The Macroeconomics of Liquidity in Financial Intermediation[a]

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Abstract

The liquidity premium on US Treasuries correlates positively with credit spreads. To explain this, we develop a theory of endogenous bank fragility arising from a coordination friction among bank creditors and embed it in a macroeconomic model. Adverse shocks to bank net worth exacerbate the friction. Thus, banks lend less and demand more liquid assets, driving up credit spreads and the liquidity premium. By mitigating the friction, expansions of public liquidity reduce spreads and boost output. Using high-frequency data and exploiting the lag between auction and issuance of US Treasuries, we identify liquidity-supply shocks and confirm negative effects on spreads.

Keywords: bank runs, bank-lending channel, liquidity.

JEL Codes: E40, E41, E44, E50, E51, G01, G21.

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1 Introduction

Disruptions to financial intermediation make credit more expensive and thereby harm the economy. This has motivated adding a specific banking friction to macroeconomic models. In their seminal contribution, Gertler and Kiyotaki (2010) introduce a problem of moral hazard between banks and their creditors. In consequence, banks' ability to fund themselves is limited by the value of their equity. The resulting leverage constraint leads to a powerful impact of bank net worth on macroeconomic outcomes via credit spreads. This explains the general observation of plummeting bank values, higher bank funding costs, and increased credit spreads in financial crises. However, this approach to banking is silent on why we see soaring demands for liquidity and hence liquidity premiums in times of financial stress.

We observe a heightened liquidity premium, defined as the difference between the 3-month general-collateral repo rate and the 3-month treasury-bill rate, during banking crises.[1] Figure 1 shows this for the 2007–8 financial crisis.[2] More systematically, we document a positive relationship over time between the liquidity premium and banks' funding costs as measured by the difference between 3-month LIBOR and the 3-month repo rate. Figure 2 shows the positive correlation between these two variables.[3]

Policymakers often react to banking crises with expansions of liquidity. Evaluation of how such policies work requires an understanding of the causes of the empirical relationship between the liquidity premium and banks' funding costs.[4] Existing research (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016) has shown the liquidity premium does respond to government policies. The facts documented here suggest that liquid assets are scarce when bank funding is tight. This is consistent with a view that scarce liquidity impairs bank lending in times of stress, pointing to a channel through which a greater supply of public liquidity can benefit the economy.

Motivated by this, we do two things in this paper. First, we develop a theory of a novel financial friction based on coordination failure among bank creditors. Liquid-asset holdings and bank net worth both mitigate the coordination friction and are substitutes. Hence, when net worth is scarce, as in a financial crisis, banks demand more liquidity. This explains a high liquidity premium at such times. It also implies policy can stabilize the economy by appropriately supplying liquid assets.

Second, we test whether the data support the mechanism in the theory. In particular, the model implies that an increase in the liquidity premium pushes up the bank-funding spread. This is because it induces banks to economize on holding liquid

[1] This definition is standard in the literature (Nagel, 2016). See section 7 for further discussion.

[2] Figure 9 in appendix A zooms in on the recent period between 2019 and 2023.

[3] Figure 10 shows that the positive correlation holds both in expansions and recessions. A scatterplot with data at monthly frequency rather than binned is available in the supplementary appendix.

[4] There is a debate in the literature on the real effects of liquidity policies and the channels through which they operate (Kuttner, 2018).

Figure 1: 2007–8 financial crisis.

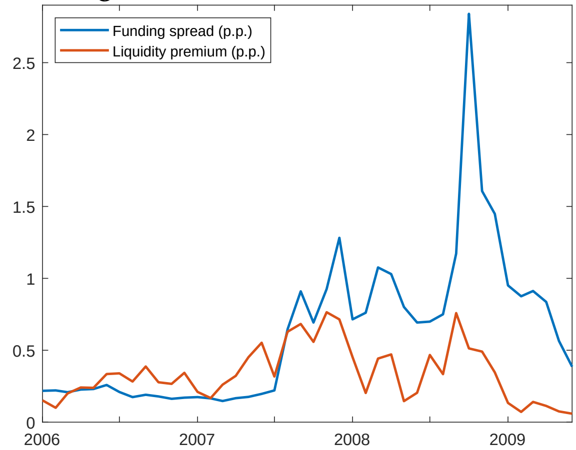
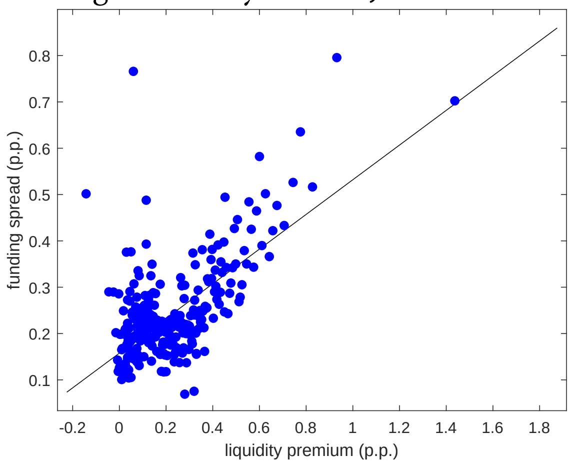


Figure 2: May 1991 – June 2023.



Note 1: The funding spread is 3-month (3M) LIBOR minus the 3M general-collateral (GC) repo rate. The liquidity premium is the 3M GC repo rate minus the 3M T-bill rate.
Note 2: US daily data. Figure 1 plots data at monthly frequency. Figure 2 plots a binned scatterplot with 300 quantile-based bins. Data sources are found in appendix B.

assets. To identify exogenous variation in the liquidity premium, we use the quantity of outstanding US Treasuries as an instrumental variable. The instrument is strongly relevant and predetermined at daily frequency given the lag of a few days between the auction and issuance of Treasury securities. We find a significant positive effect.

Maturity transformation, a core function of financial intermediation, results in a mismatch on banks' balance sheets.[5] This creates conditions for coordination failures in the market for deposits (Diamond and Dybvig, 1983).[6] Such coordination failures take the form of "runs" by panicked creditors and played a key role in the global financial crisis in 2007, the crisis of US money-market funds in 2020 and the 2023 regional banking crisis (Shin, 2009; Bernanke, 2010; Li et al., 2021; Choi et al., 2023).

This paper models the deposits market as a coordination game. Strategic complementarities imply that under perfect information there are multiple equilibria. However, a deviation from common knowledge across depositors, which we introduce following the large literature on global games (Morris and Shin, 2003), leads to a unique equilibrium. Intuitively, without common knowledge it is impossible for depositors to coordinate on arbitrary equilibria. In the resulting unique equilibrium, depositors demand a level of compensation that is commensurate to a bank's fragility, defined as the minimum share of depositors that must not run for the bank to survive. If a bank offers an insufficiently low interest rate on deposits, then depositors run even though the bank is solvent because they fear a run by other households. On the other hand, as long as the bank offers a sufficiently high deposit rate, no run takes place because no depositor has an incentive to start the run that they fear.

[5] We use 'banks' as a general label for financial intermediaries and 'deposits' for their short-term debt. The analysis applies more broadly to intermediaries with maturity mismatch on their balance sheet.

[6] Perfect deposit insurance rules out coordination failures in these models. However, in the period 1984–2023Q3 deposits made up 73% of banks' liabilities and only 62% of deposits were insured on average. These values are respectively 79% and 57% in 2023Q3 (data source: FDIC QBP).

Bank fragility, at the heart of the coordination friction, is endogenous. It is a function of a bank's balance-sheet fundamentals. In particular, more levered banks and banks with fewer liquid assets as a share of total assets are more fragile. Consequently, they face higher funding costs. In other words, the coordination friction results in a mapping from higher capital and liquidity ratios to a lower funding spread. The capital and liquidity ratios are bank choices. In equilibrium, these choices trade off the higher returns on illiquid assets against the increased funding costs due to more fragility.

With the coordination friction embedded in a standard real business cycle model, we can study its role quantitatively in the transmission of macroeconomic shocks. The banking friction can be calibrated using observations on the average size of the liquidity premium, the credit spread, and banks' return on equity.

The friction amplifies the impact of shocks that affect banks' net worth. By making it more costly for banks to fund themselves, a reduction in net worth weakens the supply of credit and reduces the economy's output. The friction raises the effect on output of capital-destruction shocks, commonly studied in the literature on financial crises, by about one third on impact. At longer horizons, the amplification is greater. This persistence comes from banks' funding costs rising alongside credit spreads, implying banks' net worth is rebuilt very slowly, in contrast to models with a leverage constraint. Furthermore, the increase in fragility due to scarcer net worth gives banks an incentive to demand more liquid assets. This generates a countercyclical liquidity premium.

Monetary and fiscal liabilities of the government are the natural source of a supply of liquidity. Banks create liquid assets for other sectors of the economy, but they cannot produce assets that maintain their value in the case of a systemic run.[7] Therefore, the relevant supply of liquid assets is a policy variable. In the model, an increase in the supply of liquidity is expansionary. Extra liquid assets are absorbed on to banks' balance sheets and reduce their fragility. With lower fragility, banks have access to funding on better terms and thus find it optimal to lend more. In other words, supplying more liquidity crowds in private investment. In the calibrated model, a shock to liquid assets that reduces the liquidity premium by 15 basis points leads to an expansion of credit supply, lowering credit spreads by 24 basis points. This generates a 2-percent increase in investment on impact, with GDP also going up by a quarter of one percent. Moreover, the supply of liquidity can be used as a stabilizing policy tool in the face of shocks. If the government responds to disruptions in financial intermediation by accommodating the increased demand for liquid assets, it can dampen the amplification of shocks.

The demand for liquid assets gives rise to a fiscal benefit for the government by reducing interest rates on its bonds, analogous to 'seigniorage' in the context of money demand. Interestingly, supplying more liquid assets can have a fiscal cost because it

[7] This is related to the seminal finding in Holmström and Tirole (1998) of a role for public liquidity supply in the presence of aggregate risk.

reduces the liquidity premium on government bonds. Thus, a benevolent government may face a trade-off when deciding how much liquidity to supply. In an extension, we also study ex-post liquidity policies such as deposit insurance and lender of last resort. These policies also reduce fragility and are expansionary in the same way as the ex-ante supply of liquid assets studied in the main part of the paper. However, ex-post liquidity policies entail a larger fiscal cost because they reduce the liquidity premium without increasing the quantity of government debt outstanding.

In the empirical section, we test the key implication of the model: an increase in the liquidity premium causes an increase in the bank funding spread. The econometric challenge is to find exogenous variation in the liquidity premium. Our strategy is to perform the analysis at a daily frequency and use the quantity of outstanding US Treasuries as an instrument. This instrument is strongly relevant to the liquidity premium. As for its validity, the quantity of treasuries is predetermined at a daily frequency because there is a lag of a few days from auction, the latest point at which the quantity may respond to events, to issuance.[8] Moreover, we include as controls 80 lags of financial and economic variables available at daily frequency, such as the dollar exchange rate and the liquidity premium itself. This cleans autocorrelation from the error term. Thereby, it ensures there is no endogeneity of the instrument driven by confounding variables or reverse causality because an error term that only contains a non-autocorrelated daily shock cannot drive a variable determined on an earlier day.

The empirical finding is a robustly-significant positive effect of the liquidity premium on the bank funding spread. A 1-basis-point increase in the liquidity premium causes the funding spread to increase by about 1 basis point. This is in line with the size of the corresponding effect in the calibrated model. As a robustness check, we split the sample between expansions and recessions. We find no evidence of a different size of the effect according to the state of the economy.

Literature review. An extensive literature builds macroeconomic models around a leverage constraint on banks (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016; Mendicino et al., 2020; Di Tella and Kurlat, 2021; Karadi and Nakov, 2021; Van der Ghote, 2021; Fernández-Villaverde et al., 2023).[9] This friction, based on moral hazard on bankers' part, does not naturally give rise to a role for banks' liquid-asset holdings, unlike this paper's friction based on coordination failure. Moreover, models with the moral-hazard friction generate limited shock propagation because adverse shocks to bank net worth push up bank profitability by increasing credit spreads with little change in funding costs. Also, they struggle to match the observed procyclicality of

[8] A related high-frequency approach to treasury-market data is adopted in Ray et al. (2024).

[9] Holmström and Tirole (1997) is an early example of a model in which a leverage constraint on banks is micro-founded with a moral-hazard problem.

banks' book leverage (Nuño and Thomas, 2017). The coordination friction also makes progress on these two counts. In this paper's model, shock propagation is strong because the positive effect of higher credit spreads on bank profitability after adverse shocks is largely offset by increased funding costs. And we find that leverage is procyclical for standard shocks that affect credit demand, such as productivity shocks.

In this paper, banks demand liquid assets to mitigate the risk of coordination failures among their creditors. This is a novel source of demand for liquid assets in the macroeconomic literature.[10] The existing literature posits an exogenous risk that bank creditors withdraw their funds. Banks then demand liquid assets as a precaution to limit the amount they must borrow from the central bank at a punitive interest rate (Poole, 1968; Arce et al., 2020; Bianchi and Bigio, 2022) or the amount of assets they must sell at fire-sale prices (Drechsler et al., 2018; D'Avernas and Vandeweyer, 2024; Li, 2025) if hit by an adverse liquidity shock. In our model, the risk of depositor withdrawals is a fully endogenous function of bank fundamentals.[11]

Studies evaluating the effects of quantitative-easing programmes, recent examples of policies that increased the supply of liquid assets, find interest-rate reductions in line with our model (Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Chodorow-Reich, 2014).[12] More recently, Acharya and Rajan (2024) and Diamond et al. (2024) have sounded a cautionary note on the effects of liquid-asset supply in the context of QE. The former paper stresses that some of the benefit to bank fragility of additional liquidity supply is undone by banks taking on extra leverage. That result conforms to the mechanism in the paper here. The latter contribution finds empirically that liquid-asset holdings increase banks' marginal cost of lending. The authors suggest the reason for this may be limited balance-sheet space due to regulation. While the effect of regulation is beyond the scope of our paper, the driving force behind our paper's results, the positive effect of liquid-asset holdings by banks on the demand for their debt, is not considered in Diamond et al. (2024).

Banks' vulnerability to runs was first formalized in Diamond and Dybvig (1983). That paper illustrates the possibility of runs, but it does not speak to their determinants because it has multiple equilibria. A literature in macroeconomics has adopted the multiple-equilibrium approach to study the effects of bank runs (Gertler and Kiyotaki, 2015; Gertler et al., 2016, 2020; Amador and Bianchi, 2024). A limitation is the need to assume an arbitrary relationship between the probability of runs and fundamentals. Consequently, the role of liquidity in the determination of run risk does not emerge.

[10]A strand of the banking literature formalizes this in static partial-equilibrium models (Rochet and Vives, 2004; Ahnert, 2016).

[11]A reduced-form approach to the demand for liquid assets is common in studies of the effects of liquidity supply (Krishnamurthy and Vissing-Jorgensen, 2012; Benigno and Benigno, 2022; Angeletos et al., 2023). This may miss important features of the demand for liquid assets such as the substitutability of liquidity and bank capital, a feature of our model, which DeYoung et al. (2018) finds empirically.

[12]Ray et al. (2024) develop a theory with segmented asset markets in which QE has real effects.

Leveraging theoretical results from Carlsson and van Damme (1993), Goldstein and Pauzner (2005) show that a small departure from perfect information produces a unique equilibrium in a bank-run game. This is an attractive feature because the evidence points to a strong relationship between poor bank fundamentals and banking crises (Gorton, 1988; Baron et al., 2021). A large literature in banking uses variations of such second-generation bank-run models to study optimal policy (Vives, 2014; Kashyap et al., 2024; Ikeda, 2024). Our paper is the first to integrate a second-generation bank-run model in a macroeconomic framework.[13]

Outline of the paper. The coordination game among depositors is laid out in section 2. This results in a constraint on bank behaviour that the following two sections (3 and 4) integrate into a standard macroeconomic model. The model is calibrated and quantitative experiments are carried out in section 5. Section 6 discusses normative implications of the model, and section 7 reports the empirical results. The appendices contain: (A) figures, (B) details about data sources, (C) proofs and (D) steady-state results.

2 Coordination game

This section sets up the coordination game played by bank depositors. It solves for the unique equilibrium, which implies a relationship between banks' balance sheets and the interest rates required to induce households to hold deposits. Since banks anticipate the outcome of the coordination game, in the remainder of the paper this relationship constrains the choices made by banks.

The economy contains a unit continuum of banks (more generally, financial intermediaries) indexed by $b \in [0,1]$. Deposits at bank b pay interest rate j_b if held until the next time period, but with an option to withdraw on demand. While referred to as 'demand deposits', this bank debt can be interpreted more broadly as short-term unsecured borrowing in money markets that is frequently rolled over.

