To Own or to Rent? The Effects of Transaction Taxes on Housing Markets*

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Abstract

Using sales and leasing data, this paper finds three novel effects of a higher property transaction tax: higher buy-to-rent transactions alongside lower buy-to-own transactions despite both being taxed, a lower sales-to-leases ratio, and a lower price-to-rent ratio. This paper explains these facts by developing a search model with entry of investors and households, households choosing to own or rent in the presence of credit frictions, and homeowners deciding when to move house. A higher transaction tax reduces homeowners' mobility and increases demand for rental properties, which explains the empirical facts and leads to a lower homeownership rate. The deadweight loss is large at 111% of tax revenue, with more than half of this due to distorting decisions to own or rent.

JEL CLASSIFICATIONS: D83; E22; R21; R28; R31.

KEYWORDS: rental market, buy-to-rent investors, homeownership rate, transaction taxes.

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1 Introduction

Real-estate transaction taxes are a common feature of tax systems around the world. A large and growing literature points to the distorting effects of such taxes on owner occupiers.[1] However, little is known about the implications of transaction taxes for households' tenure choices and landlords' investment decisions, which jointly determine the allocation of properties between the markets for ownership and rentals, and hence the homeownership rate. At least a third of the housing stock is allocated to rental markets, and the homeownership rate is the focus of many policy debates.[2] This paper offers a comprehensive understanding of the impact of transaction taxes on households' decisions along both the intensive margin (moving and transacting) and the extensive margin (owning or renting), and on investors' decisions to buy property.

The paper makes two contributions to the literature. Empirically, it documents the different way buy-to-rent investors respond to a transaction tax compared to owner-occupiers, even when the tax applies to both, and the relative effects of the tax on markets for property ownership and rentals as measured by the leases-to-sales and price-to-rent ratios. These facts demonstrate the importance of considering the extensive margin and entry of investors. Theoretically, to explain the new facts, the paper develops and quantifies a model of housing with both a rental and an ownership market subject to search and credit frictions. The model features housing tenure decisions across the two markets, endogenous moving decisions within the ownership market, and entry decisions by investors and households.

The new empirical evidence comes from using a unique dataset of Multiple Listing Service records on housing sales and leasing transactions for the Greater Toronto Area (GTA) between 2006 and 2018. With observations of leases and rents in addition to sales and prices, the data make it possible to examine both owner-occupied and rental markets and to distinguish purchases made by buy-to-rent investors from those of owner-occupiers.

In 2008, the City of Toronto introduced a new city-level transaction tax, known in Canada as Land Transfer Tax (LTT). Importantly, the new tax covered only the City of Toronto but not other parts of the GTA. This makes it possible to estimate the effects of the tax by comparing housing transactions and homeowner mobility before and after the new LTT across neighbourhoods that are adjacent to but on opposite sides of the city border. The counterfactual is supported by evidence showing that homes on opposite sides of the border are similar in their

[1] This includes, but is not limited to, Benjamin, Coulson and Yang (1993), Slemrod, Weber and Shan (2017), and Kopczuk and Munroe (2015) for the US, Besley, Meads and Surico (2014), Hilber and Lyytikäinen (2017), and Best and Kleven (2018) for the UK, Dachis, Duranton and Turner (2012) for Canada, Eerola, Harjunen, Lyytikäinen and Saarimaa (2021) and Määttänen and Terviö (2022) for Finland, Fritzsche and Vandrei (2019) for Germany, Davidoff and Leigh (2013) for Australia, Van Ommeren and Van Leuvensteijn (2005) for the Netherlands, Agarwal, Chau, Hu and Wan (2022) for Hong Kong, and Huang, Li and Yang (2021) for Singapore.

[2] See Gabriel and Rosenthal (2015) and Goodman and Mayer (2018). See also the literature that seeks to understand changes in homeownership rates, such as Chambers, Garriga and Schlagenhauf (2009), Fisher and Gervais (2011), and Floetotto, Kirker and Stroebel (2016), and the literature on the extent of flows between the rental market and owner-occupation (Glaeser and Gyourko, 2007, Bachmann and Cooper, 2014, Greenwald and Guren, 2021).

attributes. For the years spanning the tax change and in neighbourhoods along the city border, the tax effects on transactions in rental and owner-occupied markets and on the price-to-rent ratio are estimated using monthly data at the neighbourhood level. The tax effects on mobility are estimated at the household level.

The estimations yield three novel facts about the effects of transaction taxes. For ownership-market transactions, a 1.3 percentage points higher effective LTT rate causes purchases made by buy-to-rent investors to increase by 9.3%. By definition, buy-to-rent investors are those who acquire properties from the ownership market and make them available in the rental market. The increase in buy-to-rent transactions is in stark contrast to the 9.6% fall in owner-occupier purchases, even though the LTT applies to both. Across the ownership and rental markets, the LTT causes the ratio of leases to sales to rise by 26%, and the ratio of prices to rents to decline by 3.8%. Together, these findings are consistent with the recent fall in the homeownership rate in Toronto.[3] They shed new light on the consequences of transaction taxes. The heterogeneous treatment effects of the LTT on sales versus leases and on home-buyers versus investors indicate that a careful evaluation of transaction taxes must consider flows of properties between the owner-occupied market and the rental market.

This paper develops and calibrates a search model that incorporates the economic forces highlighted by these new findings with the goal of better understanding housing-market transaction taxes. There is strong evidence on the importance of search frictions in the housing market, such as the time taken to sell or buy, and the number of property viewings occurring with a sale or purchase (Genesove and Han, 2012, Ngai and Sheedy, 2020).[4] The prevalence of real-estate agents as middlemen is another piece of evidence pointing to the importance of search frictions. Thus, a search approach is a natural starting point, and is adopted here including both frictions in locating properties to view and ex-ante uncertainty about match quality.

The model allows for endogenous population flows between two regions, the city and elsewhere, which aligns with the empirical strategy of comparing transactions on opposite sides of the city border. To analyse jointly the ownership and rental markets, the model features households who choose in which market to participate, subject to paying a credit cost to access the market for property ownership. These credit costs represent the costs of mortgage financing or the difficulty of obtaining credit, which are heterogeneous across households. Setting the benefits of homeownership against its costs gives rise to an entry decision on the 'buy' side of the rental market. On the 'sell' side, there is free entry of buy-to-rent investors. The equilibrium homeownership rate is the one consistent with the behaviour of both households and investors.

Along the extensive margin, the model predicts the LTT leads simultaneously to a decrease in purchases by owner-occupiers and an increase in buy-to-rent purchases and leases. The explanation for this hinges on the difference between home-buyers and investors. Owing to

[3]The homeownership rate, defined as the fraction of properties that are lived in by their owners, is reported by Statistics Canada only at a five-year frequency. In Toronto, it steadily increased from 51% to 54.5% between 1996 and 2006, followed by a gradual decline to 52.3% in 2016.

[4]This paper documents similar patterns for the Toronto housing markets.

idiosyncratic match-quality shocks and the indivisible nature of property, households desire to move between different properties on a number of occasions throughout their lives. Hence, choosing to be an owner-occupier rather than a renter means expecting to pay the LTT every time a new property is purchased. This dissuades some potential home-buyers from incurring a credit cost and entering the ownership market. Since these households must still live somewhere, there is an increase in demand for properties in the rental market.

Investors also face paying the LTT, which reduces the return from purchasing a property. However, landlords do not need to transact again in the ownership market just because a tenant no longer finds their property suitable and moves out. This implies that investors have less need to transact compared to owner-occupiers who face match-quality shocks.[5] So while the LTT has a direct negative effect on supply in the rental market, this is relatively smaller than the increase in demand for rental properties. In equilibrium, the LTT causes the price-to-rent ratio to fall by enough to attract more buy-to-rent investors in spite of the tax. These investor purchases of properties from owner-occupiers lead to a decline in the homeownership rate. Buy-to-rent purchases and leases increase, while purchases by owner-occupiers decline, consistent with the empirical evidence.

Turning to the intensive margin, the LTT makes existing owner-occupiers more tolerant of poor match quality, so moving rates decline as households remain in properties for longer on average — a 'lock-in' effect. As match quality with a property has some persistence, households can mitigate the increased tax costs of moving by becoming more 'picky', that is, requiring higher match quality when making a property purchase, and thus reducing the need to move again in the future.

The model's parameters are calibrated to the City of Toronto housing market for the years 2006–2008. Toronto has an active rental market, and the homeownership rate in the city was then around 54%. The model is used to simulate both the transitional dynamics and steady-state effects of a 1.3 percentage-point increase in the LTT rate, calibrating to match the estimated LTT effect on homeowners' hazard rate of moving over the four-year period (2008–2012) studied in the econometric analysis.

Over the four-year post-tax-change period, the model predicts transactions by owner occupiers fall by 14%, while buy-to-rent transactions rise by 35%. Consistent with the rise in investor transactions, there is an increase in the number of leases as more households choose to be tenants rather than home-buyers, leading to a rise in the leases-to-sales ratio of 15% and a fall in the homeownership rate of 0.23 percentage points. The price-to-rent ratio falls by 1.6%. These numbers are broadly consistent in magnitude with the estimated LTT effects that were not directly targeted in the calibration.

The four-year effects on owner-occupier transactions and the price-to-rent ratio are close

[5] Section 4.2 explains why the fact that investors' transactions are a smaller fraction of total transactions than the share of rental properties in the total housing stock implies investors have longer average holding periods than owner-occupiers.

to the variables' new steady states in the model. However, those variables related to housing tenure choice are very slow to adjust, with the ultimate steady-state effects on the homeownership rate (down 2.4 percentage points) and the leases-to-sales ratio (up by 23%) being larger than the four-year effects. This is because flows in any year are small in relation to stocks. On the other hand, buy-to-rent transactions overshoot their steady-state level (up 5.1%) because of the one-off effect arising from entry of new investors.

The model spells out two facets of the welfare costs of transaction taxes that are closely related to its positive predictions. The first is a novel effect on misallocation of properties across the rental and ownership markets working through the entry of buy-to-rent investors. Intuitively, since owner-occupiers expect to transact more frequently, the same transaction tax falls more heavily on owner-occupiers than buy-to-rent investors. This means the cost of credit paid by the marginal home-buyer must fall, displacing some creditworthy households into the rental market. Transaction taxes therefore distort housing tenure choices.

Second, within the ownership market, there are two consequences for welfare. There is a 'lock-in' effect of reduced mobility, giving rise to misallocation of properties among owner-occupiers, with match quality falling on average as households move less frequently to renew it. While greater pickiness of buyers means that newly matched owner-occupiers enjoy better initial match quality, more costs are incurred from the extra time spent searching.

The implied welfare cost of the higher transaction tax is substantial. The new LTT generates a welfare loss equivalent to 111% of the extra revenue it raises. The distortions to flows between the rental and ownership markets account for a loss equal to 60% of extra revenue raised. Distortions within the rental and ownership markets lead to losses of 13% and 38% of tax revenue, respectively. Overall, the presence of the rental market in the analysis accounts for a loss equivalent to 73% of tax revenue, which is two thirds of the total loss.

In light of the implicit tax advantage enjoyed by buy-to-rent investors in a tax system where they pay the same rate as owner-occupiers, the paper studies an alternative with a higher tax rate on investors that nullifies this advantage. By raising barriers to entry for investors, it reduces the across-market welfare losses from lower homeownership. However, an important caveat is that ever further rises in buy-to-rent investors' tax rate to boost homeownership would ultimately lead to large welfare costs as uncreditworthy households are displaced into the ownership market by a lack of rental properties. Deep-pocketed investors play an important role in providing access to housing without everyone needing to pay the costs of accessing credit.

The plan of the paper is as follows. Related literature is discussed below. Section 2 presents the data and the estimation of the effects of the LTT in Toronto. Section 3 develops a two-region dual ownership and rental markets model of housing. Section 4 presents the model's qualitative predictions when the transaction tax rises. Section 5 calibrates the model and derives the quantitative effects of the tax and the associated welfare losses due to misallocation across the two markets and distortions within each market.

Related literature In the last two decades, concerns about the costs of real-estate transaction taxes have grown among policymakers and in academic research. Two prominent examples are the 'Henry Review' established by the Australian government and the 'Mirrlees Review' by the UK government. Both reviews found significant costs of transaction taxes owing to reduced mobility and distortions associated with ad valorem taxes. The reviews proposed reforms to replace stamp duty with a land value tax or a tax on housing consumption (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond *et al.*, 2010).

These findings are confirmed by economists studying housing markets using data from Australia, Canada, Finland, Germany, the UK, and the US. The majority of the literature has focused on the effects of transaction taxes on mobility, transaction volumes, or house prices. Among these papers, a few have also computed the welfare costs of transaction taxes per unit of tax revenue raised, such as Dachis, Duranton and Turner (2012) for Canada, Hilber and Lyytikäinen (2017) and Best and Kleven (2018) for the UK, Eerola, Harjunen, Lyytikäinen and Saarimaa (2021) and Määttänen and Terviö (2022) for Finland, Fritzsche and Vandrei (2019) for Germany, and Schmidt (2022) for the Netherlands. These losses are solely due to effects on the intensive margin of fewer transactions and reduced mobility of homeowners. However, as Poterba (1992) noted, "*finding the ultimate behavioral effects requires careful study of how tax parameters affect each household's decision of whether to rent or own as well as the decision of how much housing to consume conditional on tenure.*" The contribution of this paper is to study how transaction taxes affect choices of housing tenure, and to quantify the welfare effects of such taxes along both the extensive and intensive margins.

The empirical strategy of this paper is closest to Dachis, Duranton and Turner (2012) in studying the effects of the 2008 LTT change in Toronto. This paper differs in that it examines a completely new angle of the impact of transaction taxes on renting versus owning. By merging data across rental and ownership markets for property, it documents novel facts about the effects of transaction taxes on the housing tenure choice margin. While the former paper produced a reduced-form calculation of the welfare loss in the ownership market, this paper uses the empirical findings to calibrate a general-equilibrium model with both ownership and rental markets to quantify welfare losses across the two markets in addition to within each market.[6]

Recent work with a related objective to this paper in analysing the effects of transaction taxes on the homeownership rate and their implications for welfare are Cho, Li and Uren (2024), Kaas, Kocharkov, Preugschat and Siassi (2021), and Schmidt (2022). This paper's key advantage is in identifying the differential effects of transaction taxes on buy-to-rent investors and owner-occupiers using micro data on leasing and transaction records. On the theory side,

[6] A key feature of the analysis here is the general-equilibrium effect of households' tenure choices on the price-to-rent ratio following the transaction tax, which attracts entry of buy-to-rent investors. It is similar in spirit to Sommer and Sullivan (2018), who point to the general-equilibrium effect on homeownership of removing mortgage-interest deductibility through a fall in house prices, which encourages more credit-constrained households to become owner-occupiers as downpayment constraints slacken. This illustrates the importance of a framework where house prices, rents, tenure choices, and entry of investors are all endogenous in general equilibrium.

this paper allows for free entry of buy-to-rent investors in a search model that highlights the indivisible nature of housing. The model rationalizes the empirical finding of opposite effects of transaction taxes on buy-to-rent investors and owner-occupiers.

2 Empirical analysis

2.1 Data

The data on residential real-estate sales and leasing transactions come from Multiple Listing Service (MLS) transaction records for the period 2006–2018 in the Greater Toronto Area (GTA) (TRREB, 2019), the fourth largest metropolitan area in North America. Each sale has observations of the property price, the length of time it was on the market, the transaction date, and its exact address and neighbourhood. Sales transactions are documented in the MLS system once buyers and sellers sign a purchase agreement. Similar records are kept for leases, except that the time on the market is not observed. For each transaction, the sales price or rent reflect the amounts agreed upon between the parties; time-on-the-market is measured as the number of days from the initial listing to the signing of the purchase agreement. MLS data also record detailed property characteristics such as the numbers of bedrooms, washrooms, and kitchens, the lot size (except for condominiums/apartments), the styles of the house and the family room, the basement structure/style, and the heating types/sources.

The Canadian Multiple Listing Service (MLS) is a centralized database used by real-estate professionals to record listings and transactions of properties they are marketing for sale or lease. It includes comprehensive records of residential real-estate transactions facilitated by all licenced agents. According to a 2021 survey conducted in Ontario, 88% of sellers and 89% of buyers indicated their intention to use a licenced agent to assist them in buying or selling a property (OREA, 2021).

A comparison between MLS data and transaction records from the Toronto Land Registry Database shows that from 2006 to 2018, the Toronto MLS captured 79% of detached house transactions, 90% of semi-detached house transactions, and 64% of condominium transactions recorded by the Land Registry, with stable coverage throughout this period.[7] The lower coverage of condominiums is to be expected as newly constructed units are often sold directly by developers' internal sales teams. Besides directly marketed condominium units, transactions recorded by the Land Registry but not covered by the MLS typically consist of non-arm's-length transfers, such as family transfers, and for-sale-by-owner transactions.[8] Excluding non-

[7]This comparison is made available courtesy of a senior economist at the Bank of Canada. MLS data have been recognized as a standard data source in various Canadian government reports (e.g., CMHC, 2015, 2023, BoC, 2023) owing to their consistently high and stable coverage of transactions.

[8]Non-arm's-length transactions are less relevant to the issues studied in this paper. For-sale-by-owner transactions are unlikely to involve disproportionately either buy-to-own or buy-to-rent purchases. So excluding these transactions should not substantially affect the main findings here.

arm's-length and for-sale-by-owner transactions, the correlation between sales transactions for detached and semi-detached houses in the MLS and Land Registry data at the FSA $\times$ month level is nearly 0.99.[9] Given the incomplete coverage of the condominium sector and the relative scarcity of semi-detached houses, the analysis mainly focuses on detached houses, with robustness checks for other property types.[10]

A key advantage of the analysis comes from the ability to match rental transactions to sales transactions. The MLS is the largest rental listing platform and offers an unusually high coverage of verifiable long-term rental listings in Toronto. Appendix A.1.1 shows through webscraping and geocoding that MLS rental listings capture over 90% of properties for rent in the City of Toronto that were listed on alternative platforms such as *Toronto Rentals*, the second-most popular rental website serving the GTA since 1995.[11]

Properties that appear in both the sales and leases datasets within an 18-month window are identified by their transaction dates and detailed address information. This process generates a novel measure that links the markets for property ownership and rentals. If the sale of a property is followed by it being listed on the rental market between 0 and 18 months after the sale, the purchase is identified as a *buy-to-rent* transaction. Alternatively, if a sale is followed by the property being listed again for sale between 0 and 18 months after the original sale, it is identified as a *buy-to-sell* transaction.[12] The remaining sales transactions are considered to be purchases by owner-occupiers and are designated as *buy-to-own* transactions.

Between 2006 and 2017, the fraction of buy-to-own transactions in the City of Toronto declined from 89% to 84%, while the fraction of buy-to-rent transactions tripled from 4% to 12%.[13] In contrast, buy-to-sell transactions remained stable at around 4% throughout most of the period.[14] Given the small and stable fraction of buy-to-sell transactions, these are excluded from the sample used for estimation.

A market segment is defined by *community* $\times$ *year* $\times$ *month*.[15] For each market segment, the housing-market outcome variables include the number of sales transactions, which is broken down into buy-to-own (BTO) and buy-to-rent (BTR) sales, the number of leases, the ratio of the

[9] FSA refers to a Forward Sortation Area, the first three characters of a Canadian postal code that identify a specific geographic area.

[10] In the market for detached houses, MLS records cover approximately 78% of the transactions recorded in Land Registry data between 2006 and 2012. At the start of the sample period, MLS transactions account for 76.3% of deed transactions, indicating the usual concern about coverage in earlier years is less of a problem here.

[11] *Urbanation*, a third-party service that independently collects data on rentals in the Greater Toronto Area, estimates that approximately 75–80% of condominium lease activity is captured in MLS data. These estimates are based on examining MLS leasing transactions volumes relative to the size and change in the overall stock of rental properties each year as reported by the Canada Mortgage and Housing Corporation (CMHC).

[12] As a robustness check, changing the 18-month threshold to 6, 12, or 24 months does not significantly affect the estimation results.

[13] An increase in buy-to-rent transactions in recent years has been seen in other countries, including the US and Norway (Mills, Molloy and Zarutskie, 2019, Bø, 2021).

[14] In suburban areas, these fractions change from 88% to 80%, 5% to 14%, and remain stable at 5%, respectively.

[15] There are 296 communities in the GTA, including 140 in the City of Toronto. See `www.toronto.ca/city-government/data-research-maps/neighbourhoods-communities/neighbourhood-profiles/`.

numbers of leases to sales, and the average price-to-rent ratio. In addition, for each homeowner, the number of months since the property was purchased is known, irrespective of whether the property is currently listed for sale.[16]

Real-estate transaction taxes are common across Canada, where they are known as Land Transfer Tax (LTT). The tax is paid by buyers, and in spite of the name, LTT is applied to the whole property price. Before 2008, residential transactions in the province of Ontario, which includes the whole of the GTA, were subject to a provincial-level land transfer tax, but there was no additional city-level LTT. The City of Toronto experienced a housing boom in the years following 2000 and usually maintained a budget close to balance. Following an unexpected budget shortfall in late 2007, the city council approved a land transfer tax on property transactions within the city that close after 1st February 2008. The extra tax revenue was collected to meet municipal workers' demands for higher wages. The institutional background to the LTT is discussed in detail in Dachis, Duranton and Turner (2012). Table A.1 in the Appendix summarizes the city- and provincial-level LTT schedules.[17]

The effective LTT rate is defined as the mean transfer tax as a percentage of the sales price, combining provincial- and city-level taxes, averaged over detached-house transactions in the City of Toronto. Using the same set of transactions in the pre-policy period from January 2006 to January 2008 to control for compositional effects, effective LTT rates are imputed based on the tax rates before and after the new LTT is introduced, and the change in the effective LTT rate is taken to be the difference between the two. To account for partial exemptions received by first-time home-buyers, effective LTT rates are imputed assuming 0%, 40%, or 100% of home-buyers are first-time buyers, and the resulting effective LTT changes under these different assumptions are reported in the Appendix in column (1) of Table A.2. With 40% being first-time buyers, the effective LTT rate rises by 1.33 percentage points after the city-level LTT is introduced.[18] This number is unaffected by restricting the sample to transactions within 5km of the city border, and drops slightly to 1.31 percentage points for a sample within 3km, as seen in columns (2) and (3) of Table A.2. Given the consistency of the effective LTT rate across these samples, the increase in the effective LTT rate is taken to be 1.3 percentage points.

In the baseline estimation, the pre-policy period is January 2006–January 2008 and the post-

[16]The MLS sample covers Toronto listings and transactions records in the period 2000–2018. For each property, its transaction history is tracked between 2000 and 2018. The time since a homeowner purchased a property is calculated as the number of months since the previous transaction date, with the original purchase price corresponding to the sales price for the previous transaction. MLS databases are maintained by local real-estate boards that ensure consistency and accuracy in record keeping.

[17]The new city-level LTT has an exemption for first-time buyers of properties priced under $400,000, and the existing provincial-level LTT had a similar exemption for properties under $227,500. However, since first-time buyers will expect to move again in the future and then face the new tax, the effects of the LTT predicted by the model still apply in the presence of this exemption.

[18]As noted in the evidence used for calibrating the model from appendix A.5, the Canadian Association of Accredited Mortgage Professionals (now Mortgage Professionals Canada) conducted a 2015 survey finding a 45% fraction of first-time buyers, consistent with the 44% found in the 2018 Canadian Household Survey for the Greater Toronto Area. In contrast, data from the Canada Mortgage and Housing Corporation suggest the fraction is about one third. Based on these sources, a 40% fraction of first-time home-buyers is assumed here.

policy period is February 2008–February 2012. The top two panels of Table A.3 present descriptive statistics for variables within the Greater Toronto Area and within the City of Toronto before and after the introduction of the city-level LTT.

To ensure the housing stock and neighbourhoods are relatively homogeneous, the baseline sample is restricted to properties on either side of the city border within 3 or 5 kilometres of the boundary line determining whether the new LTT is applicable. The geography of the sample used for the baseline estimation is depicted in Figure A.2. Importantly, the possibility that housing-market outcome variables make a discrete jump at the border while neighbourhoods continue to change in a smooth manner allows the relationship between the LTT and housing-market outcomes to be isolated. The bottom two panels of Table A.3 present summary statistics for the 3- and 5-kilometres border samples during the pre- and post-LTT periods. Appendix A.1.3 further shows that most property characteristics do not vary significantly across the border, and that cross-border differences, if any, do not change significantly after the new LTT.

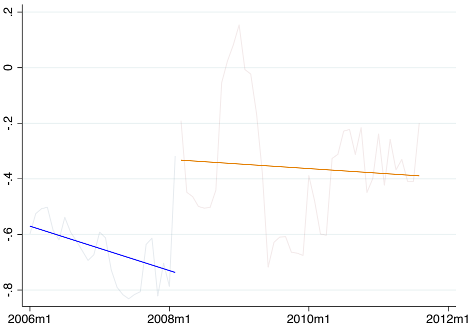
2.2 Estimating the effects of transaction taxes

The main empirical strategy resembles a variant of the regression discontinuity design in Dachis, Duranton and Turner (2012). While they estimate the six-month effects of the new LTT on sales transaction volumes and prices in the market for detached houses, this paper extends the sample to cover not only a longer time period but also a wider range of residential property types. Most importantly, benefiting from a unique combination of rental and sales data, this paper examines an array of market outcomes above and beyond prices and volumes, which reveal a detailed picture about flows of properties between and within the owner-occupied and rental markets. Between the two markets, the paper makes a new contribution by estimating the effects of the LTT on transaction volumes and costs in the rental market relative to the ownership market. Within the ownership market, the paper enriches previous work by estimating how the LTT affects individual homeowners' hazard rate of moving.

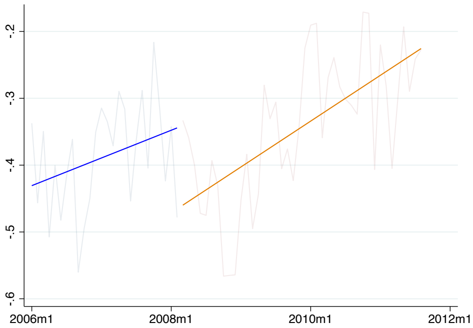
Figure 1 motivates the empirical analysis by presenting time series of cross-city-border differences in housing-market variables related to the extensive margin. The graphs cover the period before and after the new LTT is introduced in February 2008, and the difference at the City of Toronto border is calculated using neighbourhoods within five kilometres of the border on either side. Following the new LTT, there is a distinct jump in relative transaction volumes across the owner-occupied and rental sectors and relative costs as measured by the price-to-rent ratio. Compared to their nearby suburban neighbours, city residents faced lower prices relative to rents, experienced more leasing transactions relative to sales, and had fewer buy-to-own but more buy-to-rent transactions.

While illustrative, the evidence presented in Figure 1 is descriptive at best. It cannot isolate the effects of the LTT from other confounding factors. Nor can it speak to the magnitude of

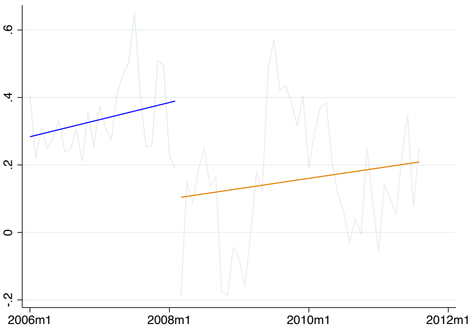
Figure 1: *Log point differences across the City of Toronto border*



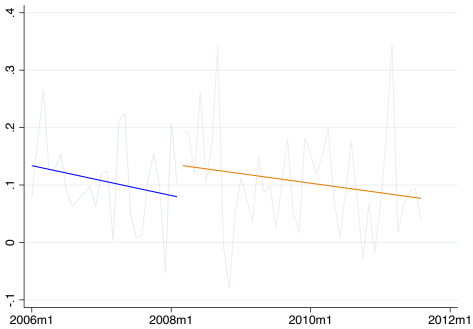
(a) Leases-to-sales ratio



(b) Price-to-rent ratio



(c) Buy-to-own sales



(d) Buy-to-rent sales

Notes: The vertical axis represents the change in the logarithm of the mean neighbourhood-level outcome variables across the Toronto border in the months before and after the imposition of the new LTT. The graphs are obtained from kernel-weighted local linear regressions of coefficients from the logarithms of outcome variables on the interaction between City of Toronto and year-month indicators, controlling for community fixed effects and year-month fixed effects. The sample comprises detached house transactions from 2006 to 2018 that are within 5km of the city border.

any jump because the shape of the time trend may be sensitive to specification. Nevertheless, it highlights two discrete changes in Toronto's housing market following the introduction of the city-level LTT: one at the city border, and one on the date the new LTT is imposed. Motivated by these discontinuities, this paper estimates the causal effects of the transaction tax using a hybrid of differences-in-differences and regression discontinuity design, focusing on housing transactions within a narrow band on both sides of the city border. By incorporating a rich set of controls and robustness checks, robust estimates of the causal effects of the LTT are derived.

Beginning with estimation at the level of a market segment ($community \times year \times month$), the dependent variable is one of the following: the log of the number of leases relative to sales, the log of the average price-to-rent ratio, the log of the number of buy-to-own (BTO) transactions, and the log of the number of buy-to-rent (BTR) transactions. Later, the effect of the LTT on individual homeowners' monthly moving hazard is estimated.

In these regressions, the key variable of interest is *LTT*, an interaction of indicators for being within the city border and after the new LTT is introduced, whose coefficient captures the impact of the new city-level transaction tax. The regressions also include the City of Toronto indicator, the post-LTT indicator, a rich set of time-varying housing characteristics (where applicable), along with a broad set of fixed effects: community, year, month, property-type (where applicable), and their interactions. These fixed effects flexibly control for housing composition, seasonality, and variation in how different housing-market segments evolve. Notably, the specification allows for separate time trends inside and outside of the city to control for city-specific trends that may be caused by factors other than the LTT. Together, these controls help account for the potential impact of households' price expectations and risk perceptions on housing transaction outcomes (see Adelino, Schoar and Severino, 2018, Kindermann, Le Blanc, Piazzesi and Schneider, 2024), which may evolve over time and vary across neighbourhoods. The detailed empirical specification is presented in appendix A.1.4.

One legitimate concern is that households may have anticipated the introduction of the new LTT and rushed to make transactions before the cost of buying a property increased. As discussed extensively in Dachis, Duranton and Turner (2012), such anticipation of the 2008 LTT in Toronto was quite limited, and would have had to occur within the three months before the new LTT was implemented. In light of this, for all specifications, indicators for transactions in the six-month period from November 2007 to April 2008 are included to condition out any run-up in sales right before the tax change and possible continuation immediately after it.[19]

2.2.1 Effects across ownership and rental markets

Consider first the market-segment level estimation of the LTT effects across the ownership and rental markets. The focus is on detached house transactions, for which the MLS data provide extensive coverage.

Leases-to-sales and price-to-rent ratios For each market segment, the leases-to-sales ratio is a measure of relative activity in the rental and ownership markets, and the price-to-rent ratio is a measure of relative cost across the markets. The top panel of Table 1 reports the estimated effects of the LTT on these measures. Column (1), the baseline specification, restricts the sample to 3km on each side of the border. It allows for anticipation effects by including indicators for transactions three months before and after the introduction of the LTT. It further allows for the presence of spatially differentiated time trends on either side of the city border.

The 1.3 percentage-point increase in the effective LTT rate causes a 26.4% increase in the numbers of leases relative to sales and a 3.8% drop in the price-to-rent ratio.[20] The LTT thus boosts activity in the rental market compared to the ownership market, and raises the rental

[19]This strategy for addressing possible anticipation effects is also consistent with Bérard and Trannoy (2018) and Benjamin, Coulson and Yang (1993), both of whom explicitly estimate anticipation effects associated with a real-estate transaction tax. Using French data, the former find that the anticipation effect is limited to one month immediately before the implementation of the tax reform, while the post-tax effects last for up to three months. Using data from Philadelphia, the latter find that anticipation effects are very small and limited to two months before the tax change.

[20]A coefficient of 0.234 indicates the LTT increases the leases-to-sales ratio by 26.4% ($100 \times (e^{0.234} - 1)$), and a coefficient of -0.039 implies the LTT decreases the price-to-rent ratio by 3.8% ($100 \times (1 - e^{-0.039})$).

Table 1: *Effects of the transaction tax across ownership and rental markets*

Dependent variable	(1)	(2)	(3)	(4)
log (*#Leases*/*#Sales*)	0.234**	0.242***	0.236**	0.264***
	(0.117)	(0.082)	(0.100)	(0.063)
Observations	1,355	2,660	1,782	7,730
log (*Price*/*Rent*)	−0.039**	−0.026*	−0.031*	−0.038***
	(0.020)	(0.015)	(0.018)	(0.013)
Observations	1,672	3,517	2,455	9,876
log (*#BTO sales*)	−0.101*	−0.097**	−0.062	−0.122***
	(0.059)	(0.045)	(0.056)	(0.033)
Observations	3,736	6,363	3,811	17,190
log (*#BTR sales*)	0.089*	0.100**	0.117**	0.110*
	(0.047)	(0.045)	(0.055)	(0.058)
Observations	531	1,031	670	2,857
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: Data comprise detached house transactions from January 2006 to February 2012. A unit of observation is a market segment defined by *community* × *year* × *month*. Repeat sales transactions taking place within 18 months of one another are discarded. Each cell of the table represents a separate regression of an outcome (specified in the left column) on the *LTT* interaction dummy. All regressions include a dummy for the post-LTT period, City of Toronto fixed effects, year fixed effects, calendar-month fixed effects, community fixed effects, and their interactions. In the specifications above, the distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trends denotes the inclusion of an interaction term between the *LTT* and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the *LTT* and the distance from the city border in column (4). Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

yield (the inverse of the price-to-rent ratio). Column (2) repeats the baseline regression of column (1) except for extending the sample to include all property transactions within 5km of the city border instead of 3km. The coefficients for the lease-to-sales ratio and the price-to-rent ratio remain close to those in column (1). One might be concerned that the rise in the leases-to-sales ratio could be due to a fall in mobility that decreases both leases and sales, but with sales falling by more. Table A.5 in the Appendix shows that this is not the case, as the LTT consistently has a negative effect on sales and a positive effect on leases. Similarly, as shown in Table A.6, the decline in the price-to-rent ratio is driven primarily by a decline in house prices.

While discontinuity design is a standard approach to estimate the effects of the tax, it requires three strong assumptions that are worth discussing and testing. The first assumption is that the leases-to-sales and price-to-rent ratios outside the city border are unaffected by the tax change. A potential sorting bias is that some buyers may respond to the LTT by switching from making purchases inside the city border to outside, boosting property sales outside the border and hence violating the assumption that the comparison group is unaffected by the tax change. To mitigate this concern, column (3) applies a 'donut approach', repeating the estimation in column (2) with a distance threshold of 5km, but excluding properties within 2km of each side of the city border. The rationale is that sorting across the border, if it occurs, would most likely happen immediately adjacent to the border. However, the coefficients in column (3) are very close to those in column (2), mitigating this concern.[21] A reason for the robustness of the estimates with respect to sorting is offered in the theoretical framework developed later.

Second, for regression discontinuity to offer an appropriate estimate of the LTT effect, the LTT impact must be uniform for all city properties irrespective of their distance to the border. But this will not hold if, for example, people who live further away from the border are more willing to pay the tax because their location demand is less price elastic. This concern is addressed in columns (2) and (3) by extending the sample to include properties within 5km of each side of the city border and adding an interaction term between exposure to the LTT and a dummy variable for properties between 2.5km and 5km from the border. With this, the LTT effect can differ depending on the distance of a property from downtown. However, the coefficient on the interaction term is small and statistically insignificant in all specifications.[22] More importantly, the coefficients on *LTT* in both the leases-to-sales and price-to-rent regressions remain consistent across specifications.

