Delay and Deadlines: Freeriding and Information Revelation in Partnerships*

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Abstract

We study two sources of delay in teams: freeriding and lack of communication. Partners contribute to the value of a common project, but have private information about the success of their own efforts. The desire to maintain a partner's motivation leads to a reluctance to share information which in turn affects the incentives to exert effort. When the deadline for project implementation is far away, unsuccessful partners freeride on each others' efforts. When the deadline draws close, successful partners stop revealing their success to maintain the motivation of other group members to work hard. We show that there is a unique finite optimal deadline that maximizes beneficial productive efforts while avoiding unnecessary delays. As long as the deadline is set optimally, welfare is higher when information is only privately observable rather than revealed to the entire partnership. We derive comparative statics results for common measures of team performance such as expected time to implementation and expected project value. Surprisingly, setting a tighter deadline may increase the expected time until the project is implemented, but may also increase the expected value of the project.

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1 Introduction

Two common sources of delay and low performance in teams are freeriding on effort and lack of communication. A large literature in economics (Olson 1965, Holmstrom 1982, Ostrom 1990) has focused on the former aspect, while social and organizational psychologists and management scholars (Stasser and Titus 1985, Morrison and Milliken 2000) have long stressed the latter source of inefficiency. In this paper, we analyze how the incentives to free-ride on effort and to communicate information interact and formally establish the importance of both sources of delay in team work.

In joint projects the returns to a partner's effort typically depend on how successful other partners have already been. A partner's choice to communicate her private information affects the future effort provision by other partners. In particular, the desire to maintain the motivation of others can lead a partner to stay silent about her own success and delay the project. This mechanism is in line with psychology and organizational behavior research which stresses that intentional revelation and withholding of information in (decision-making) groups are deliberate processes to get higher performance from other team members (Wittenbaum, Hollingshead and Botero 2004) and that "people may be motivated to strategically withhold information that they know to be important for the group" (Steinel, Koning and Utz 2010, page 86). This interaction necessitates the joint analysis of these two sources of delay and their remedies (e.g., deadlines) which we undertake in the present paper. Our model allows for a tractable equilibrium characterization with intuitive comparative statics that are consistent with the existing empirical evidence, as well as a clean welfare analysis that yields a unique and finite optimal deadline. We find that the ability to conceal information relaxes the standard freerider problem and as such improves welfare when deadlines are set optimally.

We consider a continuous-time model where each agent of a group of two can exert unobservable effort to stochastically produce a "breakthrough" or "success" for a joint project and can unilaterally choose when to implement this joint project before a finite time horizon T. The agents have private information about the success of their contributing efforts and decide whether to disclose their success or not. We assume that there are diminishing returns to output (or breakthroughs) such that agents' efforts are substitutes.[1] This general setting using stochastic production and private information about the agents' own level of production applies to a wide range of settings.

Example 1 (Entrepreneurship)**.** *Two entrepreneurs are engaged in a common venture and are each trying to raise funding to implement their joint project. Investing effort in raising funds is privately costly to each entrepreneur as it requires convincing investors of the merits of the project and it may not necessarily result in fundraising success. Any successfully raised funds contribute to the final value of the project and, before the project is implemented, are only observable to the entrepreneurship partner who obtained them. At any point in time each entrepreneur is free to communicate how much funding she has already raised and may decide to call an end to the fundraising process and implement the project.*

Example 2 (Committee Decision)**.** *A committee or managerial board is asked to make a decision in a situation where delay is costly and information collection is endogenous. By exerting costly private effort*

[1] In our examples this corresponds to the first dollar, the first piece of information and the first idea being the most valuable.

a committee member increases the probability of acquiring private information about the right course of action. However, revealing this information reduces a partner's incentives to acquire more information if the marginal value of information is decreasing.[2] *To minimize delay each committee member can request the committee to make a final decision at any point.*

Example 3 (Research & Development)**.** *Two researchers inside an organization are tasked with solving a problem. Each researcher separately exerts effort thinking about the solution and stochastically generates ideas. When the final attempt to solve the problem is made, having more ideas increases the success probability of the solution. The longer the organization takes to solve the problem, the greater the costs of delay due to missed market opportunities.*

We characterize the symmetric equilibrium as a function of the length of the deadline. When the time available until the deadline is short, unsuccessful agents exert maximum effort and successful agents reveal no information about their success. Agents have little time to generate breakthroughs and thus have strong incentives from the start of the game to exert effort themselves rather than to count on their partner's effort. In response to this high effort choice, any agent who successfully produced a breakthrough stops exerting effort and prefers to withhold information about her own success since she benefits from the potential production of another breakthrough by a hard-working unsuccessful agent. Because successes are not revealed, the implementation of the project is delayed. These strategies are not sustainable in equilibrium when the deadline is far away. On the one hand, an unsuccessful agent believes that it is too likely that her partner will successfully produce a breakthrough prior to the deadline. As the expected return to her own effort is decreasing in this belief, she would no longer be willing to exert such a high level of effort. On the other hand, a successful agent is not prepared to incur the cost of delay until the deadline before implementing the project. Instead, each agent exerts low effort and decides to implement the project upon successful production. Thus, the equilibrium for long deadlines has two phases: a first phase of low effort intensity and immediate implementation upon success; and a second phase of high effort intensity, no information disclosure and full delay of implementation until the deadline.

The symmetric equilibrium strategies show that inefficient delay is due to two causes: lack of effort exertion far from the deadline and lack of information revelation close to deadline. The first inefficiency arises from the moral hazard in teams problem. The second inefficiency is caused by a communication problem in which each agent refuses to share information about his own success to maintain the motivation of his partner. As no information is revealed close to the deadline, partnerships are expected to implement projects early on or to wait until the deadline. Our model suggests an explanation for the common phenomenon, referred to as "Parkinson's Law" (Parkinson 1955, 1958), that partnerships or teams delay the implementation of projects without their members actually exerting effort or radically reducing effort just to fill the time until implementation of the project at the deadline.

Tight deadlines are often assumed not only to reduce the expected implementation time, but also

[2] One standard model of committee decision making that applies directly to our setup involves committee members with identical quadratic loss functions who must match the decision to the state of the world, who hold the same normally distributed prior and who can acquire normally distributed signals about the state of the world. A more general information and signal structure may be accomodated when it is assumed that the realization of the signal is not known until the time when the final decision is made.

to reduce the expected value of the implemented project. We find that the opposite may occur. A longer deadline increases the time available for production. Breakthroughs may be produced in the time before the moment when team members stop revealing information about their own success (and delay implementation until the deadline). This increases the probability of an early implementation decision, but with potentially fewer breakthroughs in expectation. We show that there is a unique finite deadline that maximizes agents' ex-ante welfare by maximizing beneficial productive efforts while avoiding unnecessary delays.

Our model highlights the importance of observability of output production. When information about private successes is immediately observable to all team members, freeriding on effort exertion will lead to a severe underprovision of productive efforts, but at least projects will always be implemented immediately once a breakthrough has been achieved. In contrast, when breakthroughs are only privately observable, team members have stronger incentives to exert effort as they can reap informational rents from not disclosing their success. However, successful agents' attempts to benefit from another team member's breakthroughs causes projects to be implemented unnecessarily late. We show that, as long as the deadline is set optimally, welfare is always higher when breakthroughs are privately observable as the benefits of higher effort outweigh the costs of delay. We show that the strong effort incentives that exist when breakthroughs are private information, may even induce teams to exert more effort and generate more breakthroughs than is socially efficient. But when the deadline is set inefficiently short, the team members may benefit from making breakthroughs publicly observable, thereby eliminating any inefficient delay from lack of information sharing.

We further investigate how simple instruments and common features of some group production settings, like bonus payments, communication frictions and third-party information intermediaries such as a committee chairperson, affect the incentives to exert effort and to reveal information. Finally, we show that our qualitative findings are robust to modifications in the production technology and how adding team members influences effort exertion and delay.

1.1 Related Literature and Evidence

Our model contributes to the literature on sequential public good provision (Admati and Perry 1991, Varian 1994, Teoh 1997, Marx and Matthews 2000) which studies voluntary (private) contributions to a joint project over time in situations where contributions deterministically influence output and the aggregate level of contributions is publicly observed. In our model, private contribution efforts only stochastically influence output and the success of production is private information. As a result, we are able to investigate the timing of voluntary information disclosure of successes by contributors and its role in delaying the completion of team projects. Our dynamic modeling approach is most closely related to Bonatti and Hörner (2011) who study effort incentives in teams in a continuous-time framework. There are two fundamental differences between our setting and theirs. First, they introduce exogenous uncertainty regarding the feasibility of the project. Second, in their model there is only one decision (effort) that the agent makes at each point in time and a success by either agent immediately ends the game so that there is no private information about successful production. The present paper instead departs from the existing literature by introducing private information about production successes when one team member chooses to conceal a breakthrough from his partner. This endogenously leads to uncertainty about the returns to effort and to interactions between the agents' decisions about effort and disclosure.

From an applied theory perspective our model shares a common concern about information revelation with the large and growing literature on decision making in groups (see Example 2). Several previous contributions have focused on the distorted incentives to reveal private information in the presence of conflicting preferences (Li, Rosen and Suen 2001, Dessein 2007, Gerardi and Yariv 2007, Damiano, Li and Suen 2009, 2010),[3] of reputation or career concerns (Ottaviani and Sorensen 2001, Levy 2007, Visser and Swank 2007) and of different voting rules (Feddersen and Pesendorfer 1998). In contrast, in our model preferences are perfectly aligned, conditional on the available information, and individuals strictly prefer to reveal their private information when a decision is made. Another strand of the literature analyzes how incentives for individual information acquisition in committees can be optimally provided by structuring the decision procedure (Persico 2004), the size of the committee (Mukhopadhaya 2003, Cai 2009), the voting rules (Li 2001, Gerardi and Yariv 2008, Lizzeri and Yariv 2011), or by restricting the action space (Szalay 2005). Gershkov and Szentes (2009) provide one notable exception in this literature as they also focus on influencing the prior information of team members to induce them to exert costly effort to acquire information.[4,5] However, in our setting lack of information arises endogenously through the actions of the players rather than it being a feature of the optimally designed contract or mechanism. To our knowledge, the present paper is the first to study the interplay of incentives to acquire and to reveal information in a dynamic setting, thus closing the gap between the two above-mentioned strands of the literature on group decision making. Furthermore, in addition to having the dual instruments of information acquisition and disclosure available to them, in our model agents can also choose *when* to search for and disclose information.

Our model's central trade-off between private effort exertion and intra-group communication also speaks to the large literature on group performance and decision making outside of economics. Our findings contribute to the understanding of why groups often fail to make decisions in a timely manner and of whether common practices like deadlines and public disclosure actually improve performance. First, in their seminal research on group decision making Stasser and Titus (1985) and Stasser (1999) show that groups do not share information effectively and that the lack of proper information sharing and integration inhibits group problem-solving effectiveness. In our model, the reluctance to share information is a result of each agent's desire to maintain the motivation of other team members and leads to delay and weakened effort incentives. Second, management scholars have long stressed that while group decision making tends to lead to more information and knowledge being available when decisions are made, the decision-making process often takes longer and is costlier than individual decision-making. In his popular textbook on management practices Griffin (2006, page 250) notes that "perhaps the

[3] Another strand of literature studies how incentives for information acquisition arise if decision-makers have different preferences (Aghion and Tirole 1997) or beliefs (Che and Kartik 2009) and thus make different decisions conditional on holding the same information.

[4] Lack of information also features in the optimal contract design of principal-agent relationships studied by Fuchs (2007) and Maestri (2012) where it sustains higher efforts or prolongs efficient relationships.

[5] Blanes-I-Vidal and Moeller (2011) also study the impact on team members' incentives from communicating private information, but their focus is on incentives to implement a common decision rather than on incentives to acquire information.

biggest drawback from group and decision making is the additional time and hence the greater expense entailed. [...] Assuming the group or team decision is better, the additional expense may be justified." In particular, delay and indecision are found to be common in top management teams and can have serious consequences including missed market opportunities (Eisenhardt 1989). In our model, group decision-making also results in excess delay and groups may even overacquire information (produce too many breakthroughs) relative to the first best, despite the presence of a freeriding problem. Third, to help promote the effectiveness of group decision-making Griffin (2006) advocates the careful use of deadlines: "Time and cost can be managed by setting a deadline by which the decision must be made final." Carrison (2003, page 122) warns that excessively tight deadlines worsen the information revelation problem because "whenever the workplace is charged with the electricity of a race against time, clear communication can suffer". Our formal analysis of the interaction between incentives for information acquisition and information sharing shows how standard team practices to incentivize group members, like the imposition of deadlines and disclosure rules, though often beneficial, can also backfire when used incorrectly. While tight deadlines increase the cost of freeriding, the resulting increase in search efforts reduces the incentives to reveal information when this information discourages group members from searching intensively. Our model also provides a formal characterization of "Parkinson's Law" (Parkinson 1955, 1958) which posits that "work fills the time available" as no projects are implemented before the final deadline.

The remainder of the paper is organized as follows. In Section 2 and 3, we introduce the model and characterize the equilibrium strategies. In Section 4, we perform a welfare analysis for different deadline length and compare outcomes between settings where information about breakthroughs is private or public. In Section 5, we derive additional comparative statics results for common measures of team performance such as expected time to implementation and expected project value as a function of the length of the game. Finally, in Section 6 we discuss robustness and extensions. Section 7 concludes. Proofs are provided in the Appendix.

2 Setup

We consider a continuous-time setup where t denotes the time of the game. Two agents are engaged in production for a joint project with common value.

Both agents can exert costly private effort $e \in [0, e_{\max}]$ to stochastically generate "breakthroughs" or "successes" which increase the value of the joint project. Each agent's effort determines the exponential rate λe at which an additional breakthrough is generated. This rate is independent of the other agent's effort. Each agent incurs a linear effort cost ce and an agent's effort is unobservable to the other agent. This results in a standard moral hazard problem within the team; both agents would like to freeride on each others' effort to produce breakthroughs. We call an agent *successful* at t if she has already produced a breakthrough at any time at or before t and *unsuccessful* if she has not produced a breakthrough at or before t.

Each agent can choose unilaterally, at any point in time, whether or not to implement the project. When the project is implemented, the common value of the project for each agent is realized and the game ends. As long as the project has not been implemented at a time t agents incur an additive delay cost δ. If the project is implemented at a time t then the total delay cost is δt. If the project has not been implemented before, the game ends at a (possibly arbitrarily large) finite horizon time T.

Successful agents may verifiably reveal, at any point in time, that they have been successful, whereas unsuccessful agents are unable to verifiably reveal that they are unsuccessful. As we will see, it will be unnecessary to draw a distinction between a successful agent revealing (not revealing) a success to the other agent and a successful agent unilaterally implementing (not implementing) the project. We discuss analogous equilibria in a setting where both types of agents may engage in cheap talk and the project requires joint implementation in Section 6.2. Throughout the paper, we use the notation ˜ (tilde) to denote the strategies of the other player and * (star) to denote equilibrium strategies and beliefs.

The common value of the project to each agent is a function of the number of breakthroughs agents generate prior to implementation. We denote this value by V_0 if no breakthroughs are achieved prior to implementation. We assume that the incremental returns to breakthroughs are decreasing where the *incremental* value of the n-th breakthrough is denoted by α_n, implying $\alpha_n \geq \alpha_{n+1}$. We further assume $[\lambda\alpha_1 - c]\, e_{\max} > \delta$ and $c \geq \lambda\alpha_2$, such that an agent would be prepared to exert effort to produce the first breakthrough but not the second breakthrough. Hence, we write $e(t)$ for the effort strategy of an unsuccessful agent, but do not include notation for the effort choice of a successful agent which is always 0. These simplifying assumptions allow us to sharpen the focus of the analysis on the fundamental trade-off between incentives to exert effort and incentives to disclose information. We illustrate the robustness of the insights regarding this trade-off when agents are willing to individually generate up to two breakthroughs in Section 6.3.

We consider symmetric perfect Bayesian equilibria of the continuous game with deadline T. The focus of this paper is on the setting where the production success of any agent is private information. This gives a successful agent the opportunity to conceal her own success to maintain motivation for her unsuccessful partner to continue exerting effort. We first analyze the incentives to exert effort when breakthroughs are publicly observable. This provides a natural benchmark for the case with private information. Notice that the first best outcome is for both agents to exert maximum effort until implementation. The project is implemented immediately after the successful production of one breakthrough if $(2\lambda\alpha_2 - c)\, e_{\max} \leq \delta$ and after the production of two breakthroughs otherwise. In the one-player version of the model, a player exerts maximum effort until she successfully produces one single breakthrough. When successful, she immediately implements the project.

2.1 Public Information

Under public information it is common knowledge when a breakthrough has been achieved. Under our assumption that $c \geq \lambda\alpha_2$ the only equilibria of the continuation game after a breakthrough involve the immediate implementation of the project. Since one success immediately triggers implementation, we do not draw a distinction between a success occuring and implementation of the project in the public information case. However, as we will see, under private information this distinction will be central.

We characterize the equilibrium effort strategy in terms of the continuation value of the game at time t for a successful and an unsuccessful agent, denoted by $V^S(t)$ and $V^U(t)$ respectively. The equilibrium effort strategy satisfies

$$e^*(t) \in \arg\max_{e\in[0,e_{\max}]} \lambda e\left(V^S(t) - V^U(t)\right) - ce. \tag{1}$$

When an agent's success is public information, the added value of a breakthrough for an agent does not depend on whether or not she produced this breakthrough herself. As soon as one of the agents successfully produces one breakthrough, the production phase is immediately ended and the project is implemented. The value of being successful at time t thus equals the value of implementing the project with one breakthrough, $V^S(t) = V_0 + \alpha_1$, where V_0 denotes the value of implementation without any breakthrough and α_1 is the incremental value of the first breakthrough.

For an unsuccessful agent, the continuation value of the game at time t depends on the probability that she or her partner will produce a breakthrough before the deadline. The probability that an agent has not produced a breakthrough by time s, provided that the production phase has not been ended before, equals

$$1 - F(s|t) = \exp\left(-\int_t^s \lambda e\left(x\right) dx\right).$$

We denote the corresponding density by $f(s|t)$. For her partner, we denote the probability and density by $\tilde{F}(s|t)$ and $\tilde{f}(s|t)$ respectively. In equation (2) the value of being unsuccessful at time t consists of three terms. The first and the second term are the expected payoffs when either the agent herself or her partner is successful prior to the deadline. The payoff is the same in both terms and depends on the value of the implemented project, the delay cost and the cost of effort exerted until the project is implemented. The density of a breakthrough by the agent herself is $f^*(s|t)(1-\tilde{F}^*(s|t))$ and the density of a breakthrough by her team member $(1-F^*(s|t))\tilde{f}^*(s|t)$. The third term is the expected payoff of reaching the deadline (without a breakthrough) which occurs with probability $(1-F^*(T|t))(1-\tilde{F}^*(T|t))$. Hence,

$$\begin{aligned} V^U(t) = &\int_t^T \left[V_0 + \alpha_1 - \delta\left(s-t\right) - c\int_t^s e^*(r)dr\right] f^*(s|t)(1-\tilde{F}^*(s|t))ds \\ &+ \int_t^T \left[V_0 + \alpha_1 - \delta\left(s-t\right) - c\int_t^s e^*(r)dr\right] (1-F^*(s|t))\tilde{f}^*(s|t)ds \\ &+ \left[V_0 - \delta\left(T-t\right) - c\int_t^T e^*(s)ds\right](1-F^*(T|t))(1-\tilde{F}^*(T|t)) \end{aligned} \tag{2}$$

The equilibrium strategies which are characterized in Proposition 1 depend on a threshold time interval Δ. When there is Δ time remaining until the deadline and both agents exert maximum effort for the time remaining, an agent is exactly indifferent about exerting effort at time $t = T - \Delta$. That is, Δ solves $\lambda\left[V^S(T-\Delta) - V^U(T-\Delta)\right] = c$ with $e^*(t) = e_{\max}$ for $t \geq T - \Delta$.

Proposition 1. *When information is public, the strategies in the unique, symmetric subgame perfect equilibrium are:*
i) *If $T \leq \Delta$, any unsuccessful player exerts maximum effort for all t.*
ii) *If $T > \Delta$, any unsuccessful player exerts low effort $e^*(t) = \frac{\delta}{c}$ for $t \leq T - \Delta$ and maximum effort for $t > T - \Delta$.*

There are two distinct phases of equilibrium behavior. In one phase, when the deadline is close, it provides strong incentives for effort because there is a significant chance that it will be reached without either agent being successful. When both agents exert maximum effort, the rate at which a breakthough occurs equals $2\lambda e_{\max}$. Hence, the difference between the continuation value of a successful and an unsuccessful agent at time t when both agents exert maximum effort until the deadline T is

$$V^S(t) - V^U(t) = \alpha_1 \exp\left[-2\lambda e_{\max}(T-t)\right] + (ce_{\max} + \delta)\frac{1-\exp\left[-2\lambda e_{\max}(T-t)\right]}{2\lambda e_{\max}}.$$

The first term captures the avoided loss of reaching the deadline without either agent being successful. The second term captures the avoided costs of effort and delay where the expected time until one of the agents is successful equals $\frac{1-\exp[-2\lambda e_{\max}(T-t)]}{2\lambda e_{\max}}$. The assumption $(\lambda\alpha_1 - c)\, e_{\max} > \delta$ implies that the difference $V^S(t) - V^U(t)$ is strictly decreasing in the time remaining until the deadline $T-t$. The time length Δ is such that at the moment $t = T - \Delta$ the agent is exactly indifferent about exerting effort. From that time until the deadline at T unsuccessful agents thus strictly prefer to exert maximum effort.

There is another phase of the equilibrium, when the deadline is far (i.e., $t \leq T - \Delta$), during which both agents are indifferent about their chosen level of effort, but set it in such a way as to maintain the indifference of the other agent. The incentives for effort are determined by the incentive to bring forward the time at which a decision is made, thereby avoiding delay costs, and by the incentive to freeride on the effort of the other agent, thereby avoiding effort costs. In equilibrium, these two effects exactly balance each other when the other agent exerts effort $\tilde{e} = \frac{\delta}{c}$. To see this, note that if an agent shifts effort by ε to the next instant, this allows her to avoid the expected effort costs $\lambda\tilde{e} \times c\varepsilon$, since the rate at which the other agent acquires information is $\lambda\tilde{e}$. On the other hand, shifting effort to the next instant increases delay costs δ at the rate $\lambda\varepsilon$, hence the additional delay cost is $\lambda\varepsilon \times \delta$. These two effects exactly offset one another when

$$\lambda\tilde{e} \times c\varepsilon = \lambda\varepsilon \times \delta \Leftrightarrow \tilde{e} = \frac{\delta}{c}.$$

In sum, when production success is public information, agents try to freeride on each others' effort provision. This results in low effort unless the deadline is close. Notice that the weak incentives to exert effort when the deadline is far could also result in asymmetric equilibria with one agent exerting $e_{\max}$ and the other agent exerting 0. While the linear cost of effort and bounded maximum effort assumptions are particularly tractable for characterizing the set of symmetric equilibria, these candidate asymmetric equilibria are not robust to the introduction of a strictly convex cost of effort function, which is typically assumed for analyzing freeriding in teams. Moreover, when one agent is exerting all the effort there is no interaction between freeriding and the incentives to disclose information. We therefore focus on symmetric equilibria.

2.2 Private Information

In the case of private information it is necessary to distinguish between an event where an agent is successful and one where the project is implemented. Either a successful or an unsuccessful agent may choose to implement the project. However, in equilibrium only the successful agent chooses to implement. To simplify our exposition we suppress the notation for the implementation strategy of an unsuccessful agent and write $d(t)$ for the mixed strategy of a successful agent to implement the project.[6] Importantly, we verify that it is an equilibrium for an unsuccessful agent to never implement the project and are thus only shortening the expressions below. We also note that if a successful agent reveals that she has been successful, the continuation game is identical to the continuation game under public information after one agent is successful. In both cases the project is implemented immediately and hence it is without consequence that we do not distinguish between a successful agent revealing a success and a successful agent (unilaterally) implementing the project. Further, upon common knowledge of a successful breakthrough both agents would agree to implement the project immediately. We discuss this issue in detail in Section 6.2.