A coordination game among depositors (bank creditors) is played in each discrete time period, but time subscripts are omitted in this section given the essentially static nature of the game. Just before the coordination game begins, all deposits $D_b \geq 0$ at bank b are held equally by a unit continuum of households indexed by $h \in [0,1]$. Expected payoffs in the next time period are discounted at a common rate ρ by all households.[14]

Bank fragility. Before households decide whether to hold deposits in the coordination game, banks have made portfolio and leverage decisions. Bank b invests in illiquid and liquid assets A_b and M_b respectively, where the notion of liquidity is defined below.

[13]A small strand of the banking literature has studied the relationship between bank runs and selected macroeconomic variables (Ennis and Keister, 2003; Martin et al., 2014; Porcellacchia, 2020; Mattana and Panetti, 2021; Leonello et al., 2025).

[14]Since there is a continuum of banks, depositor behaviour can be analysed as if households were risk neutral. The discount rate ρ is taken as given here, but in the full model, ρ is an endogenous variable.

Taking as given net worth (equity) N_b, these choices result in deposit creation up to a level of deposits D_b consistent with the balance-sheet identity $A_b + M_b = D_b + N_b$.

If a positive fraction $1 - H_b$ of households chooses not to hold deposits D_b at bank b, the bank must make a total payment $(1 - H_b)D_b$ to these households by disposing of some assets. The full value M_b of the liquid assets acquired earlier can be obtained at this point, but disposal of illiquid assets A_b during the coordination game only recovers a fraction λ of their value at acquisition.[15] If the proceeds of these asset liquidations are insufficient to cover depositor withdrawals, the bank fails. The condition for failure is

$$(1 - H_b)D_b > \lambda A_b + M_b.$$

The parameter $\lambda \in [0,1]$ measures the liquidity of the assets A_b relative to the benchmark of the perfectly liquid asset M_b. Rearranging the condition above and using the balance-sheet identity, bank b does not fail if $H_b \geq F_b$, where fragility F_b is

$$F_b = \frac{(1-\lambda)A_b - N_b}{A_b + M_b - N_b}. \tag{1}$$

The notion of bank fragility F_b is the threshold for the fraction H_b of households holding its deposits below which bank b fails. If net worth N_b is positive, fragility is a number between 0 and $1 - \lambda$, and higher net worth lowers fragility. Increased holdings of liquid assets M_b reduce a bank's fragility when it is initially positive, while holding more illiquid assets A_b raises fragility when it is below $1-\lambda$ initially. Fragility can be expressed in terms of familiar liquidity and bank capital ratios, respectively

$$m_b = \frac{M_b}{A_b + M_b} \quad \text{and} \quad n_b = \frac{N_b}{A_b + M_b}, \quad \text{as } F_b = \frac{(1-\lambda)(1-m_b) - n_b}{1 - n_b}. \tag{2}$$

Hence, a bank's scale plays no role in determining its fragility.

Structure of the game. Households make simultaneous binary choices whether to hold deposits.[16] There is a separate decision for each bank b. Households' choices are captured by indicator functions H_{bh}, which equals 1 if household h holds at bank b and 0 if h withdraws. Withdrawing households receive funds in the same time period.[17]

Holding bank deposits may expose households to credit risk because banks can fail. If this happens, those holding deposits only recover the principal after incurring a cost θ per unit of deposits. The parameter $\theta > 0$ represents losses associated with the bankruptcy process. These costs are paid at the beginning of the next time period.[18]

In this environment, banks fail because of 'runs' — too many depositors deciding to withdraw. The share of households who hold bank b's deposits is $H_b = \int_0^1 H_{bh}\,\mathrm{d}h$, and there is some minimum fraction F_b, endogenous to the bank's earlier liquidity and leverage choices (equation 1), who must hold for the bank not to fail. The indicator

[15] A literature studies transaction costs (Grossman and Miller, 1988; Brunnermeier and Pedersen, 2008) and adverse selection (Eisfeldt, 2004) as sources of asset illiquidity.

[16] To simplify the game, holding deposits is a binary choice, but households would not gain by being able to make partial withdrawals.

[17] For tractability, households must wait until the next time period to deposit funds at another bank.

[18] This timing is not essential; it is chosen for consistency with the full macroeconomic model.

function Φ_b for the failure of bank b depends on comparing H_b to bank fragility F_b:

$$\Phi_b(F_b, H_b) = \begin{cases} 0 & \text{if } H_b \geq F_b, \\ 1 & \text{otherwise.} \end{cases} \tag{3}$$

Fragility F_b is the relevant measure of bank fundamentals in the coordination game. $F_b = 1$ means that bank b needs each and every household to hold its deposits in order to survive. On the other hand, an intermediary with $F_b = 0$ is not fragile at all — it will not fail even if all households refuse to hold its deposits.

Conditional on knowing a bank's fragility and the share of households holding its deposits, the net payoff per unit of deposits from holding versus withdrawing is

$$\pi(F_b, H_b) = \frac{(j_b - \rho)(1 - \Phi_b) - \theta \Phi_b}{1 + \rho}, \tag{4}$$

with Φ_b given by (3). Households want to hold deposits at bank b if $\pi(F_b, H_b) \geq 0$ given their discount rate ρ and the interest rate $j_b \geq \rho$ offered by the bank.[19,20] The net payoff $\pi(F_b, H_b)$ is weakly decreasing in fragility F_b, representing a deterioration in the bank's fundamentals, and weakly increasing in the fraction H_b holding deposits, indicating the presence of strategic complementarity in the coordination game. With complete information, there would be multiple Nash equilibria: if $F_b \in (0, 1]$, an equilibrium with $H_{bh} = 1$ where everyone holds, and a 'bank-run' equilibrium with $H_{bh} = 0$.

Notice that the illiquidity of assets A_b is key to the existence of a coordination problem. If the full value of any assets can always be realized, the special case of $\lambda = 1$, then banks with non-negative net worth are never fragile. It is also important that banks' portfolio choices are made before households decide whether to hold deposits: once illiquid assets are funded by deposit creation, there is strategic complementarity in depositors' holding decisions. This timing assumption could capture the fact that banks create deposits when they make a loan and then someone in the economy must be willing to hold these deposits if the bank is to avoid having to dispose of assets. More generally, it could be interpreted as a mismatch between the timing of capital investment, which is typically long term, and banks' more short-term funding sources.

Incomplete information. In this model, households are subject to an information friction and cannot observe bank b's fragility F_b. As is well known in the literature on global games, such frictions can rule out sunspot-driven bank runs. Each household receives for each bank a private signal $\hat{F}_{bh} = F_b + \Omega_b + \Sigma_{bh}$ centred around the bank's true fragility with independent systematic and idiosyncratic noise. Systematic noise follows uniform distribution $\Omega_b \sim \mathrm{U}[-\omega, \omega]$ for some $\omega > 0$, and idiosyncratic noise is drawn independently from $\Sigma_{bh} \sim \mathrm{U}[-\sigma, \sigma]$ for some $\sigma > 0$.[21] The noise terms represent

[19] If indifferent, the tie-breaking assumption is that households hold deposits.

[20] Restricting attention to $j_b \geq \rho$ is without loss of generality because $j_b < \rho$ makes all households refuse to hold deposits. This is ex-ante suboptimal for a bank.

[21] The systematic noise Ω_b makes the game's equilibrium outcome stochastic conditional on F_b and j_b.

common and individual-specific errors made in analysing banks' balance sheets.

Formally, in the bank-b coordination game, all of households' prior information is $\mathcal{I}_b = \{F_b \sim \mathrm{U}_{\mathbb{R}}, D_b, j_b\}$. Household h updates this prior using signal $\hat{F}_{bh}$ to form beliefs $\mathbb{P}_{bh}[\cdot] = \mathbb{P}\left[\cdot \middle| \hat{F}_{bh}, \mathcal{I}_b\right]$ and expectations $\mathbb{E}_{bh}[\cdot] = \mathbb{E}\left[\cdot \middle| \hat{F}_{bh}, \mathcal{I}_b\right]$. The uninformative prior $F_b \sim \mathrm{U}_{\mathbb{R}}$ implies households do not use other sources of information to form beliefs. This assumption can be justified given the subsequent focus on arbitrarily precise signals.[22]

Equilibrium strategies. A household h chooses to hold deposits if and only if there is a non-negative expected net payoff $\mathbb{E}_{bh}[\pi(F_b, H_b)] \geq 0$ from doing so. We write this condition in terms of the household's belief about bank failure $\mathbb{E}_{bh}[\Phi_b]$ as

$$j_b - \rho \geq \frac{\mathbb{E}_{bh}[\Phi_b]}{1 - \mathbb{E}_{bh}[\Phi_b]}\theta, \tag{5}$$

and plot it in Figure 3 as the upward-sloping curve that separates the run region (in red) from the region in which the household holds deposits. According to (5), households demand a premium to compensate for the risk of bank failure. The novelty here is that households' equilibrium beliefs are not pinned down by an exogenous source of risk.

The probability of bank failure depends on other households' decisions. Because of a lack of common knowledge, each household is uncertain about the information held by other households and forms beliefs about this. Other households' information is their private signals $\hat{F}_{bu} = F_b + \Omega_b + \Sigma_{bu}$. In particular, household h is interested in forming a belief about the number of other households that, given their information, choose to hold deposits, and whether this number is sufficient to avoid bank failure as described by (3). Conjecturing that other households play a common threshold strategy such that household u holds deposits if $\hat{F}_{bu} \leq k_b$, the number of households holding conditional on the true fragility and systematic noise is $G_{\Sigma}(k_b - F_b - \Omega_b)$, where $G_{\Sigma}(\cdot)$ is the cdf of random variable Σ_{bu}.[23] It follows that household h's belief about bank failure is $\mathbb{P}_{bh}[\Phi_b = 1] = \mathbb{P}_{bh}[G_{\Sigma}(k_b - F_b - \Omega_b) < F_b] = \mathbb{P}\left[G_{\Sigma}\left(k_b + \Sigma_{bh} - \hat{F}_{bh}\right) < \hat{F}_{bh} - \Sigma_{bh} - \Omega_b\right]$.

Taking the limit with vanishing systematic noise, that is, $\omega \to 0$, and using the fact that $\Sigma_{bh} \sim \mathrm{U}[-\sigma, \sigma]$, we can write household h's beliefs about bank failure as[24]

$$\mathbb{E}_{bh}[\Phi_b] = \mathbb{P}_{bh}[\Phi_b = 1] = \begin{cases} 0 & \text{if } \hat{F}_{bh} \leq \frac{k_b - 2\sigma^2}{1+2\sigma}, \\ \frac{(1+2\sigma)\hat{F}_{bh} - k_b + 2\sigma^2}{2\sigma(1+2\sigma)} & \text{if } \hat{F}_{bh} \in \left(\frac{k_b - 2\sigma^2}{1+2\sigma}, \frac{k_b + 2(1+\sigma)\sigma}{1+2\sigma}\right], \\ 1 & \text{otherwise.} \end{cases} \tag{6}$$

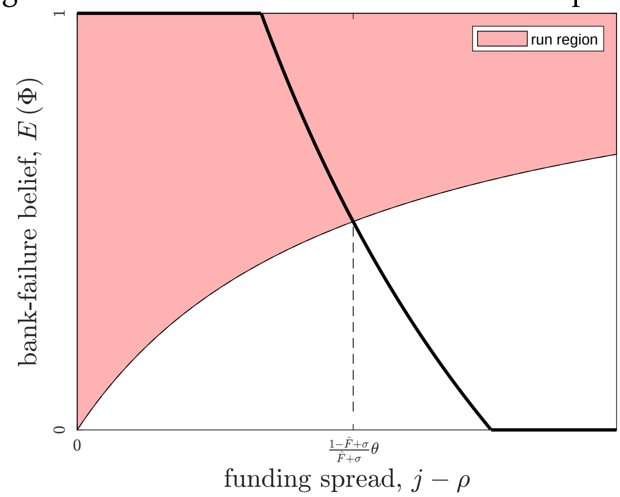
Households believe bank failure is more likely when they receive a higher signal $\hat{F}_{bh}$ for two reasons. First, it makes households believe that a bank is more fragile, increasing the number of households that must hold for the bank to survive. Second, a higher

[22]A literature studies conditions under which information provided by publicly observed endogenous variables such as policy (Angeletos et al., 2006) and prices (Atkeson, 2000; Angeletos and Werning, 2006) restores common knowledge.

[23]The proof of Lemma 1 shows a common threshold strategy is played in the unique equilibrium.

[24]A general analysis with a finite positive ω is found in the proof of Lemma 1.

Figure 3: Households' decisions to hold deposits.



Note: The run region is the graphical representation of condition (5). The downward-sloping thick line represents equation (6) with $k_b = F_b^*$ and F_b^* from equation (7).

signal shifts to the right a household's belief about the distribution of realized signals of all other households. This makes it more likely that fewer households hold deposits. By the same logic, a higher threshold k_b means each household believes bank failure is less likely because other households are more likely to hold deposits, all else equal.

Using beliefs (6) in combination with condition (5), we can define a threshold for household h's signal above which the household does not hold deposits because the offered interest rate j_b is insufficient compensation for risk. This threshold is a function of the spread of j_b over the discount rate ρ and other households' threshold k_b. In equilibrium, all households play the same threshold strategy F_b^* described below.

Lemma 1. *In equilibrium, there is a unique common threshold F_b^* such that household h holds bank b's deposits if and only if $\hat{F}_{bh} \leq F_b^*$. For $\omega \to 0$, this threshold is given by*

$$F_b^* = \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta}\sigma. \tag{7}$$

Proof. Please refer to appendix C. □

According to equation (7), a higher bank funding spread $j_b - \rho$ makes households hold deposits for higher realizations of their signals about fragility.[25]

A household's equilibrium belief about bank-failure risk is given by equation (6) after substituting in other households' equilibrium threshold strategy $k_b = F_b^*$ for their deposit-holding behaviour. The bank funding spread $j_b - \rho$ affects beliefs because it gives other households an incentive to hold deposits. Thus, higher $j_b - \rho$ reduces the risk of a coordination failure. This relationship between a household's equilibrium belief about bank-failure risk and the funding spread is depicted in Figure 3 as the downward-sloping thick black line. The lowest funding spread at which households hold deposits is where the thick line intersects the boundary of the run region.

[25]The proof of Lemma 1 gives an implicit expression for F_b^* with general distributions of noise.

With $\omega \to 0$, the bank funding spread is not a reflection of any extrinsic risk in the model. The spread itself is a driver of beliefs about bank failure because it affects others' incentives to hold deposits. Interestingly, seeing a zero funding spread means households should choose not to hold deposits for any signal $\hat{F}_{bh} > -\sigma$ because they think others do not have a sufficient incentive to hold. Hence, a bank with strictly positive fragility must offer a strictly positive funding spread to avoid failing.

Bank runs, balance sheets and funding costs. When banks make their choices of leverage, portfolio allocation and deposit rates before the coordination game, their key consideration is the endogenous response of households' deposit-holding decisions. Withdrawals of deposits force banks to dispose of assets and, if large enough, lead to bank failure with the loss of the bank's net worth.

Proposition 1. *Consider vanishingly small idiosyncratic noise $\sigma/\omega \to 0$ relative to systematic noise. Given fragility F_b and a funding spread $j_b - \rho$, the share H_b of households holding bank b's deposits in equilibrium follows a Bernoulli distribution:*

$$\mathbb{P}[H_b] = \begin{cases} \varkappa_b & \text{if } H_b = 1, \\ 1 - \varkappa_b & \text{if } H_b = 0, \end{cases} \tag{8}$$

with

$$\varkappa_b = \begin{cases} 1 & \text{if } j_b - \rho \geq \frac{F_b+\omega}{1-F_b-\omega}\theta, \\ \frac{(j_b-\rho)(1-F_b+\omega)-(F_b-\omega)\theta}{2\omega(j_b-\rho+\theta)} & \text{if } j_b - \rho \in \left[\frac{F_b-\omega}{1-F_b+\omega}\theta, \frac{F_b+\omega}{1-F_b-\omega}\theta\right), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Proof. Please refer to appendix C. □

A sufficiently small idiosyncratic component of noise rules out partial runs on banks. With effectively the same signal, all households make the same decision: either they all hold a bank's deposits or not. The proposition shows that a higher bank funding spread (or lower fragility) increases the probability $\varkappa_b$ all households hold deposits at bank b. In principle, this creates a trade-off for the bank — it can reduce its funding spread at the cost of a higher risk of a run. At $j_b - \rho = [(F_b + \omega)/(1 - F_b - \omega)]\theta$, the steepness $\partial\varkappa_b/\partial j_b$ of this trade-off is captured by the left derivative $(1 - F_b + \omega)^2/(2\omega\theta)$ of (9).