Column (4) further extends the estimation sample to cover the entire City of Toronto and the rest of the Greater Toronto Area. The estimated effects remain close to the baseline specification in magnitude and significance. Moreover, from the interaction term, distance to the border does not change the main LTT effects in any noticeable way.[23]

Given the robustness of the estimates in Table 1 and the consistency of the effective LTT rates across samples (see Table A.2), column (1) is adopted as the main specification in what follows. Expanding the geographic coverage allows for more extensive controls and specification checks, but at the cost of adding unobserved heterogeneity and thus complicating interpretation of the estimates. This is especially so for the whole sample in column (4).

Finally, the discontinuity design relies on a combination of two structural breaks in the housing market: one on the date the new LTT was imposed, and one at the city border. There

[21]The estimates are again robust if the estimation in column (1) with a distance threshold of 3km is repeated, but excluding properties within 1km of each side of the border.

[22]In column (2), the interaction term's coefficient is 1.6×10^{-5} with a standard error of 2.7×10^{-5} in the leases-to-sales regression, and 8.0×10^{-6} with a standard error of 7.0×10^{-6} in the price-to-rent regression.

[23]In column (4), the coefficient on $LTT \times distance$ in the leases-to-sales regression is -1.18×10^{-4} with a standard error of 5.24×10^{-5}. The City of Toronto covers an area of 630.2km^2 with a radius of 14.16km. Within the city, the community with the maximum distance to the border, approximately 18km, is the Waterfront neighbourhood. Hence, the LTT effect on the leases-to-sales ratio is much the same throughout the city. The corresponding coefficient in the price-to-rent regression is statistically insignificant and quantitatively irrelevant.

might be a concern that these structural breaks could pick up time variation and spatial differences in the housing market that are not necessarily related to the LTT. For example, the introduction of the LTT coincided with the global financial crisis. If the crisis affected city and suburban neighbourhoods differently, the *LTT* coefficient might inadvertently capture its effects. However, note that city-specific time trends were included to account for spatially different time trends inside and outside the city. Additionally, the financial crisis had a mild and temporary impact in Canada compared to the U.S. and other countries (Bordo, Redish and Rockoff, 2015, Haltom, 2013, Walks, 2014), with the GTA housing market experiencing a temporary slowdown starting in September 2008, followed by a quick recovery at the beginning of 2009.

To test whether this temporary slowdown drives the estimated LTT effects, the years and months around the financial crisis are excluded from the estimation sample. Specifically, the estimation of columns (2) and (3) of Table 1 is repeated after excluding transactions 21, 24, or 27 months around the financial crisis. Table A.7 in the Appendix presents the results for the leases-to-sales ratio along with other key market outcomes in these alternative sample windows. The results consistently align with those in Table 1 across samples, suggesting the estimated LTT effects are unlikely to be driven by the financial crisis.

To validate the estimated LTT effects further, a placebo test is conducted to check that the results are not simply capturing arbitrary spatial differences in housing-market trends. Using areas within the city, artificial borders are defined at distances of 3km, 4km, 5km, or 6km from the true city border and the estimations from Table 1 are repeated. These results are reported in Table A.8. The coefficients on the pseudo *LTT* dummy variables, the interaction of the pseudo-border dummies with the post-LTT indicator, are mostly small and statistically insignificant across pseudo-border choices and specifications. When significant, as in the case of the price-to-rent ratio, the coefficients show signs opposite to what would be expected. This exercise further supports that the estimated effects of the LTT originate from changes at the city border where the new LTT becomes applicable rather than arbitrary spatial differences.

Buy-to-own and buy-to-rent transactions Given the relative increase in leasing activity in the city after the LTT is introduced, it is natural to explore the breakdown of sales into buy-to-own and buy-to-rent transactions. While the literature finds that the total volume of sales decreases in response to the LTT, this aggregate effect may obscure significant differences in how owner-occupiers and investors respond to transaction taxes. Indeed, Table 1 shows that the LTT has opposite effects on buy-to-own (BTO) and buy-to-rent (BTR) transactions, in spite of the same tax rate applying to both. Column (1) shows BTO transactions fall by 9.6%, while BTR transactions rise by 9.3%. These estimates are consistent across specifications.

There are several potential concerns with the finding of opposite LTT effects on BTO and BTR transactions. First, investors and home-buyers may be treated differently in the mortgage market or in respect of taxation of capital gains. Furthermore, if some BTR transactions were not recorded in the MLS rental database, these might be mis-categorized as BTO transactions. However, such omitted variables and measurement error are less of a concern in this setting. To the extent that changes in mortgage or tax treatment or the mis-categorization of transaction types before and after the new LTT do not vary systematically between adjacent neighbourhoods across the city border, these factors would have been accounted for using the differences-in-differences approach.

Second, there may be a concern that the results are sensitive to the number of months between purchasing and leasing a property that was used to distinguish BTO and BTR transactions. To address this, Table A.9 in the Appendix shows the results are robust to changing the 18-month threshold to 6, 12, or 24 months. A third concern is that first-time buyers are more likely to benefit from partial tax exemptions compared to buy-to-rent investors. Recall that the imputation of the effective LTT rate accounts for the presence of first-time buyers. Moreover, the baseline estimation compares adjacent neighbourhoods with similar compositions of households on opposite sides of the city border before and after the new LTT, effectively conditioning out differences in the initial fraction of first-time home-buyers.[24]

So far, the analysis has focused on detached houses, given that the MLS data cover nearly the universe of transactions for this property type in Toronto after excluding non-arm's-length and for-sale-by-owner transactions. In contrast, coverage is lower for condominiums and apartments. The robustness of the main findings to other types of properties is checked by expanding the sample to include all types.[25] As shown in Table A.10, across different property types, the LTT consistently leads to an increase in the leases-to-sales ratio, a decrease in the price-to-rent ratio, a decline in buy-to-own transactions, and an increase in buy-to-rent transactions. These findings support that the results in Table 1 extend to other property types as well.

2.2.2 Effects on the owner-occupied market

The moving hazard rate This section restricts attention to flows within the ownership market and examines the effects of the transaction tax on individual homeowners' mobility. Unlike many previous studies that use transactions volume to measure mobility, here the data have precise observations of when an individual homeowner puts a property up for sale and when a sale occurs.

Homeowners' pattern of mobility is represented by a moving hazard function: the relationship between the rate at which moving occurs and the number of months since a homeowner purchased a property. This hazard function is estimated using the Kaplan-Meier (KM) method. The KM estimator computes the conditional probability of putting a property up for sale given

[24] Admittedly, the fraction of first-time home-buyers is not observed at the neighbourhood level. To the extent that there are more first-time home-buyers in border neighbourhoods, the baseline estimates from the border sample would represent a lower bound on the true effects. Note that the LTT effects estimated for both the border sample (columns 1–3 of Table 1) and the whole sample (column 4) are largely consistent.

[25] Using German data, Petkova and Weichenrieder (2017) find that an increase in real-estate transfer taxes is associated with a decline in transactions for single-family houses and a decline in prices for apartments.

the time since the homeowner moved in. Specifically, a unit of observation is each month since a homeowner has bought a property and the event is putting the property up for sale given that this has not occurred so far. The estimated hazard function is shown in Figure A.3. The mean length of time between purchasing a property and listing it for sale is 113 months.

Since the hypothesis of homogeneity of hazard rates over time is not rejected at the 1% level and the estimated hazard function shape is monotonic, the hazard function can be analysed using a Weibull model. The hazard function for homeowner j in a given year-month t is parameterized as

$$\hbar\left(t\,|\,\mathbf{x}_{jt}, LTT_{jt}\right) = \varphi t^{\varphi-1} e^{\beta_0+\mathbf{x}'_{jt}\beta_{\mathbf{x}}+LTT_{jt}\beta},$$

where t is time since the homeowner purchased their property, φ is a parameter linked to the gradient of the hazard function, and LTT_{jt} is an indicator for exposure to the new LTT. The vector $\mathbf{x}_{jt}$ is a rich set of controls, including indicators for the post-LTT period and being in the City of Toronto, time-varying property attributes, a broad range of fixed effects that flexibly control for the differential evolution of housing-market outcomes across property types and communities, and the price originally paid by the homeowner. The original purchase price proxies for non-tax-related moving costs that are positively related to a property's value, both monetary (e.g., real-estate agent commissions) and psychological (e.g., attachment to a higher-value home) (Hardman and Ioannides, 1995, Han, 2008). Controlling for the original purchase price enables the LTT effect on residential mobility to be separated from that of other transaction costs.

The estimation results are presented in Table 2. For the baseline specification in column (1), the LTT reduces an individual homeowner's moving hazard by 12.2%. Given the mean duration of remaining before the tax change is 113 months, this implies homeowners remain in their current home for 14 months longer on average after the LTT. This substantial lock-in effect is consistent with evidence from other countries.[26] The other columns allow for spatially differentiated time trends, substitution across the border, and alternative thresholds for the distance from the city border. The resulting estimates of the LTT effect are not statistically different from those in column (1).[27] Table A.11 in the Appendix shows the results of repeating the estimation for the alternative sample periods 2006–2010 and 2006–2018. The estimated LTT effect remains robust to using shorter and longer post-LTT periods. The estimated lock-in

[26]For example, using data from the Netherlands, Van Ommeren and Van Leuvensteijn (2005) find that a one percentage-point increase in transaction costs as a percentage of property prices decreases residential mobility rates by 8.1–12.7%. Using UK data, Hilber and Lyytikäinen (2017) find that a 2 percentage-point increase in stamp duty reduces the annual rate of mobility by 2.6 percentage points.

[27]As in Table 1, column (1) in Table 2 is retained as the main specification. This choice ensures relative homogeneity of the housing stock and neighbourhood characteristics. The stability of estimates from the border sample over an extended period and across different property types further underscores the advantage of using a relatively homogeneous border sample. For example, using the specification in column (1), the estimated LTT coefficient changes only slightly from −0.130 to −0.125 when extending the estimation window from 2006–2012 to 2006–2018, and to −0.110 when extending the detached house sample to cover all property types.

effect of transaction taxes on residential mobility is not only substantial but also long lasting.

Across all specifications, the estimated value of $\log \varphi$ is greater than zero, indicating a moving hazard that increases with time spent living in a property. Furthermore, the effect of the original purchase price is substantial and significant in most specifications, suggesting it is important to separate the LTT effect from that of other transaction costs.

Table 2: *Effects of the transaction tax on mobility*

	(1)	(2)	(3)	(4)
	Dependent variable: The event of moving			
LTT	−0.130**	−0.194***	−0.232**	−0.228***
	(0.064)	(0.053)	(0.088)	(0.042)
log (*Original purchase price*)	−0.095	−0.076*	−0.103**	−0.079***
	(0.065)	(0.043)	(0.048)	(0.023)
$\log \varphi$	0.513***	0.523***	0.519***	0.526***
	(0.010)	(0.007)	(0.010)	(0.005)
Observations	1,691,369	2,831,897	1,651,935	5,719,326
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
Property characteristics	Yes	Yes	Yes	Yes
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: Data comprise detached house transactions from January 2006 to February 2012. Repeat sales transactions taking place within 18 months of one another are discarded. A unit of observation is a homeowner whose property is listed on MLS. Homeowners' times between moves are assumed to follow a Weibull distribution. All regressions include an indicator for the post-LTT period, an indicator for the City of Toronto, property-type fixed effects interacted with a set of time-varying property characteristics, and *year* × *property type*, *month* × *community*, *month* × *property type*, and *community* × *property type* fixed effects. In the specifications above, the distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trends denotes the inclusion of an interaction term between the *LTT* and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the *LTT* and the distance from the city border in column (4). Standard errors clustered by community are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

While not the focus of this paper, Table A.6 in the Appendix presents the estimated effect of the new LTT on sales prices and on time-on-the-market using transactions-level data. All else equal, the LTT causes a 1.74% decline in prices, accompanied by an increase in time-on-the-market. As shown in Table A.12, the effect on the sales price estimated with transactions-level data is consistent with the average sales price effects estimated using market-segment data, and is robust to using a shorter or longer sample period.

3 A dual rental and ownership markets model of housing

This section presents a model to explain the differential effects of transaction taxes across the rental and ownership markets found in section 2, and to quantify the welfare costs of such taxes. Households make housing tenure decisions subject to credit frictions. Investors choose to enter the housing market, and households choose where to live. Within the rental and ownership markets, households make moving and housing transaction decisions subject to search frictions in locating properties to view and idiosyncratic household-property match quality.

There are two regions, the first representing the City of Toronto where the city-level LTT was introduced, and the second representing the rest of the GTA. The city has a unit measure of ex-ante identical properties and two markets for housing, a rental market and an ownership market. Time is continuous, and everyone discounts future payoffs at rate r. Households in the city exit exogenously at rate ρ, for example, for work or family reasons, and the flow of new entrants depends on the payoff from living in the city. Investors can enter freely, becoming landlords and renting out properties. Investors simply represent funds held in real estate — they could be living within the city or elsewhere. Landlords are subject to shocks arriving at rate ρ_l that force them to sell their property, for example, for liquidity reasons.

Properties are either offered for rent (measure u_l), up for sale (measure u_o), or not currently available in either market. The subscripts l and o denote the rental and ownership markets, respectively, and the dependence of variables on time t is not indicated explicitly. Properties are owned either by landlords or by those who live in them as owner-occupiers. When not for rent or sale, properties are occupied by a tenant (measure q_l) or an owner-occupier (measure q_h). The unit measure of properties in the city must each be in one of these four states:

$$q_l + q_h + u_l + u_o = 1\,. \tag{1}$$

Households are looking for a property to move into if they are not currently occupying one. They make a tenure-choice decision and search either in the rental market (measure b_l) or the ownership market (measure b_h), where the subscript h denotes those households who have chosen to be homeowners. The fraction of such households within the city is also denoted h, referred to as the homeownership rate. A household occupies at most one property at a time, so the city population n must each be in one of four states:

$$q_l + q_h + b_l + b_h = n\,, \quad \text{and } h = \frac{q_h + b_h}{n}\,. \tag{2}$$

3.1 Tenure-choice decisions

Households looking for a property decide whether to search in the rental market or as home-buyers in the ownership market. Entering the ownership market for the first time as a 'first-time buyer' requires paying an idiosyncratic credit cost K. This can be thought of as household-specific factors affecting the cost or availability of a mortgage, such as the household's credit history or wealth available for a downpayment.[28] More specifically, the distribution of K across households is calibrated using data on loan-to-value ratios and spreads between the risk-free interest rate and mortgage rates for average and marginal home-buyers. Once the credit cost has been paid, a household is free to return to the ownership market later.

After drawing its credit cost K, a household compares the value B_h of being a home-buyer to the value B_l of searching for a property to rent. Households with sufficiently low credit costs $K \leq Z$ enter the ownership market, where the credit-cost threshold Z for a marginal home-buyer indifferent between the two markets is the difference between B_h and B_l:

$$B_h - Z = B_l \,. \tag{3}$$

Credit costs are drawn from a probability distribution with cumulative distribution function $\Gamma_k(K)$ by the flow a of households who are newly arrived in the city, and are redrawn by a fraction ξ of tenants who move. Tenants who do not redraw their credit cost remain in the rental market, and owner-occupiers who have already paid the credit cost remain in the ownership market until they exit the city. The fraction κ of households drawing a credit cost below the threshold Z pay credit costs of $\bar{K}$ on average, and these variables and the flow γ of first-time buyers are

$$\kappa = \Gamma_k(Z) \,, \quad \bar{K} = \mathbb{E}[K | K \leq Z] \,, \quad \text{and} \;\; \gamma = (\xi m_l q_l + a)\kappa \,, \tag{4}$$

where the q_l existing tenants move at rate m_l.

3.2 Location choices

Households newly entering the city pay a common entry cost E, draw a credit cost K, and make a tenure-choice decision as described above. The expected value N of a new entrant is

$$N = \kappa(B_h - \bar{K}) + (1-\kappa)B_l - E \,, \tag{5}$$

where B_h and B_l are the values conditional on tenure choice, and κ and $\bar{K}$ are as defined in (4). The flow of new arrivals a to the city is positively related to a comparison of the value N and the value of being in the region outside the city, where that value is normalized to zero without loss of generality. New arrivals and the dynamics of the city population n are

$$a = \max\{\rho n + \chi N, 0\} \,, \;\; \text{where} \;\; \chi > 0 \,, \quad \text{and} \;\; \dot{n} = a - \rho n \,, \tag{6}$$

where $\dot{n}$ denotes the derivative of n with respect to time t. The parameter χ represents the sensitivity of the endogenous inflows to the value N of entering the city. When N is positive, the city attracts more new households than the exogenous outflow ρn and the total measure of households increases. When $N = 0$, new entrants are indifferent, and it is assumed inflows

[28]The credit cost K is modelled as a one-off cost, but this is equivalent in the model here to a present value of flow credit costs paid for a period of time while a household is an owner-occupier.

match exogenous outflows to leave the population stable with some turnover of households for exogenous reasons such as work and family. When $N < 0$, housing-market conditions deter some households from coming, resulting in a decline in the city population.

While living in the city, all n households receive a per-person flow benefit $g = G/n$ from public spending G on city amenities irrespective of their tenure status.

3.3 The rental market

Existing tenancies Tenants living in a landlord's property receive an idiosyncratic flow value ε specific to the match between the property and the household. Match quality ε persists until shocks arriving at rate α_l independently across tenants and across time reduce it to zero. There is no commitment or long-term contract between landlord and tenant: either can end the relationship at any subsequent time. Ongoing negotiations between landlords and tenants, both knowing ε, allow a tenant to occupy a property in return for paying rent $R(\varepsilon)$.

The value of a landlord whose property is currently occupied by a tenant with match quality ε is $L(\varepsilon)$, and the tenant's value is $W(\varepsilon)$. The landlord's surplus from the match is $\Sigma_{wl}(\varepsilon) = L(\varepsilon) - \max\{U_l, U_o\}$, where U_l is the value of having a property available to let and U_o is the value of putting a property up for sale. If a tenant were to move out, landlords choose the best of these options. Properties are ex-ante identical prior to matches forming, so U_l and U_o are independent of ε. The tenant's surplus from remaining in the property is $\Sigma_{ww}(\varepsilon) = W(\varepsilon) - B_l$, where the outside option is the value B_l of going back to the rental market to look for another property. The tenant cannot receive a new draw of the credit cost K simply by the threat to move out. Any past transaction or moving costs are sunk at this point and hence do not appear in the surpluses. The joint surplus $\Sigma_w(\varepsilon) = \Sigma_{wl}(\varepsilon) + \Sigma_{ww}(\varepsilon)$ is

$$\Sigma_w(\varepsilon) = L(\varepsilon) + W(\varepsilon) - U_l - B_l\,, \tag{7}$$

supposing $U_l \geq U_o$, as is confirmed later. This joint surplus is increasing in ε, and rent negotiations ensure matches survive for as long as it remains positive. Owing to sunk transaction costs, a landlord-tenant match ends only if the tenant receives a match quality shock (arrival rate α_l) or leaves the city (rate ρ), or if the landlord is forced to sell up by an exit shock (arrival rate ρ_l). Tenants' moving rate m_l within the city is then

$$m_l = \alpha_l + \rho_l\,. \tag{8}$$

The Bellman equation for tenants' value function $W(\varepsilon)$ is

$$rW(\varepsilon) = \varepsilon - R(\varepsilon) + g + m_l\left(\xi\kappa(B_h - \bar{K}) + (1-\xi\kappa)B_l - W(\varepsilon)\right) - \rho W(\varepsilon) + \dot{W}(\varepsilon)\,, \tag{9}$$

where $\varepsilon - R(\varepsilon)$ is the flow benefit of occupying a particular property net of the rent paid, and g is the flow benefit of residing in the city. When the tenancy ends without the household leaving the city (rate m_l from 8), the household draws a new credit cost with probability ξ, and there is a probability κ from (4) it is low enough that the household chooses to become a home-buyer (value B_h) after paying a credit cost of expected value $\bar{K}$. With probability $1-\xi\kappa$, the household keeps the same tenure status and returns to the rental market (value B_l). Households ending their tenancy because they leave the city (rate ρ) receive value zero outside the city.

The Bellman equation for landlords' value function $L(\varepsilon)$ is

$$rL(\varepsilon) = R(\varepsilon) - D - D_l + (\alpha_l + \rho)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) + \dot{L}(\varepsilon)\,, \tag{10}$$

where $R(\varepsilon)$ is the rent paid by the tenant, D is a flow maintenance cost incurred by all property owners, and D_l is an extra maintenance cost incurred when a property is let. When tenancies end because the tenant wants to move out (combined rate $\alpha_l + \rho$), the landlord decides to look for another tenant (supposing $U_l \geq U_o$, this is preferable to selling the property). If landlords are forced to exit (rate ρ_l), they sell and receive value U_o.

Rents are set through Nash bargaining, where landlords' bargaining power is ω_l. The bargaining problem maximizes the Nash product $(\Sigma_{wl}(\varepsilon))^{\omega_l}(\Sigma_{ww}(\varepsilon))^{1-\omega_l}$ of the landlord and tenant surpluses $\Sigma_{wl}(\varepsilon) = L(\varepsilon) - U_l$ and $\Sigma_{ww}(\varepsilon) = W(\varepsilon) - B_l$ with respect to $R(\varepsilon)$ without any commitment to future rent payments. Since the rent is just a transfer between the parties, $\partial\Sigma_{wl}(\varepsilon)/\partial R(\varepsilon) = -\partial\Sigma_{ww}(\varepsilon)/\partial R(\varepsilon)$, as can be seen from equations (9) and (10). The first-order condition is $\Sigma_{wl}(\varepsilon)/\Sigma_{ww}(\varepsilon) = \omega_l/(1-\omega_l)$, so the parties receive $\Sigma_{wl}(\varepsilon) = \omega_l\Sigma_w(\varepsilon)$ and $\Sigma_{ww}(\varepsilon) = (1-\omega_l)\Sigma_w(\varepsilon)$ that share the joint surplus (7) in proportion to their bargaining powers. The average rent across all surviving tenancies is denoted by $\bar{R}$.

New tenancies The rental market has a measure b_l of households looking to rent and a measure u_l of available properties offered by landlords. The ratio of these is the 'tightness' of the rental market, denoted by θ_l. Search frictions limit the speed at which households can meet landlords and view their properties, where a viewing reveals potential match quality and allows offers to be made. The meeting rates of participants on both sides of the market are determined by the constant-returns-to-scale meeting function $\Upsilon_l(b_l, u_l)$, with $v_l = \Upsilon_l(b_l, u_l)/b_l$ being the rate at which households view rental properties. The meeting function having constant returns to scale means v_l and landlords' meeting rate $\theta_l v_l$ are functions of market tightness θ_l:

$$v_l = \frac{\Upsilon_l(b_l, u_l)}{b_l} = \Upsilon_l\left(1, \theta_l^{-1}\right), \quad \text{and} \quad \frac{\Upsilon_l(b_l, u_l)}{u_l} = \theta_l v_l\,, \quad \text{where} \;\; \theta_l = \frac{b_l}{u_l}\,. \tag{11}$$

The meeting function is increasing in both b_l and u_l, hence v_l decreases with θ_l, while $\theta_l v_l$ increases with θ_l. Intuitively, if there are more 'buyers' relative to 'sellers' in the rental market, the meeting rate is lower for those viewing properties, but higher for those with property to let.

Viewings reveal potential match quality, with ε drawn from a probability distribution with CDF $\Gamma_l(\varepsilon)$ when a household views a landlord's property. If mutually agreeable, the household moves in and becomes a tenant. Prior to the revelation of ε, all landlords and households in the rental market are ex ante identical.

The Bellman equation for the value U_l of a landlord having a property available to let is

$$rU_l = -D + \theta_l v_l \int \max\{L(\varepsilon) + A(\varepsilon) - C_l - U_l, 0\} \mathrm{d}\Gamma_l(\varepsilon) + \rho_l(U_o - U_l) + \dot{U}_l\,. \tag{12}$$

A landlord meets households who are potential tenants at rate $\theta_l v_l$. If a tenant with match quality ε moves in, the landlord incurs transaction costs C_l and receives a one-off agreement fee $A(\varepsilon)$ negotiated with the tenant. After this, the landlord's value of having a tenant with match quality ε is $L(\varepsilon)$, which includes the ongoing negotiated rents. The value B_l of a household searching for a property to rent satisfies the Bellman equation:

$$rB_l = g - F_l + v_l \int \max\{W(\varepsilon) - A(\varepsilon) - C_w - B_l, 0\} \mathrm{d}\Gamma_l(\varepsilon) - \rho B_l + \dot{B}_l\,, \tag{13}$$

where F_l is the flow cost incurred while searching for a rental property, and v_l is the rate at which households make rental-market viewings. A household moving into a property directly incurs a moving cost C_w as well as paying the agreement fee $A(\varepsilon)$ to the landlord.

At the stage when a household has viewed a rental property with match quality ε, the surpluses from agreeing a tenancy are $\Sigma_{ll}(\varepsilon) = L(\varepsilon) + A(\varepsilon) - C_l - U_l$ for the landlord and $\Sigma_{lw}(\varepsilon) = W(\varepsilon) - A(\varepsilon) - C_w - B_l$ for the tenant. If the landlord agrees to the tenant moving in after paying a fee $A(\varepsilon)$ then the two parties incur costs C_l and C_w, respectively.[29] The joint surplus $\Sigma_l(\varepsilon) = \Sigma_{ll}(\varepsilon) + \Sigma_{lw}(\varepsilon)$ is

$$\Sigma_l(\varepsilon) = L(\varepsilon) + W(\varepsilon) - U_l - B_l - C_l - C_w\,, \tag{14}$$

and negotiations lead to a tenancy if this is positive. Since $\Sigma_l(\varepsilon)$ is increasing in ε, a tenancy is mutually agreeable if $\varepsilon \geq y_l$, where the leasing threshold y_l is the level of match quality ε where the joint surplus is zero:

$$\Sigma_l(y_l) = 0\,. \tag{15}$$

The probability a viewing leads to a tenancy is $\pi_l = 1 - \Gamma_l(y_l)$. There is Nash bargaining over the agreement fee $A(\varepsilon)$, with landlords having the same bargaining power ω_l as in subsequent rent negotiations, and this implies the same division of the joint surplus $\Sigma_{ll}(\varepsilon) = \omega_l \Sigma_l(\varepsilon)$ and $\Sigma_{lw}(\varepsilon) = (1 - \omega_l)\Sigma_l(\varepsilon)$ as prevails when rents are agreed.

Comparison of the joint surpluses (7) and (14) before and after a tenant moves in shows that $\Sigma_l(\varepsilon) = \Sigma_w(\varepsilon) - C_l - C_w$.[30] Since the landlord's surplus at the meeting stage is $\Sigma_{ll}(\varepsilon) = \Sigma_{wl}(\varepsilon) + A(\varepsilon) - C_l$ in terms of the surplus $\Sigma_{wl}(\varepsilon)$ after the tenancy is agreed, the fee-bargaining equation combined with the rent-bargaining equation $\Sigma_{wl}(\varepsilon) = \omega_l \Sigma_w(\varepsilon)$ imply that the outcomes of the initial negotiations when tenants move in are

$$A(\varepsilon) = A = (1 - \omega_l)C_l - \omega_l C_w\,, \quad \text{and } R = \frac{1}{\pi_l} \int_{y_l} R(\varepsilon) \mathrm{d}\Gamma_l(\varepsilon)\,, \tag{16}$$

[29] The transaction costs C_l and C_w are a type of fixed matching cost, for example, the costs of finding out about the tenant, because they are incurred before bargaining over the rent takes place (see Pissarides, 2009).

[30] This means $\Sigma_w(y_l) = C_l + C_w$ is positive when tenants move in, so moving generally occurs only after a shock.

where the agreement fee A is independent of ε, and R is the average rent on a new tenancy. The flow of new leases S_l and the rate s_l at which available rental properties are leased are

$$S_l = v_l \pi_l b_l \,, \quad \text{and} \;\; s_l = \frac{S_l}{u_l} = \theta_l v_l \pi_l \,. \tag{17}$$

The laws of motion for the stock of occupied rental properties q_l, the measure of households b_h searching in the rental market, and the stock of available properties to let u_l are

$$\dot{q}_l = s_l u_l - (m_l + \rho) q_l \,, \tag{18}$$

$$\dot{b}_l = (1 - \xi \kappa) m_l q_l + (1 - \kappa) a - (v_l \pi_l + \rho) b_l \,, \quad \text{and} \tag{19}$$

$$\dot{u}_l = (\alpha_l + \rho) q_l + S_i - (s_l + \rho_l) u_l \,, \tag{20}$$

where S_i denotes the flow of purchases by investors bringing properties into the rental market.

3.4 The ownership market

Owner-occupiers Households occupying a property they own receive a match-specific flow value ε. Match quality ε is a persistent variable subject to occasional shocks representing life events that make a property less well suited to the household occupying it than before. Shocks arrive independently across households and across time at rate α_h, and the arrival of a shock reduces match quality from ε to $\delta_h \varepsilon$, where $\delta_h < 1$ is a parameter.[31] Following a shock, owner-occupiers decide whether to move and start searching for another property to live in, putting their current property up for sale.[32] Moving is endogenous and depends on how low match quality has become relative to expectations of match quality in an alternative property.[33]

The Bellman equation for an owner-occupier's value $H(\varepsilon)$ with match quality ε is

$$rH(\varepsilon) = \varepsilon + g - D + \alpha_h \left(\max\left\{ H(\delta_h \varepsilon), B_h + U_o \right\} - H(\varepsilon) \right) + \rho \left(U_o - H(\varepsilon) \right) + \dot{H}(\varepsilon) \,. \tag{21}$$

An owner-occupier receiving a match-quality shock decides whether to remain in the property and receive value $H(\delta_h \varepsilon)$, or to move out and become both a seller and a home-buyer, which has a combined value $B_h + U_o$. Since the value function $H(\varepsilon)$ is increasing in ε, owner-occupiers decide to move if current match quality becomes sufficiently low. The condition for moving is $\delta_h \varepsilon < x_h$, where the moving threshold x_h is the level of match quality such that the value of continuing to occupy a property equals the sum of the outside options B_h and U_o of being both a buyer and a seller in the ownership market:

$$H(x_h) = B_h + U_o \,. \tag{22}$$

[31] The model has no shocks that increase match quality, but such shocks would not cause households to move.

[32] Implicitly, a decision-making cost dissuades owner-occupiers from moving if no shock has been received.

[33] In the rental market, match-quality shocks with arrival rate α_l reduce ε to zero, which effectively means having a parameter δ_l of zero there. It is possible to extend the model to allow for $\delta_l > 0$, however, it turns out that endogeneity of moving by renters within the rental market is quantitatively unimportant here, so $\delta_l = 0$ acts as a simplifying assumption. Note that the shock arrival rates α_h and α_l can differ by housing tenure.

A parameter restriction is that match-quality shocks are sufficiently large (δ_h is far enough below 1) so that some, but not all, owner-occupiers move after only one idiosyncratic shock.

Sellers and buyers The ownership market has a measure of sellers u_o and a total measure of buyers b_o comprising both home-buyers b_h and investors b_i. Just as in the rental market, search frictions limit the speed at which sellers and buyers can meet, which is necessary for buyers to view properties. There is a constant-returns-to-scale meeting function $\Upsilon_o(b_o, u_o)$ for the ownership market that determines the meeting rate v_o of buyers:

$$v_o = \frac{\Upsilon_o(b_o, u_o)}{b_o} = \Upsilon_o\left(1, \theta_o^{-1}\right), \quad \text{where } b_o = b_h + b_i \text{ and } \theta_o = \frac{b_o}{u_o}, \tag{23}$$

and this meeting rate is a decreasing function of ownership-market tightness θ_o. The meeting function here can differ from the one in the rental market. Sellers' meeting rate is $\theta_o v_o$, and the probabilities that a given meeting is with an investor or a home-buyer are ψ and $1-\psi$:

$$\frac{\Upsilon_o(b_o, u_o)}{u_o} = \Upsilon_o(\theta_o, 1) = \theta_o v_o, \quad \text{and } \psi = \frac{b_i}{b_o}, \tag{24}$$

where random search means that the probability ψ is the fraction of investors among all buyers.

Since properties are ex ante identical, those who were owner-occupiers or landlords both have a common expected value U_o from selling a property. The Bellman equation for U_o is

$$rU_o = -D + \theta_o v_o \Bigg((1-\psi) \int \max\{P_h(\varepsilon) - C_o - U_o, 0\} \, \mathrm{d}\Gamma_h(\varepsilon) + \psi \max\{P_i - C_o - U_o, 0\} \Bigg) + \dot{U}_o, \tag{25}$$

where C_o is a transaction cost paid by sellers. After meeting a potential buyer who views the property, revealing a home-buyer's match quality, the buyer and seller negotiate a price and a transaction occurs if mutually agreeable. The price may depend on the known type of the buyer and on the match quality. All investors pay the same price P_i because they face the same expected rents when their property is let, but home-buyers pay different prices $P_h(\varepsilon)$ because of idiosyncratic match quality.

The land transfer tax (LTT) is a proportional tax levied on the transaction prices paid by buyers. Home-buyers and investors face tax rates τ_h and τ_i, which in principle can differ. With a flow of S_h home-buyer purchases at average price P_h and S_i investor purchases at price P_i, the tax revenue available to spend on public goods G yielding benefit g per city resident is

$$G = \tau_h P_h S_h + \tau_i P_i S_i, \quad \text{and } g = \frac{G}{n}. \tag{26}$$

Home-buyers The Bellman equation for the expected value B_h of being a home-buyer is

$$rB_h = g - F_h + v_o \int \max\{H(\varepsilon) - C_h - (1+\tau_h)P_h(\varepsilon) - B_h, 0\} \, \mathrm{d}\Gamma_h(\varepsilon) - \rho B_h + \dot{B}_h, \tag{27}$$

where F_h is a flow search cost incurred while looking for and viewing properties. Buyers make viewings at rate v_o, revealing match quality ε drawn from distribution $\Gamma_h(\varepsilon)$, which can differ from $\Gamma_l(\varepsilon)$ in the rental market. If a transaction with price $P_h(\varepsilon)$ goes ahead, $\tau_h P_h(\varepsilon)$ is the tax paid by the home-buyer, other transaction costs C_h such as moving costs are incurred, and the home-buyer obtains value $H(\varepsilon)$ of being an owner-occupier of a property with match quality ε.

House prices are determined by Nash bargaining between buyers and sellers. If the seller of a property meets a home-buyer who draws match quality ε and were to agree to sell at price $P_h(\varepsilon)$ then the home-buyer's surplus is $\Sigma_{hh}(\varepsilon) = H(\varepsilon) - (1+\tau_h)P_h(\varepsilon) - C_h - B_h$ and the seller's surplus is $\Sigma_{hu}(\varepsilon) = P_h(\varepsilon) - C_o - U_o$. The Nash bargaining problem is to choose $P_h(\varepsilon)$ to maximize $(\Sigma_{hu}(\varepsilon))^{\omega_h}(\Sigma_{hh}(\varepsilon))^{1-\omega_h}$, where ω_h is the seller's bargaining power when facing a home-buyer. The first-order condition is $(1+\tau_h)\Sigma_{hu}(\varepsilon)/\Sigma_{hh}(\varepsilon) = \omega_h/(1-\omega_h)$, which determines how the joint surplus $\Sigma_h(\varepsilon) = \Sigma_{hh}(\varepsilon) + \Sigma_{hu}(\varepsilon)$ is to be shared.

