Online Appendix to
"To Own or to Rent? The Effects of Transaction Taxes on Housing Markets"

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A Appendices

A.1 Data, further estimation results, and robustness checks

A.1.1 Evaluating the comprehensiveness of the MLS rental listings data

Since the use of rental listings data in this paper is relatively new to the literature, it is important to examine how comprehensive are the Toronto MLS rental listings. This section shows through webscraping that MLS data provide an unusually high coverage of long-term and verifiable rental listings in the City of Toronto compared to other online rental platforms. Specifically, the MLS data capture over 90% of rental properties listed on the second-most popular rental listing platform in Toronto.

The Multiple Listing Service (MLS) is a database created by the Canadian Real Estate Association (CREA) and used by real-estate professionals to share and access information about properties for sale or lease. It enables cooperation among real-estate agents and brokers, who can pool their listings and share commissions on property transactions. An alternative popular rental listings platform is *Toronto Rentals* (hereafter referred to as TR), which is the second-largest website serving Toronto and the surrounding GTA since 1995.

For the period between 23$^{\text{rd}}$ November 2022 and 23$^{\text{rd}}$ February 2023, all rental listings from the MLS on `realtor.ca` (REALTOR.ca, 2022) and from TR (`rentals.ca/toronto`, Toronto Rentals, 2022) were webscraped. For each MLS listing, information was collected on the MLS ID, the address (as a string), the listing date, the number of bedrooms, the number of bathrooms, and the asking rent. For each TR listing, the information collected was the address (specified in terms of latitude and longitude), the listing date, the number of bedrooms, the number of bathrooms, and the asking-rent range.

To compare the two scraped datasets, MLS address strings were cleaned and parsed to apply Google Maps API to geocode the coordinates of each listing. The MLS listings were then matched with the TR listings by the geocoded address, the number of rooms, and a window around the listing date. Since a property might be listed on one platform first and later on another platform, the comparison was restricted to properties listed on TR between 25$^{\text{th}}$ November and 5$^{\text{th}}$ December 2022. The exercise then checked how many of these listings were also on the MLS during the same or surrounding time period.

Figure A.1 shows a map of the locations of rental listings in the City of Toronto. Yellow dots indicate MLS listings. Grey dots are TR listings that match with listings in the MLS data. Red dots are TR listings that are at least 200 metres away from the closest MLS listing, which is taken as an indicator that these listings were not included in the MLS.

There were 4,359 unique MLS records during the period studied, and the TR dataset includes a total of 3,516 entries. Out of all the TR listings, 294 were not matched with an MLS record, accounting for approximately 8.4% of the TR data. This fraction is likely to be overestimated because of inaccuracies in the manual matching of the MLS listings' coordinates.

There are also short-term rental websites such as *Kijiji* in Toronto. However, listings on these platforms are not included in the analysis for several reasons. First, unlike MLS or TR listings, Kijiji listings are unverified and less reliable, with most of them posted by anonymous users. Second, Kijiji users often forget to remove their listings when they are no longer active, making it questionable in what time window a listing counts as active. Third, Kijiji listings do not provide precise address information and can only be identified at neighbourhood level. Finally, unlike MLS or TR listings, most Kijiji listings are for short-term lets that are distinct from the longer-term rentals in the main analysis.

A.1.2 Descriptive statistics

Figure A.1: *Rental listings in Toronto between 25th November and 5th December 2022*

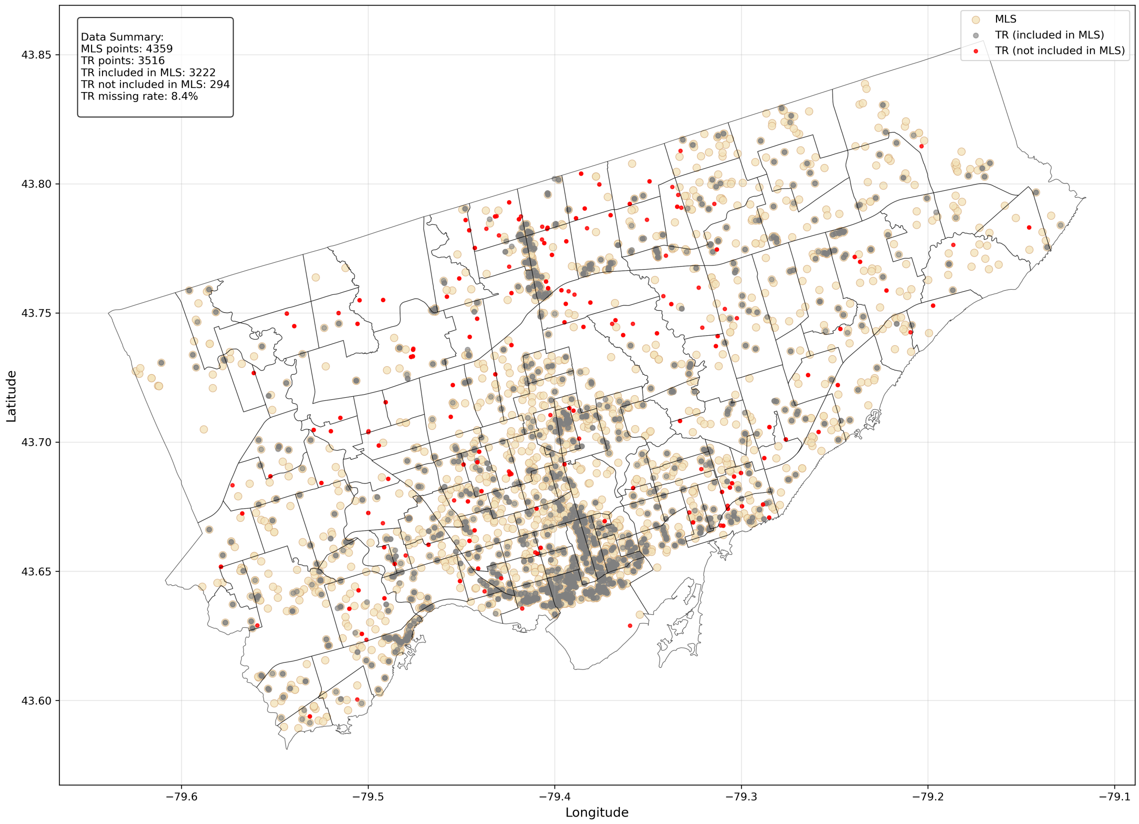


Table A.1: *Land transfer tax (LTT) rates by property value in the Greater Toronto Area*

City of Toronto (effective from 1st February 2008)		Province of Ontario (effective from 7th May 1997)	
$0–55,000	0.5%	$0–55,000	0.5%
$55,000–400,000	1.0%	$55,000–250,000	1.0%
$400,000+	2.0%	$250,000–400,000	1.5%
		$400,000+	2.0%

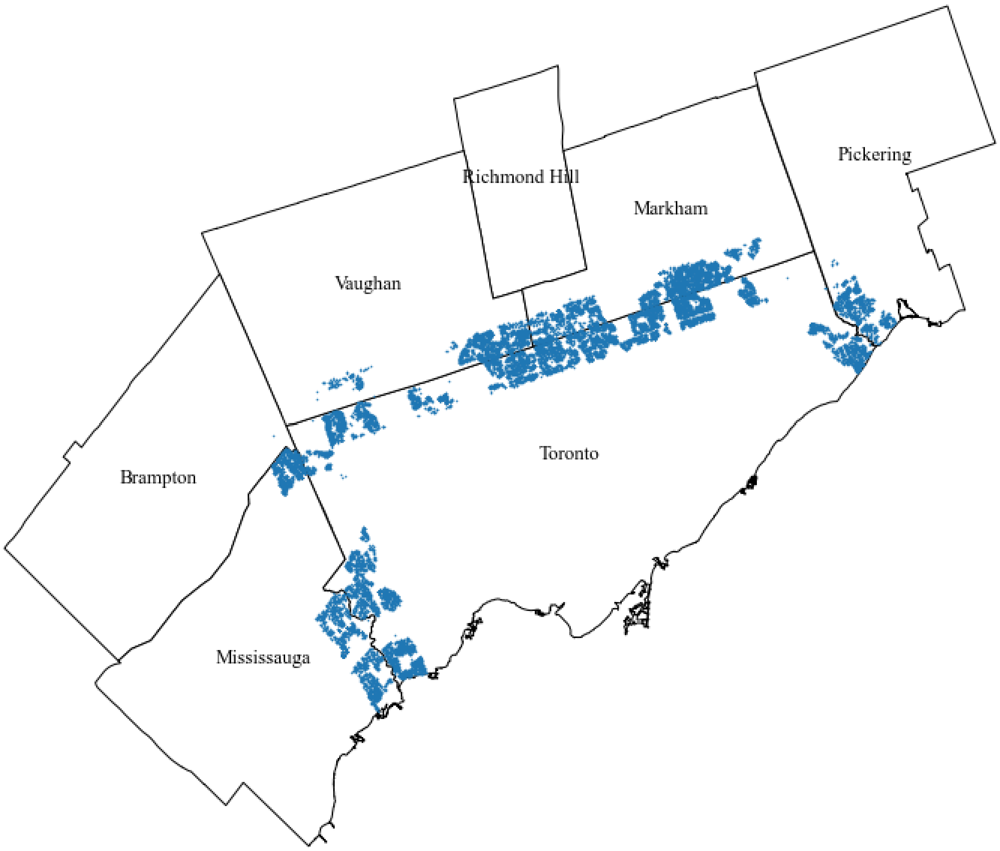
Sources: Municipal Land Transfer Tax, City of Toronto, `http://www.toronto.ca/taxes/mltt.htm`; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).
Notes: For the municipal LTT, exemptions are given to first-time buyers for purchases below a value of $400,000, while for the provincial LTT, the first-time buyer exemption value threshold is $227,500.

Table A.2: *Changes in the effective land transfer tax rate within the City of Toronto*

Fraction of first-time buyers	All of city	Within 5km of border	Within 3km of border
0%	1.521%	1.518%	1.507%
100%	1.041%	1.036%	1.014%
40%	1.329%	1.325%	1.310%

Notes: The table shows the average LTT rates before and after the new LTT for all transactions within the city, and those within 5km or 2km inside the city border. The sample is restricted to detached house transactions. The effective LTT rate is the mean transfer tax as a percentage of the sales price, combining provincial and city-level taxes, averaged over transactions between January 2006 and January 2008 to control for compositional effects. The effective LTT rates are imputed based on the tax rates before and after the new LTT is introduced, and the change in the effective LTT rate is taken to be the difference between the two.

Figure A.2: *Geography of the sample used for estimation*



A.1.3 Housing-stock composition

As a check on the assumption that there are no significant differences in housing composition potentially picked up by the coefficient on the *LTT* dummy, columns (1) and (2) of Table A.4 present evidence on property characteristics on opposite sides of the city border before the new LTT is introduced. The sample is restricted to the pre-tax-rise period, and each property characteristic is regressed on a border dummy that indicates being inside the City of Toronto, controlling for the usual factors. The border coefficients are statistically insignificant in most cases, and quantitatively small even when statistically significant. This indicates that properties transacted on opposite sides of the border were more or less similar before the new LTT.

In columns (3) and (4), each property characteristic is further regressed on the *LTT* dummy that is an interaction of the border dummy and the post-tax-rise dummy, controlling for the usual factors. The *LTT* dummy coefficients are statistically insignificant in almost all cases. As expected, cross-border differences in property characteristics, if any, remain stable before and after the new LTT. This ensures the coefficients on the *LTT* dummy in the main empirical specifications pick up the impact of the new transaction tax, rather than changes in housing-stock composition.

Table A.3: *Descriptive statistics*

	Pre-LTT 2006:1–2008:1	Post-LTT 2008:2–2010:2	2008:1–2012:2	Pre-&post-LTT 2006:1–2018:2
		Greater Toronto Area		
# BTO sales per year	53,018	45,962	46,232	52,109
# BTR sales per year	2,545	2,670	3,139	4,529
Days on the market (mean)	31.3	29.6	27.2	25.1
Days on the market (median)	21.0	20.0	18.0	16.0
Sale price (mean)	381,238	408,106	442,050	540,237
Sale price (median)	321,000	347,500	372,000	425,000
Price-rent ratio (mean)	20.3	20.6	21.9	25.8
Price-rent ratio (median)	17.4	18.2	19.0	21.6
		City of Toronto		
# BTO sales per year	27,718	23,832	24,621	27,639
# BTR sales per year	1,572	1,685	1,947	2,620
Days on the market (mean)	30.5	28.8	27.1	25.4
Days on the market (median)	20.0	18.0	17.0	15.0
Sale price (mean)	401,504	426,363	460,903	553,380
Sale price (median)	318,000	343,000	369,900	417,900
Price-rent ratio (mean)	20.7	20.9	22.2	25.7
Price-rent ratio (median)	16.9	17.9	18.8	21.1
		5km border sample		
# BTO sales per year	16,785	14,521	14,525	16,503
# BTR sales per year	908	1,015	1,155	1,548
Days on the market (mean)	33.3	30.7	28.1	26.2
Days on the market (median)	23.0	20.0	18.0	17.0
Sale price (mean)	345,754	371,534	405,536	503,184
Sale price (median)	315,000	338,000	361,000	408,000
Price-rent ratio (mean)	19.6	20.3	21.8	25.9
Price-rent ratio (median)	16.4	17.2	18.3	20.8
		3km border sample		
# BTO sales per year	8,504	7,435	7,327	8,074
# BTR sales per year	348	400	461	608
Days on the market (mean)	33.7	31.3	28.6	26.5
Days on the market (median)	24.0	21.0	19.0	17.0
Sale price (mean)	339,412	361,448	394,667	488,217
Sale price (median)	314,000	334,300	357,000	401,000
Price-rent ratio (mean)	19.0	19.5	21.1	25.7
Price-rent ratio (median)	15.9	16.1	17.3	20.1

Source: Multiple Listing Service (MLS) residential records (2006–2018).

Table A.4: *Comparison of property characteristics across the city border*

Property characteristic	(1)	(2)	(3)	(4)
Heating	0.000490	0.000320	−0.000406	−0.000120
	(0.000394)	(0.000236)	(0.000486)	(0.000329)
Observations	10,389	17,916	42,444	73,550
Basement	−0.00498	−0.00831**	−0.00133	0.00234
	(0.00391)	(0.00310)	(0.00458)	(0.00351)
Observations	10,389	17,916	42,444	73,550
Family	0.0227	−0.0907***	−0.0478	−0.0145
	(0.0344)	(0.0263)	(0.0381)	(0.0292)
Observations	10,347	17,834	42,444	73,548
Fire	0.00368	−0.0229***	−0.00655	−0.000543
	(0.00713)	(0.00562)	(0.00795)	(0.00621)
Observations	10,389	17,916	42,444	73,550
Bedrooms	0.00535	0.0157*	0.0138	0.0139
	(0.0105)	(0.00817)	(0.0110)	(0.00870)
Observations	10,389	17,916	42,444	73,550
Bathrooms	−0.115***	−0.120***	−0.0229	−0.0200
	(0.0137)	(0.0109)	(0.0157)	(0.0123)
Observations	10,389	17,916	42,444	73,550
Rooms	−0.0322	−0.0274	−0.0193	−0.0339*
	(0.0273)	(0.0185)	(0.0232)	(0.0178)
Observations	10,389	17,916	42,444	73,550
Lot	−1305.3	−918.3	1051.8	177.8
	(1006.3)	(600.3)	(967.1)	(903.3)
Observations	10,389	17,916	42,444	73,550
Distance threshold	3km	5km	3km	5km
LTT sample period	Pre	Pre	All	All

Notes: Data comprise detached house transactions from January 2006 to February 2012. A unit of observation is a transaction. In columns (1) and (2), the coefficients are from regressions of a property characteristic on a border dummy that indicates a location is in the City of Toronto. In columns (3) and (4), the coefficients are from regressions of a property characteristic on the *LTT* dummy that indicates a location in the City of Toronto and in the period after the LTT is introduced. All regressions control for other property characteristics, and year, month, and property-type fixed effects. Regressions for columns (3) and (4) include an indicator for the post-LTT period and an indicator for the City of Toronto. Distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. LTT sample period specifies whether a transaction occurred before or after the new LTT. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

A.1.4 Empirical specifications

The econometric specification is a variant of the regression discontinuity design developed by Dachis, Duranton and Turner (2012) applied to a broader set of housing-market outcomes.

Let t denote the time before ($t < 0$) or after ($t > 0$) the imposition of the new LTT, where the time unit is measured in months. Let d denote distance from the city border, with $d < 0$ meaning a location in the suburbs and $d > 0$ in the City of Toronto. Let i denote the unit of observation, *community* $\times$ *property type* $\times$ *year* $\times$ *month* in the market-segment regressions, *household* $\times$ *month* in the moving hazard regressions, and a *transaction* in the sales price and time-on-the-market regressions.

Define the following indicator variables based on time and distance:

$$\chi^{\text{POST}} = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}, \quad \text{and} \quad \chi^{\text{CT}} = \begin{cases} 1 & \text{if } d \geq 0 \\ 0 & \text{if } d < 0 \end{cases}.$$

The main variable of interest is the *LTT* dummy $\chi^{\text{POST}} \times \chi^{\text{CT}}$. Let y_{it} denote an outcome of interest, for example, buy-to-own transactions or the sales price. Let $\mathbf{x}_{it}$ denote the vector of property characteristics for unit i at time t in addition to χ^{POST} and χ^{CT}. To address anticipation effects that may arise from the announcement of the new LTT, define the following dummy variables:

$$\chi^{\tau} = \begin{cases} 1 & \text{if } t = \tau \text{ and } d \leq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \tau \in \{-3, -2, -1, 0, 1, 2, 3\}.$$

Some regressions include an interaction between the *LTT* dummy and areas away from the border, e.g, 2km away. To control for these differential effects, define the dummy variables:

$$\chi^{x > \bar{d}} = \begin{cases} 1 & \text{if } t > 0 \text{ and } d \geq \bar{d} \\ 0 & \text{otherwise} \end{cases}, \quad \text{for } \bar{d} > 0.$$

The general model is

$$y_{it} = \lambda \chi^{\text{POST}} \times \chi^{\text{CT}} + \beta' \mathbf{x}_{it} + \chi^{\tau} + \nu_t + \delta_i + \varepsilon_{it},$$

where ν_t represents *year* fixed effects and *month* fixed effects, δ_i represents *community* fixed effects, and ε_{it} is the error term. Notably, the specifications allow for separate time trends for transactions inside and outside of the city to control for Toronto-specific trends that may be caused by factors other than the LTT. In the all-properties sample, *community* $\times$ *property type*, *month* $\times$ *property type*, and *year* $\times$ *property type* fixed effects are also included.