From this point onwards, we assume that systematic noise is small with $\omega \to 0$. In this case, the gradient $\partial\varkappa_b/\partial j_b$ of the trade-off becomes vertical: a marginal reduction in the funding spread is so costly in terms of an increase in the run probability that the bank chooses the corner solution with no runs, namely $\varkappa_b = 1$, which requires

$$j_b - \rho \geq \max\left\{\frac{F_b}{1-F_b}\theta, 0\right\}. \tag{10}$$

This choice also implies that banks never fail, that is, $\mathbb{P}[\Phi_b = 1] = 0$, because bank fragility given by equation (1) is smaller than one for any level of net worth, leverage and liquid-asset holdings. Because banks find it optimal to rule out runs on the equilibrium path, in the next section we set up a bank's problem in the absence of runs and introduce the no-run condition (10) as a constraint on the bank.[26]

This paper's assumption that both systematic and idiosyncratic noise are small, respectively $\omega \to 0$ and $\sigma/\omega \to 0$, keeps the model as close as possible to the full-information paradigm and parsimonious in terms of parameters. A disadvantage is that we cannot study the effects of the realization of a bank run because on the equilibrium path banks optimally rule them out. Nonetheless, this setting allows us tractably to study those actions that all banks take to avoid bank runs, such as demanding liquid assets, limiting lending and offering a spread on their debt. These preemptive actions have important macroeconomic consequences, which are the focus of this paper.

It is worth studying the no-run condition (10) in greater depth. Substituting in the determinants of bank fragility from equation (1) yields a mapping from the bank's balance sheet to the deposit rate required to avoid a run:

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)A_b - N_b}{\lambda A_b + M_b}\theta, 0\right\}. \tag{11}$$

Further intuition is gained by substituting the familiar balance-sheet ratios (2) into the no-run condition. The mapping from these to the deposit rate required to avoid runs is

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)(1-m_b) - n_b}{\lambda + (1-\lambda)m_b}\theta, 0\right\}. \tag{12}$$

A graphical representation is provided in Figure 4. The dashed line depicts combinations of the capitalization ratio n_b and liquidity ratio m_b that rule out failure of bank b with a zero spread on deposits. If there is a positive spread on deposits, the region of fundamentals that leads to bank failure, coloured in red, is always below the dashed line. All else equal, a higher interest on deposits makes the failure region smaller. The key implication is that there is a three-way substitutability from a bank's perspective between equity, liquidity and interest on deposits. For instance, a bank can lever up while keeping its interest-rate expenses in check by boosting its liquid-asset holdings.

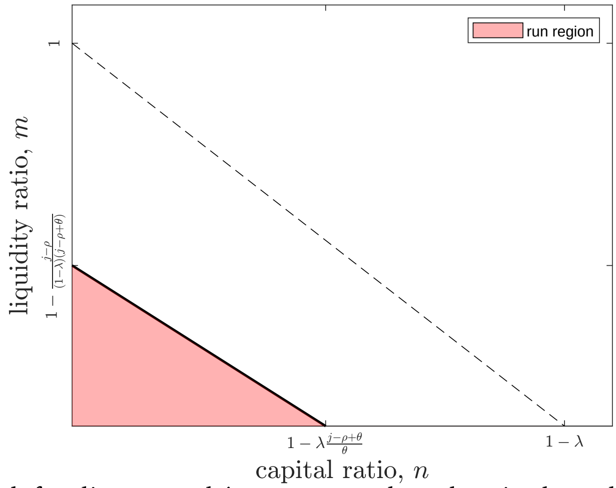
Equilibrium beliefs. According to our earlier analysis of households' equilibrium strategies, the key belief that determines households' actions is their beliefs about bank failure. Because of the information friction, there is a distribution of beliefs across households. Do some households wrongly believe that runs actually take place even though banks comply with no-run condition (10) that rules out runs in equilibrium?

Proposition 2. *Consider $\sigma/\omega \to 0$. If $\mathbb{P}[H_b = 1] = 1$ and $F_b \leq 1 - 2(\sigma + \omega)$, then household h's belief about bank failure is almost surely correct with*

$$\mathbb{P}_{bh}[\Phi_b = 1] = \mathbb{P}[\Phi_b = 1] = 0. \tag{13}$$

[26] To verify this convenient restriction on a bank's choice set is without loss of generality, once we have laid out a bank's problem in the next section we can check that, (i), the payoff function conditional on $j_b - \rho$ and H_b is continuous in $j_b - \rho$ for any given $H_b \in \{0, 1\}$, and (ii), the payoff given $H_b = 0$ and $j_b - \rho \in [0, [F_b/(1-F_b)]\theta]$ is smaller than the payoff given $H_b = 1$ and $j_b - \rho \geq [F_b/(1-F_b)]\theta$.

Figure 4: Balance-sheet fundamentals and bank runs.



Note: For a given bank funding spread $j_b - \rho$, a run takes place in the red region according to condition (12). The dashed line indicates the boundary of the run region for a zero spread.

Proof. Please refer to appendix C. □

If a bank satisfies the no-run condition and its fragility is below an upper bound, then household beliefs are correct: bank deposits are risk-free.[27] Because in the macroeconomic model σ and ω are arbitrarily small while λ is positive and net worth is non-negative, fragility is always below the upper bound as can be verified from equation (1).[28] This and the fact that banks optimally choose to rule out runs imply that on the equilibrium path, household beliefs about bank outcomes are correct.

3 Macroeconomic model

This section embeds the coordination friction faced by holders of bank deposits into a macroeconomic model. To focus on this novel friction, the core of the economy is represented by a real business cycle model (Kydland and Prescott, 1982).

Timeline. Each discrete time period $t = 0, 1, 2, \ldots$ is divided into three stages. At the first stage, perfectly competitive markets for goods, labour, physical capital, liquid assets, and illiquid bonds are open. Aggregate shocks and noise in households' signals are realized. Households choose labour supply and non-bank assets, and firms produce final goods and incomes are distributed. The government chooses a supply of liquid assets and sets fiscal policy. During this stage, banks create deposits and set deposit interest rates, select a portfolio of liquid and illiquid assets to hold, and pay dividends. At the second stage, households play the coordination game described in section 2, choosing whether to hold deposits at each bank. At the final stage, households' consumption is determined based on what happened earlier in the period.

[27]Beliefs are *almost* surely correct, because if and only if no-run condition (10) holds with equality and the worse state of the world for a bank realizes, namely $\Omega_b = \omega$, a (strictly) positive mass of households attributes a (strictly) positive probability to bank failure. This state of the world has zero probability.

[28]The role of the upper bound on fragility in the proposition is to rule out values of fragility so close to one that, despite vanishingly small noise, some households believe the bank's fragility is larger than one.

Physical capital as the illiquid asset. The illiquid assets held by banks are physical capital goods. A surviving bank b holding illiquid assets $A_{b,t-1}$ at the end of period $t-1$ has a stock of physical capital $K_{bt} = X_t A_{b,t-1}$ to rent out at price p_t for use in production of final goods. The random variable X_t, which has mean 1, represents an exogenous capital-quality shock common to all banks. Capital K_{bt} depreciates at rate δ during each time period. The ex-post return on physical capital between $t-1$ and t is

$$R_t = X_t(1 - \delta + p_t) - 1. \tag{14}$$

At the first stage of period t, final goods can be transformed into new capital through investment $I_{bt} = A_{bt} - (1-\delta)K_{bt}$ financed by banks (or existing capital transformed back into final goods if investment is negative). Only goods transformed into capital by this stage can be stored and carried into period $t+1$ as physical capital.

Capital is illiquid at the second stage of a time period in the sense that investment is partially irreversible by that point. Only a fraction λ of a bank's physical capital can be immediately converted back into goods usable for consumption without causing the bank to fail. More than this amount can be recovered, but at the cost of bank failure, with the loss of bank equity acting as a form of adjustment cost.[29] In addition, those holding deposits at the point of bank failure must incur a cost θ to recover each unit of deposits through the bankruptcy process described in section 2.

Other frictions. The macroeconomic relevance of the coordination game among bank depositors depends on three other frictions. First, households cannot directly hold physical capital (banks' illiquid assets), so financial intermediation is necessary for capital accumulation and production. Second, bank debt takes the form of the short-term demand deposits described in section 2, hence there is a mismatch between the liquidity of bank liabilities and assets. Third, banks need positive equity but face limits on accumulating equity capital, so their assets cannot be financed entirely by equity.

While the model does not speak to why such frictions are present, these are all standard features of the existing macro-banking literature. The first can be justified if holding illiquid assets requires expertise possessed only by bankers, or diversification through the scale at which banks operate. The second can come from some short-term liquidity needs of households that preclude tying up wealth in a long-term investment.

The third is often built into macro-banking models through exogenous exit of banks or bankers. Here, a simpler foundation is a problem of separation of ownership and control of banks. Suppose bank employees are able to divert bank profits to their bonus pools if these funds are not swiftly returned to shareholders. Formally, suppose a constant fraction $\gamma/(1+\gamma)$ of pre-dividend net worth is vulnerable to diversion as bonuses Ξ_{bt}, where γ is a positive parameter. Even if bank b's shareholders would otherwise prefer earnings to be retained, they need to pay out at least the funds vulnerable

[29] Banks operate with positive net worth in equilibrium.

to diversion. This motivates a minimum dividend condition

$$\Pi_{bt} \geq \gamma N_{bt}, \tag{15}$$

where Π_{bt} is bank b's dividend and N_{bt} is net worth after the dividend is distributed.

3.1 Production

A continuum of firms $f \in [0,1]$ produces homogeneous final goods for consumption or investment. They hire homogeneous labour L_{ft} at wage w_t and rent physical capital K_{ft}. Firms face a constant-returns-to-scale Cobb-Douglas production function

$$Y_{ft} = Z_t K_{ft}^{\alpha} L_{ft}^{1-\alpha}, \tag{16}$$

where Z_t is exogenous total factor productivity and α is the capital elasticity of output.

Goods and factor markets are perfectly competitive and all prices and wages are fully flexible. The price of final goods is normalized to one so that all variables are in real terms. Firms maximize profits $\Pi_{ft} = Y_{ft} - p_t K_{ft} - w_t L_{ft}$, which are immediately paid out as dividends. Profit maximization implies capital and labour are hired up to where their marginal products equal the rental price p_t and the wage w_t respectively:

$$\alpha Z_t \left(\frac{L_{ft}}{K_{ft}}\right)^{1-\alpha} = p_t, \quad \text{and} \quad (1-\alpha) Z_t \left(\frac{K_{ft}}{L_{ft}}\right)^{\alpha} = w_t. \tag{17}$$

With constant returns to scale, profits are equal to zero ($\Pi_{ft} = 0$) in equilibrium.

3.2 Households

At the beginning of period t, household $h \in [0,1]$ has expected lifetime utility

$$\mathcal{U}_{ht} = \mathbb{E}_{ht}\left[\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{C_{hs}^{1-\frac{1}{\phi}} - 1}{1-\frac{1}{\phi}} - \chi \frac{L_{hs}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right\}\right], \tag{18}$$

where C_{ht} is consumption, L_{ht} is labour supply, β is the subjective discount factor, χ is a parameter representing the disutility of labour, and ϕ and ψ are preference parameters corresponding to the elasticity of intertemporal substitution and Frisch elasticity of labour supply, respectively. All households have the same preferences and begin with equal wealth in period 0. The only heterogeneity is in their signals about bank fragility.

The information set conditioned on in expectation operator $\mathbb{E}_{ht}[\cdot]$ in (18) contains commonly known aggregate shocks, prices, and macroeconomic variables from date t and earlier. It also contains household-specific signals, and as analysed in section 2, each household uses its arbitrarily precise signals to form beliefs about banks' fragility.[30]

Because banks comply with the no-run condition (11), by Proposition 2, all households have signals that lead them correctly to believe that no runs ever occur.[31] This implies the heterogeneity in households' information sets is irrelevant for choices made

[30]They are restricted from using other information to inform their beliefs about fragility. As discussed in section 2, the use of public information could restore common knowledge and thus multiple equilibria.

[31]This follows from bank net worth being non-negative in all states of the world on the equilibrium path, as seen in section 4. Since σ and ω are arbitrarily small, $\lambda \geq 2(\sigma + \omega)$ is satisfied for positive λ.

at the competitive-markets stage, and hence the household h subscript can be dropped from the expectation operator. It is replaced by $\mathbb{E}_t[\cdot]$, which is conditional only on macroeconomic variables known by date t. Given that bank runs do not happen in equilibrium and no household assigns positive probability to one happening, this section analyses household behaviour abstracting from runs.[32]

Competitive-markets stage. Household h chooses labour supply L_{ht} and receives wage income $w_t L_{ht}$, and everyone pays a common lump-sum net tax T_t. Each household receives a non-negative dividend Π_t from owning an equal share of a non-tradable investment fund comprising all banks and non-financial firms,[33] and has an equal claim on the total bonus pool Ξ_t across all banks. Bonuses are obtained through diversion of bank net worth, though these will be zero because (15) holds in equilibrium.[34]

Household h may choose to borrow between periods t and $t+1$ an amount B_{ht} (or if negative, hold savings outside banks) in the form of a risk-free but illiquid bond with interest rate ρ_t.[35] Any past borrowing $(1+\rho_{t-1})B_{h,t-1}$ must be repaid, and a no-Ponzi condition must be respected.[36] Households may also hold a non-negative amount of non-bank liquid assets M_{ht} paying risk-free interest rate i_t.

Budget constraint and utility maximization. At the start of period t, households have deposits $(1+j_{b,t-1})D_{b,t-1}$ at bank b, including accrued interest. Bank b's net deposit creation $D_{bt} - (1+j_{b,t-1})D_{b,t-1}$ funds purchases of physical capital and liquid assets. It is implicit that bank deposits are accepted by firms and households as a means of payment and circulate at the competitive-markets stage.[37] Since non-financial firms are entirely static, paying out all sales revenue immediately as factor payments, all deposits must be in the hands of households once the competitive-markets stage is over.

Conditional on everyone holding deposits and competitive-markets-stage choices, the consumption enjoyed at the final stage of t is given by the flow budget constraint:[38]

$$C_{ht} = w_t L_{ht} + \Pi_t + \Xi_t - T_t + \int_0^1 \{(1+j_{b,t-1})D_{b,t-1} - D_{bt}\}\,\mathrm{d}b \\ - (1+\rho_{t-1})B_{h,t-1} + B_{ht} + (1+i_{t-1})M_{h,t-1} - M_{ht}. \quad (19)$$

Households directly holding physical capital is ruled out by assumption, so capital is excluded from (19). First-order conditions for maximizing utility (18) subject to (19)

[32]The supplementary appendix describes how actions taken in a run are integrated with the model.

[33]In equilibrium, there are no gains from trading shares in the investment fund among households.

[34]The total bonus pool is $\Xi_t = \int_0^1 \Xi_{bt}\,\mathrm{d}b = \frac{1}{1+\gamma}\int_0^1 \max\{0, \gamma N_{bt} - \Pi_{bt}\}\,\mathrm{d}b$.

[35]Illiquid in that no value from this asset can be realized until the $t+1$ competitive-markets stage.

[36]The no-Ponzi condition is $\lim_{s\to\infty} \frac{B_{hs}}{(1+\rho_t)\cdots(1+\rho_{s-1})} \leq 0$ in all states of the world.

[37]The medium-of-exchange role of deposits is not explicitly modelled here. Households and firms accept deposits in exchange for goods if they believe no bank failures will occur, as is true in equilibrium.

[38]At the final stage of a time period, liquid assets can be traded for consumption goods, and the government can levy additional lump-sum taxes. However, in the absence of bank runs, households do not hold liquid assets at that point and the government has no need to adjust taxes, so these possibilities can be ignored. For a full description of the final stage when runs occur, see the supplementary appendix.

with respect to B_{ht} and L_{ht} determine the optimal choices of consumption and labour supply.[39] With no heterogeneity in wealth or preferences, and no relevant heterogeneity in information sets, consumption C_{ht} and labour supply L_{ht} are the same for all $h \in [0,1]$:

$$C_t^{-\frac{1}{\phi}} = \beta(1+\rho_t)\mathbb{E}_t\left[C_{t+1}^{-\frac{1}{\phi}}\right], \quad \text{and} \quad \chi L_t^{\frac{1}{\psi}} = w_t C_t^{-\frac{1}{\phi}}. \tag{20}$$

There is effectively a representative household from a macroeconomic perspective. Households can also choose to hold liquid assets $M_{ht} \geq 0$ directly. However, since $i_t \leq \rho_t$ must hold in equilibrium, utility is maximized by choosing $M_{ht} = 0$.[40]

Coordination game. Assuming banks treat all households symmetrically when deposits are created, and since all ex-ante identical households behave the same way at the competitive-markets stage, each household carries the same amount of deposits D_{bt} at bank b into the coordination game of section 2. Households' equilibrium strategies in the coordination game maximize expected future consumption payoffs discounted at a common rate. Since there is a continuum of banks $b \in [0,1]$, this is consistent with concave utility in (18) because deposit-holding decisions and bank survival outcomes for any individual bank have only a negligible effect on a household's overall consumption. Household h therefore acts as risk neutral in respect of deposits at a particular bank, discounting payoffs expected in period $t+1$ using discount factor $\beta\mathbb{E}_{ht}\left[(C_{h,t+1}/C_{ht})^{-\frac{1}{\phi}}\right]$. The first-order condition for illiquid bonds (see 20) implies everyone's discount factor equals $1/(1+\rho_t)$. The yield ρ_t on the illiquid bond is therefore the appropriate discount rate to apply to expected payoffs during the coordination game.