Let $\tilde{G}(s|t)$ denote the implementation probability by the other team member by time s conditional on the project not being implemented before time t and $\tilde{g}(s|t)$ its corresponding density. From an agent's perspective, the ex-ante probability that the other agent will decide to implement by time t may be written as a weakly increasing function of time $\tilde{G}(t|0)$ and its corresponding density by $\tilde{g}(t|0)$. Since agents may decide not to implement the project after successfully generating a breakthrough, an agent updates her belief that her partner is still unsuccessful when the project has not been implemented. We denote this belief[7] which is the ratio of the breakthrough and implementation probabilities of the other player, by

$$\phi(t) = \frac{1 - \tilde{F}(t|0)}{1 - \tilde{G}(t|0)}.$$

In Section 3 we show that in all symmetric perfect Bayesian equilibria, subject to the earlier caveat, the equilibrium decision strategy is described by a continuous $\tilde{G}(t|0)$. In the interest of clarity, we will restrict our attention to mixing strategies which result in a continuous $\tilde{G}(t|0)$ in the main body of the paper and refer the reader to the appendix for the general specification. We describe an agent's mixed strategy at different points in time by $d(t) : [0,T] \to \{implement\} \times [0,\infty)$. If $d(t) = implement$ and $\phi(t) = 1$, the hazard rate at which the project is implemented is the rate at which unsuccessful agents become successful. Hence,

$$\frac{\tilde{g}(t|0)}{1 - \tilde{G}(t|0)} = \lambda e(t) \text{ if } d(t) = implement \text{ and } \phi(t) = 1.$$

Otherwise, for $\phi(t) < 1$, the hazard rate at which the implementation decision is made is described by $d(t) \in [0,\infty)$ and $\phi(t)$ in the following way,

$$\frac{\tilde{g}(t|0)}{1 - \tilde{G}(t|0)} = d(t)\,(1 - \phi(t)).$$

[6] In Section 3 (and in greater detail in Section B.3) we show that no symmetric equilibria involve unsuccessful agents unilaterally deciding to implement the project in equilibrium. The only exception is an equilibrium in which both types of agents decide to implement the project with certainty at a point in time, which is in effect equivalent to a deadline supported by off-equilibrium beliefs.

[7] The belief $\phi(t)$ is the same for both a successful and an unsuccessful team member. The reason for this equality is that a personal success reveals no information about the likelihood of the other team member being successful.

Bayesian updating implies that an agent's belief evolves in the following way,

$$\frac{d\phi(t)}{dt} = \begin{cases} 0 \text{ if } d(t) = \textit{implement} \text{ and } \phi(t) = 1, \\ [d(t)(1-\phi(t)) - \lambda e(t)]\,\phi(t) \ \text{ otherwise.} \end{cases} \tag{3}$$

Hence, if $d(t)(1-\phi(t)) = \lambda e(t)$ and $\phi(t) < 1$ or $d(t) = \textit{implement}$ and $\phi(t) = 1$ the belief $\phi(t)$ remains constant over time.

Perfect Bayesian Equilibrium The equilibrium strategy is the same for any continuation game starting at t and denoted by $\{e^*(t), d^*(t) \mid t \in [0,T]\}$. This is because any off-equilibrium strategy either ends the game or an earlier off-equilibrium effort choice cannot affect the beliefs of the other agent and is thus without consequence for the continuation game. The posterior belief $\phi^*(t)$ is formed according to Bayesian updating for a given strategy profile as in (3).

To characterize the equilibrium implementation strategy, we use the equilibrium continuation value $V^S(t)$ at time t for the successful player. For any t, this value consists of the payoff from a project with just one breakthrough, of the expected payoff when the other individual implements the project prior to the agent doing so herself which occurs with density $\tilde{g}^*(s|t)$, and of the expected payoff of implementing the project at time $\hat{t}$ which occurs with probability $1 - \tilde{G}^*\left(\hat{t}|t\right)$. Thus,

$$V^S(t) = V_0 + \alpha_1 + \max_{\hat{t}\in[t,T]} \left\{ \int_t^{\hat{t}} [\alpha_2 - \delta(s-t)]\tilde{g}^*(s|t)ds + [(1-\phi^*(\hat{t}))\alpha_2 - \delta(\hat{t}-t)](1-\tilde{G}^*(\hat{t}|t)) \right\} \tag{4}$$

The equilibrium implementation strategy $d^*(t)$ is determined as the solution of this optimal stopping problem. A successful player implements the project at t if her expected payoff in (4) is maximized at $\hat{t} = t$ and delays implementation if this payoff reaches a maximum at a later time $\hat{t} > t$. She may mix and implement at a rate $d^*(t) \in [0, \infty)$ only when she is indifferent between implementing and delaying implementation at time t. Compared to the public information case, the option not to disclose her success may increase the continuation value for a successful player. She is willing to delay the implementation of the project to give her partner the opportunity to produce additional breakthroughs. When deciding how long to delay the implementation, the agent trades off the potential increase in the value of the project when an unsuccessful partner produces an additional breakthrough, against the expected cost from delaying the implementation. Just before the deadline, the incentive to delay the project implementation is approximately equal to

$$\lambda\tilde{e}(T)\,\phi(T)\,\alpha_2 - \delta.$$

The return to delaying only depends on the expected increase in breakthroughs, which occurs with the probability that a still unsuccessful partner will produce additional breakthroughs. We define $\bar{\phi}_d$ as the belief for which a successful agent is indifferent between delay and implementation when the other agent exerts maximum effort, i.e.,

$$\lambda e_{\max}\bar{\phi}_d\alpha_2 = \delta.$$

To characterize the equilibrium effort strategy, we use the continuation value $V^U(t)$ at time t for the unsuccessful player. This value is represented by three simple terms in equation (5). The first term is the player's payoff in the event that her partner implements the project at time s without the player herself producing a breakthrough. In that case the team implements the project with only one breakthrough. This event occurs with density $(1 - F^*(s|t))\tilde{g}^*(s|t)$ which depends on the equilibrium effort strategy of the player herself and the implementation strategy of her team partner. The second term is the expected payoff in the event that the player produces a success at time s which happens with a density given by $f^*(s|t)(1 - \tilde{G}^*(s|t))$. In that case the player obtains the continuation value of being successful. Finally, the last term is the expected payoff in the event that the deadline is reached without the player herself producing a breakthrough. The probability of this event equals $(1 - F^*(T|t))(1 - \tilde{G}^*(T|t))$. While her partner has not implemented the project, she may still have realized a breakthrough with probability $1 - \phi^*(T)$. Thus, for any t, the equilibrium continuation value for the unsuccessful player equals

$$\begin{aligned} V^U(t) = &\int_t^T \left[V_0 + \alpha_1 - \delta(s-t) - c\int_t^s e^*(r)dr\right](1 - F^*(s|t))\tilde{g}^*(s|t)ds \\ &+ \int_t^T \left[V^S(s) - \delta(s-t) - c\int_t^s e^*(r)dr\right] f^*(s|t)(1 - \tilde{G}^*(s|t))ds \\ &+ \left[V_0 + (1-\phi^*(T))\,\alpha_1 - \delta(T-t) - c\int_t^T e^*(s)ds\right](1 - F^*(T|t))(1 - \tilde{G}^*(T|t)). \end{aligned} \tag{5}$$

In contrast to the public information case, a player values a breakthrough differently depending on whether she or her partner produced it because she can capture informational rents when she is successful herself. This is reflected by the fact that while the payoffs in the square brackets in the first and second term of equation (2) are identical, these payoffs differ in equation (5). The unsuccessful player's continuation value is written conditional on her not deciding to implement the project. To ensure that she does not have a strict incentive to decide to implement the project in the perfect Bayesian equilibrium, we verify that $V^U(t) \geq V_0 + (1 - \phi^*(t))\,\alpha_1$.

Like in the public information case, the effort decisions satisfy

$$e^*(t) \in \arg\max_{e\in[0,e_{\max}]} \lambda e\left(V^S(t) - V^U(t)\right) - ce. \tag{6}$$

An unsuccessful agent is more willing to exert effort the more valuable the breakthrough she is trying to produce. Before the deadline, the value of successfully producing a breakthrough also depends on the cost of future effort and any further delay that is avoided. At the deadline, however, the value of becoming successful only depends on the value of the achieved breakthrough. This value is lower when the partner has successfully produced a breakthrough as well. Evaluated at the deadline T, the marginal gain of effort equals

$$\lambda\left[\phi(T)\alpha_1 + (1-\phi(T))\,\alpha_2\right] - c.$$

Hence, at the deadline, an unsuccessful player is unwilling to exert any effort if the probability that her partner is unsuccessful $\phi(T)$ is smaller than the threshold $\bar{\phi}$ which is defined by

$$\lambda \left[\bar{\phi}\alpha_1 + \left(1-\bar{\phi}\right)\alpha_2\right] = c.$$

Assumption. *At the deadline T the incentives to exert effort are small relative to the incentives to delay, i.e., $\bar{\phi} > \bar{\phi}_d$.*

The above assumption implies that unsuccessful agents would stop exerting effort before successful agents stop delaying as the equilibrium belief $\phi^*(T)$ decreases. This assumption allows us to shorten the exposition by focusing on one of two cases. The equilibrium characterization, the implied welfare results and comparative statics are similar for both cases. We briefly describe the other case with $\bar{\phi} \leq \bar{\phi}_d$ in Section 6.1.

3 Equilibrium with Private Information

We now characterize the symmetric equilibrium strategies of the continuous-time game with deadline T. Equilibrium strategies change as the agents approach the deadline at T. They also depend on how large T is, that is to say how tightly the deadline is set at the start of the game. When the deadline is sufficiently tight (T is small), the unique equilibrium involves delay coupled with maximum effort throughout the game. When the deadline is sufficiently loose (T is large), any equilibrium involves no delay coupled with low effort at the start of the game and full delay coupled with high effort close to the deadline.

The equilibrium strategies characterized in Proposition 2 depend on two threshold time intervals X and Y where $Y > X$. There are three distinct phases depending on whether the time remaining until the deadline is (i) less than X, (ii) greater than X, but less than Y or (iii) greater than Y. The first threshold X equals the amount of time after which the belief $\phi^*(t)$ reaches $\bar{\phi}$ when an unsuccessful partner exerts maximum effort, but discloses no information after successfully producing a breakthrough,

$$\exp\left(-\lambda e_{\max} X\right) = \bar{\phi}.$$

This threshold thus determines the maximum amount of time for which unsuccessful players can be induced to exert maximum effort if a successful player does not disclose her success. The second threshold Y is the amount of time for which the total delay cost δY is exactly equal to the expected payoff for a successful player when his partner is successful (and thus α_2 is realized) with probability $1-\bar{\phi}$,

$$\delta Y = \left(1-\bar{\phi}\right)\alpha_2.$$

This threshold thus determines the maximum amount of time for which a successful player is willing to delay implementation if the probability ϕ that her partner is unsuccessful falls from 1 to $\bar{\phi}$.

Proposition 2. *When information is private, the equilibrium strategies are:*
(i) *If $T \leq X$, any successful player chooses not to implement, while any unsuccessful player exerts maximum effort.*
(ii) *If $X < T \leq Y$, any successful player chooses not to implement, while any unsuccessful player exerts an effort strategy which is sufficiently backloaded satisfying condition (8) and aggregates to*

$$\exp\left(-\lambda \int_0^T e^*(s)\,ds\right) = \bar{\phi}. \tag{7}$$

(iii) *If $T > Y$, any successful player decides to implement immediately, while any successful player exerts low effort $e^*(t) = \frac{\delta}{c}$ for $t < T - Y$. From $t = T - Y$ onwards, the equilibrium strategies coincide with the strategies for $T = Y$ in case* ***(ii)****.*

For ***(i)*** *and* ***(ii)*** *the players' beliefs evolve according to $\phi^*(t) = \exp(-\lambda \int_0^t e^*(s)\,ds)$ for all t. For* ***(iii)*** *they equal $\phi^*(t) = 1$ for $t < T - Y$. From $t = T - Y$ onwards, they coincide with the beliefs for $T = Y$ in case* ***(ii)****.*

Figures 1.A to 1.C illustrate the evolution over time t of the equilibrium effort e^* (lower panels), the implementation decision d^* and the equilibrium belief ϕ^* (upper panels) for different lengths of the deadline T. In Figure 1.A, the deadline T is relatively tight, corresponding to case (i) in Proposition 2 with $T \leq X$. Unsuccessful agents exert maximum effort $e_{\max}$ over the entire course of the game and successful agents delay implementation. The belief ϕ^* declines from the complete certainty at $t = 0$ that the other agent has been unsuccessful so far to $\phi^* \geq \bar{\phi}$ by the end of the game at T. This holds with equality $\phi^* = \bar{\phi}$ at T if $T = X$, as shown in Figure 1.A. Next, in Figure 1.B the length of the deadline T is longer, corresponding to case (ii) with $X < T \leq Y$. While successful agents still delay implementation, unsuccessful agents no longer exert maximum effort during the entire game. Instead, they choose to exert lower effort in such a way that the belief ϕ^* is equal to $\bar{\phi}$ at the end of the game. Effort is not fully tied down in equilibrium, but needs to be sufficiently backloaded as well, satisfying

$$\alpha_2 \left[1 - \exp\left(-\lambda \int_0^t e^*(s)\,ds\right)\right] \leq \delta t \text{ for all } t \in [0, T]. \tag{8}$$

The gray lines depict the two extreme equilibrium paths with minimal (dashed line) and maximal (full line) backloading of effort for $T = Y$. Finally, in Figure 1.C we depict the equilibrium paths for loose deadlines, corresponding to case (iii) with $T > Y$. At the beginning of the game unsuccessful agents exert effort $e^* = \frac{\delta}{c}$ and successful agents decide to implement the project immediately. Thus, during this initial phase the belief ϕ^* remains constant at 1. However, once enough time has elapsed the delay phase begins and the game proceeds as in Figure 1.B.

We now briefly discuss how the equilibrium dynamics emerge from the trade-off between the conflicting objectives of joint production and information sharing for the three cases in Proposition 2. Consider the first case where T is smaller than X. The incentives to provide effort originate from the value of additional breakthroughs when the project is implemented. Since successful individuals always delay until the deadline, this is entirely determined by the marginal value of an extra breakthrough at the

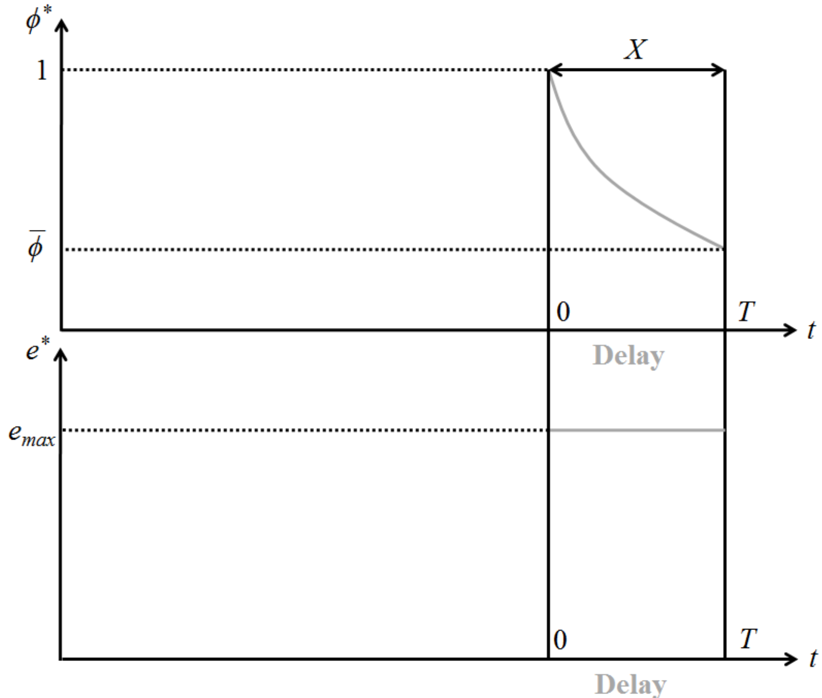

Figure 1.A: Equilibrium for $T \leq X$. The gray lines in the upper and the lower panel show the evolution of the equilibrium belief $\phi^*(t)$ and the equilibrium effort $e^*(t)$ for $t \in [0, T]$. In equilibrium, any successful agent does not reveal a breakthrough and chooses to delay until the deadline at T.

deadline, that is $\phi^*(T)\alpha_1 + (1 - \phi^*(T))\alpha_2$. As long as the pursued breakthrough is likely to be the first, the incentives for effort are sufficiently high to support maximal effort. Since T is smaller than X, the belief ϕ^* cannot fall below the threshold $\bar{\phi}$ and the agent thus chooses to exert maximum effort $e_{\max}$.[8] In response to this high effort choice any successful agent prefers to delay the implementation because she benefits from the potential production of an additional breakthrough by a hard-working agent who has not been successful before. Note that since $e^*(t) = e_{\max}$ and $d^*(t) = 0$, any agent correctly believes that as time passes it is more and more likely that the other agent has been successful.

For T larger than X (and smaller than Y), in the equilibrium outlined in the previous case with maximal effort throughout the belief ϕ^* would fall below the threshold $\bar{\phi}$ and unsuccessful agents would no longer be willing to exert such a high level of effort. Instead, in equilibrium, unsuccessful agents now choose lower effort levels in a way that ensures that at time T the belief ϕ^* is exactly at the threshold $\bar{\phi}$, i.e., $\exp\left(-\lambda \int_0^T e^*(s)\, ds\right) = \bar{\phi}$. This belief at the deadline makes unsuccessful agents indifferent with respect to the level of effort they choose, both at and before the deadline. The difference in continuation values is

$$V^S(t) - V^U(t) = \frac{c}{\lambda} + \exp\left(-\lambda \int_0^T e^*(s)\, ds\right)\left(V^S(T) - V^U(T) - \frac{c}{\lambda}\right). \tag{9}$$

Incentives to exert effort exist at t, that is $V^S(t) - V^U(t) \geq \frac{c}{\lambda}$, provided that they exist at time T, that is $V^S(T) - V^U(T) \geq \frac{c}{\lambda}$. As a result, when an unsuccessful agent is indifferent about her effort

[8] It is important to emphasize that the maximum effort exertion of agents close to the deadline is the result of the increasing importance of becoming informed as the deadline draws closer. It is not caused by any discounting motive since such an incentive is explicitly ruled out in our discounting-free setup.

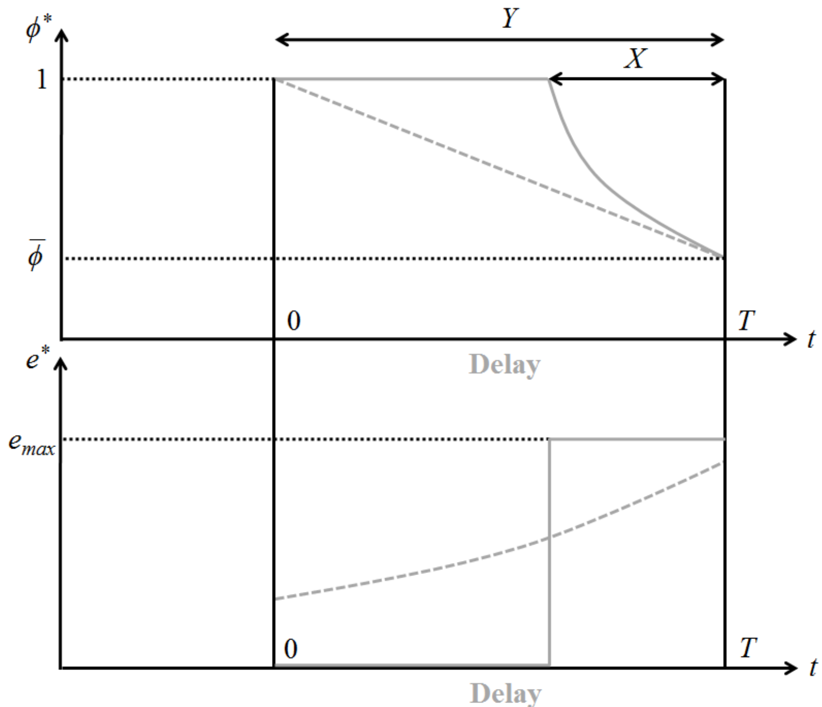

Figure 1.B: Equilibrium for $X < T \leq Y$. The gray lines in the upper and the lower panel show the evolution of the equilibrium belief $\phi^*(t)$ and the equilibrium effort $e^*(t)$ for $t \in [0, T]$. The two extreme equilibrium paths with minimal (dashed line) and maximal (full line) backloading of effort for $T = Y$ are depicted. In equilibrium, any successful agent does not reveal a breakthrough and chooses to delay until the deadline.

choice at the deadline, $\phi^*(T) = \bar{\phi}$, she is also indifferent at any time t before the deadline. Hence, when increasing the length of the game, the aggregate effort exerted by unsuccessful agents remains the same, but their average effort intensity is lower. The equilibrium path of effort is not unique, but to ensure that successful agents are willing to defer a decision until the deadline at any point during the game, unsuccessful agents need to backload their effort sufficiently. Hence, condition (8) can be re-expressed to provide a lower bound on an agent's belief that her partner has not realized a breakthrough,

$$\phi^*(t) \geq 1 - \frac{\delta}{\alpha_2} t \text{ for all } t \in [0, T]. \tag{10}$$

Figure 1.B illustrates the equilibria with minimal and maximal backloading for $T = Y$. With minimal backloading, an unsuccessful agent chooses effort such that $\phi^*(t) = 1 - \frac{\delta}{\alpha_2} t$ for all t.[9] With maximal backloading, an unsuccessful agent chooses effort $e^*(t) = 0$ for $t < T - X$ and $e^*(t) = e_{\max}$ for $t \geq T - X$.[10]

For T larger than Y, successful agents no longer prefer to delay implementing the project from the start. While the aggregate benefit of delay that accrues from the potential generation of a breakthrough by a previously unsuccessful partner remains constant at $(1 - \bar{\phi})\,\alpha_2$ as seen in equation (7), the aggregate

[9] Notice that $e^*(t)$ equals $\frac{\delta}{\lambda \alpha_2}$ and $\frac{\delta}{\lambda \alpha_2 \bar{\phi}}$ for $t = T - Y$ and $t = T$ respectively.

[10] This indeterminacy is due to our assumption that the cost of effort is linear, which affords substantial tractability. In a related context Bonatti and Hörner (2011) also assume linear effort costs. When considering convex costs they are no longer able to obtain closed-form solutions, but their numerical illustrations for cost power functions suggest that the qualitative features of our analysis would remain intact.

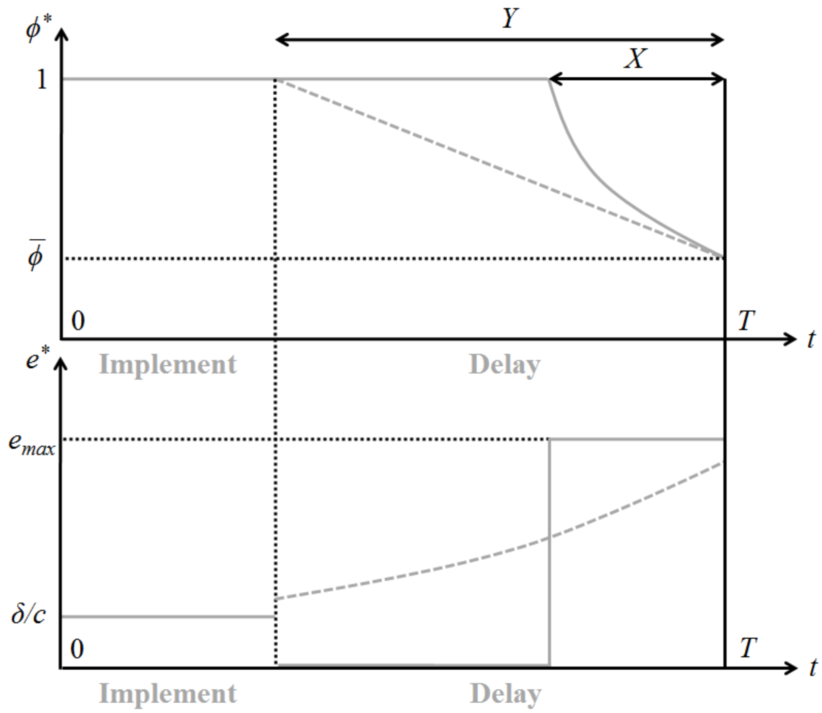

Figure 1.C: Equilibrium for $T > Y$. The gray lines in the upper and the lower panel show the evolution of the equilibrium belief $\phi^*(t)$ and the equilibrium effort $e^*(t)$ for $t \in [0, T]$. The two extreme equilibrium paths with minimal (dashed line) and maximal (full line) backloading of effort are depicted. In equilibrium, any successful agent chooses to reveal a breakthrough and implement early in the game ($t \leq T - Y$), but chooses to delay until the deadline later on ($t > T - Y$.)

cost of delay δT increases as T increases. As a result, when T exceeds Y, a successful agent will initially, that is as long as $t < T-Y$, prefer to forego any delay costs and instead choose to immediately implement the project upon the production of a first breakthrough. As in the public information case, the incentives for effort again consist of the incentive to bring forward the time of implementation, thereby avoiding delay costs, and of the incentive to freeride on the effort of the other agent, thereby avoiding effort costs. This leads to the low equilibrium effort $e^* = \frac{\delta}{c}$, which makes an unsuccessful agent indifferent about her effort choice. The effort level exerted during this phase of full disclosure is lower than the average effort level in the phase of no disclosure, reflecting the fact that a close deadline overcomes the temptation to freeride.[11] Note also that when effort is low, $\tilde{e}^* = \frac{\delta}{c}$, a successful agent is not willing to delay implementation since $\lambda \frac{\delta}{c} \alpha_2 < \delta$. If the project has not been implemented before, the equilibrium is identical from $t = T - Y$ onwards to the equilibrium for $T = Y$ in case (ii).

