(p)+m(p))$, so $1+(1-s)m(p)=a(p)(1+m(p))/(a(p)+m(p))=sa(p)$ and hence the constraint gradient (A.9) is $\partial s/\partial p|_{\text{Constraint}}=$

$s/p = \psi$. In the diagram, the tangent to the constraint at the equilibrium point is the ray through the origin with gradient equal to the incumbent income multiple ψ. Equation (A.8) gives the derivative of the first-order condition $s = (1+m(p))/(a(p)+m(p))$ in (21), which can be stated as follows using the formula for s in the first-order condition itself:

$$\left.\frac{\partial s}{\partial p}\right|_{\text{FOC}} = \frac{(1-s)m'(p) - sa'(p)}{a(p)+m(p)}. \tag{A.10}$$

The second derivative of the incumbent payoff $\partial^2 C_p/\partial p^2$ at a point where the first-order condition (21) holds (and so $\partial C_p/\partial p = 0$) is given in (A.7). At a point of intersection with the constraint (22), and hence where $1-p+\chi\theta\lambda = (1-p)pa(p)$, this second derivative is

$$\begin{aligned}\left.\frac{\partial^2 C_p}{\partial p^2}\right|_{\frac{\partial C_p}{\partial p}=0} &= -\frac{C}{p^2 a(p)}\left(2m(p)\left(\frac{1+(1-s)m(p)}{pa(p)}\right) - (1-s)m'(p)\right) \\ &= -\frac{C}{p^2 a(p)}\left((a(p)+m(p))\left.\frac{\partial s}{\partial p}\right|_{\text{Constraint}} + (m(p)-a(p))\frac{s}{p} - (1-s)m'(p)\right),\end{aligned}$$

where the second equality uses (A.9) and $\partial s/\partial p|_{\text{Constraint}} = s/p$ at a point of intersection. Noting that $a'(p) = (m(p)-a(p))/p$ and substituting (A.10) into the equation above:

$$\left.\frac{\partial^2 C_p}{\partial p^2}\right|_{\frac{\partial C_p}{\partial p}=0} = -\frac{(a(p)+m(p))C}{p^2 a(p)}\left(\left.\frac{\partial s}{\partial p}\right|_{\text{Constraint}} - \left.\frac{\partial s}{\partial p}\right|_{\text{FOC}}\right).$$

Proposition 2 shows that Assumption 2 suffices for C_p to be a quasi-concave function of p, hence this second derivative is negative at a point where the first-order condition (21) is satisfied. The coefficient of the term in parentheses above is negative, so quasi-concavity implies that $\partial s/\partial p|_{\text{FOC}} < \partial s/\partial p|_{\text{Constraint}}$ at any point of intersection between (21) and (22), that is, the first-order condition always cuts the constraint from above in Figure 1. Therefore, it follows that any point of intersection between the two is unique.

(ii) Conditional on λ, equilibrium power sharing p is found by eliminating s from (21) and (22) and solving the equation $(1+m(p))/(a(p)+m(p)) = 1-(1-p+\chi\theta\lambda)/F(p)$, assuming there is a unique solution (Assumption 2 suffices). Differentiation gives the effect of higher λ on p:

$$\frac{dp}{d\lambda} = \frac{\chi\theta}{F(p)}\left(\left.\frac{\partial s}{\partial p}\right|_{\text{Constraint}} - \left.\frac{\partial s}{\partial p}\right|_{\text{FOC}}\right)^{-1} > 0,$$

using $\partial s/\partial\lambda = -\chi\theta/F(p)$ from (22) and that $\partial s/\partial p$ is larger along the constraint than along the first-order condition. This confirms that p increases with λ. Using (22), the incumbent income multiple $\psi = s/p$ is $\psi = (1-(1-p+\chi\theta\lambda)/F(p))/p$, and comparison to (19) shows that $C_p = \psi C$. The first-order condition (21) for maximizing C_p with respect to p given λ is therefore also the first-order condition for maximizing ψ. Since $\partial\psi/\partial\lambda = -\chi\theta/(pF(p)) < 0$ holding p constant, the envelope theorem implies $d\psi/d\lambda < 0$, so ψ falls as λ increases. Finally, as λ does not appear in the first-order condition (21), but p is known to increase with λ, it follows that the direction of the effect of λ on s has the same sign as $\partial s/\partial p|_{\text{FOC}}$.

A.4 Proof of Proposition 4

Equation (25) is obtained from the derivative $C_p'(\lambda) = \psi(\lambda)C'(\lambda) + \psi'(\lambda)C(\lambda)$ of (23), using the envelope condition (24) to deduce $\psi'(\lambda)/\psi(\lambda) = -\chi\theta/(s(\lambda)F(p(\lambda)))$ by noting $s(\lambda) = p(\lambda)\psi(\lambda)$ and $1/F(p(\lambda)) = C_w(\lambda)/C(\lambda)$ from (12). Since $\chi\theta C_w(\lambda)/s(\lambda)$ is subtracted from $C'(\lambda)$ in (25), the term $\chi\theta C_w(\lambda)/s(\lambda)$ is the private marginal cost of institutional quality that incumbents compare to the marginal benefit $C'(\lambda)$ when choosing λ.

(i) Proposition 2 shows that $0 < s < 1$ for any p under Assumption 1, and consequently $1/s(\lambda) > 1$ for any $\lambda \in [0,1]$, hence $\chi\theta C_w(\lambda)/s(\lambda) > \chi\theta C_w(\lambda)$.

(ii) Proposition 1 shows that equilibrium institutions equate marginal rates of substitution between goods across all individuals, and the resulting combined resource constraint is (18). Suppose institutions feature $\lambda < 1$ with $C'(\lambda) > \chi\theta C_w(\lambda)$, and consider a small feasible increase in λ. The incentive constraint (7) initially binds given (12), and continue to assume consumption is allocated so that $C_k = (1+\theta)C_w$ and marginal rates of substitution are aligned. With the utility function (6), this means the additional and existing individuals undertaking investment opportunities are not worse off as long as C_w does not decline. As the incentive constraint (7) continues to hold, (3) implies $K = \chi\lambda$.

Substituting $C_k = (1+\theta)C_w$ into the resource constraint (18) (which assumes $K = \chi\lambda$) demonstrates that the consumption payoffs C_p and C_w are limited by $pC_p + (1 - p + \chi\theta\lambda)C_w = C$. Fixing p and differentiating with respect to λ:

$$p\frac{dC_p}{d\lambda} + (1 - p + \chi\theta\lambda)\frac{dC_w}{d\lambda} = \frac{dC}{d\lambda} - \chi\theta C_w,$$

where the right-hand side is positive if $C'(\lambda) > \chi\theta C_w(\lambda)$. It follows that either C_p or C_w (or both) can be raised without lowering the other, so a Pareto improvement is possible when λ increases. The social marginal cost of more investment is $\chi\theta C_w(\lambda)$, which is compared to $C'(\lambda)$ to judge efficiency.

(iii) The function $\mu(\lambda) = (\chi\theta C_w(\lambda)/s(\lambda))/C_p(\lambda)$ is incumbents' private marginal cost of λ as a fraction of $C_p(\lambda)$. Using (25), the expression for $C_p'(\lambda)$ in (26) follows immediately. The definition $s = pC_p/C$ and $C_w = C/F(p)$ from (12) imply $\mu(\lambda) = \chi\theta C(\lambda)/(s(\lambda)F(p(\lambda))s(\lambda)C(\lambda)/p(\lambda))$, which simplifies to $\mu(\lambda) = \chi\theta/(a(p(\lambda))s(\lambda)^2)$ using $a(p) = F(p)/p$, confirming equation (26).

(iv) The derivative of $\mu(\lambda)$ from (26) is

$$\mu'(\lambda) = -\frac{\chi\theta s(\lambda)p'(\lambda)}{(a(p(\lambda))s(\lambda)^2)^2}\left(s(\lambda)a'(p(\lambda)) + 2a(p(\lambda))\left.\frac{\partial s}{\partial p}\right|_{\text{FOC}}\right),$$

which uses $s'(\lambda) = \partial s/\partial p|_{\text{FOC}}p'(\lambda)$ because $s(\lambda)$ must satisfy the first-order condition (21). Substituting from (21) and (A.8) and simplifying (dropping the explicit dependence of p and s on λ):

$$\mu'(\lambda) = -\frac{\chi\theta s p'(\lambda)\left((1+m(p))((a(p)+m(p)) - 2a(p))a'(p) + 2a(p)(a(p)-1)m'(p)\right)}{((a(p)+m(p))a(p)s^2)^2}.$$

By using $a'(p) = (m(p) - a(p))/p$, the derivative can be written as:

$$\mu'(\lambda) = -\frac{\chi\theta s p'(\lambda)\left((1+m(p))(a(p)-m(p))^2 + 2pa(p)(a(p)-1)m'(p)\right)}{\left((a(p)+m(p))a(p)s^2\right)^2 p}.$$

The first term in the parentheses is strictly positive given the first condition $a'(p) < 0$ in Assumption 3. Since $a(p) - 1 > 0$ under Assumption 1, the second condition in Assumption 3 implies $2pa(p)(a(p)-1)m'(p) > -(1+m(p))(a(p)-m(p))^2$, and hence the whole term in parentheses is positive. Together with $p'(\lambda) > 0$ from Proposition 3, this demonstrates that $\mu'(\lambda) < 0$.

A.5 Proof of Proposition 5

(i) With π^* taken as given by a small open economy, it follows from the expression for real GDP in (18) that $C(\lambda)$ is linear in λ, and thus $C''(\lambda) = 0$. Using (27), the second derivative of $C_p(\lambda)$ evaluated at a critical point is therefore $-\psi(\lambda)\mu'(\lambda)C_p(\lambda)$, which is strictly positive under the assumption $\mu'(\lambda) < 0$. Therefore, $C_p(\lambda)$ is a strictly quasi-convex function of λ.

(ii) Given that the incumbent payoff is strictly quasi-convex in $\lambda \in [0,1]$, the maximum value of $C_p(\lambda)$ is found either at $\lambda = 0$ or $\lambda = 1$. The differences between p and ψ at these two values of λ follow immediately from Proposition 3.