3.3 Government

The government issues liabilities M_t that are liquid assets in the sense that they can be exchanged for consumption goods one-for-one at any stage of period t.[41] These liabilities are broadly interpretable as government bonds, reserves, or outside money more generally, though the model has a single type of liquid government liability for simplicity. This is an asset that offers a risk-free return i_t between periods t and $t+1$.

The government is able to levy lump-sum taxes on households or make transfers. At the competitive-markets stage of period t, the net lump-sum tax paid by all households is T_t.[42] The government can also purchase illiquid bonds B_t (or if negative, issue illiquid bonds). Changes in fiscal and monetary policy are represented through different combinations of M_t, B_t, and T_t.[43] Consolidating across all branches of government, the

[39] The transversality condition $\lim_{s\to\infty}\beta^{s-t}C_{hs}^{-\frac{1}{\phi}}(\int_0^1 D_{bs}\,\mathrm{d}b - B_{hs} + M_{hs}) \leq 0$ must also hold in all states.

[40] $M_{ht} = 0$ can be interpreted as households choosing to deposit in banks any outside money obtained from fiscal transfers, and selling any liquid financial assets to banks. Note that if $i_t > \rho_t$, there would be an unbounded demand for liquid assets, so an equilibrium must have $i_t \leq \rho_t$.

[41] The liquidity of government bonds ultimately derives from the government's ability to adjust the supply of bonds and taxes after the coordination game, as described in the supplementary appendix.

[42] If no runs occur, tax revenue can be collected at the first stage of period t without loss of generality.

[43] Purchases of illiquid assets B_t financed by issuing liquid liabilities M_t can be interpreted as a form

flow budget constraint necessary to deliver a risk-free return of i_{t-1} on M_{t-1} is

$$T_t = (1+i_{t-1})M_{t-1} - (1+\rho_{t-1})B_{t-1} - M_t + B_t. \tag{21}$$

At least one further equation is needed to specify the positive supply of liquidity M_t.

3.4 Banks

Ownership and solvency. Each bank $b \in [0,1]$ and each non-financial firm is owned by a large investment fund, and these funds are themselves owned by households. Investment funds pay out non-negative dividends Π_t to households and they direct the firms they own to act in the interests of households.

This paper focuses on the risk of bank failure caused by illiquidity and runs. To that end, we make assumptions so that, in equilibrium, banks do not fail owing to insolvency, which also simplifies the subsequent analysis. Specifically, it is necessary that investment funds recapitalize banks that would otherwise have non-positive equity E_{bt} at the beginning of time period t. Injections of capital come from dividends paid out by other banks, and each unit of equity injected costs $1+\xi$, where $\xi > 0$ is a resource cost of recapitalization.[44] This implies it is optimal for individual banks to act so that the probability of recapitalization is zero. The exogenous stochastic processes X_t and Z_t for capital quality and TFP are assumed to have finite support, which means insolvency of the whole banking system — where recapitalization is not feasible — has probability zero in equilibrium. In period 0, banks start with some positive amounts of equity $\{E_{b0}\}$.

With no recapitalizations, investment funds aggregate dividends Π_{bt} from banks and any dividends Π_{ft} from non-financial firms and distribute these to households:

$$\Pi_t = \int_0^1 \Pi_{bt}\,\mathrm{d}b + \int_0^1 \Pi_{ft}\,\mathrm{d}f. \tag{22}$$

Objectives and choices. As there is effectively a representative household in equilibrium, the objective function of bank b is $\Pi_{bt} + V_{bt}$, where V_{bt} is the present value of future dividends (the ex-dividend value of bank b) obtained using a stochastic discount factor P_{ts} given by households' common marginal rate of substitution (see 18) between consumption at date t and (state-contingent) consumption at date $s > t$:

$$V_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\Pi_{bs}\right], \quad \text{where } P_{ts} = \beta^{s-t}\left(\frac{C_s}{C_t}\right)^{-\frac{1}{\phi}}. \tag{23}$$

At each date t, bank b chooses a deposit interest rate j_{bt} and makes a deposit creation decision that results in a stock of deposits D_{bt}. There is no competitive market for deposits, so a bank can choose both the quantity and the price, but these choices affect whether households decide to hold the bank's deposits during the coordination game. The other choices are the amounts of physical capital A_{bt} and liquid assets M_{bt} to

of unconventional monetary policy. However, the government never buys physical capital, so it does not directly take on the financial intermediation role performed by banks.

[44]Equity $E_{bt} + J_{bt}$ after the injection of capital J_{bt} costing $(1+\xi)J_{bt}$ must be at least $\underline{E}$ for some $\underline{E} > 0$.

hold, and the dividend Π_{bt} to distribute.[45] Each bank is competitive in goods and asset markets and hence takes prices (R_t, i_t, ρ_t) and the stochastic discount factor P_{ts} as given.

Constraints. A bank b that reaches the beginning of period t with positive equity has after paying out dividend Π_{bt} the following net worth (bank capital) $N_{bt} = E_{bt} - \Pi_{bt}$:

$$N_{bt} = (1 + R_t)A_{b,t-1} + (1 + i_{t-1})M_{b,t-1} - (1 + j_{b,t-1})D_{b,t-1} - \Pi_{bt}, \tag{24}$$

which depends on the bank's assets and liabilities from $t-1$ and the returns on these. The equation assumes there is no diversion of funds to employee bonuses, which requires banks to satisfy the minimum-dividend condition (15), but they always find it optimal to do so.[46] Given net worth N_{bt} from (24), the balance-sheet identity of bank b is

$$A_{bt} + M_{bt} = D_{bt} + N_{bt}. \tag{25}$$

With the assumption of small systematic and idiosyncratic noise in households' signals from section 2, banks that do not satisfy the no-run condition (11) face a run with probability one causing them to fail and lose positive net worth N_{bt}. Instead, by ensuring (11) holds, runs have probability zero and banks make positive profits.[47] Hence, bank b's optimal choices maximize the present value of current and future dividends subject to (11) and (15) as constraints, along with (24) and (25).[48] Since net worth $N_{b,t+1}$ is decreasing in j_{bt}, the no-run condition (11) must bind when deposits are positive:

$$j_{bt} = \rho_t + \max\left\{\frac{1}{\lambda + \frac{\lambda N_{bt} + (1-\lambda)M_{bt}}{D_{bt}}} - 1, 0\right\}\theta. \tag{26}$$

3.5 Aggregation and market clearing

Equilibrium in factor markets requires that non-financial firms rent the physical capital owned by banks and hire the labour supplied by households:

$$\int_0^1 K_{ft}\,\mathrm{d}f = \int_0^1 K_{bt}\,\mathrm{d}b = K_t, \quad \text{and} \quad \int_0^1 L_{ft}\,\mathrm{d}f = \int_0^1 L_{ht}\,\mathrm{d}h = L_t, \tag{27}$$

where the supply of capital $K_t = X_t A_{t-1}$ depends on banks' aggregate past illiquid assets $A_{t-1} = \int_0^1 A_{b,t-1}\mathrm{d}b$ adjusted for the capital quality shock X_t. Aggregating (16) and (17) across firms implies the aggregate production function $Y_t = Z_t K_t^{\alpha} L_t^{\alpha}$ and labour demand curve $w_t = (1-\alpha)Y_t/L_t$, and the return on capital is $R_t = X_t(1 - \delta + \alpha Y_t/K_t)$.

Equilibrium in financial markets requires banks' demand for liquid assets equals the amount supplied by the government, and households' supply of illiquid bonds equals the amount purchased by the government:

$$\int_0^1 M_{bt}\,\mathrm{d}b = M_t, \quad \text{and} \quad B_t = \int_0^1 B_{ht}\,\mathrm{d}h, \tag{28}$$

noting that household choose not to hold liquid bonds and banks choose not to hold

[45] Banks do not want to hold illiquid bonds, and they cannot fund themselves by issuing such bonds.

[46] In general, $N_{bt} = E_{bt} - \Xi_{bt} - \Pi_{bt}$. Raising Π_{bt} has no negative effect on N_{bt} up to where (15) holds.

[47] For completeness, bank b's actions in the case of a run are described in the supplementary appendix.

[48] The transversality condition $\lim_{s\to\infty} P_{ts}\Pi_{bs} = 0$ in all states of the world is necessary for a maximum. Given the minimum-dividend constraint (15), this implies the restriction $\lim_{s\to\infty} P_{ts}N_{bs} = 0$ on net worth.

illiquid bonds. The market for deposits is not perfectly competitive, but households hold the amount supplied by banks as assumed in the budget constraint (19) since the no-run condition (26) holds.[49] Combining household, firm, bank, investment fund, and government budget constraints implies market clearing $C_t + I_t = Y_t$ for final goods, where $I_t = A_t - (1-\delta)K_t$ is aggregate investment financed by banks.

4 Bank behaviour

This section analyses banks' profit-maximizing choices of asset liquidity, the creation of deposits, the supply of credit to purchase physical capital, and the distribution of dividends subject to the friction developed in the coordination game of section 2.

Banks' full dynamic optimization problem stated in section 3 is solved here as a series of equivalent static problems in liquidity and leverage choices taking as given the path of net worth, and finally considering dividend policy to characterize the evolution of net worth. For bank b with net worth N_{bt}, liquid asset demand M_{bt}, credit supply A_{bt}, and the total quantity of deposits D_{bt} created must maximize the expected discounted value of $N_{b,t+1} + \mathit{\Pi}_{b,t+1}$ using a stochastic discount factor Ψ_{t+1} common to all banks that is explained below. The objective function is $W_{bt} = \mathbb{E}_t[\Psi_{t+1}(N_{b,t+1} + \mathit{\Pi}_{b,t+1})]/\mathbb{E}_t[\Psi_{t+1}]$, and using the evolution of net worth (24) and the balance-sheet identity (25) this is

$$W_{bt} = (1+r_t)N_{bt} + (r_t - j_{bt})D_{bt} - (r_t - i_t)M_{bt}, \tag{29}$$

where $r_t = \mathbb{E}_t[\Psi_{t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t+1}]$ denotes the risk-adjusted expected value of R_{t+1}. In maximizing W_{bt} with net worth N_{bt} and r_t, i_t, and ρ_t given, there are three choice variables j_{bt}, D_{bt}, and M_{bt} and one binding constraint, the no-run condition (26).

Demand for liquid assets. An increase in M_{bt} given D_{bt} and N_{bt} means switching from illiquid to liquid assets while keeping the size of bank b's balance sheet unchanged. This has a cost $r_t - i_t$, as seen from (29), reflecting the difference in (risk-adjusted) expected returns between the two assets, referred to as the credit spread since r_t is the return on supplying credit for capital accumulation. The benefit of more liquidity is the fall in bank fragility, $F_{bt} = 1 - \lambda - ((1-\lambda)M_{bt} + \lambda N_{bt})/D_{bt}$ from (1) and (25), which lowers the bank's funding cost. The binding no-run constraint (26) gives the deposit interest rate j_{bt} as a function of M_{bt} and D_{bt}, and (29) implies the marginal benefit is equal to $-\partial j_{bt}/\partial M_{bt}$ multiplied by deposits D_{bt}. If fragility is positive, the marginal benefit is

$$-D_{bt}\frac{\partial j_{bt}}{\partial M_{bt}} = (1-\lambda)\theta\left(\lambda + \frac{(1-\lambda)M_{bt} + \lambda N_{bt}}{D_{bt}}\right)^{-2} = \frac{(1-\lambda)\theta}{(1-F_{bt})^2}, \tag{30}$$

but if fragility is already negative then the marginal benefit is zero.

If $r_t - i_t > (1-\lambda)\theta$, in which case the bank's demand for liquid assets leaves it

[49] The transversality condition on households' asset holdings together with the no-Ponzi condition on household borrowing, the non-negativity of deposits, and $M_{ht} = 0$ imply the transversality condition $\lim_{s\to\infty}\mathbb{E}_t\left[P_{ts}\int_0^1 D_{bs}\,\mathrm{d}b\right] = 0$ on deposits using the formula for the stochastic discount factor in (23).

with positive fragility, the first-order condition maximizing (29) with respect to M_{bt} is $r_t - i_t = -D_{bt}\partial j_{bt}/\partial M_{bt}$.[50] If $r_t - i_t = 0$, the bank demands enough M_{bt} to ensure fragility is negative, while if $0 < r_t - i_t \leq (1-\lambda)\theta$, the bank targets zero fragility exactly when choosing liquid assets. Using (1), (25), and (30), bank b's demand for liquid assets is

$$M_{bt} \begin{cases} = \frac{1}{1-\lambda}\left(\sqrt{\frac{(1-\lambda)\theta}{r_t - i_t}} - \lambda\right) D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } r_t - i_t > (1-\lambda)\theta \\ = D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } 0 < r_t - i_t \leq (1-\lambda)\theta \\ \geq D_{bt} - \frac{\lambda}{1-\lambda} N_{bt} & \text{if } r_t - i_t = 0 \end{cases} \cdot \tag{31}$$

Demand for liquidity is decreasing in the cost $r_t - i_t$ of holding liquid assets, increasing in deposits D_{bt} because more leverage increases fragility, and decreasing in net worth N_{bt} because bank capital is a substitute for liquidity in reducing fragility.[51]

Since all banks face the same cost $r_t - i_t$ of holding liquid assets and the marginal benefit depends only on an individual bank's fragility F_{bt}, banks trade liquid assets up to the point where fragility is equalized across them.[52] This is analogous to the demand for reserves in Poole (1968) arising from payments risk. With $F_{bt} = F_t$ for all b, systemic bank fragility F_t is derived from (1) by aggregating the balance-sheet identities (25):

$$F_t = 1 - \lambda - \frac{(1-\lambda)M_t + \lambda N_t}{D_t}, \tag{32}$$

where N_t, M_t, and D_t are the aggregate amounts of equity, liquid assets, and deposits in the banking system. A consequence of $F_{bt} = F_t$ is that all banks face the same minimum funding cost $j_t = j_{bt}$ consistent with a binding no-run constraint. Using (26) and (32), this deposit rate satisfies $j_t - \rho_t + \theta = \theta/(1 - F_t)$ for non-negative fragility, and combining with (30) shows the marginal benefit of liquid assets common to all banks after trading is positively related to banks' funding cost j_t. By equating this to the cost of liquidity:

$$r_t - i_t = \frac{(1-\lambda)\theta}{(1-F_t)^2} = \frac{(1-\lambda)}{\theta}(j_t - \rho_t + \theta)^2 \quad \text{if } F_t > 0. \tag{33}$$

Deposit creation. An increase in deposits D_{bt} given M_{bt} and N_{bt} means greater leverage, with bank b increasing the size of its balance sheet (25). Since banks trade liquid assets so as to equalize fragility for any given D_{bt}, the objective function (29) can be written in terms of the common funding cost j_t and systemic fragility F_t using (32):[53]

$$W_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right) N_{bt} + \left(\frac{r_t - i_t}{1-\lambda} F_t + i_t - j_t\right) D_{bt}, \tag{34}$$

[50] The second-order condition is satisfied because (30) shows $-D_{bt}\partial j_{bt}/\partial M_{bt}$ is decreasing in M_{bt}.

[51] The non-negativity constraint $M_{bt} \geq 0$ means corner solutions must be checked. However, a corner solution for some banks but not others can be ruled out because a binding non-negativity constraint reduces the maximum attainable W_{bt}, but it will be seen that banks are indifferent about the size of D_{bt}, and the non-negativity constraint is slack for sufficiently large D_{bt} (see 31). Furthermore, a positive aggregate supply of liquidity M_t means that there cannot be a corner equilibrium for all banks.

[52] The marginal benefit depends only on fragility because the reduction in fragility is inversely proportional to deposits, but the lower funding cost is multiplied by the size of the deposit base.