In the absence of a proportional transaction tax τ_h, the surplus would be divided according to bargaining powers in line with the usual Nash rule. However, the tax skews the surplus division in favour of the buyer because the joint surplus $\Sigma_h(\varepsilon) = H(\varepsilon) - C_h - C_o - B_h - U_o - \tau_h P_h(\varepsilon)$ is raised by agreeing a lower price, and this lower price increases the buyer's share. The resulting split of the surplus is $\Sigma_{hh}(\varepsilon) = (1-\omega_h^*)\Sigma_h(\varepsilon)$ and $\Sigma_{hu}(\varepsilon) = \omega_h^*\Sigma_h(\varepsilon)$ with the seller's share ω_h^* being below the parameter ω_h. The price achieving this division is $P_h(\varepsilon) = C_o + U_o + \omega_h^*\Sigma_h(\varepsilon)$. The joint surplus after tax and the seller's share that emerge from the bargaining are

$$\Sigma_h(\varepsilon) = \frac{H(\varepsilon) - C_h - B_h - (1+\tau_h)(C_o + U_o)}{1+\tau_h\omega_h^*}, \quad \text{and } \omega_h^* = \frac{\omega_h}{1+\tau_h(1-\omega_h)}. \tag{28}$$

As match quality ε is observable and surplus is transferable, transactions go ahead if the total surplus is non-negative. Since $H(\varepsilon)$ is increasing in ε, this occurs if $\varepsilon \geq y_h$, where the transaction threshold y_h is the level of match quality where the joint surplus is zero:

$$\Sigma_h(y_h) = 0. \tag{29}$$

The probability that a viewing by a home-buyer leads to a transaction is $\pi_h = 1 - \Gamma_h(y_h)$, and the average home-buyer transaction price P_h paid is

$$P_h = \frac{1}{\pi_h}\int_{y_h} P_h(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon) = \frac{\omega_h^*\Sigma_h}{\pi_h} + C_o + U_o, \quad \text{where } \Sigma_h = \int_{y_h}\Sigma_h(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon), \tag{30}$$

with Σ_h denoting the ex-ante joint surplus from a home-buyer viewing prior to ε being realized.

The laws of motion for home-buyers b_h and the stock of owner-occupied properties q_h are

$$\dot{b}_h = m_h q_h + \gamma - (v_o\pi_h + \rho)b_h, \quad \text{and} \tag{31}$$

$$\dot{q}_h = S_h - (m_h + \rho)q_h, \quad \text{where } S_h = v_o\pi_h b_h, \tag{32}$$

and m_h is the endogenous rate at which owner-occupiers move within the city.

Investors The Bellman equation for the value I of being an investor who buys at price P_i is

$$rI = -F_i + v_o\left(U_l - (1+\tau_i)P_i - C_i - I\right) + \dot{I}\,, \tag{33}$$

where F_i is the flow search cost incurred by investors until they buy, $\tau_i P_i$ is the amount of tax paid, and C_i is any other transaction costs. Investors meet sellers at rate v_o, and because investors have no idiosyncratic match quality with properties themselves, this is also the rate at which they are able to buy. After buying, investors make properties available for rent and receive the common expected value U_l of being a landlord.

After meeting a seller, an investor's surplus is $\Sigma_{ii} = U_l - (1+\tau_i)P_i - C_i - I$ and the seller's surplus is $\Sigma_{iu} = P_i - C_o - U_o$. If there are mutual gains from a deal, the price P_i is determined by Nash bargaining, where the seller has bargaining power ω_i when facing an investor. The joint surplus $\Sigma_i = \Sigma_{ii} + \Sigma_{iu}$ is split according to $(1+\tau_i)\Sigma_{iu}/\Sigma_{ii} = \omega_i/(1-\omega_i)$, so the tax τ_i shifts the division of the surplus $\Sigma_{ii} = (1-\omega_i^*)\Sigma_i$ and $\Sigma_{iu} = \omega_i^*\Sigma_i$ in favour of investors with $\omega_i^* < \omega_i$.

Since there is no match quality in the joint surplus $\Sigma_i = U_l - C_i - C_o - U_o - I - \tau_i P_i$, either all investors are willing to buy or none is, so an equilibrium with an active rental market requiring entry of investors occurs if and only if Σ_i is non-negative. If this is true, investors buy properties at the rate v_o they meet sellers, and the price paid by all investors is

$$P_i = C_o + U_o + \omega_i^*\Sigma_i\,, \quad \text{where} \;\; \omega_i^* = \frac{\omega_i}{1+\tau_i(1-\omega_i)}\,. \tag{34}$$

The flow S_i of sales to investors, and the share i of these transactions among all sales S_o are

$$S_i = v_o b_i, \quad \text{and} \;\; i = \frac{S_i}{S_o} = \frac{\psi}{\psi + (1-\psi)\pi_h}\,, \quad \text{where} \;\; S_o = S_h + S_i\,, \tag{35}$$

the expression for i following from equations (24) and (32). The rate s_o at which sellers complete transactions and the breakdown into home-buyer and investor purchases are

$$s_o = \frac{S_o}{u_o} = \theta_o v_o\left(\psi + (1-\psi)\pi_h\right)\,, \quad \text{with} \;\; S_i = i s_o u_o \;\; \text{and} \;\; S_h = (1-i)s_o u_o\,. \tag{36}$$

The average transaction price is $P = iP_i + (1-i)P_h$.

Investors are free to enter the ownership market to buy properties and become landlords. The measure b_i adjusts so that at all times the value of entry by further investors is zero:

$$I = 0\,. \tag{37}$$

The Bellman equation (33), the price (34) for investor purchases, and the free-entry condition (37) imply that the joint surplus $\Sigma_i = U_l - C_i - C_o - I - \tau_i P_i$ satisfies

$$\Sigma_i = \frac{U_l - (1+\tau_i)U_o - (1+\tau_i)C_o - C_i}{1+\tau_i\omega_i^*} = \frac{F_i}{(1-\omega_i^*)v_o}\,. \tag{38}$$

which shows the surplus Σ_i rises with the tightness θ_o of the ownership market. Intuitively, the viewing rate v_o decreases when there are more buyers relative to sellers, so investors must be compensated in equilibrium by a higher surplus $(1-\omega_i^*)\Sigma_i$ for them to enter. Note that a non-negative joint surplus Σ_i implies the value U_l of having a property to let is always above the value of having a property for sale U_o. Thus, after purchasing a property, an investor always prefers to keep it rented out, and landlords sell properties only when hit by exit shocks.[34]

The law of motion for the stock of properties for sale u_o is

$$\dot{u}_o = (m_h+\rho)q_h + \rho_l(q_l+u_l) - s_o u_o\,. \tag{39}$$

Properties come up for sale if and only if owner-occupiers move within or exit the city, or landlords are hit by an exit shock, irrespective of whether their properties are currently occupied. A summary of the stocks and flows in the model is shown in Figure A.4 and A.5 in the Appendix.

3.5 Functional forms

Solving the model requires specifying probability distributions of credit costs and initial match qualities revealed by viewings, and the functional forms of the meeting functions. New match qualities ε are drawn from Pareto distributions indexed by $j \in \{h,l\}$ for home-buyers (h) and tenants in the rental market (l):

$$\Gamma_j(\varepsilon) = 1 - \left(\frac{\varepsilon}{\zeta_j}\right)^{-\lambda_j} \text{ for } j \in \{h,l\}, \quad \text{and hence } \pi_j = \int_{y_j} \mathrm{d}\Gamma_j(\varepsilon) = \left(\frac{y_j}{\zeta_j}\right)^{-\lambda_h}, \tag{40}$$

with ζ_j being the minimum possible draw of ε and $\lambda_j > 1$ specifying the distribution shape, in particular, how compressed are realizations of ε towards the minimum. Expected match quality from a market-j viewing is $\mathbb{E}_j[\varepsilon] = \zeta_j\lambda_j/(\lambda_j-1)$, and the transaction probabilities π_j are decreasing in the thresholds y_j relative to ζ_j. The Pareto distribution also provides a closed-form expression for owner-occupiers' endogenous moving rate m_h in the laws of motion (31) and (32).[35] As shown in appendix A.2.2, m_h depends on the moving threshold x_h and the history of past home-buyer viewings:

$$m_h = \alpha_h - \frac{\alpha_h \zeta_h^{\lambda_h} \delta_h^{\lambda_h} x_h^{-\lambda_h}}{q_h} \int_{T\to-\infty}^{t} e^{-\left(\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)(t-T)}(1-\psi(T))\theta_o(T)v_o(T)u_o(T)\mathrm{d}T\,, \tag{41}$$

where $u_o(T)$ denotes the level of u_o at time T, and similarly for other variables.

The distribution of credit costs K is log Normal:

$$\Gamma_k(K) = \Phi\left(\frac{\log K - \mu}{\sigma}\right), \quad \text{implying} \quad \bar{K} = e^{\mu+\frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log Z-\mu-\sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log Z-\mu}{\sigma}\right)}, \tag{42}$$

where μ and σ are mean and standard deviation parameters and $\Phi(\cdot)$ is the standard Normal CDF. The meeting functions $\Upsilon_j(b_j,u_j)$ indexed by $j \in \{o,l\}$ for ownership (o) and rental mar-

[34] In other words, pure 'flippers' — those who buy and sell shortly afterwards — are not present in the model.

[35] The condition that some owner-occupiers require only one shock to trigger moving is $\delta_h y_h < x_h$.

kets (l) have Cobb-Douglas functional forms:

$$\Upsilon_j(b_j, u_j) = \upsilon_j b_j^{1-\eta_j} u_j^{\eta_j}, \quad \text{and hence } \; v_j = \upsilon_j \theta_j^{-\eta_j}, \tag{43}$$

where υ_j is productivity in arranging viewings in market j, and η_j are the elasticities of buyers' and renters' viewing rates with respect to market tightnesses $\theta_j = b_j/u_j$.

3.6 Welfare

To assess the potential deadweight losses of transaction taxes both across rental and ownership markets and within each market, the welfare measure should include everyone who either pays or receives prices or rents. Welfare Ω is the sum of the value functions of all incumbents in the city (homeowners, tenants, landlords, and including owners of unsold houses who have left the city) plus the values of those who enter the city. Exit from the city (with value 0) is already accounted for in incumbents' values.

The welfare analysis takes into account that tax revenue is spent on public goods G of an equal value (see 26) because the per-person flow benefits $g = G/n$ appear in the Bellman equations of city residents. The expected payoff of someone entering the city is N from (5) and a is the flow of new arrivals from (6), so the present value Ω_a of these payoffs for all entrants satisfies the Bellman equation $r\Omega_a = aN + \dot{\Omega}_a$.

With $\bar{H}$, $\bar{L}$, and $\bar{W}$ denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of ε for all surviving matches (in q_h for $\bar{H}$, and in q_l for $\bar{L}$ and $\bar{W}$), total welfare is $\Omega = q_h\bar{H} + q_l(\bar{L} + \bar{W}) + b_h B_h + b_l B_l + b_i I + u_o U_o + u_l U_l + \Omega_a$. Appendix A.2.7 shows that the measure of welfare Ω satisfies the differential equation

$$r\Omega = q_h V_h + q_l V_l - D - q_l D_l - b_h F_h - b_i F_i - b_l F_l - S_h C_h - S_i C_i - S_o C_o \\ - S_l(C_l + C_w) - \gamma\bar{K} - aE + \dot{\Omega}, \tag{44}$$

where V_h and V_l denote average current match quality ε across the q_h owner-occupiers and q_l tenants respectively.[36] Prices and rents drop out from Ω because these are just transfers among market participants who are all included in the welfare measure. Maintenance costs D and D_l, flow search costs F_h, F_i, and F_l, non-tax transaction costs C_h, C_i, C_o, C_l, and C_w, credit costs $\bar{K}$, and entry costs E are resource costs that show up as deductions from welfare. This is because transaction costs reflect the time and resources of market participants and intermediaries that are consumed in completing transactions. Likewise, credit costs, for example, interest-rate spreads on mortgages, are treated as reflecting resources used up by banks. Transaction tax revenue is not deducted from (44) because it pays for public goods of an equivalent value.

The average match qualities V_h and V_l appearing in the welfare equation (44) are shown in

[36] This assumes all private benefits of owning or renting properties are social benefits. It is possible to envisage other policy distortions that might drive a wedge between private and social benefits such as the tax treatment of owners' implicit rental income or mortgage-interest deductibility.

appendix A.2.6 to satisfy the following pair of differential equations:

$$\dot{V}_h = \frac{(1-i)s_o u_o}{q_h}\left(\frac{\lambda_h}{\lambda_h - 1}y_h - V_h\right) - (\alpha_h - m_h)\left(V_h - \frac{\lambda_h}{\lambda_h - 1}x_h\right), \quad \text{and} \tag{45}$$

$$\dot{V}_l = \frac{s_l u_l}{q_l}\left(\frac{\lambda_l}{\lambda_l - 1}y_l - V_l\right), \tag{46}$$

which depend on differences between V_h and V_l and average new match qualities $\lambda_h y_h/(\lambda_h - 1)$ and $\lambda_l y_l/(\lambda_l - 1)$ in the two markets, and between V_h and average surviving match quality $\lambda_h x_h/(\lambda_h - 1)$ after match-quality shocks are received by owner-occupiers.

3.7 The steady state of the model

For constant tax rates τ_h and τ_i and other parameters, the model predicts convergence to a steady state for aggregate variables such as the fractions of properties and households in the various states $(q_h, q_l, u_o, u_l, b_h, b_l)$. While individual households are subject to idiosyncratic shocks affecting their match quality and housing tenure, there are stationary distributions of match quality in the rental and ownership markets. The relationships between steady-state values of variables and parameters are used to calibrate these parameters, allowing quantitative predictions about the effects of transaction taxes and their implications for welfare.

The calibration strategy described later makes use of a number of steady-state predictions of the model. For example, since home-buyers complete transactions at the same average rate, equation (31) implies the steady-state fraction of first-time buyers is $\phi = \gamma/(\gamma + m_h q_h)$, where γ and $m_h q_h$ are respectively the flows of first-time buyers and owner-occupiers who move within the city. Other steady-state moments, such as the average age difference $\aleph$ between owner-occupiers and tenants, are derived in appendix A.3.5.

Given transaction probabilities π_l and π_h for households conditional on a viewing, the average number of viewings made before renting or buying a property are $\Lambda_l = 1/\pi_l$ and $\Lambda_h = 1/\pi_h$, respectively. With renters and buyers making viewings at rates v_l and v_o, the expected times taken to find a property to live in are $T_{bl} = 1/(v_l \pi_l)$ and $T_{bh} = 1/(v_o \pi_h)$ for these two groups. Taking account of exit from the city, the expected lengths of time tenants and owner-occupiers live in a particular property are $T_{ml} = 1/(m_l + \rho)$ and $T_{mh} = 1/(m_h + \rho)$, respectively. Finally, from the perspective of property owners, the expected times on the market to lease or to sell are $T_{sl} = 1/s_l$ and $T_{so} = 1/s_o$.

4 Steady-state effects of transaction taxes

The effects of higher transaction taxes τ_h and τ_i in the model are determined by the behavioural responses of tenants, owner-occupiers, investors, and new entrants, which have implications both across and within the rental and ownership markets. This section lays out the intuition for how the model explains the empirical findings in Table 1 and 2. It focuses on the long-run tax effects, which are understood through the impact on the model's steady state, deferring discussion of short-run effects to section 5 where the model's transitional dynamics are presented.

4.1 Behaviour of households

There are three household behavioural responses to a higher tax rate τ_h. First, equation (28) shows higher τ_h has the direct effect of reducing the joint surplus Σ_h from purchases by home-buyers due to part of the surplus being absorbed by higher taxes. This happens because the cost of transactions both now and when moving again in the future becomes larger. The fall in the joint surplus reduces the value of being a home-buyer,[37] as is seen from the steady-state Bellman equation (27):

$$(r+\rho)B_h = (1-\omega_h^*)v_o\Sigma_h + g - F_h\,. \tag{47}$$

The fall in the home-buyer value B_h reduces tenants' incentive to enter the ownership market. In the indifference condition (3) for the marginal first-time buyer, the equilibrium credit-cost threshold Z falls, which means there are fewer first-time buyers γ according to (4).

Second, a higher tax rate τ_h raises the cost of moving, which makes owner-occupiers more tolerant of worse match quality, manifested in a lower moving threshold x_h. Just as the smaller joint surplus Σ_h means a fall in the value B_h of being a home-buyer, it also reduces the value U_o of being a seller, noting that the steady-state Bellman equation of a seller (25) is

$$rU_o = \theta_o v_o\left(\omega_h^*(1-\psi)\Sigma_h + \omega_i^*\psi\Sigma_i\right) - D\,. \tag{48}$$

A seller's value is a weighted average of the expected values from selling to a home-buyer or an investor. When the share of investors ψ is small, the fall in the joint surplus Σ_h is the dominant effect and U_o declines. Together with the fall in B_h, (22) shows that the moving threshold x_h is lower, which results in longer average times between moves T_{mh} given (41).

Finally, home-buyers become pickier when τ_h rises, that is, they choose a higher transaction threshold y_h. As moving decisions are endogenous and match quality has persistence, home-buyers can reduce the future incidence of moving — and lower the tax they expect to pay — by starting with better match quality. This intuition is confirmed by (29) where the joint surplus is an increasing function of y_h (21 shows owner-occupiers' value $H(\varepsilon)$ is increasing in ε). A higher tax τ_h reduces the joint surplus, requiring higher y_h for the marginal joint surplus to be zero. This higher transaction threshold results in longer average times taken to sell T_{so}.[38]

All three household responses to a higher tax rate τ_h contribute to a fall in buy-to-own transactions S_h by reducing purchases made by first-time buyers and existing owner-occupiers.

[37] While higher τ_h increases buyers' share $1-\omega_h^*$ of the joint surplus, since $(1-\omega_h^*)/(1+\tau_h\omega_h^*) = 1-\omega_h$ using (28), the overall effect of τ_h on $(1-\omega_h^*)\Sigma_h$ is unambiguously negative. Higher taxes also increase g in (47), but that affects all city residents equally and does not lead to a behavioural response of those already living in the city.

[38] As is shown empirically in the Appendix in Table A.6.

4.2 Behaviour of investors

Similar to the effect on owner-occupiers, the direct effect of a higher tax rate τ_i is to reduce entry of buy-to-rent investors. However, investors who become landlords do not have to sell their properties and pay the transaction tax again just because a tenant moves out. They are in a different position from owner-occupiers, who have to buy again and pay the tax every time they move. Buy-to-rent investors thus have an implicit tax advantage — even if they face the same tax rates. Intuitively, investors can spread the transaction tax over a longer holding period, which reduces the negative direct effect of the tax on their entry decision.

It is not straightforward directly to estimate average holding periods of investors owing to constraints on available data and the right-censoring problem arising when the sample is not long enough to observe investors' completed holding periods. However, information on the flow of buy-to-rent transactions and the stock of properties rented out can be used to derive investors' implied average holding period relative to that of owner-occupiers.

The logic is that a relatively longer holding period of investors is an implication of investors' share i of transaction flows from (35) being smaller than their share of the stock of properties, which is closely related to $1-h$, where h is the homeownership rate from (2). To see this, note that the average holding period of investors in the model is the inverse of their exit rate ρ_l. Tenants' moving rate (8), the laws of motion (18) and (20), and investor transactions from (36) imply the stock of properties $q_l + u_l$ in the rental market satisfies

$$\dot{q}_l + \dot{u}_l = i s_o u_o - \rho_l (q_l + u_l) . \tag{49}$$

This shows that buy-to-rent as a share i of all transactions governs inflows of properties into the rental market, while investors' exit rate ρ_l governs outflows of properties.

Using the homeownership rate $h = (q_h + b_h)/n$ from (2), the law of motion for the stock of owner-occupied properties in (32), and the fact that b_h and $u_l - b_l$ are very small relative to q_h, the steady-state average holding period of an investor relative to that of an owner-occupier is[39]

$$\frac{m_h + \rho}{\rho_l} \approx \left(\frac{1-h}{h}\right) \Big/ \left(\frac{i}{1-i}\right) \quad \text{as} \quad \frac{b_h}{q_h} \approx 0 \;\text{ and }\; \frac{u_l - b_l}{q_h} \approx 0 . \tag{50}$$

In Toronto before the transaction tax increase, the buy-to-rent share i of transactions was about 5%, whereas the homeownership rate h was 54%. This means that the average holding period of an investor must be much longer than the average holding period of an owner-occupier. In other words, the amortized flow cost of the same transaction tax is much smaller for investors.

An important consequence of these observations is that the direct effect of a transaction tax on investors is smaller than its direct effect on owner-occupiers. In the rental market,

[39] Using equations (32) and (49), the exact expression for (50) is

$$\frac{m_h + \rho}{\rho_l} = \left(\frac{1-h}{h} \left(1 - \frac{b_h}{q_h + b_h}\right)^{-1} + \frac{u_l - b_l}{q_h} \right) \Big/ \left(\frac{i}{1-i}\right) .$$

the direct effects of a higher tax are reduced entry of investors while entry by newly arrived households is increased and exit by existing tenants decreased. All of these developments increase the tightness of the rental market and push up the rent-to-price ratio. Through the free-entry condition (38), the equilibrium effect of the higher rent-to-price ratio creates incentives for more investors to enter. If the average investor holding period is sufficiently long so that the direct negative effect of the tax on investors is weak and dominated by the positive equilibrium effect, the model implies a rise in buy-to-rent transactions and a fall in the homeownership rate.

4.3 Behaviour of new entrants to the city

For given house prices, a higher transaction tax implies a fall in the expected value of being an owner-occupier in the city. While the imposition of the tax directly makes entry less attractive for households that might become owner-occupiers, this is offset by the extra public spending that the tax revenue finances, from which all city residents benefit.

As the housing stock of the city is fixed, and since houses must be owned or rented by someone in equilibrium, the value of living in the city must adjust through house prices or rents so as to leave the population approximately constant. This continues until households are indifferent between entering or not, that is, $N = 0$ in the new steady state (see 6).

5 Quantitative effects of transaction taxes

This section uses the model from section 3 to study quantitatively the effects of transaction taxes such as the Toronto LTT. The model quantifies the differential effects on owner-occupiers and investors, even when both groups face the same tax rate, as they did in Toronto before and after the increase in the LTT. The model also quantifies the tax effects on leasing activity in the rental market and the price-to-rent ratio. Finally, the model is used for a quantitative assessment of the welfare costs of transactions taxes.

The empirical estimates of the LTT effects from section 2 rely on comparing the City of Toronto to other areas in the GTA before and after the introduction of city-level LTT. The estimated effects inside the city were derived using a difference-in-differences approach across the city border as the aim was to isolate the impact of the LTT from other changes. Thus, the quantitative predictions of the model can be directly compared to the empirical estimates.

As explained in section 2, the effective LTT rate in the City of Toronto rises by 1.3 percentage points in February 2008 (from 1.5% to 2.8%, see Table A.2 in the Appendix). The model is calibrated so that it matches the estimated effect of the LTT increase on the hazard rate of moving for owner-occupiers. The model's predictions for other untargeted moments then serve as a basis for external validation.

The model is solved with the effective transaction tax rates prevailing in the City of Toronto before and after the new city-level LTT was introduced. A full solution of the model's dynamics is obtained to compare to the empirical results in a given window of time after the tax increase. The change in the tax rate is treated as unanticipated, and no further changes are expected, which corresponds to a perfect-foresight equilibrium of the model.

Starting from an initial steady state of the model, which can be found using the method set out in appendix A.3, the analysis derives the transitional path to the new steady state after the tax increase. The dynamics are computed by first discretizing the system of differential equations describing the model, replacing them with difference equations where a discrete time period is equal to one day. The non-linear system of difference equations is then solved computationally as explained in appendix A.4 using knowledge of the new steady state.

5.1 Calibration

The parameters of the model are calibrated so that its steady state fits features of the markets for renting or purchasing property in the City of Toronto in the period January 2006–January 2008 before the LTT change when the effective tax rates were $\tau_h = \tau_i = 0.015$. The list of calibration targets is given in Table 3 and the implied parameter values are reported in Table 4.

To give an overview, there are two broad sets of targets for the pre-tax-change steady state. The first set of targets is directly imposed and the second set comprises targets matching particular empirical observations. The data sources of all the empirical targets are detailed in appendix A.5. The calibration procedure that links targets to parameters and shows how to compute the exactly matching parameter values is set out in appendix A.6. The discussion below provides some intuition behind the procedure, though in general, individual parameters are often linked to several pieces of information.

5.1.1 Directly imposed targets

The calibration targets a city population n equal to the measure 1 of properties. The sensitivity χ of the flow of new households to the value of entering the city is set to 1, but as discussed later in section 5.4.2, this parameter has only a negligible impact on the results. In place of a direct measure of the entry cost E, this parameter is set so that the value B_l of being a tenant is zero, which is the lowest entry cost such that no entrant (or incumbent) subsequently wants to leave the city (noting that $B_h > B_l$), where the value outside the city is normalized to zero.

The elasticities η_l and η_o of the meeting functions for the two markets with respect to available properties are set equal to the respective bargaining powers of landlords and sellers. Specifically, $\eta_l = \omega_l$, and $\eta_o = \omega_o$, where $\omega_o = \psi\omega_i + (1-\psi)\omega_h$ denotes the average bargaining power of sellers facing a fraction ψ of investors and $1-\psi$ of home-buyers. This is analogous to the Hosios condition typically assumed in the labour-search literature. The bargaining powers themselves are identified by other empirical targets specified below.

There is no direct measure of the flow costs of searching F_h, F_l, and F_i. The approach taken here is to base an estimate of search costs on the opportunity cost of time spent searching. Since

Table 3: *Calibration targets*

A. Targets for pre-tax-change steady state	Notation	Value
Directly imposed targets		
Equal numbers of households and properties in the city	n	1
Speed of adjustment of the city population	χ	1
No incentive for households to choose to leave the city	B_l	0
Bargaining powers equal to meeting-function elasticities	$\omega_o/\eta_o = \omega_l/\eta_l$	1
Cost per viewing for home-buyers relative to daily income	$(F_h/v_o)/(Y/365)$	0.5
Viewings per renter relative to viewings per home-buyer	Λ_l/Λ_h	0.5
Cost of a rental viewing relative to a home-buyer viewing	$(F_l/v_l)/(F_h/v_o)$	0.5
Flow search costs of investors relative to home-buyers	F_i/F_h	1
Empirical targets		
Homeownership rate	h	54%
Buy-to-rent as a share of all transactions	i	5.4%
Average price-rent ratio for the same properties	P_i/R	14.5
Average of price paid by investors relative to home-buyers	P_i/P_h	99%
Fraction of first-time buyers among all home-buyers	ϕ	40%
Difference of average ages of owner-occupiers and tenants	$\aleph$	8.3
Risk-free real interest rate	r_g	1.86%
Average real mortgage interest rate	$\bar{r}_k$	4.93%
Real mortgage interest rate faced by marginal home-buyer	r_z	6.43%
Initial loan-to-value ratio of first-time buyers	l	80%
Mortgage term	T_k	25
Non-tax transaction costs of buyers as a fraction of price	$C_h/P_h = C_i/P_i$	0%
Property maintenance costs as a fraction of price	D/P	2.6%
Landlords' extra maintenance/management costs relative to rent	D_l/R	8%
Sellers' transaction costs as a fraction of price	C_o/P	4.5%
Landlords' transaction costs as a fraction of rent	C_l/R	8.3%
New tenancy agreement fee as a fraction of landlord costs	A/C_l	0%
Sellers' average time on the market	T_{so}	0.161
Home-buyers' average time on the market	T_{bh}	0.206
Landlords' average time on the rental market	T_{sl}	0.066
Average viewings per home-buyer	Λ_h	20.6
Average time between moves for owner-occupiers	T_{mh}	9.25
Average time between moves for tenants	T_{ml}	3.04
Ratio of house prices to income	P_h/Y	5.6
Average transaction price of a property	P	\$402k
Effective land transfer tax rate for all buyers	$\tau_h = \tau_i$	1.5%
B. Matched response to the new LTT		
Change in logarithm of moving rate of owner-occupiers	β_{mh}	−0.13

Notes: Time units are years and monetary units are 2007 Canadian dollars. See appendix A.5 for data sources.

home-buyers make viewings at rate v_o and incur flow search costs F_h per unit of time, the cost per viewing is F_h/v_o. Assuming that viewing a property entails the loss of half a day's income, this means that $F_h/v_o = 0.5 \times Y/365$, where Y denotes households' average annual income.

There is data related to the search behaviour of home-buyers, sellers, and landlords, but no information on the search behaviour of renters. Anecdotal evidence suggests that renters usually spend less time per viewing and also view fewer properties. Hence, it is assumed that viewing a rental property takes half the time needed to view a property to buy ($F_l/v_l = 0.5 \times F_h/v_o$), and the average number of viewings per renter is half the number for a home-buyer ($\Lambda_l = 0.5 \times \Lambda_h$). Finally, the per-period flow search costs F_h and F_i are assumed to be the same for home-buyers and investors, which means the cost per viewing is the same. Nonetheless, the total search costs of a home-buyer per transaction are larger than those of an investor given home-buyers' longer search times in the presence of match-specific quality.

5.1.2 Empirical targets

One group of empirical targets is related to the extensive margin across rental and ownership markets, including the homeownership rate h, the fraction i of buy-to-rent transactions among all transactions, investors' price-to-rent ratio P_i/R, the fraction ϕ of first-time buyers, and the difference $\aleph$ between the average ages of owner-occupiers and tenants.

Intuitively, P_i/R, the inverse of the gross rental yield, provides information about the discount rate r. Investors' share of transactions i, along with the homeownership rate h, is informative about landlords' turnover rate ρ_l.[40] The fraction of first-time buyers ϕ and the owner-renter age difference $\aleph$ help identify the turnover rate of households ρ in the city population and the probability ξ of drawing a new credit cost that may lead to a change in housing tenure.

A set of targets also related to the extensive margin specifies the capitalized credit costs of marginal home-buyers relative to price, Z/P_h, and the ratio of marginal to average credit costs, $Z/\bar{K}$. The calibration thus uses credit-cost information about marginal as well as average borrowers. Empirically, marginal borrowers are identified as those who do not qualify for loans from major banks and must instead borrow at higher rates from other financial institutions. Together with average credit costs, data on marginal borrowers is informative about the shape of the credit-cost distribution across households, identifying the mean and standard deviation parameters μ and σ. This plays an important role in the workings of the model because it affects how many households have a credit cost close to that of a marginal home-buyer.[41]

The capitalized credit costs themselves are derived following a procedure described in appendix A.5 from information about mortgage interest-rate spreads, the mortgage term, and the loan-to-value ratio. In short, the capitalized credit cost K of becoming a homeowner is computed from a comparison of the mortgage rate r_k a household faces relative to the risk-free interest rate r_g on government bonds. Suppose a household buys a property at price P_h by taking out a mortgage with loan-to-value ratio l, where the mortgage contract specifies an amortization

[40] See the discussion of the relative holding periods of investors and owner-occupiers in section 4.2.

[41] The sensitivity of the results to the density of the credit-cost distribution around the marginal home-buyer is discussed in section 5.4.1.

schedule such that the balance reaches zero after T_k years of a sequence of fixed repayments. The credit cost K is the present discounted value of the expected stream of repayments minus the initial amount borrowed, and as a fraction of price P_h it is given by

$$\frac{K}{P_h} = \left(1 + \frac{r_k}{r_g + \rho - r_k} e^{-(r_g+\rho)T_k} - \frac{r_g + \rho}{r_g + \rho - r_k} e^{-r_k T_k}\right) \frac{(r_k - r_g)l}{(r_g + \rho)(1 - e^{-r_k T_k})} . \tag{51}$$

This expression is used to determine Z/P_h and $\bar{K}/P_h$, and hence $Z/\bar{K}$. The information required is r_g, $\bar{r}_k$, r_z, l, and T_k, where $\bar{r}_k$ is the average real mortgage interest rate across all home-buyers and r_z is the real mortgage rate of a marginal home buyer. Data sources for these variables are given in appendix A.5.

Entry decisions of investors and households are affected by the bargaining-power parameters ω_i, ω_h, and ω_l, and these decisions affect tightness in the rental and ownership markets and the number n of people in the city relative to the housing stock. Empirically, tightness in the ownership market is identified using time-on-the-market T_{bh} for buyers relative to sellers T_{so}, and given n, this also determines rental-market tightness. Hence by using the model's free-entry condition for investors and the steady-state free-entry condition for households, targets for T_{bh} and n provide information about ω_h and ω_l, and together with the ratio P_i/P_h of prices paid by investors and home-buyers, the remaining bargaining-power parameter ω_i is identified.

Note that it is not necessary to take a stance on the presence or size of any 'warm glow' effect of homeownership in the calibration, that is, the size of the parameter ζ_h relative to ζ_l. Since the marginal home-buyer is indifferent, this ratio is determined as a residual given the calibrated costs of owning versus renting and the choices of households as manifested in the homeownership rate h. In particular, if a larger average mortgage-rate spread over the risk-free rate increased the credit costs of becoming an owner-occupier, then for a given homeownership rate and other costs, the 'warm glow' from ownership would need to be larger to match h.

Another group of empirical targets matches search behaviour and costs incurred within the ownership and rental markets. Time-to-sell T_{so} and time-to-lease T_{sl} are closely linked to incentives for home-buyers and tenants to search, which depend on the distributions of new match quality, and thus help to identity the parameters λ_h and λ_l that determine match-quality dispersion.[42] Average viewings per buyer Λ_h and per lease Λ_l, together with the information on time-on-the-market, identify the productivity parameters υ_o and υ_l of the meeting functions.

The transaction-cost parameters C_i, C_h, C_o, and C_l, and the maintenance/management-cost parameters D and D_l are directly determined by data on these variables as a fraction of prices P, P_h, P_i, or rents R. Renters' transaction cost C_w is identified indirectly by observations on the size of the agreement fee A for a new tenancy negotiated between landlords and tenants.

Data on average times between moves for owner-occupiers T_{mh} and tenants T_{ml} provide information on the arrival rates α_h and α_l of shocks to match quality. To identify the parameter

[42] The distribution of match quality across households is thus disciplined by the calibration targets adopted here, rather than being left as a free parameter.

δ_h on the size of match-quality shocks faced by owner-occupiers, the calibration matches the model's moving rate response after the new LTT to the estimated effect from section 2.

Finally, the average transaction price P in 2007 Canadian dollars provides information on the monetary value of a unit of utility in the model, and also allows welfare to be interpreted in consumption-equivalent units. This is essentially a normalization of the parameter ζ_h that completes the calibration.[43]

Table 4: *Calibrated parameters*

Parameter description	Notation	Value
Discount rate for future housing-market payoffs	r	3.28%
Households' exit rate from the city	ρ	4.27%
Investors' exit rate	ρ_l	0.700%
Property maintenance cost	D	\$10.5k
Landlords' extra maintenance/management costs	D_l	\$2.20k
Minimum new match quality in the ownership market	ζ_h	\$33.6k
Minimum new match quality in the rental market	ζ_l	\$24.6k
Shape parameter of home-buyer new match quality distribution	λ_h	33.1
Shape parameter of tenant new match quality distribution	λ_l	36.2
Arrival rate of match quality shocks in the ownership market	α_h	7.93%
Arrival rate of match quality shocks in the rental market	α_l	27.9%
Size of match quality shocks in the ownership market	δ_h	0.855
Fraction of tenants drawing a new credit cost after a shock	ξ	8.28%
Parameter for the mean of the distribution of credit costs	μ	5.05
Parameter for standard deviation of the distribution of credit costs	σ	0.674
Transaction costs of buyers excluding taxes	$C_i = C_h$	0
Transaction costs of sellers	C_o	\$18.1k
Transaction costs of landlords	C_l	\$2.28k
Transaction costs of tenants excluding tenancy-agreement fee	C_w	\$0.709k
Flow search costs of home-buyers and investors	$F_i = F_h$	\$9.84k
Flow search costs of prospective tenants in the rental market	F_l	\$9.72k
Entry cost of moving to the city	E	\$21.1k
Speed of adjustment of the city population	χ	1
Meeting function productivity parameter in the ownership market	υ_o	110
Meeting function productivity parameter in the rental market	υ_l	165
Elasticity of ownership-market meetings with respect to sellers	η_o	0.490
Elasticity of rental-market meetings with respect to landlords	η_l	0.764
Bargaining power of a seller meeting a home-buyer	ω_h	0.490
Bargaining power of a seller meeting an investor	ω_i	0.265
Bargaining power of a landlord meeting a prospective tenant	ω_l	0.764

Notes: Time units are years, and all payoff and cost parameters are measured in 2007 Canadian dollars. These parameters exactly match the targets in Table 3 using the calibration procedure from appendix A.6.