(iii) Using (18) and (23), $C_p(\lambda) = \psi(\lambda)(Q + \pi^*\chi\lambda)/(1 - \alpha + \alpha\pi^{*1-\varepsilon})^{1/(1-\varepsilon)}$ is the payoff received by those in power. The equilibrium λ maximizing $C_p(\lambda)$ is $\lambda = 1$ rather than $\lambda = 0$ if $\psi(1)(Q + \pi^*\chi) \geq \psi(0)Q$ given that the denominator of the payoff is independent of λ. This is equivalent to $\pi^*\chi \geq ((\psi(0) - \psi(1))/\psi(1))Q$ and hence to the condition stated using the definitions $\psi^\dagger = \psi(0)$ and $\tilde{\psi} = \psi(1)$.

A.6 Proof of Proposition 6

(i) The equilibrium world price is (28), and endowments are equal across countries, so $Q = Q^*$. Given a fraction γ of countries where $\lambda = 1$, the world supply of investment goods is $K^* = \chi\gamma$, which implies $\bar{\pi}^* = (\alpha Q/((1-\alpha)\chi\gamma))^{1/\varepsilon}$. Proposition 5 shows that the condition for $\lambda = 1$ to be optimal for those in power is $\pi^*\chi \geq \xi Q$, which is therefore equivalent to $(\alpha Q/((1-\alpha)\chi\gamma)) \geq (Q/\chi)^\varepsilon \xi^\varepsilon$. Rearranging to have γ on one side and all remaining terms on the other confirms the upper bound on γ given in the proposition.

(ii) Let the threshold for γ from (i) where $\lambda = 1$ is an equilibrium in a given country be denoted by $\bar{\gamma} = (\alpha/(1-\alpha))(Q/\chi)^{1-\varepsilon}(1/\xi^\varepsilon)$. The value of λ (either 0 or 1 according to Proposition 5) in each country must be an equilibrium given the world price π^*, and world markets must clear given the fraction γ of countries with $\lambda = 1$. Since $\bar{\gamma}$ is strictly positive for all parameters and prices, there cannot be an equilibrium with $\gamma = 0$ because this would imply incumbents everywhere want to choose $\lambda = 1$, resulting in $\gamma = 1$.

By using (25), (26), and $C_p(\lambda) = \psi(\lambda)C(\lambda)$, the marginal cost of institutional quality $\mu(\lambda)$ satisfies the differential equation $\mu(\lambda) = -\psi'(\lambda)/\psi(\lambda)^2$ in terms of the incumbent income multiple $\psi(\lambda)$. This differential equation is equivalent to $\mu(\lambda) = \mathrm{d}(1/\psi(\lambda))/\mathrm{d}\lambda$, hence $\psi(1)^{-1} - \psi(0)^{-1} = \int_0^1 \mu(\lambda)\mathrm{d}\lambda$. With assumptions guaranteeing $\mu'(\lambda) < 0$, it follows that $\int_0^1 \mu(\lambda)\mathrm{d}\lambda > \mu(1)$ and $\psi(1)^{-1} - \psi(0)^{-1} > \mu(1)$. Multiplying both sides by $\psi(1)$ implies $\mu(1)\psi(1) < 1 - (\psi(1)/\psi(0))$. To have Assumption 5 hold, it is therefore necessary that $\alpha \leq \overline{\alpha}$ for some $0 < \overline{\alpha} < 1$ because $\min\{\mu(1)\psi(1), 1\} < 1$ when the marginal cost of institutional quality is decreasing. Using $\mu(1)\psi(1) < 1 - (\psi(1)/\psi(0))$, it follows that $(1/\min\{\mu(1)\psi(1), 1\}) - 1 > \psi(1)/(\psi(0) - \psi(1))$. With reference to the definitions $\psi^\dagger = \psi(0)$, $\tilde{\psi} = \psi(1)$, and $\xi = (\psi^\dagger - \tilde{\psi})/\tilde{\psi}$ from Proposition 5, this means $(1/\min\{\mu(1)\psi(1), 1\}) - 1 > 1/\xi$. Combining this result with Assumption 5, it follows that $(\alpha/(1-\alpha))(Q/\chi)^{1-\varepsilon}(1/\xi^\varepsilon) < 1$ and hence $0 < \bar{\gamma} < 1$.

Since $\bar{\gamma} < 1$, if there were an equilibrium with $\gamma = 1$ then this would mean $\gamma > \bar{\gamma}$, and incumbents in all countries would have an incentive to choose $\lambda = 0$, resulting in $\gamma = 0$, and thus ruling out $\gamma = 1$ as an equilibrium. Finally, consider an equilibrium with $0 < \gamma < 1$, which requires that incumbents in some countries choose $\lambda = 0$ and others choose $\lambda = 1$. Since incumbents in all ex-ante identical countries share the same optimality condition for $\lambda = 1$, the condition from (i) must hold with equality, and thus $\gamma = \bar{\gamma}$. With $0 < \bar{\gamma} < 1$ as shown above, the existence of this equilibrium is confirmed. The equilibrium world price $\bar{\pi}^*$ follows by using (28) with $Q^* = Q$ and substituting the expression for $\bar{\gamma}$ into $K^* = \bar{\gamma}\chi$.

(iii) Since the condition for $\lambda = 1$ to be chosen by incumbents holds with equality, incumbents must receive identical payoffs $C_p^\dagger = C_p(0) = C_p(1) = \tilde{C}_p$ in equilibrium. Using the binding political constraint (12), the payoff of a worker is $C_w(\lambda) = C(\lambda)/F(p(\lambda))$. Combined with (23), this implies $C_w(\lambda) = C_p(\lambda)/(\psi(\lambda)F(p(\lambda)))$ in terms of the incumbent income multiple $\psi(\lambda)$, and $C_w(\lambda) = C_p(\lambda)/(s(\lambda)a(p(\lambda)))$ using the definitions $s = \psi/p$ and $a(p) = F(p)/p$. Rearranging equation (21) shows that the incumbent income share satisfies $sa(p) = 1 + (1-s)m(p)$, so $\mathrm{d}(s(\lambda)a(p(\lambda)))/\mathrm{d}\lambda = -(m(p(\lambda))s'(\lambda) - (1-s(\lambda))m'(\lambda))p'(\lambda)$, which is negative because $m(p) > 0$, $m'(p) \leq 0$, $p'(\lambda) > 0$, and $s'(\lambda) > 0$ (equation 26 shows that $s'(\lambda) > 0$ is necessary for $\mu'(\lambda) < 0$). It follows that $C_w^\dagger = C_w(0) = C_p^\dagger/(s(0)a(p(0))) < \tilde{C}_p/(s(1)a(p(1))) = C_w(1) = \tilde{C}_w$, so workers receive more consumption in countries where $\lambda = 1$.

The binding incentive constraint in (12) is $C_k(\lambda) = (1+\theta)C_w(\lambda)$, and thus (6) implies the utility payoff of an investor is $\log C_w(\lambda)$, which moves in line with that of a worker. Therefore, workers and investors in countries with $\lambda = 1$ are strictly better off than workers in $\lambda = 0$ countries (where there are no investors).

(iv) Using (18), it follows immediately that countries with $\lambda = 1$ have higher real GDP C than those with $\lambda = 0$ because there is an extra positive term $\pi^*\chi$ in the numerator.

From (18), the marginal benefit of institutional quality is $C'(\lambda) = \pi^*\chi/(1-\alpha+\alpha\pi^{*1-\varepsilon})^{1/(1-\varepsilon)}$, which is independent of λ and identical for countries with $\lambda = 0$ and $\lambda = 1$. Since $C_p(\lambda)$ is strictly quasi-convex and is maximized by both $\lambda = 0$ and $\lambda = 1$, it must be the case that $C'_p(1) > 0$, and hence $C'(1) > \chi\theta\tilde{C}_w/s(1)$ using (25). Using $s(1) < 1$ from Proposition 2, $C'(0) = C'(1)$, and $\tilde{C}_w > C^\dagger_w$ as shown above, it follows that $C'(0) > \chi\theta C^\dagger_w$. Hence, the criterion in Proposition 4 demonstrates that countries with $\lambda = 0$ have an inefficiently low level of investment. All other aspects of institutions are efficient given Proposition 1, so countries with $\lambda = 1$ have Pareto efficient institutions.

A.7 Proof of Proposition 7

(i) The derivative of autarky GDP $\hat{C}(\lambda)$ from (29) is the marginal benefit of institutional quality, and hence $\hat{C}'(\lambda) = \chi(\alpha+(1-\alpha)\hat{\pi}^{\varepsilon-1})^{1/(\varepsilon-1)}$ where $\hat{\pi}$ is the domestic market-clearing price also given in (29), noting that $\partial\hat{C}(\lambda)/\partial\hat{\pi} = 0$. The latter follows because $\chi\lambda - \alpha\hat{\pi}^{-\varepsilon}(Q+\hat{\pi}\chi\lambda)/(1-\alpha+\alpha\hat{\pi}^{1-\varepsilon}) = 0$ after rearranging and using (29). Imposing $\varepsilon = 1$ from Assumption 6 yields $\hat{C}'(\lambda) = \chi\hat{\pi}^{1-\alpha}$ and $\hat{\pi} = \alpha Q/((1-\alpha)\chi\lambda)$. Using $\hat{C}(\lambda) = Q^{1-\alpha}\chi^\alpha\lambda^\alpha/((1-\alpha)^{1-\alpha}\alpha^\alpha)$, the marginal benefit can be expressed as $\hat{C}'(\lambda) = \alpha\hat{C}(\lambda)/\lambda$. Together with (23) and (26), the derivative of incumbents' payoff with respect to institutional quality is $\hat{C}'_p(\lambda) = \psi(\lambda)\left((\alpha/\lambda) - \mu(\lambda)\psi(\lambda)\right)\hat{C}(\lambda)$. Since $\alpha > 0$ and $\psi(\lambda)$, $\mu(\lambda)$, and $\hat{C}(\lambda)$ are all positive and finite, this derivative is positive for λ in the neighbourhood of $\lambda = 0$. Therefore, equilibrium institutions in autarky always have $\hat{\lambda} > 0$, and the incumbent payoff derivative can be written as

$$\hat{C}'_p(\lambda) = \frac{\psi(\lambda)\hat{C}(\lambda)}{\lambda}\left(\alpha - \lambda\mu(\lambda)\psi(\lambda)\right). \tag{A.11}$$