[53] Intuitively, if D_{bt} increases by one unit, but fragility remains unchanged at F_t after adjusting liquid assets, (1) implies the composition of the increase in total assets is A_{bt} rising by $F_t/(1-\lambda)$ and M_{bt} rising by $1 - F_t/(1-\lambda)$. This delivers an additional payoff of $(r_t - i_t)F_t/(1-\lambda) + i_t$ for the bank at the cost of paying extra interest j_t, but with no further effect on overall funding costs, hence the coefficient of D_{bt} in (34).

This is linear in deposits, so if the coefficient of D_{bt} is positive, there is no limit to banks' desire to create deposits, while if negative, no deposit creation occurs. Hence, an equilibrium with a positive but finite supply of deposits requires that the coefficient on D_{bt} is zero.[54] If fragility is zero, this means that $i_t = j_t = \rho_t$. With $F_t > 0$, a rearrangement of the coefficient of D_{bt} shows that it is zero when $(r_t - \lambda i_t)/(1-\lambda) = j_t + (r_t - i_t)(1-F_t)/(1-\lambda)$. Using the implication (33) of banks' demand for liquidity, this is equivalent to:

$$\frac{r_t - \lambda i_t}{1-\lambda} = j_t + (j_t - \rho_t + \theta), \quad \text{where } j_t - \rho_t + \theta = \frac{\theta}{1-F_t}. \tag{35}$$

The terms on the right-hand side are respectively the direct funding cost of the additional deposit and the cost of holding the additional liquid assets to avoid raising fragility, which is also positively related to banks' funding costs. Taking as given r_t, i_t, and ρ_t, the aggregate supply of deposits D_t adjusts until the deposit rate j_t satisfies equation (35), with higher D_t increasing systemic fragility (32) and hence raising j_t.[55] In what follows, attention is restricted to cases where deposits D_t are strictly positive.

The liquidity premium and aggregate demand for liquidity. Equation (33) shows the difference between the returns r_t and i_t on banks' illiquid and liquid assets is positively related to banks' funding spread of j_t over the risk-free rate ρ_t. Intuitively, the funding spread reflects banks' fragility, and thus their demand for liquid assets. Together with $j_t - i_t = (r_t - i_t)F_t/(1-\lambda)$ from the zero coefficient on D_{bt} in (34) for the supply of deposits, (33) implies $j_t - i_t = (j_t - \rho_t)(j_t - \rho_t + \theta)/\theta$. Thus, a high funding spread lowers the yield i_t on liquid assets relative to other interest rates, including ρ_t on illiquid bonds. Simplifying the equation shows banks' funding spread is a geometric average of the liquidity premium $\rho_t - i_t$ and depositors' loss given default parameter θ:

$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t}. \tag{36}$$

The definitions of the funding spread and liquidity premium here are analogous to the empirical spreads between LIBOR, the GC repo, and T-bills in that j_t is unsecured, and while both ρ_t and i_t are risk-free yields, government bonds have the advantage of liquidity. Equation (36) implies a positive relationship between the funding spread and the liquidity premium consistent with the empirical evidence presented in Figure 2.

Combining equations (33) and (36) shows there is also a positive relationship between the liquidity premium and the credit spread $r_t - i_t$:

$$r_t - i_t \begin{cases} = 4(1-\lambda)\left(\frac{1}{2}\sqrt{\theta} + \frac{1}{2}\sqrt{\rho_t - i_t}\right)^2 & \text{if } F_t > 0 \\ \in [0, (1-\lambda)\theta) & \text{if } F_t = 0\,, \\ = 0 & \text{if } F_t < 0 \end{cases} \tag{37}$$

[54] Deposits are zero in equilibrium only if $r_t \leq \rho_t$. This is because fragility must be negative if $D_t = 0$, hence $r_t = i_t$ and $j_t = \rho_t$, so the coefficient of D_{bt} is $r_t - \rho_t$.

[55] Note that the exact distribution of deposits D_{bt} across banks is not uniquely determined, only the aggregate amount of deposits D_t consistent with (35). With reference to (31), this ensures that the earlier non-negativity constraint $M_{bt} \geq 0$ on liquid assets can be ignored without loss of generality.

which is a multiple $4(1-\lambda)$ of a generalized mean of $\rho_t - i_t$ and θ when fragility is positive. The term $1-\lambda$ captures the difference in liquidity of banks' assets A_t and M_t.

Conditional on the amount of credit A_t supplied by banks, the liquidity premium and other spreads are jointly determined by banks' aggregate demand for liquid assets and the supply M_t of these assets resulting from government policies. In the case $F_t > 0$, by aggregating equation (31) and using (37), banks' total demand for liquid assets is[56]

$$M_t = \frac{\sqrt{\theta}\left((1-\lambda)A_t - N_t\right)}{\sqrt{\rho_t - i_t}} - \lambda A_t, \tag{38}$$

noting that $(1-\lambda)A_t > N_t$ if and only if fragility is positive. The demand for liquidity is decreasing in the liquidity premium $\rho_t - i_t$, with a horizontal asymptote as $\rho_t - i_t$ approaches zero. The demand curve shifts to the right as bank holdings of illiquid assets A_t increase, and to the left if net worth N_t is higher. When $(1-\lambda)A_t \leq N_t$, which means fragility is non-positive, the liquidity premium must be zero, but (31) is consistent with any holdings of liquid assets, so the demand curve for M_t is horizontal at $\rho_t - i_t = 0$.

The supply curve is determined by government policy. The supply of M_t may be inelastic, or alternatively have some response to interest-rate spreads such as $\rho_t - i_t$ and $r_t - i_t$. Since spreads move together with the liquidity premium (36 and 37), the supply curve is represented as a relationship between M_t and the liquidity premium $\rho_t - i_t$:

$$M_t = M_t^* e^{\eta(\rho_t - i_t)}, \tag{39}$$

where η is the semi-elasticity of M_t with respect to the liquidity premium, and M_t^* is an exogenous variable capturing any other shifts in the supply of liquid assets.

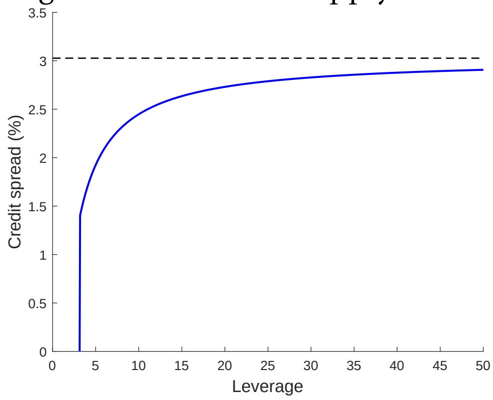
The credit supply curve. While individual banks can choose holdings of liquid assets, in equilibrium, the banking system must hold the liquidity M_t supplied by the government. Hence, banks' supply of deposits can be seen as determining the supply of credit A_t, taking as given M_t and aggregate net worth N_t. From equation (31):

$$A_t = \begin{cases} \frac{\left(\sqrt{r_t - i_t} - \sqrt{(1-\lambda)\theta}\right)M_t + \sqrt{(1-\lambda)\theta}N_t}{\sqrt{(1-\lambda)\theta} - \lambda\sqrt{r_t - i_t}} & \text{if } r_t - i_t > (1-\lambda)\theta \\ \frac{N_t}{1-\lambda} & \text{if } 0 < r_t - i_t \leq (1-\lambda)\theta \end{cases}. \tag{40}$$

This supply curve for credit implies that a higher credit spread $r_t - i_t$ induces banks to increase their leverage $1/n_t = (A_t + M_t)/N_t$, and thus increase their fragility according to (32). The credit supply curve is depicted in Figure 5 for an inelastic supply of liquid assets M_t. For low levels of the credit spread, the supply of credit is inelastic at the point where banks are not fragile and they pay the risk-free rate on deposits. Above a given credit spread, banks have an incentive to lever up and become fragile. In this region, the supply of credit is elastic. Increases in the supply of liquid assets expand credit supply in the fragile region, but they are irrelevant when banks are not fragile.

[56]This is derived by noting that (31) holds for aggregates because the coefficients are the same for all banks, and then substituting $D_t = A_t + M_t - N_t$ and $\sqrt{\frac{r_t - i_t}{(1-\lambda)\theta}} - 1 = \sqrt{\frac{\rho_t - i_t}{\theta}}$ from the formula in (37).

Figure 5: The credit supply curve.



Note 1: The credit spread is $r_t - i_t$ and leverage is $1/n_t$.
Note 2: Annualized calibrated parameter values from Table 2 are used.
Note 3: The dashed line is the spread at which credit supply is unlimited.

Dividend policy and net worth. Bank behaviour has been studied taking as given net worth N_{bt}. The remaining decision to analyse is the distribution of dividends Π_{bt}.

Since the coefficient on deposits D_{bt} in (34) is zero (or if not, deposits are zero), the static objective function W_{bt} from (29) is linear in net worth only:

$$W_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right) N_{bt}. \tag{41}$$

The evolution of net worth depends on the return on equity and banks' dividends. Defining $Q_{b,t+1}$ as the ex-post return on bank b's book equity between t and $t+1$:

$$Q_{b,t+1} = \frac{\Pi_{b,t+1} + (N_{b,t+1} - N_{bt})}{N_{bt}}, \quad \text{thus} \;\; Q_{b,t+1} = R_{t+1}\frac{A_{bt}}{N_{bt}} + i_t \frac{M_{bt}}{N_{bt}} - j_t \frac{D_{bt}}{N_{bt}}, \tag{42}$$

where the latter uses (24). The risk-adjusted expected return on bank b's book equity is $q_{bt} = \mathbb{E}_t[\Psi_{t+1} Q_{b,t+1}]/\mathbb{E}_t[\Psi_{t+1}]$ evaluated using the stochastic discount factor Ψ_{t+1} introduced earlier. The definition $W_{bt} = \mathbb{E}_t[\Psi_{t+1}(N_{b,t+1} + \Pi_{b,t+1})]/\mathbb{E}_t[\Psi_{t+1}]$ of the static objective function and (42) imply that $q_{bt} = (W_{bt} - N_{bt})/N_{bt}$, so bank behaviour analysed up to this point can be thought of as maximizing the risk-adjusted expected return on book equity conditional on initial net worth N_{bt}. Equation (41) shows this maximized expected return is the same for all banks, $q_t = q_{bt}$, and is given by

$$q_t = \frac{r_t - \lambda i_t}{1-\lambda}. \tag{43}$$

Bank b's actual objective function in choosing the path of dividends and other balance-sheet variables is the present value of current and future dividends discounted using the representative household's stochastic discount factor P_{ts}. This means maximizing $\Pi_{bt} + V_{bt}$, where V_{bt} is the present value of future dividends from (23).[57]

Optimization by banks implies the present value of future dividends V_{bt} is proportional to net worth N_{bt}, with the market-to-book ratio $v_t = V_{bt}/N_{bt}$ being common to all banks. The optimal choices of bank b's portfolio of assets and deposit creation also maximize the static objective function W_{bt} in (29) defined for some stochastic discount factor Ψ_{t+1}, hence the earlier analysis of bank behaviour correctly characterizes the

[57] The detailed solution is derived in the supplementary appendix, with the key results presented here.

solution to the full dynamic optimization problem. The appropriate stochastic discount factor Ψ_{t+1} modifies the representative-household stochastic discount factor $P_{t,t+1}$ between t and $t+1$ (from 23) depending on the future market-to-book ratio v_{t+1} of banks. An expression for Ψ_{t+1} and the expectational difference equation satisfied by v_t are

$$\Psi_{t+1} = \left(1 + \frac{v_{t+1} - 1}{1+\gamma}\right) P_{t,t+1}, \quad \text{and} \quad v_t = \left(\frac{1 + \frac{r_t - \lambda i_t}{1-\lambda}}{1+\rho_t}\right)\left(1 + \frac{\mathbb{E}_t\left[P_{t,t+1}(v_{t+1}-1)\right]}{(1+\gamma)\mathbb{E}_t[P_{t,t+1}]}\right). \tag{44}$$

One result is that the market-to-book ratio v_t is never lower than 1, and $v_t > 1$ implies the minimum dividend constraint (15) is binding. Using the ex-post return on book equity $Q_{b,t+1}$ from (42), the evolution of net worth if the constraint is binding at $t+1$ is

$$N_{b,t+1} = \left(\frac{1 + Q_{b,t+1}}{1+\gamma}\right) N_{bt}. \tag{45}$$

The minimum dividend constraint (15) binds when the parameter γ is sufficiently large. There is a range of γ values for which net worth converges to a positive steady state in the absence of shocks, and it is assumed parameters are in this range in the remainder of the paper.[58] Starting from that steady state, the minimum dividend constraint will always be binding for some bounds on the size of aggregate shocks.

5 Quantitative analysis

This section quantifies the importance of bank fragility in the transmission of shocks. The model is solved by log linearization around its non-stochastic steady state.[59]

5.1 Calibration

The banking sector of the economy is described by the three parameters, λ, θ, and γ. These are calibrated using information on the average liquidity premium, credit spread, and return on bank equity. The parameter β is calibrated using average interest rates. Parameters are chosen to match the model's implications for targeted variables in a non-stochastic steady state to the averages observed. The policy-determined supply of liquid assets consistent with the steady-state liquidity premium is inferred from the average capitalization ratio of banks. Finally, the other macroeconomic parameters α, δ, ϕ, and ψ are set to conventional values following the literature.

The model is calibrated to U.S. economy using data from 1991 up to the 2007–8 financial crisis. Data availability for banking variables determines the start of the sample in 1991Q3, and stopping in 2008Q4 accounts for the substantially different provision of liquidity after 2008 resulting from the many policy responses to the crisis.

The liquidity premium is defined with reference to the 3-month Treasury bill as the most liquid asset. The average T-bill yield over the period 1991Q3–2008Q4 is 3.7% in nominal terms. In the model, all interest rates are in real terms, so the average 2.2% rate

[58]The range of values of γ is analysed in appendix D.

[59]The steady state is studied in appendix D and the log linearization is in the supplementary appendix.

of inflation over the same period according to the personal consumption expenditure (PCE) price index is subtracted, leaving a real yield of 1.5%. The macroeconomic model is formulated in discrete time, and it is natural to align the length of one period with the 3-month maturity of the T-bill. The steady-state quarterly real interest rate on the liquid asset is i, so $i = 1.5\%/4$, where a variable without a time subscript denotes its non-stochastic steady-state value. The liquidity premium $\rho - i$ as measured by the 3-month GC repo rate minus the T-bill yield is 28 basis points on average, thus $\rho = i + 0.28\%/4$.

The credit spread $r - i$ for illiquid bank assets is proxied by the yield on Moody's seasoned Baa-rated corporate bonds over 10-year Treasuries, which is 2.2% annual, hence $r = i + 2.2\%/4$. In the model, the steady-state real return on bank equity coincides with the dividend-net worth ratio. Hence, the return on bank equity q is measured by the average ratio of cash dividends to equity for commercial banks covered by the Federal Deposit Insurance Corporation (FDIC), which is 8.4% annual, giving $q = 8.4\%/4$.[60]

Since $r = (1 - \lambda)q + \lambda i$ from (43), the parameter λ measuring the liquidity of bank assets is calibrated as $\lambda = (q - r)/(q - i)$. As the formula shows, a low value of λ arises if r is large relative to i because the illiquidity of assets makes it challenging for banks to supply credit without increasing fragility. The calibration targets imply $\lambda = 0.681$.

The parameter θ measuring the costs of bank failure for depositors is calibrated with information on ρ, i, and q. Using equations (37) and (43), $q - i = (\sqrt{\theta} + \sqrt{\rho - i})^2$, so θ is set as $\theta = (\sqrt{q - i} - \sqrt{\rho - i})^2$. High values of θ arise when the return on bank equity q is far above the risk-free interest rate ρ because a more severe credit friction increases spreads. The value resulting from the calibration targets is $\theta = 4.4\%/4$.

In a steady state where the return on bank equity exceeds the risk-free rate, the return on equity q is equal to the minimum fraction γ of equity distributed as dividends. This immediately implies $\gamma = 8.4\%/4$. In steady state, households' Euler equation from (20) implies the discount factor β satisfies $\beta = 1/(1 + \rho)$. With $\rho = 1.78\%/4$, the implied discount factor is $\beta = 0.996$. In summary, the calibration makes use of the following equations linking the model parameters to the targets:

$$\lambda = \frac{r - i}{q - i}, \quad \theta = \left(\sqrt{q - i} - \sqrt{\rho - i}\right)^2, \quad \gamma = q, \quad \text{and} \quad \beta = \frac{1}{1 + \rho}.$$

The targets are collected in Table 1, and the implied parameters are shown in Table 2.