Uniqueness In the Online Appendix we establish the uniqueness of the equilibrium described in Proposition 2 in the set of symmetric perfect Bayesian equilibria.[12] We first show that the symmetric

[11] That is, $\frac{\delta}{c} < \frac{X}{Y} e_{\max}$. This follows since $\lambda \frac{\delta}{c} \alpha_2 Y < \delta Y = \left(1 - \bar{\phi}\right) \alpha_2 = \int_0^X \lambda e_{\max} \phi^*(t)\, dt \alpha_2 < \int_0^X \lambda e_{\max} dt \alpha_2 = \lambda e_{\max} X \alpha_2$.

[12] Note, however, that in addition to the equilibrium discussed previously, there are also symmetric equilibria which involve strategies whereby both uninformed and informed agents decide to implement at the same instant of time with probability 1 conditional on reaching that time. We do not discuss these equilibria where individuals implement the project with certainty at a point in time because they are in effect equivalent to deadlines which are enforced by appropriately specified off-equilibrium beliefs.

equilibria involving unsuccessful team members implementing the project involve both team members (irrespective of type) coordinating on implementing the project at a moment in time prior to the deadline. This is the only moment an unsuccessful team member is willing to implement the project. This is made possible in equilibrium by appropriately specifying off-equilibrium beliefs at the times immediately after this time. In effect this time acts as a de facto deadline and the on-equilibrium beliefs of agents are the same as if it is a deadline. The following proposition states that the set of equilibria we find is unique amongst the set of symmetric equilibria where unsuccessful agents do not implement the project. Amongst the symmetric equilibria where unsuccessful agents do implement the project our proposition characterizes the possible on-equilibrium behavior.

Proposition 3. *Suppose unsuccessful team members do not implement the project. The set of equilibria described in Proposition 2 are then the unique sets of symmetric perfect Bayesian equilibria.*

Thus, the withholding of information through delay in the lead-up to the deadline is a characteristic of all symmetric equilibria. This further allows us to consider comparative statics as well as the welfare implications of (optimal) deadlines.

4 Welfare

In this section we turn our attention to the effect of the deadline on the welfare of the agents. We find that there exists a finite and unique welfare maximizing deadline. Furthermore, we show that when information is private and the deadline is set optimally, the expected welfare at the start of the game is higher than when the breakthroughs are publicly observed.

4.1 Optimal Deadline

In the baseline case the optimal deadline is set such that agents always delay the implementation of the project until the deadline, but still exert maximum effort. The following proposition formally characterizes the effect of the deadline on welfare.

Proposition 4. *The expected utility of each partnership member is maximized for $T = X$. The expected utility is strictly increasing in the length of the deadline T for $0 \leq T \leq X$, strictly decreasing in T for $X < T \leq Y$ and independent of T for $T > Y$.*

The gray line in Figure 2 graphically illustrates how expected welfare at the start of the game varies with the choice of the deadline T and reaches a maximum at $T = X$. For very tight deadlines, $T \leq X$, agents exert maximum effort and implement the project only when the deadline arrives. Thus, increasing the deadline improves welfare because even though it increases the time until implementation, it also allows the agents more time to intensely exert effort to generate breakthroughs. However, once T is larger than X, the aggregate effort exerted and the expected number of breakthroughs achieved prior to the deadline remain unchanged. The team members simply choose their effort in such a way that they are equally likely to be successful at the deadline, no matter whether the deadline occurs at X or Y or at any time in between. Consequently, any increase in the deadline over and above

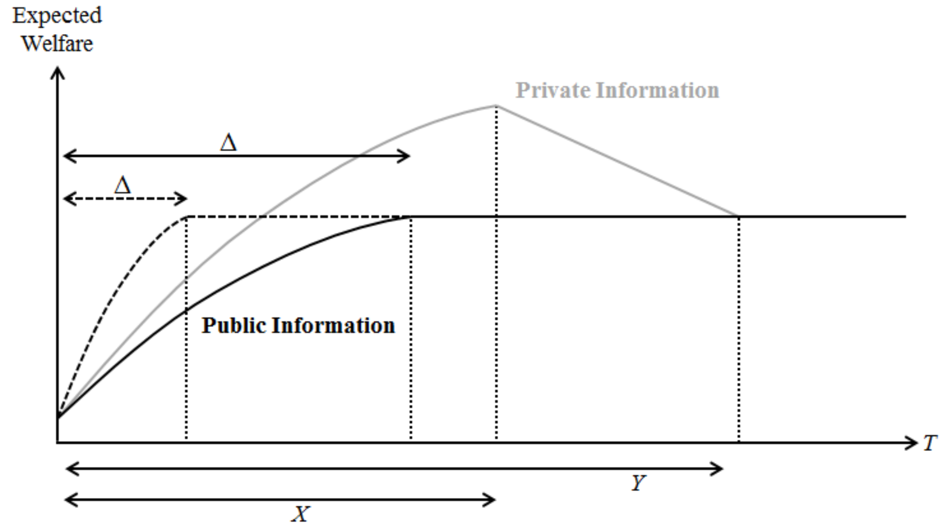

Figure 2: Expected welfare at $t = 0$ as a function of T under private (gray line) and public (full and dashed black lines) information. The full black line shows the case where welfare under public information is lower than welfare under private information for any T. The dashed black line shows the case where for (inefficiently) short deadlines welfare under public information is higher than under private information. In both cases, Δ denotes the time interval for which agents exert maximum effort under public information.

X only introduces additional costly delay. For T between X and Y, the expected utility decreases linearly in T at rate δ. Finally, for loose deadlines, $T > Y$, the welfare of the agents is independent of the deadline.[13] Shortening the length of the game reduces the probability that one's partner will be successful in producing a breakthrough. Her effort level during the phase of no delay is such that the value of this lost opportunity, $\lambda e^* (0) \left[V_T^S (0) - V_T^U (0)\right]$, is exactly offset by the foregone delay δ.[14]

Related to our findings that emphasize the beneficial incentive effects of short deadlines as well as the lack of delay associated with them, are the influential studies of strategic decision making of executive teams in the microcomputer industry by Bourgeois and Eisenhardt (1988) and Eisenhardt (1989). The authors document that indecision and delay can cost firms their technical and market advantages and even lead to bankruptcy. They further show that management teams that make fast decisions due to strict deadlines also use high levels of information and develop many problem-solving alternatives. This fast and informed decision making avoids delay and is positively related to superior firm performance.

Corollary 1. *The optimal length of the deadline $T = X$ is decreasing in $e_{\max}$ and c, increasing in α_1 and α_2, and ambiguous with respect to λ.*

The optimal deadline is affected by changes in the underlying model parameters. Remember that the threshold X equals the time it takes to reduce the probability of still being unsuccessful to $\bar{\phi}$ when

[13] Notice that the expected utility in the long horizon game, $V_0 + \alpha_1 - \frac{c}{\lambda}$, exceeds the highest achievable expected utility in the one-player version, $V_0 + \alpha_1 - \frac{[ce_{\max}+\delta]}{\lambda e_{\max}}$. This is achieved when no deadline is imposed.

[14] Since the project is immediately implemented when one agent becomes successful, the value of being successful, $V_T^S (0) = V_0 + \alpha_1$, does not depend on the deadline in this stage. Moreover, both agents are indifferent with respect to the effort level they exert,

$$\lambda \left[V_T^S (0) - V_T^U (0)\right] = c \text{ for any } T > Y.$$

Hence, the value of being unsuccessful, $V_T^U (0) = V_T^I (0) - \frac{c}{\lambda}$, does not depend on the deadline either.

exerting the maximal effort level,

$$\exp\left(-\lambda e_{\max} X\right) = \bar{\phi},$$

where $\bar{\phi}$ is the belief that makes an agent indifferent about the level of effort,

$$\lambda\left[\bar{\phi}\alpha_1 + \left(1-\bar{\phi}\right)\alpha_2\right] = c.$$

When the maximal effort $e_{\max}$ increases, it takes less time to reach this critical probability. Hence, the longest time that maximal effort can be induced decreases. An increase in the marginal productivity of effort, λ, has a similar effect in that it reduces the time needed for the equilibrium belief ϕ^* to reach the threshold $\bar{\phi}$. However, an increase in λ has a countervailing second effect because it also reduces $\bar{\phi}$ by making it more beneficial to exert effort. The aggregate effect on the optimal deadline of a change in λ is therefore ambiguous. In contrast, an increase in the marginal cost c unambiguously shortens the optimal deadline by increasing $\bar{\phi}$. When it is more costly to exert effort, an unsuccessful agent will only be indifferent between exerting and not exerting effort when it is more likely that the other agent has not been successful. Thus, maximal effort can be sustained for a shorter period of time. Conversely, increases in the value of the first breakthrough, α_1, and of the second breakthrough, α_2, both raise the marginal benefit of effort and thus lengthen the optimal deadline by decreasing $\bar{\phi}$. Finally, note that the optimal deadline is independent of the delay cost δ. This independence result is due to our initial assumption that δ is sufficiently small for successful agents to be willing to delay implementation of the project.

4.2 Private versus Public Information

The ability to withhold information increases the private returns to successful production of a breakthrough and thus the incentives for effort. When information is private and the deadline is set optimally–trading off effort incentives and inefficient delay–the expected welfare at the start of the game is higher than the highest achievable welfare when breakthroughs are publicly observed.

Proposition 5. *The highest welfare achieved by optimally setting the deadline when information about breakthroughs is private exceeds the highest welfare when such information is public.*

As long as the deadline is sufficiently far away, the effort strategies coincide, since agents fully disclose their breakthroughs, whether they are required to or not. For long games, welfare does not depend on the deadline, nor does it depend on whether breakthroughs are publicly or privately observed. For close deadlines, the incentives to exert effort are different in the two cases, since successful agents no longer disclose information about their production success when they are not required to. A priori, it may seem that the change in the observability of production successes can increase or decrease the effort incentives. The private nature of this information provides stronger incentives for unsuccessful players by allowing successful players to 'rest on their laurels'. However, the fact that other players may already have been successful, decreases the value of additional breakthroughs and thus the incentives to exert effort. The first effect dominates the second effect in equilibrium. When breakthroughs are private information, incentives for maximum effort can be sustained throughout for games with longer deadlines than when such information is publicly available, i.e., $X > \Delta$. This also implies that for games with long deadlines, the initial stage of low effort, $e(t) = \frac{\delta}{c}$, lasts longer when breakthroughs are publicly observable.[15]

Corollary 2. *Incentives for maximum effort can be sustained for a longer period of time when information is private than when it is public.*

Clearly, the higher effort incentives that arise from the private nature of information about production successes, increase welfare by mitigating the inefficiency due to freeriding. However, for short deadlines, $T \leq \Delta$, incentives are sufficiently strong for players to exert maximum effort, regardless of whether production successes are publicly or privately observed. Moreover, the option to withhold information about production successes also affects welfare through the delay of decisions. Successful players stop exerting effort and may delay implementation in the hope that their partner will be successful. Hence, both players may have stopped exerting effort, but may still delay implementation not knowing that their partner has already been successful in generating a breakthrough. This is clearly inefficient ex-post. However, as any unsuccessful player would stop exerting effort if the information were disclosed, any opportunity to produce a second breakthrough is lost when production successes are publicly observable. This opportunity can only be valuable if it is socially efficient to produce a second breakthrough, $(2\lambda\alpha_2 - c)\, e_{\max} > \delta$. Hence, when the deadline is short and it is socially inefficient to produce a second breakthrough, the team partners' welfare is higher under public information.

Corollary 3. *Welfare under public information exceeds welfare under private information if $T \leq \Delta$ and $(2\lambda\alpha_2 - c)\, e_{\max} < \delta$.*

In Figure 2, we also plot the expected welfare at $t = 0$ for different lengths of the deadline T under the public information case. The full black line shows the case where welfare under public information is always lower than under private information. In contrast, when production of a second breakthrough is socially inefficient and the deadline is set inefficiently short, welfare under public information can be higher than under private information as illustrated by the dashed black line. Still, keeping information about production successes private is always preferable for the team when the deadline can be set optimally. That is, the stronger effort incentives that arise when breakthroughs are only privately known, can be harnessed at the relatively low cost of inefficient delay by setting the appropriate deadline. As a result, partners would lose from public disclosure rules or any commitment to reveal successes they achieve.

It is instructive to relate our finding to other settings where deadlines are beneficial. Deadlines have been previously highlighted as beneficial commitment devices to solve self control problems (Ariely and Wertenbroch (2002), O'Donoghue and Rabin (1999, 2001), Brocas, Carillo and Dewatripont (2004)). In contrast, in our setting deadlines resolve a freeriding problem. This is not sufficient to improve welfare as illustrated by our public information case in which deadlines necessarily decrease welfare. Bonatti and Hörner (2011) deviate from our public information case by introducing exogenous uncertainty about the feasibility of the project. In their setting agents learn about the feasibility of the project too slowly

[15] Notice that equilibria exist for which $e^{*,priv}(t) \geq e^{e^*,pub}(t)$ for any t. However, $e^{*,priv}(t) = 0$ ($< e^{e^*,pub}(t)$) for some t may well be part of an equilibrium strategy for an unsuccessful player.

due to suboptimal effort and a deadline raises welfare by increasing effort and hence the rate of learning. The learning component is instrumental. In contrast, in our model with dual decisions about effort and disclosure (in the private information case), the beneficial effect of a deadline is different. Our results show that the ability to conceal information allows a deadline to provide stronger incentives for effort than in the public information case and improve welfare despite introducing some inefficient delay.

5 Setting Deadlines

We now analyze how the deadline affects outcomes other than welfare. We focus on two common measures of team performance: the expected time until the project is implemented and the expected value of the project. These measures are of interest to an outside party who is able to choose the length of the deadline and may have different preferences than the team members do. This further allows us to relate our results to existing evidence on the relationship between deadlines, delay and performance.

5.1 Time to Implementation

The natural intuition is that tighter deadlines lead to less delay. We show that this need not be the case and that instead the expected implementation time may be non-monotonic in the length of the deadline.

Proposition 6. *For $T \leq Y$, the expected time to implementation is increasing in T. For $T > Y$, the expected time to implementation is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1-\bar{\phi}}$ and increasing otherwise.*

For tight deadlines, $T \leq Y$, successful partners never disclose their breakthrough and the team always delays project implementation until the deadline. The time until the project is implemented thus equals T and a shorter deadline will always reduce the expected implementation time. The delay of implementation is strongly reminiscent of Parkinson's Law. This widely accepted behavioral law, as stated in its original source (Parkinson 1955, 1958), posits that "the amount of time which one has to perform a task is the amount of time it will take to complete the task". In our context, Parkinson's Law manifests itself by the fact that the amount of time the team takes to implement the project is exactly equal to the amount of time it is given.

For loose deadlines, $T > Y$, implementation may also occur earlier, namely after a successful breakthrough in the first phase of the game. Increasing the deadline T now decreases the probability that agents reach the second phase where implementation decisions are delayed until the deadline. The expected time until the project is implemented equals

$$\tau = \int_0^{T-Y} th(t)\,dt + [1 - H(T-Y)]\,T,$$

where $h(t) = 2\lambda\frac{\delta}{c}\exp\left(-2\lambda\frac{\delta}{c}t\right)$ is the probability that a breakthrough is produced at time t, when the two agents are exerting the low equilibrium effort level $e^*(t) = \frac{\delta}{c}$. The derivative of the expected time to implementation with respect to the deadline is then given by

$$\frac{\partial \tau}{\partial T} = h\left(T-Y\right)\left[\frac{1-H\left(T-Y\right)}{h\left(T-Y\right)} - Y\right].$$

The overall effect of increasing the deadline is ambiguous as the combination of immediate implementation upon success (despite the slow arrival of successes) during the first phase may be a slower or faster process than incurring the fixed delay of Y upon reaching the second phase. The expected implementation time τ is decreasing in the length of the game T if and only if $\frac{1-H(T-Y)}{h(T-Y)} < Y$. Using the expression for Y, we find that this is the case if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1-\bar{\phi}}$.

Taken together, our model predicts that teams either complete and implement projects relatively early on in the process or right at the deadline. Tight deadlines may increase the expected time to implementation by reducing the probability that projects are implemented early on. The relationship between delay, performance and deadlines has been extensively studied both in laboratory and field settings in psychology with a myriad of tasks (Bryan and Locke 1967, Peters et al. 1984, Locke et al. 1981 for an overview of this large literature). In addition to delay, their findings also indicate that with longer deadlines work intensity and performance may suffer. We address these issues in the next subsection.

5.2 Project Value

In addition to influencing how long it will take a group to make a decision, the choice of deadline also affects the expected number of breakthroughs available to the agents when a project is implemented. When more breakthroughs are produced, the project has a higher value from which both partners benefit. One would expect that a longer deadline would always allow for more breakthroughs to be generated until the project is implemented, but this line of reasoning ignores that agents may choose to implement the project before the deadline.

Proposition 7. *The expected value of the project at implementation is increasing in T for $T \leq X$ and it is constant for $X < T \leq Y$. For $T > Y$, it is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$ and increasing otherwise.*

For short deadlines, $T \leq X$, an increase in the length of the game strictly increases the expected number of breakthroughs produced until the deadline at which time the project will be implemented. Agents exert maximum effort throughout the game as long as they are unsuccessful and thus have more time to become successful if the game lasts longer. The expected value of the project when it is implemented is equal to

$$V_0 + (1-\phi^*\left(T\right)^2)\alpha_1 + \left(1-\phi^*\left(T\right)\right)^2\alpha_2,$$

where the probability that an agent is still unsuccessful at the deadline, $\phi^*\left(T\right) = \exp\left(-\lambda e_{\max} T\right)$, is decreasing in T. For intermediate deadlines, $X < T \leq Y$, an increase in the length of the game has no impact on the expected value of the implemented project. The probability that an agent is still unsuccessful at the deadline equals $\phi^*\left(T\right) = \bar{\phi}$, regardless of the length of the game. This is an even stronger expression of Parkinson's Law, alternatively stated as: "Work expands so as to fill the time available for its completion." The additional time granted to the agents has no impact at all on their total effort. This result is also found in a plethora of field and experimental studies that find no change in task completion time and demonstrate that effort (or work pace) is adjusted to the time available or the difficulty of the task (Bassett 1979, Locke et al. 1981).

For long deadlines, $T > Y$, an increase in the length of the game may decrease the expected number of breakthroughs available at implementation. Agents may implement the project before the deadline is reached to avoid the costly delay that occurs when waiting for the other agent to produce additional breakthroughs. When T is large, it becomes more likely that a project will be implemented by a successful agent who has only one breakthrough. The key comparison is therefore whether the value of a single breakthrough is greater or smaller than the expected value of the project for a game with short deadline $T \leq Y$, i.e.,

$$\alpha_1 \gtrless (1 - \phi^* (T)^2)\alpha_1 + (1 - \phi^* (T))^2 \alpha_2.$$

When α_2 is small compared to α_1 the second breakthrough is of relatively little value, but the agents run a high risk of ending up with no breakthrough at all for short deadlines. Thus, the expected value of the project is increasing in T. Conversely, when the second breakthrough is sufficiently valuable, the expected value of the project is higher for $T \leq Y$ and hence the expected value is decreasing in T for $T > Y$ if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{(1-\bar{\phi})^2}$.

Corollary 4. *When $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{(1-\bar{\phi})^2}$ and $(2\lambda\alpha_2 - c)\, e_{\max} < \delta$, the team produces too many breakthroughs in expectation for $T \in [X, Y]$.*

Interestingly, when $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{(1-\bar{\phi})^2}$ and $X \leq T \leq Y$, the expected value of the project can be inefficiently high. Agents may implement projects that use more breakthroughs than is efficient from an ex-ante perspective. In particular, if it is socially efficient for the team to produce only a single breakthrough, i.e., $(2\lambda\alpha_2 - c)\, e_{\max} < \delta$, the agents will overgenerate breakthroughs in expectation. While it is not surprising that agents may end up producing too many breakthroughs ex-post given our assumption of private information about effort exertion and production success, it is quite surprising that despite the agents' freeriding problem, ex-ante overproduction can occur in our model. Ex-ante overproduction of breakthroughs may occur even when the deadline T is set optimally at X.

6 Discussion and Extensions

Our basic framework identified a strong tension between the incentives to exert effort and to reveal information. In equilibrium, this resulted in two potential phases depending on the proximity of the deadline: a first phase of low effort and full disclosure and a second phase of high effort and no disclosure. In this section we provide a brief description of the case in which the incentives to exert effort for the unsuccessful agent exceed the incentives to delay for the successful agent at the deadline, i.e., $\bar{\phi}_d \geq \bar{\phi}$. We then discuss the robustness of the qualitative equilibrium characteristics to modifications of the nature of communication, the value of additional breakthroughs and the number of players. We also briefly consider some extensions of our basic framework such as the introduction of explicit contracts and reputational rewards to being successful, the presence of communication frictions, and the potential role of a third party intermediary. Some of these extensions are particularly relevant in group decision-making processes. To simplify the presentation we discuss each new element separately and keep the mathematical formalities to a minimum.

6.1 Case with Large Effort Incentives ($\bar{\phi}_d \geq \bar{\phi}$)

We briefly consider the case in which the incentives to exert effort for the unsuccessful agent exceed the incentives to delay for the successful agent at the deadline, i.e., $\bar{\phi}_d \geq \bar{\phi}$. A complete discussion of this case is contained in Section B.2 of the Online Appendix.

Like in the baseline case, there is a lower bound on the equilibrium belief $\phi^*(t)$. The only difference is that this lower bound is now determined by the incentives to delay implementation rather than the incentives to exert effort. That is, in equilibrium, the belief $\phi^*(t)$ does not fall below $\bar{\phi}_d$. Otherwise, a successful player would strictly prefer to implement the project as it is too likely that her partner is already successful. However, a belief $\phi(t) < 1$ cannot be consistent with a successful player's pure strategy to implement at time t.[16] By deciding to implement at a rate $d(t)$ such that $(1-\phi(t))\,d(t) = \lambda e_{\max}$, a successful player keeps her partner's belief constant at $\phi(t)$. A successful player, however, is only indifferent about implementing the project when her belief equals exactly $\bar{\phi}_d$. This interplay introduces a new phase in equilibrium for games with length exceeding X_d where $\exp(-\lambda e_{\max} X_d) = \bar{\phi}_d$. After unsuccessful agents have exerted maximum effort for a length of time X_d and the belief $\phi^*(t)$ has decreased to $\bar{\phi}_d$, successful players start implementing at rate $d^*(t) = \frac{\lambda e_{\max}}{1-\bar{\phi}_d}$ keeping the belief constant at $\bar{\phi}_d$.

The characterization of the equilibrium is thus very similar as in the baseline case except for this final phase of mixing delay coupled with maximum effort exerted by unsuccessful agents. This final phase maximally lasts up to a length of time Z. The length of time $X_d + Z$ is the longest possible time for which maximum effort can be sustained throughout the game. When the length of the game exceeds $X_d + Z$, players will reduce their average effort during the first stage of the game such that the belief $\phi^*(t)$ equals exactly $\bar{\phi}_d$ for $t = T - Z$.