With the functional form $F(p) = \beta + \delta p$ in Assumption 6, the marginal and average political products are $m(p) = \delta$ and $a(p) = (\beta+\delta p)/p$. Substituting into (21) implies $s = (1+\delta)p/(\beta+2\delta p)$ and hence the incumbent income multiple is $\psi = s/p = (1+\delta)/(\beta+2\delta p)$. Further substituting for s and $a(p)$ in the marginal cost of institutional quality $\mu(\lambda)$ from (26) and evaluating at $p = p(\lambda)$:

$$\mu(\lambda) = \frac{\chi\theta(\beta+2\delta p(\lambda))^2}{(1+\delta)^2(\beta+\delta p(\lambda))p(\lambda)}, \quad \text{and} \quad \psi(\lambda) = \frac{1+\delta}{\beta+2\delta p(\lambda)}. \tag{A.12}$$

From the constraint (22), it follows that $1+\chi\theta\lambda = p + (1-s)F(p) = p(1+(1-s)a(p))$. Since (21) implies $sa(p) = 1+(1-s)m(p)$, this leads to $1+\chi\theta\lambda = p(a(p)-m(p)+sm(p)) = \beta+\delta ps$ by using $m(p) = \delta$ and $a(p) = (\beta+\delta p)/p$. Hence, by using $s = (1+\delta)p/(\beta+2\delta p)$ again, the inverse of the function $p(\lambda)$ is

$$\lambda = \frac{\delta(1+\delta)p^2 - (1-\beta)(\beta+2\delta p)}{\chi\theta(\beta+2\delta p)}. \tag{A.13}$$

Combining (A.12) and (A.13) yields the following formulas in terms of $p = p(\lambda)$:

$$\mu(\lambda)\psi(\lambda) = \frac{\chi\theta(\beta+2\delta p)}{(1+\delta)(\beta+\delta p)p}, \quad \lambda\mu(\lambda)\psi(\lambda) = \frac{\delta(1+\delta)p^2 - (1-\beta)(\beta+2\delta p)}{(1+\delta)(\beta+\delta p)p}. \tag{A.14}$$

Therefore, the derivative of the incumbent payoff (A.11) is

$$\hat{C}'_p(\lambda) = \frac{\psi(\lambda)\hat{C}(\lambda)J(p(\lambda))}{(1+\delta)(\beta+\delta p(\lambda))p(\lambda)\lambda}, \quad \text{so } \hat{C}'_p(\lambda) = 0 \text{ only if } J(p(\lambda)) = 0, \tag{A.15}$$

where the function $J(p)$ is defined by:

$$J(p) = (1-\beta)\beta + (2\delta(1-\beta) + \alpha\beta(1+\delta))p - (1-\alpha)\delta(1+\delta)p^2. \tag{A.16}$$

Using (A.15), the second derivative of the incumbent payoff (A.11) evaluated at a critical point is

$$\hat{C}''_p(\lambda) = \frac{\psi(\lambda)\hat{C}(\lambda)J'(p(\lambda))}{(1+\delta)(\beta+\delta p(\lambda))p(\lambda)\lambda} \quad \text{where } \hat{C}'_p(\lambda) = 0. \tag{A.17}$$

Since $0 < \alpha < 1$ and $0 < \beta < 1$, the quadratic equation (A.16) has a positive and a negative root, and $J''(p) < 0$. Given that $\hat{C}'_p(0) > 0$, equation (A.15) implies $J(p^\dagger) > 0$ for $p^\dagger = p(0)$, so it follows that for any $\lambda \in [0,1]$ where $J(p(\lambda)) = 0$, it must be the case that $J'(p(\lambda)) < 0$ because $p(\lambda) \geq p^\dagger$. Using (A.17), this means $\hat{C}''_p(\lambda) < 0$ for any $\lambda \in [0,1]$ where $\hat{C}'_p(\lambda) = 0$, which establishes that $\hat{C}_p(\lambda)$ is a strictly quasi-concave function of λ.

(ii) The condition in Assumption 5 with $\varepsilon = 1$ is $(\alpha/(1-\alpha))((1/\min\{\mu(1)\psi(1), 1\}) - 1) < 1$. With $\mu'(\lambda) < 0$, the proof of Proposition 6 shows that $\mu(1)\psi(1) < 1$, so this can be further simplified to $(1/(\mu(1)\psi(1))) - 1 < (1/\alpha) - 1$ and hence to $\alpha < \mu(1)\psi(1)$. Referring to (A.11), it follows that $\hat{C}'_p(1) < 0$, so institutional quality must satisfy $\hat{\lambda} < 1$. It is already shown that $\hat{\lambda} > 0$, so there must be an interior equilibrium $\hat{\lambda} \in (0,1)$. Since $\hat{C}_p(\lambda)$ is a quasi-concave function, this equilibrium is the unique solution of the first-order condition $\alpha = \lambda\mu(\lambda)\phi(\lambda)$ where $\hat{C}'_p(\lambda) = 0$ according to (A.11). With $0 < \hat{\lambda} < 1$, the results of Proposition 3 imply that $\hat{p} = p(\hat{\lambda})$ and $\hat{\psi} = \psi(\hat{\lambda})$ respectively lie between $p^\dagger = p(0)$ and $\tilde{p} = p(1)$, and between $\tilde{\psi} = \psi(1)$ and $\psi^\dagger = \psi(0)$.

(iii) Since Q does not appear in the equation $\alpha = \hat{\lambda}\mu(\hat{\lambda})\phi(\hat{\lambda})$ for equilibrium institutional quality, the equilibrium value $\hat{\lambda}$ is independent of the endowment Q.

(iv) Now allow for the possibility of trade and take an arbitrary world price π^*. For each $\lambda \in [0,1]$, let the functions $\hat{C}(\lambda)$ and $C(\lambda)$ respectively denote the levels of real GDP in autarky and with free trade in an open economy. Note that in an open economy with a particular λ, it is possible to obtain the same consumption outcomes as autarky (with the same λ) by setting a tariff τ (see A.1) that results in net exports of zero. Using the formulas from (A.5) with $\varepsilon = 1$ and $K = \chi\lambda$, the required tariff is $\hat{\tau} = (\alpha Q/((1-\alpha)\pi^*\chi\lambda)) - 1$, which can be written as $\hat{\tau} = (\hat{\pi}/\pi^*) - 1$ in terms of the autarky price $\hat{\pi}$ from (29). With $X_E = 0$ and $X_I = 0$ and the same λ, real GDP would be equal to its autarky value $\hat{C}(\lambda)$. This level of real GDP can also be compared to the free-trade ($\tau = 0$) open-economy level $C(\lambda)$ using the relationship $\hat{C}(\lambda) = D(\hat{\tau})C(\lambda)$ derived in the proof of Proposition 1 in terms of the function $D(\tau)$ from (A.6).

It follows that $\hat{C}_p(\lambda) = D(\hat{\tau})C_p(\lambda)$ using incumbents' consumption levels $\hat{C}_p(\lambda) = \psi(\lambda)\hat{C}(\lambda)$ and $C_p(\lambda) = \psi(\lambda)C(\lambda)$ respectively under autarky and in an open economy with free trade. Since

$D(\tau) \leq 1$ for all τ using the properties of $D(\tau)$ from (A.6), this implies $\hat{C}_p(\lambda) \leq C_p(\lambda)$ for all $0 \leq \lambda \leq 1$. If $\pi^* = \hat{\pi}$ for some particular λ then $\hat{\tau} = 0$ and $D(\hat{\tau}) = 1$, in which case $\hat{C}_p(\lambda) = C_p(\lambda)$.

The strict quasi-convexity of $C_p(\lambda)$ implies $\max\{C_p(0), C_p(1)\} > C_p(\hat{\lambda})$, where $0 < \hat{\lambda} < 1$ is equilibrium institutional quality under autarky. Together with $C_p(\hat{\lambda}) \geq \hat{C}_p(\hat{\lambda})$, it follows that incumbents' consumption with international trade, either $C_p(0)$ or $C_p(1)$, is greater than their consumption $\hat{C}_p = \hat{C}_p(\hat{\lambda})$ in autarky. Therefore, those in power always strictly gain from the ability to trade with the rest of the world irrespective of world prices.

If $\lambda = 1$ is chosen with trade, it must be the case that $\tilde{C}_p = C_p(1) > \hat{C}_p$. Since $\tilde{C}_p = \psi(1)\tilde{C}$ with $\tilde{C} = C(1)$, it follows that $\tilde{C} = \tilde{C}_p/\psi(1) > \hat{C}_p/\psi(1) = (\psi(\hat{\lambda})/\psi(1))\hat{C} > \hat{C}$ because $\psi(\hat{\lambda}) > \psi(1)$. The real value of the economy's output is thus increased by trade if those in power choose $\lambda = 1$. The proof of Proposition 6 shows that $C_w(\lambda) = C_p(\lambda)/(s(\lambda)a(p(\lambda)))$, where $s(\lambda)a(p(\lambda))$ is decreasing in λ, and this relationship between C_w and C_p holds in autarky as well as in an open economy. As $\tilde{C}_p > \hat{C}_p$ and $\hat{\lambda} < 1$, this means that $\tilde{C}_w > \hat{C}_w$, so workers in economies with $\lambda = 1$ gain from trade. The same is true for investors who receive a utility payoff that moves in line with workers' consumption. With $\tilde{p} = p(1) > p(\hat{\lambda}) = \hat{p}$, there are also more members of the group in power, who receive higher payoffs than workers ($C_p > C_w$, as can be shown by noting $sa(p) = 1 + (1-s)m(p) > 1$). Hence, for economies that move to $\lambda = 1$, opening up to trade is a Pareto improvement.