A.1.5 Additional results

Figure A.3: *Kaplan-Meier estimate of homeowners' moving hazard function*

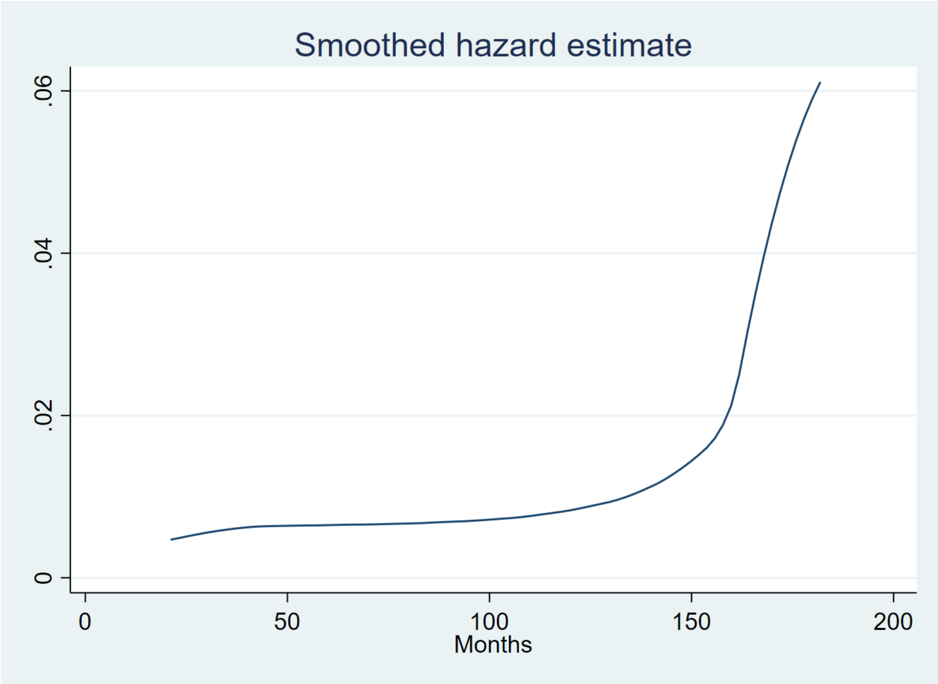


Table A.5: *The effects of the transaction tax on sales and leases separately*

	(1)	(2)	(3)
log (#*Sales*)	−0.177***	−0.0937**	−0.108**
	(0.0519)	(0.0405)	(0.0361)
Observations	6,540	10,798	13,437
log (#*Leases*)	0.0765	0.0985*	0.109**
	(0.0470)	(0.0550)	(0.0516)
Observations	2,660	5,545	6,006
Sample	Border	Border	Border
Distance threshold	5km	5km	5km
Border	Yes	Yes	Yes
Semi-detached	No	No	Yes
Detached	Yes	Yes	Yes
Condo apartments	No	Yes	Yes

Notes: The sample comprises transactions from 2006 to 2012 that are within 5km of the City of Toronto border. Given the limited sample size for the detached houses rental market (column 1), the sample is expanded to include also condominiums and semi-detached houses (columns 2 and 3). Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. All regressions include a post-LTT dummy, and city, community, year, and calendar month fixed effects, as well as their interactions with the property type. Six dummy variables are included for transactions inside the city of Toronto during the last three months of 2007 and the first three of 2008. City time trends and distance LTT trends are included. Property type rows indicate whether the property type was included in the regression. The border row indicates if a border sample was used, and the distance threshold indicates the distance threshold defining the sample. Robust standard errors are in parentheses and *, **, and *** indicate significance at the 10%, 5%, and 1% levels.

Table A.6: *Effects of the transaction tax on prices and time-on-the-market*

	(1)	(2)	(3)	(4)
	Dependent variable: log (*Sales price*)			
LTT	−0.0176**	−0.0232**	−0.0235	−0.0123***
	(0.00824)	(0.00953)	(0.0147)	(0.00342)
Observations	14,702	24,970	14,808	110,952
	Dependent variable: log (*Time-on-the-market*)			
LTT	0.426***	0.420***	0.347***	0.396***
	(0.0589)	(0.0623)	(0.0542)	(0.0474)
Observations	14,704	24,973	14,809	110,961
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
Property characteristics	Yes	Yes	Yes	Yes
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: The data comprise detached house transactions from 2006 to 2012. The unit of observation is a transaction. Repeat sales transactions taking place within 18 months of one another are discarded. All regressions include an indicator for the post-LTT period, an indicator for the city of Toronto, community fixed effects, calendar month fixed effects, a rich set of time-varying property characteristics, as well as separate time trends for transactions inside and outside the City of Toronto. The distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. Distance LTT trend denotes the inclusion of an interaction term between the LTT and a dummy variable for properties between 2.5km and 5km away from the city border in columns (2)–(3) and the interaction between the LTT and the distance from the city border in column (4). Standard errors clustered by community are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

A.1.6 Robustness checks

Table A.7: *Estimated tax effects excluding the financial-crisis period*

	(1)	(2)	(3)	(4)	(5)	(6)
	Excluding 2008:4–2009:12		Excluding 2008:1–2009:12		Excluding 2007:10–2009:12	
log (#*Leases*/#*Sales*)	0.508**	0.590**	0.492**	0.581**	0.489**	0.568*
	(0.238)	(0.291)	(0.238)	(0.291)	(0.238)	(0.291)
Observations	1,827	1,221	1,774	1,186	1,672	1,114
log (#*BTO sales*)	−0.404***	−0.258*	−0.405***	−0.259*	−0.404***	−0.257*
	(0.121)	(0.155)	(0.121)	(0.155)	(0.121)	(0.155)
Observations	4,484	2,685	4,319	2,584	4,062	2,432
log (#*BTR sales*)	0.107**	0.129**	0.106**	0.129**	0.100*	0.128**
	(0.0509)	(0.0626)	(0.0508)	(0.0625)	(0.0516)	(0.0637)
Observations	751	486	727	467	675	432
Event of moving	−0.204***	−0.182*	−0.190***	−0.196**	−0.137**	−0.157
	(0.0542)	(0.0954)	(0.0548)	(0.0963)	(0.0559)	(0.0978)
log (*Original price*)	−0.0498	−0.0785	−0.0401	−0.0621	−0.0284	−0.0589
	(0.0447)	(0.0511)	(0.0453)	(0.0523)	(0.0460)	(0.0537)
log φ	0.397***	0.391***	0.398***	0.392***	0.399***	0.392***
	(0.00888)	(0.0113)	(0.00910)	(0.0116)	(0.00936)	(0.0119)
Observations	2,025,845	1,182,656	1,921,838	1,122,344	1,820,454	1,063,626
log (*Sales price*)	−0.0400**	−0.0666***	−0.0379**	−0.0562**	−0.0404**	−0.0561**
	(0.0153)	(0.0188)	(0.0139)	(0.0173)	(0.0141)	(0.0174)
Observations	32,822	21,108	30,226	19,404	28,474	18,273
log (*Time-on-market*)	0.189***	0.270***	0.241***	0.234**	0.176**	0.189**
	(0.0549)	(0.0689)	(0.0618)	(0.0826)	(0.0740)	(0.0924)
Observations	20,873	13,652	18,894	12,341	17,810	11,664
Sample	Border	Border	Border	Border	Border	Border
Distance threshold	5km	5km	5km	5km	5km	5km
Donut hole		2km		2km		2km
Months removed	21	21	24	24	27	27

Notes: The table shows the results of the robustness checks for the crisis period removing 21, 24, or 27 months. The sample comprises transactions from 2006 to 2012. The first three rows present the estimated coefficients for the leases-to-sales ratio, BTO sales, and BTR sales. The market segment regressions in the first three rows use detached house transactions. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. For the moving hazard regressions, a unit of observation is a homeowner whose property is listed on MLS. For the transaction level regressions, a unit of observation is a property transaction. Repeat sales transactions taking place within 12 months of one another are discarded for the moving hazard model and 18 months for the transaction level regressions. All regressions include a post-LTT dummy, controls for time-varying house characteristics, community, year, month, property type fixed effects, and their interactions. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008 whenever this applies. The border row indicates if a border sample was used, and the distance threshold indicates the distance radius used in the sample. The donut hole row indicates the number of kilometres from the city border excluded from the sample. Standard errors are in parentheses and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.8: *Placebo test based on pseudo-borders within Toronto*

	Pseudo-border at 3km				Pseudo-border at 5km			
log (#*Leases*/#*Sales*)	0.161	0.0967	−0.0341	0.155	−0.0984	0.0325	0.157	0.0376
	(0.141)	(0.120)	(0.228)	(0.115)	(0.162)	(0.101)	(0.143)	(0.0976)
Observations	1,200	1,746	546	2,081	1,041	2,371	1,330	2,544
log (*Price*/*Rent*)	0.0200	0.00898	−0.0235	0.0128	0.0258	0.0729**	0.0987**	0.0654**
	(0.0337)	(0.0306)	(0.0778)	(0.0305)	(0.0669)	(0.0340)	(0.0391)	(0.0329)
Observations	1,832	2,411	565	2,649	1,303	2,958	1,625	3,234
log (#*BTO sales*)	0.0606	0.0205	0.0655	0.00614	−0.0693	−0.0527	−0.0462	−0.0389
	(0.0734)	(0.0611)	(0.0760)	(0.0576)	(0.0847)	(0.0534)	(0.0607)	(0.0509)
Observations	2,605	3,845	2,605	4,481	2,171	5,032	3,801	5,572
log (#*BTR sales*)	−0.0160	−0.157	−0.109	−0.159	−0.0534	−0.0895	−0.0349	−0.0793
	(0.102)	(0.112)	(0.137)	(0.105)	(0.108)	(0.0875)	(0.103)	(0.0843)
Observations	523	780	494	883	418	1,000	744	1,083
	Pseudo-border at 4km				**Pseudo-border at 6km**			
log (#*Leases*/#*Sales*)	−0.0374	0.0442	0.155	0.00374	0.228	0.0836	−0.0195	0.0434
	(0.146)	(0.109)	(0.171)	(0.0994)	(0.155)	(0.107)	(0.149)	(0.0987)
Observations	1,233	2,081	848	2,520	1,032	2,134	1,102	2,591
log (*Price*/*Rent*)	−0.000958	0.00544	0.0401	0.0372	0.0588	0.0833**	0.106**	0.0854**
	(0.0405)	(0.0346)	(0.0781)	(0.0281)	(0.0861)	(0.0382)	(0.0402)	(0.0323)
Observations	1,658	2,649	965	3,216	1,047	2,557	1,480	3,244
log (#*BTO sales*)	−0.0913	−0.0292	−0.0305	−0.0377	−0.0227	−0.0552	−0.0885	−0.00809
	(0.0759)	(0.0564)	(0.0667)	(0.0516)	(0.0829)	(0.0568)	(0.0626)	(0.0499)
Observations	2,471	4,481	3,190	5,392	2,139	4,470	3,590	5,846
log (#*BTR sales*)	−0.0468	−0.0977	−0.189	−0.0596	−0.0469	0.0298	−0.0462	0.0273
	(0.0961)	(0.0948)	(0.118)	(0.0876)	(0.113)	(0.0907)	(0.103)	(0.0785)
Observations	527	883	598	1,075	395	866	706	1,074
Distance threshold	2km	4km	4km	5km	2km	4km	4km	5km
City indicators ±3 m.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes		Yes	Yes	Yes
Donut hole			1km				1km	

Notes: Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. The sample comprises detached house transactions from 2006 to 2012. There are four panels, each showing the results of a placebo test with different distances from the Toronto border. All regressions include a post-LTT dummy, and city, community, year, and calendar month fixed effects. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008 whenever this applies. Distance threshold indicates the radius used in the sample. Robust standard errors are in parentheses, and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.9: *Robustness checks on threshold to distinguish BTO and BTR transactions*

Dependent variable	(1)	(2)	(3)	(4)
	6-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.115**	−0.135**	−0.117**	−0.158***
	(0.0577)	(0.0433)	(0.0546)	(0.0322)
log (#*BTR sales*)	0.194**	0.200***	0.206***	0.0956**
	(0.0739)	(0.0518)	(0.0612)	(0.0477)
	12-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.0835	−0.0972**	−0.0799	−0.128***
	(0.0580)	(0.0438)	(0.0554)	(0.0326)
log (#*BTR sales*)	0.167**	0.144**	0.148**	0.0478
	(0.0637)	(0.0472)	(0.0588)	(0.0431)
	24-month cutoff to distinguish BTO and BTR			
log (#*BTO sales*)	−0.110*	−0.116**	−0.0917	−0.125***
	(0.0592)	(0.0447)	(0.0566)	(0.0333)
log (#*BTR sales*)	0.139**	0.113**	0.114**	0.0298
	(0.0602)	(0.0442)	(0.0526)	(0.0411)
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: See the footnote to Table 1. Standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.10: *Effect of the transaction tax by property type*

Dependent variable	(1)	(2)	(3)	(4)	(5)
$\log(\#Leases/\#Sales)$	0.147**	0.105**	0.101**	0.0719*	0.0730*
	(0.0503)	(0.0478)	(0.0436)	(0.0408)	(0.0391)
Observations	10,233	14,713	17,216	19,817	21,719
$\log(Price/Rent)$	−0.0695**	−0.0451**	−0.0440**	−0.0386**	−0.0367**
	(0.0338)	(0.0194)	(0.0188)	(0.0177)	(0.0176)
Observations	3,745	7,660	8,383	9,313	9,876
$\log(\#BTO\ sales)$	−0.0722**	−0.0467*	−0.0507**	−0.0529**	−0.0478**
	(0.0256)	(0.0276)	(0.0233)	(0.0208)	(0.0191)
Observations	26,639	27,304	36,753	45,585	52,598
$\log(\#BTR\ sales)$	0.0822**	0.0439*	0.0458*	0.0415*	0.0376*
	(0.0389)	(0.0258)	(0.0242)	(0.0230)	(0.0219)
Observations	3,550	5,376	6,069	6,945	7,475
Distance threshold	20km	20km	20km	20km	20km
Detached	Yes	Yes	Yes	Yes	Yes
Semi-detached	Yes	No	Yes	Yes	Yes
Condo apartments	No	Yes	Yes	Yes	Yes
Condo townhouse	No	No	No	Yes	Yes
Row/attached/townhouse	No	No	No	No	Yes

Notes: The table shows estimates of the tax effects for different types of properties. The sample comprises transactions from 2006 to 2012. Given the limited sample size for condos and apartments, the border sample radius is extended from 5km to 20km and different property segments are combined flexibly. In addition to the extensive controls in Table 1, *property type* × *neighbourhood* and *property type* × *year* × *month* fixed effects are included to control for differences in housing stock composition and variations in how different property type segments evolve over time. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. All regressions include a post-LTT dummy, and city, community, year, and calendar-month fixed effects, as well as the interaction between these fixed effects and property types. Six dummy variables are included for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends and distance LTT trends are included. Property type rows indicate whether the property type was included in the regression. Border indicates if a border sample was used, and the distance threshold indicates the distance radius used in the sample. Standard errors are clustered at the neighbourhood, year, and property type levels using three-way clustering. *, **, and *** indicate significance at the 10%, 5%, and 1% levels.

Table A.11: *Robustness checks on the moving hazard rate*

	(1)	(2)	(3)	(4)
		Sample period 2006–2010		
Event of moving	−0.156**	−0.218***	−0.243**	−0.286***
	(0.0736)	(0.0636)	(0.111)	(0.0520)
Observations	1,012,969	1,690,705	982,110	3,395,033
		Sample period 2016–2018		
Event of moving	−0.125**	−0.179***	−0.213**	−0.259***
	(0.0597)	(0.0476)	(0.0722)	(0.0357)
Observations	4,327,556	7,306,558	4,296,732	14,969,191
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes
Distance LTT trends		Yes	Yes	Yes
Donut hole			2km	

Notes: The table shows the estimated effect of the LTT on the moving hazard rate. The sample comprises detached house transactions in the GTA. The first panel shows the estimates for the period 2006–2010 and the second panel shows the estimates for the period 2016–2018. Each cell of the table represents a separate regression of an outcome on the *LTT* interaction dummy. See the footnote to Table 2 for more details. Standard errors clustered by the community are in parentheses, and *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A.12: *Robustness checks on sales prices at the market-segment level*

Dependent variable	(1)	(2)	(3)	(4)
		Sample period 2006–2010		
$\log(Price)$	−0.0186**	−0.0172***	−0.0122**	−0.0125**
	(0.00610)	(0.00488)	(0.00613)	(0.00442)
Observations	7,515	12,939	7,949	37,698
		Sample period 2006–2018		
$\log(Price)$	−0.0200***	−0.0174***	−0.0125**	−0.0155***
	(0.00525)	(0.00418)	(0.00524)	(0.00378)
Observations	11,169	19,227	11,802	55,895
Sample	Border	Border	Border	All
Distance threshold	3km	5km	5km	All
City indicators ±3 m.	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	
Donut hole			2km	

Notes: The estimation sample covers four types of properties: detached houses, townhouses, condominiums, and apartments. A unit of observation is a market segment defined by *community* × *property type* × *year* × *month*. The dependent variable is the average sales price within each market segment. Each cell of the table represents a separate regression on the *LTT* interaction dummy. All regressions include a dummy for the post-LTT period, and *city* × *property type*, *year* × *property type*, *month* × *property type*, and *community* × *property type* fixed effects. See the footnote to Table 1. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Figure A.4: *Household stocks and flows in the model*

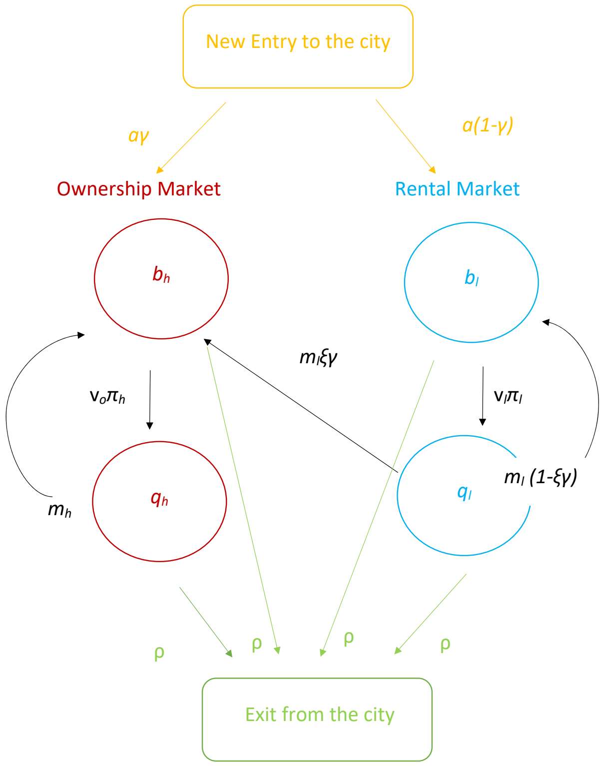
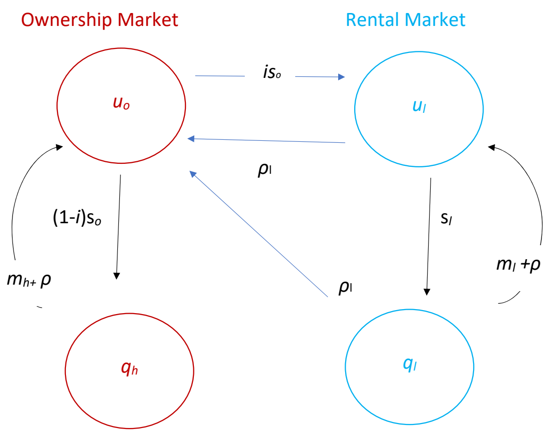


Figure A.5: *Property stocks and flows in the model*



A.2 Deriving the equations of the model

This section shows how to derive equations exactly characterizing the non-household-specific aggregate variables of the model with a finite-dimensional state space. In particular, value functions in match quality such as $L(\varepsilon)$, $W(\varepsilon)$, and $H(\varepsilon)$ are replaced by a finite number of variables that describe the aggregate outcomes in the model, and similarly for the distribution functions of the endogenous distribution of match quality ε. The solution for these variables is obtained from a finite number of equations.