Figure 3 graphically illustrates the evolution of the strategies and beliefs over the course of the game. Equilibrium behavior of successful agents is divided into three distinct phases. As in the baseline case, agents decide to implement the project immediately upon successfully producing a breakthrough when the deadline is far away. However, once the deadline is sufficiently close, i.e., $t \geq T - (Y_d + Z)$,[17] successful agents prefer to delay the implementation and thus the equilibrium belief ϕ^* falls until it reaches $\bar{\phi}_d$. At that point, successful agents are indifferent between implementing and delaying the implementation and thus probabilistically choose one or the other until the conclusion of the game in such a way that ϕ^* remains constant at $\bar{\phi}_d$. As in Figures 1.B and 1.C the full and dashed gray lines for ϕ^* are associated with the two extreme equilibrium paths for e^* involving minimal and maximal backloading of effort. The evolution of effort in Figure 3 mirrors Figure 1.C. Unsuccessful agents choose $e^* = \frac{\delta}{c}$ during the initial decision phase and then start increasing their effort until they exert effort $e_{\max}$

[16] If $d(t) = \textit{implement}$ and no player has decided to implement at t, the belief should be reset at 1 and thus a successful player would again strictly prefer to delay the decision.

[17] Y_d is the maximum length of time a successful agent is (strictly) willing to delay to allow her partner to produce a second breakthrough with probability $1-\bar{\phi}_d$.

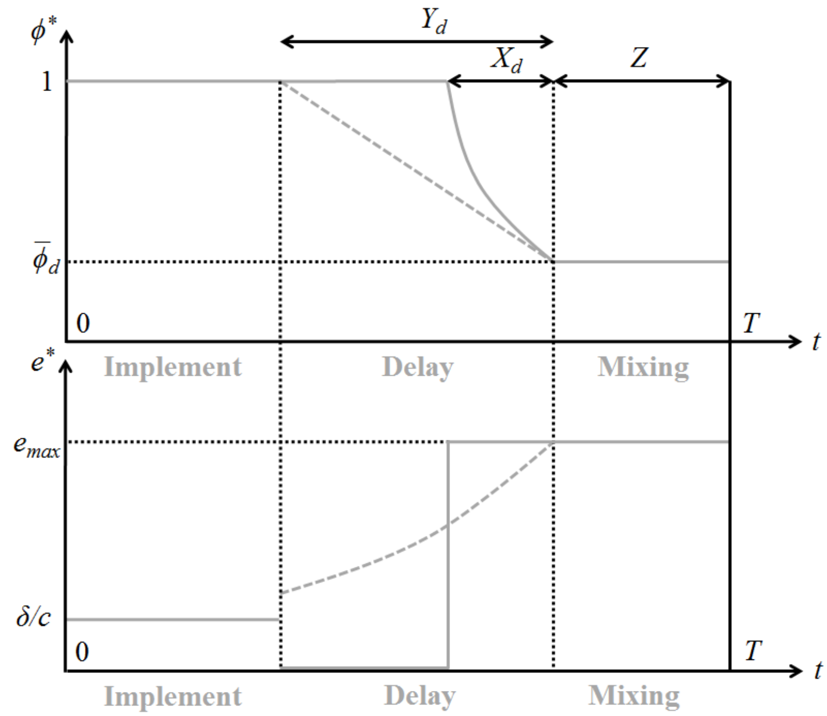

Figure 3: Equilibrium for $T > Y_d + Z$ when $\bar{\phi} < \bar{\phi}_d$. The gray lines in the upper and the lower panel show the evolution of the equilibrium belief $\phi^*(t)$ and the equilibrium effort $e^*(t)$ for $t \in [0, T]$. The two extreme equilibrium paths with minimal (dashed line) and maximal (full line) backloading of effort are depicted. In equilibrium, any successful agent chooses to reveal a breakthrough and implement early in the game ($t \leq T - (Y_d + Z)$), chooses to delay in the middle of the game ($T - Z \geq t > T - (Y_d + Z)$) and to mix between delaying and implementing at the end ($t > T - Z$).

at the end of the game.

The equilibrium description, both in the baseline case with small incentives for effort and the case with large incentives for effort, formally establishes that the implementation of the joint project may be significantly delayed. This is due to a lack of productive effort when the deadline is far away and due to a lack of information sharing when the deadline is close. All remaining results regarding uniqueness, welfare and deadlines continue to hold as well.

6.2 Communication and Implementation

In Section 2 we proceeded by not distinguishing between an agent revealing a success to the other agent and deciding to implement the project. We also assumed that one agent can decide to implement the project unilaterally. Both assumptions can be relaxed as long as both agents agree to implement the project once they believe there has been a success, which is satisfied when $c \geq \lambda\alpha_2$. To illustrate this we describe cheap talk equilibria which are analogous to the equilibria we find in our earlier setting. On the equilibrium path the timing of effort by successful and unsuccessful agents, the time at which projects are implemented, and the beliefs are the same.

Consider allowing each agent to send a non-verifiable message $m \in \{s, u\}$ ("successful", "unsuccessful") at any point in time and requiring that both agents must agree (choose $d = implement$) in order to implement the project. Suppose a successful agent sends s after the histories for which a decision to implement is made in the earlier setting. Otherwise, the agent announces nothing. Upon receiving a message, in a perfect Bayesian equilibrium, an agent will update her belief to $\phi = 0$. It is then common knowledge between the successful agent, who sent the message, and the agent who received the message that there has been at least one success. At this point it is an equilibrium of the continuation game for neither agent to exert further effort, since $\lambda\alpha_2 < c$, and hence both players prefer to implement the project immediately. A successful agent who sends s, therefore, induces both agents to choose $d = \mathit{implement}$ and the project is implemented. Hence, a successful agent has identical incentives to announce s in this setting and to unilaterally decide to implement the project in the earlier setting. Also the incentives for effort by both successful and unsuccessful agents are identical in this setting to our earlier setting given that projects are implemented at the same times in both settings. Finally, an unsuccessful agent has no incentive to announce s as it will result in the project's implementation, which she does not want for the same reason that she does not implement the project in our earlier setting. For the off-equilibrium-path history in which she sends s but chooses not to implement the project herself (thereby prolonging the game), we specify that the other agent's beliefs revert to the agent's beliefs as if no message had been sent.

Here allowing communication–either through cheap talk or verifiable disclosure of success–and requiring both agents to agree for the project to be implemented is a straightforward extension of our earlier setting because once both agents believe there has been at least one success then the project is immediately implemented.

6.3 Value of Breakthroughs

We continue to investigate the robustness of the qualitative characteristics of the earlier model to a setting where agents would consider producing more than one breakthrough. We consider a setting where an individual is willing to produce a second, but not a third breakthrough, $\alpha_2 > \frac{c}{\lambda} > \alpha_3$. In contrast to the previous setting, agents may want to communicate that they have successfully produced a breakthrough without deciding to implement the project. Therefore, we allow agents to take an action $w_i \in N$ at any instance of time, which verifiably reveals that an agent has produced at least w_i breakthroughs.

We consider the equilibrium for two extreme cases: a short deadline and an infinite deadline. We describe only the main results here and refer the interested reader to the appendix for an explicit characterization of the setting and propositions. We show that a symmetric equilibrium of a game with a short deadline (below a threshold level) exhibits no information revelation $w_i^* = 0$ and pure delay $d_i^* = 0$. An agent exerts high effort when she has produced not more than one breakthrough before, $e_i^*(t, n_i) = e_{\max}$ for $n_i = 0, 1$, and no effort when she has produced two breakthroughs, $e_i^*(t, 2) = 0$. These are characteristics shared by our earlier model where an unsuccessful agent exerts high effort provision and a successful agent exerts no effort. Similarly, pure delay (and hence lack of timely information sharing) is also observed in our earlier model close to a deadline.

To gain some intuition for what may occur a long way from the deadline we also consider an equilibrium of the infinite horizon game. The infinite horizon case allows us to focus on a setting where there is no effect of a future deadline and thus avoids the complication of specifying the changes in behavior for the subgames as we transition from behavior far away from the deadline to close to the deadline. We show that the symmetric equilibrium strategies are similar to the equilibrium strategies in our earlier model far away from the deadline (i.e., for $t < T - Y$). We find that there is immediate revelation of breakthroughs by each individual, $w_i^* = n_i$. Hence, the project is implemented whenever the number of breakthroughs reaches two, $d_i^* = \textit{implement}$ if $n_i + w_{-i} \geq 2$. This is similar to the earlier model where the agents immediately disclose their success. Individuals also exert less than the maximum effort level $e_i^* = \frac{\delta}{c} < e_{\max}$ as in the earlier model, trading off freeriding incentives with incentives to bring forward the implementation. Although we do not have a complete characterization of this setting, we conjecture on the basis of these two cases that the qualitative characteristics of our earlier model close to the deadline and far away from a deadline remain.

6.4 Number of Players

We now consider how the number of players influences the interactions of the members of the group. We find that adding more players decreases X, the longest deadline which can sustain maximum effort by all players as an equilibrium. However, the effect on Y, the longest deadline for which the project is not implemented until the deadline, is ambiguous.

The intuition for the first result is that as more team members are added to the group and are exerting maximum effort in the face of a close deadline ($T < X$), it becomes more likely that other team members have been successful. As a result, an unsuccessful player has muted incentives for effort. Maximum effort can thus only be sustained for a shorter period before the deadline. We illustrate this by contrasting the two-player case where the relationship which determines X is given by

$$\lambda \exp\left(-\lambda e_{\max} X\right) \alpha_1 + \lambda \left(1 - \exp\left(-\lambda e_{\max} X\right)\right) \alpha_2 = c,$$

to the three-player case where the relevant threshold X' is defined by the following equation

$$\lambda \left(\exp\left(-\lambda e_{\max} X'\right)\right)^2 \alpha_1 + \lambda 2 \exp\left(-\lambda e_{\max} X'\right)\left(1 - \exp\left(-\lambda e_{\max} X'\right)\right) \alpha_2 \\ + \lambda \left(1 - \exp\left(-\lambda e_{\max} X'\right)\right)^2 \alpha_3 = c.$$

Since the probability weights on α_1, α_2 and α_3 sum to one in both equations and $\alpha_1 > \alpha_2 > \alpha_3$, it is the case that $\left(\exp\left(-\lambda e_{\max} X'\right)\right)^2 > \exp\left(-\lambda e_{\max} X\right)$ and hence $X' < X$.[18]

While the relationship between the amount of time during which unsuccessful agents exert maximum effort and the number of team members is unambiguous, the relationship between the length of time during which no decision is made until the deadline and the number of team members is ambiguous. The

[18] More generally, consider any group of n members and add an additional group member. The relevant equations are now

$$\lambda \sum_{k=0}^{n-1} \binom{n-1}{k} \left(1 - \exp\left(-\lambda e_{\max} X^n\right)\right)^{n-1-k} \left(\exp\left(-\lambda e_{\max} X^n\right)\right)^k \alpha_{n-k} = c$$

and

$$\lambda \sum_{k=0}^{n} \binom{n}{k} \left(1 - \exp\left(-\lambda e_{\max} X^{n+1}\right)\right)^{n-k} \left(\exp\left(-\lambda e_{\max} X^{n+1}\right)\right)^k \alpha_{n+1-k} = c$$

Note that $\lambda \sum_{k=0}^{n} \binom{n}{k} (1-\phi)^{n-k} \phi^k \alpha_{n+1-k}$ is strictly decreasing in n since $\alpha_1 > \alpha_2 > \ldots > \alpha_n$. Note further that this expression is increasing in ϕ. For both indifference conditions to be satisfied we again require that $X^{n+1} < X^n$.

delay phase may increase or decrease when the team grows larger depending on the value of additional breakthroughs. Consider the case of two and three team members where the thresholds are defined in the following way

$$Y = \frac{(1 - \exp(-\lambda e_{\max} X))\, \alpha_2}{\delta}$$

and

$$Y' = \frac{\left(1 - \exp(-\lambda e_{\max} X')^2\right) \alpha_2 + (1 - \exp(-\lambda e_{\max} X'))^2\, \alpha_3}{\delta}.$$

When $\alpha_3 = 0$, we know from the indifference conditions for X and X' that $(\exp(-\lambda e_{\max} X'))^2 > \exp(-\lambda e_{\max} X)$ and hence $Y' < Y$. The introduction of additional players makes it more likely that effort-exerting agents are not looking to produce the very first breakthrough for the group. When the value of additional successes is very low, effort incentives are only maintained if the probability that one is exerting effort for a valuable first success is increased. The aggregate effort by the team members must, therefore, decrease if the number of team members increases. This, in turn, decreases the incentives to delay implementation by successful agents. On the other hand, when $\alpha_3 = \alpha_2$ then from the indifference conditions for X and X' we have $\exp(-\lambda e_{\max} X) = (\exp(-\lambda e_{\max} X'))^2$ and so $Y' > Y$. This is the flipside of the previous result. When obtaining a third success is almost as valuable as finding a second success, the effort incentives for each unsuccessful team member do not decrease by much, but the presence of an additional team member who is exerting effort makes it more attractive for a successful agent to delay implementation.

6.5 Alternative Mechanisms

Our model captures settings in which there may exist severe restrictions on the types of mechanisms and contracts that are available. There is a broad range of assumptions one can make about what can be contracted on to help address the incentive distortions of our model. However, our analysis has identified a strong tension between incentives for effort provision and incentives for revealing success. By fixing the incentives in one dimension, the incentives in the other dimension may be reduced. The incentives for an agent to reveal a success in our model are determined by the trade-off between the benefits of waiting and the associated costs of delay, or more formally

$$\lambda \tilde{e}^*(t) \phi(t) \alpha_2 \gtreqless \delta.$$

Ceteris paribus, the higher the equilibrium effort $\tilde{e}^*(t)$ of an unsuccessful partner, the greater are the incentives for a successful agent to delay the sharing of information. A lesson we can draw from this result is that bonuses or reputational rewards for bringing a breakthrough to the table during implementation will induce agents to exert more effort, but may result in more delay. This shifts the source of inefficiency from one of limited effort provision to limited information sharing. Consider a bonus b paid to any agent who has successfully produced a breakthrough at the time a project is implemented. This bonus would increase each agent's incentives to exert effort and thus decrease the belief $\bar{\phi}_b < \bar{\phi}$. This makes an agent indifferent about how much effort to exert,

$$\lambda \left[\bar{\phi}_b \alpha_1 + \left(1 - \bar{\phi}_b\right) \alpha_2 + b\right] = c.$$

Hence, the longest time that maximum effort can be induced in equilibrium increases,

$$X_b = \frac{-\log \bar{\phi}_b}{\lambda e_{\max}} > X.$$

However, since more information is expected to be acquired, agents are also more willing to delay implementation. In equilibrium, they stop disclosing successes earlier on,

$$Y_b = \frac{\left(1 - \bar{\phi}_b\right) \alpha_2}{\delta} > Y.$$

A second lesson we can draw is that the ability of agents to withhold information about their private success may be beneficial for welfare since it improves their incentives to become successful. Organizational hierarchies or communication frictions that inhibit the disclosure of earlier successes at the time of project implementation may induce agents to disclose their success immediately after successful production. Yet these frictions may well decrease the incentives for effort. Consider a situation where agents implement projects unilaterally and can only communicate and transfer a breakthrough (e.g., successful discovery of an informative signal to aid decision-making), but cannot obtain any breakthroughs generated by the other agent due to communication frictions. That is, when implementing a project, an agent cannot consult her partner and is limited to utilizing only the previously revealed successes during implementation of the project. In this setting agents have strong incentives to disclose successes immediately to avoid successes being left unused in the event that their partner implements the project. The analysis of this setting is in fact identical to the public information case analyzed earlier. As we showed before, this is not necessarily welfare-improving. In fact, with an optimally set deadline it is welfare-decreasing.

To guarantee a welfare increase, contracts or mechanisms should increase incentives in one dimension, without hurting incentives in the other dimension. One type of contract or mechanism that may achieve this in our setting is a payment that depends on the order in which successes are revealed or that simply depends on time. An example of this is a decreasing payment (increasing punishment) to the agents depending on when the project is implemented. Essentially, this type of scheme will increase the delay cost δ for each agent. If we take the case of a long deadline then the freeriding effort level is $\frac{\delta}{c}$. A relatively simple way to obtain maximum effort and immediate implementation would be to set the rate of decrease (increase) of a reward (punishment) for implementing the project at a particular time equal to $ce_{\max} - \delta$. The effective delay cost an agent then faces is $ce_{\max}$. Hence, this contract fixes the freeriding effort at $e_{\max}$ and successes are revealed immediately as $ce_{\max} > \lambda e_{\max} \alpha_2$.[19]

[19] However, when implementing this type of scheme one must also check that agents still have an incentive to produce any breakthrough at all. This is satisfied in our original model by the assumption that $\alpha_1 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$. By adjusting the effective delay cost, this constraint may no longer hold and agents will simply immediately implement the project. If $\alpha_1 > \frac{2c}{\lambda}$, this will not occur.

6.6 Moderator

A clear inefficiency from keeping successes private arises when both agents have acquired a success prior to the deadline and thus have stopped exerting effort, but the implementation of the project is nonetheless delayed as neither player shares her success with the other. There are potential gains from the introduction of a moderator who cannot exert effort (or whose effort decision is unaffected by any successes of the other agents), but is able to facilitate communication between the team members. The role of this moderator would be to avoid situations in which both team members are successful, but neither is willing to reveal her success to the other agent.

We previously showed that there is a unique optimal deadline $T = X$ which maximizes the ex-ante welfare of the players. It is straightforward to show that for this optimal deadline ex-ante welfare can be further increased by introducing a moderator to whom a success can be revealed. The moderator will choose when to implement the project, maximizing her own private value from the project and accounting for the delay cost. Team members are willing to disclose their information to the moderator because the moderator will decide to implement the project only when both agents have revealed their information, but will not implement the project before. The reason is that the expected gain for the moderator when an unsuccessful partner exerts maximum effort outweighs the cost of delaying the decision. In the game of length X without a moderator, unsuccessful agents are exerting maximum effort throughout in equilibrium. With a moderator, the incentive to exert effort for an unsuccessful agent is even higher because an additional success may induce the moderator to implement the project before the deadline and thus reduces the expected time to implementation.[20] Notice that a benevolent moderator who tries to maximize the social surplus, may not be able to convince agents to reveal their information. If it is socially inefficient to invest effort to produce a second breakthrough, the benevolent moderator will already implement the project after only one agent discloses her success. As a result, successful agents will be unwilling to reveal their information. Thus, a privately motivated moderator is more effective at inducing communication.

7 Conclusion

In private and public organizations, teams are often allocated the dual task of investing in joint projects and deciding when to implement the project. In this paper we have investigated the link between the incentive to exert effort and the incentive to share implementation-relevant information. One clear lesson that emerges from our analysis is that team members are reluctant to disclose information that undermines the incentives of fellow team members to exert more effort. As a result, although a strict deadline provides strong incentives for agents to exert effort, it also mutes the incentives to reveal information and to facilitate timely implementation. Therefore, it is not too surprising that strict deadlines may sometimes be counterproductive when swift implementation is required. In our setting

[20] In the presence of a moderator, the optimal deadline is no longer equal to X. Without a deadline, an equilibrium exists in which both agents exert maximum effort from the start until they are successful which they immediately reveal to the moderator. The moderator implements the project only when both players have acquired a success. The option to freeride on the efforts of a partner is effectively eliminated if a moderator is committed not to take a decision until both agents are successful. Surprisingly, in the presence of a moderator, an approaching deadline could undermine incentives to exert maximum effort, since the deadline provides an alternative way to end the game.

the optimal deadline strikes a balance between providing strong effort incentives and limiting delay. Furthermore, we have shown that mutual monitoring in partnerships is not a panacea to solving incentive problems. In fact, the non-observability to the partner of one's success in conjunction with joint control of the length of the deadline is precisely what allows the partnership to circumvent the moral hazard in teams problem. Finally, we emphasized that the design of communication and incentive structures of a partnership requires attention to detail: features that sharpen effort incentives may blunt disclosure incentives and vice versa.

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A Appendix

A.1 Proofs

Proof of Proposition 1.

Since $\lambda\alpha_2 < c$, no player exerts effort to produce a second breakthrough and hence the game ends as soon as one player succeeds in producing one breakthrough. At time t, the value of being successful equals

$$V^S(t) = V_0 + \alpha_1.$$

The value of being unsuccessful at t, if all agents exert maximum effort until the deadline, equals

$$V^U(t) = V_0 + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\right]\{1 - \exp[-2\lambda e_{\max}(T-t)]\}.$$

Observe that $\alpha_1 > \frac{c}{\lambda} > \frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}$. Hence, $\frac{dV^U(t)}{dt} < 0$. Since $V^S(t)$ is constant, this implies that $V^S(t) - V^U(t)$ is increasing in t. Define Δ by

$$V^S(T-\Delta) - V^U(T-\Delta) = \frac{c}{\lambda}.$$

At any time $t > \max\{T-\Delta, 0\}$, unsuccessful agents exert maximum effort, since $V^S(t) - V^U(t) > \frac{c}{\lambda}$. Notice that these equilibrium strategies are unique, since for a lower effort level by the other agent, the incentives to exert effort increase further. At any time t before the threshold $T-\Delta$, each agents exerts $e^*(t) = \frac{\delta}{c}$, which makes every agent indifferent about how much effort to exert. This equilibrium strategy corresponds to the first stage in the private information case for long deadlines when successful players disclose their successes. The proof for uniqueness of these equilibrium strategies is the same as for the private information case.■

Proof of Proposition 2.

We consider the three different cases separately.

i) Case 1: $T \leq X$.

We start by writing out the implied continuation values of successful and unsuccessful agents on the equilibrium path for the proposed equilibrium strategies. At the deadline, $t = T$, these are equal to the expected value of a project at that time,

$$\begin{aligned} V^S(T) &= V_0 + \alpha_1 + (1 - \exp(-\lambda e_{\max} T))\alpha_2, \\ V^U(T) &= V_0 + (1 - \exp(-\lambda e_{\max} T))\alpha_1. \end{aligned}$$

For any $t \in [0, T]$, the continuation values are

$$\begin{aligned} V^S(t) &= V^S(T) - \delta(T-t), \\ V^U(t) &= [1 - \exp(-\lambda e_{\max}(T-t))]V^S(T) + \exp(-\lambda e_{\max}(T-t))V^U(T) \\ &\quad -\frac{c}{\lambda}[1 - \exp(-\lambda e_{\max}(T-t))] - \delta(T-t). \end{aligned}$$

The difference in continuation values at and before the deadline equals

$$\begin{aligned} V^S(T) - V^U(T) &= \exp(-\lambda e_{\max} T)\alpha_1 + [1 - \exp(-\lambda e_{\max} T)]\alpha_2, \\ V^S(t) - V^U(t) &= \frac{c}{\lambda} + \exp(-\lambda e_{\max}(T-t))\left[V^S(T) - V^U(T) - \frac{c}{\lambda}\right]. \end{aligned}$$

Strategy of successful player: Check that the successful individual's implementation strategy $d^*(t) = 0$ is optimal by noting:

$$V^S(t) > V_0 + \alpha_1 + (1 - \exp(-\lambda e_{\max} t))\alpha_2 \text{ for all } t < X,$$

since $X < Y = \frac{[(1-\bar{\phi})\alpha_2]}{\delta}$. The right-hand side above is the value of implementing the project at time t for a successful player.

Strategy of unsuccessful player: We check that the unsuccessful agent's choice of effort $e^*(t) = e_{\max}$ is optimal by noting that

$$\begin{aligned} V^S(t) - V^U(t) - \frac{c}{\lambda} &= \exp(-\lambda e_{\max}[T-t])\left[\exp(-\lambda e_{\max} T)\alpha_1 + (1 - \exp(-\lambda e_{\max} T))\alpha_2 - \frac{c}{\lambda}\right] \\ &> 0 \text{ for all } t < T \leq X, \end{aligned}$$

since $\phi^*(T) = \exp(-\lambda e_{\max} T) > \bar{\phi}$. We then check that the unsuccessful individual will not implement the project. Note that because exerting effort is optimal, we have $V^U(t) \geq V^U(T) - \delta(T-t)$, where the right hand side equals the expected utility when not exerting any effort and not implementing the project either. Moreover,

$$\begin{aligned} V^U(T) - \delta(T-t) &= V_0 + (1 - \exp(-\lambda e_{\max} T))\,\alpha_1 - \delta(T-t) \\ &> V_0 + (1 - \exp(-\lambda e_{\max} t))\,\alpha_1 \text{ for all } t < X, \end{aligned}$$

since $X < Y = \frac{(1-\bar{\phi})\alpha_2}{\delta}$ and $\alpha_1 > \alpha_2$. The right-hand side of the inequality above is the value of implementing the project at time t for an unsuccessful player.

ii) Case 2: $X < T \leq Y$.