A.8 Proof of Proposition 8

An open economy with real GDP from (18) has $C''(\lambda) = 0$, so equation (27) implies $C_p''(\lambda) = -\mu'(\lambda)\psi(\lambda)C_p(\lambda)$ for any λ with $C_p'(\lambda) = 0$. The assumption $\mu'(\lambda) > 0$ thus implies $C_p''(\lambda) < 0$ at a critical point, so $C_p(\lambda)$ is a strictly quasi-concave function of λ. This is maximized at a unique value of λ. With $Q(n) = Q$ for all $n \in [0,1]$, the function $C(\lambda)$ is the same for all countries, and hence so is $C_p(\lambda)$ in (23). Therefore, the same level of institutional quality maximizes $C_p(\lambda)$ in all countries, so there is a degenerate global distribution of λ.

Since $Q(n) = Q$ and $\lambda(n) = \lambda$ for all countries $n \in [0,1]$, the global supplies of the endowment and investment goods are $Q^* = Q$ and $K^* = \chi\lambda$. The world market-clearing relative price $\bar{\pi}^*$ from (28) thus reduces to the same function of λ as the autarky market-clearing price $\hat{\pi}$ within a country from (29). Using (23) and (26), the first-order condition that characterizes the unique equilibrium value of λ across all countries is $C'(\lambda)/C(\lambda) = \mu(\lambda)\psi(\lambda)$. From real GDP (18) evaluated at $\pi^* = \bar{\pi}^*$, it follows that $C'(\lambda)/C(\lambda) = \bar{\pi}^*\lambda/(Q + \bar{\pi}^*\chi\lambda)$. In autarky where $\hat{C}_p(\lambda)$ is a quasi-concave function, the first-order condition uniquely characterizing equilibrium institutional quality $\hat{\lambda}$ is $\hat{C}'(\lambda)/\hat{C}(\lambda) = \mu(\lambda)\psi(\lambda)$. With $\hat{C}'(\lambda)/\hat{C}(\lambda) = \hat{\pi}\lambda/(Q + \hat{\pi}\chi\lambda)$ and $\hat{\pi}$ being the same function of λ as $\bar{\pi}^*$, the equation for the equilibrium λ in open economies is the same as in autarky, so $\lambda = \hat{\lambda}$.

A.9 Proof of Proposition 9

(i) If the partial openness constraints $-\sigma\chi \leq X_I \leq \sigma\chi$ do not bind then the equilibrium institutions have net exports (17) consistent with free trade ($\pi = \pi^*$). With $K = \chi\lambda$ and $\varepsilon = 1$ under Assumption 6, (17) implies $X_I = (1-\alpha)\chi\lambda - \alpha Q/\pi^*$, so the partial openness constraints are slack if $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ where

$$\underline{\lambda} = \frac{\alpha Q}{(1-\alpha)\pi^*\chi} - \frac{\sigma}{1-\alpha} \text{ and } \overline{\lambda} = \frac{\alpha Q}{(1-\alpha)\pi^*\chi} + \frac{\sigma}{1-\alpha}. \tag{A.18}$$

These bounds on λ satisfy $\underline{\lambda} < \overline{\lambda}$ because $\sigma > 0$ and $0 < \alpha < 1$, so the interval $[\underline{\lambda}, \overline{\lambda}]$ always contains a continuum of λ values, though it is possible that $\underline{\lambda} < 0$ or $\overline{\lambda} > 1$, so it may not be contained entirely within the unit interval of valid $\lambda \in [0,1]$ values.

If $\lambda \in [0, \underline{\lambda})$ then the constraint on imports of the investment good binds, hence $X_I = -\sigma\chi$, and if $\lambda \in (\overline{\lambda}, 1]$ then the constraint on exports binds, hence $X_I = \sigma\chi$. The argument from Proposition 1 that equilibrium institutions feature free exchange domestically still applies in these cases, so real GDP is given by (A.4) with $\varepsilon = 1$, that is, $C(\lambda) = ((Q - X_E) + \pi(\chi\lambda - X_I))/\pi^\alpha$, where the domestic market-clearing price is $\pi = (\alpha(Q - X_E))/((1-\alpha)(\chi\lambda - X_I))$ from (16). It follows that $C(\lambda) = (Q - X_E)^{1-\alpha}(\chi\lambda - X_I)^\alpha/((1-\alpha)^{1-\alpha}\alpha^\alpha)$ in these cases, and hence by using the binding partial openness constraints and the international budget constraint (2):

$$C(\lambda) = \begin{cases} \frac{\chi^\alpha(Q-\sigma\pi^*\chi)^{1-\alpha}(\lambda+\sigma)^\alpha}{(1-\alpha)^{1-\alpha}\alpha^\alpha} & \text{if } \lambda \in [0, \underline{\lambda}] \\ \frac{Q+\pi^*\chi\lambda}{\pi^{*\alpha}} & \text{if } \lambda \in [\underline{\lambda}, \overline{\lambda}] \\ \frac{\chi^\alpha(Q+\sigma\pi^*\chi)^{1-\alpha}(\lambda-\sigma)^\alpha}{(1-\alpha)^{1-\alpha}\alpha^\alpha} & \text{if } \lambda \in [\overline{\lambda}, 1] \end{cases}, \tag{A.19}$$

noting that real GDP $C(\lambda)$ is identical to (18) in the range $\lambda \in [\underline{\lambda}, \overline{\lambda}]$. Net exports implied by (17) with no restrictions on trade are consistent with $X_I = -\sigma\chi$ and $X_I = \sigma\chi$ respectively at $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$, so the expression for real GDP $C(\lambda)$ in (A.19) is continuous at the boundaries of the interval $[\underline{\lambda}, \overline{\lambda}]$. Furthermore, since $\partial C/\partial \pi = 0$ for real GDP $C = ((Q - X_E) + \pi(\chi\lambda - X_I))/\pi^\alpha$ using (16), it follows that $C'(\lambda) = \pi^{1-\alpha}\chi$ in all cases, whether or not the partial openness constraints are binding. Since $\pi = \pi^*$ at $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$, as well as for all $\lambda \in (\underline{\lambda}, \overline{\lambda})$, this demonstrates that $C(\lambda)$ is differentiable for all $\lambda \in (0,1)$, even across the boundaries of the interval $[\underline{\lambda}, \overline{\lambda}]$. Therefore, the incumbent objective $C_p(\lambda)$ is continuous and differentiable for all λ.

Curvature of the incumbent payoff

In the range $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, the incumbent payoff $C_p(\lambda) = \psi(\lambda)C(\lambda)$ is a strictly quasi-convex function following the proof in Proposition 5. To establish its properties outside this range, note that (A.19) implies the marginal benefit of institutional quality satisfies

$$\frac{C'(\lambda)}{C(\lambda)} = \begin{cases} \frac{\alpha}{\lambda+\sigma} & \text{if } \lambda \in [0, \underline{\lambda}] \\ \frac{\pi^*\chi}{Q+\pi^*\chi\lambda} & \text{if } \lambda \in [\underline{\lambda}, \overline{\lambda}] \\ \frac{\alpha}{\lambda-\sigma} & \text{if } \lambda \in [\overline{\lambda}, 1] \end{cases}. \tag{A.20}$$

Together with (23) and (26), it follows that

$$C_p'(\lambda) = \begin{cases} \psi(\lambda)\left(\frac{\alpha}{\lambda+\sigma} - \mu(\lambda)\psi(\lambda)\right)C(\lambda) & \text{if } \lambda \in [0, \underline{\lambda}] \\ \psi(\lambda)\left(\frac{\pi^*\chi}{Q+\pi^*\lambda} - \mu(\lambda)\psi(\lambda)\right)C(\lambda) & \text{if } \lambda \in [\underline{\lambda}, \overline{\lambda}] \\ \psi(\lambda)\left(\frac{\alpha}{\lambda-\sigma} - \mu(\lambda)\psi(\lambda)\right)C(\lambda) & \text{if } \lambda \in [\overline{\lambda}, 1] \end{cases}. \tag{A.21}$$

As $F(p) = \beta + \delta p$ under Assumption 6, the formulas for $\mu(\lambda)\psi(\lambda)$ and $\lambda\mu(\lambda)\psi(\lambda)$ from (A.14) in the proof of Proposition 7 can be used here. Substituting these, the derivative of the incumbent payoff can be expressed as $C_p'(\lambda) = \psi(\lambda)C(\lambda)\underline{J}(p(\lambda))/((1+\delta)(\beta+\delta p(\lambda))p(\lambda)(\lambda+\sigma))$ if $\lambda \in [0, \underline{\lambda}]$ or $C_p'(\lambda) = \psi(\lambda)C(\lambda)\overline{J}(p(\lambda))/((1+\delta)(\beta+\delta p(\lambda))p(\lambda)(\lambda-\sigma))$ if $\lambda \in [\overline{\lambda}, 1]$ for functions $\underline{J}(p)$ and $\overline{J}(p)$ given by:

$$\underline{J}(p) = \beta(1-\beta-\sigma\chi\theta) + (2\delta(1-\beta-\sigma\chi\theta) + \alpha\beta(1+\delta))p - (1-\alpha)\delta(1+\delta)p^2, \quad \text{and}$$
$$\overline{J}(p) = \beta(1-\beta+\sigma\chi\theta) + (2\delta(1-\beta+\sigma\chi\theta) + \alpha\beta(1+\delta))p - (1-\alpha)\delta(1+\delta)p^2.$$

Critical points of the objective function $C_p(\lambda)$ in the two cases correspond to roots of the quadratic equations $\underline{J}(p) = 0$ and $\overline{J}(p) = 0$ for $p = p(\lambda)$. The functions both satisfy $\underline{J}''(p) < 0$ and $\overline{J}''(p) < 0$. Since $0 < \alpha < 1$ and $0 < \beta < 1$ under Assumption 6, the quadratic equation $\overline{J}(p) = 0$ has a positive and a negative root. As $p^\dagger > 0$, any $p \geq p^\dagger$ where $\overline{J}(p) = 0$ must have $\overline{J}'(p) < 0$. This establishes that $C_p(\lambda)$ is quasi-concave for $\lambda \in [\overline{\lambda}, 1]$. For values of σ less than the positive number $(1-\beta)/(\chi\theta)$, the quadratic $\underline{J}(p)$ also has a positive and a negative root because $1-\beta-\sigma\chi\theta > 0$. This means that any $p \geq p^\dagger$ where $\underline{J}(p) = 0$ has $\underline{J}'(p) < 0$, demonstrating that $C_p(\lambda)$ is quasi-concave for $\lambda \in [0, \underline{\lambda}]$.