A.2.1 The value functions and thresholds for owner-occupiers and home-buyers

The value function $H(\varepsilon)$ from (21) is increasing in ε. Assuming $\delta_h y_h < x_h$ for all t, by taking ε in a neighbourhood above y_h or any value below, the Bellman equation (21) reduces to the following as $H(\delta_h \varepsilon) < B_h + U_o$:

$$rH(\varepsilon) = \varepsilon + g - D + \alpha_h(B_h + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon).$$

This simplifies to

$$(r+\rho+\alpha_h)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon + g - D + \alpha_h B_h + (\rho+\alpha_h)U_o, \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range described above:

$$(r+\rho+\alpha_h)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1.$$

For a given ε, this specifies a first-order differential equation in time t for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+\alpha_h)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows that the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+\alpha_h}, \quad \text{with } \dot{H}(\varepsilon) = \dot{\underline{H}}, \tag{A.2}$$

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_h and all values below. Substituting into (A.1) shows that $\underline{H}$ satisfies

$$(r+\rho+\alpha_h)\underline{H} - \dot{\underline{H}} = \alpha_h B_h + (\rho+\alpha_h)U_o + g - D. \tag{A.3}$$

Since $x_h < y_h$, equation (22) together with (A.2) implies that

$$x_h = (r+\rho+\alpha_h)(B_h + U_o - \underline{H}). \tag{A.4}$$

The surplus in (28) and the definition of the transaction threshold (29) imply y_h satisfies

$$H(y_h) = H(x_h) + C_h + (1+\tau_h)C_o + \tau_h U_o, \tag{A.5}$$

and combining (A.2) with (A.5) yields

$$y_h = x_h + (r+\rho+\alpha_h)(C_h + (1+\tau_h)C_o + \tau_h U_o). \tag{A.6}$$

The joint surplus $\Sigma_h(\varepsilon)$ is given in (28) and $1-\omega_h^*$ is the share received by buyers. Equation (30) defines the expected surplus Σ_h, thus the Bellman equation for a buyer (27) can be expressed as

$$(r+\rho)B_h - \dot{B}_h = (1-\omega_h^*)v_o\Sigma_h + g - F_h. \tag{A.7}$$

The joint surplus from trade with an investor is given in (38) and ω_i^* is sellers' share. Together with the surplus from trade with a home-buyer, the Bellman equation of a seller (25) is

$$rU_o - \dot{U}_o = \theta_o v_o\left(\omega_h^*(1-\psi)\Sigma_h + \omega_i^*\psi\Sigma_i\right) - D. \tag{A.8}$$

Using equations (28), (29), and (40), the expected surplus Σ_h in (30) can be written as

$$\Sigma_h = \int_{y_h}^{\infty} \lambda_h \zeta_h^{\lambda_h} \varepsilon^{-(\lambda_h+1)}\Sigma_h(\varepsilon)\mathrm{d}\varepsilon = \int_{y_h}^{\infty} \frac{\lambda_h \zeta_h^{\lambda_h} \varepsilon^{-(\lambda_h+1)}(H(\varepsilon) - H(y_h))}{1+\tau_h\omega_h^*}\mathrm{d}\varepsilon. \tag{A.9}$$

Defining $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε and noting the link with Σ_h:

$$\bar{H}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)} (H(w) - H(\varepsilon))\mathrm{d}w, \quad \text{where } \Sigma_h = \frac{\zeta_h^{\lambda_h} y_h^{-\lambda_h} \bar{H}(y_h)}{1 + \tau_h \omega_h^*}. \tag{A.10}$$

Now restrict attention to ε such that $\delta_h \varepsilon < x_h$, so (21) implies $rH(\varepsilon) = \varepsilon + g - D + \alpha_h(B_h + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_h y_h < x_h$, this limits ε to a neighbourhood above y_h and all values below. Using equation (22):

$$r(H(w) - H(\varepsilon)) = (w - \varepsilon) + \alpha_h (\max\{H(\delta_h w), H(x_h)\} - H(w)) - \alpha_h (H(x_h) - H(\varepsilon)) \\ - \rho(H(w) - H(\varepsilon)) + (\dot{H}(w) - \dot{H}(\varepsilon)),$$

which holds for any $w \geq \varepsilon$. This simplifies to

$$(r + \rho + \alpha_h)(H(w) - H(\varepsilon)) - (\dot{H}(w) - \dot{H}(\varepsilon)) = (w - \varepsilon) + \alpha_h \max\{H(\delta_h w) - H(x_h), 0\},$$

and multiplying both sides by $\lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)}$, integrating over w, and using (A.10):

$$(r + \rho + \alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)} \left((w - \varepsilon) + \alpha_h \max\{H(\delta_h w) - H(x_h), 0\}\right) \mathrm{d}w, \tag{A.11}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon) = \int_{w=\varepsilon}^{\infty} \lambda_h \varepsilon^{\lambda_h} w^{-(\lambda_h+1)} (\dot{H}(w) - \dot{H}(\varepsilon))\mathrm{d}w.$$

In (A.11), the term in $(w - \varepsilon)$ integrates to $\varepsilon/(\lambda_h - 1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $w < x_h/\delta_h$ because $H(\delta_h w)$ is increasing in w. Hence, equation (A.11) becomes

$$(r + \rho + \alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_h - 1} + \alpha_h \varepsilon^{\lambda_h} \int_{w=x_h/\delta_h}^{\infty} \lambda_h w^{-(\lambda_h+1)} (H(\delta_h w) - H(x_h))\mathrm{d}w,$$

and with the change of variable $w' = \delta_h w$ in the second integral, this can be written as

$$(r + \rho + \alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_h - 1} + \alpha_h \delta_h^{\lambda_h} \varepsilon^{\lambda_h} \int_{w'=x_h}^{\infty} \lambda_h w'^{-(\lambda_h+1)} (H(w') - H(x_h))\mathrm{d}w'. \tag{A.12}$$

Make the following definition of a new variable X_h:

$$X_h(t) = \left((\lambda_h - 1)\left(r + \rho + \alpha_h(1 - \delta_h^{\lambda_p})\right) \int_{T=t}^{\infty} (r + \rho + \alpha_h) e^{-(r+\rho+\alpha_h)(T-t)} \left(\\ \int_{\varepsilon=x_h(T)}^{\infty} \lambda_h \varepsilon^{-(\lambda_h+1)} (H(\varepsilon, T) - H(x_h(T), T))\mathrm{d}\varepsilon \right) \mathrm{d}T \right)^{\frac{1}{1-\lambda_h}}. \tag{A.13}$$

By differentiating with respect to time t, this variable must satisfy the differential equation

$$(r + \rho + \alpha_h) X_h^{1-\lambda_h} - (1 - \lambda_h)\dot{X}_h X_h^{-\lambda_h} = (\lambda_h - 1)(r + \rho + \alpha_h)\left(r + \rho + \alpha_h(1 - \delta_h^{\lambda_h})\right) x_h^{-\lambda_h} \bar{H}(x_h) \\ = (\lambda_h - 1)(r + \rho + \alpha_h)\left(r + \rho + \alpha_h(1 - \delta_h^{\lambda_h})\right) \int_{\varepsilon=x_h}^{\infty} \lambda_h \varepsilon^{-(\lambda_h+1)} (H(\varepsilon) - H(x_h))\mathrm{d}\varepsilon, \tag{A.14}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r + \rho + \alpha_h)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{1}{\lambda_h - 1}\left(\varepsilon + \frac{\alpha_h \delta_h^{\lambda_h} \varepsilon^{\lambda_h} \left((r + \rho + \alpha_h) X_h^{1-\lambda_h} - (1 - \lambda_h)\dot{X}_h X_h^{-\lambda_h}\right)}{(r + \rho + \alpha_h)\left(r + \rho + \alpha_h(1 - \delta_h^{\lambda_h})\right)} \right),$$

and by collecting terms this can be written as

$$(r+\rho+\alpha_h)\left(\bar{H}(\varepsilon)-\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)}X_h^{1-\lambda_h}\right)$$
$$-\left(\dot{\bar{H}}(\varepsilon)-\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)}(1-\lambda_h)\dot{X}_hX_h^{-\lambda_h}\right)=\frac{\varepsilon}{\lambda_h-1}\,.$$

Noting that $\mathrm{d}X_h(t)^{1-\lambda_h}/\mathrm{d}t=(1-\lambda_h)\dot{X}_hX_h^{-\lambda_h}$ and observing the right-hand side of the equation above is time invariant and none of the variables is predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon)-\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}X_h^{1-\lambda_h}/((\lambda_h-1)(r+\rho+\alpha_h)(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})))$ that is time invariant and equal to $\varepsilon/((\lambda_h-1)(r+\rho+\alpha_h))$. This demonstrates that for any given ε in a neighbourhood above y_h or any value below it, the function $\bar{H}(\varepsilon)$ is given by

$$\bar{H}(\varepsilon)=\frac{1}{(\lambda_h-1)(r+\rho+\alpha_h)}\left(\varepsilon+\frac{\alpha_h\delta_h^{\lambda_h}\varepsilon^{\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}X_h^{1-\lambda_h}\right). \tag{A.15}$$

Evaluating (A.15) at $\varepsilon=x_h$ and multiplying by $(\lambda_h-1)(r+\rho+\alpha_h)(r+\rho+\alpha_h(1-\delta_h^{\lambda_h}))x_h^{-\lambda_h}$:

$$(\lambda_h-1)(r+\rho+\alpha_h)\left(r+\rho+\alpha_h(1-\delta_h^{\lambda_h})\right)x_h^{-\lambda_h}\bar{H}(x_h)=\left(r+\rho+a_h(1-\delta_h^{\lambda_h})\right)x_h^{1-\lambda_h}+\alpha_h\delta_h^{\lambda_h}X_h^{1-\lambda_h}\,,$$

and then by substituting into (A.14) shows that X_h is related to the moving threshold x_h as follows:

$$\frac{\dot{X}_h}{X_h}=\left(\frac{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}{\lambda_h-1}\right)\left(\left(\frac{x_h}{X_h}\right)^{1-\lambda_h}-1\right). \tag{A.16}$$

Finally, evaluating (A.15) at $\varepsilon=y_h$ and substituting into (A.10) yields an equation for the joint surplus:

$$\Sigma_h=\frac{\zeta_h^{\lambda_h}}{(1+\tau_h\omega_h^*)(\lambda_h-1)(r+\rho+\alpha_h)}\left(y_h^{1-\lambda_h}+\frac{\alpha_h\delta_h^{\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}X_h^{1-\lambda_h}\right). \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_h, x_h, X_h, Σ_h, $\underline{H}$, B_h, and U_o, taking as given Σ_k, v_o, ψ, and g.

A.2.2 The moving rate of owner-occupiers

The flow of owner-occupiers who move within the city is M_h and the moving rate is $m_h=M_h/q_h$. The group of existing owner-occupiers q_h is made up of matches that formed at various points in the past and that have survived to the present. Given a sufficiently large inconvenience cost of moving in the absence of a shock, moving occurs only if owner-occupiers receive an idiosyncratic shock with arrival rate α_h independent of history. A measure α_hq_h of households might therefore decide to move.

All matches began as a viewing with some initial match quality ε. Using (23), the flow of viewings v_h done by home-buyers in the ownership market at a point in time is

$$v_h=v_ob_h=(1-\psi)\theta_ov_ou_o\,. \tag{A.18}$$

Initial match quality drawn in viewings is from a Pareto(ζ_h,λ_h) distribution (see 40). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving owner-occupiers where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon},\lambda_h)$. Among this group, denote current match quality as a multiple Ξ of original match quality ε, where Ξ is equal to δ_h raised to the power of the number of past shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon'=\delta_h\Xi\varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon'<x_h$, so only those with initial match quality $\varepsilon\geq$

$x_h/(\delta_h \Xi)$ remain. Since $\delta_h < 1$ and $\delta_h y_h < x_h$, there is a range of variation in thresholds y_h and x_h that ensures $x_h/(\delta_h \Xi) > \underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_h}(x_h/(\delta_h \Xi))^{-\lambda_h} = x_h^{-\lambda_h}\delta_h^{\lambda_h}\Xi^{\lambda_h}\underline{\varepsilon}^{\lambda_h}$. Since that surviving group is a fraction $\zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_h^{-\lambda_h}\delta_h^{\lambda_h}\Xi^{\lambda_h}\underline{\varepsilon}^{\lambda_h} \times \zeta_h^{\lambda_h}\underline{\varepsilon}^{-\lambda_h} = (\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}) \times \Xi^{\lambda_h}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Ξ of past idiosyncratic shocks. Let Θ_h be the integral of Ξ^{λ_h} over the measure of current and past viewings done by home-buyers who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}$ of Ξ^{λ_h}, the measure of those choosing not to move after a new shock is $\alpha_h\zeta_h^{\lambda_h}x_h^{-\lambda_h}\delta_h^{\lambda_h}\Theta_h$. Therefore, the size of the group of movers is

$$M_h = \alpha_h q_h - \alpha_h \zeta_h^{\lambda_h} x_h^{-\lambda_h} \delta_h^{\lambda_h} \Theta_h \,. \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction α_h of the group used to define Θ_h have Ξ^{λ_h} reduced to $\delta_h^{\lambda_h}\Xi^{\lambda_h}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Xi^{\lambda_h} = 1$ with measure ν_h from (A.18). The equation for Θ_h is therefore

$$\dot{\Theta}_h = \nu_h + \alpha_h(\delta_h^{\lambda_h}\Theta_h - \Theta_h) - \rho\Theta_h \,. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings ν_h:

$$\bar{\nu}_h(t) = \int_{T\to-\infty}^{t} (\rho + \alpha_h(1-\delta_h^{\lambda_h}))e^{-(\rho+\alpha_h(1-\delta_h^{\lambda_h}))(t-T)}\nu_h(T)\mathrm{d}T \,, \tag{A.21}$$

and note that it satisfies the differential equation

$$\dot{\bar{\nu}}_h + (\rho + \alpha_h(1-\delta_h^{\lambda_h}))\bar{\nu}_h = (\rho + \alpha_h(1-\delta_h^{\lambda_h}))\nu_h \,. \tag{A.22}$$

A comparison of (A.20) and (A.22) shows that $\Theta_h = \bar{\nu}_h/(\rho + \alpha_h(1-\delta_h^{\lambda_h}))$, and substituting this into (A.19) yields an equation for the moving rate $m_h = M_h/q_h$:

$$m_h = \alpha_h - \frac{\alpha_h \zeta_h^{\lambda_h}\delta_h^{\lambda_h}x_h^{-\lambda_h}\bar{\nu}_h}{(\rho + \alpha_h(1-\delta_h^{\lambda_h}))q_h} \,. \tag{A.23}$$

Using the definition of $\bar{\nu}_h(t)$ in (A.21) and (A.18), this confirms equation (41) for the moving rate m_h.

A.2.3 The transaction threshold and value functions in the rental market

By adding the Bellman equations (9) and (10) for the tenant and landlord value functions:

$$\begin{aligned} r(L(\varepsilon)+W(\varepsilon)) = \varepsilon + g - D - D_l + (\alpha_l + \rho)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) \\ + m_l(\xi\kappa(B_h - \bar{K}) + (1-\xi\kappa)B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon) \,. \end{aligned}$$

Letting $J(\varepsilon) = L(\varepsilon) + W(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting that $B_h - B_l = Z$ from (3) and $m_l = \alpha_l + \rho_l$ from (8):

$$(r + \rho + m_l)J(\varepsilon) = \varepsilon + g - D - D_l + (\rho + \alpha_l)U_l + \rho_l U_o + m_l B_l + \xi m_l \kappa(Z - \bar{K}) + \dot{J}(\varepsilon) \,. \tag{A.24}$$

Differentiating with respect to ε leads to the differential equation

$$(r + \rho + m_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon) \,,$$

and this equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r + \rho + m_l} \,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r+\rho+m_l}\,, \tag{A.25}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.24) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation

$$(r+\rho+m_l)\underline{J} = m_l B_l + (\rho+\alpha_l)U_l + \rho_l U_o + g - D - D_l + \xi m_l \kappa(Z-\bar{K}) + \dot{\underline{J}}\,. \tag{A.26}$$

The joint rental surplus from (14) prior to a tenant moving in is linked to $J(\varepsilon)$ by

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l\,, \tag{A.27}$$

and together with (A.25), the definition of the rental transaction threshold y_l in (15) implies

$$y_l = (r+\rho+m_l)(B_l + U_l - \underline{J} + C_l + C_w)\,. \tag{A.28}$$

Using (15), (A.25), and (A.27), it follows that the surplus $\Sigma_l(\varepsilon)$ is

$$\Sigma_l(\varepsilon) = \frac{\varepsilon - y_l}{r+\rho+m_l}\,, \quad \text{and } \Sigma_l = \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon) = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{(\lambda_l - 1)(r+\rho+m_l)}\,, \tag{A.29}$$

where the second equation uses the Pareto distribution in (40) to derive the expected rental surplus Σ_l. Landlords' share of the joint surplus is ω_l, so $\Sigma_{ll}(\varepsilon) = L(\varepsilon) + A(\varepsilon) - C_l - U_l = \omega_l \Sigma_l(\varepsilon)$. Together with (A.29), equation (12) for U_l becomes

$$(r+\rho_l)U_l - \dot{U}_l = \omega_l \theta_l v_l \Sigma_l - D + \rho_l U_o\,. \tag{A.30}$$

Similarly, with $\Sigma_{lw}(\varepsilon) = W(\varepsilon) - A(\varepsilon) - C_w - B_l = (1-\omega_l)\Sigma_l$, equation (13) for B_l becomes

$$(r+\rho)B_l - \dot{B}_l = (1-\omega_l)v_l \Sigma_l + g - F_l\,. \tag{A.31}$$

In summary, equations (A.26), (A.28), (A.29), (A.30), and (A.31) determine y_l, Σ_l, $\underline{J}$, B_l, and U_l, taking as given Z, $\bar{K}$, κ, and g.

A.2.4 Rents

With $m_l = \alpha_l + \rho_l$ from (8), the Bellman equation (10) can be written as follows:

$$(r+\rho+m_l)(L(\varepsilon) - U_l) = R(\varepsilon) - D - D_l - (r+\rho_l)U_l + \rho_l U_o + \dot{L}(\varepsilon)\,,$$

and substituting from (A.30) implies that the rent $R(\varepsilon)$ for a property with match quality ε is

$$R(\varepsilon) = D_l + \omega_l \theta_l v_l \Sigma_l + (r+\rho+m_l)(L(\varepsilon) - U_l) - (\dot{L}(\varepsilon) - \dot{U}_l)\,.$$

Using the landlord's surplus $\Sigma_{wl}(\varepsilon) = L(\varepsilon) - U_l$ after a tenant has moved in, and its derivative with respect to time $\dot{\Sigma}_{wl}(\varepsilon) = \dot{L}(\varepsilon) - \dot{U}_l$, the surplus division $\Sigma_{wl}(\varepsilon) = \omega_l \Sigma_w(\varepsilon)$ implies

$$R(\varepsilon) = D_l + \omega_l \theta_l v_l \Sigma_l + \omega_l \left((r+\rho+m_l)\Sigma_w(\varepsilon) - \dot{\Sigma}_w(\varepsilon)\right)\,.$$

Noting that $\Sigma_l(\varepsilon) = \Sigma_w(\varepsilon) - (C_l + C_w)$ from (7) and (14) and substituting for $\Sigma_w(\varepsilon)$ in the above:

$$R(\varepsilon) = D_l + \omega_l(r+\rho+m_l)(C_l + C_w) + \omega_l \theta_l v_l \Sigma_l + \omega_l \left((r+\rho+m_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon)\right)\,.$$

Using the first equation in (A.29), it follows that $\dot{\Sigma}_l(\varepsilon) = -\dot{y}_l/(r+\rho+m_l)$ for all ε, and hence:

$$R(\varepsilon) = D_l + \omega_l(r+\rho+m_l)(C_l + C_w) + \omega_l \theta_l v_l \Sigma_l + \omega_l(\varepsilon - y_l) + \frac{\omega_l}{r+\rho+m_l}\dot{y}_l\,. \tag{A.32}$$

With expected surplus Σ_l from (A.29), average new rents R from (16) are given by

$$R = D_l + \omega_l(r+\rho+m_l)(C_l + C_w) + \omega_l(r+\rho+m_l+\theta_l v_l \pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r+\rho+m_l}\dot{y}_l\,. \tag{A.33}$$

Since (A.32) implies $R'(\varepsilon) = \omega_l$ for all ε, rents are linear in match quality, so the average $\bar{R}$ of all rents on current tenancies is

$$\bar{R} = R + \omega_l \left(V_l - \frac{\lambda_l}{\lambda_l - 1} y_l \right), \tag{A.34}$$

where V_l is the average ε over all current tenancies (see 46), and $\lambda_l y_l/(\lambda_l - 1)$ is the average of ε over new tenancies using (40).