[43]There are 32 parameters, excluding tax rates τ_h and τ_i, and 38 targets. The targets exactly identify the parameters. There are more targets than parameters because τ_h and τ_i must be included in the list of targets, the credit-cost calibration is based on five pieces of information that are ultimately collapsed to an average and a marginal credit cost, and the price-to-income ratio must be included to calculate the size of the opportunity costs of search.

5.2 Quantitative effects of transaction taxes

The effects of increasing the transaction tax rates τ_h and τ_i for both home-buyers and investors from 1.5% to 2.8% are reported in Table 5. The first column gives the baseline empirical estimates of the LTT effects from section 2 expressed as percentage changes. The second column reports the model's predictions averaged over the same four-year time window used in the econometric analysis, and expressed as a comparable percentage change relative to the initial steady state. The third column gives the model-implied long-run percentage changes once the new steady state is reached.

Table 5: *Simulations of the model following an increase in the transaction tax rate*

Variable	Econometric evidence 2008–2012	Predictions of the model 4-year average	Steady state
Owners' moving rate (T_{mh}^{-1})	−12%	−12% (matched)	−12%
Buy-to-own (BTO) sales (S_h)	−9.6%	−14%	−16%
Buy-to-rent (BTR) sales (S_i)	9.3%	35%	5.1%
Leases-to-sales ratio (S_l/S_o)	26%	15%	23%
Price-to-rent ratio (P_i/R)	−3.8%	−1.6%	−1.6%
Average sales price (P)	−1.7%	−1.6%	−1.6%
Homeownership rate (h)	-	−0.23 p.p.	−2.4 p.p.
Effective LTT tax rate ($\tau_h = \tau_i$)	Increased from 1.5% to 2.8% (1.3 p.p.)		

Notes: The solution procedure to find the predictions of the model is described in appendix A.3 and A.4. The econometric evidence on the average sales price response is taken from Table A.6 in the Appendix.

Consistent with the empirical results and the discussion in section 4, the model predicts that buy-to-own (BTO) sales S_h and buy-to-rent (BTR) sales S_i move in opposite directions when the transaction tax rises. Sales to home-buyers fall, while sales to investors rise, despite the two groups facing the same tax rates. Quantitatively, on average over the first four years, the model predicts a 14% fall in BTO sales and a 35% rise in BTR sales. Consistent with the increase in property investors, there is a rise in the number of leases as more households choose to be tenants rather than home-buyers, leading to an increase in the leases-to-sales ratio by 15% and a fall in the homeownership rate by 0.23 percentage points. Data on the homeownership rate in Toronto is not available at the micro level or at high frequencies, so the causal effect of the LTT change cannot be estimated. However, the empirical findings for BTR sales and leases indicate that the homeownership rate would fall after the LTT increase, all else equal.[44]

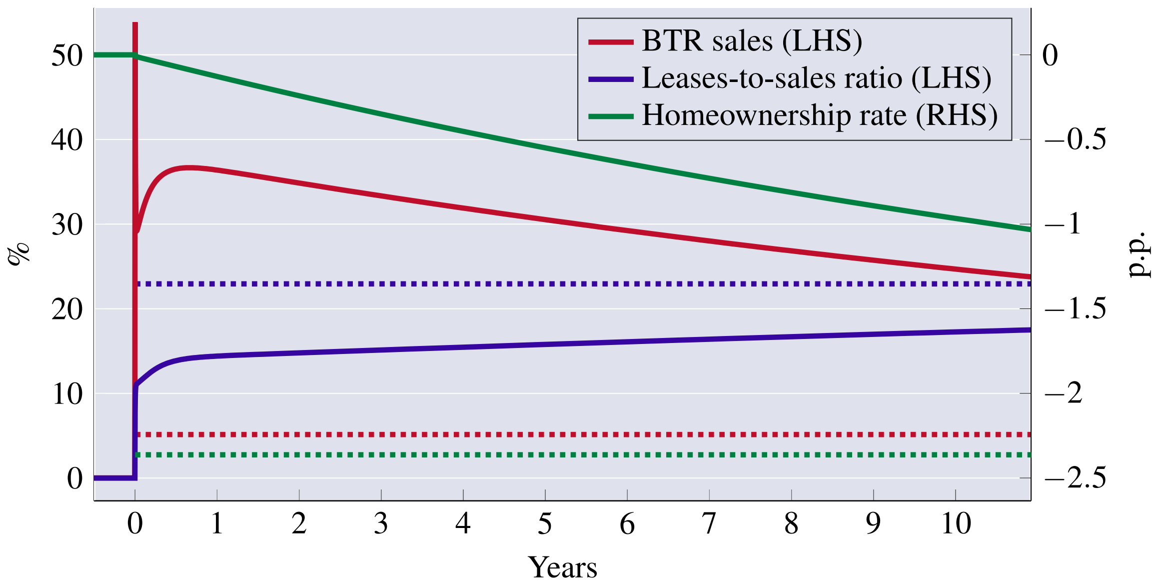
The model-predicted increase in the leases-to-sales ratio may appear low in light of the large increase in BTR sales. However, the initial level of BTR sales is much smaller than the initial level of leases owing to the much faster turnover of tenants moving between rental properties than the turnover of landlords owning rental properties. Hence, the effect is much larger as a

[44] Simply looking at the aggregate data on the homeownership rate in Toronto reveals a rising trend prior to the LTT increase and a flattening out afterwards. The period of stagnation in the homeownership rate coincides with a rising fraction of BTR sales in the aggregate.

percentage for BTR sales. In the long run, once the new steady state is reached, the percentage change in the flow of BTR sales is much smaller and the change in the leases-to-sales ratio is much larger. The difference between the four-year and long-run effects on these two variables reflects slow convergence to the new steady-state homeownership rate.

The transitional dynamics of the variables related to tenure choice are shown in Figure 2. BTR sales overshoot the new steady state persistently for more than a decade because more BTR sales are needed to get up to the higher stock of investor-owned properties than are needed subsequently to sustain that level. The path of BTR sales is non-monotonic, with a large initial spike. Investors enter according to the free-entry condition and thus act very quickly after the tax change. However, as only a small fraction of all households makes a tenure-choice decision in a given year, households overall are slow to adjust relative to investors, as seen in the very gradual change in the homeownership rate and the leases-to-sales ratio in the years after the tax change. This acts as a force to slow down entry of investors, otherwise the rental market would be flooded with vacant properties. These effects interact to produce the overshooting seen in the BTR sales impulse response. For the other variables in the model, transitional dynamics are relatively short and thus are not reported.

Figure 2: *Impulse responses of tenure-choice variables to the transaction tax increase*



Notes: The responses of BTR transactions and the leases-to-sales ratio are percentage changes to be read from the left scale. The response of the homeownership rate is the percentage-point change shown on the right scale. The dotted lines with matching colours are the corresponding levels of the variables in the new steady state. The daily impulse responses are smoothed to a monthly frequency of observation.

The predicted average price paid drops by 1.6%, which is very close to matching the decrease found empirically in Table A.6 in the Appendix. Interestingly, the percentage decline in prices is larger than the 1.3 percentage-point rise in the tax rate.[45] The impact on the average

[45] A simple analysis of tax incidence might suggest that prices should change by less than the tax rate because

price reflects the expectation that a given property will be subject to the transaction tax each time it is sold, and thus the expected future incidence of the tax is capitalized into property prices. The decline in the average price also drives a 1.6% reduction in the price-to-rent ratio, capturing a substantial fraction of the decrease found empirically. This prediction is in line with the general-equilibrium role of the price-to-rent ratio in increasing BTR sales.

5.3 Welfare effects of transactions taxes

This section evaluates the welfare costs of the transaction tax effects. The extra tax revenue is spent on delivering extra public services to all city residents, as represented by the per-person public services g in all residents' value functions. All individuals' utility is linear, so the changes in their value functions are measured in consumption-equivalent units. The calibration matches the average sales price of properties from January 2006 to January 2008 in Canadian dollars, so a change of 1 is approximately equivalent to a 2007 Canadian dollar. The total welfare effects of the policy are then obtained by summing the individual value functions in (44). In light of the slow rate of convergence of tenure-choice variables, the analysis focuses on the change in the steady-state level of welfare.[46]

Using the calibrated model, the predicted welfare losses from the new LTT are shown in Table 6. The changes in present-value welfare are expressed as per-year, per city-resident amounts by multiplying by $r+\rho$. The welfare costs are substantial, especially relative to the extra tax revenue being raised. The new LTT causes total welfare to fall by an amount equivalent to 111% of the extra tax revenue it generates.[47]

The importance of the tenure-choice variables for the welfare costs of the LTT can be understood by decomposing the welfare function (44) into terms related to across- and within-market effects. The change in welfare $\Delta\Omega$ between steady states can be expressed as

$$r\Delta\Omega = \{(V_h+\Delta V_h)\Delta q_h + (V_l+\Delta V_l - D_l)\Delta q_l - F_i\Delta b_i - C_i\Delta S_i - \Delta(\gamma\bar{K})\} - \rho E\Delta n$$
$$+ \{(q_l\Delta V_l - F_l\Delta b_l - (C_w+C_l)\Delta S_l) + (q_h\Delta V_h - F_h\Delta b_h - C_h\Delta S_h - C_o\Delta S_o)\}\,. \quad (52)$$

The first line groups terms related to tenure-choice decisions that affect welfare across markets inside the curly brackets. The term on the second line groups the within-market effects for both rental and ownership markets. The remaining term captures the effect of any changes in population across the two regions. Quantitatively, the inter-regional effect is very close to zero

buyers have some bargaining power — see equation (28). The equation also shows that a proportional transaction tax reduces the effective bargaining power of sellers, contributing to a lower price.

[46]The welfare of a 'newborn', a new entrant to the city, is always zero given the steady-state free-entry condition $N=0$. Thus, this measure hides the welfare losses borne by incumbents within the city. The key point is that the free-entry condition for entrants shifts the welfare costs entirely on to incumbents through the mechanism of price or rent adjustment at the point when the new LTT is introduced.

[47]The model predicts that the percentage change in steady-state tax revenue $G=\tau_h PS_h + \tau_i P_i S_i$ is only 56%, while the percentage change in the tax rates is 87% (from 1.5% to 2.8%). This discrepancy is explained by erosion of the tax base: total transactions go down and the average price drops, so the tax base shrinks.

because there is only a negligible change in the city population, and so this is not reported below. This point is discussed in more detail in section 5.4.2.

The welfare losses arising from distortions across and within rental and ownership markets are both large as seen in Table 6. Distortions across the two markets generate a loss equivalent to 60% of the extra tax revenue, which accounts for more than half of the total loss. Within markets, rental- and ownership-market distortions generate losses of 13% and 38% of tax revenue, respectively. Overall, the presence of the rental market and tenure choice in the analysis account for a welfare loss of 73% of the extra tax revenue, which is two thirds of the total loss.

The welfare loss across the two markets results from the drop in the homeownership rate. Some households with low enough credit costs who would otherwise have gained from being owner-occupiers decide to remain renters owing to the extra costs imposed by the transaction tax both now and expected again when they move in the future. The size of this welfare loss largely depends on the distribution of credit costs, which is calibrated using data on mortgage spreads. This is because the credit-cost distribution across households is the relevant source of heterogeneity for the owing-versus-renting decision — everyone shares the same ex-ante expectation of housing utility in the two markets, so there is no lack of substitutability between owner-occupied and rental properties in terms of preferences. The decline in homeownership also adds to the welfare loss through an increase in rental management costs.[48]

Within the ownership market, the welfare loss is mainly due to the fall in match quality, partly offset by lower non-tax transaction costs saved because moving is less frequent. It is also offset by home-buyers becoming more picky, though that entails costs of having to search for longer. Quantitatively, the large size of the welfare loss relates to the indivisibility of housing: households are taxed on the whole value of a property purchase, not the marginal improvement in match quality that comes from moving. The welfare loss within the rental market is much smaller and mainly reflects increased transaction costs from more leases being arranged.

The distributional effects of the tax on welfare of different groups can be evaluated using their value functions, averaging $H(\varepsilon)$, $W(\varepsilon)$, $L(\varepsilon)$ over the endogenous distribution of match quality for owner-occupiers and tenants.[49] The biggest losers from the LTT are owner-occupiers, who suffer a loss of \$601 per person per year (the same units as the Table 6 results). These households face paying the tax each time they move, the same reason the tax affects the tenure-choice decision as discussed earlier. In addition, they suffer from a fall in the value of their property given that they bought at a price reflecting the initial lower tax rate. This fall in house prices also implies a large loss for sellers (\$501). Home-buyers lose (\$239) for the first reason given above for owner-occupiers, but not the second. Landlords lose \$122 and tenants gain \$173, whether or not they are in a match as rental-market matching is relatively quick.

[48]It is important to note that the model does not imply a monotonic relationship between the homeownership rate and welfare. This can be seen from the term $\gamma\bar{K}$ in the expression for welfare (44), where credit costs associated with increasing homeownership have a negative impact on welfare, all else equal (see section 5.4.3).

[49]The results given here refer to per-person welfare effects. These are not the same as the contributions to the total welfare loss because the sizes of the various groups change.

The gain for tenants derives from the extra public expenditure, noting that a large fraction of households remain tenants throughout their time in the city. Finally, owing to the free-entry conditions, there is no change in the welfare of investors or new households entering the city.

Table 6: *Welfare losses from the new transaction tax*

Welfare loss	Canadian dollars per person per year	Percent of extra tax revenue
Total	\$534	111%
Across market	\$290	60%
Within markets	\$243	51%
Rental market	\$61	13%
Ownership market	\$182	38%
Increase in tax revenue	\$480	

Notes: The change in welfare is equal to the change in the steady-state value of (44) multiplied by $r+\rho$. The decomposition into across- and within-market effects is explained in the text. The monetary units are 2007 Canadian dollars.

5.4 Discussion of the quantitative results

5.4.1 Credit-cost heterogeneity and the size of the tax effects across markets

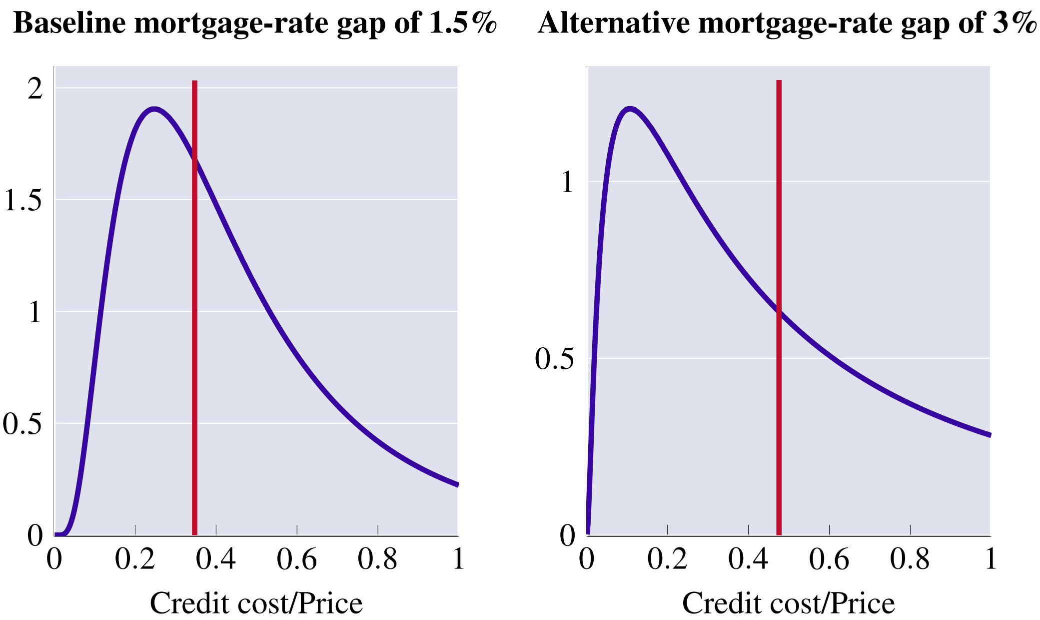
The extent of the reallocation of households and properties from the ownership market to the rental market after the higher transaction tax depends crucially on the mass of marginal home-buyers prior to the tax increase. Marginal households have a credit cost at the threshold Z. A higher transaction tax lowers the threshold Z, turning buyers with credit costs just below the original threshold away from the ownership market and leaving them as tenants.

Intuitively, the closer the threshold Z is to the mode of the probability distribution of credit costs K, the higher is the mass of marginal buyers. Thus, an important empirical target is the mortgage interest rate gap between the marginal and average buyers, which is what determines the probability mass of credit costs near the threshold. As explained in appendix A.5, the gap used in the baseline calibration is 1.5%. This is based on micro-level mortgage data from the Bank of Canada showing that the interest-rate gap between the average borrower and those with low credit scores is around 3% for a typical 5-year mortgage loan. Given that a marginal buyer is likely to be able to pay a lower interest rate after the first five years, the baseline calibration assumes a smaller 1.5% gap to apply to the whole period of mortgage borrowing.

If the gap were increased to 3%, essentially assuming marginal buyers cannot refinance at a better rate after the first five years, the mass of marginal buyers would be smaller. This is depicted in the credit-cost distribution in the right panel of Figure 3 compared to the baseline case in the left panel. A higher transaction tax lowers the credit-cost threshold Z, but when the mass of marginal buyers is smaller, this has less impact on the owning-versus-renting margin. In equilibrium, this leads to a smaller increase in entry of buy-to-rent investors, with BTR transactions rising by 13% in the four years after the tax change (see Table A.13 in the Appendix). This is closer to the empirical finding of a 9.3% increase (Table 1). Since it is a strong assumption that marginal buyers cannot ever refinance at lower spreads, the model prediction in this case can be thought of as a lower bound on the tax effects on BTR transactions.

The smaller impact on BTR transactions in this alternative calibration translates into smaller welfare losses. The total welfare cost of the LTT is lower but still substantial at 76% of the extra tax revenue. Distortions across the two markets imply losses of 24% of revenue, distortions within the rental market 4%, and distortions within the ownership market 47%. In this case, the presence of the rental market in the analysis accounts for about 40% of the total loss. The smaller across-markets loss is due to the smaller predicted increase in buy-to-rent transactions compared to the baseline. The full results can be found in Table A.13 in the Appendix. This case provides a lower bound on the quantitative impact of the LTT on owning versus renting and its implications for welfare.

Figure 3: *Distributions of credit costs under different calibrations of mortgage-rate gaps*



Notes: The two panels show the distributions of credit costs implied by different calibrations of the mortgage interest rate gap between marginal and average home-buyers. The red lines show the equilibrium threshold Z/P_h.

5.4.2 Mobility across the two regions

To examine the role played by mobility across the two regions, this section considers the case where households cannot choose to enter the city. This is done by setting the sensitivity parameter of the inflows of new households to the city to zero, that is, $\chi = 0$ in (6). As shown in Table A.13 in the Appendix, the tax effects on quantities are similar to the baseline. Empirical support for this prediction is seen in the findings from Table 1 and Table 2 where the 'donut' specifications yield broadly similar results to the non-'donut' specifications. The intuition for why mobility across regions does not have a large impact on quantities is as follows. If the housing stock in the city is fixed, since houses must be owned or rented by someone in equilibrium, the value of living inside the city must adjust through changes in house prices or rents. Since the population does not change by much in equilibrium, the analysis of quantities and welfare is very similar to the baseline model.[50]

5.4.3 A tax on investors

A key feature of the analysis in this paper is in allowing for free entry of buy-to-rent investors, which helps to understand why the LTT has different effects on BTO and BTR transactions. It also has implications for the welfare costs of transaction taxes. Since homeowners are more heavily affected by the same transaction tax rate than investors, a higher tax rate increases distortions in the allocation of housing across the ownership and rental markets.

This novel effect can be isolated by considering a hypothetical tax regime with different tax rates for owner-occupiers and investors. Taking the same increase in τ_h from 1.5% to 2.8% as before, the tax rate τ_i on investors can be raised to such a level where there is no change in the equilibrium homeownership rate. The required change in τ_i for this is from 1.5% to 5.8%. This alternative tax system raises slightly more revenue (up by 70% instead of 56%), but not much more because buy-to-rent investors are a small minority and do not transact frequently on average. Importantly, the welfare loss in this case is considerably smaller, being only 40% of the extra revenue raised instead of 111% with an equal increase in the tax rates τ_h and τ_i.

Intuitively, this exercise shuts down the extensive margin, keeping the homeownership rate unchanged by putting up higher barriers to entry for investors. This offsets the implicit advantage investors receive from not needing to pay the LTT as often as owner-occupiers do when tax rates rise by the same amount. The welfare loss is smaller because the unequal tax rates undo the effects of this distortion.

However, increasing τ_i ever further to raise the homeownership rate would ultimately lead to large welfare costs because uncreditworthy households would be forced into the ownership market owing to a lack of rental properties. This would result in them paying very high borrowing costs, reducing welfare through the term $\gamma\bar{K}$ in (44). Deep-pocketed investors play an important role in providing access to housing without everyone having to incur credit costs.

6 Conclusions

Using a unique dataset of property sales and leasing transactions, this paper documents two novel effects of a higher transaction tax. First, there is a rise in buy-to-rent transactions and a

[50]The total welfare cost of the new LTT in the case of no mobility is 112% of the extra tax revenue. Distortions across the two markets imply losses of 61% of tax revenue, distortions within the rental market 13%, and distortions within the ownership market 38%.

fall in owner-occupier transactions, despite the same tax applying to both. Second, there is a simultaneous fall in the sales-to-leases and price-to-rent ratios.

This paper builds a tractable model with free entry of investors and where households choose owning or renting, with entry to the ownership market incurring a heterogeneous cost of accessing credit. The calibrated model explains the empirical findings and points to a novel welfare cost of transaction taxes. A higher transaction tax distorts the allocation of properties across the ownership and rental markets by reducing the homeownership rate, as well as distorting the allocation within the ownership market by reducing mobility. The calibrated model implies a substantial welfare loss equivalent to 111% of revenue from the transaction tax, with about two thirds of this due to the analysis allowing for the presence of a rental market.

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A Appendices

A.1 Data, further estimation results, and robustness checks

A.1.1 Evaluating the comprehensiveness of the MLS rental listings data

Since the use of rental listings data in this paper is relatively new to the literature, it is important to examine how comprehensive are the Toronto MLS rental listings. This section shows through webscraping that MLS data provide an unusually high coverage of long-term and verifiable rental listings in the City of Toronto compared to other online rental platforms. Specifically, the MLS data capture over 90% of rental properties listed on the second-most popular rental listing platform in Toronto.

The Multiple Listing Service (MLS) is a database created by the Canadian Real Estate Association (CREA) and used by real-estate professionals to share and access information about properties for sale or lease. It enables cooperation among real-estate agents and brokers, who can pool their listings and share commissions on property transactions. An alternative popular rental listings platform is *Toronto Rentals* (hereafter referred to as TR), which is the second-largest website serving Toronto and the surrounding GTA since 1995.

For the period between 23rd November 2022 and 23rd February 2023, all rental listings from the MLS on `realtor.ca` (REALTOR.ca, 2022) and from TR (`rentals.ca/toronto`, Toronto Rentals, 2022) were webscraped. For each MLS listing, information was collected on the MLS ID, the address (as a string), the listing date, the number of bedrooms, the number of bathrooms, and the asking rent. For each TR listing, the information collected was the address (specified in terms of latitude and longitude), the listing date, the number of bedrooms, the number of bathrooms, and the asking-rent range.

To compare the two scraped datasets, MLS address strings were cleaned and parsed to apply Google Maps API to geocode the coordinates of each listing. The MLS listings were then matched with the TR listings by the geocoded address, the number of rooms, and a window around the listing date. Since a property might be listed on one platform first and later on another platform, the comparison was restricted to properties listed on TR between 25th November and 5th December 2022. The exercise then checked how many of these listings were also on the MLS during the same or surrounding time period.

Figure A.1 shows a map of the locations of rental listings in the City of Toronto. Yellow dots indicate MLS listings. Grey dots are TR listings that match with listings in the MLS data. Red dots are TR listings that are at least 200 metres away from the closest MLS listing, which is taken as an indicator that these listings were not included in the MLS.

There were 4,359 unique MLS records during the period studied, and the TR dataset includes a total of 3,516 entries. Out of all the TR listings, 294 were not matched with an MLS record, accounting for approximately 8.4% of the TR data. This fraction is likely to be overestimated because of inaccuracies in the manual matching of the MLS listings' coordinates.

There are also short-term rental websites such as *Kijiji* in Toronto. However, listings on these platforms are not included in the analysis for several reasons. First, unlike MLS or TR listings, Kijiji listings are unverified and less reliable, with most of them posted by anonymous users. Second, Kijiji users often forget to remove their listings when they are no longer active, making it questionable in what time window a listing counts as active. Third, Kijiji listings do not provide precise address information and can only be identified at neighbourhood level. Finally, unlike MLS or TR listings, most Kijiji listings are for short-term lets that are distinct from the longer-term rentals in the main analysis.

A.1.2 Descriptive statistics

Figure A.1: *Rental listings in Toronto between 25^{th} November and 5^{th} December 2022*

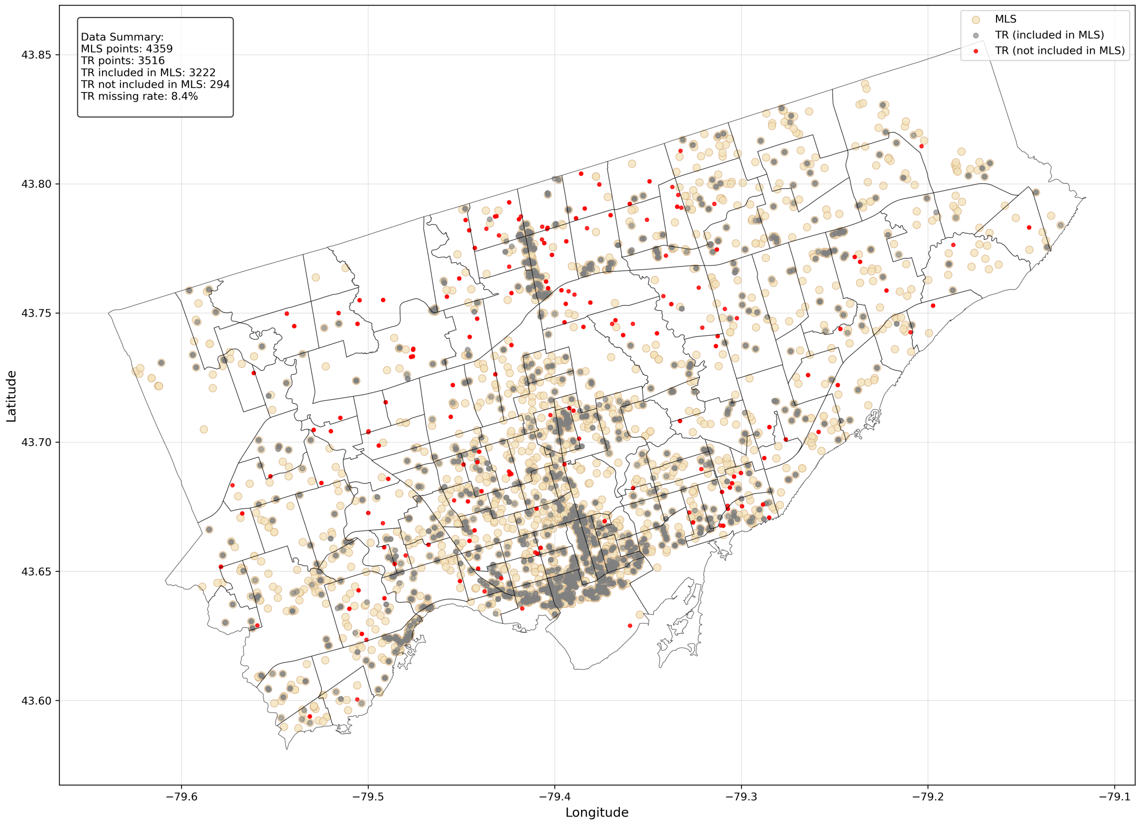


Table A.1: *Land transfer tax (LTT) rates by property value in the Greater Toronto Area*

City of Toronto (effective from 1^{st} February 2008)		Province of Ontario (effective from 7^{th} May 1997)	
\$0–55,000	0.5%	\$0–55,000	0.5%
\$55,000–400,000	1.0%	\$55,000–250,000	1.0%
\$400,000+	2.0%	\$250,000–400,000	1.5%
		\$400,000+	2.0%

Sources: Municipal Land Transfer Tax, City of Toronto, `http://www.toronto.ca/taxes/mltt.htm`; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).

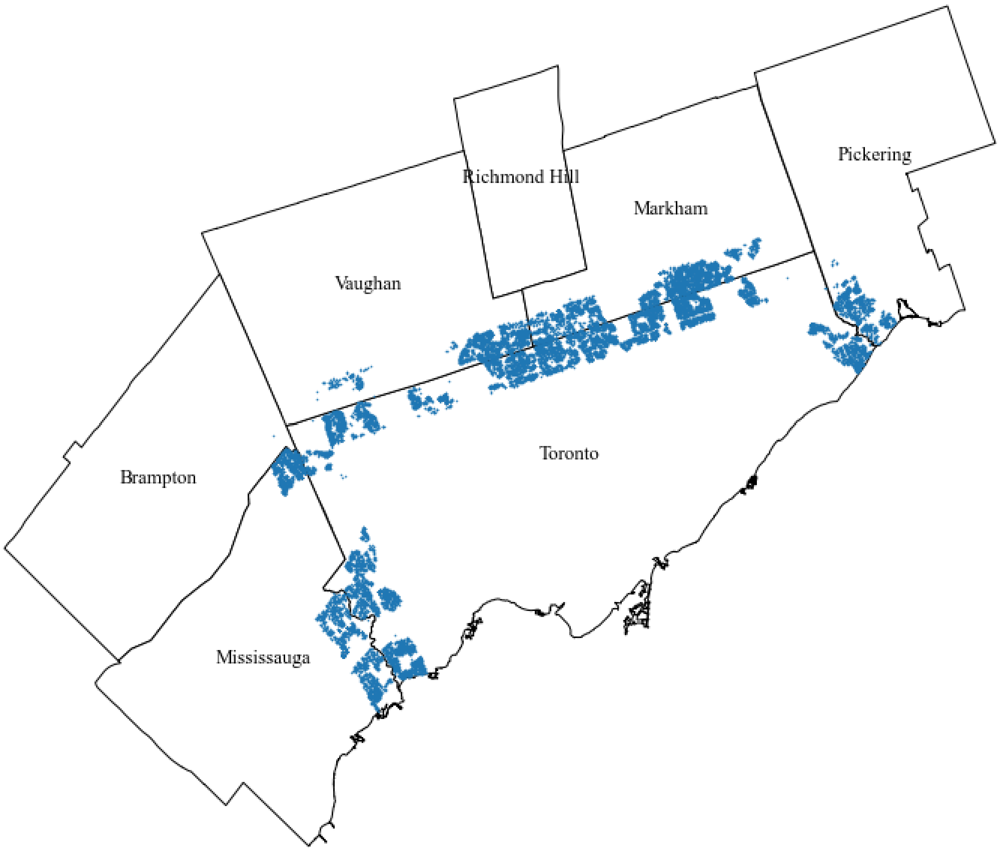
Notes: For the municipal LTT, exemptions are given to first-time buyers for purchases below a value of \$400,000, while for the provincial LTT, the first-time buyer exemption value threshold is \$227,500.

Table A.2: *Changes in the effective land transfer tax rate within the City of Toronto*

Fraction of first-time buyers	All of city	Within 5km of border	Within 3km of border
0%	1.521%	1.518%	1.507%
100%	1.041%	1.036%	1.014%
40%	1.329%	1.325%	1.310%

Notes: The table shows the average LTT rates before and after the new LTT for all transactions within the city, and those within 5km or 2km inside the city border. The sample is restricted to detached house transactions. The effective LTT rate is the mean transfer tax as a percentage of the sales price, combining provincial and city-level taxes, averaged over transactions between January 2006 and January 2008 to control for compositional effects. The effective LTT rates are imputed based on the tax rates before and after the new LTT is introduced, and the change in the effective LTT rate is taken to be the difference between the two.

Figure A.2: *Geography of the sample used for estimation*



A.1.3 Housing-stock composition

As a check on the assumption that there are no significant differences in housing composition potentially picked up by the coefficient on the *LTT* dummy, columns (1) and (2) of Table A.4 present evidence on property characteristics on opposite sides of the city border before the new LTT is introduced. The sample is restricted to the pre-tax-rise period, and each property characteristic is regressed on a border dummy that indicates being inside the City of Toronto, controlling for the usual factors. The border coefficients are statistically insignificant in most cases, and quantitatively small even when statistically significant. This indicates that properties transacted on opposite sides of the border were more or less similar before the new LTT.

In columns (3) and (4), each property characteristic is further regressed on the *LTT* dummy that is an interaction of the border dummy and the post-tax-rise dummy, controlling for the usual factors. The *LTT* dummy coefficients are statistically insignificant in almost all cases. As expected, cross-border differences in property characteristics, if any, remain stable before and after the new LTT. This ensures the coefficients on the *LTT* dummy in the main empirical specifications pick up the impact of the new transaction tax, rather than changes in housing-stock composition.

Table A.3: *Descriptive statistics*

	Pre-LTT	Post-LTT		Pre-&post-LTT
	2006:1–2008:1	2008:2–2010:2	2008:1–2012:2	2006:1–2018:2
		Greater Toronto Area		
# BTO sales per year	53,018	45,962	46,232	52,109
# BTR sales per year	2,545	2,670	3,139	4,529
Days on the market (mean)	31.3	29.6	27.2	25.1
Days on the market (median)	21.0	20.0	18.0	16.0
Sale price (mean)	381,238	408,106	442,050	540,237
Sale price (median)	321,000	347,500	372,000	425,000
Price-rent ratio (mean)	20.3	20.6	21.9	25.8
Price-rent ratio (median)	17.4	18.2	19.0	21.6
		City of Toronto		
# BTO sales per year	27,718	23,832	24,621	27,639
# BTR sales per year	1,572	1,685	1,947	2,620
Days on the market (mean)	30.5	28.8	27.1	25.4
Days on the market (median)	20.0	18.0	17.0	15.0
Sale price (mean)	401,504	426,363	460,903	553,380
Sale price (median)	318,000	343,000	369,900	417,900
Price-rent ratio (mean)	20.7	20.9	22.2	25.7
Price-rent ratio (median)	16.9	17.9	18.8	21.1
		5km border sample		
# BTO sales per year	16,785	14,521	14,525	16,503
# BTR sales per year	908	1,015	1,155	1,548
Days on the market (mean)	33.3	30.7	28.1	26.2
Days on the market (median)	23.0	20.0	18.0	17.0
Sale price (mean)	345,754	371,534	405,536	503,184
Sale price (median)	315,000	338,000	361,000	408,000
Price-rent ratio (mean)	19.6	20.3	21.8	25.9
Price-rent ratio (median)	16.4	17.2	18.3	20.8
		3km border sample		
# BTO sales per year	8,504	7,435	7,327	8,074
# BTR sales per year	348	400	461	608
Days on the market (mean)	33.7	31.3	28.6	26.5
Days on the market (median)	24.0	21.0	19.0	17.0
Sale price (mean)	339,412	361,448	394,667	488,217
Sale price (median)	314,000	334,300	357,000	401,000
Price-rent ratio (mean)	19.0	19.5	21.1	25.7
Price-rent ratio (median)	15.9	16.1	17.3	20.1

Source: Multiple Listing Service (MLS) residential records (2006–2018).