Specialization in institutional quality

If there were no specialization in the global equilibrium then all ex-ante identical countries would choose a common λ^*. The global supply of investment goods would be $K^* = \chi\lambda^*$, and hence the world equilibrium price (28) with $Q = Q^*$ and $\varepsilon = 1$ is $\bar{\pi}^* = \alpha Q/((1-\alpha)\chi\lambda^*)$. Comparison with (A.18) shows that $\underline{\lambda} = \lambda^* - \sigma/(1-\alpha)$ and $\overline{\lambda} = \lambda^* + \sigma/(1-\alpha)$, so λ^* lies in the interior of the interval $[\underline{\lambda}, \overline{\lambda}]$. Since $\lambda^* \in [\underline{\lambda}, \overline{\lambda}]$, (A.19) and (A.21) imply the derivative of the incumbent payoff is $C_p'(\lambda) = \psi(\lambda)(\pi^*\chi - \mu(\lambda)\psi(\lambda)(Q+\pi^*\chi\lambda))/\pi^{*\alpha}$. Evaluating this at $\pi^* = \bar{\pi}^*$ and $\lambda = \lambda^*$ gives $C_p'(\lambda^*) = (\bar{\pi}^*\chi/\alpha)(\alpha - \lambda^*\mu(\lambda^*)\psi(\lambda^*))/\bar{\pi}^{*\alpha}$. If $\lambda^* = 0$ then this is positive, while if $\lambda^* = 1$ then this is negative under Assumption 5 because $\alpha < \mu(1)\psi(1)$ as demonstrated in the proof of Proposition 7. Therefore, it is not possible to have a common choice of $\lambda^* = 0$ or $\lambda^* = 1$ as this is not consistent with maximization of the incumbent payoff. With $\lambda^* \in (0,1)$, the interval $[\underline{\lambda}, \overline{\lambda}]$ would include a continuum of values inside $[0,1]$, so the quasi-convexity of $C_p(\lambda)$ in this range means that $\lambda = \lambda^*$ cannot maximize $C_p(\lambda)$. This rules out a global equilibrium without specialization.

In equilibrium, the world price π^* must be such that $\underline{\lambda}$ and $\overline{\lambda}$ in (A.18) satisfy $\underline{\lambda} < 1$ or $\overline{\lambda} > 0$. If not, $C_p(\lambda)$ would be strictly quasi-concave on the whole unit interval, and there would be a unique value of λ maximizing $C_p(\lambda)$ for all countries. Such an equilibrium without specialization has already been ruled out, so the interval $[\underline{\lambda}, \overline{\lambda}]$ must overlap with a unit interval $[0,1]$ for a continuum of λ values. Since $C_p(\lambda)$ is strictly quasi-convex for $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, the only possible values of λ in this range that could maximize $C_p(\lambda)$ are $\lambda = \underline{\lambda}$ or $\lambda = \overline{\lambda}$, or the endpoints of $[0,1]$ if $\underline{\lambda} < 0$ or $\overline{\lambda} > 1$. The strict quasi-concavity of $C_p(\lambda)$ for $\lambda \in [0, \underline{\lambda}]$ and $\lambda \in [\overline{\lambda}, 1]$ means there is a unique value of λ that maximizes $C_p(\lambda)$ within each of the intervals $[0, \underline{\lambda}]$ and $[\overline{\lambda}, 1]$. Therefore, there are at most two possible values of λ that are local maximums of $C_p(\lambda)$, and hence could maximize $C_p(\lambda)$ over the whole unit interval. As the incumbent payoff function $C_p(\lambda)$ is the same in all countries, let these common values of λ be denoted by $\lambda^\dagger$ and $\tilde{\lambda}$, which satisfy $0 \leq \lambda^\dagger < \tilde{\lambda} \leq 1$. Equilibrium specialization in λ across countries must therefore take on a 'high-low' pattern.

The width of the interval $[\underline{\lambda}, \overline{\lambda}]$ is $2\sigma/(1-\alpha)$ using (A.18), and it is known that this interval overlaps with $[0,1]$ for a continuum of λ values. Hence, for a range of sufficiently small positive values of σ, it must be the case that $0 < \underline{\lambda} < \overline{\lambda} < 1$. As the endpoints of $[\underline{\lambda}, \overline{\lambda}]$ are strictly inside the unit interval, $C_p(\lambda)$ cannot be maximized where it is strictly quasi-convex. It follows that $\lambda^\dagger \in [0, \underline{\lambda})$ and $\tilde{\lambda} \in (\overline{\lambda}, 1]$, so one of the partial openness constraints $X_I \leq \sigma\chi$ or $X_I \geq -\sigma\chi$ is binding in each country.

Using (A.19), since the terms involving π^* in the expression for $C(\lambda)$ are multiplicative and independent of λ, it follows that the local maximums $\lambda^\dagger$ and $\tilde{\lambda}$ of $C_p(\lambda) = \psi(\lambda)C(\lambda)$ in the ranges $[0, \underline{\lambda}]$ and $[\overline{\lambda}, 1]$ are independent of π^*. As the function $C_p(\lambda)$ is the same in all countries, equilibrium where some choose $\lambda^\dagger$ and some $\tilde{\lambda}$ requires $C_p(\lambda^\dagger) = C_p(\tilde{\lambda})$. From (A.19), this occurs at the following unique equilibrium world price:

$$\bar{\pi}^* = \frac{1}{\sigma}\left(\frac{\left(\psi(\lambda^\dagger)(\lambda^\dagger+\sigma)^\alpha\right)^{\frac{1}{1-\alpha}} - \left(\psi(\tilde{\lambda})(\tilde{\lambda}+\sigma)^\alpha\right)^{\frac{1}{1-\alpha}}}{\left(\psi(\lambda^\dagger)(\lambda^\dagger+\sigma)^\alpha\right)^{\frac{1}{1-\alpha}} + \left(\psi(\tilde{\lambda})(\tilde{\lambda}+\sigma)^\alpha\right)^{\frac{1}{1-\alpha}}}\right)\frac{Q}{\chi}.$$

As $X_I = -\sigma\chi$ for countries with $\lambda = \lambda^\dagger$ and $X_I = \sigma\chi$ for countries with $\lambda = \tilde{\lambda}$, equilibrium in world markets (1) requires a fraction $\bar{\gamma} = 1/2$ of countries choose $\lambda = \tilde{\lambda}$ and a fraction $1/2$ choose $\lambda = \lambda^\dagger$.

Interior equilibria for institutional quality

The $\lambda = \lambda^\dagger$ maximizing $C_p(\lambda)$ in the range $[0, \underline{\lambda}]$ has $\lambda^\dagger > 0$ if $C'_p(0) > 0$ because $C_p(\lambda)$ is quasi-concave in that range. Likewise, $\tilde{\lambda} < 1$ if $C'_p(1) < 0$. Using (A.21), these require $\alpha/\sigma > \mu(0)\psi(0)$ and $\alpha/(1-\sigma) < \mu(1)\psi(1)$. Both conditions are satisfied when $\sigma \leq \min\{\alpha/(\mu(0)\psi(0)), 1 - (\alpha/(\mu(1)\psi(1)))\}$. The minimum value is positive because $\alpha < \mu(1)\psi(1)$, as shown under Assumption 5 in the proof of Proposition 7.

(ii) The expressions for $C_p'(\lambda)$ in (A.21) imply that $C_p'(\lambda)$ is decreasing in σ for $\lambda \in [0, \underline{\lambda}]$, and increasing in σ for $\lambda \in [\overline{\lambda}, 1]$. Since $C_p(\lambda)$ is strictly quasi-concave in these ranges, and $C_p'(\lambda^\dagger) = 0$ and $C_p'(\tilde{\lambda}) = 0$ where $0 < \lambda^\dagger < \underline{\lambda} < \overline{\lambda} < \tilde{\lambda} < 1$, it follows that $\lambda^\dagger$ is decreasing in σ and $\tilde{\lambda}$ is increasing in σ.

A.10 Proof of Proposition 10

(i) A fraction $\varkappa$ of countries has $\lambda = 1$ imposed, so the fraction γ of countries in the world with $\lambda = 1$ must satisfy $\gamma \geq \varkappa$. In the remaining fraction $1 - \varkappa$ of countries, the value of λ is chosen to maximize the payoff of those in power. For these countries Proposition 5 continues to apply, with either $\lambda = 0$ or $\lambda = 1$ being the equilibrium. For a particular value of γ, Proposition 6 shows that $\lambda = 1$ is an equilibrium only if $\gamma \leq \bar{\gamma}_0$, where $\bar{\gamma}_0 = (\alpha/(1-\alpha))(Q/\chi)^{1-\varepsilon}/\xi^\varepsilon$ is the equilibrium fraction of rule-of-law countries in the absence of intervention.