A.2.5 The relationship between market tightnesses across the two markets

Subtracting the total measures of properties in (1) from the total measure of households in (2), and using the definitions market tightnesses θ_l and θ_o from (11) and (23), and the fraction ψ of investors among all buyers from (24) leads to the following equation:

$$((1-\psi)\,\theta_o - 1)\,u_o + (\theta_l - 1)\,u_l = n - 1\,. \tag{A.35}$$

A.2.6 Average match quality and value functions averaged over surviving match quality

Let $\mathscr{E}_h$ denote the integral of ε over all current owner-occupiers. There is a flow S_h of new owner-occupier matches. Since the transaction threshold is y_h, the Pareto distribution (40) implies the average value of ε in these new matches is $\lambda_h y_h/(\lambda_h - 1)$, so these new matches add to $\mathscr{E}_h$ at rate $S_h \lambda_h y_h/(\lambda_h - 1)$ over time.

Owner-occupier matches are destroyed (sending the contribution to $\mathscr{E}_h$ to zero) if households exit the city or match-quality shocks arrive and households choose to move. Households exit the city at rate ρ, reducing $\mathscr{E}_h$ by $\rho\mathscr{E}_h$. Match-quality shocks arrive randomly at rate α_h for the measure q_h of owner-occupiers, leading to a flow M_h of movers out of the group $\alpha_h q_h$ receiving a shock, which reduces the contribution to $\mathscr{E}_h$ of those M_h to zero. For the group of size $\alpha_h q_h - M_h$ that receives a shock but does not move, the conditional distribution of surviving match quality ε is truncated at x_h, which is a Pareto distribution with shape parameter λ_h across all cohorts within that group, which has mean $\lambda_h x_h/(\lambda_h - 1)$. Putting together all these effects on $\mathscr{E}_h$, the following differential equation must hold:

$$\dot{\mathscr{E}}_h = S_h \frac{\lambda_h y_h}{\lambda_h - 1} + \left(M_h \times 0 + (\alpha_h q_h - M_h) \times \frac{\lambda_h x_h}{\lambda_h - 1} - \alpha_h \mathscr{E}_h \right) - \rho \mathscr{E}_h\,.$$

Average match quality among owner-occupiers is $V_h = \mathscr{E}_h/q_h$, thus $\dot{V}_h = \dot{\mathscr{E}}_h/q_h - (\dot{q}_h/q_h)V_h = \dot{\mathscr{E}}_h/q_h - ((S_h/q_h) - (m_h + \rho))V_h$, where the second equation uses the differential equation for q_h in (32). Together with the equation for $\dot{\mathscr{E}}_h$ above and the definition of the moving rate $m_h = M_h/q_h$, average match quality V_h must satisfy the differential equation in (45).

Let $\mathscr{E}_l$ denote the equivalent summation of surviving match quality for tenants. There is a flow $s_l u_l$ of new rental matches. Since the transaction threshold is y_l, these new matches add to $\mathscr{E}_l$ at rate $s_l u_l \lambda_l y_l/(\lambda_l - 1)$. Matches are destroyed if households exit the city (rate ρ), if landlords must sell up (rate ρ_l), or if match quality falls to zero owing to an idiosyncratic shock (rate α_l). The differential equation for $\mathscr{E}_l$ is thus $\dot{\mathscr{E}}_l = s_l u_l(\lambda_l y_l/(\lambda_l - 1)) - (\alpha_l + \rho_l + \rho)\mathscr{E}_l$. Average match quality for tenants is $V_l = \mathscr{E}_l/q_l$, hence $\dot{V}_l = (\dot{\mathscr{E}}_l/q_l) - (\dot{q}_l/q_l)V_l$, and by substituting $\dot{q}_l/q_l = (s_l u_l/q_l) - (m_l + \rho)$ from (18), the differential equation for V_l is (46), which uses $m_l = \alpha_l + \rho_l$ from (8).

Let $\Gamma_\varepsilon(\varepsilon)$ denote the distribution function of current match quality ε for owner-occupiers. The average value of $H(\varepsilon)$ across all q_h matches and the integral of these values are denoted by $\bar{H}$ and Q:

$$\bar{H} = \int_\varepsilon H(\varepsilon)\mathrm{d}\Gamma_\varepsilon(\varepsilon), \quad \text{and} \quad Q = q_h \bar{H} = \int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon, \quad \text{where} \quad \varsigma(\varepsilon) = q_h \Gamma'_\varepsilon(\varepsilon)\,. \tag{A.36}$$

The function $\varsigma(\varepsilon)$ is the density function $\Gamma'_\varepsilon(\varepsilon)$ of the distribution of surviving match quality ε multiplied by q_h. Differentiating Q with respect to time implies $\dot{Q} = \int_\varepsilon \left(\dot{H}(\varepsilon)\varsigma(\varepsilon) + H(\varepsilon)\dot{\varsigma}(\varepsilon)\right)\mathrm{d}\varepsilon$ and hence

$$rQ - \dot{Q} = \int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon - \int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon\,. \tag{A.37}$$

Shocks scaling down match quality ε to $\delta_h\varepsilon$ occur with arrival rate α_h, which triggers moving if match quality falls below x_h. There is also exogenous exit from the city at rate ρ. New matches form at rate S_h and begin with ε having distribution function $\Gamma_h(\varepsilon)/\pi_h$ for $\varepsilon \geq y_h$, where $\pi_h = 1-\Gamma_h(y_h)$. The dynamics of $\varsigma(\varepsilon) = q_h\Gamma_\varepsilon'(\varepsilon)$ describing the distribution of ε across all surviving owner-occupier matches are thus:

$$\dot{\varsigma}(\varepsilon) = \begin{cases} -(\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } \varepsilon < x_h \\ \alpha_h\delta_h^{-1}\varsigma(\delta_h^{-1}\varepsilon) - (\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } x_h \leq \varepsilon < y_h \\ (S_h/\pi_h)\Gamma_h'(\varepsilon) + \alpha_h\delta_h^{-1}\varsigma(\delta_h^{-1}\varepsilon) - (\alpha_h+\rho)\varsigma(\varepsilon) & \text{if } y_h \leq \varepsilon \end{cases}.$$

It follows that

$$\begin{aligned}\int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon &= \frac{S_h}{\pi_h}\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon) + \frac{\alpha_h}{\delta_h}\int_{\varepsilon=x_h} H(\varepsilon)\varsigma\left(\frac{\varepsilon}{\delta_h}\right)\mathrm{d}\varepsilon - (\alpha_h+\rho)q_h\bar{H} \\ &= v_o b_h \int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon) + \alpha_h\int_{\varepsilon=x_h/\delta_h} H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon - (\alpha_h+\rho)q_h\bar{H}, \end{aligned} \tag{A.38}$$

which uses $S_h = v_o\pi_h b_h$ from (32) and a change of variable $\varepsilon' = \varepsilon/\delta_h$ in the second term on the second line. Using the Bellman equation (21) for $H(\varepsilon)$ and the definitions of V_h and $\bar{H}$ from (A.36):

$$\begin{aligned}&\int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon = \int_\varepsilon(\varepsilon+g-D)\mathrm{d}\varsigma(\varepsilon) + \alpha_h(B_h+U_o)\int_{\varepsilon=0}^{\frac{x_h}{\delta_h}}\varsigma(\varepsilon)\mathrm{d}\varepsilon - \alpha_h\int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &+\alpha_h\int_{\varepsilon=\frac{x_h}{\delta_h}}^\infty H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon + \rho\int_\varepsilon(U_o - H(\varepsilon))\varsigma(\varepsilon)\mathrm{d}\varepsilon = (V_h+g-D)q_h + \alpha_h\int_{\varepsilon=\frac{x_h}{\delta_h}}^\infty H(\delta_h\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &+m_h(B_h+U_o)q_h - \alpha_h\bar{H}q_h + \rho(U_o-\bar{H})q_h, \quad \text{where } \int_{\varepsilon=0}^{\frac{x_h}{\delta_h}}\alpha_h\varsigma(\varepsilon) = m_h q_h. \end{aligned} \tag{A.39}$$

where $\varsigma(\varepsilon)$ integrates to q_h over all ε, and to the number of moves $m_h q_h$ within the city over the range up to $\varepsilon = x_h/\delta_h$. Substituting equations (A.38) and (A.39) into (A.37) yields:

$$rQ - \dot{Q} = (V_h+g-D)q_h + m_h(B_h+U_o)q_h + \rho U_o q_h - v_o b_h\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon).$$

Since $\bar{H} = Q/q_h$ implies $\dot{\bar{H}} = \dot{Q}/q_h - \bar{H}\dot{q}_h/q_h$, the equation above and (32) for $\dot{q}_h$ imply $\bar{H}$ satisfies:

$$r\bar{H} = V_h + g - D + m_h(B_h+U_o-\bar{H}) + \rho(U_o-\bar{H}) + \frac{v_o\pi_h b_h}{q_h}(\bar{H}-H) + \dot{\bar{H}}. \tag{A.40}$$

where H is defined as the average of $H(\varepsilon)$ over ε for new owner-occupier matches:

$$H = \frac{1}{\pi_h}\int_{\varepsilon=y_h} H(\varepsilon)\mathrm{d}\Gamma_h(\varepsilon), \quad L = \frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon) \quad \text{and } W = \frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}\Gamma_l(\varepsilon), \tag{A.41}$$

with L and W defined similarly as the averages of $L(\varepsilon)$ and $W(\varepsilon)$ over new rental-market matches. Analogous to the definition of $\bar{H}$, let $\bar{L}$ and $\bar{W}$ be the average values of $L(\varepsilon)$ and $W(\varepsilon)$ across the distribution of match quality ε for all surviving matches in the rental market. The same method used to derive (A.40) can be applied to show the equivalent for $\bar{W}$ of the Bellman equation (9) for $W(\varepsilon)$ is

$$r\bar{W} = V_l - \bar{R} + g + m_l\left(\xi\kappa(B_h-\bar{K}) + (1-\xi\kappa)B_l - \bar{W}\right) - \rho\bar{W} + \frac{v_l\pi_l b_l}{q_l}(\bar{W}-W) + \dot{\bar{W}}, \tag{A.42}$$

where V_l and $\bar{R}$ are averages of ε and $R(\varepsilon)$ for surviving rental-market matches, and the equivalent of the Bellman equation (10) for $L(\varepsilon)$ in terms of $\bar{L}$ is

$$r\bar{L} = \bar{R} - D - D_l + (\alpha_l+\rho)(U_l-\bar{L}) + \rho_l(U_o-\bar{L}) + \frac{v_l\pi_l b_l}{q_l}(\bar{L}-L) + \dot{\bar{L}}. \tag{A.43}$$

A.2.7 Welfare

With $\bar{H}$, $\bar{L}$, and $\bar{W}$ denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of surviving owner-occupier and rental-market matches, total welfare Ω is defined as follows:

$$\Omega = q_h\bar{H} + q_l(\bar{L}+\bar{W}) + b_hB_h + b_lB_l + b_iI + u_oU_o + u_lU_l + \Omega_a\,, \tag{A.44}$$

where Ω_a is the expected present values N of new entrants to the city, which by using (5) satisfies:

$$r\Omega_a = a\left(\kappa(B_h-\bar{K}) + (1-\kappa)B_l - E\right) + \dot{\Omega}_a\,. \tag{A.45}$$

Differentiating total welfare Ω from (A.44) with respect to t and subtracting from $r\Omega$:

$$r\Omega = q_h(r\bar{H}-\dot{\bar{H}}) - \bar{H}\dot{q}_h + q_l(r\bar{L}-\dot{\bar{L}}) + q_l(r\bar{W}-\dot{\bar{W}}) - (\bar{L}+\bar{W})\dot{q}_l + b_h(rB_h-\dot{B}_h) - B_h\dot{b}_h + \dot{\Omega}$$
$$+ b_l(rB_l-\dot{B}_l) - B_l\dot{b}_l + b_i(rI-\dot{I}) - I\dot{b}_i + u_o(rU_o-\dot{U}_o) - U_o\dot{u}_o + u_l(rU_l-\dot{U}_l) - U_l\dot{u}_l + (r\Omega_a-\dot{\Omega}_a)\,.$$

Substituting Bellman equations (12), (13), (25), (27), (33), (A.40), (A.42), (A.43), (A.45), and laws of motion (18), (19), (20), (31), (32), and (39) into the equation above:

$$\begin{aligned}
r\Omega = {}& q_h\left(V_h + g - D + m_h(B_h+U_o-\bar{H}) + \rho(U_o-\bar{H}) + \frac{v_o\pi_hb_h}{q_h}(\bar{H}-H)\right)\\
&-\bar{H}\left(v_o\pi_hb_h - (m_h+\rho)q_h\right) + q_l\left(\bar{R} - D - D_l + (\alpha_l+\rho)(U_l-\bar{L}) + \rho_l(U_o-\bar{L}) + \frac{v_l\pi_lb_l}{q_l}(\bar{L}-L)\right)\\
&+q_l\left(V_l - \bar{R} + g + m_l\left(\xi\kappa(B_h-\bar{K}) + (1-\xi\kappa)B_l - \bar{W}\right) - \rho\bar{W} + \frac{v_l\pi_lb_l}{q_l}(\bar{W}-W)\right)\\
&-(\bar{L}+\bar{W})\left(s_lu_l - (m_l+\rho)q_l\right) + b_h\left(g - F_h + v_o\pi_h(H - C_h - (1+\tau_h)P_h - B_h) - \rho B_h\right)\\
&-B_h\left(m_hq_h + \gamma - (v_o\pi_h+\rho)b_h\right) + b_l\left(g - F_l + v_l\pi_l(W - A - C_w - B_l) - \rho B_l\right)\\
&-B_l\left((1-\xi\kappa)m_lq_l + (1-\kappa)a - (v_l\pi_l+\rho)b_l\right) + b_i\left(-F_i + v_o(U_l - (1+\tau_i)P_i - C_i - I)\right) - I\dot{b}_i\\
&+u_o\left(-D + \theta_ov_o(1-\psi)\pi_h(P_h - C_o - U_o) + \theta_ov_o\psi(P_i - C_o - U_o)\right)\\
&-U_o\left((m_h+\rho)q_h + \rho_l(q_l+u_l) - s_ou_o\right) + u_l\left(-D + \theta_lv_l\pi_l(L + A - C_l - U_l) + \rho_l(U_o-U_l)\right)\\
&\qquad -U_l\left((\alpha_l+\rho)q_l + S_i - (s_l+\rho_l)u_l\right) + a\left(\kappa(B_h-\bar{K}) + (1-\kappa)B_l - E\right) + \dot{\Omega}\,,
\end{aligned} \tag{A.46}$$

which also uses the transactions probabilities $\pi_j = 1-\Gamma_j(y_j)$ for $j\in\{h,l\}$, the constant value of A from (16), the average price P_h paid by home-buyers from (30), and the definitions of the average values H, L, and W for new matches from (A.41). This expression for welfare can be simplified in a number of ways. First, observe that by collecting terms multiplying the values $\bar{H}$, H, $\bar{L}$, L, $\bar{W}$, W, B_h, and B_l, all of the coefficients of these values are zero, reflecting transitions of particular individuals between different states. This can be seen directly for $\bar{H}$, H, W, and B_l. Noting $v_l\pi_lb_l = \theta_lv_l\pi_lu_l$ using (11) gives a zero coefficient on L, and $v_l\pi_lb_l = s_lu_l$ from (17) a zero coefficient on $\bar{W}$. These observations together with $m_l = \alpha_l+\rho_l$ from (8) yield a zero coefficient on $\bar{L}$. A zero coefficient on B_h follows from the expression for first-time buyers γ in (4), a zero coefficient on U_l from $s_l = \theta_lv_l\pi_l$ in (17) and $S_i = v_ob_i$ in (35), and a zero coefficient on U_o from $s_o = \theta_ov_o(\psi+(1-\psi)\pi_h)$ in (36). Using investors' free-entry condition (37), the terms that are multiplied by I are also zero.[51]

Next, note that rent payments $\bar{R}$ and tenancy agreement fees A cancel out, the latter using the definition of market tightness $\theta_l = b_l/u_l$ from (11). This is because such payments are simply transfers among individuals that net out from total welfare. The terms in average prices P_h and P_i simplify to $-\tau_hP_hS_h$ and $-\tau_iP_iS_i$ respectively using (32), (35), and (36), where net payments equal the tax revenue transferred to the government. Collecting all terms in g from (A.46) yields $(q_l+q_h+b_l+b_h)g = ng = G = \tau_hP_hS_h + \tau_iP_iS_i$ using (2) and (26). Hence, these terms and those in prices cancel out overall, reflecting the assumption that tax revenue is used to provide public goods of an equivalent value.

The only terms that remain on the right-hand side of (A.46) are $\dot{\Omega}$ are those involving average match qualities V_h and V_l and costs D, D_l, F_h, F_i, F_l, C_h, C_i, C_o, C_l, C_w, $\bar{K}$, and E. The coefficient of D is

[51] This holds even at points in time where $\dot{b}_i$ is not well defined owing to jumps in b_i.

-1 using $q_l + q_h + u_l + u_o = 1$ from (1) and the coefficient of $\bar{K}$ is $-\gamma$ using (4). The coefficients on transaction costs C_h, C_i, C_o, C_l, and C_w can be expressed in terms of the flows of various types of transactions S_h, S_i, S_o, and S_l by using equations (17), (32), (35), and (36). This completes the derivation of the expression for total welfare given in (44).

A.3 Solving for the steady state

The solution method is based on a numerical search over the fraction ψ of investors among buyers and ownership-market tightness θ_o that satisfy two equations representing equilibrium in the ownership and rental markets. Within this search, given a (ψ, θ_o), the ownership-market thresholds (y_h, x_h) and rental-market and credit-cost thresholds (y_l, Z) are found by solving two equations numerically.

A.3.1 Ownership-market thresholds

This part of the solution method derives an equation satisfied by the ownership-market transaction threshold y_h, which can be solved taking as given (ψ, θ_o). Once y_h is known, the moving threshold x_h is determined, along with other variables related to the ownership market.