The observed liquidity premium as the price of liquidity effectively pins down, along with the other spreads, the quantity of liquidity supplied in the steady state by the government. Using equation (32) for bank fragility, the steady-state liquidity ratio is

$$m = 1 - \left(\frac{q - i}{r - i}\right)\left(n + (1 - n)\sqrt{\frac{\rho - i}{q - i}}\right),$$

where n is the steady-state capital ratio. Using data on total equity capital and total

[60]The nominal return on book equity for FDIC banks is 11.6%, implying an annual real return of 9.4%, which is close to the dividend-equity ratio.

Table 1: Targets used to calibrate the parameters of the model.

Description	Notation	Value
Liquidity premium	$\rho - i$	0.28%/4
Credit spread	$r - i$	2.2%/4
Real return on bank equity	q	8.4%/4
Real Treasury Bill rate	i	1.5%/4
Bank capital ratio	n	8.8%

assets from the FDIC, the average bank capital ratio is 8.8%. This implies $m = 0.148$.

The macroeconomic parameters are set following the literature. The elasticity of intertemporal substitution ϕ is 1 and the Frisch elasticity of labour supply ψ is 3. The capital elasticity of output α is set to 1/3 to match the capital share of national income. The depreciation parameter δ is chosen to give a 7.5% annualized depreciation rate.

5.2 Results

Capital destruction shocks. We simulate the model to show the effects of a one-off capital destruction shock, that is, an unexpected negative shock to X_t. Formally, $X_t = 1 + v_t$, where v_t is a zero-mean i.i.d. shock with support on $[-\varsigma, \varsigma]$ for some $\varsigma > 0$. To begin with, we assume government policy is completely passive and the supply of liquid assets is not adjusted, that is, $\eta = 0$ and $M_t^* = M$ in (39). Impulse response functions of key macroeconomic and banking variables for 10 years after a 5% shock ($v_t = -0.05$) are shown as the red solid lines in Figure 6 labelled 'Banks'. Variables such as interest rates, spreads, and ratios are percentage-point deviations from steady state (annualized for interest rates and spreads), with 1 meaning 1 percentage point. All other variables are percentage deviations from steady state, with 1 denoting 1%.

As a point of comparison, consider an RBC model with the same macroeconomic features but no banking sector. In that model, households directly hold physical capital, but to make the steady states comparable, there is an exogenous but time-invariant spread between the risk-adjusted return on capital $\hat{r}_t$ and the risk-free rate ρ_t:

$$\hat{r}_t = \rho_t + (r - \rho), \quad \text{where } \hat{r}_t = \frac{\mathbb{E}_t[P_{t,t+1} R_{t+1}]}{\mathbb{E}_t[P_{t,t+1}]}, \tag{46}$$

and $r - \rho$ is the steady-state spread between r_t and ρ_t in the model with banks. This equation replaces banks' credit supply function (40), but all other equations for household and firm behaviour in the RBC model are also found in the banking model.

The responses of macroeconomic variables in the RBC model to the capital destruction shock are shown in Figure 6 as the blue dashed lines labelled 'RBC'. Note that spreads are either constant or absent from the RBC model, as are variables related to banks. The shock directly reduces the capital stock by 5%, which brings down GDP. The RBC model effectively captures the frictionless response to the shock, hence investment

Table 2: Calibrated parameters of the model.

Description	Notation	Value
Bank-asset liquidity relative to T-bills	λ	0.681
Loss given bank default	θ	4.4%/4
Minimum dividend distribution	γ	8.4%/4
Subjective discount factor	β	0.996
Elasticity of intertemporal substitution	ϕ	1
Frisch elasticity of labour supply	ψ	3
Capital elasticity of output	α	1/3
Depreciation	δ	7.5%/4
Steady-state liquidity ratio	m	0.148

rises so that the marginal product of capital is tied to the risk-free interest rate.

In the model with banks, the loss of some of the assets held by banks reduces their equity and capital ratios (leverage is countercyclical for this shock). Banks' fragility rises and this causes them to demand more liquid assets, pushing up the liquidity premium $\rho_t - i_t$ by 11 basis points. Greater fragility means banks must offer a higher interest rate on deposits to avoid runs, with the funding spread $j_t - \rho_t$ rising by 21 basis points. The increase in funding costs reduces banks' supply of credit, causing the credit spread $r_i - i_t$ to rise by 17 basis points. This results in less investment and a slower recovery of the capital stock compared to the RBC model. Consequently, GDP is lower and returns to its steady state at a slower rate. The amplification of the shock to GDP is quantitatively important, being around one third on impact and larger at longer horizons.

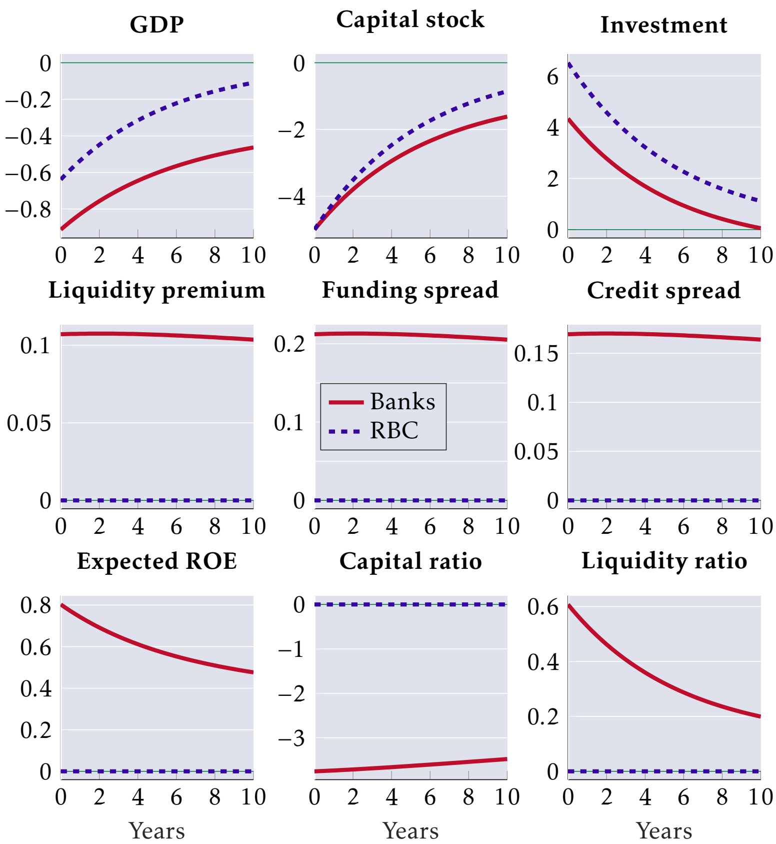
Endogenous persistence. One notable feature of the quantitative results is the slow return of variables to their steady states after a transitory shock. This endogenous persistence comes from the behaviour of net worth, which approaches its steady state only gradually. Using the expected return on equity $q_t = \mathbb{E}_t[Q_{b,t+1}]$ common to all banks, (45) implies the expected path of aggregate net worth is $\mathbb{E}_t N_{t+1} = (1+q_t)N_t/(1+\gamma)$. The low rate of convergence to the steady state is accounted for by q_t rising by a relatively small amount when banks' equity N_t falls after a shock.

Aggregating equation (42) for banks' returns on equity and balance sheets (25):

$$q_t = \rho_t + (r_t - \rho_t)\frac{(1-m_t)}{n_t} - (j_t - \rho_t)\frac{(1-n_t)}{n_t} - (\rho_t - i_t)\frac{m_t}{n_t}, \tag{47}$$

which shows that the difference between the expected return on bank equity q_t and the risk-free rate ρ_t can be decomposed into terms that depend on the spread between r_t and ρ_t, banks' funding spread $j_t - \rho_t$, and the liquidity premium $\rho_t - i_t$. These spreads are scaled by terms that depend on the aggregate bank capitalization and liquidity ratios $n_t = N_t/(A_t + M_t)$ and $m_t = M_t/(A_t + M_t)$.

Figure 6: Impulse response functions following a capital destruction shock.

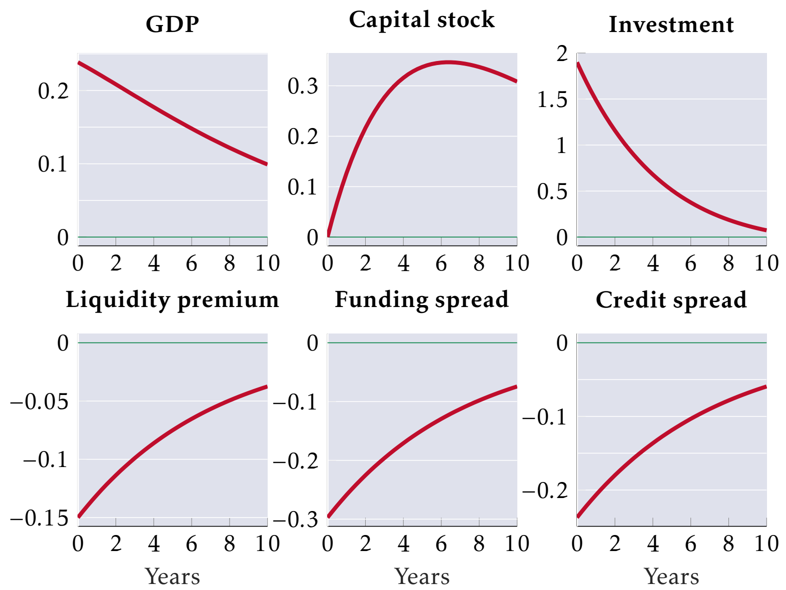


In an environment with no liquidity premium and where bank funding costs j_t are equal to the yield on government bonds i_t and the risk-free rate ρ_t, banks would generate an expected return on equity of $q_t = i_t + ((1-m_t)/n_t)(r_t - i_t)$. The credit spread $r_t - i_t$ is multiplied by a factor $(1-m_t)/n_t$, reflecting the magnifying effect of leverage ratio $1/n_t$ on the expected return r_t from a fraction $1-m_t$ of bank assets. With a lower supply of credit after a shock, the credit spread $r_t - i_t$ rises, which leads to a large increase in the return on equity given high bank leverage. Models that abstract from bank funding spreads and the liquidity premium therefore imply that equity can be rebuilt rapidly after a negative shock, limiting endogenous persistence.

When banks are fragile as in the model here, lower equity also means an increase in funding costs j_t, and a higher funding spread $j_t - \rho_t$ reduces their return on equity according to (47). The funding spread is multiplied by $(1-n_t)/n_t$, which is large given bank leverage. Moreover, since the demand for liquid assets increases, the liquidity premium $\rho_t - i_t$ rises, which is multiplied by m_t/n_t in (47), also reducing the return on equity through a lower overall return on banks' portfolio of assets. Taken together, these novel effects significantly reduce the rise in the expected return on bank equity after a capital destruction shock, resulting in a very high degree of endogenous persistence.

Liquidity shocks. The no-run constraint (11) implies that the quantity of liquid assets held by banks matters for their fragility in addition to their net worth. Since government

Figure 7: Impulse response functions following an expansion of liquid assets.



policies affect the aggregate supply of liquid assets, this opens up a channel through which fiscal or monetary policy can affect the banking system and the economy.

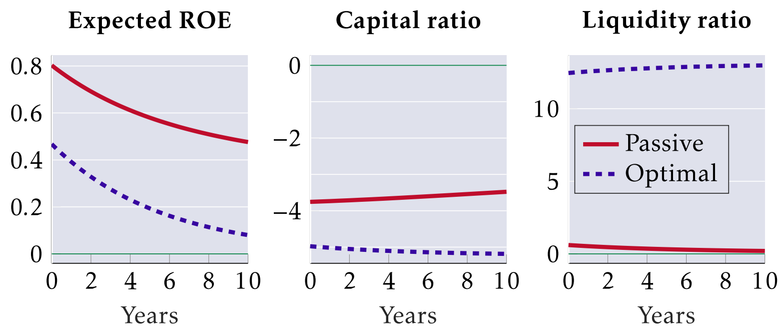
We simulate the effects of an increase in liquidity by considering an exogenous shift in policy such that there is an unexpected 15 basis points decline in the liquidity premium $l_t = \rho_t - i_t$ with a half-life of 5 years. Formally, the quantity of liquid assets M_t is adjusted to target a liquidity premium equal to l_t^*, with the deviation of l_t^* from the steady-state liquidity premium l following the AR(1) process $l_t^* - l = a(l_{t-1}^* - l) + v_t$, where v_t is a zero-mean i.i.d. shock with support $[-\varsigma, \varsigma]$.[61] The autoregressive parameter a is set so that the persistence of $l_t^* - l$ matches the 5-year half-life. The impulse response functions of macroeconomic variables and spreads are shown in Figure 7.

The reduction in fragility due to the expansion of liquidity causes banks' funding costs to fall and leads them to lever up. The funding spread falls by 30 basis points, and the credit spread by 24 basis points. There is a rise in investment, which boosts GDP. Observe that leverage is procyclical for liquidity shocks due to government policy.

Stabilizing the liquidity premium. We can also study the supply of liquidity as a systematic response to shocks. Optimal policy is discussed formally in section 6, but it is natural to think of an elastic response of liquid assets to accommodate changes in demand as desirable. Suppose the government supplies sufficient M_t to keep the liquidity premium $\rho_t - i_t$ constant at its initial steady state l after the capital destruction shock considered earlier. Since other spreads are linked to the liquidity premium (see 36 and 37), this policy also completely stabilizes the bank funding spread and credit spread by offsetting the effect of the shock on bank fragility. Consequently, to a first-

[61] This is implemented with a perfectly elastic supply of liquidity ($\eta \to \infty$ in 39). The supply of M_t is what is consistent with the aggregate demand for liquidity (38) at the target liquidity premium $\rho_t - i_t = l_t^*$.

Figure 8: Stabilizing the liquidity premium after a capital destruction shock.



order approximation, the response to the shock is now the same as in the benchmark RBC model (see 46), and the difference between a passive and elastic supply of liquidity can be seen by comparing the 'Banks' and 'RBC' impulse responses in Figure 6.

To stabilize spreads, the supply of liquid assets must increase significantly and persistently, with banks' liquidity ratio rising by 12 percentage points (Figure 8). The persistence is necessary because absent changes to spreads, bank equity does not recover.

6 Liquidity policy

This section studies the supply of liquidity from a normative perspective.

First best. As a benchmark, a social planner assigns equal consumption C_t and labour supply L_t to each household to maximize expected lifetime utility (18) subject only to resource constraints, namely the aggregate production function $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$, the constraint $C_t + I_t = Y_t$ on utilization of the economy's output, and the capital accumulation equation $K_{t+1} = X_{t+1}(I_t + (1-\delta)K_t)$. The first-order conditions of this problem are $(1-\alpha)Y_t/L_t = \chi C_t^{-\frac{1}{\phi}} L_t^{\frac{1}{\psi}}$ and $\beta \mathbb{E}_t\left[(C_{t+1}/C_t)^{-\frac{1}{\phi}} X_{t+1}(\alpha Y_{t+1}/K_{t+1} + 1 - \delta)\right] = 1$. Except for the final one, all of these constraints and first-order conditions are equilibrium conditions of the market economy with banks (see section 3 and equations 17 and 20).

To judge whether the planner's first-order condition with respect to capital holds in the economy with banks, consider the expected return $\hat{r}_t$ on physical capital, risk-adjusted using the representative household's stochastic discount factor $P_{t,t+1}$, as defined for the benchmark RBC economy in (46). The first best is attained in the market economy with banks if and only if $\hat{r}_t = \rho_t$, where ρ_t is the yield on an illiquid but risk-free bond.

The liquidity premium as a capital wedge. From the credit spread formula in (37) with positive fragility, $r_t - \rho_t = (1-\lambda)\left(\sqrt{\theta} + \sqrt{l_t}\right)^2 - l_t$, where $l_t = \rho_t - i_t$ is the liquidity premium and $r_t = \mathbb{E}_t[\Psi_{t,t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t,t+1}]$ is the risk-adjusted expected return on physical capital using banks' stochastic discount factor Ψ_{t+1} from (44). The spread between $r_t - \rho_t$ is increasing in the liquidity premium l_t. Taking a second-order approximation around a steady state with no aggregate risk, $\hat{r}_t \approx r_t$, and hence the wedge between $\hat{r}_t$ and ρ_t is approximately equal to $r_t - \rho_t$, which is increasing in the liquidity premium.

Therefore, in an economy with banks, a large liquidity premium acts as a wedge between the expected return on capital and households' discount rate. Government policy that increases the supply of liquidity and reduces the liquidity premium thus acts to move the economy closer to first best by reducing the size of the capital wedge.