We again start by writing out the implied continuation values of successful and unsuccessful agents on the equilibrium path for the proposed equilibrium strategies. At the deadline, we find

$$\begin{aligned} V^S(T) &= V_0 + \alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2, \\ V^U(T) &= V_0 + \left(1 - \bar{\phi}\right)\alpha_1. \end{aligned}$$

For general $t \in [0, T]$, the continuation values are

$$\begin{aligned} V^S(t) &= V^S(T) - \delta(T-t), \\ V^U(t) &= \left(1 - \exp\left(-\lambda \int_t^T e^*(s)\,ds\right)\right) V^S(T) + \exp\left(-\lambda \int_t^T e^*(s)\,ds\right) V^U(T) \\ &\quad - \frac{c}{\lambda}\left(1 - \exp\left(-\lambda \int_t^T e^*(s)\,ds\right)\right) - \delta(T-t). \end{aligned}$$

Strategy of successful player: We check that the successful individual's implementation decision strategy $d^*(t) = 0$ is optimal by noting that $V^S(t) \geq V_0 + \alpha_1 + (1 - \phi^*(t))\,\alpha_2$ when $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right) \geq \frac{\delta}{\alpha_2}(T-t) + \bar{\phi}$, which is true given the effort strategy specified.

Strategy of unsuccessful player: We check that the unsuccessful agent is indifferent about the level of effort to exert for all $t \in [0, T]$ by noting that for $\phi^*(T) = \bar{\phi}$:

$$\begin{aligned} V^S(t) - V^U(t) - \frac{c}{\lambda} &= \exp\left(-\lambda \int_t^T e^*(s)\,ds\right)\left[V^S(T) - V^U(T) - \frac{c}{\lambda}\right] \\ &= \exp\left(-\lambda \int_t^T e^*(s)\,ds\right)\left[\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2 - \frac{c}{\lambda}\right] \\ &= 0. \end{aligned}$$

Finally, the argument for the unsuccessful individual not to implement the project is the same as in Case 1.

iii) Case 3: $T > Y$.

Since for $t \in [0, T-Y]$ projects are implemented as soon as one player has successfully produced a breakthrough, the belief is $\phi^*(t) = 1$ for the subgame starting at $t = T - Y$. Hence, the proof for the strategies being equilibria of that subgame are encompassed in case 2. It remains to show that the strategies specified for $t \in [0, T-Y]$ also constitute an equilibrium. We write out the implied continuation values of successful and unsuccessful agents on the equilibrium path for the proposed equilibrium strategies for $t \in [0, T-Y]$,

$$\begin{aligned} V^S(t) &= V_0 + \alpha_1, \\ V^U(t) &= V_0 + \alpha_1 - \frac{c}{\lambda}, \end{aligned}$$

where the continuation value of the unsuccessful player follows from

$$V^U(t) = \int_t^{T-Y} \left[V^S(s) - c\int_t^s e^*(r)\,dr - \delta(s-t) \right] 2\lambda e^*(s)\exp(-2\lambda\int_t^s e^*(r)\,dr)ds$$
$$+\exp(-2\lambda\int_t^{T-Y} e^*(r)\,dr)\left[V^U(T-Y) - c\int_t^{T-Y} e^*(r)\,dr - \delta(T-Y-t)\right].$$

Using the fact that $e^*(s) = \frac{\delta}{c}$ and $V^S(s) = V^S(0)$ for $s \in [0, T-Y]$, we can further simplify the expression and find

$$V^U(t) = \left[1 - \exp\left(-2\lambda e^*(t)(T-Y-t)\right)\right]\left(V^S(t) - \frac{c}{\lambda}\right)$$
$$+\exp\left(-2\lambda e^*(t)(T-Y-t)\right)V^U(T-Y).$$

From case 2, we know that

$$\begin{aligned} V^U(T-Y) &= V^S(T-Y) - \frac{c}{\lambda} - \exp\left(-\lambda\int_t^T e^*(s)\,ds\right)\left[\bar{\phi}\alpha_1 + \left(1-\bar{\phi}\right)\alpha_2 - \frac{c}{\lambda}\right] \\ &= V^S(T-Y) - \frac{c}{\lambda}. \end{aligned}$$

Hence,

$$V^U(t) = V^S(t) - \frac{c}{\lambda} \text{ for } t \in [0, T-Y].$$

Strategy of successful player: We prove that the successful agent's implementation decision strategy $d^*(t) =$ *implement* is optimal. The payoff from waiting until $\widehat{t} < T-Y$ upon becoming successful at time t which we denote by $V^S\left(\widehat{t}|t\right)$ is:

$$V^S\left(\widehat{t}|t\right) = \int_t^{\widehat{t}} \left(V_0 + \alpha_1 + \alpha_2 - \delta(s-t)\right)\lambda e^*(s)\exp\left(-\lambda\int_t^s e^*(r)\,dr\right)ds$$
$$+\exp\left(-\lambda\int_t^{\widehat{t}} e^*(s)\,ds\right)\left(V_0 + \alpha_1 - \delta\left(\widehat{t} - t\right)\right),$$

where the equilibrium effort is constant at $\frac{\delta}{c}$ over the time interval $\left[t, \widehat{t}\right]$ Using partial integration to solve the integral, this simplifies to

$$V^S\left(\widehat{t}|t\right) = V_0 + \alpha_1 + \left[1 - \exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t} - t\right)\right)\right]\left(\alpha_2 - \frac{c}{\lambda}\right).$$

This is strictly decreasing in $\hat{t}$ since $\frac{c}{\lambda} > \alpha_2$. Conditional on not waiting beyond $T-Y$, the optimal decision is therefore to implement at

$$t = \arg\max_{\widehat{t} \geq t} V^S\left(\widehat{t}|t\right).$$

Moreover, given the results in case 2, the agent will not prefer to wait beyond $T-Y$. Hence, a successful agent optimally decides to implement, $d^*(t) =$ *implement* for $t \leq T-Y$.

Strategy of unsuccessful player: Note that the unsuccessful agent is indifferent about the level of effort to exert for all $t \in [0, T-Y]$ by noting that $V^U(t) = V^S(t) - \frac{c}{\lambda}$. Furthermore, $V^S(t) - \frac{c}{\lambda} > V_0$, so it is never optimal for an unsuccessful agent to choose to implement the project.

All times are reached with positive probability in equilibrium and the only action an agent observes by the other agent is the action to implement the project which ends the game. The only off-equilibrium scenario agents may find themselves in is one in which they have not followed their own effort strategy previously. These effort costs are sunk and conditional on being successful or unsuccessful these off-equilibrium continuation games are identical to the continuation games for successful and unsuccessful agents on the equilibrium path. The strategies and beliefs on the equilibrium path thus also describe these continuation games off the equilibrium path.

For completeness, note that $X < Y$ is implied by $\bar{\phi} > \bar{\phi}_d$. When $\bar{\phi} > \bar{\phi}_d$, a successful partner is willing to delay locally when $\phi^*(t) \geq \bar{\phi}$ and an unsuccessful partner exerts $e = e_{\max}$. That is, for $T = X$,

$$\delta < \lambda e_{\max} \phi^*(t) \alpha_2 \text{ for all } t \leq T.$$

Hence,

$$\delta X = \int_0^X \delta dt < \int_0^X \lambda e_{\max} \phi^*(t) \alpha_2 dt = \left(1 - \bar{\phi}\right) \alpha_2 = \delta Y,$$

where we used the expression $\phi^*(t) = \exp(-\lambda e_{\max} t)$ to solve the integral.■

Proof of Proposition 3.

See Online Appendix.■

Proof of Proposition 4.

Denote by $V_T^U(0)$ the expected utility (when unsuccessful) at the start $t = 0$ of the game with a deadline at T,

$$V_T^U(0) = \begin{cases} V_0 + \alpha_1 - \frac{c}{\lambda} \text{ for } T \geq Y, \\ V_0 + \left(1 - \bar{\phi}^2\right) \alpha_1 + \left(1 - \bar{\phi}\right)^2 \alpha_2 - \delta T - \left(1 - \bar{\phi}\right) \frac{c}{\lambda} \text{ for } Y \geq T > X, \\ V_0 + (1 - \phi^*(T)^2)\alpha_1 + (1 - \phi^*(T))^2 \alpha_2 - \delta T - (1 - \phi^*(T)) \frac{c}{\lambda} \text{ for } 0 \leq T \leq X, \end{cases}$$

where $\phi^*(T) = \exp(-\lambda e_{\max} T)$ and $\bar{\phi} = \exp(-\lambda e_{\max} X)$. The linear decrease in expected utility $V_T^U(0)$ when increasing T for $X < T \leq Y$ is immediate since the aggregate effort and hence expected number of breakthroughs are independent of the deadline, but the delay cost increases with the length of the deadline. For $T \geq Y$, the expected utility is independent of T, hence the result is immediate as well. Now, for $0 \leq T \leq X$, we find that

$$\frac{dV_T^U(0)}{dT} = 2\lambda e_{\max} \left[\alpha_1 \phi^*(T) + \alpha_2 (1 - \phi^*(T)) - \frac{c}{2\lambda}\right] \exp(-\lambda e_{\max} T) - \delta.$$

At $T = X$, we have $\phi^*(T) = \bar{\phi}$. Hence, we find that $\lim_{T \to X} \frac{dV_T^U(0)}{dT} = e_{\max} c \bar{\phi} - \delta$. Moreover, since $e_{\max} \bar{\phi}_d \alpha_2 > \frac{\delta}{\lambda} > \frac{\delta}{c} \alpha_2$, we find $e_{\max} c \bar{\phi}_d > \delta$. By assumption, in the case of small incentives, we have $\bar{\phi} > \bar{\phi}_d$ and thus $\lim_{T \to X} \frac{dV_T^U(0)}{dT} > 0$. Note further that

$$\frac{\partial}{\partial T}\left(\frac{dV_T^U(0)}{dT}\right) = 2\lambda e_{\max} \times \\ \left\{-\lambda e_{\max} \phi^*(T) \left[\alpha_1 \phi^*(T) + \alpha_2 (1 - \phi^*(T)) - \frac{c}{2\lambda}\right] - \left(\lambda e_{\max} \phi^*(T)\right)^2 (\alpha_1 - \alpha_2)\right\} < 0.$$

Hence, $\frac{dV_T^U(0)}{dT} > 0$ for all $T \in [0, X)$.■

Proof of Proposition 5.

For the case of public information, the continuation values at the beginning of the game are given by

$$V_T^{U,pub}(0) = \begin{cases} V_0 + \alpha_1 - \frac{c}{\lambda} \text{ for } T \geq \Delta, \\ V_0 + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\right] \left[1 - \exp(-2\lambda e_{\max} T)\right] \text{ for } T < \Delta. \end{cases}$$

Together with the results in Proposition 4, this implies the following relations for any $T' > \max\{Y, \Delta\}$:

$$\max_T V_T^U(0) > V_{T'}^U(0) = V_{T'}^{U,pub}(0) = \max_T V_T^{U,pub}(0).$$

Hence, $\max_T V_T^U(0) > \max_T V_T^{U,pub}(0)$.■

Proof of Proposition 6.

Denote by τ the expected time at which the project is implemented. For $T \leq Y$, $\tau = T$. Hence, $\frac{d\tau}{dT} > 0$. For

$T > Y$,

$$\begin{aligned}\tau &= \int_0^{T-Y} t2\lambda\frac{\delta}{c}\exp\left(-2\lambda\frac{\delta}{c}t\right)dt + \exp\left(-2\lambda\frac{\delta}{c}(T-Y)\right)T \\ &= \frac{1}{2\lambda\frac{\delta}{c}}\left[1-\exp\left(-2\lambda\frac{\delta}{c}(T-Y)\right)\right] + \exp\left(-2\lambda\frac{\delta}{c}(T-Y)\right)Y.\end{aligned}$$

Hence,

$$\frac{d\tau}{dT} = 2\lambda\frac{\delta}{c}\exp\left(-2\lambda\frac{\delta}{c}(T-Y)\right)\left(\frac{1}{2\lambda\frac{\delta}{c}} - Y\right).$$

It follows that $\frac{d\tau}{dT} < 0$ if and only if $\frac{1}{2\lambda\frac{\delta}{c}} < Y$. Substituting for $Y = \frac{(1-\bar{\phi})\alpha_2}{\delta}$, this condition can be restated as $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}}{1-\bar{\phi}}$, or, in terms of the primitives of the model, as $\alpha_1 - \frac{c}{\lambda} > \frac{\alpha_1}{\alpha_2}\left(\frac{c}{\lambda} - \alpha_2\right)$. Notice that since $\alpha_1 > \frac{c}{\lambda} > \alpha_2$, by assumption, both sides of the inequality are positive and the relationship is satisfied for $\frac{c}{\lambda}$ close to α_2 and violated for $\frac{c}{\lambda}$ close to α_1.■

Proof of Proposition 7.

If $T \leq X$, agents only decide to implement at the deadline and unsuccessful agents are exerting maximum effort until the deadline. Hence, the expected number of generated breakthroughs is strictly increasing in T. If $X < T \leq Y$, agents still only decide to implement at the deadline and unsuccessful agents are exerting the same aggregate amount of effort until the deadline. Thus, the expected number of breakthroughs is the same for any T in this range. Finally, if $T > Y$ agents may decide to implement before the deadline. They exert effort $\frac{\delta}{c}$ and immediately decide to implement when successful. Thus, for large T the expected number of breakthroughs when the project is implemented is approximately equal to 1 and the associated expected value is $V_0 + \alpha_1$. In contrast, for $T \leq Y$, the team may produce a second breakthrough, but they may also end up without any breakthrough. The associated expected utility when implementing the project at the deadline is equal to

$$\left(1-\bar{\phi}\right)^2(V_0+\alpha_1+\alpha_2) + 2\left(1-\bar{\phi}\right)\bar{\phi}(V_0+\alpha_1) + \bar{\phi}^2 V_0.$$

Thus, for $T > Y$, the expected value of the project is a weighted sum of the expression above and $V_0 + \alpha_1$. By increasing T, weight is shifted from the first to the second term. Hence, the expected value is increasing in T for $T > Y$ if and only if

$$\left(1-\bar{\phi}\right)^2(V_0+\alpha_1+\alpha_2) + 2\left(1-\bar{\phi}\right)\bar{\phi}(V_0+\alpha_1) + \bar{\phi}^2 V_0 < V_0 + \alpha_1.$$

This simplifies to $\frac{\alpha_2}{\alpha_1} < \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$. This condition can be satisfied given the restrictions on the parameters. For, $\alpha_1 = \frac{2c}{\lambda}$ and $\alpha_2 = \frac{c}{2\lambda}$, we have $\frac{\alpha_2}{\alpha_1} = \frac{1}{4}$ and $\bar{\phi} = \frac{\frac{c}{\lambda}-\alpha_2}{\alpha_1-\alpha_2} = \frac{1}{3}$. Hence, $\frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2} = \frac{1}{4} = \frac{\alpha_2}{\alpha_1}$. Thus, for slightly smaller (larger) values of α_1 the expected value of all available breakthroughs when the project is implemented will be decreasing (increasing) in T for $T > Y$.[21]■

[21] Note that $\bar{\phi}_d = \frac{\delta}{\lambda e_{\max}\alpha_2}$, so let δ be small such that $\bar{\phi}_d < \bar{\phi}$ is satisfied.

B ONLINE APPENDIX

B.1 Proofs of Corollaries

Proof of Corollary 1.

This follows immediately from

$$\exp\left(-\lambda e_{\max} X\right) = \bar{\phi} \text{ and } \lambda\left[\bar{\phi}\alpha_1 + \left(1-\bar{\phi}\right)\alpha_2\right] = c.$$

If either c decreases, α_1 increases or α_2 increases, $\bar{\phi}$ decreases and thus X increases. If $e_{\max}$ increases, clearly X decreases. Finally, using $X = -\frac{1}{\lambda e_{\max}} \log \frac{\frac{c}{\lambda}-\alpha_2}{\alpha_1-\alpha_2}$, we find that $\frac{dX}{d\lambda} > 0$ if and only if $-\log\bar{\phi} < \frac{c}{\bar{\phi}\lambda} = e_{\max}\alpha_2$.■

Proof of Corollary 2.

For $T > X$, an equilibrium with private information exists in which $e^{*,priv}(t) = 0$ for $t < T - X$ and $e^{*,priv}(t) = e_{\max}$ for $t \geq T - X$. In the unique equilibrium with public information $e^{*,pub}(t) = e_{\max}$ if and only if $t \geq T - \Delta$. We show that $X > \Delta$ by contradiction. By definition, X is the deadline T solving

$$V_T^{S,priv}(0) - V_T^{U,priv}(0) = \frac{c}{\lambda} \Leftrightarrow \alpha_1\phi^*(T) + \alpha_2\left(1-\phi^*(T)\right) = \frac{c}{\lambda},$$

where $\phi^*(T) = \exp\left(-\lambda e_{\max} T\right)$. Also, by definition, Δ is the deadline T solving

$$V_T^{S,pub}(0) - V_T^{U,pub}(0) = \frac{c}{\lambda} \Leftrightarrow \alpha_1\phi^*(T)^2 + \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\left(1-\phi^*(T)^2\right) = \frac{c}{\lambda},$$

where we use $\exp\left(-2\lambda e_{\max} T\right) = \phi^*(T)^2$. Since $\phi^*(T)^2 \leq \phi^*(T)$, a necessary condition for $X < \Delta$ is

$$\alpha_2 < \frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}, \tag{11}$$

which implies that exerting effort to produce a second breakthrough is socially inefficient. We now show that when this inequality holds, welfare under public information exceeds the welfare under private information for short deadlines,

$$V_T^{U,pub}(0) - V_T^{U,priv}(0) > 0 \text{ for } T \leq \min\{X, \Delta\}. \tag{12}$$

However, from Proposition 5, we have

$$V_X^{U,priv}(0) \geq V_\Delta^{U,pub}(0) = \max_T V_T^{U,pub}(0) \geq V_X^{U,pub}(0).$$

Hence, this implies that $X > \Delta$, which is a contradiction. To establish the inequality (12), we use that for $T \leq \Delta$,

$$V_T^{U,pub}(0) = V_0 + \left(\alpha_1 - \frac{c}{2\lambda} - \frac{\delta}{2\lambda e_{\max}}\right)\left(1 - \exp\left(-2\lambda e_{\max} T\right)\right),$$

and for $T < X$,

$$V_T^{U,priv}(0) = V_0 + \alpha_1\left(1-\phi^*(T)^2\right) + \alpha_2\left(1-\phi^*(T)\right)^2 - \frac{c}{\lambda}\left(1-\phi^*(T)\right) - \delta T.$$

Using $\exp\left(-2\lambda e_{\max} T\right) = \phi^*(T)^2$, we find that higher welfare is achieved in the public information case if

$$\alpha_2\left(1-\phi^*(T)\right)^2 - \frac{c}{\lambda}\left(1-\phi^*(T)\right) - \delta T \leq -\frac{c}{2\lambda}\left(1-\phi^*(T)^2\right) - \frac{\delta}{2\lambda e_{\max}}\left(1-\phi^*(T)^2\right).$$

Rearranging, we find

$$\left(\alpha_2 - \frac{c}{2\lambda} - \frac{\delta}{2\lambda e_{\max}}\right)\left(1-\phi^*(T)\right)^2 \leq \delta\left(T - \frac{1-\phi^*(T)}{\lambda e_{\max}}\right).$$

The term $\frac{1-\phi^*(T)}{\lambda e_{\max}}$ corresponds to the expected duration of a game with maximum length T when the project is implemented at the rate $\lambda e_{\max}$ and is thus smaller than T. Hence, the right-hand side has a positive sign. Moreover, from inequality (11), we know that the left-hand side has a negative sign. This establishes the inequality.■

Proof of Corollary 3.

Knowing that $T < \Delta$ implies $T < X$ by Corollary 2, this follows immediately from the second part of the proof of that Corollary.■

Proof of Corollary 4.

The condition $(2\lambda\alpha_2 - c)\, e_{\max} < \delta$ implies that it is socially efficient for the team to produce only one breakthrough. The efficient level of the value of the project is thus bounded above by $V_0 + \alpha_1$. However, for $T \in [X, Y]$, the expected value of the project equals $V_0 + \left(1-\bar{\phi}\right)^2 (\alpha_1 + \alpha_2) + 2\left(1-\bar{\phi}\right)\bar{\phi}\,(V_0 + \alpha_1)$. When $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}^2}{\left(1-\bar{\phi}\right)^2}$, this exceeds $V_0 + \alpha_1$.■

B.2 Large Effort Incentives ($\bar{\phi}_d \geq \bar{\phi}$)

In this section, we describe the case where effort incentives are large ($\bar{\phi}_d \geq \bar{\phi}$, see Section 6.1) in more detail. We also state the equilibrium strategies and beliefs formally and provide a proof.

We consider deadlines of different length T of which there are four distinct cases. We define two thresholds X_d and Y_d, similar to X and Y, and an additional threshold Z. The threshold X_d denotes the length of time necessary such that an agent exerting maximum effort is successful with probability $1 - \bar{\phi}_d$. Thus, the threshold solves

$$\exp\left(-\lambda e_{\max} X_d\right) = \bar{\phi}_d.$$

Similarly, the threshold Y_d denotes the length of time that an agent would be willing to delay implementation in exchange for the additional benefit of a second breakthrough with probability $1 - \bar{\phi}_d$. Thus, the threshold solves

$$\left(1 - \bar{\phi}_d\right) \alpha_2 = \delta Y_d.$$

The characterization of the equilibrium is very similar as before, with the exception of a final stage which lasts up to Z for games with length exceeding X_d. Once the length of the game exceeds X_d and unsuccessful agents have exerted maximum effort until $t = X_d$, successful players will implement projects at a rate $d^*(t)$ such that $\left(1 - \bar{\phi}_d\right) d^*(t) = \lambda e_{\max}$ keeping the belief constant at $\bar{\phi}_d$. The threshold Z is the maximum length of this mixing stage which maintains maximum incentives to exert effort throughout the game,

$$Z = -\frac{1}{2\lambda e_{\max}} \log\left(-\frac{\left(1-\bar{\phi}_d\right)\alpha_2 - \frac{c e_{\max} - \delta}{\lambda e_{\max}}}{\bar{\phi}_d 2\alpha_1 + \left(1-\bar{\phi}_d\right)\alpha_2 - \frac{c e_{\max} + \delta}{\lambda e_{\max}}}\right).$$

Proposition 8. *If $\bar{\phi}_d \geq \bar{\phi}$, then the equilibrium strategies and beliefs are as follows:*
i) If $T \leq X_d$, any successful player chooses not to implement, $d^*(t) = 0$, for all t while any unsuccessful player chooses to exert maximum effort, $e^*(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda e_{\max} t\right)$.
ii) If $X_d < T \leq X_d + Z$, any successful player chooses not to implement for $t < X_d$ and decides to implement at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max}\alpha_2 - \delta}$ for $t \geq X_d$. Any unsuccessful player chooses to exert maximum effort, $e^*(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda e_{\max} t\right)$ for $t \leq X_d$ and $\phi^*(t) = \bar{\phi}_d$ for $t > X_d$.
iii) If $X_d + Z < T \leq Y_d + Z$, any successful agent chooses not to implement for $t < t_d \equiv T - Z$, and decides to implement at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max}\alpha_2 - \delta}$ for $t \geq t_d$. Any unsuccessful player chooses to exert effort $e^*(t)$ for $0 \leq t < t_d$ which is not uniquely determined, but the effort choice must satisfy the following conditions:

$$\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right) \leq \frac{\delta}{\alpha_2}\left[t - t_0\right] \;\; \textit{for } t \in [t_0, t_d] \tag{13}$$

and

$$\phi(t_d) = \exp\left(-\lambda \int_{t_0}^{t_d} e^*(s)\, ds\right) = \frac{\delta}{\lambda\alpha_2}, \tag{14}$$

for $t_0 = 0$. For $t \geq t_d$ the unsuccessful agent exerts maximal effort $e^*(t) = e_{\max}$. The agents' beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right)$ for $0 \leq t \leq t_d$ and $\phi^*(t) = \bar{\phi}_d$ for $t > t_d$.

iv) If $T > Y_d + Z$, any successful player chooses to implement immediately for $t < t_d - Y_d$, not to implement, $d^*(t) = 0$ for $t_d - Y_d \leq t < t_d$, and to implement at the mixing rate $d^*(t) = \frac{(\lambda e_{\max})^2 \alpha_2}{\lambda e_{\max}\alpha_2 - \delta}$ for $t \geq t_d$. Any unsuccessful agent chooses to exert effort $e^*(t) = \frac{\delta}{c}$ for $t < t_d - Y_d$, and to exert effort $e^*(t)$ for $t_d - Y_d \leq t < t_d$ which is not uniquely determined but must satisfy the following conditions (13) and (7) for $t_0 = t_d - Y_d$, and to exert maximal effort $e^*(t) = e_{\max}$ for $t \geq t_d$.