First consider the case where $\varkappa \leq \bar{\gamma}_0$. The equilibrium must be unchanged at $\bar{\gamma} = \bar{\gamma}_0$. If $\gamma < \bar{\gamma}_0$, this would imply all countries would have $\lambda = 1$, that is, $\gamma = 1$, but $\bar{\gamma}_0 < 1$. If $\gamma > \bar{\gamma}_0$ then no country would have $\lambda = 0$ except those where it is imposed, hence $\gamma = \varkappa$, but $\varkappa \leq \bar{\gamma}_0$. This leaves $\gamma = \bar{\gamma}_0$, which is an equilibrium because incumbents are indifferent between $\lambda = 0$ and $\lambda = 1$, so a fraction $\bar{\gamma}_0 - \varkappa$ of countries have rulers that choose $\lambda = 1$, and there is no change in $\bar{\gamma}$. Next, consider the case $\varkappa > \bar{\gamma}_0$. The equilibrium must be $\bar{\gamma} = \varkappa$ because $\gamma < \varkappa$ is not feasible and $\gamma > \varkappa$ would mean rulers would not choose $\lambda = 1$ unless it is imposed on them.

(ii) Now suppose a subsidy $\tau < 0$ to the investment good is exogenously imposed in a fraction $\upsilon > 0$ of countries, meaning the domestic market-clearing price in those countries is $\pi = (1+\tau)\pi^*$, as in (A.1). The remaining fraction $1 - \upsilon$ of countries chooses institutions with free trade ($\tau = 0$, see Proposition 1). Since the imposition of τ has a multiplicative effect on real GDP as shown in the proof of Proposition 1, the argument in Proposition 5 that equilibrium institutional quality is either $\lambda = 0$ or $\lambda = 1$ still applies to all countries, and the same criterion $\pi^* \geq \xi Q/\chi$ for $\lambda = 1$ to be chosen remains valid for all. It is not possible to have world market clearing (1) with $\pi^* < \xi Q/\chi$ because all countries would have $\lambda = 0$, $K = 0$, and $X_I < 0$ using (A.5). If the subsidy results in $\pi^* > \xi Q/\chi$ then all countries would have $\lambda = 1$ and $\bar{\gamma}$ is increased. The remaining case to consider is where the equilibrium world price remains at $\bar{\pi}^* = \xi Q/\chi$.

With $\pi^* = \xi Q/\chi$, a fraction γ of countries have $\lambda = 1$ and a fraction $1 - \gamma$ have $\lambda = 0$. Differentiating net exports of investment goods X_I from (A.5) with respect to τ and K:

$$\frac{\partial X_I}{\partial \tau} = \frac{\alpha(1-\alpha)\varepsilon(1+\tau)^{\varepsilon-1}\pi^{*-\varepsilon}}{\left((1-\alpha)(1+\tau)^\varepsilon + \alpha\pi^{*1-\varepsilon}\right)^2} > 0, \quad \text{and} \quad \frac{\partial X_I}{\partial K} = \frac{(1-\alpha)(1+\tau)^\varepsilon}{(1-\alpha)(1+\tau)^\varepsilon + \alpha\pi^{*1-\varepsilon}} > 0,$$

where the signs of these partial derivatives do not depend on the initial value of τ. Following the imposition of the subsidy $\tau < 0$, X_I declines in a positive measure of countries υ. Given $\pi^* = \xi Q/\chi$, world market clearing (1) therefore requires an increase in K from $K = 0$ to $K = \chi$ in a positive measure of countries, raising $\int_0^1 X_I(n)\mathrm{d}n$ to restore equilibrium. This shows that the equilibrium fraction $\bar{\gamma}$ of countries with good institutions is increased by the subsidy.

(iii) If all countries impose the subsidy $\tau < 0$, and the goal is that all will have $\lambda = 1$ in equilibrium ($\bar{\gamma} = 1$), then all will have the same $K = \chi$ and net exports X_I from (A.5). Equilibrium in world markets (1) therefore requires $X_I = 0$. The minimum world price π^* consistent with $\lambda = 1$ in equilibrium is $\pi^* = \xi Q/\chi$. Substituting $\bar{\pi}^* = \xi Q/\chi$ and $K = \chi$ into (A.5) and solving for the $\tau = \tilde{\tau}$ such that $X_I = 0$ yields:

$$\tilde{\tau} = \left(\frac{\alpha}{1-\alpha}\left(\frac{Q}{\chi}\right)\left(\xi\frac{Q}{\chi}\right)^{-\varepsilon}\right)^{\frac{1}{\varepsilon}} - 1.$$

This confirms the expression given for $\tilde{\tau}$.

A.11 Proof of Proposition 11

(i) The finding of Proposition 5 that equilibrium institutions in open economies have either $\lambda = 0$ or $\lambda = 1$ still applies here. If Q is an arbitrary country-specific endowment, the condition derived in Proposition 5 shows that $\lambda = 1$ is optimal only if $q \leq \bar{q}$, where $q = Q/Q^*$ is the endowment measured relative to the global mean Q^*, and the threshold for q is given by $\bar{q} = (\chi/Q^*)(\pi^*/\xi)$.

(ii) If a fraction γ of countries have institutions with $\lambda = 1$, the market-clearing world price from (28) is $\bar{\pi}^* = ((\alpha/(1-\alpha))(Q^*/\chi)/\gamma)^{1/\varepsilon}$. Hence, the threshold is $\bar{q} = (\alpha/(1-\alpha))^{1/\varepsilon}(Q^*/\chi)^{(1-\varepsilon)/\varepsilon}$ for extractive institutions versus the rule of law. Since $q < \bar{q}$ is necessary for $\lambda = 1$, the fraction of countries with $\lambda = 1$ must satisfy $\gamma = G(\bar{q})$. Note that the threshold can be written as $\bar{q} = (\bar{\gamma}_0/\gamma)^{1/\varepsilon}$, where $\bar{\gamma}_0 = (\alpha/(1-\alpha))(Q^*/\chi)^{1-\varepsilon}/\xi^\varepsilon$ is the equilibrium fraction of countries with $\lambda = 1$ in the case of homogeneous endowments ($Q = Q^*$) as given in Proposition 6. The equilibrium $\bar{\gamma}$ with heterogeneous endowments must therefore satisfy the equation $\bar{\gamma} = G\left((\bar{\gamma}_0/\bar{\gamma})^{1/\varepsilon}\right)$ as claimed.

This equation for $\bar{\gamma}$ can be stated as $H(\bar{\gamma}) = 0$, where $H(\bar{\gamma}) = \bar{\gamma} - G\left((\bar{\gamma}_0/\bar{\gamma})^{1/\varepsilon}\right)$. The positive term $\bar{\gamma}_0$ depends only on parameters. Since the cumulative distribution function $G(q)$ is weakly increasing in q and as $0 < \varepsilon < 1$, the function $H(\bar{\gamma})$ is strictly increasing in $\bar{\gamma}$. Any solution of the equation $H(\bar{\gamma}) = 0$ must therefore be unique. A property of the cumulative distribution function is $G(\infty) = 1$, which implies $H(0) = -1$. Proposition 6 shows that $\bar{\gamma}_0 < 1$, and since $G(1) < 1$ is assumed (the fraction of countries above the mean is strictly positive), it follows that $G(\bar{\gamma}_0^{1/\varepsilon}) \leq G(1) < 1$ and hence $H(1) = 1 - G(\bar{\gamma}_0^{1/\varepsilon}) > 0$. Since q has a continuous distribution, the function $H(\bar{\gamma})$ must be continuous. With $H(0) < 0$ and $H(1) > 0$, the intermediate value theorem implies there exists a $\bar{\gamma}$ such that $H(\bar{\gamma}) = 0$ satisfying $0 < \bar{\gamma} < 1$.

(iii) Observe that $H(\bar{\gamma}_0) = \bar{\gamma}_0 - G((\bar{\gamma}_0/\bar{\gamma}_0)^{1/\varepsilon}) = \bar{\gamma}_0 - G(1) = \bar{\gamma}_0 - \gamma^*$, where $\gamma^* = G(1)$ denotes the fraction of countries with an endowment below than the global mean. Furthermore, note $H(\gamma^*) =$

$\gamma^* - G((\bar{\gamma}_0/\gamma^*)^{1/\varepsilon}) = G(1) - G((\bar{\gamma}_0/\gamma^*)^{1/\varepsilon})$. Since $G(q)$ is weakly increasing, $G((\bar{\gamma}_0/\gamma^*)^{1/\varepsilon}) \leq G(1)$ if $\bar{\gamma}_0/\gamma^* < 1$ and $G((\bar{\gamma}_0/\gamma^*)^{1/\varepsilon}) \geq G(1)$ if $\gamma_0/\gamma^* > 1$. Together with the expressions for $H(\bar{\gamma}_0)$ and $H(\gamma^*)$ above, it follows that $H(\bar{\gamma})$ changes sign between $\bar{\gamma}_0$ and γ^* (irrespective of the ordering of the terms). The unique solution for $\bar{\gamma}$ must therefore lie between $\bar{\gamma}_0$ and γ^* (or coincide if equal).

A.12 Proof of Proposition 12

(i) For the $1-\varsigma$ countries that are price takers in world markets, the condition for $\lambda(n) = 1$ to be the equilibrium in country n is that derived in Proposition 5, namely $\pi^*\chi \geq \xi Q(n)$. This gives a threshold $\bar{q} = \pi^*\chi/(\xi Q^*)$ for endowments $q = Q/Q^*$ relative to the global mean Q^* such that those countries choosing $\lambda(n) = 1$ are those with $q(n) \leq \bar{q}$. The small open economies have a continuous probability distribution of relative endowments with cumulative distribution function $G(q)$. Since $\hat{\lambda} = 0$ is assumed to be the equilibrium within the cartel, the fraction of economies with the rule of law is $\gamma = (1-\varsigma)G(\bar{q})$.