Liquidity policy cannot implement the first best. While a lower liquidity premium improves efficiency, policies affecting liquidity cannot implement a first-best allocation of resources. Even if the liquidity premium were zero, the wedge $r_t - \rho_t$ remains positive (assuming bank net worth is scarce, so fragility is not negative, see 37). Furthermore, the shape of the aggregate demand curve for liquidity (38) shows the liquidity premium cannot be reduced to zero with any large but finite supply of liquid assets. Therefore, the capital wedge cannot be entirely eliminated by the government supplying liquidity.

Stabilizing spreads. While the capital wedge cannot be closed with liquidity policy, the government can stabilize the size of the wedge with an elastic supply of liquid assets. By adjusting M_t to target the steady-state positive liquidity premium l, banks' funding spread, the credit spread, and the capital wedge also remain at their steady-state levels (see 36 and 37). This generally requires permanent changes in the supply of liquidity following temporary shocks because the expected return on bank equity (43) is also held at its steady-state level, so bank equity does not revert to its mean after a shock.

Substitutability between liquidity and bank capital. The policy described above is based on there being some substitutability between liquid assets and bank capital in managing bank fragility. The credit supply curve (40) can be expressed equivalently as follows using (38) for a given supply of liquid assets M_t:

$$A_t = \frac{\sqrt{\rho_t - i_t} M_t + (1-\lambda)\sqrt{\theta} N_t}{(1-\lambda)\sqrt{\theta} - \lambda\sqrt{\rho_t - i_t}}.$$

If a shock changes net worth N_t, the adjustment of liquidity supply M_t needed to maintain the same supply of credit at the same liquidity premium and other spreads is

$$\left.\frac{\partial M_t}{\partial N_t}\right|_{\rho_t - i_t, A_t} = -\frac{(1-\lambda)\sqrt{\theta}}{\sqrt{\rho_t - i_t}}. \tag{48}$$

The required size of the liquidity response to falls in net worth is decreasing in $\rho_t - i_t$. When liquidity is abundant and the premium is low, a larger response of liquid assets is needed to stabilize spreads, reflecting a form of diminishing returns to liquidity.

Fiscal implications of liquidity policy. Iterating forwards the government's flow budget constraint (21), and using $\lim_{s\to\infty} \mathbb{E}_t[P_{ts}(M_s - B_s)] = 0$ implied by the transversality and no-Ponzi conditions, yields a present-value government budget constraint:

$$\sum_{s=t}^{\infty} \mathbb{E}_t[P_{ts} T_s] = (1+i_{t-1})M_{t-1} - (1+\rho_{t-1})B_{t-1} - \sum_{s=t}^{\infty} \mathbb{E}_t[P_{ts}\tau_s], \quad \text{where } \tau_t = \frac{(\rho_t - i_t)M_t}{1+\rho_t}.$$

This states that the present value of current and future taxes T_t must equal initial government liabilities $(1+i_{t-1})M_{t-1}$ net of initial government assets $(1+\rho_{t-1})B_{t-1}$, minus the present-value of the fiscal gain τ_t from the government's ability to supply liquid assets.[62] This fiscal gain derives from a positive liquidity premium $\rho_t - i_t$, which means the government is able to borrow at a lower rate than issuers of illiquid bonds.

Policies that reduce the liquidity premium $\rho_t - i_t$, which move the economy closer to first best, can have a fiscal cost. If the present value of τ_t falls, the present value of taxes T_t must increase to satisfy the government's budget constraint.[63]

The liquidity Laffer curve. Using the aggregate demand curve for liquid assets (38):

$$(\rho_t - i_t)M_t = \sqrt{\theta}\sqrt{\rho_t - i_t}((1-\lambda)A_t - N_t) - \lambda(\rho_t - i_t)A_t.$$

This shows that as M_t rises and moves $\rho_t - i_t$ towards zero, the fiscal gain τ_t to the government approaches zero. In other words, the elasticity of liquid asset demand with respect to the liquidity premium is less than one, so eventually a large enough supply of liquidity pushes the government's total fiscal gain towards zero. Higher government borrowing costs are thus a drawback of large expansions in the supply of liquid assets.

Note however that more liquidity does not necessarily mean lower fiscal gains. As M_t approaches zero, the liquidity premium $\rho_t - i_t$ rises, but only by a finite amount, which implies τ_t would also approach zero. Therefore, there is a 'Laffer curve' for the fiscal gains deriving from the government's supply of liquid assets.

Ex-ante versus ex-post liquidity supply. The liquidity policies studied so far are essentially changes in the supply of liquid assets held ex ante by banks before run risk materializes. But ex-post provision of liquidity can also be analysed in this framework.

Suppose the government or central bank offers a discount window facility whereby banks can exchange illiquid assets for liquid assets. Assume the central bank applies a 'haircut' λ_t^* at date t, where $\lambda_t^* > \lambda$.[64] All the analysis of section 2 and 4 goes through as before with the parameter λ replaced by the policy variable λ_t^*.[65]

Alternatively, suppose the government sets up a system of deposit insurance whereby those holding deposits at failing banks now suffer a loss θ_t^* smaller than θ.[66] This is analysed by replacing parameter θ with the policy variable θ_t^* in all equations.

With these new policy instruments, the aggregate supply of credit (40) is now

$$A_t = \frac{\sqrt{\rho_t - i_t}M_t + (1-\lambda_t^*)\sqrt{\theta_t^*}N_t}{(1-\lambda_t^*)\sqrt{\theta_t^*} - \lambda_t^*\sqrt{\rho_t - i_t}},$$

[62] Ricardian equivalence holds in respect of tax policy T_t and the government's supply or purchase of illiquid bonds B_t. Only policies that change M_t have an impact on the economy, and whatever combination of T_t and B_t is implemented to satisfy the government budget constraint does not matter.

[63] While this model has no distortions arising from (lump-sum) taxes T_t, more realistic representations of the tax system entail deadweight losses from increases in the government's fiscal needs.

[64] Implicitly, this facility is backed by the government's tax-raising powers at the final stage of period t. See the supplementary appendix on bank runs in the macroeconomic model for a formal treatment.

[65] Since banks still want to avoid runs, the facility is not used on the equilibrium path, but its presence affects the outcome of the coordination game.

[66] Again, backed by the government's tax-raising powers. The cost θ_t^* is paid at the beginning of $t+1$. On the equilibrium path, deposit insurance is not used, but its presence affects the coordination game.

which is increasing in λ_t^* and decreasing in θ_t^*. Hence, a rise in credit supply, or equivalently, a lower credit spread and liquidity premium, can also be achieved with more ex-post liquidity, that is, higher λ_t^* or lower θ_t^*, as well as more ex-ante liquidity M_t.

Since the ex-post liquidity facilities are not used on the equilibrium path, there is no direct change to the government's budget constraint, and $\tau_t = (\rho_t - i_t)M_t/(1+\rho_t)$ remains the fiscal gain derived from supplying liquid assets. But as all policies affect the equilibrium liquidity premium $\rho_t - i_t$, ex-post liquidity is not a free lunch for the government, even though the facilities are not used. Their availability reduces banks' desire to hold liquid assets ex ante, lowering the liquidity premium. By effectively raising the government's borrowing costs, ex-post liquidity policies have a fiscal cost.

Moreover, if the same reduction in $\rho_t - i_t$ is achieved through λ_t^* or θ_t^* without an increase in M_t, the reduction in τ_t is larger than when higher M_t is used to lower $\rho_t - i_t$. Thus, ex-post liquidity provision to reduce $\rho_t - i_t$ is more expensive to the government than the same change brought about through an expansion of liquidity ex ante.

7 Empirical analysis

In this section, we empirically test the key prediction that distinguishes our model from other macroeconomic models with financial frictions. Our model predicts that liquidity is an important factor in banks' ability to fund lending. Specifically, it predicts that an increase in the liquidity premium increases banks' funding spread.

Specification. Equation (36) in the model describes the equilibrium relationship between the liquidity premium and the funding spread. Linearizing and adding an error term ϵ_t to capture possible drivers of the funding spread outside the model:

$$\mathrm{FS}_t = \alpha + \beta\,\mathrm{LP}_t + \epsilon_t. \tag{49}$$

We allow the error term to be correlated with L lags of a data vector $\mathbf{y}_t$, to contain time fixed effects $\boldsymbol{d}_t$ and a linear trend.[67] Thus, we can re-write the empirical specification as

$$\mathrm{FS}_t = \alpha + \beta\,\mathrm{LP}_t + \sum_{l=1}^{L} \mathbf{y}_{t-l}^{\top}\zeta_l + \mathbf{d}_t^{\top}\boldsymbol{\eta} + \kappa t + \nu_t, \tag{50}$$

where ν_t is a stochastic innovation that is not autocorrelated but is potentially heteroskedastic. Vectors ζ_l and $\boldsymbol{\eta}$ and the scalar κ contain parameters.

Data. We include in the data vector $\mathbf{y}_t$ eleven variables at daily frequency with the first observation on 3 January 2006 and the last on 30 June 2023.[68] (1) The funding spread measured as the difference between 3-month LIBOR and the 3-month general-collateral (GC) repo rate. (2) The liquidity premium measured as the difference between the

[67] The data vector also contains the funding spread and liquidity premium.

[68] Before 2006, we do not have daily data on the dollar's trade-weighted exchange rate. The dataset's end date coincides with the final discontinuation date of LIBOR in the US. After merging the series, we are left with 4,157 observations over the period. Data sources are reported in appendix B.

3-month GC repo rate and the 3-month T-bill rate.[69] (3) The log-transformed quantity of outstanding treasuries. (4) The log-transformed balance on the Treasury General Account. (5) The spread between Moody's seasoned Baa corporate bond yield and the 10-year treasury yield. (6) The log-transformed value of the S&P 500 stockmarket index. (7) The log-transformed value of the S&P 500 financials stockmarket index. (8) The log-transformed VIX. (9) The level of the 3-month GC repo rate. (10) The level of the 10-year treasury yield. (11) The trade-weighted exchange rate of the US dollar. We set $L = 80$ to ensure we control for at least one quarter of data as lags. Our vector $\mathbf{d}_t$ includes time dummies for (1) weekdays, (2) days of the month, (3) months, and (4) NBER recessions. The linear time trend does not allow for gaps in the observed dates.[70]

Identification. The econometric challenge is to find exogenous variation in the liquidity premium to estimate our coefficient of interest β. Because of omitted variables, measurement error and reverse causality, OLS estimates are unlikely to be consistent. For example, it is possible that unobserved shocks to uncertainty are driving both the funding spread and the liquidity premium. Or perhaps the GC repo rate is a noisy measure of the risk-free rate, and measurement error is driving a correlation between the measured liquidity premium and funding spread. It is also possible that shocks to the funding spread are driving demand for liquidity and thus the liquidity premium.[71]

Our identification strategy is to instrument the liquidity premium with the quantity of outstanding treasury debt. The quantity of treasuries is relevant to the liquidity premium as shown in a vast literature studying the convenience yield on treasuries (Krishnamurthy and Li, 2023), and we confirm its relevance in the first-stage regression.

As for the instrument's validity, treasury debt is issued a few days after it is auctioned with a median lag of three days.[72] This institutional feature makes outstanding treasury debt predetermined at daily frequency. This rules out confounding variables in the error term ν_t that would make it invalid. It also rules out reverse causality.

Another threat to the instrument's validity are alternative mechanisms through which the quantity of treasuries affects the funding spread for a given liquidity premium. We can assuage this concern by noting that an implication of outstanding treasuries being predetermined at daily frequency is that they are perfectly anticipated. In other words, there is no new information revealed when treasuries are issued and mature. All the information, for instance regarding fiscal policy, is revealed at the latest during the auction. This rules out a direct information effect of the quantity of treasuries.

[69] Our adopted measure of the liquidity premium is standard in the literature (Nagel, 2016; Krishnamurthy and Li, 2023). The funding spread is the difference between the rate at which banks can borrow without collateral and the risk-free rate as measured by the GC repo rate.

[70] On average, our dataset contains 59 observations per quarter, nearly the universe of business days.

[71] The results from an OLS regression, reported in the supplementary appendix, are consistent with measurement error in the risk-free rate as a driver of endogeneity.

[72] Data on time from auction to issuance are reported in the supplementary appendix.

Table 3: Regression table.

	Funding spread
Liquidity premium	0.99**
	(0.45)
Lags	Y
Time dummies	Y
Linear trend	Y
R-squared	97%
Observations	4,077
1st-stage F statistic	15

Note 1: Outstanding treasuries as external instrument.
Note 2: Heteroskedasticity-consistent standard errors in parentheses.
Note 3: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Finally, treasury debt is a highly persistent variable. To rule out a persistent omitted variable driving both treasury debt and the funding spread, it is important that the controls included in the regression succeed in removing the autocorrelation from the residual. For that, a rich lag structure is needed. Suppose we omitted lags of an element of the true data vector $\mathbf{y}_t$ from the analysis. Then, the residual would contain the omitted lags as well as the stochastic innovation. If in addition to driving the funding spread the omitted lags are also driving treasury debt, because for instance they drive fiscal policy, then the instrument is no longer valid.[73] As described above, we include as controls 80 lags of eleven variables available at daily frequency. As a result, the estimated residuals are not autocorrelated.[74]

Key result. Table 3 contains the results of the benchmark IV regression. An exogenous one basis-point increase in the liquidity premium increases banks' funding spread by 1 basis point.[75] The effect is robustly significant with a p-value of 2.8%.[76]

The instrument is highly relevant as confirmed by the first-stage F statistic of 15. In the first-stage regression, we find that a one-percent increase in treasuries reduces the liquidity premium by 2.1 basis points (p-value is 0.3%). The direction is consistent with a movement along the downward-sloping demand for treasuries.

To check the robustness of the results, we look for evidence of state-dependence in the effect of the liquidity premium on the funding spread. We add as regressor

[73]For example, the policymaker could use private information available to him at the auction date to anticipate the funding spread on the issuance date. If the policymaker used this private information to stabilize the funding spread with their treasury issuance, then our estimates would be biased downwards.

[74]We report the partial autocorrelation function of the error term in the supplementary appendix.

[75]The effect in the calibrated model is 2 basis points, which is in the 99% confidence interval.

[76]We use heteroskedasticity-consistent standard errors although a Pagan-Hall general test overwhelmingly fails to reject homoskedasticity of the residuals (the test's p-value is 100%). With regular standard errors, the p-value is 0.4%.

an interaction term of the liquidity premium with the recession dummy to see to what extent the effect differs according to the state of the economy. As an additional instrument, we use the interaction of treasuries with the recession dummy. As reported in Table 4 in appendix A, the effect of the liquidity premium on the funding spread in recessions is not significantly different from the same effect in expansions.

In Table 4 in appendix A, we check alternative specifications and find that excluding the time dummies or the lag structure does not affect the results.[77]

8 Conclusion

This paper has developed a novel financial friction based on coordination failure in the market for bank deposits. The friction implies that fragile banks borrow on worse terms. Liquid-asset holdings and net worth are substitutable factors that keep banks' fragility in check. Hence, when net worth is scarce, banks demand more liquid assets. Introducing this friction in a canonical macroeconomic model, we have found that the model matches the positive correlation of the liquidity premium with indicators of financial stress. This is a fact that current macroeconomic models with financial frictions do not speak to. Moreover, the model has a role for policy to adjust the supply of liquid assets and thus stabilize the economy. Empirically, we have tested a key prediction of the model: a high liquidity premium leads to high funding costs for banks. Exploiting exogenous variation in the liquidity premium at daily frequency due to predetermined changes in the supply of treasuries, we find a robustly-significant positive effect. The corresponding effect in the calibrated model is within the 99% confidence interval of the empirical estimate.

The paper provides a quantitative framework to understand and evaluate policies that change the quantity of liquid assets in the economy. A case in point is quantitative easing, as enacted in response to the financial disruptions of the global financial crisis. The current generation of macroeconomic models largely appraise such policy as a credit policy: QE is effective because the central bank makes loans that banks cannot make on account of a binding leverage constraint. In this paper's framework, the real effects of QE stem from the liability-side of the central-bank balance sheet regardless of its asset holdings. Lots of liquid reserves on banks' balance sheets make creditors willing to lend to banks at more favourable conditions. The two effects are not exclusive. Hence, there is scope for studying moral-hazard and coordination frictions together for a rounder account of central-bank balance-sheet policies. More generally, the interaction of liquid-asset supply with other policy levers warrants further investigation. For this, the introduction of additional frictions from the literature, such as distortionary taxes or nominal rigidities, is necessary.

[77] In the supplementary appendix, we also report the estimate of interest for different numbers of lags.

A Figures

Figure 9: Pandemic and tightening cycle.