With $\dot{B}_h = 0$ and $\dot{U}_o = 0$ in the steady state, the Bellman equations (A.7) and (A.8) become

$$(r+\rho)B_h = g - F_h + (1-\omega_h^*)v_o\Sigma_h\,, \quad \text{and} \tag{A.47}$$

$$rU_o = \theta_o v_o((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i) - D\,. \tag{A.48}$$

Substituting from (A.48) into equation (A.6) that links y_h and x_h:

$$y_h = x_h + (r+\rho+\alpha_h)\left(C_h + C_o + \tau_h\left(C_o - \frac{D}{r} + \frac{\theta_o v_o((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i)}{r}\right)\right)\,. \tag{A.49}$$

With $\dot{\underline{H}} = 0$ in the steady state, equation (A.3) becomes $(r+\rho+\alpha_h)\underline{H} = \alpha_h B_h + (\rho+\alpha_h)U_o + g - D$. Substituting into (A.4) implies $x_h = (r+\rho+\alpha_h)(B_h+U_o) - \alpha_h B_h - (\rho+\alpha_h)U_o + D - g$ and hence

$$x_h = D - g + (r+\rho)B_h + rU_o\,.$$

Then substituting for B_h and U_o from (A.47) and (A.48) yields

$$x_h + F_h = (1-\omega_h^* + (1-\psi)\omega_h^*\theta_o)\,v_o\Sigma_h + \theta_o v_o\psi\omega_i^*\Sigma_i\,. \tag{A.50}$$

Equation (A.16) implies that the steady state must have $X_h = x_h$ since $\dot{X}_h = 0$. Substituting into (A.17):

$$\Sigma_h = \frac{\zeta_h^{\lambda_h}}{(r+\rho+\alpha_h)(\lambda_h-1)(1+\tau_h\omega_h^*)}\left(y_h^{1-\lambda_h} + \frac{\alpha_h\delta_h^{\lambda_h}x_h^{1-\lambda_h}}{r+\rho+\alpha_h\left(1-\delta_h^{\lambda_h}\right)}\right)\,. \tag{A.51}$$

The next step is to reduce these equations to a single equation that can be solved numerically for y_h. Equation (A.50) implies $v_o\Sigma_h = (x_h + F_h - \psi\theta_h v_o\omega_i^*\Sigma_i)/(1-\omega_h^* + (1-\psi)\omega_h^*\theta_h)$, and together with $v_o\Sigma_i = F_i/(1-\omega_i^*)$ from (38), it follows that:

$$\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right) = \frac{\omega_h^*\theta_o}{1-\omega_h^* + (1-\psi)\omega_h^*\theta_o}\left((1-\psi)(x_h+F_h) + \psi\frac{(1-\omega_h^*)\omega_i^*}{\omega_h^*(1-\omega_i^*)}F_i\right)\,.$$

Substituting the above into (A.49) yields a linear equation for x_h that can be solved in terms of y_h:

$$x_h = \frac{y_h - (r+\rho+\alpha_h)\left(C_h + (1+\tau_h)C_o - \tau_h\frac{D}{r} + \tau_h\frac{\theta_o\omega_h^*}{1-\omega_h^*+(1-\psi)\omega_h^*\theta_o}\left(\frac{(1-\psi)F_h}{r} + \frac{\psi(1-\omega_h^*)\omega_i^*F_i}{\omega_h^*(1-\omega_i^*)r}\right)\right)}{1+\tau_h\left(\frac{(1-\psi)\omega_h^*\theta_o}{1-\omega_h^*+(1-\psi)\omega_h^*\theta_o}\right)\left(\frac{r+\rho+\alpha_h}{r}\right)}\,. \tag{A.52}$$

Combining equations (38), (A.50), (A.51) and $v_o = \upsilon_o\theta_o^{-\eta_o}$ from (43):

$$x_h + F_h - \frac{(1-\omega_h^* + (1-\psi)\omega_h^*\theta_o)\upsilon_o\theta_o^{-\eta_o}\zeta_h^{\lambda_h}}{(1+\tau_h\omega_h^*)(r+\rho+\alpha_h)(\lambda_h-1)}\left(y_h^{1-\lambda_h} + \frac{\alpha_h\delta_h^{\lambda_h}x_h^{1-\lambda_h}}{r+\rho+\alpha_h(1-\delta_h^{\lambda_h})}\right) - \frac{\psi\theta_o\omega_i^*F_i}{1-\omega_i^*} = 0\,. \tag{A.53}$$

Taking any y_h, the implied x_h is obtained from (A.52), and x_h is seen to be a strictly increasing function of y_h because the denominator is positive. Substituting the relationship between x_h and y_h into (A.53) yields a single equation in y_h that can be evaluated given (ψ, θ_o). Since the left-hand side of (A.53) is strictly increasing in both x_h and y_h, the single equation for y_h is strictly monotonic in y_h, which means any solution for x_h and y_h is unique. As the left-hand side of (A.53) is sure to be positive for large y_h and x_h because $\lambda_h > 1$, existence of a solution is confirmed by checking whether the left-hand side is negative at $y_h = \zeta_h$, the minimum value of y_h. Once the solution for y_h is found numerically, x_h is obtained from (A.52), and it can be verified whether $\delta_h y_h < x_h$ is satisfied.

A.3.2 Other ownership-market variables

Once y_h is found, $\pi_h = (\zeta_h/y_h)^{\lambda_h}$ is obtained using (40). This yields i from (35) given the value of ψ. Moreover, given that $v_o = \upsilon_o \theta_o^{-\eta_o}$ is known conditional on θ_o, the sales rate s_o is found using (36). The surplus Σ_h is found by substituting the thresholds into (A.51), and $\Sigma_i = F_i/((1-\omega_i^*)v_o)$ comes from (38). The average price P_h from (30) can be written as follows by using (A.48) for U_o:

$$P_h = \left(\frac{r + \theta_o v_o (1-\psi)\pi_h}{r}\right)\left(\frac{\omega_h^* \Sigma_h}{\pi_h}\right) + \frac{\theta_o v_o \psi \omega_i^* \Sigma_i}{r} + C_o - \frac{D}{r}, \tag{A.54}$$

and the price P_i is obtained from (34) and (A.48):

$$P_i = C_o + \frac{\theta_o v_o ((1-\psi)\omega_h^* \Sigma_h + \psi \omega_i^* \Sigma_i) - D}{r} + \omega_i^* \Sigma_i. \tag{A.55}$$

The stock-flow accounting (32) and (39) in the steady state together with (36) require that

$$(1-i)s_o u_o = (m_h + \rho) q_h, \quad \text{and} \tag{A.56}$$

$$s_o u_o = (m_h + \rho) q_h + \rho_l (q_l + u_l). \tag{A.57}$$

Evaluating equation (41) for the moving rate in the steady state and substituting $\zeta_h^{\lambda_h} = \pi_h y_h^{\lambda_o}$ from (40):

$$m_h = \alpha_h - \frac{\alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h} \pi_h}{q_h} \frac{(1-\psi)\theta_o v_o u_o}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right)}.$$

Equations (35) and (36) imply that $(1-i)s_o = (1-\psi)\theta_o v_o \pi_h$, and hence by using equation (A.56) it follows that $(1-\psi)\theta_o v_o \pi_h u_o / q_h = m_h + \rho$. By substituting this into the above and solving for m_h:

$$m_h = \alpha_h \left(\frac{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right) - \rho \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right) + \alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}\right). \tag{A.58}$$

Dividing both sides of (A.57) by $\rho_l > 0$ and substituting for $q_l + u_l = 1 - q_h - u_o$ from (1) implies $u_o + q_h - ((m_h + \rho)/\rho_l) q_h + (s_o/\rho_l) u_o = 1$. Equation (A.56) shows that $q_h = ((1-i)s_o/(m_h + \rho))u_o$, and substituting this into the previous equation and solving for u_o yields:

$$u_o = \frac{1}{1 + \frac{(1-i)s_o}{m_h + \rho} + \frac{i s_o}{\rho_l}}, \quad \text{and} \quad q_h = \frac{(1-i)s_o}{m_h + \rho} u_o. \tag{A.59}$$

With $b_h = (1-\psi)\theta_o u_o$ and $b_i = \psi \theta_o u_o$ by using (23) and (24), equations (32) and (35) give S_h and S_i because v_o and π_h are known. Together with prices P_h and P_i from (A.54) and (A.55), total tax revenue $G = \tau_h P_h S_h + \tau_i P_i S_i$ is determined.

A.3.3 The rental-market and credit-cost thresholds, and the city population

The next part of the solution method derives an equation to solve numerically for the rental-market transaction threshold y_l, taking as given (ψ, θ_o) and the solution for ownership-market variables, which are also found conditional on (ψ, θ_o). Solving for y_l depends on finding the credit-cost threshold Z and the city population n. Given y_l, there is a single equation that can be solved numerically for the fraction κ of households choosing to pay the credit cost, which directly determines Z and n.

The moving rate $m_l = \alpha_l + \rho_l$ is given by parameters according to (8). With $\dot{B}_l = 0$ and $\dot{U}_l = 0$ in the steady state, the Bellman equations (A.30) and (A.31) become

$$(r+\rho)B_l = g - F_l + (1-\omega_l)v_l\Sigma_l\,, \text{ and} \tag{A.60}$$

$$(r+\rho_l)U_l = \omega_l\theta_l v_l\Sigma_l - D + \rho_l U_o\,. \tag{A.61}$$

In steady state, $\dot{\underline{J}} = 0$, which using (A.26) yields the equation $(r+\rho+m_l)\underline{J} = m_l B_l + (\rho+\alpha_l)U_l + \rho_l U_o + g - D - D_l + \xi m_l \kappa(Z-\bar{K})$. Substituting this into (A.28) with $m_l = \alpha_l + \rho_l$ implies

$$y_l = D + D_l - g + (r+\rho)B_l + (r+\rho_l)U_l - \rho_l U_o + (r+\rho+m_l)(C_l + C_w) - \xi m_l\kappa(Z-\bar{K})\,,$$

and by using (A.60) and (A.61), the equation becomes

$$y_l - D_l + F_l - (r+m_l+\rho)(C_l+C_w) + \xi m_l\kappa(Z-\bar{K}) - (1-\omega_l+\omega_l\theta_l)v_l\Sigma_l = 0\,. \tag{A.62}$$

This equation links the transaction threshold y_l to the surplus Σ_l and the credit-cost threshold Z (and hence κ and $\bar{K}$). A further equation involving Σ_l is obtained by multiplying both sides of the first equation in (38) by $r+\rho_l$ and substituting for $(r+\rho_l)U_l$ from (A.61):

$$\omega_l\theta_l q_l\Sigma_l = D - \rho_l U_o + (r+\rho_l)(1+\tau_i)U_o + (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right)\,.$$

Using $(r+\rho_l)(1+\tau_i)U_o - \rho_l U_o = (1+\tau_i(1+(\rho_l/r))rU_o$ and substituting from (A.48) leads to:

$$\omega_l\theta_l v_l\Sigma_l = \left(1+\tau_i\left(1+\frac{\rho_l}{r}\right)\right)\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right) + (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right) - \tau_i\left(1+\frac{\rho_l}{r}\right)D\,. \tag{A.63}$$

Rental-market variables conditional on the transactions threshold The numerical procedure to solve for y_l depends on checking whether one equation holds. Taking a particular value of the threshold y_l, the implied transaction probability from (40) is $\pi_l = (\zeta_l/y_l)^{\lambda_l}$. Using the formula (A.29) for the surplus Σ_l to state it in terms of y_l and π_l:

$$\Sigma_l = \frac{\pi_l y_l}{(\lambda_l-1)(r+\rho+m_l)}\,.$$

Observe that this implies $\omega_l\theta_l v_l\Sigma_l = \omega_l y_l s_l/((\lambda_l-1)(r+\rho+m_l))$, where $s_l = \theta_l v_l\pi_l$ is the letting rate from (17). As another equation for the left-hand side is given by (A.63), the rate s_l implied by y_l is

$$s_l = \frac{(\lambda_l-1)(r+\rho+m_l)}{\omega_l y_l}\left(\left(1+\tau_i\left(1+\frac{\rho_l}{r}\right)\right)\theta_o v_o\left((1-\psi)\omega_h^*\Sigma_h + \psi\omega_i^*\Sigma_i\right) + (r+\rho_l)\left((1+\tau_i)C_o + C_i + (1+\tau_i\omega_i^*)\Sigma_i\right) - \tau_i\left(1+\frac{\rho_l}{r}\right)D\right), \tag{A.64}$$

which can be evaluated conditional on (ψ,θ_o), from which the ownership-market variables v_o, Σ_h, and Σ_i are also known. Equation (43) gives the meeting rate $v_l = \upsilon_l\theta_l^{-\eta_l}$, and hence the letting rate $s_l = \theta_l v_l\pi_l$ satisfies $s_l = \upsilon_l\pi_l\theta_l^{1-\eta_l}$. The implied market tightness in the rental market is

$$\theta_l = \left(\frac{s_l}{\upsilon_l\pi_l}\right)^{\frac{1}{1-\eta_l}}\,, \tag{A.65}$$

and this also determines $v_l = \upsilon_l\theta_l^{-\eta_l}$.

Solving for the credit-cost threshold and city population With $\dot{q}_l = 0$ and $\dot{u}_l = 0$ in the steady state, equations (18), (20), and (36) require

$$s_l u_l = (m_l + \rho) q_l \,, \quad \text{and} \tag{A.66}$$

$$(s_l + \rho_l) u_l = (\alpha_l + \rho) q_l + i s_o u_o \,. \tag{A.67}$$

Equations (1) and (A.66) imply $q_l + u_l = 1 - q_h - u_o$ and $q_l = (s_l/(m_l+\rho))u_l$. Combining these and using the known values of q_h and u_o to solve for u_l and q_l:

$$u_l = \frac{1 - q_h - u_o}{1 + \frac{s_l}{m_l+\rho}} \,, \quad \text{and} \quad q_l = \frac{s_l}{m_l+\rho} u_l \,. \tag{A.68}$$

Since the stock-flow accounting equations are consistent with (1), and as the solution already satisfies (A.56), (A.57), and (A.66), this means equation (A.67) holds automatically. The steady state also has $\dot{n} = 0$, $\dot{b}_h = 0$, and $\dot{b}_l = 0$. Using (4), (6), (19), and (31), this means that $a = \rho n$ and the following equations must hold:

$$(v_o \pi_h + \rho) b_h = m_h q_h + (\xi m_l q_l + \rho n)\kappa \,, \quad \text{and} \tag{A.69}$$

$$(v_l \pi_l + \rho) b_l = (1-\xi) m_l q_l + (\xi m_l q_l + \rho n)(1-\kappa) \,. \tag{A.70}$$

Since $b_h = (1-\psi)\theta_o u_o$, v_o, π_h, m_h, q_h, and q_l are known at this point, equation (A.69) can be rearranged to solve for the city population n as a function of κ:

$$n = \frac{1}{\rho}\left(\frac{(v_o\pi_h + \rho)(1-\psi)\theta_o u_o - m_h q_h}{\kappa} - \xi m_l q_l\right) . \tag{A.71}$$

Note that since the stock-flow accounting equations are consistent with (2), and as the solution already satisfies (A.56), (A.66), and (A.69) given equation (A.71), it follows that (A.70) automatically holds. To have $\dot{n} = 0$ in (6), it is necessary that $a = \rho n > 0$, which requires $N = 0$ since $\chi > 0$. Using (5) and (3), this requires $B_l + \kappa(Z - \bar{K}) - E = 0$. Substituting for B_l from (A.60) and noting $g = G/n$ from (26), this equation is equivalent to:

$$\frac{(G/n) - F_l + (1-\omega_l) v_l \Sigma_l}{r + \rho} + \kappa(Z - \bar{K}) - E = 0 \,. \tag{A.72}$$

Now consider the numerical search over the fraction κ conditional on y_l. Equation (A.69) shows that $(v_o\pi_h + \rho)(1-\psi)\theta_o u_o > m_h q_h$ is necessary to have a positive κ. After checking this condition for the given value of y_l, equation (A.71) is used to obtain n as a function of κ, and it can be seen that n is decreasing in κ. The credit-cost threshold Z is obtained by inverting equation (42) using the given κ:

$$Z = e^{\mu + \sigma \Phi^{-1}(\kappa)} \,, \tag{A.73}$$

which is an increasing function of κ. The average credit cost $\bar{K}$ follows immediately from (42) using the Z obtained from (A.73). Since $\kappa(Z - \bar{K}) = \int_{K=0}^{Z} (Z - K)\mathrm{d}\Gamma_k(K)$, the left-hand side of equation (A.72) is increasing in κ after taking account of the effects on Z and $\bar{K}$. Moreover, since n falls as κ rises, and as $G > 0$, higher κ has an unambiguously positive effect on the left-hand side of (A.72). A solution for κ is therefore unique, and existence is verified by checking (A.72) changes sign over the interval $\kappa \in [0,1]$.

Solving for the transaction threshold The method described above finds the unique solution for κ, Z, $\bar{K}$, and n conditional on the rental-market transaction threshold y_l (and ψ, θ_o, and other ownership-market variables). The value of y_l itself is found numerically as the solution of equation (A.62), taking account of the effect of y_l on Σ_l, v_l, θ_l, and the term $\kappa(Z - \bar{K})$. From (A.29) and $\lambda_l > 1$, it follows that higher y_l reduces Σ_l. Equation (A.64) shows that s_l is proportional to $1/y_l$, and since $\pi_l = \zeta_l^{\lambda_l} y_l^{-\lambda_l}$, the ratio s_l/π_l and hence θ_l from (A.65) are increasing in y_l because $\lambda_l > 1$. This means that v_l is decreasing in y_l, and from (A.68), q_l is decreasing in y_l because s_l is negatively related to y_l.

Higher y_l directly increases the left-hand side of (A.62), and indirectly increases it through lower Σ_l. Moreover, since higher y_l lowers $v_l\Sigma_l$, and reduces G/n because of higher n from falling q_l in (A.71), the required value of $\kappa(Z - \bar{K})$ consistent with (A.72) increases (as does κ and Z, because $\kappa(Z - \bar{K})$ is increasing in Z). This implies another positive effect on the left-hand side of (A.62). The only term that is not unambiguously increasing in y_l is $(1-\omega_l+\omega_l\theta_l)v_l$ because θ_l rises, while $v_l = \upsilon_l\theta_l^{-\eta_l}$ falls.[52]

Equation (A.62) is solved numerically for the transaction threshold y_l, verifying uniqueness if necessary. Since the left-hand side becomes arbitrarily large as y_l increases, existence is confirmed by checking whether the left-hand side is negative at $y_l = \zeta_l$. Given the solution for y_l, the associated solutions for Z, κ, and n are obtained as explained earlier.

A.3.4 Solving for the fraction of investors and market tightnesses

The steps above are used to derive values of all the variables in the model conditional on a given pair of values for (ψ,θ_o). The fraction ψ of investors and the tightness θ_o of the ownership market are found by a numerical solution of the two remaining equations of the model.

The first equation is (A.35), which links the fraction of investors ψ, market tightnesses θ_o and θ_l, properties on the market u_o and u_l, and the city population n. All of these variables are known conditional on (ψ,θ_o). The second equation is the marginal home-buyer indifference condition (3) in steady state. Substituting the expressions for B_h and B_l from (A.47) and (A.60) into (3):

$$(1-\omega_h^*)v_o\Sigma_h - (1-\omega_l)v_l\Sigma_l = (r+\rho)Z + F_h - F_l\,, \tag{A.74}$$

where v_o, v_l, Σ_h, Σ_l, and Z are determined above for given ψ and θ_o.

Searching over ψ and θ_o to find values satisfying (A.35) and (A.74), the steady-state equilibrium of the model is found. Since the search is only over two dimensions, existence and uniqueness can be confirmed numerically. Once (ψ,θ_o) is known, all other variables are derived using the earlier methods.

A.3.5 Steady-state values of other variables

This section shows how to compute steady-state values of other variables of interest, including those used as part of the calibration strategy. With $\dot{y}_l = 0$ in steady state, the new rent equation (A.33) becomes

$$R = D_l + \omega_l(r+\rho+m_l)(C_l+C_w) + \omega_l(r+\rho+m_l+\theta_l v_l\pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.75}$$

Steady-state match qualities in the two markets ($\dot{V}_h = 0$ and $\dot{V}_l = 0$) are derived from (45) and (46):

$$V_h = \frac{\lambda_h}{\lambda_h-1}\left(\frac{m_h+\rho}{\alpha_h+\rho}y_h + \frac{\alpha_h-m_h}{\alpha_h+\rho}x_h\right)\,, \quad \text{and } V_l = \frac{\lambda_l}{\lambda_l-1}y_l\,, \tag{A.76}$$

where the first equation also makes use of (32) with $\dot{q}_h = 0$. As steady-state match quality V_l is the same as average match quality $\lambda_l y_l/(\lambda_l-1)$ for new leases, steady-state new rents R are equal to average rents $\bar{R}$ for existing tenants (see equation A.34).