Proof. **i) Case 1:** $T \leq X_d$

The proof is here exactly the same as for Case 1 in Proposition 1, since $\bar{\phi}_d = \exp(-\lambda e_{\max} X_d) > \bar{\phi}$ in this case with large effort incentives.

ii) **Case 2:** $X_d < T \leq X_d + Z$

We again start by writing out the implied continuation values of successful and unsuccessful agents on the equilibrium path for the proposed equilibrium strategies.

At the deadline, the continuation values equal

$$\begin{aligned} V^S(T) &= V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2, \\ V^U(T) &= V_0 + \left(1 - \bar{\phi}_d\right)\alpha_1. \end{aligned}$$

For $t \in [X_d, T]$, the continuation values equal

$$V^S(t) = V^S(T)$$

and

$$V^U(t) = \int_t^T \left(V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (ce_{\max} + \delta)(s - t)\right) 2\lambda e^{-2\lambda e_{\max}(s-t)} ds + e^{-2\lambda e_{\max}(T-t)}\left(V^U(T) - (ce_{\max} + \delta)(T - t)\right).$$

Note that the 2 in the density function comes from the probability of one of two events occuring, "agent successfully produces a breakthrough" or "the other agent decides to implement". In this time interval, the rate of breakthrough production is the same as the rate at which the other agent is deciding to implement the project. The payoff contains the term $\left(1 - \bar{\phi}_d\right)\alpha_2/2$ because with $1/2$ probability the agent will produce a breakthrough before the other agent decides to implement the project in which case the payoff is increased by $\left(1 - \bar{\phi}_d\right)\alpha_2$. The continuation value simplifies further to

$$V^U(t) = V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (ce_{\max} + \delta)\frac{1}{2\lambda e_{\max}} - e^{-2\lambda e_{\max}(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (ce_{\max} + \delta)\frac{1}{2\lambda e_{\max}}\right].$$

The difference between being successful and unsuccessful is given by

$$V^S(t) - V^U(t) = \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} + (ce_{\max} + \delta)\frac{1}{2\lambda e_{\max}} + e^{-2\lambda e_{\max}(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (ce_{\max} + \delta)\frac{1}{2\lambda e_{\max}}\right].$$

We define Z such that this difference equals exactly $\frac{c}{\lambda}$ when $t = T - Z$. Notice that $\frac{d\left(V^S(t) - V^U(t)\right)}{dt}|_{t=T-Z} > 0$, since $\frac{dV^S(t)}{dt}|_{t=T-Z} = 0$ and $\frac{dV^U(t)}{dt}|_{t=T-Z} < 0$.

For $t \in [0, X_d]$, the continuation values equal

$$\begin{aligned} V^S(t) &= V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta\left(X_d - t\right), \\ V^U(t) &= \left[1 - \exp\left(-\lambda e_{\max}\left(X_d - t\right)\right)\right]\left(V^S\left(X_d\right) - \frac{c}{\lambda}\right) + \exp\left(-\lambda e_{\max}\left(X_d - t\right)\right)V^U\left(X_d\right) - \delta\left(\hat{t} - t\right). \end{aligned}$$

The difference between the two equals

$$V^S(t) - V^U(t) = \frac{c}{\lambda} + e^{-\lambda(X_d - t)}\left(V^S\left(X_d\right) - V^U\left(X_d\right) - \frac{c}{\lambda}\right).$$

Strategy of successful player: Check first that the successful individual's implementation decision strategy $d^*(t) = 0$ is optimal for $t \in [0, X_d]$ by verifying whether $V^S(t) \geq V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2$, where the right hand side equals the payoff from immediate implementation. Rearranging , we obtain

$$\phi^*(t) - \bar{\phi}_d \geq \frac{\delta}{\alpha_2}\left(X_d - t\right),$$

which holds with equality for $t = X_d$. Furthermore, the derivative of the LHS is strictly less than the RHS, $-\lambda e_{\max}\phi^*(t) < -\frac{\delta}{\alpha_2}$, for $t < X_d$. Hence, the relation holds for $t < X_d$. Next, we check that the successful individual is indifferent about a decision now versus delaying a decision any amount Δt into the future for $t \in (X_d, T]$, so that she is willing to implement at a positive rate. The expected utility of immediate implementation at t equals

$$V(t|t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

This is exactly the same as the expected utility of waiting until $t + \Delta t$ to implement, when $d^*(t) = \frac{\lambda e_{\max}}{1 - \bar{\phi}_d}$ for $t \geq X_d$,

$$\begin{aligned} V(t + \Delta t|t) = V_0 + \alpha_1 + \int_t^{t+\Delta t} \left(\alpha_2 - \delta(s - t)\right)d^*(s)\left(1 - \phi(s)\right)\exp\left(-\int_t^s d^*(r)\left(1 - \phi^*(r)\right)dr\right)ds \\ + \exp\left(-\int_t^{t+\Delta t} d^*(r)\left(1 - \phi^*(r)\right)dr\right)\left(\left(1 - \bar{\phi}_d\right)\alpha_2 - \delta\Delta t\right). \end{aligned}$$

Using that $\phi^*(r) = \bar{\phi}_d$ for $r \geq X_d$, this simplifies further to

$$\begin{aligned} V(t + \Delta t|t) = V_0 + \alpha_1 + \left(1 - \exp\left(-d^*(t)\left(1 - \bar{\phi}_d\right)\Delta t\right)\right)\left(\alpha_2 - \frac{\delta}{d^*(t)\left(1 - \bar{\phi}_d\right)}\right) \\ + \exp\left(-d^*(t)\left(1 - \bar{\phi}_d\right)\Delta t\right)\left(\left(1 - \bar{\phi}_d\right)\alpha_2\right), \end{aligned}$$

where

$$\frac{\delta}{d^*(t)\left(1 - \bar{\phi}_d\right)} = \frac{\delta}{\lambda e_{\max}} = \bar{\phi}_d\alpha_2.$$

Hence, we find indeed that

$$V(t + \Delta t|t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

and thus the mixing strategy is an equilibrium strategy.

Strategy of unsuccessful player: Check that the unsuccessful agent's choice of effort $e^*(t) = e_{\max}$ is optimal by noting that $V^S(t) - V^U(t) > \frac{c}{\lambda}$ for all t provided $T < X_d + Z$ and $V^S(t) - V^U(t) = \frac{c}{\lambda}$ for $t \in [0, X_d]$ when $T = X_d + Z$. The argument that an unsuccessful agent will not decide to implement is again the same as in Case 1 of Proposition 1. The expected utility when following the equilibrium strategy exceeds the expected utility when exerting no effort, but delaying implementation, which exceeds the expected utility of implementing the project immediately.

iii) **Case 3:** $X_d + Z < T \leq Y_d + Z$

Notice first that the subgames for $t \geq T - Z$ are identical to those described above in case 2 for $t \geq X_d$ and the proof is identical. Turning to $t < T - Z$. For $t \in [0, T - Z]$, the continuation value for the successful individual is

$$V^S(t) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta(T - Z - t)$$

and for the unsuccessful agent

$$V^U(t) = \left(1 - \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right)\right) V^S(T-Z) + \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right) V^U(T-Z)$$
$$- \frac{c}{\lambda}\left(1 - \exp\left(-\lambda \int_t^{T-Z} e^*(s)\,ds\right)\right) - \delta(T - Z - t).$$

Hence, the difference equals

$$V^S(t) - V^U(t) = \frac{c}{\lambda} \text{ for } 0 \leq t \leq T - Z.$$

Strategy of successful player: This corresponds to the proof in Case 2 of Proposition 1. Check that the successful individual's decision strategy $d^*(t) = 0$ is optimal by noting that $V^S(t) \geq V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2$ when $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\,ds\right) \geq \bar{\phi}_d + \frac{\delta}{\alpha_2}(X_d - t)$, which is true given the equilibrium effort strategy specified.

Strategy of unsuccessful player: Check that the unsuccessful agent is indifferent about the level of exerted effort for all $t \in [0, T - Z]$, since $V^S(t) - V^U(t) = \frac{c}{\lambda}$. An unsuccessful agent will not implement the project provided that

$$V^U(t) > V_0 + (1 - \phi^*(t))\alpha_1$$

$\Leftrightarrow$

$$V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{c}{\lambda} - \delta(X_d - t) >$$
$$V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2 - (\phi^*(t)\alpha_1 + (1 - \phi^*(t))\alpha_2).$$

We know from the successful player's strategy that

$$\left(1 - \bar{\phi}_d\right)\alpha_2 - \delta(X_d - t) \geq (1 - \phi^*(t))\alpha_2.$$

Hence, it remains to show that

$$\phi^*(t)\alpha_1 + (1 - \phi^*(t))\alpha_2 > \frac{c}{\lambda}$$

which is true because $\bar{\phi}\alpha_1 + \left(1 - \bar{\phi}\right)\alpha_2 = \frac{c}{\lambda}$ and $\phi^*(t) > \bar{\phi}_d > \bar{\phi}$.

iv) **Case 4:** $T > Y_d + Z$

Here again we note that all subgames starting from $t = T - Y_d + Z$ are encompassed by the proof of Case 3 above, and the continuation values at $t = T - Y_d + Z$ are

$$\begin{aligned} V^S(t) &= V_0 + \alpha_1 \\ V^U(t) &= V_0 + \alpha_1 - \frac{c}{\lambda} \end{aligned}$$

which are exactly the same continuation values as in **Case 3** of Proposition 1 for $t = T - Y$. In this case the strategies and the proof for the subgames $t < T - (Y_d + Z)$ are identical to that for **Case 3** of Proposition 1 for $t < T - Y$. ☐

For $T < X_d$, the equilibrium strategies are exactly like before. For $T \geq X_d$, the marginal value of a breakthrough at and close to the deadline is strictly greater than $\frac{c}{\lambda}$, unlike in the small incentives case. This also continues to be the case for all longer deadlines. As the length of the game T increases, however, the incentives for effort at a given time decrease. To see this, consider the incentives for effort at $t = 0$ which are given by

$$V^S(0) - V^U(0) = \frac{c}{\lambda} + \bar{\phi}_d\left(V^S(X_d) - V^U(X_d) - \frac{c}{\lambda}\right).$$

The only part of the expression which changes with T, is $V^U(X_d)$ since all the other terms above are constants and

$$V^S(X_d) = V^S(T) = V_0 + \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

$V^U(X_d)$ can be rewritten as

$$V^U(X_d) = \exp\left(-2\lambda e_{\max}(T - X_d)\right) V^U(T) \\ + \left(1 - \exp\left(-2\lambda e_{\max}(T - X_d)\right)\right)\left[V_0 + \alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (ce_{\max} + \delta)\frac{1}{2\lambda e_{\max}}\right].$$

This is a weighted sum of the expected payoff conditional on either producing a breakthrough or the other agent implementing the project prior to the deadline and the payoff from being unsuccessful at the deadline. Both of these payoffs are independent of T and it is only the relative likelihood of each which is affected by T. The likelihood of being unsuccessful at the deadline $\exp\left(-2\lambda e_{\max}(T - X_d)\right)$ decreases in T. Hence, the continuation value of being unsuccessful at X_d is increasing in T. There exists a deadline $T = X_d + Z$ where $V^S(X_d) - V^U(X_d) = \frac{c}{\lambda}$ and an agent is indifferent about exerting effort at $t = 0$, so $V^S(0) - V^U(0) = \frac{c}{\lambda}$. For T larger than $X_d + Z$, that is case iii) above, maximal effort by unsuccessful agents can no longer be sustained throughout the entire game. In equilibrium, an unsuccessful agent reduces her average effort intensity before $t_d = T - Z$ such that $\phi(t_d) = \bar{\phi}_d$, while the successful agent fully delays. For T larger than $Y_d + Z$, successful agents will no longer prefer to delay their implementation decision at the beginning of the game, exactly as in the case with small incentives.

B.3 Uniqueness

In this section, we prove uniqueness of our proposed equilibrium. We first provide a more general description of our model and then state a sequence of lemmas that we use to prove Proposition 3 which establishes the uniqueness of our equilibria described in Proposition 2 and Proposition 8.

B.3.1 General Model Description

To establish uniqueness we introduce some additional notation that describes the agents' strategies. Define the decision strategy of the unsuccessful agent by $\mu(t)$ which is the conditional probability of implementing a project with no breakthroughs produced by time t given that the other agent does not implement prior to t. Define the decision strategy of the successful agent by $\rho(t)$ as the conditional probability of implementing the project with one breakthrough produced by time t given that the other agent does not implement prior to t. Define $e(t)$ as the effort strategy of the unsuccessful agent. Define $\sigma(t)$ as the conditional probability of being unsuccessful by time t given that the other agent does not implement prior to t. Hence, $1 - \sigma(t)$ is the conditional probability of being successful by that time. $\sigma(t)$ changes over time according to

$$\frac{d\sigma}{dt} = -\lambda e(t)\left(\sigma(t) - \mu(t)\right)$$

The effort strategy of the unsuccessful agent influences $\frac{d\sigma}{dt}$. The implementation decision strategy of an unsuccessful agent influences both $\frac{d\sigma}{dt}$ and $\mu(t)$. The implementation decision strategy of the successful agent controls $\rho(t)$. The following relationships between these functions hold

$$\mu(t) + \rho(t) \leq 1$$

and

$$\mu(t) \leq \sigma(t)$$

and

$$\rho(t) \leq 1 - \sigma(t)$$

We will use the notation ˜ (tilde) to denote the strategies of the other player and * (star) to denote equilibrium strategies. The Bayesian belief $\phi(t)$ at a time t that the other agent is unsuccessful conditional on the project not being implemented prior to that time is

$$\phi(t) = \frac{\tilde{\sigma}(t) - \tilde{\mu}(t)}{1 - \tilde{\mu}(t) - \tilde{\rho}(t)}$$

A strategy for an agent maps into a path for $e(t), \sigma(t), \mu(t), \rho(t)$. We restrict our attention to strategies which result in piecewise continuously differentiable functions of $e(t)$, $\sigma(t)$, $\mu(t)$ and $\rho(t)$. Clearly, given the nature of the model $\frac{d\sigma}{dt} \leq 0$ since agents do not lose or destroy breakthroughs and $\frac{d\mu}{dt}, \frac{d\rho}{dt} \geq 0$ since deciding to implement the project is irreversible. The upper bound on $e(t)$ also ensures that $\sigma(t)$ is continuous.

We have assumed that $\rho(t)$ and $\mu(t)$ are continuous and differentiable at all but a finite number of points. Denote the set of points where the strategy is discontinuous by $\chi_\rho = \{t_1^\rho, ... t_\nu^\rho\}$, $\chi_\mu = \{t_1^\mu, ... t_\nu^\mu\}, \tilde{\chi}_\rho = \{\tilde{t}_1^\rho, ... \tilde{t}_{\tilde{\nu}}^\rho\}$, $\tilde{\chi}_\mu = \{\tilde{t}_1^\mu, ... \tilde{t}_{\tilde{\nu}}^\mu\}$ and define $\chi = \chi_\rho \cup \chi_\mu$ and $\tilde{\chi} = \tilde{\chi}_\rho \cup \tilde{\chi}_\mu$. Also, define

$$D_\rho(t) = \lim_{s \to t^+} \rho(s) - \lim_{r \to t^-} \rho(r)$$

and

$$D_\mu(t) = \lim_{s \to t^+} \mu(s) - \lim_{r \to t^-} \mu(r)$$

These are non-zero only at points in χ_ρ and χ_μ respectively and represent the probability that a decision to implement at that moment conditional on the other agent not implementing the project prior to that time. The objective function of the agent is:

$$\begin{aligned}
&\max_{e(t),\mu(t),\rho(t)} V_0 + \int_0^T \left[\alpha_1 + \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right] \frac{d\rho}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right] dt \\
&+ \int_0^T \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1 \frac{d\mu}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right] dt \\
&+ \int_0^T \left[\alpha_1 + \left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right] \frac{d\tilde{\rho}}{dt}\left[1-\mu(t)-\rho(t)\right] dt \\
&+ \int_0^T \left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1 \frac{d\tilde{\mu}}{dt}\left[1-\mu(t)-\rho(t)\right] dt \\
&+ \sum_{t\in\chi\cap\tilde{\chi}} \left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right] \left\{ \begin{array}{c} D_\rho(t)\left[\alpha_1 + \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right] \\ +D_\mu(t)\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1 \end{array} \right\} \\
&+ \sum_{t\in\chi\cap\tilde{\chi}} \left[1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)\right] \left\{ \begin{array}{c} D_{\tilde{\rho}}(t)\left[\alpha_1 + \left(\frac{1-\sigma(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_2\right] \\ D_{\tilde{\mu}}(t)\left(\frac{1-\sigma(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_1 \end{array} \right\} \\
&+ \sum_{t\in\chi\backslash\tilde{\chi}} \left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right] \left\{D_\rho(t)\left[\alpha_1 + \left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right] + D_\mu(t)\left(\frac{1-\tilde{\sigma}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\right\} \\
&+ \sum_{t\in\tilde{\chi}\backslash\chi} \left[1-\mu(t)-\rho(t)\right] \left\{D_{\tilde{\rho}}(t)\left[\alpha_1 + \left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right] + D_{\tilde{\mu}}(t)\left(\frac{1-\sigma(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1\right\} \\
&+ \left(1-\mu(T)-\rho(T)\right)\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right)\left[\alpha_1 + \left(\frac{1-\tilde{\sigma}(T)-\tilde{\rho}(T)}{1-\tilde{\mu}(T)-\tilde{\rho}(T)}\right)\alpha_2\right] \\
&- \delta T\left(1-\mu(T)-\rho(T)\right)\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right) \\
&- \int_0^T \delta\left(1-\mu(t)-\rho(t)\right)\left(1-\tilde{\mu}(t)-\tilde{\rho}(t)\right) dt \\
&- c\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right)\left(\sigma(T)-\mu(T)\right)\int_0^T e(t)\, dt \\
&- c\int_0^T e(t)\left(1-\tilde{\mu}(t)-\tilde{\rho}(t)\right)\left(\sigma(t)-\mu(t)\right) dt
\end{aligned}$$

$$\begin{aligned} s.t.\ \rho(t) &\leq 1-\sigma(t),\ \mu(t) \leq \sigma(t) \\ \sigma(0) &= 1,\ \frac{d\sigma}{dt} = -\lambda e(t)(\sigma(t)-\mu(t)) \end{aligned}$$

where $\tilde{\mu}(t), \tilde{\rho}(t), \tilde{\sigma}(t)$ denote the strategy of the other player. To simplify notation we will continue using the definition of $\phi(t) = \frac{\tilde{\sigma}(t)-\tilde{\mu}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}$.

We are interested in perfect Bayesian equilibria of the model so strategies must be an equilibrium for all subgames starting at each time t. We describe these by writing out the problem in terms of continuation values for the successful $V^S(t)$ and unsuccessful agent $V^U(t)$ upon reaching time t. Also, write $\tilde{\rho}(s|t)$ $\tilde{\mu}(s|t)$ for the perceived probabilities that a successful and unsuccessful agent implement the project at $s \geq t$ given the agent is at t. At on-equilibrium times these are $\tilde{\rho}(s|t) = \frac{\tilde{\rho}(s)-\tilde{\rho}(t)}{1-\tilde{\rho}(t)}$ and $\tilde{\mu}(s|t) = \frac{\tilde{\mu}(s)-\tilde{\mu}(t)}{1-\tilde{\mu}(t)}$. However, if an off-equilibrium time is reached then this is no longer the case and separate equilibrium strategies and beliefs must be specified for these subgames. As we will show below the set of symmetric perfect Bayesian equilibria we focus on involve all times being reached with positive probability.

The continuation value of being successful is just the payoff from implementing the optimal stopping policy $\widehat{t}_\rho$ from that moment forward:

$$\begin{aligned} V^S(t) &= \max_{\widehat{t}\in[t,T]} V_0 + \alpha_1 + \int_t^{\widehat{t}} (\alpha_2 - \delta(r-t)) \frac{d\tilde{\rho}(r|t)}{dt} dr + \int_t^{\widehat{t}} (-\delta(r-t)) \frac{d\tilde{\mu}(r|t)}{dt} dr \\ &\quad + \sum_{\substack{r\in\tilde{\chi} \\ t<r<\widehat{t}}} D_{\tilde{\rho}}(r|t)(\alpha_2 - \delta(r-t)) + D_{\tilde{\mu}}(r|t)(-\delta(r-t)) \\ &\quad + \left(1-\tilde{\mu}\left(\widehat{t}|t\right) - \tilde{\rho}\left(\widehat{t}|t\right)\right)\left(\left(1-\phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t}-t\right]\right) \end{aligned} \tag{15}$$

where $\hat{t}^*_\rho(t)$ is the optimizer or set of optimizers of equation 15. The implementation decision strategy is optimal provided that:

$$\begin{aligned} \lim_{s\to r^+} \rho^*(s|t) &= \sigma(r) \text{ if } r = \hat{t}^*_\rho(t) \\ \frac{d\rho^*(r|t)}{dt} &\geq 0 \text{ or } D_{\rho^*}(r|t) \geq 0 \text{ if } r \in \hat{t}^*_\rho(t) \\ \frac{d\rho^*(r|t)}{dt} &= 0 \text{ if } r \notin \hat{t}^*_\rho(t) \end{aligned}$$

and these conditions ensure ρ satisfies the adding up constraint:

$$\int_{\hat{t}^*_\rho(t)} \frac{d\rho^*(t)}{dt} dt = \sigma\left(\max\left(\hat{t}^*(t)\right)\right) - \rho(t) + (\sigma(T) - \rho(T)) \times \mathbf{1}\left(\max\left(\hat{t}^*(t)\right) = T\right)$$

The payoff from being unsuccessful is defined by a joint effort and stopping problem given by:

$$\begin{aligned}
V^U(t) = \max_{\substack{\widehat{t}\in[t,T] \\ e(r|t) \text{ for } r\in\left[t,\widehat{t}\right]}} & V_0 + \int_t^{\widehat{t}} \left[\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)\,dw\right] \frac{d\tilde{\rho}(r|t)}{dt}\left[1-\exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right] dr \\
&+ \int_t^{\widehat{t}} \left[-\delta(r-t) - c\int_t^r e(w|t)\,dw\right] \frac{d\tilde{\mu}(r)}{dt}\left[1-\exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right] dr \\
&+ \int_t^{\widehat{t}} \left[V^S(t) - \delta(r-t) - c\int_t^r e(w|t)\,dw\right] \lambda e(r)\exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\left(1-\tilde{\mu}(r|t)-\tilde{\rho}(r|t)\right) dr \qquad (16) \\
&+ \sum_{\substack{r\in\tilde{\chi}\setminus\chi \\ t<r<\widehat{t}_\mu(t)}} \frac{1-\mu(r)-\sigma(r)}{1-\mu(t)-\sigma(t)} \left[\begin{array}{c} D_{\tilde{\rho}}(r|t)\left(\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)\,dw\right) \\ +D_{\tilde{\mu}}(r|t)\left(-\delta(r-t) - c\int_t^r e(w|t)\,dw\right) \end{array}\right] \\
&+ \left(1-\exp\left(-\int_t^{\widehat{t}} \lambda e(r)\,dr\right)\right)\left(1-\tilde{\mu}\left(\widehat{t}|t\right)-\tilde{\rho}\left(\widehat{t}|t\right)\right)\left[\left(1-\phi^*\left(\widehat{t}|t\right)\right)\alpha_1 - \delta\left(\widehat{t}-t\right) - c\int_t^{\widehat{t}} e(r|t)\,dr\right]
\end{aligned}$$

where $\hat{t}^*_\mu(t)$ is the optimizer or set of optimizers of equation 16. The condition for the effort strategy profile to be an equilibrium satisfies

$$e^*(t) = \arg\max_{e\in[0,1]} \lambda e\left(V^S(t) - V^U(t)\right) - ce$$

and the implementation decision strategy is an equilibrium provided that

$$\begin{aligned}
\lim_{s\to r^+} \mu^*(s|t) &= 1-\sigma(r) \text{ if } r = \hat{t}^*_\mu(t) \\
\frac{d\mu^*(r|t)}{dt} &\geq 0 \text{ or } D_{\rho^*}(r|t) \geq 0 \text{ if } r \in \hat{t}^*_\mu(t) \\
\frac{d\mu^*(r|t)}{dt} &= 0 \text{ if } r \notin \hat{t}^*_\mu(t)
\end{aligned}$$

where $\hat{t}^*(t)$ solves (16) the unsuccessful agent's effort and stopping problem. These conditions also ensure that it satisfies the adding up constraint

$$\int\limits_{\hat{t}^*_\mu(t)} \frac{d\mu^*(t)}{dt}dt = 1-\sigma\left(\max\left(\hat{t}^*_\mu(t)\right)\right) - \mu(t) + (1-\sigma(T)-\mu(T)) \times \mathbf{1}\left(\max\left(\hat{t}^*_\mu(t)\right) = T\right)$$

A symmetric perfect Bayesian equilibrium may be described by a tuple $(e^*(t), \rho^*(t), \mu^*(t), \phi^*(t))$ if $\rho^*(t) + \mu^*(t) < 1$ for all $t < T$, where $\phi^*(t)$ is the Bayesian belief an agent has at time t that the other agent is unsuccessful conditional on the project not having been implemented prior to that time. If $\exists t' < T : \rho^*(t') + \mu^*(t') = 1$ then it must also include off-equilibrium strategies and beliefs $(e^*(r|t), \rho^*(r|t), \mu^*(r|t), \phi^*(r|t))$ for all times t where $\rho^*(t) + \mu^*(t) = 1$ which themselves are equilibria of those subgames, where $\phi^*(r|t)$ is the Bayesian belief an agent has at time r that the other agent is unsuccessful conditional on no project implementation prior to that time in a subgame starting at time t. We now rule out some types of decision strategies at on-equilibrium times by the unsuccessful agent. The following lemma rules out a continuously increasing $\mu^*(t)$.