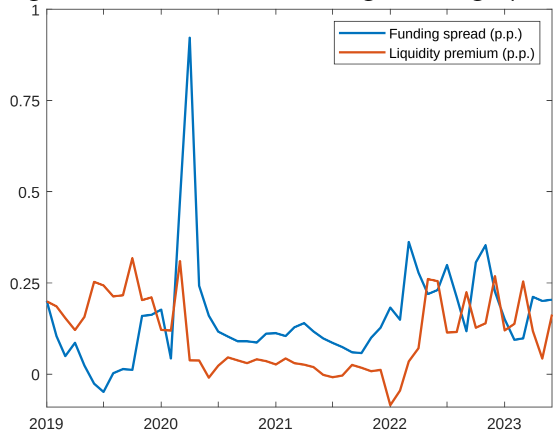
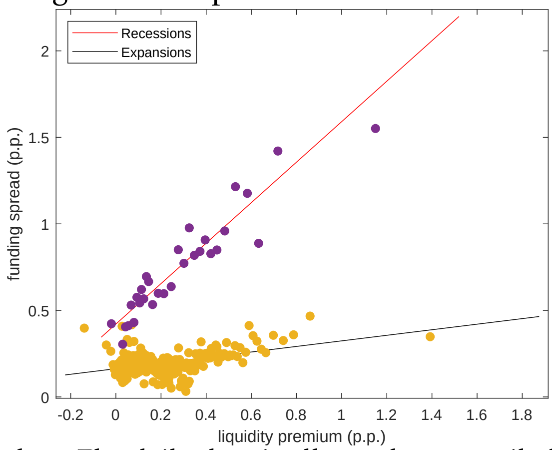


Figure 10: Expansions vs recessions.



Note: Scatterplot of binned data. The daily data is allocated to quantile-based bins according to the liquidity premium. There are 270 bins for expansions and 30 bins for recessions.

Table 4: IV with alternative specifications

Funding spread	IV	IV	IV	IV	IV
Liquidity premium	1.4	1.0**	0.31***	1.28***	0.99**
	(1.0)	(0.48)	(0.04)	(0.06)	(0.45)
Liquidity premium × Recession	-0.54				
	(1.0)				
Lags	Y	Y	N	N	Y
Time dummies	Y	N	Y	N	Y
Linear trend	Y	Y	Y	Y	Y
R-squared	96%	96%	57%	17%	97%
Observations	4077	4077	4157	4157	4077
1^{st}-stage F statistic	3.9	13	1560	1823	15

Note 1: IV estimation uses outstanding treasuries as external instrument.
Note 2: In first column, outstanding treasuries × recession used as additional external instrument.
Note 3: Heteroskedasticity-consistent standard errors in parentheses.
Note 4: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

B Data sources

We obtain the 3-month GC repo rate (mid-price from ticker ”USRGCGC ICUS Currncy”) and the 3-month LIBOR from Bloomberg. Daily data on quantity of outstanding treasuries (series ”Debt held by the public” in dataset ”Debt to the Penny”) and on the TGA closing balance (series ”Treasury General Account (TGA) Closing Balance” in dataset ”Daily Treasury Statement (DTS)”) is available on the website Fiscaldata maintained by the US Treasury Department. From the website FRED maintained by the Federal Reserve Bank of St. Louis, we retrieve the 3-month T-bill rate (series ”DTB3”), the spread between Moody's seasoned Baa corporate bond yield and the 10-year treasury rate (series ”BAA10Y”), the 10-year treasury rate (series ”DGS10”), the VIX (series ”VIXCLS”), and the nominal broad US dollar index (series ”DTWEXBGS”). The closing values of the S&P 500 stockmarket index and the S&P 500 financials stockmarket index are downloaded from the website Yahoo! Finance.

C Proofs

Proof of Lemma 1. A strategy in the coordination game is a correspondence that maps a household's signal $\hat{F}_{bh}$ into the deposit-holding decision H_{bh}.

Consider other households playing the same threshold strategy such that they hold a bank's deposits with $H_{bh} = 1$ if they receive signal $\hat{F}_{bh} \leq k_b$ and do not hold the deposits otherwise. Given household h's (improper) uniform prior and signal $\hat{F}_{bh}$ about bank fragility, its expected net payoff of holding deposits can be written as

$$\tilde{\pi}^* \left(\hat{F}_{bh}, k_b\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\pi}\left(\hat{F}_{bh}, k_b, \Sigma_{bh}, \Omega_b\right) g_{\Sigma}(\Sigma_{bh}) g_{\Omega}(\Omega_b)\, \mathrm{d}\Sigma_{bh}\, \mathrm{d}\Omega_b, \tag{51}$$

where $\tilde{\pi}(\hat{F}_{bh}, k_b, \Sigma_{bh}, \Omega_b)$ is the net payoff from holding deposits given the unknown noise and $g_x(\cdot)$ is a general pdf for a random variable x.

Conditional on the noise, the share of households holding the deposits of bank b is $H_b = G_{\Sigma}\left(k_b - \hat{F}_{bh} + \Sigma_{bh}\right)$. Together with the definition of bank failure (3), this result implies that we can write a condition for Σ_{bh} such that the bank fails if and only if $\Sigma_{bh} < \underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)$ with $\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b)$ solving the following implicit equation:

$$\underline{\Sigma} = \hat{F}_{bh} - \Omega_b - G_{\Sigma}\left(k_b - \hat{F}_{bh} + \underline{\Sigma}\right). \tag{52}$$

Importantly, the solution to this equation is unique because the left-hand side is continuous and increasing in $\underline{\Sigma}$, while the right-hand side is continuous and decreasing in $\underline{\Sigma}$. We can now re-write the expected net payoff from holding deposits as

$$\tilde{\pi}^* \left(\hat{F}_{bh}, k_b\right) = \mathbb{E}\left\{G_{\Sigma}\left[\underline{\Sigma}\left(\Omega_b, \hat{F}_{bh}, k_b\right)\right]\right\}(-\theta) + \left(1 - \mathbb{E}\left\{G_{\Sigma}\left[\underline{\Sigma}\left(\Omega_b, \hat{F}_{bh}, k_b\right)\right]\right\}\right)(j_b - \rho), \tag{53}$$

where the expectation is taken with respect to the unknown systematic noise Ω_b.

Now, we start iteratively to delete dominated strategies. First, we consider a strategy of holding deposits if and only if $\hat{F}_{bh} \leq \underline{F}$ with $\underline{F}$ large enough. This strategy implies holding when $\hat{F}_{bh} \to +\infty$ and in this case we have that $\tilde{\pi}^* \left(\hat{F}_{bh}, k_b\right) = -\theta$ for any k_b. Hence,

this is a dominated strategy and no household will play it.
We can extend this logic by studying $\hat{\tilde{\pi}}^*(z) = \tilde{\pi}^*(z,z)$. If other households play threshold strategy z, does household h have an incentive to hold if it receives $\hat{F}_{bh} = z$? Without loss of generality, we consider threshold strategies for other households because, due to strategic complementarities, they are the non-dominated strategy that makes household h's expected net payoff from holding deposits highest. In other words, given that holding deposits for $\hat{F}_{bh} > z$ is dominated if other households play threshold strategy z and household h is better off not holding for $\hat{F}_{bh} = z$, then holding deposits for $\hat{F}_{bh} = z$ is also dominated. Function $\hat{\tilde{\pi}}^*$ is monotonically decreasing and crosses zero once at z^* with

$$j_b - \rho = \frac{\mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\}}{1 - \mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\}}\theta. \tag{54}$$

With this, we can delete as dominated strategies such that a household holds deposits with $\hat{F}_{bh} > z^*$. We can apply this analysis in reverse to delete as dominated all strategies that set $H_{bh} = 0$ for $\hat{F}_{bh} < z^*$. Hence, we are left with a unique equilibrium strategy.
Furthermore, using the fact that idiosyncratic noise Σ_{bh} follows a uniform distribution $U[-\sigma, \sigma]$, we obtain

$$\underline{\Sigma}(\Omega_b, \hat{F}_{bh}, k_b) = \begin{cases} \hat{F}_{bh} - \Omega_b & \text{if } \Omega_b > k_b + \sigma, \\ \frac{(1+2\sigma)\hat{F}_{bh} - 2\sigma\Omega_b - k_b - \sigma}{1+2\sigma} & \text{if } \Omega_b \in (k_b - 1 - \sigma, k_b + \sigma], \\ \hat{F}_{bh} - \Omega_b - 1 & \text{otherwise.} \end{cases} \tag{55}$$

Because Ω_b is uniformly distributed, we have that

$$\mathbb{E}\{G_\Sigma[\underline{\Sigma}(\Omega_b, z^*, z^*)]\} =$$
$$= \begin{cases} 0 & \text{if } z^* \le -\omega - \sigma, \\ \frac{z^*+\sigma+\omega}{2\omega}\frac{z^*+\sigma}{1+2\sigma} - \frac{\sigma}{2\omega(1+2\sigma)}\left[(z^*+\sigma)^2 - \omega^2\right] & \text{if } z^* \in (-\omega-\sigma, \omega-\sigma], \\ \frac{z^*+\sigma}{1+2\sigma} & \text{if } z^* \in (\omega-\sigma, 1-\omega+\sigma], \\ \frac{z^*-1+\omega-\sigma}{2\omega} + \frac{\omega-z^*+1+\sigma}{2\omega}\frac{z^*+\sigma}{1+2\sigma} + \frac{\sigma}{2\omega(1+2\sigma)}\left[\omega^2 - (z^*-1-\sigma)^2\right] & \text{if } z^* \in (1-\omega+\sigma, 1+\sigma+\omega], \\ 1 & \text{otherwise.} \end{cases} \tag{56}$$

In combination with equation (54), this pins down the equilibrium threshold z^* with finite variances of both the idiosyncratic and systematic noise.
Finally, under $\omega \to 0$ we find that the strategy played by all households in the unique Bayesian Nash equilibrium of the coordination game is

$$H^*_{bh} = \begin{cases} 1 & \text{if } \hat{F}_{bh} \ge \frac{j_b-\rho}{j_b-\rho+\theta} + \frac{j_b-\rho-\theta}{j_b-\rho+\theta}\sigma, \\ 0 & \text{otherwise.} \end{cases} \tag{57}$$

□

Proof of Proposition 1. The probability that no one runs given F_b and j_b is given by the probability that even the household drawing the highest signal is (weakly) below the run threshold:

$$\mathbb{P}[H_b = 1] = \mathbb{P}\left[F_b + \sigma + \Omega_b \leq F_b^*\right] = \mathbb{P}\left[\Omega_b \leq F_b^* - \sigma - F_b\right]. \tag{58}$$

This is the cdf of systematic noise Ω_b evaluated at $F_b^* - \sigma - F_b$. Hence, $\mathbb{P}[H_b = 1] = \varkappa_b$ where $\varkappa_b$ is given by equation (9).

Now we prove that $\mathbb{P}[H_b = 0] = 1 - \mathbb{P}[H_b = 1]$. Partial runs with $H_b \in (0, 1)$ imply that the household receiving the lowest signal holds deposits while the household receiving the highest signal does not. This event has probability $\mathbb{P}\left[F_b - \sigma + \Omega_b \leq F_b^* \cap F_b + \sigma + \Omega_b > F_b^*\right]$, which we can re-write as

$$\mathbb{P}\left\{\Omega_b \in \left[F_b^* - F_b - \sigma, F_b^* - F_b + \sigma\right)\right\} = G_\Omega\left(F_b^* - F_b + \sigma\right) - G_\Omega\left(F_b^* - F_b - \sigma\right) \leq \frac{\sigma}{\omega}, \tag{59}$$

where $G_\Omega(\cdot)$ is the cdf of random variable Ω_b. For $\sigma/\omega \to 0$, the proposition holds. □

Proof of Proposition 2. If a household has the belief that all households hold deposits with certainty $\mathbb{P}_{bh}[H_b = 1] = 1$ and that a bank's fragility is not greater than one $\mathbb{P}_{bh}[F_b \leq 1] = 1$, then by the condition that determines bank failure (3), the household must also believe that the bank does not fail $\mathbb{P}_{bh}[\Phi_b = 1] = 0$.

A household is sure that all households hold, that is, $H_b = 1$, if $\hat{F}_{bh} \leq F_b^* - 2\sigma$. And a household is sure that a bank's fragility F_b is smaller than one if $\hat{F}_{bh} \leq 1 - \sigma - \omega$. This condition holds for all possible signals under $F_b \leq 1 - 2(\sigma + \omega)$. Hence, the sufficient condition for household h to believe for sure that $\Phi_b = 0$ is simply given by $\hat{F}_{bh} \leq F_b^* - 2\sigma$. With this, we can write that for a given fragility F_b:

$$\mathbb{P}\left[\mathbb{P}_{bh}[\Phi_b = 1] = 0\right] \geq \mathbb{P}\left[\hat{F}_{bh} \leq F_b^* - 2\sigma\right] = \mathbb{P}\left[F_b + \Sigma_{bh} + \Omega_b \leq F_b^* - 2\sigma\right]. \tag{60}$$

Proposition 1 implies that a necessary and sufficient condition for $\mathbb{P}[H_b = 1] = 1$ is $F_b \leq F_b^* - \sigma - \omega$. Substituting this into the equation above, we obtain

$$\mathbb{P}\left[\mathbb{P}_{bh}[\Phi_b = 1] = 0\right] \geq \mathbb{P}\left[\Sigma_{bh} + \Omega_b \leq \omega - \sigma\right]. \tag{61}$$

Under $\sigma/\omega \leq 1$, we can compute the right-hand side as $1 - \sigma/(2\omega)$. Hence, if $\sigma/\omega \to 0$, then the proposition holds. □

D Steady state

In this section, we analyse the long-run dynamics of the model by studying its steady state. The model's steady state is a constant sequence for prices and quantities that satisfies the model's equilibrium conditions.

We look for a steady state with a strictly positive liquidity premium $\rho - i > 0$ and bank net worth $N > 0$. Combining equations (37) and (43), we obtain

$$q - \rho = \theta + 2\sqrt{\theta(\rho - i)} > 0. \tag{62}$$

Evaluating the formula for the banks' market-to-book ratio (44) in steady state, we obtain

$$v = \frac{\gamma(1+q)}{(1+\gamma)(1+\rho) - (1+q)} > 1, \tag{63}$$

which implies that the minimum dividend constraint is binding in steady state so that $\Pi = \gamma N$. Together with the law of motion for banks' net worth in (45), a binding minimum dividend constraint implies that in a steady state

$$q = \gamma \tag{64}$$

for $N > 0$.[78] First, we notice from (62) that the parametric restriction $\gamma > \rho + \theta$ is necessary for (64) to be sustained with a strictly positive liquidity premium.[79] Under this restriction, we pin down the steady-state liquidity premium as

$$\rho - i = \frac{[\gamma - (\rho + \theta)]^2}{4\theta}. \tag{65}$$

This liquidity premium creates the right level of returns on bank net worth so that bank net worth is stable. Interestingly, it is independent of policy. Also, (37) implies

$$r - i = 4(1-\lambda)\left(\frac{1}{2}\sqrt{\theta} + \frac{1}{2}\sqrt{\rho - i}\right)^2. \tag{66}$$

Moreover, we can determine the steady-state balance-sheet structure of banks with equations (25), (26) and (36) as

$$N = \left[1 - \lambda - \frac{\gamma - (\rho + \theta)}{2\theta}\left(\lambda + \frac{M}{K}\right)\right]K. \tag{67}$$

To have positive net worth in steady state, we need to restrict the equity friction with

$$\gamma \leq \rho + \theta\frac{2-\lambda}{\lambda} \tag{68}$$

and policy with

$$M < \left[\frac{2\theta(1-\lambda)}{\gamma - (\rho + \theta)} - \lambda\right]K. \tag{69}$$

An excessively strong equity friction makes it impossible to sustain positive net worth in steady state even with no liquidity. Excessively large liquid-asset supply rules out a fragile steady state with positive liquidity premium for any positive level of net worth.

The key finding that in the long run liquidity policy has no effect on the liquidity premium, and thus fragility, is due to the endogenous structure of banks' balance sheet. Increases in liquid-asset supply crowd out bank net worth in the long run to the point where fragility is unchanged.

As is standard in a real business cycle model, the steady-state risk-free rate is pinned down by the Euler equation in (20) as $\rho = (1-\beta)/\beta$ and the steady-state level of capital is the unique strictly-positive solution to the system of equations given by

$$(1-\alpha)K^{\alpha\left(1+\frac{1}{\phi}\right)} = L^{\alpha+\frac{1}{\psi}}\left(ZL^{1-\alpha} - \delta\right)^{-\frac{1}{\phi}} \tag{70}$$

and

$$K = \left(\frac{\alpha}{r-\delta}\right)^{\frac{1}{1-\alpha}}L. \tag{71}$$

[78] The upper limit on M identified below rules out $N = 0$ and $q < \gamma$ in the steady state with strictly positive liquidity premium.

[79] If this is violated, then net worth grows up to the point where there is no fragility and the liquidity premium is zero.

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