The cartel has positive measure ς in world markets and chooses net exports $\hat{X}_E$ of the endowment good. The cartel's endowment is $\hat{Q}$, and let $\check{Q}$ denote the average endowment of price-taking economies, so the global mean is $Q^* = \varsigma\hat{Q} + (1-\varsigma)\check{Q}$ (the average value of q for small open economies is $\check{Q}/Q^*$). With net exports given by $X_E = \alpha Q - (1-\alpha)\pi^* K$ for the small open economies (17 with $\varepsilon = 1$), and only those economies producing capital, world markets clear (1) if

$$\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q} - (1-\alpha)\bar{\pi}^*\chi\gamma = 0.$$

It follows that the threshold $\bar{q}$ for the choice of $\lambda(n) = 0$ or $\lambda(n) = 1$ in small open economies is

$$\bar{q} = \frac{\bar{\pi}^*\chi}{\xi Q^*} = \frac{1}{\gamma}\left(\frac{\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q}}{(1-\alpha)\xi Q^*}\right), \tag{A.22}$$

and combined with $\gamma = (1-\varsigma)G(\bar{q})$, the equilibrium threshold $\bar{q}$ is therefore determined by

$$\bar{q}G(\bar{q}) = \frac{\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q}}{(1-\alpha)(1-\varsigma)\xi Q^*}. \tag{A.23}$$

The implied elasticity of the equilibrium threshold $\bar{q}$ with respect to the cartel's net exports $\hat{X}_E$ is

$$\nu = \frac{\partial\bar{q}}{\partial\hat{X}_E}\frac{\hat{X}_E}{\bar{q}} = \left(\frac{\varsigma\hat{X}_E}{\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q}}\right)\Big/\left(1 + \frac{\bar{q}G'(\bar{q})}{G(\bar{q})}\right). \tag{A.24}$$

Since the cartel chooses $\hat{K} = 0$, the quantity of investment goods available for consumption is $-\hat{X}_I = \hat{X}_E/\bar{\pi}^*$ using (2). As the cartel cannot choose $\hat{X}_E < 0$, and with $\check{Q} > 0$ and $0 < \varsigma < 1$, it follows from (A.22) that $\bar{\pi}^*$ must be strictly positive. The cartel must therefore choose $\hat{X}_E > 0$. It further follows from (A.23) that $\bar{q}$ must be positive and finite, and $G(\bar{q})$ must be positive. Since q has a continuous probability distribution, $G'(\bar{q})$ is finite, and together with the other observations, the elasticity in (A.24) therefore satisfies $0 < \nu < 1$. With (A.22) showing that $\bar{q}$ and $\bar{\pi}^*$ are proportional for given parameters and (A.23) determining $\bar{q}$ for each $\hat{X}_E$, it follows that the equilibrium world price is a function $\bar{\pi}^*(\hat{X}_E)$ of the cartel's net exports, and the elasticity of $\bar{\pi}^*$ with respect to $\hat{X}_E$ is equal to ν.

Conditional on $\hat{\lambda} = 0$, and hence on an incumbent income multiple $\psi(0)$ of GDP, the cartel's equilibrium trade policy is to choose $\hat{X}_E$ and $\hat{X}_I$ to maximize real GDP. With $\varepsilon = 1$, substituting $\pi = (\alpha/(1-\alpha))((Q - X_E)/(K - X_I))$ from (16) into $C = ((Q - X_E) + \pi(K - X_I))/\pi^\alpha$ from (A.4), real GDP is $C = (Q - X_E)^{1-\alpha}(K - X_I)^\alpha/((1-\alpha)^{1-\alpha}\alpha^\alpha)$. This is maximized subject to the international budget constraint (2), where the world price $\bar{\pi}^*$ is now a function of the cartel's $\hat{X}_E$. Using $\hat{X}_I = -\hat{X}_E/\bar{\pi}^*$ to eliminate $\hat{X}_I$ and noting $\hat{K} = 0$, the objective function is $(\hat{Q} - \hat{X}_E)^{1-\alpha}(\hat{X}_E/\pi^*(\hat{X}_E))^\alpha/((1-\alpha)^{1-\alpha}\alpha^\alpha)$. The first-order condition with respect to $\hat{X}_E$ is

$$\frac{\alpha}{\hat{X}_E/\pi^*(\hat{X}_E)}\left(\frac{1}{\pi^*(\hat{X}_E)} - \frac{\hat{X}_E\pi^{*\prime}(\hat{X}_E)}{(\pi^*(\hat{X}_E))^2}\right) - \frac{1-\alpha}{\hat{Q} - \hat{X}_E} = 0.$$

The domestic market-clearing price (16) in the cartel is $\hat{\pi} = \alpha(\hat{Q} - \hat{X}_E)/((1-\alpha)(\hat{X}_E/\pi^*(\hat{X}_E)))$, and using $\hat{X}_E\pi^{*\prime}(\hat{X}_E)/\pi^*(\hat{X}_E) = \nu$ from (A.24), the first-order condition can be expressed as

$$\hat{\pi} = \frac{\pi^*(\hat{X}_E)}{1-\nu}, \quad \text{and hence (A.1) holds with } \tau = \frac{\nu}{1-\nu}.$$

The cartel's trade policy is thus equivalent to a positive tariff τ on the investment good as $0 < \nu < 1$. Substituting into (A.5) with $\hat{K} = 0$ shows that $\hat{X}_E = ((1-\nu)/(1-\alpha\nu))\alpha\hat{Q} < \alpha\hat{Q}$, so the countries of the cartel export less of the endowment good than they would have done as small open economies.

(ii) With the cartel, equation (A.22) implies $\bar{q}$ and the equilibrium fraction $\bar{\gamma}$ of countries with $\lambda = 1$ jointly satisfy $\bar{\gamma}\bar{q} = (\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q})/((1-\alpha)\xi Q^*)$. Since $\bar{\gamma}/(1-\varsigma) = G(\bar{q})$ and $G(q)$ is strictly increasing, it follows that $\bar{q} = G^{-1}(\bar{\gamma}/(1-\varsigma))$ and hence an equation for $\bar{\gamma}$ is

$$\bar{\gamma}G^{-1}\left(\frac{\bar{\gamma}}{1-\varsigma}\right) = \frac{\varsigma\hat{X}_E + \alpha(1-\varsigma)\check{Q}}{(1-\alpha)\xi Q^*}. \tag{A.25}$$

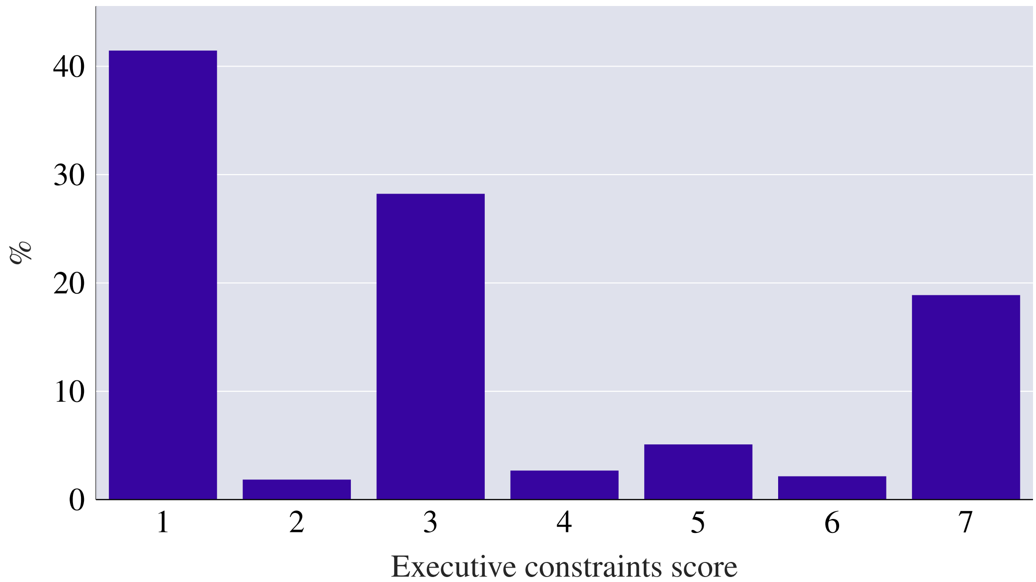
If the cartel were broken up and its members acted instead as small open economies then their optimal trade policy is $\tau = 0$ (Proposition 1), implying net exports $\hat{X}_E$ would rise to $\alpha\hat{Q}$ if a country continued to choose $\lambda(n) = 0$. Conditional on the fraction of countries $\bar{\gamma}$ choosing the rule of law (which may now include some of the former cartel members), the derivation of equation (A.22) is unaffected. As it possible former cartel members might choose $\lambda(n) = 1$ whereas they all previously had $\hat{\lambda} = 0$, the fraction of economies with the rule of law now satisfies $\bar{\gamma} \geq (1-\varsigma)G(\bar{q})$. This implies $\bar{q} \leq G^{-1}(\bar{\gamma}/(1-\varsigma))$, and therefore the new value of $\bar{\gamma}$ must have the left-hand side of (A.25) be no less than the right-hand side. The right-hand side increases as $\hat{X}_E$ is replaced by $\alpha\hat{Q}$, which is more than its previous value. As the left-hand side of (A.25) is a strictly increasing function of $\bar{\gamma}$, it follows that the new equilibrium value of $\bar{\gamma}$ must be greater than with the cartel.

B Further information about the empirical analysis

B.1 Description of the data

The empirical analysis uses data from the Center for Systemic Peace's Polity IV Project (http://www.systemicpeace.org/inscrdata.html) on 'Executive Constraints' (XCONST, a score between 1 and 7). Figure B.1 plots the frequency distribution of the executive constraint scores after pooling this annual data over the period 1841–1905. Note that approximately 60% of the observations are an extreme classification (1 or 7), and about 88% of all observations are in $\{1,3,7\}$.

Figure B.1: *Frequency distribution of executive constraints scores*



The time series of countries' executive constraints scores are very persistent, though the measured degree of persistence depends on exactly how missing data are treated. Since missing data usually reflect some political uncertainty, it is reasonable to treat missing observations as an eighth possible score. Doing this, the probability of a change in the score for a given country from one year to the next is less than 4%. On average, it takes somewhat more than 25 years for there to be a (usually not very large) change in a country's score.

Table B.1 gives the list of countries used in the empirical analysis from section 5. The reported executive constraints scores are averages over the 1841–1860 and 1881–1900 sub-periods.

The trade shock for each country is calculated using the predicted trade time series from Pascali (2017), which is available at a 5-yearly frequency. A country's trade shock is defined as the difference between the logarithms of average predicted trade in the two sub-periods. The trade shocks are reported in Table B.1, which orders countries by the size of their trade shocks.