Lemma 1. *$\nexists\mu^*(t), r > 0, \varepsilon > 0 : \frac{d\mu^*(t)}{dt} > 0$ for $t \in [r-\varepsilon, r]$.*

Proof. Suppose not and $\exists\mu^*(t) : \frac{d\mu^*(t)}{dt} > 0$ for $t \in [r-\varepsilon, r]$. If an unsuccessful agent weakly prefers to implement then a successful agent strictly prefers to implement hence

$$\rho^*(t) = \sigma^*(t) \text{ for } t \in (r-\varepsilon, r)$$

and

$$\phi^*(t) = 1 \text{ for } t \in (r-\varepsilon, r)$$

if not then $\exists r' > t$ such that

$$V_0 + \alpha_1 + (1 - \phi^*(t))\alpha_2 \leq V_0 + \alpha_1 + (1 - \tilde{\mu}^*(r'|t) - \tilde{\rho}^*(r'|t))\left[(1 - \phi^*(r'))\alpha_2 - \delta(r' - t)\right]$$
$$- \delta\left[\int_t^{r'} (y - t)\left[\frac{d\tilde{\mu}^*(y|t)}{dy} + \frac{d\tilde{\rho}^*(y|t)}{dy}\right]dy + \sum_{\substack{y \in \tilde{\mathcal{X}} \\ t<y<r'}} (y - t)\left[D_{\tilde{\mu}}(y|t) + D_{\tilde{\rho}}(y|t)\right]\right]$$
$$+ \alpha_2\left[\int_t^{r'} \frac{d\tilde{\rho}^*(r'|t)}{dy}dy + \sum_{\substack{y \in \tilde{\mathcal{X}} \\ t<y<r'}} D_{\tilde{\rho}}(y|t)\right]$$

This can be rewritten as

$$\delta\left[\int_t^{r'} (y - t)\left[\frac{d\tilde{\mu}^*(y|t)}{dy} + \frac{d\tilde{\rho}^*(y|t)}{dy}\right]dy + \sum_{\substack{y \in \tilde{\mathcal{X}} \\ t<y<r'}} \left[D_{\tilde{\mu}}(y|t) + D_{\tilde{\rho}}(y|t)\right](-\delta(y - t)) + (r' - t)(1 - \tilde{\mu}^*(r'|t))\right]$$
$$\leq \alpha_2\left[(1 - \tilde{\mu}^*(r'|t) - \tilde{\rho}^*(r'|t))(1 - \phi^*(r')) - (1 - \phi^*(t)) + \int_t^{r'} \frac{d\tilde{\rho}^*(r'|t)}{dy}dy + \sum_{\substack{y \in \tilde{\mathcal{X}} \\ t<y<r'}} D_{\tilde{\rho}}(y|t)\right].$$

However, if this inequality holds then an unsuccessful agent could do strictly better by delaying implementation until r' since the comparison of payoffs would result in the same expression except with α_1 replacing α_2. The inequality would then be strict and this would be a contradiction of the unsuccessful agent mixing at t. Hence, $\phi^*(t) = 1$ for $t \in (r - \varepsilon, r)$. However, if $\phi^*(t) = 1$ then an uniformed agent can do strictly better by delaying implementation and exerting effort $e_{\max}$ over a period of time since

$$\lambda e_{\max}\alpha_1 > ce_{\max} + \delta$$

by assumption. □

The following lemma rules out a jump in the implementation decision function $\mu^*(t)$ at a time on the equilibrium path as long as that jump does not occur when both types, successful and unsuccessful, decide to implement with certainty at that instant.

Lemma 2. $\nexists \mu^*(t), 0 < s < T : D_{\mu^*}(s) > 0$ *and* $\mu^*(s) + \rho^*(s) < 1$

Proof. Suppose not and $\exists s : D_{\mu^*}(s) > 0$ and $\mu^*(s) + \rho^*(s) < 1$. As above, this implies

$$\lim_{r \to s^+} \phi^*(r) = 1$$

by the same reasoning as before. Hence, an unsuccessful agent at s can do better than deciding to implement immediately by delaying and putting in effort since

$$\lambda e_{\max}\alpha_1 > ce_{\max} + \delta$$

by assumption. □

The only equilibria involving $D_{\mu^*}(s) > 0$ also have $\mu^*(s) + \rho^*(s) = 1$ whereby beliefs at times later than s are off the equilbrium path. In this case it may be possible to support unsuccessful agents implementing the project with appropriately specified off-equilibrium-path beliefs. However, we will exclude this type of equilibrium as we feel for all intents and purposes that it is equivalent to imposing a deadline at time s. We thus continue the analysis under the assumption that $\mu^*(t) = 0$ for all t. This also means that all times are reached on the equilibrium path. The following lemma rules out jumps in the implementation decision function of the successful type $\rho^*(t)$.

Lemma 3. $\nexists \rho^*(t), 0 < t^\wedge < T : D_{\rho^*}(t^\wedge) > 0.$

Proof. We proceed with a proof by contradiction. Say there is an equilibrium with a mass point at a time $t^\wedge$ where a mass of $D_{\rho^*}(t^\wedge) = (1-\phi^*(t^\wedge))\beta > 0$ decisions to implement are made. For this to be the case then $\phi < 1$. If $\phi^* = 1$ $\rho^*(t) = \sigma^*(t)$ successful agents may only decide to implement at the rate at which unsuccessful agents are becoming successful. Consider $\lim_{t\to t^{\wedge -}} e^*(t)$ and $\lim_{t\to t^{\wedge +}} e^*(t)$. For there to be a mass point the following conditions need to hold for an agent not to implement the project earlier or later:

$$\lim_{t\to t^{\wedge -}} \lambda e^*(t)\,\alpha_2(1-\phi^*(t)) > \delta - \varepsilon \text{ for any } \varepsilon > 0$$

and

$$\lim_{t\to t^{\wedge +}} \lambda e^*(t)\,\alpha_2(1-\phi^*(t)) < \delta + \varepsilon \text{ for any } \varepsilon > 0$$

The first of these inequalities implies that a successful agent will be willing not to implement in the neighborhood immediately prior to $t^\wedge$. The second guarantees that a successful agent cannot do better by waiting at time $t^\wedge$. Also note that $\lim_{t\to t^{\wedge +}} \phi^*(t^\wedge) > \lim_{t\to t^{\wedge -}} \phi^*(t^\wedge)$ due to the mass point. This implies there is a discontinuous change in the effort level at $t^\wedge$ if the above two conditions are to be satisfied. We show that this cannot be maintained in equilibrium. We can write the continuation value from being unsuccessful at time $t = t^\wedge - \Delta t$ as:

$$\begin{aligned}
V^U(t^\wedge - \Delta t) = \int_{t^\wedge-\Delta t}^{t^{\wedge -}} &\left(\frac{\frac{d\sigma^*(s|t^\wedge-\Delta t)}{ds}}{1-\sigma^*(s|t^\wedge-\Delta t)} V^S(t) + (V_0+\alpha_1)\frac{\frac{d\tilde{\rho}^*(s|t^\wedge-\Delta t)}{ds}}{1-\tilde{\rho}^*(s|t^\wedge-\Delta t)} - ce^*(s) - \delta \right) \times \\
&(1-\sigma^*(s|t^\wedge-\Delta t))(1-\tilde{\rho}^*(s|t^\wedge-\Delta t))\,ds \\
&+ (1-\sigma^*(t^\wedge|t^\wedge-\Delta t))\,D_{\tilde{\rho}^*}(t^\wedge|t^\wedge-\Delta t)\left(V_0+\alpha_1-\delta\Delta t - c\int_{t^\wedge-\Delta t}^{t^\wedge} e^*(s)\,ds\right) \\
&\left(\begin{array}{c} \int_{t^{\wedge +}}^{t^\wedge+\Delta t}\left(\frac{\frac{d\sigma^*(s|t^\wedge-\Delta t)}{ds}}{1-\sigma^*(s|t^\wedge-\Delta t)} V^S(t) + (V_0+\alpha_1)\frac{\frac{d\tilde{\rho}^*(s|t^\wedge-\Delta t)}{ds}}{1-\tilde{\rho}^*(s|t^\wedge-\Delta t)} - ce^*(s)-\delta\right)\times \\ (1-\sigma^*(s|t^\wedge-\Delta t))(1-\tilde{\rho}^*(s|t^\wedge-\Delta t))\,ds \\ +(1-\sigma^*(t^\wedge+\Delta t|t^\wedge-\Delta t))(1-\tilde{\rho}^*(t^\wedge+\Delta t|t^\wedge-\Delta t))\,V^U(t^\wedge+\Delta t) \end{array}\right)
\end{aligned}$$

Where Δt may always be chosen small enough such that there are no other points of discontinuity of $\rho^*(t)$ for $t \in [t^\wedge - \Delta t, t^\wedge + \Delta t]$ other than at $t = t^\wedge$. Now consider moving a unit of effort from $t^\wedge - \varepsilon$ to $t^\wedge + \varepsilon$ by augmenting the strategy $e^*(t)$ as follows:

$$\begin{aligned}
e^{**}(t) &= e^*(t) - \varepsilon \text{ for } t \in [t^\wedge - \Delta t, t^\wedge) \\
e^{**}(t) &= e^*(t) + \varepsilon \text{ for } t \in [t^\wedge, t^\wedge + \Delta t]
\end{aligned}$$

The strategies are piecewise continuous so we can always find a Δt such that they are continuous over the intervals $[t-\Delta t, t)$ and $(t, t+\Delta t]$. Using a Taylor series expansion

$$\begin{aligned}
\lim_{\Delta t\to 0}\frac{V^U(t^\wedge-\Delta t|e^{**}) - V^U(t^\wedge-\Delta t|e^*)}{\Delta t} = &-\varepsilon\left(\lambda\left(\lim_{t\to t^{\wedge -}} V^S(t)\right) - c\right) \\
&+ \lambda\varepsilon D_{\rho^*}(t^\wedge|t^\wedge-\Delta t)(V_0+\alpha_1) \\
&+ \varepsilon(1-D_{\rho^*}(t^\wedge|t^\wedge-\Delta t))\left(\lambda\left(\lim_{t\to t^{\wedge +}} V^S(t)\right) - c\right) - O(\Delta t)
\end{aligned}$$

$$\begin{aligned}
\lim_{\Delta t\to 0}\frac{V^U(t^\wedge-\Delta t|e^{**}) - V^U(t^\wedge-\Delta t|e^*)}{\Delta t} \geq &-\varepsilon(\lambda(V_0+\alpha_1+(1-\phi(t^\wedge))\alpha_2) - c) \\
&+ \lambda\varepsilon D_{\rho^*}(t^\wedge|t^\wedge-\Delta t)(V_0+\alpha_1) \\
&+ \varepsilon(1-D_{\rho^*}(t^\wedge|t^\wedge-\Delta t))(\lambda(V_0+\alpha_1+(1-\phi(t^\wedge)-D_{\rho^*}(t^\wedge|t^\wedge-\Delta t))\alpha_2) - c) - O(\Delta t)
\end{aligned}$$

Define the right-hand side by R then

$$\begin{aligned} R &= -D_{\rho^*}\left(t^{\hat{}} | t^{\hat{}} - \Delta t\right) \lambda \alpha_2 \varepsilon + D_{\rho^*}\left(t^{\hat{}} | t^{\hat{}} - \Delta t\right) c \varepsilon - O\left(\Delta t\right) \\ &= \varepsilon D_{\rho^*}\left(t^{\hat{}} | t^{\hat{}} - \Delta t\right)\left(c - \lambda \alpha_2\right) - O\left(\Delta t\right) \\ &> 0 \end{aligned}$$

Hence, there exists $\Delta t > 0$ such that this change in strategy is profitable which is a contradiction that the original effort $e^*(t)$ is optimal and can be part of an equilibrium. □

This along with the earlier lemmas that unsuccessful individuals do not decide to implement implies that $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ are all continuous.

Lemma 4. *$V^S(t)$ is continuous.*

Proof. The continuity of $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ ensures that

$$f\left(t, \widehat{t}\right) = V_0 + \alpha_1 + \int_t^{\widehat{t}} \left(\alpha_2 - \delta\left(r - t\right)\right) \frac{d\tilde{\rho}^*\left(r|t\right)}{dt} dr + \left(1 - \tilde{\rho}^*\left(\widehat{t}|t\right)\right)\left(\left(1 - \phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t} - t\right]\right)$$

is continuous in $\hat{t}$. Hence, $V^S(t) = \max_{\hat{t}\in[t,T]} f(t)$ is continuous in t by the theorem of the maximum (Berge 1963). □

Lemma 5. *$V^U(t)$ is continuous.*

Proof. The continuity of $\phi^*(t)$, $\rho^*(t)$ and $\sigma^*(t)$ ensures that

$$\begin{aligned} f\left(t, e\left(r|t\right)\right) = V_0 &+ \int_t^T \left(\alpha_1 - \delta\left(r - t\right) - c\int_t^r e\left(w|t\right) dw\right) \frac{d\tilde{\rho}^*\left(r|t\right)}{dt}\left[1 - \exp\left(-\lambda \int_t^r e\left(w|t\right) dw\right)\right] dr \\ &+ \int_t^T \left(V^S\left(t\right) - \delta\left(r - t\right) - c\int_t^r e\left(w|t\right) dw\right) \lambda e\left(r\right) \exp\left(-\lambda \int_t^r e\left(w|t\right) dw\right)\left[1 - \tilde{\rho}^*\left(r|t\right)\right] dr \\ &+ \left(1 - \exp\left(-\int_t^T e\left(r\right) dr\right)\right)\left(1 - \tilde{\rho}^*\left(T|t\right)\right)\left[\left(1 - \phi^*\left(T|t\right)\right)\alpha_1 - \delta\left(T - t\right) - c\int_t^T e\left(r|t\right) dr\right] \end{aligned}$$

is continuous in $e(r|t)$. Hence,

$$V^S(t) = \max_{e(r|t)\in C_1([t,T],[0,e_{\max}])} f\left(t, e\left(r|t\right)\right)$$

where $C_1([t,T],[0,e_{\max}])$ are piecewise continuous functions with domain $[t,T]$ and range $[0,e_{\max}]$ that are continuous in t by the theorem of the maximum (Berge 1963). □

Lemma 6. *Suppose $\rho^*(t)$ and $e^*(t)$ constitute equilibrium strategies and $\exists s, \Delta s > 0 : \frac{d\rho^*(t)}{dt} > 0$ and $0 < e^*(t) \leq e_{\max}$ for $t \in [s, s+\Delta s]$, then $e^*(t) = \frac{\delta}{\lambda\phi(t)\alpha_2}$ for $t \in [s+\Delta s]$.*

Proof. We use the relation $d^*(t)(1-\phi^*(t)) = \frac{\frac{d\rho^*(t)}{dt}}{1-\rho^*(t)}$. Also, note that for $\mu^*(t) = 0$

$$\begin{aligned}
\phi(t) &= \frac{\tilde{\sigma}(t)}{1-\tilde{\rho}(t)} \\
\frac{d\phi}{dt} &= \frac{\frac{d\tilde{\sigma}}{dt} + \frac{\frac{d\tilde{\rho}}{dt}}{1-\tilde{\rho}(t)}\tilde{\sigma}(t)}{1-\tilde{\rho}(t)} \\
&= \frac{-\lambda\tilde{e}(t)\tilde{\sigma}(t) + \tilde{d}(t)(1-\phi(t))\tilde{\sigma}(t)}{1-\tilde{\rho}(t)} \\
&= \left[\tilde{d}(t)(1-\phi(t)) - \lambda\tilde{e}(t)\right]\frac{\tilde{\sigma}(t)}{1-\tilde{\rho}(t)} \\
&= \left[\tilde{d}(t)(1-\phi(t)) - \lambda\tilde{e}(t)\right]\phi(t)
\end{aligned}$$

The incentives for delaying rather than implementing are equal if the agent is mixing, that is

$$\begin{aligned}
V_0 + \alpha_1 + (1-\phi^*(t))\alpha_2 = \int_t^{t+\Delta t} &-\delta(r-t)\tilde{d}^*(r)(1-\phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)(1-\phi^*(s))\,ds\right)dr \\
&+ (V_0+\alpha_1+\alpha_2)\left(1-\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\right) \\
&+ \exp\left(-\int_t^r \tilde{d}^*(s)(1-\phi^*(s))\,ds\right)(V_0+\alpha_1+(1-\phi^*(t+\Delta t))\alpha_2 - \Delta t\delta)
\end{aligned}$$

This can be rearranged to obtain

$$\begin{aligned}
\delta\int_t^{t+\Delta t}(r-t)\tilde{d}^*(r)(1-\phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)(1-\phi^*(s))\,ds\right)dr \\
+ \delta\Delta t\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right) \\
= \alpha_2\left(\phi(t) - \phi(t+\Delta t)\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\right)
\end{aligned}$$

Apply a Taylor series expansion to $\phi^*(t+\Delta t)$ and $\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)$

$$\phi^*(t+\Delta t) = \phi^*(t) - \int_t^{t+\Delta t}\left[\lambda\tilde{e}^*(s) - \tilde{d}^*(s)(1-\phi^*(s))\right]\phi^*(s)\,ds$$

$\Leftrightarrow$

$$\phi^*(t+\Delta t) = \phi^*(t)\left\{1-\Delta t\left[\lambda\tilde{e}^*(t) - \tilde{d}^*(t)(1-\phi^*(t))\right]\right\} + O\left((\Delta t)^2\right)$$

and apply it also to $\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right)$:

$$\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right) = 1-\Delta t\tilde{d}^*(t)(1-\phi^*(t)) + O\left((\Delta t)^2\right)$$

We combine these expressions and denote the expression inside of the brackets on the right-hand side by $R = \phi^*(t) - \phi^*(t+\Delta t)\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)$

$$R = \phi^*(t)\alpha_2\left\{\Delta t\left[\lambda\tilde{e}^*(t) - \tilde{d}^*(t)(1-\phi^*(t))\right]\right\} + \Delta t\tilde{d}^*(t)(1-\phi^*(t)) + O\left((\Delta t)^2\right)$$

Simplifying this expression yields

$$R = \Delta t \left[\lambda \tilde{e}^* (t) \phi^* (t) \alpha_2\right] + O\left((\Delta t)^2\right)$$

Denote the left-hand side by L and apply a Taylor series expansion:

$$L = \delta \int_t^{t+\Delta t} (r-t) \tilde{d}^* (r) (1-\phi^* (r)) \exp\left(-\int_t^r \tilde{d}^* (s) (1-\phi^* (s)) ds\right) dr$$
$$+ \delta \Delta t \exp\left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1-\phi^* (s)) ds\right)$$

$$L = \delta \Delta t + O\left(\Delta t^2\right)$$

Equating the Δt terms from the left- and right-hand sides leads one to conclude that $\delta = \lambda \tilde{e}^* (t) \phi^* (t) \alpha_2$ ☐

The indifference condition implies $\lambda \tilde{e}^* (t) \phi^* (t) \alpha_2 = \delta$ when $\tilde{d}^* (t) > 0$ for all t.

Lemma 7. *Suppose $\rho^* (t)$, $e^* (t)$ and $\phi^* (t)$ constitute equilibrium strategies and beliefs, and $\exists s, \Delta s > 0$: $\frac{d\rho^*(t)}{dt} > 0, \phi^* (t) < 1$ and $0 < e^* (t) \leq e_{\max}$ for $t \in [s, s+\Delta s]$, then $e^* (t) = e_{\max}, \phi^* (t) = \bar{\phi}_d, d^* (t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \in [s+\Delta s]$.*

Proof. Suppose not. This implies $e^* (t) < e_{\max}$, $\phi^* (t) > \bar{\phi}_d$ from lemma 6. The following condition must hold

$$V^S (s) - V^U (s) = \frac{c}{\lambda} \text{ for all } s \in [t, t+\Delta t]$$

for effort to be optimal. Hence, we also require

$$\begin{aligned} V^S (t) - V^S (t+\Delta t) &= V^U (t) - V^U (t+\Delta t) \\ \left[\phi^* (t+\Delta t) - \phi^* (t)\right] \alpha_2 &= V^U (t) - V^U (t+\Delta t) \end{aligned}$$

since this is true for all s. Recall

$$V^S (t) = \int_t^{t+\Delta t} -\delta (r-t) \tilde{d}^* (r) (1-\phi^* (r)) \exp\left(-\int_t^r \tilde{d}^* (s) (1-\phi^* (s)) ds\right) dr$$
$$+ (V_0 + \alpha_1 + \alpha_2) \left[1 - \exp\left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1-\phi^* (s)) ds\right)\right]$$
$$+ \exp\left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1-\phi^* (s)) ds\right) \left[V^S (t+\Delta t) - \Delta t \delta\right]$$

and

$$V^S (t) - V^S (t+\Delta t) = \int_t^{t+\Delta t} -\delta (r-t) \tilde{d}^* (r) (1-\phi^* (r)) \exp\left(-\int_t^r \tilde{d}^* (s) (1-\phi^* (s)) ds\right) dr$$
$$+ \left(V_0 + \alpha_1 + \alpha_2 - V^S (t+\Delta t)\right) \left[1 - \exp\left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1-\phi^* (s)) ds\right)\right]$$
$$- \Delta t \delta \exp\left(-\int_t^{t+\Delta t} \tilde{d}^* (s) (1-\phi^* (s)) dsds\right)$$

Now we can write $V^U(t)$ as follows

$$V^U(t) = \int_t^{t+\Delta t} -\delta(r-t)\tilde{d}^*(r)(1-\phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)(1-\phi^*(s))\,ds\right)dr$$
$$+ (V_0+\alpha_1)\left(1-\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\right)$$
$$+ \exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\left[V^U(t+\Delta t)-\Delta t\delta\right]$$

since the agent is indifferent about which level of effort to exert we can write it out assuming $e_i(t)=0$. We can further calculate $V^U(t)-V^U(t+\Delta t)$ using $V^U(t+\Delta t)=V^S(t+\Delta t)-\frac{c}{\lambda}$

$$V^U(t)-V^U(t+\Delta t) = \int_t^{t+\Delta t} -\delta(r-t)\tilde{d}^*(r)(1-\phi^*(r))\exp\left(-\int_t^r \tilde{d}^*(s)(1-\phi^*(s))\,ds\right)dr$$
$$+\left(V_0+\alpha_1-V^S(t+\Delta t)+\frac{c}{\lambda}\right)\left[1-\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\right]$$
$$-\Delta t\delta\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)$$

$$V^U(t)-V^U(t+\Delta t) = V^S(t)-V^S(t+\Delta t)+\left(\frac{c}{\lambda}-\alpha_2\right)\left[1-\exp\left(-\int_t^{t+\Delta t}\tilde{d}^*(s)(1-\phi^*(s))\,ds\right)\right]$$

Hence, we have a contradiction $V^U(t)-V^U(t+\Delta t) > V^S(t)-V^S(t+\Delta t)$ for $\tilde{d}^*(s)>0$ and $\phi^*(s)<1$. ☐