Table A.4: *Comparison of property characteristics across the city border*

Property characteristic	(1)	(2)	(3)	(4)
Heating	0.000490	0.000320	−0.000406	−0.000120
	(0.000394)	(0.000236)	(0.000486)	(0.000329)
Observations	10,389	17,916	42,444	73,550
Basement	−0.00498	−0.00831**	−0.00133	0.00234
	(0.00391)	(0.00310)	(0.00458)	(0.00351)
Observations	10,389	17,916	42,444	73,550
Family	0.0227	−0.0907***	−0.0478	−0.0145
	(0.0344)	(0.0263)	(0.0381)	(0.0292)
Observations	10,347	17,834	42,444	73,548
Fire	0.00368	−0.0229***	−0.00655	−0.000543
	(0.00713)	(0.00562)	(0.00795)	(0.00621)
Observations	10,389	17,916	42,444	73,550
Bedrooms	0.00535	0.0157*	0.0138	0.0139
	(0.0105)	(0.00817)	(0.0110)	(0.00870)
Observations	10,389	17,916	42,444	73,550
Bathrooms	−0.115***	−0.120***	−0.0229	−0.0200
	(0.0137)	(0.0109)	(0.0157)	(0.0123)
Observations	10,389	17,916	42,444	73,550
Rooms	−0.0322	−0.0274	−0.0193	−0.0339*
	(0.0273)	(0.0185)	(0.0232)	(0.0178)
Observations	10,389	17,916	42,444	73,550
Lot	−1305.3	−918.3	1051.8	177.8
	(1006.3)	(600.3)	(967.1)	(903.3)
Observations	10,389	17,916	42,444	73,550
Distance threshold	3km	5km	3km	5km
LTT sample period	Pre	Pre	All	All

Notes: Data comprise detached house transactions from January 2006 to February 2012. A unit of observation is a transaction. In columns (1) and (2), the coefficients are from regressions of a property characteristic on a border dummy that indicates a location is in the City of Toronto. In columns (3) and (4), the coefficients are from regressions of a property characteristic on the *LTT* dummy that indicates a location in the City of Toronto and in the period after the LTT is introduced. All regressions control for other property characteristics, and year, month, and property-type fixed effects. Regressions for columns (3) and (4) include an indicator for the post-LTT period and an indicator for the City of Toronto. Distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. LTT sample period specifies whether a transaction occurred before or after the new LTT. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

A.1.4 Empirical specifications

The econometric specification is a variant of the regression discontinuity design developed by Dachis, Duranton and Turner (2012) applied to a broader set of housing-market outcomes.

Let t denote the time before ($t < 0$) or after ($t > 0$) the imposition of the new LTT, where the time unit is measured in months. Let d denote distance from the city border, with $d < 0$ meaning a location in the suburbs and $d > 0$ in the City of Toronto. Let i denote the unit of observation, *community* × *property type* × *year* × *month* in the market-segment regressions, *household* × *month* in the moving hazard regressions, and a *transaction* in the sales price and time-on-the-market regressions.

Define the following indicator variables based on time and distance:

$$\chi^{\text{POST}} = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}, \quad \text{and} \quad \chi^{\text{CT}} = \begin{cases} 1 & \text{if } d \geq 0 \\ 0 & \text{if } d < 0 \end{cases}.$$

The main variable of interest is the *LTT* dummy $\chi^{\text{POST}} \times \chi^{\text{CT}}$. Let y_{it} denote an outcome of interest, for example, buy-to-own transactions or the sales price. Let $\mathbf{x}_{it}$ denote the vector of property characteristics for unit i at time t in addition to χ^{POST} and χ^{CT}. To address anticipation effects that may arise from the announcement of the new LTT, define the following dummy variables:

$$\chi^{\tau} = \begin{cases} 1 & \text{if } t = \tau \text{ and } d \leq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \tau \in \{-3, -2, -1, 0, 1, 2, 3\}.$$

Some regressions include an interaction between the *LTT* dummy and areas away from the border, e.g, 2km away. To control for these differential effects, define the dummy variables:

$$\chi^{x > \bar{d}} = \begin{cases} 1 & \text{if } t > 0 \text{ and } d \geq \bar{d} \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \bar{d} > 0.$$

The general model is

$$y_{it} = \lambda \chi^{\text{POST}} \times \chi^{\text{CT}} + \beta' \mathbf{x}_{it} + \chi^{\tau} + \nu_t + \delta_i + \varepsilon_{it},$$

where ν_t represents *year* fixed effects and *month* fixed effects, δ_i represents *community* fixed effects, and ε_{it} is the error term. Notably, the specifications allow for separate time trends for transactions inside and outside of the city to control for Toronto-specific trends that may be caused by factors other than the LTT. In the all-properties sample, *community* × *property type*, *month* × *property type*, and *year* × *property type* fixed effects are also included.

A.1.5 Additional results

Figure A.3: *Kaplan-Meier estimate of homeowners' moving hazard function*

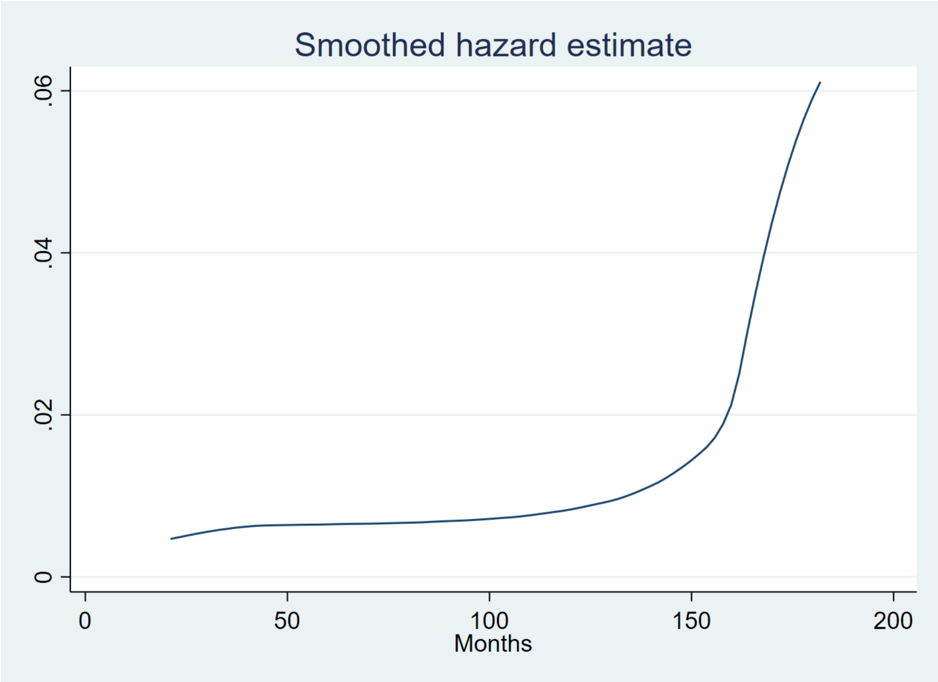


Table A.5: *The effects of the transaction tax on sales and leases separately*

	(1)	(2)	(3)
log (#*Sales*)	−0.177***	−0.0937**	−0.108**
	(0.0519)	(0.0405)	(0.0361)
Observations	6,540	10,798	13,437
log (#*Leases*)	0.0765	0.0985*	0.109**
	(0.0470)	(0.0550)	(0.0516)
Observations	2,660	5,545	6,006
Sample	Border	Border	Border
Distance threshold	5km	5km	5km
Border	Yes	Yes	Yes
Semi-detached	No	No	Yes
Detached	Yes	Yes	Yes
Condo apartments	No	Yes	Yes

Notes: The sample comprises transactions from 2006 to 2012 that are within 5km of the City of Toronto border. Given the limited sample size for the detached houses rental market (column 1), the sample is expanded to include also condominiums and semi-detached houses (columns 2 and 3). Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. All regressions include a post-LTT dummy, and city, community, year, and calendar month fixed effects, as well as their interactions with the property type. Six dummy variables are included for transactions inside the city of Toronto during the last three months of 2007 and the first three of 2008. City time trends and distance LTT trends are included. Property type rows indicate whether the property type was included in the regression. The border row indicates if a border sample was used, and the distance threshold indicates the distance threshold defining the sample. Robust standard errors are in parentheses and *, **, and *** indicate significance at the 10%, 5%, and 1% levels.

Table A.6: *Effects of the transaction tax on prices and time-on-the-market*

	(1)	(2)	(3)	(4)
		Dependent variable: log (*Sales price*)		
LTT	−0.0176**	−0.0232**	−0.0235	−0.0123***
	(0.00824)	(0.00953)	(0.0147)	(0.00342)
Observations	14,702	24,970	14,808	110,952
		Dependent variable: log (*Time-on-the-market*)		
LTT	0.426***	0.420***	0.347***	0.396***
	(0.0589)	(0.0623)	(0.0542)	(0.0474)
Observations	14,704	24,973	14,809	110,961
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
Property characteristics	Yes	Yes	Yes	Yes
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: The data comprise detached house transactions from 2006 to 2012. The unit of observation is a transaction. Repeat sales transactions taking place within 18 months of one another are discarded. All regressions include an indicator for the post-LTT period, an indicator for the city of Toronto, community fixed effects, calendar month fixed effects, a rich set of time-varying property characteristics, as well as separate time trends for transactions inside and outside the City of Toronto. The distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. Distance LTT trend denotes the inclusion of an interaction term between the LTT and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the LTT and the distance from the city border in column (4). Standard errors clustered by community are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

A.1.6 Robustness checks

Table A.7: *Estimated tax effects excluding the financial-crisis period*

	(1)	(2)	(3)	(4)	(5)	(6)
	Excluding 2008:4–2009:12		Excluding 2008:1–2009:12		Excluding 2007:10–2009:12	
log (#*Leases*/#*Sales*)	0.508**	0.590**	0.492**	0.581**	0.489**	0.568*
	(0.238)	(0.291)	(0.238)	(0.291)	(0.238)	(0.291)
Observations	1,827	1,221	1,774	1,186	1,672	1,114
log (#*BTO sales*)	−0.404***	−0.258*	−0.405***	−0.259*	−0.404***	−0.257*
	(0.121)	(0.155)	(0.121)	(0.155)	(0.121)	(0.155)
Observations	4,484	2,685	4,319	2,584	4,062	2,432
log (#*BTR sales*)	0.107**	0.129**	0.106**	0.129**	0.100*	0.128**
	(0.0509)	(0.0626)	(0.0508)	(0.0625)	(0.0516)	(0.0637)
Observations	751	486	727	467	675	432
Event of moving	−0.204***	−0.182*	−0.190***	−0.196**	−0.137**	−0.157
	(0.0542)	(0.0954)	(0.0548)	(0.0963)	(0.0559)	(0.0978)
log (*Original price*)	−0.0498	−0.0785	−0.0401	−0.0621	−0.0284	−0.0589
	(0.0447)	(0.0511)	(0.0453)	(0.0523)	(0.0460)	(0.0537)
$\log\varphi$	0.397***	0.391***	0.398***	0.392***	0.399***	0.392***
	(0.00888)	(0.0113)	(0.00910)	(0.0116)	(0.00936)	(0.0119)
Observations	2,025,845	1,182,656	1,921,838	1,122,344	1,820,454	1,063,626
log (*Sales price*)	−0.0400**	−0.0666***	−0.0379**	−0.0562**	−0.0404**	−0.0561**
	(0.0153)	(0.0188)	(0.0139)	(0.0173)	(0.0141)	(0.0174)
Observations	32,822	21,108	30,226	19,404	28,474	18,273
log (*Time-on-market*)	0.189***	0.270***	0.241***	0.234**	0.176**	0.189**
	(0.0549)	(0.0689)	(0.0618)	(0.0826)	(0.0740)	(0.0924)
Observations	20,873	13,652	18,894	12,341	17,810	11,664
Sample	Border	Border	Border	Border	Border	Border
Distance threshold	5km	5km	5km	5km	5km	5km
Donut hole		2km		2km		2km
Months removed	21	21	24	24	27	27

Notes: The table shows the results of the robustness checks for the crisis period removing 21, 24, or 27 months. The sample comprises transactions from 2006 to 2012. The first three rows present the estimated coefficients for the leases-to-sales ratio, BTO sales, and BTR sales. The market segment regressions in the first three rows use detached house transactions. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. For the moving hazard regressions, a unit of observation is a homeowner whose property is listed on MLS. For the transaction level regressions, a unit of observation is a property transaction. Repeat sales transactions taking place within 12 months of one another are discarded for the moving hazard model and 18 months for the transaction level regressions. All regressions include a post-LTT dummy, controls for time-varying house characteristics, community, year, month, property type fixed effects, and their interactions. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008 whenever this applies. The border row indicates if a border sample was used, and the distance threshold indicates the distance radius used in the sample. The donut hole row indicates the number of kilometres from the city border excluded from the sample. Standard errors are in parentheses and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.8: *Placebo test based on pseudo-borders within Toronto*

	Pseudo-border at 3km				Pseudo-border at 5km			
log (#*Leases*/#*Sales*)	0.161	0.0967	−0.0341	0.155	−0.0984	0.0325	0.157	0.0376
	(0.141)	(0.120)	(0.228)	(0.115)	(0.162)	(0.101)	(0.143)	(0.0976)
Observations	1,200	1,746	546	2,081	1,041	2,371	1,330	2,544
log (*Price*/*Rent*)	0.0200	0.00898	−0.0235	0.0128	0.0258	0.0729**	0.0987**	0.0654**
	(0.0337)	(0.0306)	(0.0778)	(0.0305)	(0.0669)	(0.0340)	(0.0391)	(0.0329)
Observations	1,832	2,411	565	2,649	1,303	2,958	1,625	3,234
log (#*BTO sales*)	0.0606	0.0205	0.0655	0.00614	−0.0693	−0.0527	−0.0462	−0.0389
	(0.0734)	(0.0611)	(0.0760)	(0.0576)	(0.0847)	(0.0534)	(0.0607)	(0.0509)
Observations	2,605	3,845	2,605	4,481	2,171	5,032	3,801	5,572
log (#*BTR sales*)	−0.0160	−0.157	−0.109	−0.159	−0.0534	−0.0895	−0.0349	−0.0793
	(0.102)	(0.112)	(0.137)	(0.105)	(0.108)	(0.0875)	(0.103)	(0.0843)
Observations	523	780	494	883	418	1,000	744	1,083
	Pseudo-border at 4km				**Pseudo-border at 6km**			
log (#*Leases*/#*Sales*)	−0.0374	0.0442	0.155	0.00374	0.228	0.0836	−0.0195	0.0434
	(0.146)	(0.109)	(0.171)	(0.0994)	(0.155)	(0.107)	(0.149)	(0.0987)
Observations	1,233	2,081	848	2,520	1,032	2,134	1,102	2,591
log (*Price*/*Rent*)	−0.000958	0.00544	0.0401	0.0372	0.0588	0.0833**	0.106**	0.0854**
	(0.0405)	(0.0346)	(0.0781)	(0.0281)	(0.0861)	(0.0382)	(0.0402)	(0.0323)
Observations	1,658	2,649	965	3,216	1,047	2,557	1,480	3,244
log (#*BTO sales*)	−0.0913	−0.0292	−0.0305	−0.0377	−0.0227	−0.0552	−0.0885	−0.00809
	(0.0759)	(0.0564)	(0.0667)	(0.0516)	(0.0829)	(0.0568)	(0.0626)	(0.0499)
Observations	2,471	4,481	3,190	5,392	2,139	4,470	3,590	5,846
log (#*BTR sales*)	−0.0468	−0.0977	−0.189	−0.0596	−0.0469	0.0298	−0.0462	0.0273
	(0.0961)	(0.0948)	(0.118)	(0.0876)	(0.113)	(0.0907)	(0.103)	(0.0785)
Observations	527	883	598	1,075	395	866	706	1,074
Distance threshold	2km	4km	4km	5km	2km	4km	4km	5km
City indicators ±3 m.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes		Yes	Yes	Yes
Donut hole			1km				1km	

Notes: Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. The sample comprises detached house transactions from 2006 to 2012. There are four panels, each showing the results of a placebo test with different distances from the Toronto border. All regressions include a post-LTT dummy, and city, community, year, and calendar month fixed effects. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008 whenever this applies. Distance threshold indicates the radius used in the sample. Robust standard errors are in parentheses, and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.9: *Robustness checks on threshold to distinguish BTO and BTR transactions*

Dependent variable	(1)	(2)	(3)	(4)
	6-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.115**	−0.135**	−0.117**	−0.158***
	(0.0577)	(0.0433)	(0.0546)	(0.0322)
log (#*BTR sales*)	0.194**	0.200***	0.206***	0.0956**
	(0.0739)	(0.0518)	(0.0612)	(0.0477)
	12-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.0835	−0.0972**	−0.0799	−0.128***
	(0.0580)	(0.0438)	(0.0554)	(0.0326)
log (#*BTR sales*)	0.167**	0.144**	0.148**	0.0478
	(0.0637)	(0.0472)	(0.0588)	(0.0431)
	24-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.110*	−0.116**	−0.0917	−0.125***
	(0.0592)	(0.0447)	(0.0566)	(0.0333)
log (#*BTR sales*)	0.139**	0.113**	0.114**	0.0298
	(0.0602)	(0.0442)	(0.0526)	(0.0411)
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: See the footnote to Table 1. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.10: *Effect of the transaction tax by property type*

Dependent variable	(1)	(2)	(3)	(4)	(5)
$\log(\#Leases/\#Sales)$	0.147**	0.105**	0.101**	0.0719*	0.0730*
	(0.0503)	(0.0478)	(0.0436)	(0.0408)	(0.0391)
Observations	10,233	14,713	17,216	19,817	21,719
$\log(Price/Rent)$	−0.0695**	−0.0451**	−0.0440**	−0.0386**	−0.0367**
	(0.0338)	(0.0194)	(0.0188)	(0.0177)	(0.0176)
Observations	3,745	7,660	8,383	9,313	9,876
$\log(\#BTO\ sales)$	−0.0722**	−0.0467*	−0.0507**	−0.0529**	−0.0478**
	(0.0256)	(0.0276)	(0.0233)	(0.0208)	(0.0191)
Observations	26,639	27,304	36,753	45,585	52,598
$\log(\#BTR\ sales)$	0.0822**	0.0439*	0.0458*	0.0415*	0.0376*
	(0.0389)	(0.0258)	(0.0242)	(0.0230)	(0.0219)
Observations	3,550	5,376	6,069	6,945	7,475
Distance threshold	20km	20km	20km	20km	20km
Detached	Yes	Yes	Yes	Yes	Yes
Semi-detached	Yes	No	Yes	Yes	Yes
Condo apartments	No	Yes	Yes	Yes	Yes
Condo townhouse	No	No	No	Yes	Yes
Row/attached/townhouse	No	No	No	No	Yes

Notes: The table shows estimates of the tax effects for different types of properties. The sample comprises transactions from 2006 to 2012. Given the limited sample size for condos and apartments, the border sample radius is extended from 5km to 20km and different property segments are combined flexibly. In addition to the extensive controls in Table 1, *property type* × *neighbourhood* and *property type* × *year* × *month* fixed effects are included to control for differences in housing stock composition and variations in how different property type segments evolve over time. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. All regressions include a post-LTT dummy, and city, community, year, and calendar-month fixed effects, as well as the interaction between these fixed effects and property types. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends and distance LTT trends are included. Property type rows indicate whether the property type was included in the regression. Border indicates if a border sample was used, and the distance threshold indicates the distance radius used in the sample. Standard errors are clustered at the neighbourhood, year, and property type levels using three-way clustering. *, **, and *** indicate significance at the 10%, 5%, and 1% levels.

Table A.11: *Robustness checks on the moving hazard rate*

	(1)	(2)	(3)	(4)
		Sample period 2006–2010		
Event of moving	−0.156**	−0.218***	−0.243**	−0.286***
	(0.0736)	(0.0636)	(0.111)	(0.0520)
Observations	1,012,969	1,690,705	982,110	3,395,033
		Sample period 2016–2018		
Event of moving	−0.125**	−0.179***	−0.213**	−0.259***
	(0.0597)	(0.0476)	(0.0722)	(0.0357)
Observations	4,327,556	7,306,558	4,296,732	14,969,191
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: The table shows the estimated effect of the LTT on the moving hazard rate. The sample comprises detached house transactions in the GTA. The first panel shows the estimates for the period 2006–2010 and the second panel shows the estimates for the period 2016–2018. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. See the footnote to Table 2 for more details. Standard errors clustered by the community are in parentheses, and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.12: *Robustness checks on sales prices at the market-segment level*

Dependent variable	(1)	(2)	(3)	(4)
		Sample period 2006–2010		
$\log(Price)$	−0.0186**	−0.0172***	−0.0122**	−0.0125**
	(0.00610)	(0.00488)	(0.00613)	(0.00442)
Observations	7,515	12,939	7,949	37,698
		Sample period 2006–2018		
$\log(Price)$	−0.0200***	−0.0174***	−0.0125**	−0.0155***
	(0.00525)	(0.00418)	(0.00524)	(0.00378)
Observations	11,169	19,227	11,802	55,895
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	
Donut hole			2km	

Notes: The estimation sample covers four types of properties: detached houses, townhouses, condominiums, and apartments. A unit of observation is a market segment defined by *community* × *property type* × *year* × *month*. The dependent variable is the average sales price within each market segment. Each cell of the table represents a separate regression on the *LTT* interaction dummy. All regressions include a dummy for the post-LTT period, and *city* × *property type*, *year* × *property type*, *month* × *property type*, and *community* × *property type* fixed effects. See the footnote to Table 1. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Figure A.4: *Household stocks and flows in the model*

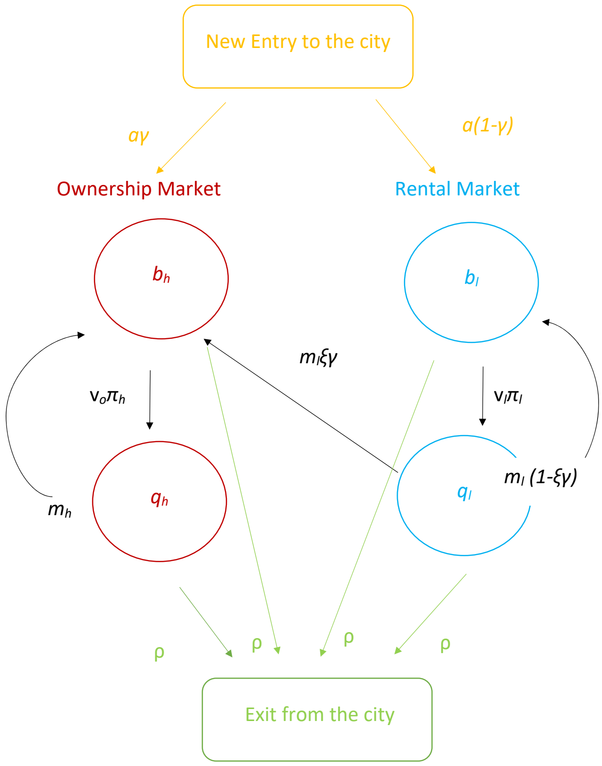
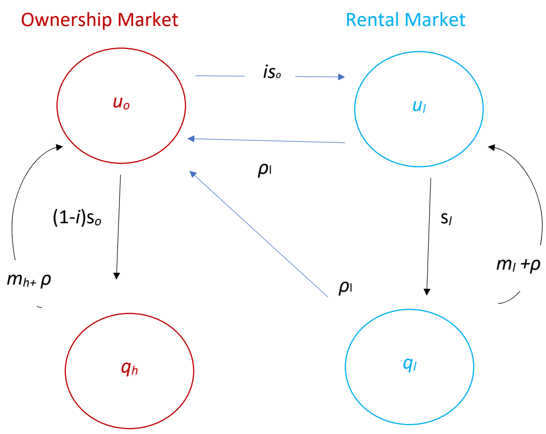


Figure A.5: *Property stocks and flows in the model*



A.2 Deriving the equations of the model

This section shows how to derive equations exactly characterizing the non-household-specific aggregate variables of the model with a finite-dimensional state space. In particular, value functions in match quality such as $L(\varepsilon)$, $W(\varepsilon)$, and $H(\varepsilon)$ are replaced by a finite number of variables that describe the aggregate outcomes in the model, and similarly for the distribution functions of the endogenous distribution of match quality ε. The solution for these variables is obtained from a finite number of equations.

A.2.1 The value functions and thresholds for owner-occupiers and home-buyers

The value function $H(\varepsilon)$ from (21) is increasing in ε. Assuming $\delta_h y_h < x_h$ for all t, by taking ε in a neighbourhood above y_h or any value below, the Bellman equation (21) reduces to the following as $H(\delta_h \varepsilon) < B_h + U_o$:

$$rH(\varepsilon) = \varepsilon + g - D + \alpha_h(B_h + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)\,.$$

This simplifies to

$$(r+\rho+\alpha_h)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon + g - D + \alpha_h B_h + (\rho+\alpha_h)U_o\,, \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range described above:

$$(r+\rho+\alpha_h)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1\,.$$

For a given ε, this specifies a first-order differential equation in time t for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+\alpha_h)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows that the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+\alpha_h}\,, \quad \text{with } \dot{H}(\varepsilon) = \dot{\underline{H}}\,, \tag{A.2}$$

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_h and all values below. Substituting into (A.1) shows that $\underline{H}$ satisfies

$$(r+\rho+\alpha_h)\underline{H} - \dot{\underline{H}} = \alpha_h B_h + (\rho+\alpha_h)U_o + g - D\,. \tag{A.3}$$

Since $x_h < y_h$, equation (22) together with (A.2) implies that

$$x_h = (r+\rho+\alpha_h)(B_h + U_o - \underline{H})\,. \tag{A.4}$$

The surplus in (28) and the definition of the transaction threshold (29) imply y_h satisfies

$$H(y_h) = H(x_h) + C_h + (1+\tau_h)C_o + \tau_h U_o\,, \tag{A.5}$$

and combining (A.2) with (A.5) yields

$$y_h = x_h + (r+\rho+\alpha_h)(C_h + (1+\tau_h)C_o + \tau_h U_o)\,. \tag{A.6}$$

The joint surplus $\Sigma_h(\varepsilon)$ is given in (28) and $1-\omega_h^*$ is the share received by buyers. Equation (30) defines the expected surplus Σ_h, thus the Bellman equation for a buyer (27) can be expressed as

$$(r+\rho)B_h - \dot{B}_h = (1-\omega_h^*)v_o\Sigma_h + g - F_h\,. \tag{A.7}$$

The joint surplus from trade with an investor is given in (38) and ω_i^* is sellers' share. Together with the surplus from trade with a home-buyer, the Bellman equation of a seller (25) is

$$rU_o - \dot{U}_o = \theta_o v_o\left(\omega_h^*(1-\psi)\Sigma_h + \omega_i^*\psi\Sigma_i\right) - D\,. \tag{A.8}$$

Using equations (28), (29), and (40), the expected surplus Σ_h in (30) can be written as

$$\Sigma_h = \int_{y_h}^{\infty} \lambda_h \zeta_h^{\lambda_h} \varepsilon^{-(\lambda_h+1)}\Sigma_h(\varepsilon)\mathrm{d}\varepsilon = \int_{y_h}^{\infty} \frac{\lambda_h \zeta_h^{\lambda_h} \varepsilon^{-(\lambda_h+1)}(H(\varepsilon) - H(y_h))}{1+\tau_h\omega_h^*}\mathrm{d}\varepsilon\,. \tag{A.9}$$

Defining $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε and noting the link with Σ_h:

$$\bar{H}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)}(H(w) - H(\varepsilon))\mathrm{d}w, \quad \text{where } \Sigma_h = \frac{\zeta_h^{\lambda_h} y_h^{-\lambda_h} \bar{H}(y_h)}{1+\tau_h \omega_h^*}. \tag{A.10}$$

Now restrict attention to ε such that $\delta_h \varepsilon < x_h$, so (21) implies $rH(\varepsilon) = \varepsilon + g - D + \alpha_h(B_h + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_h y_h < x_h$, this limits ε to a neighbourhood above y_h and all values below. Using equation (22):

$$\begin{aligned} r(H(w) - H(\varepsilon)) = (w-\varepsilon) + \alpha_h\left(\max\{H(\delta_h w), H(x_h)\} - H(w)\right) - \alpha_h(H(x_h) - H(\varepsilon)) \\ - \rho(H(w) - H(\varepsilon)) + (\dot{H}(w) - \dot{H}(\varepsilon)), \end{aligned}$$

which holds for any $w \geq \varepsilon$. This simplifies to

$$(r+\rho+\alpha_h)(H(w) - H(\varepsilon)) - (\dot{H}(w) - \dot{H}(\varepsilon)) = (w-\varepsilon) + \alpha_h \max\{H(\delta_h w) - H(x_h), 0\},$$

and multiplying both sides by $\lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)}$, integrating over w, and using (A.10):

$$(r+\rho+\alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)} \left((w-\varepsilon) + \alpha_h \max\{H(\delta_h w) - H(x_h), 0\}\right)\mathrm{d}w, \tag{A.11}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)}(\dot{H}(w) - \dot{H}(\varepsilon))\mathrm{d}w.$$

In (A.11), the term in $(w-\varepsilon)$ integrates to $\varepsilon/(\lambda_h - 1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $w < x_h/\delta_h$ because $H(\delta_h w)$ is increasing in w. Hence, equation (A.11) becomes

$$(r+\rho+\alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_h - 1} + \alpha_h \varepsilon^{\lambda_h} \int_{w=x_h/\delta_h}^{\infty} \lambda_h w^{-(\lambda_h+1)}(H(\delta_h w) - H(x_h))\mathrm{d}w,$$

and with the change of variable $w' = \delta_h w$ in the second integral, this can be written as

$$(r+\rho+\alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_h - 1} + \alpha_h \delta_h^{\lambda_h} \varepsilon^{\lambda_h} \int_{w'=x_h}^{\infty} \lambda_h w'^{-(\lambda_h+1)}(H(w') - H(x_h))\mathrm{d}w'. \tag{A.12}$$

Make the following definition of a new variable X_h:

$$\begin{aligned} X_h(t) = \Bigg((\lambda_h - 1)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_p})\right) \int_{T=t}^{\infty} (r+\rho+\alpha_h) e^{-(r+\rho+\alpha_h)(T-t)} \Bigg(\\ \int_{\varepsilon=x_h(T)}^{\infty} \lambda_h \varepsilon^{-(\lambda_h+1)}(H(\varepsilon, T) - H(x_h(T), T))\mathrm{d}\varepsilon \Bigg) \mathrm{d}T \Bigg)^{\frac{1}{1-\lambda_h}}. \end{aligned} \tag{A.13}$$

By differentiating with respect to time t, this variable must satisfy the differential equation

$$\begin{aligned} (r+\rho+\alpha_h)X_h^{1-\lambda_h} - (1-\lambda_h)\dot{X}_h X_h^{-\lambda_h} &= (\lambda_h - 1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right) x_h^{-\lambda_h} \bar{H}(x_h) \\ &= (\lambda_h - 1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right) \int_{\varepsilon=x_h}^{\infty} \lambda_h \varepsilon^{-(\lambda_h+1)}(H(\varepsilon) - H(x_h))\mathrm{d}\varepsilon, \end{aligned} \tag{A.14}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r+\rho+\alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{1}{\lambda_h - 1}\left(\varepsilon + \frac{\alpha_h \delta_h^{\lambda_h} \varepsilon^{\lambda_h}\left((r+\rho+\alpha_h)X_h^{1-\lambda_h} - (1-\lambda_h)\dot{X}_h X_h^{-\lambda_h}\right)}{(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)}\right),$$

and by collecting terms this can be written as

$$(r+\rho+\alpha_h)\left(\bar{H}(\varepsilon)-\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)}X_h^{1-\lambda_h}\right)$$
$$-\left(\dot{\bar{H}}(\varepsilon)-\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)}(1-\lambda_h)\dot{X}_hX_h^{-\lambda_h}\right)=\frac{\varepsilon}{\lambda_h-1}\,.$$

Noting that $\mathrm{d}X_h(t)^{1-\lambda_h}/\mathrm{d}t=(1-\lambda_h)\dot{X}_hX_h^{-\lambda_h}$ and observing the right-hand side of the equation above is time invariant and none of the variables is predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon)-\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}X_h^{1-\lambda_h}/((\lambda_h-1)(r+\rho+\alpha_h)(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})))$ that is time invariant and equal to $\varepsilon/((\lambda_h-1)(r+\rho+\alpha_h))$. This demonstrates that for any given ε in a neighbourhood above y_h or any value below it, the function $\bar{H}(\varepsilon)$ is given by

$$\bar{H}(\varepsilon)=\frac{1}{(\lambda_h-1)(r+\rho+\alpha_h)}\left(\varepsilon+\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}X_h^{1-\lambda_h}\right)\,. \tag{A.15}$$

Evaluating (A.15) at $\varepsilon=x_h$ and multiplying by $(\lambda_h-1)(r+\rho+\alpha_h)(r+\rho+\alpha_h(1-\delta_h^{\lambda_h}))x_h^{-\lambda_h}$:

$$(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)x_h^{-\lambda_h}\bar{H}(x_h)=\left(r+\rho+a_h(1-\delta_h^{\lambda_h})\right)x_h^{1-\lambda_h}+\alpha_h\delta_h^{\lambda_h}X_h^{1-\lambda_h}\,,$$

and then by substituting into (A.14) shows that X_h is related to the moving threshold x_h as follows:

$$\frac{\dot{X}_h}{X_h}=\left(\frac{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}{\lambda_h-1}\right)\left(\left(\frac{x_h}{X_h}\right)^{1-\lambda_h}-1\right)\,. \tag{A.16}$$

Finally, evaluating (A.15) at $\varepsilon=y_h$ and substituting into (A.10) yields an equation for the joint surplus:

$$\Sigma_h=\frac{\zeta_h^{\lambda_h}}{(1+\tau_h\omega_h^*)(\lambda_h-1)(r+\rho+\alpha_h)}\left(y_h^{1-\lambda_h}+\frac{\alpha_h\delta_h^{\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}X_h^{1-\lambda_h}\right)\,. \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_h, x_h, X_h, Σ_h, $\underline{H}$, B_h, and U_o, taking as given Σ_k, v_o, ψ, and g.

A.2.2 The moving rate of owner-occupiers

The flow of owner-occupiers who move within the city is M_h and the moving rate is $m_h=M_h/q_h$. The group of existing owner-occupiers q_h is made up of matches that formed at various points in the past and that have survived to the present. Given a sufficiently large inconvenience cost of moving in the absence of a shock, moving occurs only if owner-occupiers receive an idiosyncratic shock with arrival rate α_h independent of history. A measure α_hq_h of households might therefore decide to move.