The table also reports the numbers used to construct Figure 5. Countries are divided into two

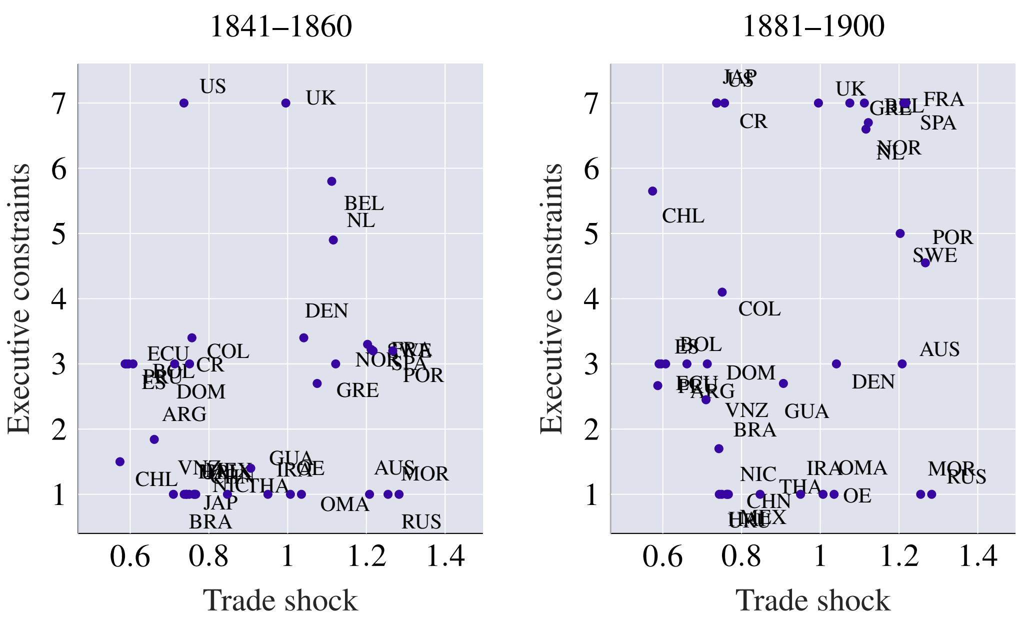
Table B.1: *Executive constraints scores and trade shocks for countries in the sample*

Country	1841–1860	1880–1900	Trade shock	Weight	Data available
Chile	1.5	5.7	0.574	0.088	1841
Peru	3.0	2.7	0.587	0.084	1841
El Salvador	3.0	3.0	0.591	0.084	1841
Ecuador	3.0	3.0	0.596	0.082	1841
Bolivia	3.0	3.0	0.607	0.079	1841
Argentina	1.8	3.0	0.661	0.064	1841
Venezuela	1.0	2.5	0.710	0.051	1841
Dominican Republic	3.0	3.0	0.713	0.050	1844
United States	7.0	7.0	0.736	0.044	1841
Japan	1.0	7.0	0.737	0.044	1841
Brazil	1.0	1.7	0.742	0.043	1841
Haiti	1.0	1.0	0.743	0.042	1841
Uruguay	1.0	1.0	0.750	0.041	1841
Colombia	3.0	4.1	0.751	0.040	1841
Costa Rica	3.4	7.0	0.757	0.039	1841
Nicaragua	1.0	1.0	0.762	0.037	1841
China	1.0	1.0	0.764	0.037	1841
Mexico	1.0	1.0	0.766	0.036	1841
Siam	1.0	1.0	0.847	0.014	1841
Guatemala	1.4	2.7	0.906	0.002	1841
Persia	1.0	1.0	0.950	0.013	1841
United Kingdom	7.0	7.0	0.995	0.026	1841
Ottoman Empire	1.0	1.0	1.007	0.029	1841
Oman	1.0	1.0	1.035	0.036	1841
Denmark	3.4	3.0	1.041	0.038	1841
Greece	2.7	7.0	1.075	0.047	1841
Belgium	5.8	7.0	1.112	0.057	1841
Netherlands	4.9	6.6	1.116	0.058	1841
Norway	3.0	6.7	1.122	0.060	1841
Sweden	3.3	5.0	1.203	0.082	1841
Austria-Hungary	1.0	3.0	1.208	0.083	1841
France	3.2	7.0	1.212	0.084	1841
Spain	3.2	7.0	1.217	0.086	1841
Morocco	1.0	1.0	1.255	0.096	1841
Portugal	3.2	4.6	1.267	0.099	1841
Russia	1.0	1.0	1.283	0.103	1841
Mean	2.4	3.6	0.900	0.056	1841

groups, small-shock and large-shock, based on whether their trade shocks are respectively below or above the mean. The cumulative distribution functions in Figure 5 weight each observation by the absolute value of the difference between the country's trade shock and the mean trade shock. These weights are normalized to sum to 1 within the two groups of countries.

Figure B.2 plots the relationship between executive constraints scores and the size of the trade shock before and after the shock. In the pre-shock period 1841–1860, the distribution of Polity scores does not seem to depend on the size of the shock. Several European and Latin American countries have scores around 3, a few European and many Latin American and Asian countries have scores close to 1, and a small number of countries were at the maximum score of 7. Matters look different by the post-shock period 1881–1900. In the set of countries exposed to large shocks, few of them have intermediate scores with most being close to 1 or 7. In contrast, in the set of countries exposed to small shocks, there is a substantial number of countries with scores close to 3.

Figure B.2: *Executive constraints and trade shock relationships in the two sub-periods*



Sources: Predicted trade data from Pascali (2017); Executive Constraints data from the Polity IV Project, Center for Systemic Peace (`http://www.systemicpeace.org/inscrdata.html`).

B.2 Robustness exercises

This section repeats the estimation of (30) using different specifications of the pre- and post-shock periods and the transitional period between the two. Table B.2 shows specifications with narrower and wider pre- and post-shock periods. Table B.3 has specifications with shorter transitional periods. Finally, Table B.4 shortens both the transitional and pre- and post-shock periods.

Table B.2: *Regression results with narrower and wider pre- and post-shock periods*

P_{jA}	Pre: 1846–1860	Post: 1881–1895		Pre: 1841–1865	Post: 1881–1905	
	OLS	Tobit	Probit	OLS	Tobit	Probit
1	1.79 (1.93) [0.36]	1.58 (2.29) [0.49]	1.78 (1.84) [0.33]	3.47 (2.31) [0.14]	3.60 (2.60) [0.18]	4.71 (2.52) [0.06]
P_{jB}	0.21 (0.57) [0.72]	0.30 (0.82) [0.71]	−1.26 (0.76) [0.10]	−0.41 (0.69) [0.56]	−0.44 (0.98) [0.66]	−2.30 (1.04) [0.03]
Z_j	−0.46 (2.44) [0.85]	−0.50 (2.86) [0.86]	−3.33 (2.62) [0.20]	−2.64 (2.40) [0.28]	−3.52 (2.69) [0.20]	−6.22 (3.02) [0.04]
P_{jB} $\times Z_j$	0.84 (0.70) [0.23]	1.00 (1.01) [0.33]	2.13 (1.12) [0.06]	1.58 (0.74) [0.04]	2.09 (1.10) [0.07]	3.07 (1.27) [0.02]
N	37	37	34	36	36	34

Notes: Standard errors are in parentheses and p-values are in brackets under the coefficients.

Table B.3: *Regression results with shorter transitional periods*

P_{jA}	Pre: 1846–1865	Post: 1881–1900		Pre: 1846–1865	Post: 1876–1895	
	OLS	Tobit	Probit	OLS	Tobit	Probit
1	2.07 (2.09) [0.33]	1.81 (2.50) [0.48]	2.00 (1.98) [0.30]	1.39 (1.65) [0.40]	1.13 (2.00) [0.58]	1.28 (1.49) [0.40]
P_{jB}	0.14 (0.62) [0.83]	0.26 (0.90) [0.78]	−1.14 (0.82) [0.17]	0.47 (0.48) [0.34]	0.61 (0.72) [0.40]	−0.95 (0.63) [0.13]
Z_j	−0.78 (2.69) [0.77]	−0.77 (3.20) [0.81]	−3.77 (2.73) [0.17]	−0.01 (2.54) [1.00]	0.00 (3.05) [1.00]	−2.77 (2.46) [0.26]
P_{jB} $\times Z_j$	0.92 (0.76) [0.23]	1.05 (1.11) [0.35]	2.07 (1.23) [0.09]	0.60 (0.70) [0.40]	0.72 (1.04) [0.50]	1.92 (1.08) [0.08]
N	37	37	34	37	37	34

Notes: Standard errors are in parentheses and p-values are in brackets under the coefficients.

Table B.4: *Regression results with narrower transitional and pre- and post-shock periods*

P_{jA}	Pre: 1851–1865	Post: 1876–1890		Pre: 1851–1865	Post: 1881–1895	
	OLS	Tobit	Probit	OLS	Tobit	Probit
1	0.30	−0.28	−0.51	0.65	0.04	0.30
	(1.44) [0.84]	(1.81) [0.88]	(1.17) [0.66]	(1.69) [0.70]	(2.11) [0.99]	(1.32) [0.82]
P_{jB}	0.94	1.24	−0.11	0.78	1.08	−0.31
	(0.41) [0.03]	(0.62) [0.05]	(0.47) [0.81]	(0.47) [0.11]	(0.72) [0.15]	(0.55) [0.58]
Z_j	2.58	3.36	0.09	1.88	2.65	−1.51
	(3.10) [0.41]	(3.89) [0.40]	(2.25) [0.97]	(3.19) [0.56]	(4.00) [0.51]	(2.33) [0.52]
P_{jB}	−0.31	−0.52	0.73	−0.04	−0.23	1.09
$\times Z_j$	(0.79) [0.70]	(1.18) [0.66]	(0.91) [0.43]	(0.82) [0.96]	(1.22) [0.85]	(1.00) [0.28]
N	37	37	34	37	37	34

Notes: Standard errors are in parentheses and p-values are in brackets under the coefficients.