Viewings and time-on-the-market If home-buyers have a constant probability π_h of making a purchase conditional on a viewing, the expected number of viewings per home-buyer purchase is $\Lambda_h = 1/\pi_h$. Similarly, those searching for property to rent have constant probability π_l of transacting, so their expected number of viewings is $\Lambda_l = 1/\pi_l$, which is also the expected viewings required for a landlord to lease a property. If properties on the rental market are leased at a constant rate s_l, the expected time-on-the-market for properties in the rental market is $T_{sl} = 1/s_l$. In the ownership market, properties are sold at a constant rate s_o in steady state, implying an expected time-on-the-market for sellers of $T_{so} = 1/s_o$. Home-buyers complete a purchase at rate $v_o\pi_h$ over time, so their expected time-on-the-market is $T_{bh} = 1/(v_o\pi_h)$, and similarly $T_{bl} = 1/(v_l\pi_l)$ for renters. In summary:

$$\Lambda_h = \frac{1}{\pi_h}\,,\quad \Lambda_l = \frac{1}{\pi_l}\,,\quad T_{so} = \frac{1}{s_o}\,,\quad T_{sl} = \frac{1}{s_l}\,,\quad T_{bh} = \frac{1}{v_o\pi_h}\,,\quad \text{and } T_{bl} = \frac{1}{v_l\pi_l}\,. \tag{A.77}$$

[52] If the Hosios condition $\omega_l = \eta_l$ holds, $(1-\omega_l+\omega_l\theta_l)\upsilon_l\theta_l^{-\eta_l}$ is decreasing in θ_l (and hence y_l) if $\theta_l < 1$, as is the case given the calibration in Table 3.

The moving hazard rate and expected time between moves Tenants move house within the city at an constant exogenous rate $m_l = a_l + \rho_l$. The expected time a tenant remains in a property is therefore $T_{ml} = 1/(m_l + \rho)$ after accounting for moves outside the city (exit rate ρ). For homeowners, moving is endogenous, so even in steady state where the moving threshold x_h and transaction threshold y_h remain constant, the hazard rate of moving depends on how long a household lived in a property. Let $\Psi(T)$ denote the steady-state survival function of matches in the ownership market. This gives the fraction of matches that remain in existence T years after households first moved into their properties.

In order for a match to survive for T years, first, a household must not leave the city during that time. With constant exit rate ρ, this has probability $e^{-\rho T}$. Second, the household must choose to remain in a property after any shocks to idiosyncratic match quality have occurred. These shocks arrive independently at rate α_h, so the number of shocks j that occur over a period of time T has a Poisson$(\alpha_h T)$ distribution. The probability of receiving exactly j shocks is $e^{-\alpha_h T}(\alpha_h T)^j/j!$ for $j = 0, 1, 2, \ldots$.

If initial match quality is ε, match quality becomes $\varepsilon' = \delta_h^j \varepsilon$ after j shocks. The household chooses not to move if $\varepsilon' \geq x_h$, which is equivalent to $\varepsilon \geq x_h/\delta_h^j$ in terms of initial match quality ε (and if this condition holds for some j then it also holds for any smaller j because $\delta_h < 1$ and x_h remains constant in the steady state). New match quality has a Pareto(y_h, λ_h) distribution, so the probability that $\varepsilon \geq x_h/\delta_h^j$ is $((x_h/\delta_h^j)/y_h)^{-\lambda_h}$. This is well defined if $x_h/\delta_h^j > y_h$, which is true for all $j \geq 1$ because $\delta_h y_h < x_h$. With zero shocks ($j = 0$), households remain in the same property unless they leave the city.

The fraction of households who remain in the same property for T years is therefore

$$\Psi(T) = e^{-\rho T}\left(e^{-\alpha_h T} + \sum_{j=1}^{\infty} e^{-\alpha_h T}\frac{(\alpha_h T)^j}{j!}\left(\frac{x_h/\delta_h^j}{y_h}\right)^{-\lambda_h}\right)$$

$$= e^{-(\alpha_h+\rho)T}\left(1 + \left(\frac{y_h}{x_h}\right)^{\lambda_h}\sum_{j=1}^{\infty}\frac{(\alpha_h\delta_h^{\lambda_h}T)^j}{j!}\right) = e^{-(\alpha_h+\rho)T}\left(1 + \left(\frac{y_h}{x_h}\right)^{\lambda_h}\left(e^{\alpha_h\delta_h^{\lambda_h}T} - 1\right)\right)$$

$$= \left(\frac{y_h}{x_h}\right)^{\lambda_h} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - \left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}.$$

The moving hazard $\hbar(T)$ as a function of match duration T is defined by the percentage decline in the proportion of surviving matches, that is, $\hbar(T) = -\mathrm{d}\log\Psi(T)/\mathrm{d}T = -\Psi'(T)/\Psi(T)$. This follows immediately from the expression for $\Psi(T)$ above:

$$\hbar(T) = \frac{\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)\left(\frac{y_h}{x_h}\right)^{\lambda_o} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - (\alpha_h+\rho)\left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}}{\left(\frac{y_h}{x_h}\right)^{\lambda_h} e^{-\left(\alpha_h(1-\delta_h^{\lambda_h})+\rho\right)T} - \left(\left(\frac{y_h}{x_h}\right)^{\lambda_h} - 1\right)e^{-(\alpha_h+\rho)T}}.$$

The density function of the probability distribution of moving times T is $\hbar(T)\Psi(T) = -\Psi'(T)$, and hence the expected time until moving is $T_{mh} = \int_{T=0}^{\infty} -T\Psi'(T)\mathrm{d}T = \int_{T=0}^{\infty}\Psi(T)\mathrm{d}T$, where the second expression for T_{mh}, the area under the survival function, is derived from integration by parts (and $\lim_{T\to\infty} T\Psi(T) = 0$). In the cross-section of households at a point in time, the steady-state distribution of time spent in the same property has density function $\Psi(T)/T_{mh}$, and the hazard rate $\hbar(T)$ averaged over the cross-section of homeowners is $\int_{T=0}^{\infty}\hbar(T)(\Psi(T)/T_{mh})\mathrm{d}T = 1/T_{mh}$ because $\hbar(T)\Psi(T) = -\Psi'(T)$, $\Psi(0) = 1$, and $\lim_{T\to\infty}\Psi(T) = 0$. Since the within-city moving rate averaged over the cross-section of homeowners is m_h from (41), it follows that $T_{mh} = 1/(m_h + \rho)$. In summary:

$$T_{mh} = \frac{1}{m_h + \rho}, \text{ and } T_{ml} = \frac{1}{m_l + \rho}. \tag{A.78}$$

The demographics of owners versus renters As can be seen from (4) and the law of motion (31) for home-buyers, there is a flow of first-time buyers $(\xi m_l q_l + a)\kappa$ coming from the rental market or outside the city, and a flow $m_h q_h$ of existing homeowners returning to the market when they decide the move house. Since these two groups of home-buyers subsequently transact at the same rate $v_o\pi_h$, or leave the city at the same rate ρ, the steady-state fraction ϕ of first-time buyers can be calculated as the ratio of the inflow of first-time buyers to the inflow of all buyers entering b_h:

$$\phi = \frac{(\xi m_l q_l + a)\kappa}{m_h q_h + (\xi m_l q_l + a)\kappa} = \frac{(v_o\pi_h + \rho)b_h - m_h q_h}{(v_o\pi_h + \rho)b_h},$$

where the second expression for ϕ follows from (A.69) because b_h is a steady state. In steady state, (A.57) implies $(m_h + \rho)q_h = (1-i)s_o u_o$, and (32) and (36) imply $(1-i)s_o u_o = v_o\pi_h b_h$. Dividing numerator and denominator of the expression for ϕ by q_h and substituting $v_o\pi_h b_h/q_h = m_h + \rho$:

$$\phi = \frac{\left(1 + \frac{\rho}{v_o\pi_h}\right)(m_h + \rho) - m_h}{\left(1 + \frac{\rho}{v_o\pi_h}\right)(m_h + \rho)}. \tag{A.79}$$

Now consider the steady-state demographics of homeowners compared to renters. Let $\digamma_{qh}$, $\digamma_{ql}$, $\digamma_{bh}$, and $\digamma_{bl}$ be the average ages of the household heads of those in q_h, q_l, b_h, and b_l, and $\digamma_h$ and $\digamma_l$ the average ages of those in $q_h + b_h$ and $q_l + b_l$. Furthermore, let $\digamma_a$ and $\digamma_\gamma$ denote the average age of new entrants to the city a and first-time buyers γ respectively, and the difference between the average ages of homeowners and renters is denoted by $\aleph = \digamma_h - \digamma_l$.

Taking the group in $q_h + b_h$, the laws of motion (31) and (32) imply $\dot{q}_h + \dot{b}_h = (\xi m_l q_l + a)\kappa - \rho(q_h + b_h)$, noting $\gamma = (\xi m_l q_l + a)\kappa$ and $(1-i)s_o u_o = v_o\pi_h b_h$. Exit occurs at rate ρ, with first-time buyers $\rho(q_h + b_h)$ arriving in steady state to ensure $\dot{q}_h + \dot{b}_h = 0$. The differential equation for the average age in this group is thus $\dot{\digamma}_h = 1 - \rho\digamma_h + \rho\digamma_\gamma$, and a steady-state age distribution therefore has $\digamma_h = \digamma_\gamma + \rho^{-1}$. It is convenient to consider all average ages relative to the average age at first entry to the city, which are denoted by $\aleph_h = \digamma_h - \digamma_a$, $\aleph_l = \digamma_l - \digamma_a$, and similarly for the other groups. In terms of these variables, the definition of the average owner-renter age difference $\aleph$ and the equation for the steady-state homeowner versus first-time-buyer age difference are:

$$\aleph = \aleph_h - \aleph_l, \quad \text{and} \quad \aleph_h = \aleph_\gamma + \rho^{-1}. \tag{A.80}$$

Now consider the group q_l. There is exit at rate $m_l + \rho$ and entry $v_l\pi_l b_l/q_l = m_l + \rho$ from b_l as a proportion of the group q_l in steady state (see 18 with $v_l\pi_l b_l = s_l u_l$ from 11 and 17), and the average age of entrants is $\digamma_{bl}$. Thus, in steady state, $1 = (m_l + \rho)(\digamma_{ql} - \digamma_{bl})$ and hence:

$$\aleph_{ql} = \aleph_{bl} + (m_l + \rho)^{-1}. \tag{A.81}$$

Since $\digamma_l = (q_l/(q_l + b_l))\digamma_{ql} + (b_l/(q_l + b_l))\digamma_{bl}$ by definition of the average age of the whole group $q_l + b_l$, it follows that $\aleph_{ql} - \aleph_l = (b_l/(q_l + b_l))(\aleph_{ql} - \aleph_{bl})$. With $b_l/(q_l + b_l) = (m_l + \rho)/(m_l + \rho + v_l\pi_l)$ from $v_l\pi_l b_l/q_l = m_l + \rho$ in steady state, this can be used together with (A.81) to deduce:

$$\aleph_{ql} = \aleph_l + (\rho + m_l + v_l\pi_l)^{-1}. \tag{A.82}$$

For the group b_l, given the law of motion (19), there are outflows at rate $v_l\pi_l + \rho$, and as a proportion of b_l, inflows $a(1-\kappa)/b_l$ from outside the city with average age $\digamma_a$ and $(1-\xi\kappa)m_l q_l/b_l$ from q_l where the average age is $\digamma_{ql}$. Thus, at the steady-state age distribution:

$$1 + \frac{a(1-\kappa)}{b_l}\digamma_a + \frac{m_l(1-\xi\kappa)q_l}{b_l}\digamma_{ql} = (v_l\pi_l + \rho)\digamma_{bl}.$$

Using $a(1-\kappa) = (v_l\pi_l + \rho)b_l - (1-\xi\kappa)m_l q_l$ in steady state from (A.70) with $a = \rho n$, the equation above can be written as $b_l + (1-\xi\kappa)m_l q_l\aleph_{ql} = (v_l\pi_l + \rho)b_l\aleph_{bl}$. Substituting from (A.81) and using (A.70) again implies that $a(1-\kappa)\aleph_{ql} = b_l + (v_l\pi_l + \rho)b_l(m_l + \rho)^{-1}$. With $s_l u_l = v_l\pi_l b_l$ using (11) and (17), it follows that $v_l\pi_l b_l(m_l + \rho)^{-1} = q_l$, and hence $\aleph_{ql}$ is given by:

$$\aleph_{ql} = \frac{(q_l + b_l) + \rho b_l(m_l + \rho)^{-1}}{a(1-\kappa)}. \tag{A.83}$$

Finally, consider the average age $\digamma_\gamma$ of first-time buyers. Using (4), a fraction $\xi m_l q_l\kappa/((\xi m_l q_l + a)\kappa)$

come from q_l where the average age is $\digamma_{ql}$, and a fraction $a\kappa/((\xi m_l q_l + a)\kappa)$ are new entrants to the city with average age $\digamma_a$. Therefore, $\digamma_\gamma = (\xi m_l q_l/(\xi m_l q_l + a))\digamma_{ql} + (a/(\xi m_l q_l + a))\digamma_a$, and hence:

$$\aleph_\gamma = \left(1 - \frac{a}{\xi m_l q_l + a}\right)\aleph_{ql} = \aleph_{ql} - \frac{(q_l + b_l) + \rho b_l (m_l + \rho)^{-1}}{(\xi m_l q_l + a)(1-\kappa)}\,, \tag{A.84}$$

where the second expression substitutes from (A.83). Using $v_l \pi_l b_l = s_l u_l = m_l q_l + \rho q_l$ from (A.66) together with (A.70) implies $(\xi m_l q_l + a)(1-\kappa) = \rho(q_l + b_l) + \xi m_l q_l$ and substituting into (A.84):

$$\aleph_\gamma = \aleph_{ql} - \frac{(q_l + b_l) + \rho b_l (m_l + \rho)^{-1}}{\rho(q_l + b_l) + \xi m_l q_l} = \aleph_{ql} - \frac{1 + \rho(\rho + m_l + v_l \pi_l)^{-1}}{\rho + \xi v_l \pi_l m_l (\rho + m_l + v_l \pi_l)^{-1}}\,, \tag{A.85}$$

where $b_l = (m_l + \rho)(\rho + m_l + v_l \pi_l)^{-1}(q_l + b_l)$ and $q_l = v_l \pi_l (\rho + m_l + v_l \pi_l)^{-1}(q_l + b_l)$ are used to derive the second expression. Combining (A.80), (A.82), and (A.85) and factorizing leads to the following expression for the average age difference between owners and renters:

$$\aleph = \left(1 + \frac{\rho}{\rho + m_l + v_l \pi_l}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + \frac{\xi m_l v_l \pi_l}{\rho + m_l + v_l \pi_l}}\right). \tag{A.86}$$

A.4 Solving for the transitional dynamics

This section describes how the transitional path to the new steady state in a perfect-foresight equilibrium is found numerically. There is an unanticipated change to the tax rates τ_h and τ_i at a point in time, $t = 0$, without loss of generality. No further changes are anticipated. For state variables such as q_h, the measure of owner-occupiers, the left-derivative of $q_h(t)$ with respect to time t must exist at all points along the transitional path — the variable cannot 'jump'. For non-predetermined variables such as B_h, the value of being a home-buyer, the left derivative may not be well defined at $t = 0$, but the right-derivative of $B_h(t)$ with respect to time t must exist at all points along the transitional path to satisfy Bellman equations such as (27). The size of any jumps in values such as B_h is determined by the requirement that values cannot grow faster than the discount rate r, and hence these

An approximate solution of the differential equations of the model is obtained by discretization. Dividing continuous time into a small discrete periods of length ℓ, the time derivative of state variables such as q_h is approximated by $(q_h(t) - q_h(t-\ell))/\ell$, which converges to $\dot{q}_h$ as $\ell \to 0$ because the left-derivative of $q_h(t)$ exists. This means differential equations such as the law of motion (32) are replaced by difference equations of the form:

$$\frac{q_h(t) - q_h(t-\ell)}{\ell} = v_o(t)\pi_h(t)b_h(t) - (m_h(t) + \rho)q_h(t)\,,$$

where continuity of the time path of $q_h(t)$ means the right-hand side can be evaluated at t. For non-predetermined variables such as B_h, the time derivative is approximated by $(B_h(t+\ell) - B_h(t))/\ell$, which converges to $\dot{B}_h$ because the right-derivative of $B_h(t)$ exists. This means that differential equations such as the Bellman equation (27) are replaced by difference equations of the form:

$$\frac{B_h(t+\ell) - B_h(t)}{\ell} = (r + \rho)B_h(t) - g(t) + F_h - (1 - \omega_h^*)v_o(t)\Sigma_h(t)\,,$$

which is based on the equation in (A.7) that is equivalent to (27).

With the differential equations of the model replaced by difference equations, the transitional dynamics of the non-linear system of difference equations can be found using the perfect-foresight solver in the Dynare MATLAB package, together with knowledge of the original and new steady states computed using the procedure described in appendix A.3. The discretization is based on a time period of one day, so $\ell = 1/365$ when the model is calibrated in annual time units.

A.5 Calibration targets

The parameters of the model are chosen to match the City of Toronto housing market in the pre-policy period (January 2006–January 2008). The average sales price P taken from Table A.3 is \$402,000 during that period. The initial effective LTT rate is 1.5%, so $\tau_h = \tau_i = 0.015$.

Housing tenure and entry of investors Based on the 2006 City of Toronto Profile Report, the homeownership rate is $h = 54\%$, the average age of homeowners is 53.3, and the average age of tenants is 45.0. Hence the target for the difference between the average ages of homeowners and renters is $\aleph = 8.3$. There is no survey that specifically captures the proportion of first-time buyers ϕ in Toronto. The Canadian Association of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 finding that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the Greater Toronto Area. On the other hand, data from the Canada Mortgage and Housing Corporation suggests the fraction of first-time buyers is about a third. Based on this information, the calibration target is $\phi = 0.4$.

Using Toronto MLS data on sales and rental transactions, the fraction of purchases by buy-to-rent investors is 5.4% during the pre-policy period, so $i = 0.054$. The price-to-rent ratio for the same property is 14.5 in 2007, and the ratio of average prices paid by investors to prices paid by home-buyers is 0.99. Hence, $P_i/R = 14.5$ and $P_i/P_h = 0.99$ are used as targets.

Credit costs The credit cost K of becoming an owner-occupier is computed from a comparison of the mortgage interest rate r_k the household would face relative to the risk-free interest rate r_g on government bonds. The interest rates r_k and r_g are real interest rates. There is a spread between them due to unmodelled financial frictions. The risk-free real rate r_g used to discount future cashflows need not be the same as the discount rate r applied to future utility flows from owning property, allowing for an unmodelled housing risk premium between r and r_g. It is assumed all these interest rates are expected to remain constant over the mortgage term.