All matches began as a viewing with some initial match quality ε. Using (23), the flow of viewings v_h done by home-buyers in the ownership market at a point in time is

$$v_h=v_ob_h=(1-\psi)\theta_ov_ou_o\,. \tag{A.18}$$

Initial match quality drawn in viewings is from a Pareto(ζ_h,λ_h) distribution (see 40). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving owner-occupiers where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon},\lambda_h)$. Among this group, denote current match quality as a multiple Ξ of original match quality ε, where Ξ is equal to δ_h raised to the power of the number of past shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon'=\delta_h\Xi\varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon'<x_h$, so only those with initial match quality $\varepsilon\geq$

$x_h/(\delta_h \Xi)$ remain. Since $\delta_h < 1$ and $\delta_h y_h < x_h$, there is a range of variation in thresholds y_h and x_h that ensures $x_h/(\delta_h \Xi) > \underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_h}(x_h/(\delta_h \Xi))^{-\lambda_h} = x_h^{-\lambda_h}\delta_h^{\lambda_h}\Xi^{\lambda_h}\underline{\varepsilon}^{\lambda_h}$. Since that surviving group is a fraction $\zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_h^{-\lambda_h}\delta_h^{\lambda_h}\Xi^{\lambda_h}\underline{\varepsilon}^{\lambda_h} \times \zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h} = (\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}) \times \Xi^{\lambda_h}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Ξ of past idiosyncratic shocks. Let Θ_h be the integral of Ξ^{λ_h} over the measure of current and past viewings done by home-buyers who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}$ of Ξ^{λ_h}, the measure of those choosing not to move after a new shock is $\alpha_h\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}\Theta_h$. Therefore, the size of the group of movers is

$$M_h = \alpha_h q_h - \alpha_h \zeta_h^{\lambda_h} x_h^{-\lambda_h} \delta_h^{\lambda_h} \Theta_h \,. \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction α_h of the group used to define Θ_h have Ξ^{λ_h} reduced to $\delta_h^{\lambda_h}\Xi^{\lambda_h}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Xi^{\lambda_h} = 1$ with measure v_h from (A.18). The equation for Θ_h is therefore

$$\dot{\Theta}_h = v_h + \alpha_h(\delta_h^{\lambda_h}\Theta_h - \Theta_h) - \rho\Theta_h \,. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings v_h:

$$\bar{v}_h(t) = \int_{T\to-\infty}^{t} (\rho + \alpha_h(1-\delta_h^{\lambda_h}))e^{-(\rho+\alpha_h(1-\delta_h^{\lambda_h}))(t-T)} v_h(T)\mathrm{d}T \,, \tag{A.21}$$

and note that it satisfies the differential equation

$$\dot{\bar{v}}_h + (\rho + \alpha_h(1-\delta_h^{\lambda_h}))\bar{v}_h = (\rho + \alpha_h(1-\delta_h^{\lambda_h}))v_h \,. \tag{A.22}$$

A comparison of (A.20) and (A.22) shows that $\Theta_h = \bar{v}_h/(\rho + \alpha_h(1-\delta_h^{\lambda_h}))$, and substituting this into (A.19) yields an equation for the moving rate $m_h = M_h/q_h$:

$$m_h = \alpha_h - \frac{\alpha_h \zeta_h^{\lambda_h} \delta_h^{\lambda_h} x_h^{-\lambda_h} \bar{v}_h}{(\rho + \alpha_h(1-\delta_h^{\lambda_h}))q_h} \,. \tag{A.23}$$

Using the definition of $\bar{v}_h(t)$ in (A.21) and (A.18), this confirms equation (41) for the moving rate m_h.

A.2.3 The transaction threshold and value functions in the rental market

By adding the Bellman equations (9) and (10) for the tenant and landlord value functions:

$$\begin{aligned} r(L(\varepsilon)+W(\varepsilon)) = \varepsilon + g - D - D_l + (\alpha_l+\rho)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) \\ + m_l\left(\xi\kappa(B_h - \bar{K}) + (1-\xi\kappa)B_l - W(\varepsilon)\right) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon) \,. \end{aligned}$$

Letting $J(\varepsilon) = L(\varepsilon) + W(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting that $B_h - B_l = Z$ from (3) and $m_l = \alpha_l + \rho_l$ from (8):

$$(r+\rho+m_l)J(\varepsilon) = \varepsilon + g - D - D_l + (\rho+\alpha_l)U_l + \rho_l U_o + m_l B_l + \xi m_l \kappa(Z - \bar{K}) + \dot{J}(\varepsilon) \,. \tag{A.24}$$

Differentiating with respect to ε leads to the differential equation

$$(r+\rho+m_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon) \,,$$

and this equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r+\rho+m_l} \,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r + \rho + m_l}\,, \tag{A.25}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.24) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation

$$(r + \rho + m_l)\underline{J} = m_l B_l + (\rho + \alpha_l)U_l + \rho_l U_o + g - D - D_l + \xi m_l \kappa(Z - \bar{K}) + \dot{\underline{J}}\,. \tag{A.26}$$

The joint rental surplus from (14) prior to a tenant moving in is linked to $J(\varepsilon)$ by

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l\,, \tag{A.27}$$

and together with (A.25), the definition of the rental transaction threshold y_l in (15) implies

$$y_l = (r + \rho + m_l)(B_l + U_l - \underline{J} + C_l + C_w)\,. \tag{A.28}$$

Using (15), (A.25), and (A.27), it follows that the surplus $\Sigma_l(\varepsilon)$ is

$$\Sigma_l(\varepsilon) = \frac{\varepsilon - y_l}{r + \rho + m_l}\,, \quad \text{and } \Sigma_l = \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon) = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{(\lambda_l - 1)(r + \rho + m_l)}\,, \tag{A.29}$$

where the second equation uses the Pareto distribution in (40) to derive the expected rental surplus Σ_l. Landlords' share of the joint surplus is ω_l, so $\Sigma_{ll}(\varepsilon) = L(\varepsilon) + A(\varepsilon) - C_l - U_l = \omega_l \Sigma_l(\varepsilon)$. Together with (A.29), equation (12) for U_l becomes

$$(r + \rho_l)U_l - \dot{U}_l = \omega_l \theta_l v_l \Sigma_l - D + \rho_l U_o\,. \tag{A.30}$$

Similarly, with $\Sigma_{lw}(\varepsilon) = W(\varepsilon) - A(\varepsilon) - C_w - B_l = (1 - \omega_l)\Sigma_l$, equation (13) for B_l becomes

$$(r + \rho)B_l - \dot{B}_l = (1 - \omega_l)v_l \Sigma_l + g - F_l\,. \tag{A.31}$$

In summary, equations (A.26), (A.28), (A.29), (A.30), and (A.31) determine y_l, Σ_l, $\underline{J}$, B_l, and U_l, taking as given Z, $\bar{K}$, κ, and g.

A.2.4 Rents

With $m_l = \alpha_l + \rho_l$ from (8), the Bellman equation (10) can be written as follows:

$$(r + \rho + m_l)(L(\varepsilon) - U_l) = R(\varepsilon) - D - D_l - (r + \rho_l)U_l + \rho_l U_o + \dot{L}(\varepsilon)\,,$$

and substituting from (A.30) implies that the rent $R(\varepsilon)$ for a property with match quality ε is

$$R(\varepsilon) = D_l + \omega_l \theta_l v_l \Sigma_l + (r + \rho + m_l)(L(\varepsilon) - U_l) - (\dot{L}(\varepsilon) - \dot{U}_l)\,.$$

Using the landlord's surplus $\Sigma_{wl}(\varepsilon) = L(\varepsilon) - U_l$ after a tenant has moved in, and its derivative with respect to time $\dot{\Sigma}_{wl}(\varepsilon) = \dot{L}(\varepsilon) - \dot{U}_l$, the surplus division $\Sigma_{wl}(\varepsilon) = \omega_l \Sigma_w(\varepsilon)$ implies

$$R(\varepsilon) = D_l + \omega_l \theta_l v_l \Sigma_l + \omega_l \left((r + \rho + m_l)\Sigma_w(\varepsilon) - \dot{\Sigma}_w(\varepsilon)\right)\,.$$

Noting that $\Sigma_l(\varepsilon) = \Sigma_w(\varepsilon) - (C_l + C_w)$ from (7) and (14) and substituting for $\Sigma_w(\varepsilon)$ in the above:

$$R(\varepsilon) = D_l + \omega_l(r + \rho + m_l)(C_l + C_w) + \omega_l \theta_l v_l \Sigma_l + \omega_l \left((r + \rho + m_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon)\right)\,.$$

Using the first equation in (A.29), it follows that $\dot{\Sigma}_l(\varepsilon) = -\dot{y}_l/(r + \rho + m_l)$ for all ε, and hence:

$$R(\varepsilon) = D_l + \omega_l(r + \rho + m_l)(C_l + C_w) + \omega_l \theta_l v_l \Sigma_l + \omega_l(\varepsilon - y_l) + \frac{\omega_l}{r + \rho + m_l}\dot{y}_l\,. \tag{A.32}$$

With expected surplus Σ_l from (A.29), average new rents R from (16) are given by

$$R = D_l + \omega_l(r + \rho + m_l)(C_l + C_w) + \omega_l(r + \rho + m_l + \theta_l v_l \pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r + \rho + m_l}\dot{y}_l\,. \tag{A.33}$$

Since (A.32) implies $R'(\varepsilon) = \omega_l$ for all ε, rents are linear in match quality, so the average $\bar{R}$ of all rents on current tenancies is

$$\bar{R} = R + \omega_l \left(V_l - \frac{\lambda_l}{\lambda_l - 1} y_l \right), \tag{A.34}$$

where V_l is the average ε over all current tenancies (see 46), and $\lambda_l y_l/(\lambda_l - 1)$ is the average of ε over new tenancies using (40).

A.2.5 The relationship between market tightnesses across the two markets

Subtracting the total measures of properties in (1) from the total measure of households in (2), and using the definitions market tightnesses θ_l and θ_o from (11) and (23), and the fraction ψ of investors among all buyers from (24) leads to the following equation:

$$((1-\psi)\,\theta_o - 1)\,u_o + (\theta_l - 1)\,u_l = n - 1\,. \tag{A.35}$$

A.2.6 Average match quality and value functions averaged over surviving match quality

Let $\mathscr{E}_h$ denote the integral of ε over all current owner-occupiers. There is a flow S_h of new owner-occupier matches. Since the transaction threshold is y_h, the Pareto distribution (40) implies the average value of ε in these new matches is $\lambda_h y_h/(\lambda_h - 1)$, so these new matches add to $\mathscr{E}_h$ at rate $S_h \lambda_h y_h/(\lambda_h - 1)$ over time.

Owner-occupier matches are destroyed (sending the contribution to $\mathscr{E}_h$ to zero) if households exit the city or match-quality shocks arrive and households choose to move. Households exit the city at rate ρ, reducing $\mathscr{E}_h$ by $\rho\mathscr{E}_h$. Match-quality shocks arrive randomly at rate α_h for the measure q_h of owner-occupiers, leading to a flow M_h of movers out of the group $\alpha_h q_h$ receiving a shock, which reduces the contribution to $\mathscr{E}_h$ of those M_h to zero. For the group of size $\alpha_h q_h - M_h$ that receives a shock but does not move, the conditional distribution of surviving match quality ε is truncated at x_h, which is a Pareto distribution with shape parameter λ_h across all cohorts within that group, which has mean $\lambda_h x_h/(\lambda_h - 1)$. Putting together all these effects on $\mathscr{E}_h$, the following differential equation must hold:

$$\dot{\mathscr{E}}_h = S_h \frac{\lambda_h y_h}{\lambda_h - 1} + \left(M_h \times 0 + (\alpha_h q_h - M_h) \times \frac{\lambda_h x_h}{\lambda_h - 1} - \alpha_h \mathscr{E}_h \right) - \rho \mathscr{E}_h\,.$$

Average match quality among owner-occupiers is $V_h = \mathscr{E}_h/q_h$, thus $\dot{V}_h = \dot{\mathscr{E}}_h/q_h - (\dot{q}_h/q_h)V_h = \dot{\mathscr{E}}_h/q_h - ((S_h/q_h) - (m_h + \rho))V_h$, where the second equation uses the differential equation for q_h in (32). Together with the equation for $\dot{\mathscr{E}}_h$ above and the definition of the moving rate $m_h = M_h/q_h$, average match quality V_h must satisfy the differential equation in (45).

Let $\mathscr{E}_l$ denote the equivalent summation of surviving match quality for tenants. There is a flow $s_l u_l$ of new rental matches. Since the transaction threshold is y_l, these new matches add to $\mathscr{E}_l$ at rate $s_l u_l \lambda_l y_l/(\lambda_l - 1)$. Matches are destroyed if households exit the city (rate ρ), if landlords must sell up (rate ρ_l), or if match quality falls to zero owing to an idiosyncratic shock (rate α_l). The differential equation for $\mathscr{E}_l$ is thus $\dot{\mathscr{E}}_l = s_l u_l(\lambda_l y_l/(\lambda_l - 1)) - (\alpha_l + \rho_l + \rho)\mathscr{E}_l$. Average match quality for tenants is $V_l = \mathscr{E}_l/q_l$, hence $\dot{V}_l = (\dot{\mathscr{E}}_l/q_l) - (\dot{q}_l/q_l)V_l$, and by substituting $\dot{q}_l/q_l = (s_l u_l/q_l) - (m_l + \rho)$ from (18), the differential equation for V_l is (46), which uses $m_l = \alpha_l + \rho_l$ from (8).

Let $\Gamma_\varepsilon(\varepsilon)$ denote the distribution function of current match quality ε for owner-occupiers. The average value of $H(\varepsilon)$ across all q_h matches and the integral of these values are denoted by $\bar{H}$ and Q:

$$\bar{H} = \int_\varepsilon H(\varepsilon)\mathrm{d}\Gamma_\varepsilon(\varepsilon), \quad \text{and} \quad Q = q_h \bar{H} = \int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon, \quad \text{where} \quad \varsigma(\varepsilon) = q_h \Gamma'_\varepsilon(\varepsilon)\,. \tag{A.36}$$

The function $\varsigma(\varepsilon)$ is the density function $\Gamma'_\varepsilon(\varepsilon)$ of the distribution of surviving match quality ε multiplied by q_h. Differentiating Q with respect to time implies $\dot{Q} = \int_\varepsilon \left(\dot{H}(\varepsilon)\varsigma(\varepsilon) + H(\varepsilon)\dot{\varsigma}(\varepsilon)\right)\mathrm{d}\varepsilon$ and hence

$$rQ - \dot{Q} = \int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon - \int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon\,. \tag{A.37}$$

Shocks scaling down match quality ε to $\delta_h\varepsilon$ occur with arrival rate α_h, which triggers moving if match quality falls below x_h. There is also exogenous exit from the city at rate ρ. New matches form at rate S_h and begin with ε having distribution function $\Gamma_h(\varepsilon)/\pi_h$ for $\varepsilon \geq y_h$, where $\pi_h = 1-\Gamma_h(y_h)$. The dynamics of $\varsigma(\varepsilon) = q_h\Gamma'_\varepsilon(\varepsilon)$ describing the distribution of ε across all surviving owner-occupier matches are thus:

$$\dot{\varsigma}(\varepsilon) = \begin{cases} -(\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } \varepsilon < x_h \\ \alpha_h\delta_h^{-1}\varsigma(\delta_h^{-1}\varepsilon)-(\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } x_h \leq \varepsilon < y_h \\ (S_h/\pi_h)\Gamma'_h(\varepsilon)+\alpha_h\delta_h^{-1}\varsigma(\delta_h^{-1}\varepsilon)-(\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } y_h \leq \varepsilon \end{cases}.$$

It follows that

$$\begin{aligned}\int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon &= \frac{S_h}{\pi_h}\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon)+\frac{\alpha_h}{\delta_h}\int_{\varepsilon=x_h} H(\varepsilon)\varsigma\left(\frac{\varepsilon}{\delta_h}\right)\mathrm{d}\varepsilon-(\alpha_h+\rho)q_h\bar{H} \\ &= v_ob_h\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon)+\alpha_h\int_{\varepsilon=x_h/\delta_h} H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon-(\alpha_h+\rho)q_h\bar{H}\,, \quad \text{(A.38)}\end{aligned}$$

which uses $S_h = v_o\pi_hb_h$ from (32) and a change of variable $\varepsilon' = \varepsilon/\delta_h$ in the second term on the second line. Using the Bellman equation (21) for $H(\varepsilon)$ and the definitions of V_h and $\bar{H}$ from (A.36):

$$\begin{aligned}&\int_\varepsilon \left(rH(\varepsilon)-\dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon = \int_\varepsilon(\varepsilon+g-D)\mathrm{d}\varsigma(\varepsilon)+\alpha_h(B_h+U_o)\int_{\varepsilon=0}^{\frac{x_h}{\delta_h}}\varsigma(\varepsilon)\mathrm{d}\varepsilon-\alpha_h\int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &+\alpha_h\int_{\varepsilon=\frac{x_h}{\delta_h}}^{\infty} H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon+\rho\int_\varepsilon(U_o-H(\varepsilon))\varsigma(\varepsilon)\mathrm{d}\varepsilon = (V_h+g-D)q_h+\alpha_h\int_{\varepsilon=\frac{x_h}{\delta_h}}^{\infty} H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &+m_h(B_h+U_o)q_h-\alpha_h\bar{H}q_h+\rho(U_o-\bar{H})q_h\,, \quad \text{where } \int_{\varepsilon=0}^{\frac{x_h}{\delta_h}}\alpha_h\varsigma(\varepsilon) = m_hq_h\,. \quad \text{(A.39)}\end{aligned}$$

where $\varsigma(\varepsilon)$ integrates to q_h over all ε, and to the number of moves m_hq_h within the city over the range up to $\varepsilon = x_h/\delta_h$. Substituting equations (A.38) and (A.39) into (A.37) yields:

$$rQ-\dot{Q} = (V_h+g-D)q_h+m_h(B_h+U_o)q_h+\rho U_oq_h-v_ob_h\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon)\,.$$

Since $\bar{H} = Q/q_h$ implies $\dot{\bar{H}} = \dot{Q}/q_h-\bar{H}\dot{q}_h/q_h$, the equation above and (32) for $\dot{q}_h$ imply $\bar{H}$ satisfies:

$$r\bar{H} = V_h+g-D+m_h(B_h+U_o-\bar{H})+\rho(U_o-\bar{H})+\frac{v_o\pi_hb_h}{q_h}(\bar{H}-H)+\dot{\bar{H}}\,. \quad \text{(A.40)}$$

where H is defined as the average of $H(\varepsilon)$ over ε for new owner-occupier matches:

$$H = \frac{1}{\pi_h}\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon)\,, \quad L = \frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon) \quad \text{and } W = \frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon)\,, \quad \text{(A.41)}$$

with L and W defined similarly as the averages of $L(\varepsilon)$ and $W(\varepsilon)$ over new rental-market matches. Analogous to the definition of $\bar{H}$, let $\bar{L}$ and $\bar{W}$ be the average values of $L(\varepsilon)$ and $W(\varepsilon)$ across the distribution of match quality ε for all surviving matches in the rental market. The same method used to derive (A.40) can be applied to show the equivalent for $\bar{W}$ of the Bellman equation (9) for $W(\varepsilon)$ is

$$r\bar{W} = V_l-\bar{R}+g+m_l(\xi\kappa(B_h-\bar{K})+(1-\xi\kappa)B_l-\bar{W})-\rho\bar{W}+\frac{v_l\pi_lb_l}{q_l}(\bar{W}-W)+\dot{\bar{W}}\,, \quad \text{(A.42)}$$

where V_l and $\bar{R}$ are averages of ε and $R(\varepsilon)$ for surviving rental-market matches, and the equivalent of the Bellman equation (10) for $L(\varepsilon)$ in terms of $\bar{L}$ is

$$r\bar{L} = \bar{R}-D-D_l+(\alpha_l+\rho)(U_l-\bar{L})+\rho_l(U_o-\bar{L})+\frac{v_l\pi_lb_l}{q_l}(\bar{L}-L)+\dot{\bar{L}}\,. \quad \text{(A.43)}$$

A.2.7 Welfare

With $\bar{H}$, $\bar{L}$, and $\bar{W}$ denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of surviving owner-occupier and rental-market matches, total welfare Ω is defined as follows:

$$\Omega = q_h\bar{H} + q_l(\bar{L}+\bar{W}) + b_hB_h + b_lB_l + b_iI + u_oU_o + u_lU_l + \Omega_a\,, \tag{A.44}$$

where Ω_a is the expected present values N of new entrants to the city, which by using (5) satisfies:

$$r\Omega_a = a\left(\kappa(B_h-\bar{K}) + (1-\kappa)B_l - E\right) + \dot{\Omega}_a\,. \tag{A.45}$$

Differentiating total welfare Ω from (A.44) with respect to t and subtracting from $r\Omega$:

$$r\Omega = q_h(r\bar{H}-\dot{\bar{H}}) - \bar{H}\dot{q}_h + q_l(r\bar{L}-\dot{\bar{L}}) + q_l(r\bar{W}-\dot{\bar{W}}) - (\bar{L}+\bar{W})\dot{q}_l + b_h(rB_h-\dot{B}_h) - B_h\dot{b}_h + \dot{\Omega}$$
$$+b_l(rB_l-\dot{B}_l) - B_l\dot{b}_l + b_i(rI-\dot{I}) - I\dot{b}_i + u_o(rU_o-\dot{U}_o) - U_o\dot{u}_o + u_l(rU_l-\dot{U}_l) - U_l\dot{u}_l + (r\Omega_a-\dot{\Omega}_a)\,.$$

Substituting Bellman equations (12), (13), (25), (27), (33), (A.40), (A.42), (A.43), (A.45), and laws of motion (18), (19), (20), (31), (32), and (39) into the equation above:

$$\begin{aligned}
r\Omega = {} & q_h\left(V_h + g - D + m_h(B_h+U_o-\bar{H}) + \rho(U_o-\bar{H}) + \frac{v_o\pi_hb_h}{q_h}(\bar{H}-H)\right) \\
& - \bar{H}\left(v_o\pi_hb_h - (m_h+\rho)q_h\right) + q_l\left(\bar{R} - D - D_l + (\alpha_l+\rho)(U_l-\bar{L}) + \rho_l(U_o-\bar{L}) + \frac{v_l\pi_lb_l}{q_l}(\bar{L}-L)\right) \\
& + q_l\left(V_l - \bar{R} + g + m_l\left(\xi\kappa(B_h-\bar{K}) + (1-\xi\kappa)B_l - \bar{W}\right) - \rho\bar{W} + \frac{v_l\pi_lb_l}{q_l}(\bar{W}-W)\right) \\
& - (\bar{L}+\bar{W})\left(s_lu_l - (m_l+\rho)q_l\right) + b_h\left(g - F_h + v_o\pi_h(H - C_h - (1+\tau_h)P_h - B_h) - \rho B_h\right) \\
& - B_h\left(m_hq_h + \gamma - (v_o\pi_h+\rho)b_h\right) + b_l\left(g - F_l + v_l\pi_l(W - A - C_w - B_l) - \rho B_l\right) \\
& - B_l\left((1-\xi\kappa)m_lq_l + (1-\kappa)a - (v_l\pi_l+\rho)b_l\right) + b_i\left(-F_i + v_o(U_l - (1+\tau_i)P_i - C_i - I)\right) - I\dot{b}_i \\
& + u_o\left(-D + \theta_ov_o(1-\psi)\pi_h(P_h - C_o - U_o) + \theta_ov_o\psi(P_i - C_o - U_o)\right) \\
& - U_o\left((m_h+\rho)q_h + \rho_l(q_l+u_l) - s_ou_o\right) + u_l\left(-D + \theta_lv_l\pi_l(L + A - C_l - U_l) + \rho_l(U_o-U_l)\right) \\
& \qquad - U_l\left((\alpha_l+\rho)q_l + S_i - (s_l+\rho_l)u_l\right) + a\left(\kappa(B_h-\bar{K}) + (1-\kappa)B_l - E\right) + \dot{\Omega}\,,
\end{aligned} \tag{A.46}$$

which also uses the transactions probabilities $\pi_j = 1-\Gamma_j(y_j)$ for $j\in\{h,l\}$, the constant value of A from (16), the average price P_h paid by home-buyers from (30), and the definitions of the average values H, L, and W for new matches from (A.41). This expression for welfare can be simplified in a number of ways. First, observe that by collecting terms multiplying the values $\bar{H}$, H, $\bar{L}$, L, $\bar{W}$, W, B_h, and B_l, all of the coefficients of these values are zero, reflecting transitions of particular individuals between different states. This can be seen directly for $\bar{H}$, H, W, and B_l. Noting $v_l\pi_lb_l = \theta_lv_l\pi_lu_l$ using (11) gives a zero coefficient on L, and $v_l\pi_lb_l = s_lu_l$ from (17) a zero coefficient on $\bar{W}$. These observations together with $m_l = \alpha_l+\rho_l$ from (8) yield a zero coefficient on $\bar{L}$. A zero coefficient on B_h follows from the expression for first-time buyers γ in (4), a zero coefficient on U_l from $s_l = \theta_lv_l\pi_l$ in (17) and $S_i = v_ob_i$ in (35), and a zero coefficient on U_o from $s_o = \theta_ov_o(\psi+(1-\psi)\pi_h)$ in (36). Using investors' free-entry condition (37), the terms that are multiplied by I are also zero.[51]

Next, note that rent payments $\bar{R}$ and tenancy agreement fees A cancel out, the latter using the definition of market tightness $\theta_l = b_l/u_l$ from (11). This is because such payments are simply transfers among individuals that net out from total welfare. The terms in average prices P_h and P_i simplify to $-\tau_hP_hS_h$ and $-\tau_iP_iS_i$ respectively using (32), (35), and (36), where net payments equal the tax revenue transferred to the government. Collecting all terms in g from (A.46) yields $(q_l+q_h+b_l+b_h)g = ng = G = \tau_hP_hS_h + \tau_iP_iS_i$ using (2) and (26). Hence, these terms and those in prices cancel out overall, reflecting the assumption that tax revenue is used to provide public goods of an equivalent value.

The only terms that remain on the right-hand side of (A.46) are $\dot{\Omega}$ are those involving average match qualities V_h and V_l and costs D, D_l, F_h, F_i, F_l, C_h, C_i, C_o, C_l, C_w, $\bar{K}$, and E. The coefficient of D is

[51] This holds even at points in time where $\dot{b}_i$ is not well defined owing to jumps in b_i.

-1 using $q_l + q_h + u_l + u_o = 1$ from (1) and the coefficient of $\bar{K}$ is $-\gamma$ using (4). The coefficients on transaction costs C_h, C_i, C_o, C_l, and C_w can be expressed in terms of the flows of various types of transactions S_h, S_i, S_o, and S_l by using equations (17), (32), (35), and (36). This completes the derivation of the expression for total welfare given in (44).

A.3 Solving for the steady state

The solution method is based on a numerical search over the fraction ψ of investors among buyers and ownership-market tightness θ_o that satisfy two equations representing equilibrium in the ownership and rental markets. Within this search, given a (ψ, θ_o), the ownership-market thresholds (y_h, x_h) and rental-market and credit-cost thresholds (y_l, Z) are found by solving two equations numerically.

A.3.1 Ownership-market thresholds

This part of the solution method derives an equation satisfied by the ownership-market transaction threshold y_h, which can be solved taking as given (ψ, θ_o). Once y_h is known, the moving threshold x_h is determined, along with other variables related to the ownership market.

With $\dot{B}_h = 0$ and $\dot{U}_o = 0$ in the steady state, the Bellman equations (A.7) and (A.8) become

$$(r+\rho)B_h = g - F_h + (1-\omega_h^*)v_o\Sigma_h\,, \quad \text{and} \tag{A.47}$$

$$rU_o = \theta_o v_o((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i) - D\,. \tag{A.48}$$

Substituting from (A.48) into equation (A.6) that links y_h and x_h:

$$y_h = x_h + (r+\rho+\alpha_h)\left(C_h + C_o + \tau_h\left(C_o - \frac{D}{r} + \frac{\theta_o v_o((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i)}{r}\right)\right). \tag{A.49}$$

With $\dot{\underline{H}} = 0$ in the steady state, equation (A.3) becomes $(r+\rho+\alpha_h)\underline{H} = \alpha_h B_h + (\rho+\alpha_h)U_o + g - D$. Substituting into (A.4) implies $x_h = (r+\rho+\alpha_h)(B_h+U_o) - \alpha_h B_h - (\rho+\alpha_h)U_o + D - g$ and hence

$$x_h = D - g + (r+\rho)B_h + rU_o\,.$$

Then substituting for B_h and U_o from (A.47) and (A.48) yields

$$x_h + F_h = (1-\omega_h^* + (1-\psi)\omega_h^*\theta_o)\,v_o\Sigma_h + \theta_o v_o\psi\omega_i^*\Sigma_i\,. \tag{A.50}$$

Equation (A.16) implies that the steady state must have $X_h = x_h$ since $\dot{X}_h = 0$. Substituting into (A.17):

$$\Sigma_h = \frac{\zeta_h^{\lambda_h}}{(r+\rho+\alpha_h)(\lambda_h-1)(1+\tau_h\omega_h^*)}\left(y_h^{1-\lambda_h} + \frac{\alpha_h\delta_h^{\lambda_h}x_h^{1-\lambda_h}}{r+\rho+\alpha_h\left(1-\delta_h^{\lambda_h}\right)}\right). \tag{A.51}$$

The next step is to reduce these equations to a single equation that can be solved numerically for y_h. Equation (A.50) implies $v_o\Sigma_h = (x_h + F_h - \psi\theta_h v_o\omega_i^*\Sigma_i)/(1-\omega_h^* + (1-\psi)\omega_h^*\theta_h)$, and together with $v_o\Sigma_i = F_i/(1-\omega_i^*)$ from (38), it follows that:

$$\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right) = \frac{\omega_h^*\theta_o}{1-\omega_h^* + (1-\psi)\omega_h^*\theta_o}\left((1-\psi)(x_h+F_h) + \psi\frac{(1-\omega_h^*)\omega_i^*}{\omega_h^*(1-\omega_i^*)}F_i\right).$$

Substituting the above into (A.49) yields a linear equation for x_h that can be solved in terms of y_h:

$$x_h = \frac{y_h - (r+\rho+\alpha_h)\left(C_h + (1+\tau_h)C_o - \tau_h\frac{D}{r} + \tau_h\frac{\theta_o\omega_h^*}{1-\omega_h^*+(1-\psi)\omega_h^*\theta_o}\left(\frac{(1-\psi)F_h}{r} + \frac{\psi(1-\omega_h^*)\omega_i^*F_i}{\omega_h^*(1-\omega_i^*)r}\right)\right)}{1 + \tau_h\left(\frac{(1-\psi)\omega_h^*\theta_o}{1-\omega_h^*+(1-\psi)\omega_h^*\theta_o}\right)\left(\frac{r+\rho+\alpha_h}{r}\right)}. \tag{A.52}$$

Combining equations (38), (A.50), (A.51) and $v_o = \upsilon_o\theta_o^{-\eta_o}$ from (43):

$$x_h + F_h - \frac{(1-\omega_h^* + (1-\psi)\omega_h^*\theta_o)\upsilon_o\theta_o^{-\eta_o}\zeta_h^{\lambda_h}}{(1+\tau_h\omega_h^*)(r+\rho+\alpha_h)(\lambda_h-1)}\left(y_h^{1-\lambda_h} + \frac{\alpha_h\delta_h^{\lambda_h}x_h^{1-\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}\right) - \frac{\psi\theta_o\omega_i^*F_i}{1-\omega_i^*} = 0\,. \tag{A.53}$$

Taking any y_h, the implied x_h is obtained from (A.52), and x_h is seen to be a strictly increasing function of y_h because the denominator is positive. Substituting the relationship between x_h and y_h into (A.53) yields a single equation in y_h that can be evaluated given (ψ, θ_o). Since the left-hand side of (A.53) is strictly increasing in both x_h and y_h, the single equation for y_h is strictly monotonic in y_h, which means any solution for x_h and y_h is unique. As the left-hand side of (A.53) is sure to be positive for large y_h and x_h because $\lambda_h > 1$, existence of a solution is confirmed by checking whether the left-hand side is negative at $y_h = \zeta_h$, the minimum value of y_h. Once the solution for y_h is found numerically, x_h is obtained from (A.52), and it can be verified whether $\delta_h y_h < x_h$ is satisfied.

A.3.2 Other ownership-market variables

Once y_h is found, $\pi_h = (\zeta_h/y_h)^{\lambda_h}$ is obtained using (40). This yields i from (35) given the value of ψ. Moreover, given that $v_o = \upsilon_o \theta_o^{-\eta_o}$ is known conditional on θ_o, the sales rate s_o is found using (36). The surplus Σ_h is found by substituting the thresholds into (A.51), and $\Sigma_i = F_i/((1-\omega_i^*)v_o)$ comes from (38). The average price P_h from (30) can be written as follows by using (A.48) for U_o:

$$P_h = \left(\frac{r + \theta_o v_o (1-\psi)\pi_h}{r}\right)\left(\frac{\omega_h^* \Sigma_h}{\pi_h}\right) + \frac{\theta_o v_o \psi \omega_i^* \Sigma_i}{r} + C_o - \frac{D}{r}, \tag{A.54}$$

and the price P_i is obtained from (34) and (A.48):

$$P_i = C_o + \frac{\theta_o v_o((1-\psi)\omega_h^* \Sigma_h + \psi \omega_i^* \Sigma_i) - D}{r} + \omega_i^* \Sigma_i. \tag{A.55}$$

The stock-flow accounting (32) and (39) in the steady state together with (36) require that

$$(1-i)s_o u_o = (m_h + \rho) q_h, \quad \text{and} \tag{A.56}$$

$$s_o u_o = (m_h + \rho) q_h + \rho_l (q_l + u_l). \tag{A.57}$$

Evaluating equation (41) for the moving rate in the steady state and substituting $\zeta_h^{\lambda_h} = \pi_h y_h^{\lambda_o}$ from (40):

$$m_h = \alpha_h - \frac{\alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h} \pi_h}{q_h} \frac{(1-\psi)\theta_o v_o u_o}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right)}.$$

Equations (35) and (36) imply that $(1-i)s_o = (1-\psi)\theta_o v_o \pi_h$, and hence by using equation (A.56) it follows that $(1-\psi)\theta_o v_o \pi_h u_o / q_h = m_h + \rho$. By substituting this into the above and solving for m_h:

$$m_h = \alpha_h \left(\frac{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right) - \rho \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right) + \alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}\right). \tag{A.58}$$

Dividing both sides of (A.57) by $\rho_l > 0$ and substituting for $q_l + u_l = 1 - q_h - u_o$ from (1) implies $u_o + q_h - ((m_h + \rho)/\rho_l) q_h + (s_o/\rho_l) u_o = 1$. Equation (A.56) shows that $q_h = ((1-i)s_o/(m_h+\rho))u_o$, and substituting this into the previous equation and solving for u_o yields:

$$u_o = \frac{1}{1 + \frac{(1-i)s_o}{m_h + \rho} + \frac{i s_o}{\rho_l}}, \quad \text{and} \quad q_h = \frac{(1-i)s_o}{m_h + \rho} u_o. \tag{A.59}$$

With $b_h = (1-\psi)\theta_o u_o$ and $b_i = \psi \theta_o u_o$ by using (23) and (24), equations (32) and (35) give S_h and S_i because v_o and π_h are known. Together with prices P_h and P_i from (A.54) and (A.55), total tax revenue $G = \tau_h P_h S_h + \tau_i P_i S_i$ is determined.