Suppose a household buys a property at price P_h at date $t = 0$ by taking out a mortgage with loan-to-value ratio l. Assume the mortgage has term T_k and a constant real repayment ι until maturity. Let $\mathscr{D}(t)$ denote the outstanding mortgage balance at date t, which has initial condition $\mathscr{D}(0) = lP_h$ and terminal condition $\mathscr{D}(T_k) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{\mathscr{D}}(t) = r_k\mathscr{D}(t) - \iota \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_k t}\mathscr{D}(t))}{\mathrm{d}t} = -\iota e^{-r_k t}.$$

Solving this differential equation using the initial condition $\mathscr{D}(0) = lP_h$ implies:

$$\mathscr{D}(t) = e^{r_k t} lP_h - \frac{\iota}{r_k}(e^{r_k t} - 1). \tag{A.87}$$

The terminal condition $\mathscr{D}(T_k) = 0$ requires that the constant real repayment ι satisfies:

$$\iota = \frac{r_k lP_h}{1 - e^{-r_k T_k}}. \tag{A.88}$$

In the model, owner-occupiers exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds from selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment ι will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $\mathscr{D}(t)$ is repaid at date t. The credit cost K is defined as the present value of the expected stream of repayments discounted at rate r_g minus the amount borrowed (which would equal the present value of the repayments if $r_k = r_g$ in the absence of an interest-rate spread):

$$K = \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \iota \,\mathrm{d}t + \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \rho \mathscr{D}(t) \mathrm{d}t - lP_h.$$

To derive an explicit formula for K, first observe that

$$\int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} \mathrm{d}t = \frac{1 - e^{-(r_g+\rho)T_k}}{r_g + \rho} \quad \text{and} \quad \int_{t=0}^{T_k} e^{-r_g t} e^{-\rho t} e^{r_k t} \mathrm{d}t = \frac{1 - e^{-(r_g+\rho-r_k)T_k}}{r_g + \rho - r_k}.$$

Together with equations (A.87) and (A.88) for $\mathscr{D}(t)$ and ι, the credit cost can be expressed as follows:

$$\begin{aligned}
K &= \frac{\left(\iota + \frac{\rho\iota}{r_k}\right)}{(r_g+\rho)}(1 - e^{-(r_g+\rho)T_k}) + \frac{\rho\left(lP_h - \frac{\iota}{r_k}\right)}{(r_g+\rho-r_k)}(1 - e^{-(r_g+\rho-r_k)T_k}) - lP_h \\
&= \left(\frac{(r_k+\rho)(1-e^{-(r_g+\rho)T_k})}{(r_g+\rho)(1-e^{-r_k T_k})} + \frac{\rho\left(1 - \frac{1}{1-e^{-r_k T_k}}\right)(1 - e^{-(r_g+\rho-r_k)T_k})}{(r_g+\rho-r_k)} - 1\right) lP_h \\
&= \frac{\left((r_k+\rho)(1-e^{-(r_g+\rho)T_k}) - \frac{\rho(r_g+\rho)}{r_g+\rho-r_k}(e^{-r_k T_k} - e^{-(r_g+\rho)T_k}) - (r_g+\rho)(1-e^{-r_k T_k})\right) lP_h}{(r_g+\rho)(1-e^{-r_k T_k})} \\
&= \frac{\left((r_k - r_g) + \frac{\rho(r_g+\rho)-(r_g+\rho)(r_g+\rho-r_k)}{r_g+\rho-r_k} e^{-(r_g+\rho)T_k} - \frac{(r_g+\rho)(r_g+\rho-r_k)-\rho(r_g+\rho)}{r_g+\rho-r_k} e^{-r_k T_k}\right) lP_h}{(r_g+\rho)(1-e^{-r_k T_k})},
\end{aligned}$$

and dividing both sides by price P_h and simplifying leads to the equation given in (51). That equation is used to determine calibration targets for the credit cost Z of a marginal home-buyer relative to the average price P_h, and for the marginal credit cost Z relative to the average credit cost $\bar{K}$ conditional on becoming an owner-occupier.

A mortgage term of 25 years ($T_k = 25$) and an average loan-to-value ratio of 80% ($l = 0.8$) are assumed. Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Statistics Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate $\bar{r}_k$ is 4.93% for an average homeowner. Since the average mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate comes from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate r_g is 1.86%.

Information on different mortgage rates is then used to compute credit costs for a marginal home-buyer. Based on micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores who did not qualify for loans from major banks could obtain mortgages from trust companies or private lenders at mortgage rates of around 6.15%, suggesting an interest rate gap of 3% between the marginal and average home-buyer.

But households faced with a high mortgage rate when they first buy a property do not necessarily continue with that rate for the whole time they have a mortgage. They can build up equity and improve their credit score, and thus obtain a mortgage rate closer to the average when they refinance. The baseline calibration assumes that a marginal home-buyer is able to close half of the initial gap with the average home-buyer over the whole term of the mortgage loan. This translates into an interest rate gap of 1.5%, implying the real mortgage rate r_z for a marginal buyer is 6.43%.

In summary, Z/P_h is derived from (51) using $T_k = 25$, $l = 0.8$, $r_g = 1.86\%$, and $r_k = r_z = 6.43\%$, together with the value of ρ obtained from the calibration method. The value of $Z/\bar{K}$ is derived by taking the ratio of Z/P_h and $\bar{K}/P_h$ from (51) with $r_k = \bar{r}_k = 4.93\%$ and the other terms being the same.

Non-tax transaction costs in the ownership market Apart from the land transfer tax, the only other cost buyers may pay is a home inspection cost of about \$500, but this is very small relative to average house prices. Hence, buyer non-tax transaction costs C_h and C_i are set to zero in the calibration.

From the side of sellers of a property, the primary cost is the real-estate agent commission. Using Multiple Listing Service sales data, the average commission rate is about 4.5% of price. There are some other costs such as legal fees of around \$1,000, but these are negligible in comparison. Sellers may sometimes spend approximately \$2,500 on staging, but the seller's agent might cover this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_o is set to be 4.5% of the average house price P.

Maintenance costs The maintenance cost D paid by owners of a property is set so that it is 2.6% of the average price P. This cost is made up of a 2% physical maintenance cost and a 0.6% property tax in Toronto. The additional maintenance cost D_l for properties that are rented out is set to be 8% of the average rent R. There are two parts to this cost: approximately 5–7% that a landlord spends on hiring a property manager, and approximately 1% paying for services such as taking out garbage, shovelling snow, and salting walkways.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real-estate agents to lease their properties. Hence, C_l is set to be $1/12$ of average annual rent R. Tenants in Toronto do not typically pay a fee when arranging to rent a property, so the calibration targets a zero tenancy agreement fee A.

Flows within ownership and rental markets Flows within the two housing markets are related to the average time between moves, times on the market, and viewings per sale and lease. Information on time-to-move, time-to-sell, and time-to-lease is derived from Toronto MLS data on sales and rental transactions during the pre-policy period. Estimates of the moving hazard function imply that owner-occupiers move after $T_{mh} = 9.25$ years on average The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 3.04$ years. The average time-to-sell for property owners is 30.5 days and the average time-to-lease is 18.7 days. During this period, the fraction of withdrawals from for-sale listings is 48% and from for-lease listings is 22%. In light of these withdrawals, the targets are $T_{so} = (30.5/365)/(1-0.48) = 0.161$ and $T_{sl} = (18.7/365)/(1-0.22) = 0.066$. Adjustments for withdrawals are made because measures of time-on-the-market are calculated from the final successful listing without accounting for earlier unsuccessful attempts, so true time-on-the-market is longer.

Data on buyers' time-on-the-market and viewings per sale and per lease are not available for Toronto. Using the 'Profile of Buyers and Sellers' survey collected by NAR in the United States, Genesove and Han (2012) report that for the period 2006–2009 the ratio of average time-to-buy to average time-to-sell is 1.28, and the average number of homes viewed by home-buyers is 10.7. Using this information, the targets used are $T_{bh} = 1.28 \times T_{so} = 0.206$ and $\Lambda_h = 10.7/(1-0.48) = 20.6$, where the latter adjusts the number of viewings to account for the withdrawal rate seen in Toronto. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than reported viewings in the final successful listing. There is no data on the number of properties that renters view on average. According to an industry expert, renters view fewer properties than buyers, so the target adopted is half the number of viewings per sale ($\Lambda_l/\Lambda_h = 1/2$).

Flow search costs The estimated search costs are based on the opportunity cost of time spent searching. This approach requires data on the ratio of house prices to income. Taking the median household-level income from Statistics Canada implies a price-to-income ratio of $P_h/Y = 5.6$ in Toronto in 2007.

A.6 Calibration method

The calibration targets in Table 3 identify the parameters in Table 4. The logic behind the exact identification of the parameters is explained here, along with a method for computing the parameter values from the targets.

Parameters implied directly by the targets Some parameters can be deduced directly from the calibration targets. A value of χ is set directly. Given P and targets for D/P and C_o/P, the values of $D = (D/P) \times P$ and $C_o = (C_o/P) \times P$ follow immediately. The average transactions price is $P = iP_i + (1-i)P_h$, so $P_h = P/(1 - i + iP_i/P_h)$. Hence, C_h and C_i are obtained directly from the targets:

$$C_h = \frac{\frac{C_h}{P_h}P}{1-i+i\frac{P_i}{P_h}}, \quad \text{and } C_i = \frac{\frac{C_i}{P_i}\frac{P_i}{P_h}P}{1-i+i\frac{P_i}{P_h}}. \tag{A.89}$$

Since $R = (P_i/P_h) \times P/((P_i/R) \times (1 - i + iP_i/P_h)$, the cost parameters D_l and C_l also follow directly from the targets:

$$D_l = \frac{\frac{D_l}{R}\frac{P_i}{P_h}P}{\frac{P_i}{R}\left(1 - i + i\frac{P_i}{P_h}\right)}, \quad \text{and } C_l = \frac{\frac{C_l}{R}\frac{P_i}{P_h}P}{\frac{P_i}{R}\left(1 - i + i\frac{P_i}{P_h}\right)}. \tag{A.90}$$

Ratios and quantities implied by the targets The targets also provide some direct information about market tightness, the fraction of investors, transaction probabilities and selling/leasing rates, and the quantities of properties and households in different states. The fraction i of purchases made by buy-to-rent investors is given in (35). Using $\pi_h = \Lambda_h^{-1}$ from (A.77), it follows that $i = \psi\Lambda_h/(1 - \psi + \psi\Lambda_h)$, with which the fraction ψ of investors among all buyers is obtained from the targets for i and average viewings Λ_h per home-buyer:

$$\psi = \frac{i}{i + (1 - i)\Lambda_h}. \tag{A.91}$$

Using (A.56) with $(1 - i)s_o u_o = v_o \pi_h b_h$ from (32) and (36), it follows that $u_o = (T_{so}/T_{mh})q_h/(1 - i)$ and $b_h = (T_{bh}/T_{mh})q_h$ with reference to (A.77) and (A.78). Hence, using the definition of the homeownership rate h from (2):

$$q_h = \frac{nhT_{mh}}{T_{mh} + T_{bh}}, \quad \text{and } u_o = \frac{nhT_{so}}{(1 - i)(T_{mh} + T_{bh})}. \tag{A.92}$$

Equation (A.67) together with (A.77) and (A.78) implies $u_l = (T_{sl}/T_{ml})q_l$, and combining this with the total measure of properties from (1) leads to:

$$q_l = \frac{(1 - q_h - u_o)T_{ml}}{T_{ml} + T_{sl}}, \quad \text{and } u_l = \frac{(1 - q_h - u_o)T_{sl}}{T_{ml} + T_{sl}}. \tag{A.93}$$

Using (17), (36), and (A.77), it follows that $T_{bl}/T_{sl} = \theta_l$ and $T_{bh}/T_{so} = \theta_o(1 - \psi + \psi\pi_h^{-1})$, where the latter is solved for ownership-market tightness θ_o. Combining the definitions from (23) and (24), $b_h = (1 - \psi)\theta_o u_o$, and hence (2) provides an equation for rental-market tightness θ_l:

$$\theta_o = \frac{1}{1 - \psi + \psi\Lambda_h}\frac{T_{bh}}{T_{so}}, \quad \theta_l = \frac{n - q_h - q_l - (1 - \psi)\theta_o u_o}{u_l}, \quad \text{and } T_{bl} = \theta_l T_{sl}, \tag{A.94}$$

where ψ, q_h, u_o, q_l, and u_l are taken from (A.91), (A.92), and (A.93), and the other terms are known targets. Note that the value of T_{bl} cannot be chosen freely given the other targets.

Exit rate of investors Substituting $(m_h + \rho)q_h = (1 - i)s_o u_o$ from (A.56) into $s_o u_o = (m_h + \rho)q_h + \rho_l(q_l + u_l)$ from (A.57) implies $\rho_l = is_o u_o/(q_l + u_l)$. Using the formulas from (A.77), (A.92), and (A.93), this can be expressed as follows in terms of the targets:

$$\rho_l = \frac{i/(1 - i)}{(1 - nh)/nh}\frac{1}{T_{mh}}\left(\frac{1}{1 + \frac{T_{bh} - nhT_{so}/(1 - i)}{(1 - nh)T_{mh}}}\right). \tag{A.95}$$

Intuitively, this is identified from a comparison of the flow of investor transactions i relative to the stock of properties in the rental market.

Demographics and transitions to homeownership The target for the fraction ϕ of first-time buyers among all home-buyers provides information about the turnover rate ρ of households in the city. Equation (A.79) for the steady-state fraction ϕ can be written in terms of ρ and home-buyers' time-on-the-market T_{bh} and owner-occupiers' expected time between moves T_{mh} from (A.77) and (A.78):

$$\phi = \frac{\rho\left(1 + \frac{m_h + \rho}{v_o \pi_h}\right)}{m_h + \rho\left(1 + \frac{m_h + \rho}{v_o \pi_h}\right)} = \frac{\rho\left(1 + \frac{T_{bh}}{T_{mh}}\right)}{\frac{1}{T_{mh}} + \rho\frac{T_{bh}}{T_{mh}}}.$$

Intuitively, ϕ identifies ρ because being an owner-occupier is an absorbing state for households that remain in the city, so a flow of first-time buyers depends on new arrivals to the city. The equation above can be solved explicitly for ρ, and once ρ is known, m_h is inferred from $m_h = (1/T_{mh}) - \rho$ with (A.78):

$$\rho = \frac{\phi}{T_{mh} + (1-\phi)T_{bh}}, \quad \text{and } m_h = \frac{(1-\phi)(T_{mh}+T_{bh})}{T_{mh}(T_{mh}+(1-\phi)T_{bh})}. \tag{A.96}$$

Taking ρ from (A.96) and using the formula for T_{ml} in (A.78) yields $m_l = T_{ml}^{-1} - \rho$, and it can be checked whether this is positive. With equation (8) and ρ_l from (A.95), the parameter $\alpha_l = m_l - \rho_l$ is obtained.

The target $\aleph$ for the difference between the average ages of owners and renters provides information about the probability ξ that renters draw a new credit cost when moving. Intuitively, if credit costs were drawn once and for all when households entered the city, there would be no reason in the model why the average ages of the two groups would differ. Using equation (A.86), the value of $\aleph$ is therefore informative about how long it is expected to take for a renter to make the transition to being an owner-occupier. This equation is rearranged to show $\xi m_l v_l \pi_l(\rho + (1-\rho\aleph)(\rho + m_l + v_l\pi_l)) = \rho^2\aleph(\rho + m_l + v_l\pi_l)$, which can be solved explicitly for ξ with reference to (A.77) and (A.78):

$$\xi = \frac{\rho^2\aleph(\rho + m_l + v_l\pi_l)}{m_l v_l \pi_l(\rho + (1-\rho\aleph)(\rho + m_l + v_l\pi_l))} = \frac{\rho^2\aleph\left(1+\frac{T_{ml}}{T_{bl}}\right)^2}{\frac{T_{ml}}{T_{bl}}\left(\frac{1}{T_{ml}} - \rho\right)\left(\rho T_{ml} + (1-\rho\aleph)\left(1+\frac{T_{ml}}{T_{bl}}\right)\right)}, \tag{A.97}$$

and this is known given the targets and the values of T_{bl} and ρ from (A.94) and (A.96).

By substituting $v_l\pi_l b_l = s_l u_l$ into (A.70) and using $s_l u_l = (m_l + \rho)q_l$ from (A.67), it follows that $(\xi m_l q_l + \rho n)\kappa = \rho(n - q_l - b_l)$. Together with (2) and q_l from (A.93), the value of κ is:

$$\kappa = \frac{\rho n h}{\xi m_l q_l + \rho n}. \tag{A.98}$$

Distribution of credit costs The calibration targets related to the mortgage interest rates and other aspects of mortgage contracts determine the present-discounted value of total credit costs K relative to house prices P_h for an average and a marginal home-buyer. The implied values of $\bar{K}/P_h$ and Z/P_h are given by the formula in (51), which determine $\bar{K}$ and Z using $P_h = P/(1 - i + iP_i/P_h)$. These provide information about the mean and standard-deviation parameters μ and σ of the log-Normal credit-cost distribution from (42). Since $\kappa = \Gamma_k(Z)$, the marginal credit cost Z is at a known percentile κ of the distribution from (A.98), and $\bar{K} = \mathbb{E}[K|K \le Z]$ is the mean credit cost K conditional on being below the threshold Z.

Using (42), the marginal credit cost Z and the parameters μ and σ satisfy $\log Z = \mu + \sigma\Phi^{-1}(\kappa)$, where $\Phi(\cdot)$ is the standard Normal cumulative distribution function, and the conditional mean satisfies $\log\bar{K} = \mu + \sigma^2/2 + \log\Phi((\log Z - \mu - \sigma^2)/\sigma) - \log\Phi((\log Z - \mu)/\sigma)$. Subtracting the first equation from the second, noting that μ cancels out and using $\kappa = \Phi((\log Z - \mu)/\sigma)$ and $(\log Z - \mu)/\sigma = \Phi^{-1}(\kappa)$:

$$\log\left(\frac{Z}{\bar{K}}\right) - \log\kappa - \sigma\Phi^{-1}(\kappa) + \frac{\sigma^2}{2} + \log\Phi\left(\Phi^{-1}(\kappa) - \sigma\right) = 0. \tag{A.99}$$

Evaluating at $\sigma = 0$ shows that the left-hand side is $\log(Z/\bar{K})$, which is strictly positive because $Z > \bar{K}$. The derivative with respect to σ is $-(\Phi'(\Phi^{-1}(\kappa)-\sigma)/\Phi(\Phi^{-1}(\kappa)-\sigma) + (\Phi^{-1}(\kappa)-\sigma))$. This is strictly negative because the Normal CDF satisfies $\Phi'(w)/\Phi(w) > -w$ for any w, hence the left-hand side of (A.99) is strictly decreasing in σ. Moreover, by L'Hôpital's rule, the left-hand side behaves like $\log(Z/\bar{K}) - \log\kappa - (\sigma/2)\Phi^{-1}(\kappa) - (\sigma/2)(\Phi'(\Phi^{-1}(\kappa)-\sigma)/\Phi(\Phi^{-1}(\kappa)-\sigma) - (\Phi^{-1}(\kappa)-\sigma))$ for very large σ. The first two terms are constants, the final term is strictly negative, and the third term is negative and linear in σ. It follows that the left-hand side of (A.99) becomes negative for sufficiently large σ, hence there always exists a unique solution of the equation for σ, which can be found numerically. Given this solution, the other parameter of the credit-cost distribution is $\mu = \log Z - \sigma\Phi^{-1}(\kappa)$.

Search costs and entry costs The steady-state value of N in (5) is zero, hence $E = \kappa(B_h - \bar{K}) + (1-\kappa)B_l$. Using the equation for Z from (3), the entry cost parameter is $E = B_l + \kappa(Z - \bar{K})$, which is identified by the target for B_l, the known values of Z and $\bar{K}$, and κ from (A.98).

The flow search-cost parameters F_h, F_i, F_l are obtained from the following:

$$F_h = \frac{P}{365(1-i+iP_i/P_h)(P_h/Y)} \frac{\Lambda_h}{T_{bh}} \frac{F_h/v_o}{Y/365}, \quad F_i = \frac{F_i}{F_h} F_h, \quad \text{and } F_l = \frac{\Lambda_l}{\Lambda_h} \frac{T_{bh}}{T_{bl}} \frac{F_l/v_l}{F_h/v_o} F_h, \quad \text{(A.100)}$$

which are stated in terms of the calibration targets (and T_{bl} from A.94) by using (A.77).

Discount rate and bargaining powers Information on prices, rents, costs, and time-on-the-market is used to identify the discount rate r for future housing payoffs and the bargaining-power parameters ω_h, ω_i, and ω_l. Taking the equations (A.54) and (A.55) and dividing both sides of the first by P_h, and similarly for the difference between the two equations:

$$1 = \frac{r + \theta_o v_o (1-\psi)\pi_h}{r} \frac{\omega_h^* \Sigma_h}{\pi_h P_h} + \frac{\theta_o v_o \psi}{r} \frac{\omega_i^* \Sigma_i}{P_h} + \frac{C_o - (D/r)}{P_h}, \quad \text{and} \quad 1 - \frac{P_i}{P_h} = \frac{\omega_h^* \Sigma_h}{\pi_h P_h} - \frac{\omega_i^* \Sigma_i}{P_h}.$$

Solving these simultaneous equations for the surpluses yields an expression for Σ_h:

$$\frac{\omega_h^* \Sigma_h}{\pi_h P_h} = \frac{\left(1 - \frac{C_o}{P_h}\right) r + \frac{D}{P_h} + \theta_o \psi \frac{\Lambda_h}{T_{bh}} \left(1 - \frac{P_i}{P_h}\right)}{r + T_{so}^{-1}}, \quad \text{(A.101)}$$

which uses formulas for T_{so}, T_{bh}, and Λ_h from (A.77), and an expression for Σ_i:

$$\frac{\omega_i^* \Sigma_i}{P_h} = \frac{\left(1 - \frac{C_o}{P_h}\right) r + \frac{D}{P_h} - \left(r + \theta_o (1-\psi) T_{bh}^{-1}\right) \left(1 - \frac{P_i}{P_h}\right)}{r + T_{so}^{-1}}. \quad \text{(A.102)}$$

Using equation (16) for the equilibrium tenancy agreement fee A, the sum of the total transaction costs incurred by landlords and tenants is

$$C_l + C_w = \frac{1}{\omega_l} \left(1 - \frac{A}{C_l}\right) C_l. \quad \text{(A.103)}$$

Dividing both sides of the steady-state average rent equation (A.75) by R, substituting for $\omega_l(C_l + C_w)$ using (A.103), and rearranging to write an equation for the rental-market surplus Σ_l:

$$\frac{\omega_l \Sigma_l}{\pi_l R} = \frac{1 - \frac{D_l}{R} - (r + m_l + \rho)\omega_l \frac{(C_l + C_w)}{R}}{r + m_l + \rho + \theta_l v_l \pi_l} = \frac{1 - \frac{D_l}{R} - \left(r + T_{ml}^{-1}\right)\left(1 - \frac{A}{C_l}\right)\frac{C_l}{R}}{r + T_{ml}^{-1} + T_{sl}^{-1}}, \quad \text{(A.104)}$$

which makes use of the formulas for T_{sl} and T_{ml} from (A.77) and (A.78).

Investors' surplus Σ_i satisfies (38), and this equation can be written as follows:

$$\frac{\omega_i^*}{1 - \omega_i^*} = \frac{\frac{\Lambda_h}{T_{bh}} \frac{\omega_i^* \Sigma_i}{P_h}}{\frac{F_i}{F_h} \frac{F_h}{P_h}}. \quad \text{(A.105)}$$

Dividing both sides of the free-entry condition (A.63) by P_i and using the investor price equation (A.55) to substitute $\theta_o v_o((1-\psi)\omega_h^* \Sigma_h + \psi \omega_i^* \Sigma_i)/P_i = (D/P_i) + r(1 - (C_o/P_i) - (\omega_i^* \Sigma_i/P_i))$:

$$\theta_l v_l \pi_l \left(\frac{\omega_l \Sigma_l}{\pi_l P_i}\right) = \left(1 + \tau_i \left(1 + \frac{\rho_l}{r}\right)\right) \left(D + r\left(1 - \frac{C_o}{P_i} - \frac{\omega_i^* \Sigma_i}{P_i}\right)\right)$$
$$+ (r + \rho_l)\left((1+\tau_i)\frac{C_o}{P_i} + \frac{C_i}{P_i} + (1 + \tau_i \omega_i^*)\frac{\Sigma_i}{P_i}\right) - \tau_i \left(1 + \frac{\rho_l}{r}\right)\frac{D}{P_i},$$

which simplifies to the following by noting that $r + \rho_l - r\omega_i^* = (r+\rho_l)(1-\omega_i^*) + \rho_l \omega_i^*$ and $\theta_l v_l \pi_l = T_{sl}^{-1}$:

$$\frac{1}{T_{sl}} \frac{R}{P_i} \frac{\omega_l \Sigma_l}{\pi_l R} = \frac{D}{P_i} + r + \tau_i(r + \rho_l) + \rho_l \frac{C_o}{P_i} + (r + \rho_l)\frac{C_i}{P_i} + (r+\rho_l)\frac{(1-\omega_i^*)}{\omega_i^*} \frac{\omega_i^* \Sigma_i}{P_h} \frac{P_h}{P_i} + \rho_l \frac{P_h}{P_i} \frac{\omega_i^* \Sigma_i}{P_h}.$$

Substituting from (A.102), (A.104), and (A.105) leaves an equation in just one unknown r:

$$\frac{D}{P_i}+r+\rho_l\frac{C_o}{P_i}+(r+\rho_l)\left(\tau_i+\frac{C_i+F_i\frac{T_{bh}}{\Lambda_h}}{P_i}\right)+\rho_l\left(\frac{\left(1-\frac{C_o}{P_i}\right)r+\frac{D}{P_i}-\theta_o(1-\psi)T_{bh}^{-1}\left(\frac{P_h}{P_i}-1\right)}{r+T_{so}^{-1}}\right)$$
$$=\frac{1}{T_{sl}}\frac{R}{P_i}\left(\frac{1-\frac{D_l}{R}-\left(r+T_{ml}^{-1}\right)\left(1-\frac{A}{C_l}\right)\frac{C_l}{R}}{r+T_{ml}^{-1}+T_{sl}^{-1}}\right). \quad \text{(A.106)}$$

The right-hand side is strictly decreasing in r, while the second and fourth terms on the left-hand side are linear in r with positive coefficients. Under a weak restriction that time-to-sell T_{so} is not too long, specifically $T_{so}<(1-(C_o/P_i))/((D/P_i)-\theta_o(1-\psi)T_{bh}^{-1}((P_h/P_i)-1))$, the final term on the left-hand side is also increasing in r, and hence any solution of (A.106) is unique and r is identified.[53] The left-hand side exceeds the right-hand side for large r, so existence of a solution is verified by checking the left-hand side is below the right-hand side at $r=0$, which is true for sufficiently high rental yields R/P_i. The value of r consistent with the calibration targets is then found by solving (A.106) numerically.

Dividing both sides of the steady-state Bellman equations (A.47) and (A.60) by P_h, using $B_h=B_l+Z$ from (3), and rearranging to solve for $\omega_h^*/(1-\omega_h^*)$ and $\omega_l/(1-\omega_l)$:

$$\frac{\omega_h^*}{1-\omega_h^*}=\frac{\frac{1}{T_{bh}}\frac{\omega_h^*\Sigma_h}{\pi_h P_h}}{\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}+(r+\rho)\frac{Z}{P_h}}, \quad \text{and} \quad \frac{\omega_l}{1-\omega_l}=\frac{\frac{1}{T_{bl}}\frac{R}{P_i}\frac{P_i}{P_h}\frac{\omega_l\Sigma_l}{\pi_l R}}{\frac{F_l}{F_h}\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}}, \quad \text{(A.107)}$$

which use $T_{bl}=1/(v_l\pi_l)$ and $T_{bh}=1/(v_o\pi_h)$ from (A.77). Tax revenue is given in (26), and by using (36) and (A.77), per-person expenditure on public services $g=G/n$ relative to house prices P_h is

$$\frac{g}{P_h}=\frac{\left(\tau_h(1-i)+\tau_i i\frac{P_i}{P_h}\right)u_o}{nT_{so}}, \quad \text{(A.108)}$$

which can be calculated from the targets and the value of u_o given in (A.92). Note that (28) and (34) imply $\omega_h^*/(1-\omega_h^*)=(\omega_h/(1-\omega_h))/(1+\tau_h)$ and $\omega_i^*/(1-\omega_i^*)=(\omega_i/(1-\omega_i))/(1+\tau_i)$. Hence, by combining equations (A.101), (A.102), and (A.104) with (A.105) and (A.107), the bargaining powers ω_h, ω_l, and ω_l are determined by the following equations after knowing r by solving equation (A.106):

$$\frac{\omega_h}{1-\omega_h}=\frac{(1+\tau_h)\frac{1}{T_{bh}}\left(\left(1-\frac{C_o}{P_h}\right)r+\frac{D}{P_h}+\theta_o\psi\frac{\Lambda_h}{T_{bh}}\left(1-\frac{P_i}{P_h}\right)\right)}{\left(\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}+(r+\rho)\frac{Z}{P_h}\right)\left(r+\frac{1}{T_{so}}\right)}, \quad \text{(A.109)}$$

$$\frac{\omega_l}{1-\omega_l}=\frac{\frac{1}{T_{bl}}\frac{R}{P_i}\frac{P_i}{P_h}\left(1-\frac{D_l}{R}-\left(r+T_{ml}^{-1}\right)\left(1-\frac{A}{C_l}\right)\frac{C_l}{R}\right)}{\left(\frac{F_l}{F_h}\frac{F_h}{P_h}-\frac{g}{P_h}+(r+\rho)\frac{B_l}{P_h}\right)\left(r+\frac{1}{T_{ml}}+\frac{1}{T_{sl}}\right)}, \quad \text{(A.110)}$$

$$\frac{\omega_i}{1-\omega_i}=\frac{(1+\tau_i)\frac{\Lambda_h}{T_{bh}}\left(\left(1-\frac{C_o}{P_h}\right)r+\frac{D}{P_h}-\left(1-\frac{P_i}{P_h}\right)\left(r+\theta_o(1-\psi)\frac{1}{T_{bh}}\right)\right)}{\frac{F_i}{F_h}\frac{F_h}{P_h}\left(r+\frac{1}{T_{so}}\right)}. \quad \text{(A.111)}$$

Hence, $\omega_h=(\omega_h/(1-\omega_h))/(1+\omega_h/(1-\omega_h))$ and similarly for the other parameters ω_l and ω_i using (A.109)–(A.111). Once ω_l is known, the implied tenant moving cost C_w is deduced from (A.103):

$$C_w=\left(\frac{1}{\omega_l}\left(1-\frac{A}{C_l}\right)-1\right)C_l, \quad \text{(A.112)}$$

and it can be verified whether C_w is positive.

Meeting functions With ω_h, ω_l, and ω_i known, the meeting-function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l, where $\omega_o=(1-\psi)\omega_h+\psi\omega_i$ is the average

[53] This condition is satisfied for the calibration targets in Table 3.

bargaining power of sellers in the ownership market. Since market tightnesses θ_o and θ_l are known from (A.94) and the viewing rates v_o and v_l can be deduced from the targets (and T_{bl} from A.94) using (A.77), the meeting-function productivity parameters υ_o and υ_l are set to be consistent with (43):

$$\upsilon_o = \theta_o^{\eta_o} \frac{\Lambda_h}{T_{bh}}, \quad \text{and} \quad \upsilon_l = \theta_l^{\eta_l} \frac{\Lambda_l}{\Lambda_h} \frac{\Lambda_h}{T_{bl}}. \tag{A.113}$$

Rental-market parameters Using (40) and (A.77), equation (A.29) for the rental-market surplus can rearranged as follows:

$$\lambda_l = 1 + \frac{\omega_l \frac{y_l}{R}}{\left(r + \frac{1}{T_{ml}}\right) \frac{\omega_l \Sigma_l}{\pi_l R}}.$$

By using (A.62) to obtain an expression for y_l/R and substituting for $\omega_l \Sigma_l/(\pi_l R)$ from (A.104):

$$\lambda_l = 1 + \frac{\omega_l \left(\frac{D_l - F_l}{R} + \left(r + \frac{1}{T_{ml}}\right)\frac{C_l + C_w}{R} - \xi m_l \kappa \frac{(Z - \bar{K})}{R}\right)\left(r + \frac{1}{T_{ml}} + \frac{1}{T_{sl}}\right)}{\left(1 - \frac{D_l}{R} - \left(r + \frac{1}{T_{ml}}\right)\left(1 - \frac{A}{C_l}\right)\frac{C_l}{R}\right)\left(r + \frac{1}{T_{ml}}\right)} + \frac{(1 - \omega_l + \omega_l \theta_l)\frac{1}{T_{bl}}}{r + \frac{1}{T_{ml}}}. \tag{A.114}$$

Knowing λ_l, ζ_l is found using $\zeta_l = y_l \pi_l^{1/\lambda_l}$ implied by (40) along with (A.62), (A.77), and (A.104):

$$\zeta_l = \frac{D_l - F_l + \left(r + \frac{1}{T_{ml}}\right)(C_l + C_w) - \xi m_l \kappa (Z - \bar{K}) + \frac{1}{T_{bl}}\left(\frac{1 - \omega_l + \omega_l \theta_l}{\omega_l}\right)\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) R}{\Lambda_l^{\frac{1}{\lambda_l}}}. \tag{A.115}$$

Moving decisions and the size of idiosyncratic shocks The response β_{mh} of the log moving hazard rate to the increase in the transaction tax in the subsequent four years is one of the calibration targets. In the model, this hazard rate is the combined moving rate within and outside the city, namely $m_h + \rho$, so the model prediction to match to the econometric estimate of β_{mh} is the average response of $\log(m_h + \rho)$ in the four years after the tax change. The response must be computed using the numerical solution of the model's dynamics for given parameters. The endogenous response of the moving rate is most closely connected to the parameter δ_h that governs the size of the idiosyncratic shocks to match quality (if $\delta_h = 0$ then the moving rate would be exogenous and not respond to the tax change, as can be seen from 41). The value of δ_h is set to match β_{mh}, but it is convenient to search numerically over the following transformation $\varkappa$ of the parameter δ_h:

$$\varkappa = \frac{\alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h}\right)^{\lambda_h}}{\rho + \alpha_h \left(1 - \delta_h^{\lambda_h}\right)}. \tag{A.116}$$

The remaining parameters of the model can be found conditional on $\varkappa$ in what follows, and once these are known, δ_h is inferred from (A.116):

$$\delta_h = \left(\frac{(\rho + \alpha_h)\varkappa}{\alpha_h \left(\varkappa + (y_h/x_h)^{\lambda_h}\right)}\right)^{\frac{1}{\lambda_h}}, \tag{A.117}$$

where y_h/x_h can also be calculated from the calibration targets as explained below. Identification is confirmed by verifying numerically that there is a unique $\varkappa$ matching the moving hazard response.

Ownership-market match quality and the arrival of idiosyncratic shocks Using the definition of $\varkappa$ in (A.116) and equation (A.58) for m_h:

$$\alpha_h = (1 + \varkappa) m_h + \rho \varkappa, \tag{A.118}$$

and hence α_h is determined conditional on $\varkappa$ and the known values of m_h and ρ from (A.96). The steady-state value of x_h is found given the calibration targets by using (A.50), (A.77), (A.101), and (A.102):

$$x_h = \left(\frac{1}{T_{bh}} \left(\frac{1 - \omega_h^* + (1-\psi)\omega_h^* \theta_o}{\omega_h^*} \right) \left(\frac{\omega_h^* \Sigma_h}{\pi_h P_h} \right) + \psi \theta_o \frac{\Lambda_h}{T_{bh}} \left(\frac{\omega_i^* \Sigma_i}{P_h} \right) \right) P_h - F_h \,, \tag{A.119}$$

and the value of y_h is then computed using (A.49) together with (A.101), (A.102), (A.118), and (A.119):

$$y_h = x_h + (r+\rho+\alpha_h) \left(C_h + C_o + \tau_h \left(C_o - \frac{D}{r} + \frac{\theta_o}{r} \frac{P_h}{T_{bh}} \left((1-\psi) \frac{\omega_h^* \Sigma_h}{\pi_h P_h} + \psi \Lambda_h \frac{\omega_i^* \Sigma_i}{P_h} \right) \right) \right) . \tag{A.120}$$

Equation (A.51) for owner-occupiers' expected surplus can be rearranged as follows using (40):

$$\lambda_h = 1 + \frac{\omega_h^* \frac{y_h}{P_h}}{(r+\rho+\alpha_h)(1+\tau_h \omega_h^*) \frac{\omega_h^* \Sigma_h}{\pi_h P_h}} \left(1 + \frac{x_h}{y_h} \frac{\alpha_h \delta_h^{\lambda_h} \left(\frac{y_h}{x_h} \right)^{\lambda_h}}{r + \rho + \alpha_h (1 - \delta_h^{\lambda_h})} \right) .$$

Since the procedure is to search over $\varkappa$ rather δ_h, equation (A.117) is used to write $\alpha_h \delta_h^{\lambda_h} (y_h/x_h)^{\lambda_h} = (\rho+\alpha_h)\varkappa(y_h/x_h)^{\lambda_h}/(\varkappa + (y_h/x_h)^{\lambda_h})$ and $\alpha_h(1-\delta_h^{\lambda_h}) = (\alpha_h (y_h/x_h)^{\lambda_h} - \rho\varkappa)/(\varkappa + (y_h/x_h)^{\lambda_h})$. Substituting into the equation above and making use of (A.101) yields an equation for λ_h:

$$\lambda_h = 1 + \frac{\omega_h^* \frac{y_h}{P_h} \left(r + \frac{1}{T_{so}} \right) \left(1 + \frac{x_h}{y_h} \frac{\varkappa(\rho+\alpha_h)}{(r+\rho+\alpha_h) + \varkappa r (y_h/x_h)^{-\lambda_h}} \right)}{(r+\rho+\alpha_h)(1+\tau_h \omega_h^*) \left(\left(1 - \frac{C_o}{P_h} \right) r + \frac{D}{P_h} + \theta_o \psi \frac{\Lambda_h}{T_{bh}} \left(1 - \frac{P_i}{P_h} \right) \right)} . \tag{A.121}$$

Apart from λ_h, all terms in the above equation are known given the calibration targets (including $\varkappa$, chosen to target the moving hazard response), so the equation can be solved numerically for λ_h. As the left-hand side is below the right-hand side at $\lambda_h = 1$, but rises above the right-hand side as λ_h becomes large (the right-hand side is bounded, but the left-hand side increases linearly with λ_h), there exists a solution with $\lambda_h > 1$. While $1/((r+\rho+\alpha_h) + \varkappa r (y_h/x_h)^{-\lambda_h})$ is increasing in λ_h along with the left-hand side of (A.121), for $\varkappa < 1 + (\rho+\alpha_h)/r$, the right-hand side is a strictly concave function of λ_h, in which case the solution is unique.[54] Once λ_h is found, the parameter δ_h is obtained from (A.117) for a given value of $\varkappa$, and it can be checked whether $\delta_h y_h < x_h$. Finally, using (40), (A.77), and (A.120) the parameter ζ_h is given by $\zeta_h = y_h \pi_h^{1/\lambda_h} = y_h / \Lambda_h^{1/\lambda_h}$.

A.7 Additional quantitative results

[54] The condition that $\varkappa$ or r is sufficiently low is satisfied for the calibration targets in Table 3.

Table A.13: *Tax effects with 3% mortgage interest rate gap and no mobility across regions*

Variable	Model predictions average over 4 years Baseline	3% gap	No mobility
Owners' moving rate (T_{mh}^{-1})	-12% (matched)	-12% (matched)	-12% (matched)
Buy-to-own (BTO) sales (S_h)	-14%	-13%	-14%
Buy-to-rent (BTR) sales (S_i)	35%	13%	35%
Total sales (S)	-12%	-11%	-12%
Time-to-sell (T_{so})	6.0%	7.7%	6.4%
Leases-to-sales ratio (S_l/S_o)	15%	13%	14%
Price-to-rent ratio (P_i/R)	-1.6%	-1.6%	-1.6%
Average sales price (P)	-1.6%	-1.5%	-1.9%
Homeownership rate (h)	-0.23 p.p.	-0.089 p.p.	-0.23 p.p.
City population (n)	0.0%	0.0%	0.0%
Transaction tax revenue (G)	61%	62%	61%
Effective LTT tax rate ($\tau_h = \tau_i$)	Increased from 1.5% to 2.8% (1.3 p.p.)		

Notes: The responses of variables are reported as percentage changes.