A.3.3 The rental-market and credit-cost thresholds, and the city population

The next part of the solution method derives an equation to solve numerically for the rental-market transaction threshold y_l, taking as given (ψ, θ_o) and the solution for ownership-market variables, which are also found conditional on (ψ, θ_o). Solving for y_l depends on finding the credit-cost threshold Z and the city population n. Given y_l, there is a single equation that can be solved numerically for the fraction κ of households choosing to pay the credit cost, which directly determines Z and n.

The moving rate $m_l = \alpha_l + \rho_l$ is given by parameters according to (8). With $\dot{B}_l = 0$ and $\dot{U}_l = 0$ in the steady state, the Bellman equations (A.30) and (A.31) become

$$(r+\rho)B_l = g - F_l + (1-\omega_l)v_l\Sigma_l\,, \text{ and} \tag{A.60}$$

$$(r+\rho_l)U_l = \omega_l\theta_l v_l\Sigma_l - D + \rho_l U_o\,. \tag{A.61}$$

In steady state, $\dot{\underline{J}} = 0$, which using (A.26) yields the equation $(r+\rho+m_l)\underline{J} = m_l B_l + (\rho+\alpha_l)U_l + \rho_l U_o + g - D - D_l + \xi m_l \kappa(Z-\bar{K})$. Substituting this into (A.28) with $m_l = \alpha_l + \rho_l$ implies

$$y_l = D + D_l - g + (r+\rho)B_l + (r+\rho_l)U_l - \rho_l U_o + (r+\rho+m_l)(C_l + C_w) - \xi m_l \kappa(Z-\bar{K})\,,$$

and by using (A.60) and (A.61), the equation becomes

$$y_l - D_l + F_l - (r+m_l+\rho)(C_l+C_w) + \xi m_l \kappa(Z-\bar{K}) - (1-\omega_l+\omega_l\theta_l)v_l\Sigma_l = 0\,. \tag{A.62}$$

This equation links the transaction threshold y_l to the surplus Σ_l and the credit-cost threshold Z (and hence κ and $\bar{K}$). A further equation involving Σ_l is obtained by multiplying both sides of the first equation in (38) by $r+\rho_l$ and substituting for $(r+\rho_l)U_l$ from (A.61):

$$\omega_l\theta_l q_l\Sigma_l = D - \rho_l U_o + (r+\rho_l)(1+\tau_i)U_o + (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right)\,.$$

Using $(r+\rho_l)(1+\tau_i)U_o - \rho_l U_o = (1+\tau_i(1+(\rho_l/r))rU_o$ and substituting from (A.48) leads to:

$$\omega_l\theta_l v_l\Sigma_l = \left(1+\tau_i\left(1+\frac{\rho_l}{r}\right)\right)\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right)$$
$$+ (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right) - \tau_i\left(1+\frac{\rho_l}{r}\right)D\,. \tag{A.63}$$

Rental-market variables conditional on the transactions threshold The numerical procedure to solve for y_l depends on checking whether one equation holds. Taking a particular value of the threshold y_l, the implied transaction probability from (40) is $\pi_l = (\zeta_l/y_l)^{\lambda_l}$. Using the formula (A.29) for the surplus Σ_l to state it in terms of y_l and π_l:

$$\Sigma_l = \frac{\pi_l y_l}{(\lambda_l-1)(r+\rho+m_l)}\,.$$

Observe that this implies $\omega_l\theta_l v_l\Sigma_l = \omega_l y_l s_l/((\lambda_l-1)(r+\rho+m_l))$, where $s_l = \theta_l v_l \pi_l$ is the letting rate from (17). As another equation for the left-hand side is given by (A.63), the rate s_l implied by y_l is

$$s_l = \frac{(\lambda_l-1)(r+\rho+m_l)}{\omega_l y_l}\Bigg(\left(1+\tau_i\left(1+\frac{\rho_l}{r}\right)\right)\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right)$$
$$+ (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right) - \tau_i\left(1+\frac{\rho_l}{r}\right)D\Bigg)\,, \tag{A.64}$$

which can be evaluated conditional on (ψ,θ_o), from which the ownership-market variables v_o, Σ_h, and Σ_i are also known. Equation (43) gives the meeting rate $v_l = \upsilon_l\theta_l^{-\eta_l}$, and hence the letting rate $s_l = \theta_l v_l \pi_l$ satisfies $s_l = \upsilon_l\pi_l\theta_l^{1-\eta_l}$. The implied market tightness in the rental market is

$$\theta_l = \left(\frac{s_l}{\upsilon_l\pi_l}\right)^{\frac{1}{1-\eta_l}}\,, \tag{A.65}$$

and this also determines $v_l = \upsilon_l\theta_l^{-\eta_l}$.

Solving for the credit-cost threshold and city population With $\dot{q}_l = 0$ and $\dot{u}_l = 0$ in the steady state, equations (18), (20), and (36) require

$$s_l u_l = (m_l + \rho) q_l\,, \quad \text{and} \tag{A.66}$$

$$(s_l + \rho_l) u_l = (\alpha_l + \rho) q_l + i s_o u_o\,. \tag{A.67}$$

Equations (1) and (A.66) imply $q_l + u_l = 1 - q_h - u_o$ and $q_l = (s_l/(m_l + \rho))u_l$. Combining these and using the known values of q_h and u_o to solve for u_l and q_l:

$$u_l = \frac{1 - q_h - u_o}{1 + \frac{s_l}{m_l + \rho}}\,, \quad \text{and} \quad q_l = \frac{s_l}{m_l + \rho} u_l\,. \tag{A.68}$$

Since the stock-flow accounting equations are consistent with (1), and as the solution already satisfies (A.56), (A.57), and (A.66), this means equation (A.67) holds automatically. The steady state also has $\dot{n} = 0$, $\dot{b}_h = 0$, and $\dot{b}_l = 0$. Using (4), (6), (19), and (31), this means that $a = \rho n$ and the following equations must hold:

$$(v_o \pi_h + \rho) b_h = m_h q_h + (\xi m_l q_l + \rho n)\kappa\,, \quad \text{and} \tag{A.69}$$

$$(v_l \pi_l + \rho) b_l = (1 - \xi) m_l q_l + (\xi m_l q_l + \rho n)(1 - \kappa)\,. \tag{A.70}$$

Since $b_h = (1 - \psi)\theta_o u_o$, v_o, π_h, m_h, q_h, and q_l are known at this point, equation (A.69) can be rearranged to solve for the city population n as a function of κ:

$$n = \frac{1}{\rho}\left(\frac{(v_o \pi_h + \rho)(1 - \psi)\theta_o u_o - m_h q_h}{\kappa} - \xi m_l q_l\right). \tag{A.71}$$

Note that since the stock-flow accounting equations are consistent with (2), and as the solution already satisfies (A.56), (A.66), and (A.69) given equation (A.71), it follows that (A.70) automatically holds. To have $\dot{n} = 0$ in (6), it is necessary that $a = \rho n > 0$, which requires $N = 0$ since $\chi > 0$. Using (5) and (3), this requires $B_l + \kappa(Z - \bar{K}) - E = 0$. Substituting for B_l from (A.60) and noting $g = G/n$ from (26), this equation is equivalent to:

$$\frac{(G/n) - F_l + (1 - \omega_l) v_l \Sigma_l}{r + \rho} + \kappa(Z - \bar{K}) - E = 0\,. \tag{A.72}$$

Now consider the numerical search over the fraction κ conditional on y_l. Equation (A.69) shows that $(v_o \pi_h + \rho)(1 - \psi)\theta_o u_o > m_h q_h$ is necessary to have a positive κ. After checking this condition for the given value of y_l, equation (A.71) is used to obtain n as a function of κ, and it can be seen that n is decreasing in κ. The credit-cost threshold Z is obtained by inverting equation (42) using the given κ:

$$Z = e^{\mu + \sigma \Phi^{-1}(\kappa)}\,, \tag{A.73}$$

which is an increasing function of κ. The average credit cost $\bar{K}$ follows immediately from (42) using the Z obtained from (A.73). Since $\kappa(Z - \bar{K}) = \int_{K=0}^{Z} (Z - K)\,\mathrm{d}\Gamma_k(K)$, the left-hand side of equation (A.72) is increasing in κ after taking account of the effects on Z and $\bar{K}$. Moreover, since n falls as κ rises, and as $G > 0$, higher κ has an unambiguously positive effect on the left-hand side of (A.72). A solution for κ is therefore unique, and existence is verified by checking (A.72) changes sign over the interval $\kappa \in [0, 1]$.

Solving for the transaction threshold The method described above finds the unique solution for κ, Z, $\bar{K}$, and n conditional on the rental-market transaction threshold y_l (and ψ, θ_o, and other ownership-market variables). The value of y_l itself is found numerically as the solution of equation (A.62), taking account of the effect of y_l on Σ_l, v_l, θ_l, and the term $\kappa(Z - \bar{K})$. From (A.29) and $\lambda_l > 1$, it follows that higher y_l reduces Σ_l. Equation (A.64) shows that s_l is proportional to $1/y_l$, and since $\pi_l = \zeta_l^{\lambda_l} y_l^{-\lambda_l}$, the ratio s_l/π_l and hence θ_l from (A.65) are increasing in y_l because $\lambda_l > 1$. This means that v_l is decreasing in y_l, and from (A.68), q_l is decreasing in y_l because s_l is negatively related to y_l.

Higher y_l directly increases the left-hand side of (A.62), and indirectly increases it through lower Σ_l. Moreover, since higher y_l lowers $v_l \Sigma_l$, and reduces G/n because of higher n from falling q_l in (A.71), the required value of $\kappa(Z - \bar{K})$ consistent with (A.72) increases (as does κ and Z, because $\kappa(Z - \bar{K})$ is increasing in Z). This implies another positive effect on the left-hand side of (A.62). The only term that is not unambiguously increasing in y_l is $(1-\omega_l+\omega_l\theta_l)v_l$ because θ_l rises, while $v_l=\upsilon_l\theta_l^{-\eta_l}$ falls.[52]

Equation (A.62) is solved numerically for the transaction threshold y_l, verifying uniqueness if necessary. Since the left-hand side becomes arbitrarily large as y_l increases, existence is confirmed by checking whether the left-hand side is negative at $y_l=\zeta_l$. Given the solution for y_l, the associated solutions for Z, κ, and n are obtained as explained earlier.

A.3.4 Solving for the fraction of investors and market tightnesses

The steps above are used to derive values of all the variables in the model conditional on a given pair of values for (ψ,θ_o). The fraction ψ of investors and the tightness θ_o of the ownership market are found by a numerical solution of the two remaining equations of the model.

The first equation is (A.35), which links the fraction of investors ψ, market tightnesses θ_o and θ_l, properties on the market u_o and u_l, and the city population n. All of these variables are known conditional on (ψ,θ_o). The second equation is the marginal home-buyer indifference condition (3) in steady state. Substituting the expressions for B_h and B_l from (A.47) and (A.60) into (3):

$$(1-\omega_h^*)v_o\Sigma_h-(1-\omega_l)v_l\Sigma_l=(r+\rho)Z+F_h-F_l\,, \tag{A.74}$$

where v_o, v_l, Σ_h, Σ_l, and Z are determined above for given ψ and θ_o.

Searching over ψ and θ_o to find values satisfying (A.35) and (A.74), the steady-state equilibrium of the model is found. Since the search is only over two dimensions, existence and uniqueness can be confirmed numerically. Once (ψ,θ_o) is known, all other variables are derived using the earlier methods.

A.3.5 Steady-state values of other variables

This section shows how to compute steady-state values of other variables of interest, including those used as part of the calibration strategy. With $\dot{y}_l=0$ in steady state, the new rent equation (A.33) becomes

$$R=D_l+\omega_l(r+\rho+m_l)(C_l+C_w)+\omega_l(r+\rho+m_l+\theta_l v_l\pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.75}$$

Steady-state match qualities in the two markets ($\dot{V}_h=0$ and $\dot{V}_l=0$) are derived from (45) and (46):

$$V_h=\frac{\lambda_h}{\lambda_h-1}\left(\frac{m_h+\rho}{\alpha_h+\rho}y_h+\frac{\alpha_h-m_h}{\alpha_h+\rho}x_h\right),\quad\text{and}\quad V_l=\frac{\lambda_l}{\lambda_l-1}y_l\,, \tag{A.76}$$

where the first equation also makes use of (32) with $\dot{q}_h=0$. As steady-state match quality V_l is the same as average match quality $\lambda_l y_l/(\lambda_l-1)$ for new leases, steady-state new rents R are equal to average rents $\bar{R}$ for existing tenants (see equation A.34).

Viewings and time-on-the-market If home-buyers have a constant probability π_h of making a purchase conditional on a viewing, the expected number of viewings per home-buyer purchase is $\Lambda_h=1/\pi_h$. Similarly, those searching for property to rent have constant probability π_l of transacting, so their expected number of viewings is $\Lambda_l=1/\pi_l$, which is also the expected viewings required for a landlord to lease a property. If properties on the rental market are leased at a constant rate s_l, the expected time-on-the-market for properties in the rental market is $T_{sl}=1/s_l$. In the ownership market, properties are sold at a constant rate s_o in steady state, implying an expected time-on-the-market for sellers of $T_{so}=1/s_o$. Home-buyers complete a purchase at rate $v_o\pi_h$ over time, so their expected time-on-the-market is $T_{bh}=1/(v_o\pi_h)$, and similarly $T_{bl}=1/(v_l\pi_l)$ for renters. In summary:

$$\Lambda_h=\frac{1}{\pi_h},\quad\Lambda_l=\frac{1}{\pi_l},\quad T_{so}=\frac{1}{s_o},\quad T_{sl}=\frac{1}{s_l},\quad T_{bh}=\frac{1}{v_o\pi_h},\quad\text{and}\quad T_{bl}=\frac{1}{v_l\pi_l}\,. \tag{A.77}$$

[52]If the Hosios condition $\omega_l=\eta_l$ holds, $(1-\omega_l+\omega_l\theta_l)\upsilon_l\theta_l^{-\eta_l}$ is decreasing in θ_l (and hence y_l) if $\theta_l<1$, as is the case given the calibration in Table 3.

The moving hazard rate and expected time between moves Tenants move house within the city at an constant exogenous rate $m_l = a_l + \rho_l$. The expected time a tenant remains in a property is therefore $T_{ml} = 1/(m_l + \rho)$ after accounting for moves outside the city (exit rate ρ). For homeowners, moving is endogenous, so even in steady state where the moving threshold x_h and transaction threshold y_h remain constant, the hazard rate of moving depends on how long a household lived in a property. Let $\Psi(T)$ denote the steady-state survival function of matches in the ownership market. This gives the fraction of matches that remain in existence T years after households first moved into their properties.

In order for a match to survive for T years, first, a household must not leave the city during that time. With constant exit rate ρ, this has probability $e^{-\rho T}$. Second, the household must choose to remain in a property after any shocks to idiosyncratic match quality have occurred. These shocks arrive independently at rate α_h, so the number of shocks j that occur over a period of time T has a Poisson$(\alpha_h T)$ distribution. The probability of receiving exactly j shocks is $e^{-\alpha_h T}(\alpha_h T)^j/j!$ for $j = 0, 1, 2, \ldots$.

If initial match quality is ε, match quality becomes $\varepsilon' = \delta_h^j \varepsilon$ after j shocks. The household chooses not to move if $\varepsilon' \geq x_h$, which is equivalent to $\varepsilon \geq x_h/\delta_h^j$ in terms of initial match quality ε (and if this condition holds for some j then it also holds for any smaller j because $\delta_h < 1$ and x_h remains constant in the steady state). New match quality has a Pareto(y_h, λ_h) distribution, so the probability that $\varepsilon \geq x_h/\delta_h^j$ is $((x_h/\delta_h^j)/y_h)^{-\lambda_h}$. This is well defined if $x_h/\delta_h^j > y_h$, which is true for all $j \geq 1$ because $\delta_h y_h < x_h$. With zero shocks ($j = 0$), households remain in the same property unless they leave the city.

The fraction of households who remain in the same property for T years is therefore

$$\Psi(T) = e^{-\rho T}\left(e^{-\alpha_h T} + \sum_{j=1}^{\infty} e^{-\alpha_h T}\frac{(\alpha_h T)^j}{j!}\left(\frac{x_h/\delta_h^j}{y_h}\right)^{-\lambda_h}\right)$$
$$= e^{-(\alpha_h+\rho)T}\left(1 + \left(\frac{y_h}{x_h}\right)^{\lambda_h}\sum_{j=1}^{\infty}\frac{(\alpha_h\delta_h^{\lambda_h}T)^j}{j!}\right) = e^{-(\alpha_h+\rho)T}\left(1 + \left(\frac{y_h}{x_h}\right)^{\lambda_h}\left(e^{\alpha_h\delta_h^{\lambda_h}T} - 1\right)\right)$$
$$= \left(\frac{y_h}{x_h}\right)^{\lambda_h} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - \left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}.$$

The moving hazard $\hbar(T)$ as a function of match duration T is defined by the percentage decline in the proportion of surviving matches, that is, $\hbar(T) = -\mathrm{d}\log\Psi(T)/\mathrm{d}T = -\Psi'(T)/\Psi(T)$. This follows immediately from the expression for $\Psi(T)$ above:

$$\hbar(T) = \frac{\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)\left(\frac{y_h}{x_h}\right)^{\lambda_o} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - (\alpha_h+\rho)\left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}}{\left(\frac{y_h}{x_h}\right)^{\lambda_h} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - \left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}}.$$

The density function of the probability distribution of moving times T is $\hbar(T)\Psi(T) = -\Psi'(T)$, and hence the expected time until moving is $T_{mh} = \int_{T=0}^{\infty} -T\Psi'(T)\mathrm{d}T = \int_{T=0}^{\infty}\Psi(T)\mathrm{d}T$, where the second expression for T_{mh}, the area under the survival function, is derived from integration by parts (and $\lim_{T\to\infty} T\Psi(T) = 0$). In the cross-section of households at a point in time, the steady-state distribution of time spent in the same property has density function $\Psi(T)/T_{mh}$, and the hazard rate $\hbar(T)$ averaged over the cross-section of homeowners is $\int_{T=0}^{\infty}\hbar(T)(\Psi(T)/T_{mh})\mathrm{d}T = 1/T_{mh}$ because $\hbar(T)\Psi(T) = -\Psi'(T)$, $\Psi(0) = 1$, and $\lim_{T\to\infty}\Psi(T) = 0$. Since the within-city moving rate averaged over the cross-section of homeowners is m_h from (41), it follows that $T_{mh} = 1/(m_h + \rho)$. In summary:

$$T_{mh} = \frac{1}{m_h + \rho}, \quad \text{and } T_{ml} = \frac{1}{m_l + \rho}. \tag{A.78}$$

The demographics of owners versus renters As can be seen from (4) and the law of motion (31) for home-buyers, there is a flow of first-time buyers $(\xi m_l q_l + a)\kappa$ coming from the rental market or outside the city, and a flow $m_h q_h$ of existing homeowners returning to the market when they decide the move house. Since these two groups of home-buyers subsequently transact at the same rate $v_o\pi_h$, or leave the city at the same rate ρ, the steady-state fraction ϕ of first-time buyers can be calculated as the ratio of the inflow of first-time buyers to the inflow of all buyers entering b_h:

$$\phi = \frac{(\xi m_l q_l + a)\kappa}{m_h q_h + (\xi m_l q_l + a)\kappa} = \frac{(v_o\pi_h + \rho)b_h - m_h q_h}{(v_o\pi_h + \rho)b_h},$$

where the second expression for ϕ follows from (A.69) because b_h is a steady state. In steady state, (A.57) implies $(m_h+\rho)q_h = (1-i)s_o u_o$, and (32) and (36) imply $(1-i)s_o u_o = v_o\pi_h b_h$. Dividing numerator and denominator of the expression for ϕ by q_h and substituting $v_o\pi_h b_h/q_h = m_h + \rho$:

$$\phi = \frac{\left(1+\frac{\rho}{v_o\pi_h}\right)(m_h+\rho) - m_h}{\left(1+\frac{\rho}{v_o\pi_h}\right)(m_h+\rho)}. \tag{A.79}$$

Now consider the steady-state demographics of homeowners compared to renters. Let $\digamma_{qh}$, $\digamma_{ql}$, $\digamma_{bh}$, and $\digamma_{bl}$ be the average ages of the household heads of those in q_h, q_l, b_h, and b_l, and $\digamma_h$ and $\digamma_l$ the average ages of those in q_h+b_h and q_l+b_l. Furthermore, let $\digamma_a$ and $\digamma_\gamma$ denote the average age of new entrants to the city a and first-time buyers γ respectively, and the difference between the average ages of homeowners and renters is denoted by $\aleph = \digamma_h - \digamma_l$.

Taking the group in q_h+b_h, the laws of motion (31) and (32) imply $\dot{q}_h + \dot{b}_h = (\xi m_l q_l + a)\kappa - \rho(q_h + b_h)$, noting $\gamma = (\xi m_l q_l + a)\kappa$ and $(1-i)s_o u_o = v_o\pi_h b_h$. Exit occurs at rate ρ, with first-time buyers $\rho(q_h+b_h)$ arriving in steady state to ensure $\dot{q}_h + \dot{b}_h = 0$. The differential equation for the average age in this group is thus $\dot{\digamma}_h = 1 - \rho\digamma_h + \rho\digamma_\gamma$, and a steady-state age distribution therefore has $\digamma_h = \digamma_\gamma + \rho^{-1}$. It is convenient to consider all average ages relative to the average age at first entry to the city, which are denoted by $\aleph_h = \digamma_h - \digamma_a$, $\aleph_l = \digamma_l - \digamma_a$, and similarly for the other groups. In terms of these variables, the definition of the average owner-renter age difference $\aleph$ and the equation for the steady-state homeowner versus first-time-buyer age difference are:

$$\aleph = \aleph_h - \aleph_l, \quad \text{and} \quad \aleph_h = \aleph_\gamma + \rho^{-1}. \tag{A.80}$$

Now consider the group q_l. There is exit at rate $m_l + \rho$ and entry $v_l\pi_l b_l/q_l = m_l + \rho$ from b_l as a proportion of the group q_l in steady state (see 18 with $v_l\pi_l b_l = s_l u_l$ from 11 and 17), and the average age of entrants is $\digamma_{bl}$. Thus, in steady state, $1 = (m_l+\rho)(\digamma_{ql} - \digamma_{bl})$ and hence:

$$\aleph_{ql} = \aleph_{bl} + (m_l+\rho)^{-1}. \tag{A.81}$$

Since $\digamma_l = (q_l/(q_l+b_l))\digamma_{ql} + (b_l/(q_l+b_l))\digamma_{bl}$ by definition of the average age of the whole group q_l+b_l, it follows that $\aleph_{ql} - \aleph_l = (b_l/(q_l+b_l))(\aleph_{ql} - \aleph_{bl})$. With $b_l/(q_l+b_l) = (m_l+\rho)/(m_l+\rho+v_l\pi_l)$ from $v_l\pi_l b_l/q_l = m_l+\rho$ in steady state, this can be used together with (A.81) to deduce:

$$\aleph_{ql} = \aleph_l + (\rho + m_l + v_l\pi_l)^{-1}. \tag{A.82}$$

For the group b_l, given the law of motion (19), there are outflows at rate $v_l\pi_l + \rho$, and as a proportion of b_l, inflows $a(1-\kappa)/b_l$ from outside the city with average age $\digamma_a$ and $(1-\xi\kappa)m_l q_l/b_l$ from q_l where the average age is $\digamma_{ql}$. Thus, at the steady-state age distribution:

$$1 + \frac{a(1-\kappa)}{b_l}\digamma_a + \frac{m_l(1-\xi\kappa)q_l}{b_l}\digamma_{ql} = (v_l\pi_l+\rho)\digamma_{bl}.$$

Using $a(1-\kappa) = (v_l\pi_l+\rho)b_l - (1-\xi\kappa)m_l q_l$ in steady state from (A.70) with $a = \rho n$, the equation above can be written as $b_l + (1-\xi\kappa)m_l q_l \aleph_{ql} = (v_l\pi_l+\rho)b_l\aleph_{bl}$. Substituting from (A.81) and using (A.70) again implies that $a(1-\kappa)\aleph_{ql} = b_l + (v_l\pi_l+\rho)b_l(m_l+\rho)^{-1}$. With $s_l u_l = v_l\pi_l b_l$ using (11) and (17), it follows that $v_l\pi_l b_l(m_l+\rho)^{-1} = q_l$, and hence $\aleph_{ql}$ is given by:

$$\aleph_{ql} = \frac{(q_l+b_l) + \rho b_l(m_l+\rho)^{-1}}{a(1-\kappa)}. \tag{A.83}$$

Finally, consider the average age $\digamma_\gamma$ of first-time buyers. Using (4), a fraction $\xi m_l q_l\kappa/((\xi m_l q_l + a)\kappa)$

come from q_l where the average age is $\digamma_{ql}$, and a fraction $a\kappa/((\xi m_l q_l + a)\kappa)$ are new entrants to the city with average age $\digamma_a$. Therefore, $\digamma_\gamma = (\xi m_l q_l/(\xi m_l q_l + a))\digamma_{ql} + (a/(\xi m_l q_l + a))\digamma_a$, and hence:

$$\aleph_\gamma = \left(1 - \frac{a}{\xi m_l q_l + a}\right) \aleph_{ql} = \aleph_{ql} - \frac{(q_l + b_l) + \rho b_l (m_l + \rho)^{-1}}{(\xi m_l q_l + a)(1 - \kappa)}, \tag{A.84}$$

where the second expression substitutes from (A.83). Using $v_l \pi_l b_l = s_l u_l = m_l q_l + \rho q_l$ from (A.66) together with (A.70) implies $(\xi m_l q_l + a)(1 - \kappa) = \rho(q_l + b_l) + \xi m_l q_l$ and substituting into (A.84):

$$\aleph_\gamma = \aleph_{ql} - \frac{(q_l + b_l) + \rho b_l (m_l + \rho)^{-1}}{\rho(q_l + b_l) + \xi m_l q_l} = \aleph_{ql} - \frac{1 + \rho(\rho + m_l + v_l \pi_l)^{-1}}{\rho + \xi v_l \pi_l m_l (\rho + m_l + v_l \pi_l)^{-1}}, \tag{A.85}$$

where $b_l = (m_l + \rho)(\rho + m_l + v_l \pi_l)^{-1}(q_l + b_l)$ and $q_l = v_l \pi_l (\rho + m_l + v_l \pi_l)^{-1}(q_l + b_l)$ are used to derive the second expression. Combining (A.80), (A.82), and (A.85) and factorizing leads to the following expression for the average age difference between owners and renters:

$$\aleph = \left(1 + \frac{\rho}{\rho + m_l + v_l \pi_l}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + \frac{\xi m_l v_l \pi_l}{\rho + m_l + v_l \pi_l}}\right). \tag{A.86}$$

A.4 Solving for the transitional dynamics

This section describes how the transitional path to the new steady state in a perfect-foresight equilibrium is found numerically. There is an unanticipated change to the tax rates τ_h and τ_i at a point in time, $t = 0$, without loss of generality. No further changes are anticipated. For state variables such as q_h, the measure of owner-occupiers, the left-derivative of $q_h(t)$ with respect to time t must exist at all points along the transitional path — the variable cannot 'jump'. For non-predetermined variables such as B_h, the value of being a home-buyer, the left derivative may not be well defined at $t = 0$, but the right-derivative of $B_h(t)$ with respect to time t must exist at all points along the transitional path to satisfy Bellman equations such as (27). The size of any jumps in values such as B_h is determined by the requirement that values cannot grow faster than the discount rate r, and hence these

An approximate solution of the differential equations of the model is obtained by discretization. Dividing continuous time into a small discrete periods of length ℓ, the time derivative of state variables such as q_h is approximated by $(q_h(t) - q_h(t - \ell))/\ell$, which converges to $\dot{q}_h$ as $\ell \to 0$ because the left-derivative of $q_h(t)$ exists. This means differential equations such as the law of motion (32) are replaced by difference equations of the form:

$$\frac{q_h(t) - q_h(t - \ell)}{\ell} = v_o(t)\pi_h(t)b_h(t) - (m_h(t) + \rho)q_h(t),$$

where continuity of the time path of $q_h(t)$ means the right-hand side can be evaluated at t. For non-predetermined variables such as B_h, the time derivative is approximated by $(B_h(t + \ell) - B_h(t))/\ell$, which converges to $\dot{B}_h$ because the right-derivative of $B_h(t)$ exists. This means that differential equations such as the Bellman equation (27) are replaced by difference equations of the form:

$$\frac{B_h(t + \ell) - B_h(t)}{\ell} = (r + \rho)B_h(t) - g(t) + F_h - (1 - \omega_h^*)v_o(t)\Sigma_h(t),$$

which is based on the equation in (A.7) that is equivalent to (27).

With the differential equations of the model replaced by difference equations, the transitional dynamics of the non-linear system of difference equations can be found using the perfect-foresight solver in the Dynare MATLAB package, together with knowledge of the original and new steady states computed using the procedure described in appendix A.3. The discretization is based on a time period of one day, so $\ell = 1/365$ when the model is calibrated in annual time units.

A.5 Calibration targets

The parameters of the model are chosen to match the City of Toronto housing market in the pre-policy period (January 2006–January 2008). The average sales price P taken from Table A.3 is $\$402,000$ during that period. The initial effective LTT rate is 1.5%, so $\tau_h = \tau_i = 0.015$.

Housing tenure and entry of investors Based on the 2006 City of Toronto Profile Report, the homeownership rate is $h = 54\%$, the average age of homeowners is 53.3, and the average age of tenants is 45.0. Hence the target for the difference between the average ages of homeowners and renters is $\aleph = 8.3$. There is no survey that specifically captures the proportion of first-time buyers ϕ in Toronto. The Canadian Association of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 finding that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the Greater Toronto Area. On the other hand, data from the Canada Mortgage and Housing Corporation suggests the fraction of first-time buyers is about a third. Based on this information, the calibration target is $\phi = 0.4$.

Using Toronto MLS data on sales and rental transactions, the fraction of purchases by buy-to-rent investors is 5.4% during the pre-policy period, so $i = 0.054$. The price-to-rent ratio for the same property is 14.5 in 2007, and the ratio of average prices paid by investors to prices paid by home-buyers is 0.99. Hence, $P_i/R = 14.5$ and $P_i/P_h = 0.99$ are used as targets.

Credit costs The credit cost K of becoming an owner-occupier is computed from a comparison of the mortgage interest rate r_k the household would face relative to the risk-free interest rate r_g on government bonds. The interest rates r_k and r_g are real interest rates. There is a spread between them due to unmodelled financial frictions. The risk-free real rate r_g used to discount future cashflows need not be the same as the discount rate r applied to future utility flows from owning property, allowing for an unmodelled housing risk premium between r and r_g. It is assumed all these interest rates are expected to remain constant over the mortgage term.

Suppose a household buys a property at price P_h at date $t = 0$ by taking out a mortgage with loan-to-value ratio l. Assume the mortgage has term T_k and a constant real repayment ι until maturity. Let $\mathscr{D}(t)$ denote the outstanding mortgage balance at date t, which has initial condition $\mathscr{D}(0) = lP_h$ and terminal condition $\mathscr{D}(T_k) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{\mathscr{D}}(t) = r_k\mathscr{D}(t) - \iota \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_k t}\mathscr{D}(t))}{\mathrm{d}t} = -\iota e^{-r_k t}.$$

Solving this differential equation using the initial condition $\mathscr{D}(0) = lP_h$ implies:

$$\mathscr{D}(t) = e^{r_k t} lP_h - \frac{\iota}{r_k}(e^{r_k t} - 1). \tag{A.87}$$

The terminal condition $\mathscr{D}(T_k) = 0$ requires that the constant real repayment ι satisfies:

$$\iota = \frac{r_k lP_h}{1 - e^{-r_k T_k}}. \tag{A.88}$$

In the model, owner-occupiers exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds from selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment ι will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $\mathscr{D}(t)$ is repaid at date t. The credit cost K is defined as the present value of the expected stream of repayments discounted at rate r_g minus the amount borrowed (which would equal the present value of the repayments if $r_k = r_g$ in the absence of an interest-rate spread):

$$K = \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \iota \,\mathrm{d}t + \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \rho \mathscr{D}(t) \mathrm{d}t - lP_h.$$

To derive an explicit formula for K, first observe that

$$\int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \mathrm{d}t = \frac{1 - e^{-(r_g+\rho)T_k}}{r_g + \rho} \quad \text{and} \quad \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} e^{r_k t} \mathrm{d}t = \frac{1 - e^{-(r_g+\rho-r_k)T_k}}{r_g + \rho - r_k}.$$

Together with equations (A.87) and (A.88) for $\mathscr{D}(t)$ and ι, the credit cost can be expressed as follows:

$$\begin{aligned}
K &= \frac{\left(\iota + \frac{\rho\iota}{r_k}\right)}{(r_g+\rho)}(1 - e^{-(r_g+\rho)T_k}) + \frac{\rho\left(lP_h - \frac{\iota}{r_k}\right)}{(r_g+\rho-r_k)}(1 - e^{-(r_g+\rho-r_k)T_k}) - lP_h \\
&= \left(\frac{(r_k+\rho)(1-e^{-(r_g+\rho)T_k})}{(r_g+\rho)(1-e^{-r_k T_k})} + \frac{\rho\left(1 - \frac{1}{1-e^{-r_k T_k}}\right)(1 - e^{-(r_g+\rho-r_k)T_k})}{(r_g+\rho-r_k)} - 1\right) lP_h \\
&= \frac{\left((r_k+\rho)(1-e^{-(r_g+\rho)T_k}) - \frac{\rho(r_g+\rho)}{r_g+\rho-r_k}(e^{-r_k T_k} - e^{-(r_g+\rho)T_k}) - (r_g+\rho)(1-e^{-r_k T_k})\right) lP_h}{(r_g+\rho)(1-e^{-r_k T_k})} \\
&= \frac{\left((r_k - r_g) + \frac{\rho(r_g+\rho)-(r_g+\rho)(r_g+\rho-r_k)}{r_g+\rho-r_k} e^{-(r_g+\rho)T_k} - \frac{(r_g+\rho)(r_g+\rho-r_k)-\rho(r_g+\rho)}{r_g+\rho-r_k} e^{-r_k T_k}\right) lP_h}{(r_g+\rho)(1-e^{-r_k T_k})},
\end{aligned}$$

and dividing both sides by price P_h and simplifying leads to the equation given in (51). That equation is used to determine calibration targets for the credit cost Z of a marginal home-buyer relative to the average price P_h, and for the marginal credit cost Z relative to the average credit cost $\bar{K}$ conditional on becoming an owner-occupier.

A mortgage term of 25 years ($T_k = 25$) and an average loan-to-value ratio of 80% ($l = 0.8$) are assumed. Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Statistics Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate $\bar{r}_k$ is 4.93% for an average homeowner. Since the average mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate comes from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate r_g is 1.86%.

Information on different mortgage rates is then used to compute credit costs for a marginal home-buyer. Based on micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores who did not qualify for loans from major banks could obtain mortgages from trust companies or private lenders at mortgage rates of around 6.15%, suggesting an interest rate gap of 3% between the marginal and average home-buyer.

But households faced with a high mortgage rate when they first buy a property do not necessarily continue with that rate for the whole time they have a mortgage. They can build up equity and improve their credit score, and thus obtain a mortgage rate closer to the average when they refinance. The baseline calibration assumes that a marginal home-buyer is able to close half of the initial gap with the average home-buyer over the whole term of the mortgage loan. This translates into an interest rate gap of 1.5%, implying the real mortgage rate r_z for a marginal buyer is 6.43%.

In summary, Z/P_h is derived from (51) using $T_k = 25$, $l = 0.8$, $r_g = 1.86\%$, and $r_k = r_z = 6.43\%$, together with the value of ρ obtained from the calibration method. The value of $Z/\bar{K}$ is derived by taking the ratio of Z/P_h and $\bar{K}/P_h$ from (51) with $r_k = \bar{r}_k = 4.93\%$ and the other terms being the same.

Non-tax transaction costs in the ownership market Apart from the land transfer tax, the only other cost buyers may pay is a home inspection cost of about \$500, but this is very small relative to average house prices. Hence, buyer non-tax transaction costs C_h and C_i are set to zero in the calibration.

From the side of sellers of a property, the primary cost is the real-estate agent commission. Using Multiple Listing Service sales data, the average commission rate is about 4.5% of price. There are some other costs such as legal fees of around \$1,000, but these are negligible in comparison. Sellers may sometimes spend approximately \$2,500 on staging, but the seller's agent might cover this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_o is set to be 4.5% of the average house price P.

Maintenance costs The maintenance cost D paid by owners of a property is set so that it is 2.6% of the average price P. This cost is made up of a 2% physical maintenance cost and a 0.6% property tax in Toronto. The additional maintenance cost D_l for properties that are rented out is set to be 8% of the average rent R. There are two parts to this cost: approximately 5–7% that a landlord spends on hiring a property manager, and approximately 1% paying for services such as taking out garbage, shovelling snow, and salting walkways.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real-estate agents to lease their properties. Hence, C_l is set to be $1/12$ of average annual rent R. Tenants in Toronto do not typically pay a fee when arranging to rent a property, so the calibration targets a zero tenancy agreement fee A.

Flows within ownership and rental markets Flows within the two housing markets are related to the average time between moves, times on the market, and viewings per sale and lease. Information on time-to-move, time-to-sell, and time-to-lease is derived from Toronto MLS data on sales and rental transactions during the pre-policy period. Estimates of the moving hazard function imply that owner-occupiers move after $T_{mh} = 9.25$ years on average The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 3.04$ years. The average time-to-sell for property owners is 30.5 days and the average time-to-lease is 18.7 days. During this period, the fraction of withdrawals from for-sale listings is 48% and from for-lease listings is 22%. In light of these withdrawals, the targets are $T_{so} = (30.5/365)/(1-0.48) = 0.161$ and $T_{sl} = (18.7/365)/(1-0.22) = 0.066$. Adjustments for withdrawals are made because measures of time-on-the-market are calculated from the final successful listing without accounting for earlier unsuccessful attempts, so true time-on-the-market is longer.

Data on buyers' time-on-the-market and viewings per sale and per lease are not available for Toronto. Using the 'Profile of Buyers and Sellers' survey collected by NAR in the United States, Genesove and Han (2012) report that for the period 2006–2009 the ratio of average time-to-buy to average time-to-sell is 1.28, and the average number of homes viewed by home-buyers is 10.7. Using this information, the targets used are $T_{bh} = 1.28 \times T_{so} = 0.206$ and $\Lambda_h = 10.7/(1-0.48) = 20.6$, where the latter adjusts the number of viewings to account for the withdrawal rate seen in Toronto. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than reported viewings in the final successful listing. There is no data on the number of properties that renters view on average. According to an industry expert, renters view fewer properties than buyers, so the target adopted is half the number of viewings per sale ($\Lambda_l/\Lambda_h = 1/2$).

Flow search costs The estimated search costs are based on the opportunity cost of time spent searching. This approach requires data on the ratio of house prices to income. Taking the median household-level income from Statistics Canada implies a price-to-income ratio of $P_h/Y = 5.6$ in Toronto in 2007.

A.6 Calibration method

The calibration targets in Table 3 identify the parameters in Table 4. The logic behind the exact identification of the parameters is explained here, along with a method for computing the parameter values from the targets.

Parameters implied directly by the targets Some parameters can be deduced directly from the calibration targets. A value of χ is set directly. Given P and targets for D/P and C_o/P, the values of $D = (D/P) \times P$ and $C_o = (C_o/P) \times P$ follow immediately. The average transactions price is $P = iP_i + (1-i)P_h$, so $P_h = P/(1-i+iP_i/P_h)$. Hence, C_h and C_i are obtained directly from the targets:

$$C_h = \frac{\frac{C_h}{P_h}P}{1-i+i\frac{P_i}{P_h}}, \quad \text{and } C_i = \frac{\frac{C_i}{P_i}\frac{P_i}{P_h}P}{1-i+i\frac{P_i}{P_h}}. \tag{A.89}$$

Since $R = (P_i/P_h) \times P/((P_i/R) \times (1 - i + iP_i/P_h)$, the cost parameters D_l and C_l also follow directly from the targets:

$$D_l = \frac{\frac{D_l}{R}\frac{P_i}{P_h}P}{\frac{P_i}{R}\left(1 - i + i\frac{P_i}{P_h}\right)}, \quad \text{and } C_l = \frac{\frac{C_l}{R}\frac{P_i}{P_h}P}{\frac{P_i}{R}\left(1 - i + i\frac{P_i}{P_h}\right)}. \tag{A.90}$$

Ratios and quantities implied by the targets The targets also provide some direct information about market tightness, the fraction of investors, transaction probabilities and selling/leasing rates, and the quantities of properties and households in different states. The fraction i of purchases made by buy-to-rent investors is given in (35). Using $\pi_h = \Lambda_h^{-1}$ from (A.77), it follows that $i = \psi\Lambda_h/(1 - \psi + \psi\Lambda_h)$, with which the fraction ψ of investors among all buyers is obtained from the targets for i and average viewings Λ_h per home-buyer:

$$\psi = \frac{i}{i + (1 - i)\Lambda_h}. \tag{A.91}$$

Using (A.56) with $(1 - i)s_o u_o = v_o \pi_h b_h$ from (32) and (36), it follows that $u_o = (T_{so}/T_{mh})q_h/(1 - i)$ and $b_h = (T_{bh}/T_{mh})q_h$ with reference to (A.77) and (A.78). Hence, using the definition of the homeownership rate h from (2):

$$q_h = \frac{nhT_{mh}}{T_{mh} + T_{bh}}, \quad \text{and } u_o = \frac{nhT_{so}}{(1 - i)(T_{mh} + T_{bh})}. \tag{A.92}$$

Equation (A.67) together with (A.77) and (A.78) implies $u_l = (T_{sl}/T_{ml})q_l$, and combining this with the total measure of properties from (1) leads to:

$$q_l = \frac{(1 - q_h - u_o)T_{ml}}{T_{ml} + T_{sl}}, \quad \text{and } u_l = \frac{(1 - q_h - u_o)T_{sl}}{T_{ml} + T_{sl}}. \tag{A.93}$$

Using (17), (36), and (A.77), it follows that $T_{bl}/T_{sl} = \theta_l$ and $T_{bh}/T_{so} = \theta_o(1 - \psi + \psi\pi_h^{-1})$, where the latter is solved for ownership-market tightness θ_o. Combining the definitions from (23) and (24), $b_h = (1 - \psi)\theta_o u_o$, and hence (2) provides an equation for rental-market tightness θ_l:

$$\theta_o = \frac{1}{1 - \psi + \psi\Lambda_h}\frac{T_{bh}}{T_{so}}, \quad \theta_l = \frac{n - q_h - q_l - (1 - \psi)\theta_o u_o}{u_l}, \quad \text{and } T_{bl} = \theta_l T_{sl}, \tag{A.94}$$

where ψ, q_h, u_o, q_l, and u_l are taken from (A.91), (A.92), and (A.93), and the other terms are known targets. Note that the value of T_{bl} cannot be chosen freely given the other targets.

Exit rate of investors Substituting $(m_h + \rho)q_h = (1 - i)s_o u_o$ from (A.56) into $s_o u_o = (m_h + \rho)q_h + \rho_l(q_l + u_l)$ from (A.57) implies $\rho_l = is_o u_o/(q_l + u_l)$. Using the formulas from (A.77), (A.92), and (A.93), this can be expressed as follows in terms of the targets:

$$\rho_l = \frac{i/(1 - i)}{(1 - nh)/nh}\frac{1}{T_{mh}}\left(\frac{1}{1 + \frac{T_{bh} - nhT_{so}/(1-i)}{(1-nh)T_{mh}}}\right). \tag{A.95}$$

Intuitively, this is identified from a comparison of the flow of investor transactions i relative to the stock of properties in the rental market.

Demographics and transitions to homeownership The target for the fraction ϕ of first-time buyers among all home-buyers provides information about the turnover rate ρ of households in the city. Equation (A.79) for the steady-state fraction ϕ can be written in terms of ρ and home-buyers' time-on-the-market T_{bh} and owner-occupiers' expected time between moves T_{mh} from (A.77) and (A.78):

$$\phi = \frac{\rho\left(1 + \frac{m_h + \rho}{v_o \pi_h}\right)}{m_h + \rho\left(1 + \frac{m_h + \rho}{v_o \pi_h}\right)} = \frac{\rho\left(1 + \frac{T_{bh}}{T_{mh}}\right)}{\frac{1}{T_{mh}} + \rho\frac{T_{bh}}{T_{mh}}}.$$

Intuitively, ϕ identifies ρ because being an owner-occupier is an absorbing state for households that remain in the city, so a flow of first-time buyers depends on new arrivals to the city. The equation above can be solved explicitly for ρ, and once ρ is known, m_h is inferred from $m_h = (1/T_{mh}) - \rho$ with (A.78):

$$\rho = \frac{\phi}{T_{mh} + (1-\phi)T_{bh}}, \quad \text{and } m_h = \frac{(1-\phi)(T_{mh}+T_{bh})}{T_{mh}(T_{mh}+(1-\phi)T_{bh})}. \tag{A.96}$$

Taking ρ from (A.96) and using the formula for T_{ml} in (A.78) yields $m_l = T_{ml}^{-1} - \rho$, and it can be checked whether this is positive. With equation (8) and ρ_l from (A.95), the parameter $\alpha_l = m_l - \rho_l$ is obtained.

The target $\aleph$ for the difference between the average ages of owners and renters provides information about the probability ξ that renters draw a new credit cost when moving. Intuitively, if credit costs were drawn once and for all when households entered the city, there would be no reason in the model why the average ages of the two groups would differ. Using equation (A.86), the value of $\aleph$ is therefore informative about how long it is expected to take for a renter to make the transition to being an owner-occupier. This equation is rearranged to show $\xi m_l v_l \pi_l(\rho + (1-\rho\aleph)(\rho + m_l + v_l\pi_l)) = \rho^2\aleph(\rho + m_l + v_l\pi_l)$, which can be solved explicitly for ξ with reference to (A.77) and (A.78):

$$\xi = \frac{\rho^2\aleph(\rho + m_l + v_l\pi_l)}{m_l v_l \pi_l(\rho + (1-\rho\aleph)(\rho + m_l + v_l\pi_l))} = \frac{\rho^2\aleph\left(1+\frac{T_{ml}}{T_{bl}}\right)^2}{\frac{T_{ml}}{T_{bl}}\left(\frac{1}{T_{ml}} - \rho\right)\left(\rho T_{ml} + (1-\rho\aleph)\left(1+\frac{T_{ml}}{T_{bl}}\right)\right)}, \tag{A.97}$$

and this is known given the targets and the values of T_{bl} and ρ from (A.94) and (A.96).

By substituting $v_l\pi_l b_l = s_l u_l$ into (A.70) and using $s_l u_l = (m_l + \rho)q_l$ from (A.67), it follows that $(\xi m_l q_l + \rho n)\kappa = \rho(n - q_l - b_l)$. Together with (2) and q_l from (A.93), the value of κ is:

$$\kappa = \frac{\rho n h}{\xi m_l q_l + \rho n}. \tag{A.98}$$

Distribution of credit costs The calibration targets related to the mortgage interest rates and other aspects of mortgage contracts determine the present-discounted value of total credit costs K relative to house prices P_h for an average and a marginal home-buyer. The implied values of $\bar{K}/P_h$ and Z/P_h are given by the formula in (51), which determine $\bar{K}$ and Z using $P_h = P/(1 - i + iP_i/P_h)$. These provide information about the mean and standard-deviation parameters μ and σ of the log-Normal credit-cost distribution from (42). Since $\kappa = \Gamma_k(Z)$, the marginal credit cost Z is at a known percentile κ of the distribution from (A.98), and $\bar{K} = \mathbb{E}[K|K \leq Z]$ is the mean credit cost K conditional on being below the threshold Z.

Using (42), the marginal credit cost Z and the parameters μ and σ satisfy $\log Z = \mu + \sigma\Phi^{-1}(\kappa)$, where $\Phi(\cdot)$ is the standard Normal cumulative distribution function, and the conditional mean satisfies $\log \bar{K} = \mu + \sigma^2/2 + \log\Phi((\log Z - \mu - \sigma^2)/\sigma) - \log\Phi((\log Z - \mu)/\sigma)$. Subtracting the first equation from the second, noting that μ cancels out and using $\kappa = \Phi((\log Z - \mu)/\sigma)$ and $(\log Z - \mu)/\sigma = \Phi^{-1}(\kappa)$:

$$\log\left(\frac{Z}{\bar{K}}\right) - \log\kappa - \sigma\Phi^{-1}(\kappa) + \frac{\sigma^2}{2} + \log\Phi\left(\Phi^{-1}(\kappa) - \sigma\right) = 0. \tag{A.99}$$

Evaluating at $\sigma = 0$ shows that the left-hand side is $\log(Z/\bar{K})$, which is strictly positive because $Z > \bar{K}$. The derivative with respect to σ is $-(\Phi'(\Phi^{-1}(\kappa) - \sigma)/\Phi(\Phi^{-1}(\kappa) - \sigma) + (\Phi^{-1}(\kappa) - \sigma))$. This is strictly negative because the Normal CDF satisfies $\Phi'(w)/\Phi(w) > -w$ for any w, hence the left-hand side of (A.99) is strictly decreasing in σ. Moreover, by L'Hôpital's rule, the left-hand side behaves like $\log(Z/\bar{K}) - \log\kappa - (\sigma/2)\Phi^{-1}(\kappa) - (\sigma/2)(\Phi'(\Phi^{-1}(\kappa) - \sigma)/\Phi(\Phi^{-1}(\kappa) - \sigma) - (\Phi^{-1}(\kappa) - \sigma))$ for very large σ. The first two terms are constants, the final term is strictly negative, and the third term is negative and linear in σ. It follows that the left-hand side of (A.99) becomes negative for sufficiently large σ, hence there always exists a unique solution of the equation for σ, which can be found numerically. Given this solution, the other parameter of the credit-cost distribution is $\mu = \log Z - \sigma\Phi^{-1}(\kappa)$.

Search costs and entry costs The steady-state value of N in (5) is zero, hence $E = \kappa(B_h - \bar{K}) + (1-\kappa)B_l$. Using the equation for Z from (3), the entry cost parameter is $E = B_l + \kappa(Z - \bar{K})$, which is identified by the target for B_l, the known values of Z and $\bar{K}$, and κ from (A.98).

The flow search-cost parameters F_h, F_i, F_l are obtained from the following:

$$F_h = \frac{P}{365(1-i+iP_i/P_h)(P_h/Y)}\frac{\Lambda_h}{T_{bh}}\frac{F_h/v_o}{Y/365}, \quad F_i = \frac{F_i}{F_h}F_h, \quad \text{and } F_l = \frac{\Lambda_l}{\Lambda_h}\frac{T_{bh}}{T_{bl}}\frac{F_l/v_l}{F_h/v_o}F_h, \tag{A.100}$$

which are stated in terms of the calibration targets (and T_{bl} from A.94) by using (A.77).

Discount rate and bargaining powers Information on prices, rents, costs, and time-on-the-market is used to identify the discount rate r for future housing payoffs and the bargaining-power parameters ω_h, ω_i, and ω_l. Taking the equations (A.54) and (A.55) and dividing both sides of the first by P_h, and similarly for the difference between the two equations:

$$1 = \frac{r + \theta_o v_o(1-\psi)\pi_h}{r}\frac{\omega_h^*\Sigma_h}{\pi_h P_h} + \frac{\theta_o v_o \psi}{r}\frac{\omega_i^*\Sigma_i}{P_h} + \frac{C_o - (D/r)}{P_h}, \quad \text{and} \quad 1 - \frac{P_i}{P_h} = \frac{\omega_h^*\Sigma_h}{\pi_h P_h} - \frac{\omega_i^*\Sigma_i}{P_h}.$$

Solving these simultaneous equations for the surpluses yields an expression for Σ_h:

$$\frac{\omega_h^*\Sigma_h}{\pi_h P_h} = \frac{\left(1 - \frac{C_o}{P_h}\right)r + \frac{D}{P_h} + \theta_o\psi\frac{\Lambda_h}{T_{bh}}\left(1 - \frac{P_i}{P_h}\right)}{r + T_{so}^{-1}}, \tag{A.101}$$

which uses formulas for T_{so}, T_{bh}, and Λ_h from (A.77), and an expression for Σ_i:

$$\frac{\omega_i^*\Sigma_i}{P_h} = \frac{\left(1 - \frac{C_o}{P_h}\right)r + \frac{D}{P_h} - \left(r + \theta_o(1-\psi)T_{bh}^{-1}\right)\left(1 - \frac{P_i}{P_h}\right)}{r + T_{so}^{-1}}. \tag{A.102}$$

Using equation (16) for the equilibrium tenancy agreement fee A, the sum of the total transaction costs incurred by landlords and tenants is

$$C_l + C_w = \frac{1}{\omega_l}\left(1 - \frac{A}{C_l}\right)C_l. \tag{A.103}$$

Dividing both sides of the steady-state average rent equation (A.75) by R, substituting for $\omega_l(C_l + C_w)$ using (A.103), and rearranging to write an equation for the rental-market surplus Σ_l:

$$\frac{\omega_l\Sigma_l}{\pi_l R} = \frac{1 - \frac{D_l}{R} - (r + m_l + \rho)\omega_l\frac{(C_l+C_w)}{R}}{r + m_l + \rho + \theta_l v_l \pi_l} = \frac{1 - \frac{D_l}{R} - \left(r + T_{ml}^{-1}\right)\left(1 - \frac{A}{C_l}\right)\frac{C_l}{R}}{r + T_{ml}^{-1} + T_{sl}^{-1}}, \tag{A.104}$$

which makes use of the formulas for T_{sl} and T_{ml} from (A.77) and (A.78).

Investors' surplus Σ_i satisfies (38), and this equation can be written as follows:

$$\frac{\omega_i^*}{1-\omega_i^*} = \frac{\frac{\Lambda_h}{T_{bh}}\frac{\omega_i^*\Sigma_i}{P_h}}{\frac{F_i}{F_h}\frac{F_h}{P_h}}. \tag{A.105}$$

Dividing both sides of the free-entry condition (A.63) by P_i and using the investor price equation (A.55) to substitute $\theta_o v_o((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i)/P_i = (D/P_i) + r(1 - (C_o/P_i) - (\omega_i^*\Sigma_i/P_i))$:

$$\theta_l v_l \pi_l\left(\frac{\omega_l\Sigma_l}{\pi_l P_i}\right) = \left(1 + \tau_i\left(1 + \frac{\rho_l}{r}\right)\right)\left(D + r\left(1 - \frac{C_o}{P_i} - \frac{\omega_i^*\Sigma_i}{P_i}\right)\right)$$
$$+ (r+\rho_l)\left((1+\tau_i)\frac{C_o}{P_i} + \frac{C_i}{P_i} + (1 + \tau_i\omega_i^*)\frac{\Sigma_i}{P_i}\right) - \tau_i\left(1 + \frac{\rho_l}{r}\right)\frac{D}{P_i},$$

which simplifies to the following by noting that $r + \rho_l - r\omega_i^* = (r+\rho_l)(1-\omega_i^*) + \rho_l\omega_i^*$ and $\theta_l v_l \pi_l = T_{sl}^{-1}$:

$$\frac{1}{T_{sl}}\frac{R}{P_i}\frac{\omega_l\Sigma_l}{\pi_l R} = \frac{D}{P_i} + r + \tau_i(r+\rho_l) + \rho_l\frac{C_o}{P_i} + (r+\rho_l)\frac{C_i}{P_i} + (r+\rho_l)\frac{(1-\omega_i^*)}{\omega_i^*}\frac{\omega_i^*\Sigma_i}{P_h}\frac{P_h}{P_i} + \rho_l\frac{P_h}{P_i}\frac{\omega_i^*\Sigma_i}{P_h}.$$

Substituting from (A.102), (A.104), and (A.105) leaves an equation in just one unknown r:

$$\frac{D}{P_i}+r+\rho_l\frac{C_o}{P_i}+(r+\rho_l)\left(\tau_i+\frac{C_i+F_i\frac{T_{bh}}{\Lambda_h}}{P_i}\right)+\rho_l\left(\frac{\left(1-\frac{C_o}{P_i}\right)r+\frac{D}{P_i}-\theta_o(1-\psi)T_{bh}^{-1}\left(\frac{P_h}{P_i}-1\right)}{r+T_{so}^{-1}}\right)$$
$$=\frac{1}{T_{sl}}\frac{R}{P_i}\left(\frac{1-\frac{D_l}{R}-\left(r+T_{ml}^{-1}\right)\left(1-\frac{A}{C_l}\right)\frac{C_l}{R}}{r+T_{ml}^{-1}+T_{sl}^{-1}}\right). \quad \text{(A.106)}$$

The right-hand side is strictly decreasing in r, while the second and fourth terms on the left-hand side are linear in r with positive coefficients. Under a weak restriction that time-to-sell T_{so} is not too long, specifically $T_{so}<(1-(C_o/P_i))/((D/P_i)-\theta_o(1-\psi)T_{bh}^{-1}((P_h/P_i)-1))$, the final term on the left-hand side is also increasing in r, and hence any solution of (A.106) is unique and r is identified.[53] The left-hand side exceeds the right-hand side for large r, so existence of a solution is verified by checking the left-hand side is below the right-hand side at $r=0$, which is true for sufficiently high rental yields R/P_i. The value of r consistent with the calibration targets is then found by solving (A.106) numerically.

Dividing both sides of the steady-state Bellman equations (A.47) and (A.60) by P_h, using $B_h=B_l+Z$ from (3), and rearranging to solve for $\omega_h^*/(1-\omega_h^*)$ and $\omega_l/(1-\omega_l)$:

$$\frac{\omega_h^*}{1-\omega_h^*}=\frac{\frac{1}{T_{bh}}\frac{\omega_h^*\Sigma_h}{\pi_h P_h}}{\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}+(r+\rho)\frac{Z}{P_h}}, \quad \text{and} \quad \frac{\omega_l}{1-\omega_l}=\frac{\frac{1}{T_{bl}}\frac{R}{P_i}\frac{P_i}{P_h}\frac{\omega_l\Sigma_l}{\pi_l R}}{\frac{F_l}{F_h}\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}}, \quad \text{(A.107)}$$

which use $T_{bl}=1/(v_l\pi_l)$ and $T_{bh}=1/(v_o\pi_h)$ from (A.77). Tax revenue is given in (26), and by using (36) and (A.77), per-person expenditure on public services $g=G/n$ relative to house prices P_h is

$$\frac{g}{P_h}=\frac{\left(\tau_h(1-i)+\tau_i i\frac{P_i}{P_h}\right)u_o}{nT_{so}}, \quad \text{(A.108)}$$

which can be calculated from the targets and the value of u_o given in (A.92). Note that (28) and (34) imply $\omega_h^*/(1-\omega_h^*)=(\omega_h/(1-\omega_h))/(1+\tau_h)$ and $\omega_i^*/(1-\omega_i^*)=(\omega_i/(1-\omega_i))/(1+\tau_i)$. Hence, by combining equations (A.101), (A.102), and (A.104) with (A.105) and (A.107), the bargaining powers ω_h, ω_l, and ω_l are determined by the following equations after knowing r by solving equation (A.106):

$$\frac{\omega_h}{1-\omega_h}=\frac{(1+\tau_h)\frac{1}{T_{bh}}\left(\left(1-\frac{C_o}{P_h}\right)r+\frac{D}{P_h}+\theta_o\psi\frac{\Lambda_h}{T_{bh}}\left(1-\frac{P_i}{P_h}\right)\right)}{\left(\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}+(r+\rho)\frac{Z}{P_h}\right)\left(r+\frac{1}{T_{so}}\right)}, \quad \text{(A.109)}$$

$$\frac{\omega_l}{1-\omega_l}=\frac{\frac{1}{T_{bl}}\frac{R}{P_i}\frac{P_i}{P_h}\left(1-\frac{D_l}{R}-\left(r+T_{ml}^{-1}\right)\left(1-\frac{A}{C_l}\right)\frac{C_l}{R}\right)}{\left(\frac{F_l}{F_h}\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}\right)\left(r+\frac{1}{T_{ml}}+\frac{1}{T_{sl}}\right)}, \quad \text{(A.110)}$$

$$\frac{\omega_i}{1-\omega_i}=\frac{(1+\tau_i)\frac{\Lambda_h}{T_{bh}}\left(\left(1-\frac{C_o}{P_h}\right)r+\frac{D}{P_h}-\left(1-\frac{P_i}{P_h}\right)\left(r+\theta_o(1-\psi)\frac{1}{T_{bh}}\right)\right)}{\frac{F_i}{F_h}\frac{F_h}{P_h}\left(r+\frac{1}{T_{so}}\right)}. \quad \text{(A.111)}$$

Hence, $\omega_h=(\omega_h/(1-\omega_h))/(1+\omega_h/(1-\omega_h)))$ and similarly for the other parameters ω_l and ω_i using (A.109)–(A.111). Once ω_l is known, the implied tenant moving cost C_w is deduced from (A.103):

$$C_w=\left(\frac{1}{\omega_l}\left(1-\frac{A}{C_l}\right)-1\right)C_l, \quad \text{(A.112)}$$

and it can be verified whether C_w is positive.

Meeting functions With ω_h, ω_l, and ω_i known, the meeting-function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l, where $\omega_o=(1-\psi)\omega_h+\psi\omega_i$ is the average

[53] This condition is satisfied for the calibration targets in Table 3.

bargaining power of sellers in the ownership market. Since market tightnesses θ_o and θ_l are known from (A.94) and the viewing rates v_o and v_l can be deduced from the targets (and T_{bl} from A.94) using (A.77), the meeting-function productivity parameters υ_o and υ_l are set to be consistent with (43):

$$\upsilon_o = \theta_o^{\eta_o} \frac{\Lambda_h}{T_{bh}}, \quad \text{and} \quad \upsilon_l = \theta_l^{\eta_l} \frac{\Lambda_l}{\Lambda_h} \frac{\Lambda_h}{T_{bl}}. \tag{A.113}$$

Rental-market parameters Using (40) and (A.77), equation (A.29) for the rental-market surplus can rearranged as follows:

$$\lambda_l = 1 + \frac{\omega_l \frac{y_l}{R}}{\left(r + \frac{1}{T_{ml}}\right) \frac{\omega_l \Sigma_l}{\pi_l R}}.$$

By using (A.62) to obtain an expression for y_l/R and substituting for $\omega_l \Sigma_l/(\pi_l R)$ from (A.104):

$$\lambda_l = 1 + \frac{\omega_l \left(\frac{D_l - F_l}{R} + \left(r + \frac{1}{T_{ml}}\right) \frac{C_l + C_w}{R} - \xi m_l \kappa \frac{(Z-\bar{K})}{R}\right) \left(r + \frac{1}{T_{ml}} + \frac{1}{T_{sl}}\right)}{\left(1 - \frac{D_l}{R} - \left(r + \frac{1}{T_{ml}}\right)\left(1 - \frac{A}{C_l}\right) \frac{C_l}{R}\right) \left(r + \frac{1}{T_{ml}}\right)} + \frac{(1 - \omega_l + \omega_l \theta_l) \frac{1}{T_{bl}}}{r + \frac{1}{T_{ml}}}. \tag{A.114}$$

Knowing λ_l, ζ_l is found using $\zeta_l = y_l \pi_l^{1/\lambda_l}$ implied by (40) along with (A.62), (A.77), and (A.104):

$$\zeta_l = \frac{D_l - F_l + \left(r + \frac{1}{T_{ml}}\right)(C_l + C_w) - \xi m_l \kappa (Z - \bar{K}) + \frac{1}{T_{bl}} \left(\frac{1 - \omega_l + \omega_l \theta_l}{\omega_l}\right) \left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) R}{\Lambda_l^{\frac{1}{\lambda_l}}}. \tag{A.115}$$

Moving decisions and the size of idiosyncratic shocks The response β_{mh} of the log moving hazard rate to the increase in the transaction tax in the subsequent four years is one of the calibration targets. In the model, this hazard rate is the combined moving rate within and outside the city, namely $m_h + \rho$, so the model prediction to match to the econometric estimate of β_{mh} is the average response of $\log(m_h + \rho)$ in the four years after the tax change. The response must be computed using the numerical solution of the model's dynamics for given parameters. The endogenous response of the moving rate is most closely connected to the parameter δ_h that governs the size of the idiosyncratic shocks to match quality (if $\delta_h = 0$ then the moving rate would be exogenous and not respond to the tax change, as can be seen from 41). The value of δ_h is set to match β_{mh}, but it is convenient to search numerically over the following transformation $\varkappa$ of the parameter δ_h:

$$\varkappa = \frac{\alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right)}. \tag{A.116}$$

The remaining parameters of the model can be found conditional on $\varkappa$ in what follows, and once these are known, δ_h is inferred from (A.116):

$$\delta_h = \left(\frac{(\rho + \alpha_h)\varkappa}{\alpha_h \left(\varkappa + (y_h/x_h)^{\lambda_h}\right)}\right)^{\frac{1}{\lambda_h}}, \tag{A.117}$$

where y_h/x_h can also be calculated from the calibration targets as explained below. Identification is confirmed by verifying numerically that there is a unique $\varkappa$ matching the moving hazard response.

Ownership-market match quality and the arrival of idiosyncratic shocks Using the definition of $\varkappa$ in (A.116) and equation (A.58) for m_h:

$$\alpha_h = (1 + \varkappa) m_h + \rho \varkappa, \tag{A.118}$$

and hence α_h is determined conditional on $\varkappa$ and the known values of m_h and ρ from (A.96). The steady-state value of x_h is found given the calibration targets by using (A.50), (A.77), (A.101), and (A.102):

$$x_h = \left(\frac{1}{T_{bh}}\left(\frac{1-\omega_h^* + (1-\psi)\omega_h^*\theta_o}{\omega_h^*}\right)\left(\frac{\omega_h^*\Sigma_h}{\pi_h P_h}\right) + \psi\theta_o\frac{\Lambda_h}{T_{bh}}\left(\frac{\omega_i^*\Sigma_i}{P_h}\right)\right)P_h - F_h\,, \tag{A.119}$$

and the value of y_h is then computed using (A.49) together with (A.101), (A.102), (A.118), and (A.119):

$$y_h = x_h + (r+\rho+\alpha_h)\left(C_h + C_o + \tau_h\left(C_o - \frac{D}{r} + \frac{\theta_o}{r}\frac{P_h}{T_{bh}}\left((1-\psi)\frac{\omega_h^*\Sigma_h}{\pi_h P_h} + \psi\Lambda_h\frac{\omega_i^*\Sigma_i}{P_h}\right)\right)\right). \tag{A.120}$$

Equation (A.51) for owner-occupiers' expected surplus can be rearranged as follows using (40):

$$\lambda_h = 1 + \frac{\omega_h^*\frac{y_h}{P_h}}{(r+\rho+\alpha_h)(1+\tau_h\omega_h^*)\frac{\omega_h^*\Sigma_h}{\pi_h P_h}}\left(1 + \frac{x_h}{y_h}\frac{\alpha_h\delta_h^{\lambda_h}\left(\frac{y_h}{x_h}\right)^{\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}\right).$$

Since the procedure is to search over $\varkappa$ rather δ_h, equation (A.117) is used to write $\alpha_h\delta_h^{\lambda_h}(y_h/x_h)^{\lambda_h} = (\rho+\alpha_h)\varkappa(y_h/x_h)^{\lambda_h}/(\varkappa+(y_h/x_h)^{\lambda_h})$ and $\alpha_h(1-\delta_h^{\lambda_h}) = (\alpha_h(y_h/x_h)^{\lambda_h} - \rho\varkappa)/(\varkappa+(y_h/x_h)^{\lambda_h})$. Substituting into the equation above and making use of (A.101) yields an equation for λ_h:

$$\lambda_h = 1 + \frac{\omega_h^*\frac{y_h}{P_h}\left(r+\frac{1}{T_{so}}\right)\left(1+\frac{x_h}{y_h}\frac{\varkappa(\rho+\alpha_h)}{(r+\rho+\alpha_h)+\varkappa r(y_h/x_h)^{-\lambda_h}}\right)}{(r+\rho+\alpha_h)(1+\tau_h\omega_h^*)\left(\left(1-\frac{C_o}{P_h}\right)r + \frac{D}{P_h} + \theta_o\psi\frac{\Lambda_h}{T_{bh}}\left(1-\frac{P_i}{P_h}\right)\right)}. \tag{A.121}$$

Apart from λ_h, all terms in the above equation are known given the calibration targets (including $\varkappa$, chosen to target the moving hazard response), so the equation can be solved numerically for λ_h. As the left-hand side is below the right-hand side at $\lambda_h = 1$, but rises above the right-hand side as λ_h becomes large (the right-hand side is bounded, but the left-hand side increases linearly with λ_h), there exists a solution with $\lambda_h > 1$. While $1/((r+\rho+\alpha_h)+\varkappa r(y_h/x_h)^{-\lambda_h})$ is increasing in λ_h along with the left-hand side of (A.121), for $\varkappa < 1+(\rho+\alpha_h)/r$, the right-hand side is a strictly concave function of λ_h, in which case the solution is unique.[54] Once λ_h is found, the parameter δ_h is obtained from (A.117) for a given value of $\varkappa$, and it can be checked whether $\delta_h y_h < x_h$. Finally, using (40), (A.77), and (A.120) the parameter ζ_h is given by $\zeta_h = y_h\pi_h^{1/\lambda_h} = y_h/\Lambda_h^{1/\lambda_h}$.

A.7 Additional quantitative results

[54]The condition that $\varkappa$ or r is sufficiently low is satisfied for the calibration targets in Table 3.

Table A.13: *Tax effects with 3% mortgage interest rate gap and no mobility across regions*

Variable	Model predictions average over 4 years Baseline	3% gap	No mobility
Owners' moving rate (T_{mh}^{-1})	-12% (matched)	-12% (matched)	-12% (matched)
Buy-to-own (BTO) sales (S_h)	-14%	-13%	-14%
Buy-to-rent (BTR) sales (S_i)	35%	13%	35%
Total sales (S)	-12%	-11%	-12%
Time-to-sell (T_{so})	6.0%	7.7%	6.4%
Leases-to-sales ratio (S_l/S_o)	15%	13%	14%
Price-to-rent ratio (P_i/R)	-1.6%	-1.6%	-1.6%
Average sales price (P)	-1.6%	-1.5%	-1.9%
Homeownership rate (h)	-0.23 p.p.	-0.089 p.p.	-0.23 p.p.
City population (n)	0.0%	0.0%	0.0%
Transaction tax revenue (G)	61%	62%	61%
Effective LTT tax rate ($\tau_h = \tau_i$)	Increased from 1.5% to 2.8% (1.3 p.p.)		

Notes: The responses of variables are reported as percentage changes.