Lemma 8. $\phi^*(t) \geq \max\left\{\bar{\phi}_d, \bar{\phi}\right\}$

Proof. Suppose $\exists \breve{t} : \phi^*\left(\breve{t}\right) < \bar{\phi}_d$ then $\exists \Delta\breve{t} > 0 : \phi^*(s) < \bar{\phi}_d$ for $s \in \left[\breve{t}-\Delta\breve{t}, \breve{t}\right]$. However, this is a contradiction as a successful player will strictly prefer to implement for all $s \in \left[\breve{t}-\Delta\breve{t}, \breve{t}\right]$ and hence $\phi^*\left(\breve{t}\right) > \bar{\phi}_d$ which is a contradiction that $\exists \breve{t} : \phi^*\left(\breve{t}\right) < \bar{\phi}_d$. Suppose $\exists \breve{t} : \phi^*\left(\breve{t}\right) < \bar{\phi}$ then $\exists \Delta\breve{t} > 0 : \phi^*\left(\breve{t}-\Delta\breve{t}\right) < \bar{\phi}$. This implies an upper bound on the value of information is $\left(1-\phi^*\left(\breve{t}\right)\right)\alpha_2 + \phi^*\left(\breve{t}\right)\alpha_1$. Thus, for any time s such that $\phi^*(s) < \bar{\phi} \Rightarrow e^*(s) = 0$ therefore $\frac{d\phi^*(s)}{ds} = 0$ so $\lim_{r\to}\phi^*\left(\breve{t}^-\right) > \bar{\phi}\left(\breve{t}\right)$ which is a violation of the continuity of $\phi^*(t)$. This is a contradiction given the upper bound on the arrival rate of information. ☐

Lemma 9. *Suppose $e^*(t) < e_{\max}$ for some t then under large incentives $\phi^*(T) = \bar{\phi}_d$ and under small incentives $\phi^*(T) = \bar{\phi}$*

Proof. Suppose not, then $\phi^*(T) > \max\left\{\bar{\phi}_d, \bar{\phi}\right\}$. Let $\hat{t} = \inf\left\{t | e^*(t) < e_{\max}\right\}$. Being successful at time $\hat{t}$ has a continuation value given by

$$V^S\left(\hat{t}\right) = V_0 + \alpha_1 + (1-\phi^*(T))\alpha_2 - \delta\left(T-\hat{t}\right)$$

since the optimal strategy for a successful individual is to delay until the deadline which is due to $e^*(t) = e_{\max}$ and $\phi^*(t) > \bar{\phi}_d$ for $t \geq \hat{t}$. The continuation value for the unsuccessful individual is

$$V^U\left(\hat{t}\right) = \left(V_0+\alpha_1+(1-\phi^*(T))\alpha_2-\frac{c}{\lambda}\right)\left[1-\exp\left(-\lambda e_{\max}\left(T-\hat{t}\right)\right)\right]$$
$$+\exp\left(-\lambda e_{\max}\left(T-\hat{t}\right)\right)\left[V_0+(1-\phi^*(T))\alpha_1\right]-\delta\left(T-\hat{t}\right)$$

Thus, incentives for effort are given by

$$V^S\left(\hat{t}\right)-V^U\left(\hat{t}\right) = \frac{c}{\lambda}+\exp\left(-\lambda e_{\max}\left(T-\hat{t}\right)\right)\left[(1-\phi^*(T))\alpha_2+\phi^*(T)\alpha_1-\frac{c}{\lambda}\right].$$

Further, by definition of $\bar{\phi}_d$ and $\bar{\phi}$.we have

$$(1-\phi^*(T))\,\alpha_2+\phi^*(T)\,\alpha_1>\left(\left(1-\bar{\phi}_d\right)\alpha_2+\bar{\phi}_d\alpha_1\right)$$
$$>\left(\left(1-\bar{\phi}\right)\alpha_2+\bar{\phi}\alpha_1\right)=\frac{c}{\lambda}$$

Hence, $V^S\left(\hat{t}\right)-V^U\left(\hat{t}\right)>\frac{c}{\lambda}$ and by the continuity of V^S and V^U $\exists\omega>0:e^*\left(\hat{t}-\omega\right)=e_{\max}$ which is a contradiction of $\hat{t}=\inf\left\{t|e^*(t)<e_{\max}\right\}$. □

The previous lemmas restrict the set of potential equilibria to those where $\phi^*(t)$ is continuous, decreasing and bounded below by $\max\left\{\bar{\phi}_d,\bar{\phi}\right\}$. Furthermore, if implementation decisions are taken prior to the deadline, then $\frac{d\phi^*}{dt}=0$ and either $\phi^*=1$ or $\phi^*=\bar{\phi}_d$ during those times.

B.3.2 Proof for Uniqueness of Symmetric Equilibria Set for Large Incentives Case

Define $V^{S*}(t)$ and $V^{U*}(t)$

$$\begin{aligned}
V^{S*}(t) &= V_0+\alpha_1+\left(1-\bar{\phi}_d\right)\alpha_2\\
V^{U*}(t) &= V_0+\alpha_1+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\\
&\quad -\exp\left(-2\lambda e_{\max}(T-t)\right)\left[\bar{\phi}_d\alpha_1+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}$$

Note further that

$$\begin{aligned}
V^{S*}(t)-V^{U*}(t) &= \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}+\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\\
&\quad +e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}$$

and

$$V^{S*}(t)-V^{U*}(t)\begin{cases}>\frac{c}{\lambda} & \text{for } T-t<Z\\ =\frac{c}{\lambda} & \text{for } T-t=Z\\ <\frac{c}{\lambda} & \text{for } T-t>Z\end{cases}$$

Also, define $\bar{t}_d(\phi)$, $V^{S*}(t,\phi)$ and $V^{U*}(t,\phi)$

$$\bar{t}_d(\phi)=\frac{1}{\lambda}\ln\frac{\phi}{\bar{\phi}_d}$$

$$V^{S*}(t,\phi) = \begin{cases} V_0+\alpha_1+\left(1-\phi(t)\exp\left(-\lambda e_{\max}(T-t)\right)\right)\alpha_2-\delta(T-t) & \text{for } T-\bar{t}_d(\phi)<t\leq T\\ V^{S*}\left(t+\tilde{t}\right)-\delta\tilde{t} & \text{for } T-\bar{t}_d(\phi)-Z\leq t\leq T-\bar{t}_d(\phi),\end{cases}$$

$$V^{U*}(t,\phi) = \begin{cases} \left[1-\exp\left(-\lambda e_{\max}(T-t)\right)\right]\left(V_0+\alpha_1+\left(1-\phi(t)\exp\left(-\lambda e_{\max}(T-t)\right)\right)\alpha_2-\frac{c}{\lambda}\right) \\ \quad +\exp\left(-\lambda e_{\max}(T-t)\right)\left(V_0+\left[1-\phi(t)\exp\left(-\lambda e_{\max}(T-t)\right)\right]\alpha_1\right)-\delta(T-t) \\ \qquad \text{for } T-\bar{t}_d(\phi)<t\leq T \\ \left(1-\exp\left(-\lambda e_{\max}\tilde{t}\right)\right)\left(V^{S*}\left(t+\tilde{t}\right)-\frac{c}{\lambda}\right)+\exp\left(-\lambda e_{\max}\tilde{t}\right)V^{U*}\left(t+\tilde{t}\right)-\delta\tilde{t} \\ \qquad \text{for } T-\bar{t}_d(\phi)-Z\leq t\leq T-\bar{t}_d(\phi).\end{cases}$$

Note that

$$V^{S*}(t,\phi)-V^{U*}(t,\phi)=\begin{cases}\frac{c}{\lambda}+\exp\left(-\lambda e_{\max}(T-t)\right)\left[\phi(t)\exp\left(-\lambda e_{\max}(T-t)\right)\alpha_1\right.\\ \quad\left.+\left(1-\phi(t)\exp\left(-\lambda e_{\max}(T-t)\right)\right)\alpha_2-\frac{c}{\lambda}\right]\\ \qquad \text{for } T-\bar{t}_d(\phi)<t\leq T\\ \frac{c}{\lambda}+\exp\left(-\lambda e_{\max}\tilde{t}\right)\left(V^{S*}\left(t+\tilde{t}\right)-V^{U*}\left(t+\tilde{t}\right)-\frac{c}{\lambda}\right)\\ \qquad \text{for } T-\bar{t}_d(\phi)-Z\leq t\leq T-\bar{t}_d(\phi).\end{cases}$$

Hence,

$$V^{S*}(t,\phi)-V^{U*}(t,\phi)\begin{cases} >\frac{c}{\lambda} \text{ for } T-\bar{t}_d(\phi)-Z<t \\ =\frac{c}{\lambda} \text{ for } T-\bar{t}_d(\phi)-Z=t \\ <\frac{c}{\lambda} \text{ for } T-\bar{t}_d(\phi)-Z>t \end{cases}.$$

Lemma 10. *The unique equilibrium strategies in any subgame starting at t with beliefs $\phi(t)\geq\bar{\phi}_d$ such that $t\geq T-Z-\bar{t}_d(\phi)$ are $e^*(s)=e_{\max}$ for $t\leq s\leq T$ and $d^*(s)=\begin{cases} 0 \text{ for } t\leq s\leq \bar{t}_d(\phi) \\ \frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta} \text{ for } \bar{t}_d(\phi)<s\leq T\end{cases}$*

Proof. Suppose $\exists s\geq t,\varepsilon>0$ such that $e^*(r)<e_{\max}$ for $r\in[s-\varepsilon,s)$. If this is the case, we can check the continuation values at $\widehat{s}$ where

$$\widehat{s}=\sup\{r|e^*(r)<1\}.$$

Given that $e^*(s)=e_{\max}$ for $r\geq\widehat{s}$ then the unique implementation decision strategy is

$$d^*(s)=\begin{cases} 0 \text{ for } \widehat{s}\leq s\leq\min\{T,\widehat{s}+\bar{t}_d(\phi(\widehat{s}))\} \\ \frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta} \text{ for } \widehat{s}+\bar{t}_d(\phi(\widehat{s}))<s\leq T \text{ if } \widehat{s}+\tilde{t}(\phi(\widehat{s}))<T\end{cases}$$

since the only belief at which a successful individual will implement is $\phi=\bar{\phi}_d$ when the unsuccessful agent is exerting maximum effort. We can therefore write the continuation values as $V^{S*}(\widehat{s},\phi)$ and $V^{U*}(\widehat{s},\phi)$. The contradiction now comes from noting that for $t>T-Z-\bar{t}_d(\phi)$ and $V^{S*}(t,\phi)-V^{U*}(t,\phi)>\frac{c}{\lambda}$. Therefore, $\exists\zeta: V^{S*}(r,\phi(r))-V^{U*}(r,\phi(r))>\frac{c}{\lambda}$ and $e^*(r)<e_{\max}$ for $r\in[\widehat{s}-\zeta,\widehat{s})$ which means $e^*(r)$ is not an equilibrium strategy. □

Lemma 11. *Suppose $T\geq X_d+Z$, then an upper bound on $\phi^*(t)$ is given by*

$$\phi^*(t)\leq \begin{matrix} \bar{\phi}_d\exp(\lambda(T-Z-t)) & \text{for } T-X_d-Z\leq t<T-Z \\ \bar{\phi}_d & \text{for } T-Z\leq t\leq T\end{matrix}$$

Proof. Suppose $\exists t',\phi^*(t')>\bar{\phi}_d\exp(\lambda(T-Z-t'))$ for $T-X_d-Z\leq t'<T-Z$ or $\phi^*(t)>\bar{\phi}_d$ for $T-Z\leq t'\leq T$ then

$$\exists s<t':(s,\phi^*(s))\in\left\{\begin{array}{c}(r(\phi),\phi)|r(\phi)=T-X_d-Z+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi} \\ +\gamma\left(t'-\left(T-X_d-Z+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi}\right)\right),\gamma\in(0,1),\phi\in[\phi(t),1]\end{array}\right\}.$$

Now $s>T-Z-\bar{t}_d(\phi^*(s))$ and thus the unique equilibrium of the subgame starting from $(s,\phi^*(s))$ is given by Lemma 10. However, the Bayesian belief $\widehat{\phi}^*(r)$ in this subgame reaches

$$\widehat{\phi}^*(r)=\phi^*(t')=(T-X_d-Z)+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}+\gamma(s)\left(t'-(T-X_d-Z)+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}\right)$$

where $\gamma(s)=\frac{s-(T-X_d-Z)+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(s)}}{t'-(T-X_d-Z)+\frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}}<1$. Thus, $r<t$ and $\widehat{\phi}^*(t')>\phi^*(t')$ and hence $\phi^*(t')$ is not part of a perfect Bayesian equilibrium. □

Together with lemma 8 this uniquely determines $\phi(t)=\bar{\phi}_d$ for $t\geq T-Z$ if $T\geq X_d+Z$.

Lemma 12. *Suppose $T\geq X_d+Z$ then a lower bound on $\phi^*(t)$ is given by*

$$\phi^*(t)\geq\begin{cases}\bar{\phi}_d \text{ for } t\geq T-Z \\ \bar{\phi}_d+\delta(T-Z-t) \text{ for } T-Y_d-Z\leq t<T-Z \\ 1 \text{ for } t\leq T-Y_d-Z\end{cases}$$

Proof. Lemma 11 and lemma 8 pin down $\phi^*(t)=\bar{\phi}_d$ for $t\geq T-Z$. Now suppose $\exists s<T-Y_d-Z:\phi^*(s)<1$ or $\exists s: T-Y_d-Z\leq s<T-Z,\phi^*(s)<\bar{\phi}_d+\delta(T-Z-s)$. If $\exists r<T-Z: d^*(t)=0$ for $t\in[r,T-Z]$ there is an immediate contradiction as successful individuals would strictly prefer to implement the project immediately. If not then using lemma 7 if $d^*(t)>0$ for $s\leq t<T-Z$ then $e^*(t)=e_{\max},\phi^*(t)=\bar{\phi}_d,d^*(t)=\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}$ for $t\in[r,T-Z]$. However in this case we also have a contradiction as $V^S(t)-V^U(t)=V^{S*}(t)-V^{U*}(t)<\frac{c}{\lambda}$ since

$t < T - Z$ and the effort strategy $e^*(t) = e_{\max}$ is not optimal and cannot be part of an equilibrium.

$\square$

These two lemmas provide an upper and lower bound on the values of $\phi^*(t)$ in equilibrium. The proof for uniqueness now proceeds by showing that the only equilibrium strategies which support values of ϕ between these bounds are the ones given in the propositions.

Proof. **i) Case 1:** $T < X_d$

Strategy of successful player: $d^*(t) = 0$ for all t. Strategy of unsuccessful player: $e^*(t) = e_{\max}$ for all t. Beliefs: $\phi^*(t) = \exp(-\lambda e_{\max} t)$ for all t.

ii) Case 2: $X_d \le T < X_d + Z$

Strategy of successful player: $d^*(t) = 0$ for $t < X_d$, $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \ge X_d$. Strategy of unsuccessful player: $e^*(t) = e_{\max}$ for all t. Beliefs: $\phi^*(t) = \exp(-\lambda e_{\max} t)$ for $t < X_d$.and $\phi(t) = \bar{\phi}_d$ for $t \ge X_d$.

Lemma 10 covers Case 1 and 2.

iii) Case 3: $X_d + Z < T < Y_d + Z$

Strategy of successful player: $d^*(t) = 0$ for $t < T - Z$ and $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \ge T - Z$. Strategy of unsuccessful player: $d^*(t) = 0$ for all t and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_0^t e^*(s)\, ds\right) \ge \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_0^{T-Z} e^*(s)\, ds\right) = \bar{\phi}_d.$$

for $t < T - Z$ and $e^*(t) = e_{\max}$ for all $t \ge T - Z$. Beliefs: $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for $t < T - Z$ and $\phi^*(t) = \bar{\phi}_d$ for $t \ge T - Z$.

Lemmas 10, 12 and 11 determine the bounds on $\phi^*(t)$ and that $\phi^*(t) = \bar{\phi}_d, e^*(t) = e_{\max}$ for $t \ge T - Z$. Lemma 7 implies that $d^*(t) = 0$ for $t < T - Z$ which suffices along with the earlier lemmas for the equilibrium strategy set.

iv) Case 4: $T > Y_d + Z$

Strategy of successful player: $d^*(t) = \textit{implement}$ for $0 \le t \le T - Y_d - Z$ and $d^*(t) = 0$ for $T - Y_d - Z < t < T - Z$ and $d^*(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \ge T - Z$. Strategy of unsuccessful player: $d^*(t) = 0$ for all t. $e^*(t) = \frac{\delta}{c}$ for $0 \le t \le T - Y_d - Z$ and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_{T-Y_d-Z}^t e^*(s)\, ds\right) \ge \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_{T-Y_d-Z}^{T-Z} e^*(s)\, ds\right) = \bar{\phi}_d.$$

for $T - Y_d - Z < t < T - Z$ and $e^*(t) = e_{\max}$ for $t \ge T - Z$.

Beliefs: $\phi^*(t) = 1$ for $0 \le t \le T - Y_d - Z$, $\phi^*(t) = \exp\left(-\lambda \int_{T-Y_d-Z}^t e^*(s)\, ds\right)$ for $T - Y_d - Z < t < T - Z$ and $\phi^*(t) = \bar{\phi}_d$ for $t \ge T - Z$.

Case 3 above covers the subgames for $t > T - Z - \widehat{T}$. It remains to show that the above strategies are unique for $t \le T - Z - \widehat{T}$. For $t \le T - Z - \widehat{T}$ we have shown that $\phi^*(t) = 1$. First, rule out that $e(t) = 0$. If this were the case the continuation payoffs would be $V^S(t) = V_0 + \alpha_1$ and $V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda} - \left(\tilde{t} - t\right)\delta$ where $\tilde{t} = \inf\{s > t : e(s) > 0\}$, therefore the strategy $e^*(t) = 0$ is not optimal as $V^S - V^U > \frac{c}{\lambda}$. Implying that $0 < e^*(t) \le \frac{\delta}{\lambda \alpha_2}$ and individuals decide to implement immediately. We have $V^S(t) = V_0 + \alpha_1$ so $V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda}$

for $t \leq T - Z - \widehat{T}$.

$$V^U(t) = \int_t^{t+\Delta t} \left(V_0 + \alpha_1 - c \int_t^s e^*(r)\, dr - \delta(s-t) \right) 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\, dr \right) ds$$
$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \left(V^U(t+\Delta t) - \delta \Delta t - c \int_t^{t+\Delta t} e^*(r)\, dr \right)$$

$$V^U(t) = \left(V_0 + \alpha_1 - \frac{c}{2\lambda} \right) \left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \right)$$
$$- \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\, dr \right) ds$$
$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \left(V^U(t+\Delta t) - \delta \Delta t \right)$$

$$V^U(t) = V_0 + \alpha_1 - \frac{c}{\lambda} + \frac{c}{2\lambda} \left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \right)$$
$$- \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\, dr \right) ds$$
$$- \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \delta \Delta t$$

hence

$$\frac{c}{2\lambda} \left(1 - \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \right) = \int_t^{t+\Delta t} \delta(s-t)\, 2\lambda e^*(s) \exp\left(-2\lambda \int_t^s e^*(r)\, dr \right) ds$$
$$+ \exp\left(-2\lambda \int_t^{t+\Delta t} e^*(r)\, dr \right) \delta \Delta t$$

$$c e^*(t)\, \Delta t + O\left(\Delta t^2\right) = \delta \Delta t + O\left(\Delta t^2\right)$$

We therefore require that $e^*(t) = \frac{\delta}{c}$. $\square$

B.3.3 Proof for Uniqueness of Symmetric Equilibria Set for Baseline Case

Define

$$\begin{aligned} \bar{t}_e(\phi) &= \frac{1}{\lambda} \ln \frac{\phi}{\bar{\phi}} \\ V_e^{S*}(t,\phi) &= V_0 + \alpha_1 + (1 - \phi \exp[-\lambda e_{\max}(T-t)])\, \alpha_2 - \delta \bar{t}_e(\phi) \end{aligned}$$

as well as

$$V_e^{U*}(t,\phi) = (1 - \exp[-\lambda e_{\max}(T-t)]) \left(V_0 + \alpha_1 + (1 - \phi \exp[-\lambda e_{\max}(T-t)])\, \alpha_2 - \frac{c}{\lambda} \right)$$
$$+ \exp[-\lambda e_{\max}(T-t)] \left(V_0 + (1 - \phi \exp[-\lambda e_{\max}(T-t)])\, \alpha_1 \right) - \delta \bar{t}_e(\phi)$$

and

$$V_e^{S*}(t,\phi) - V_e^{U*}(t,\phi) = \frac{c}{\lambda} + \exp\left[-\lambda e_{\max}(T-t)\right]\left(\phi\exp\left[-\lambda e_{\max}(T-t)\right]\alpha_1 + \left(1-\phi\exp\left[-\lambda e_{\max}(T-t)\right]\right)\alpha_2 - \frac{c}{\lambda}\right)$$

Note that

$$V_e^{S*}(t,\phi) - V_e^{U*}(t,\phi) > \frac{c}{\lambda}$$

provided that $t < \bar{t}_e(\phi)$.

Lemma 13. *The unique equilibrium strategy in any subgame starting at t with beliefs $\phi(t)$ such that $t \geq T - \bar{t}_e(\phi)$ is $e^*(s) = e_{\max}$ for $t \leq s \leq T$ and $d^*(s) = 0$.*

Proof. Suppose $\exists s \geq t, \varepsilon > 0$ such that $e^*(r) < e_{\max}$ for $r \in [s-\varepsilon, s)$. If this is the case, we can check the continuation values at $\widehat{s}$ where

$$\widehat{s} = \sup\{r | e^*(r) < 1\}.$$

Given that $e^*(r) = e_{\max}$ for $r \geq \widehat{s}$ then the unique decision strategy is $d^*(r) = 0$ since the only belief at which a successful individual will decide to implement is $\bar{\phi}_d < \bar{\phi}$ when the unsuccessful agent is exerting maximum effort. We can therefore write the continuation values as $V_e^{S*}(\widehat{s},\phi(\widehat{s}))$, $V_e^{U*}(\widehat{s},\phi(\widehat{s}))$. The contradiction now comes from noting that

$$t > T - \bar{t}_e(\phi) \Rightarrow \widehat{s} > T - \bar{t}_e(\phi(\widehat{s}))$$

hence $V_e^{S*}(\widehat{s},\phi(\widehat{s})) - V_e^{U*}(\widehat{s},\phi(\widehat{s})) > \frac{c}{\lambda}$. Thus $\exists\zeta : V_e^{S*}(r,\phi(r)) - V_e^{U*}(r,\phi(r)) > \frac{c}{\lambda}$ and $e^*(r) < e_{\max}$ for $r \in [\widehat{s}-\zeta, \widehat{s})$ which means that $e^*(r)$ is not an equilibrium strategy. □

Lemma 14. *Suppose $T \geq X$, then an upper bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \leq \begin{cases} 1 \textit{ for } \; t < T - X \\ \bar{\phi}\exp(\lambda(T-t)) \textit{ for } \; T - X \leq t < T \end{cases}$$

Proof. Suppose $\exists t'$ such that $\phi^*(t') > \bar{\phi}\exp(\lambda(T-t))$ for $T - X \leq t' < T$ then

$$\exists(s,\phi^*(s)) \in \left\{ \begin{array}{c} (r(\phi),\phi) | r(\phi) = T - X + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi} \\ +\gamma\left(t' - \left(T - X + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi}\right)\right), \gamma \in (0,1), \phi \in [\phi(t'),1] \end{array} \right\}$$

Now $s > T - \bar{t}_e(\phi^*(s))$, so the unique equilibrium of the subgame starting from $(s,\phi^*(s))$ is given by Lemma 14. However, the Bayesian belief $\widehat{\phi}^*(r)$ in this subgame reaches $\widehat{\phi}^*(r) = \phi^*(t')$ at $r = (T-X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')} + \gamma(s)\left[t' - (T-X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}\right]$ where

$$\gamma(s) = \frac{s - (T-X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(s)}}{t' - (T-X) + \frac{1}{\lambda e_{\max}}\ln\frac{1}{\phi^*(t')}} < 1$$

Thus, $r < t'$ and $\widehat{\phi}^*(t') > \phi^*(t')$ and hence $\phi^*(t')$ is not part of the perfect Bayesian equilibrium. □

This uniquely determines $\phi^*(T) = \bar{\phi}$ for $T \geq X$.

Lemma 15. *Suppose $T \geq X$ then a lower bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \geq \begin{cases} \bar{\phi} + \delta(T-t) \textit{ for } \; T - Y \leq t < T \\ 1 \textit{ for } \; t \leq T - Y \end{cases}$$

Proof. As noted above $\phi^*(T) = \bar{\phi}$, if $T \geq X$. Now suppose $\exists s : \phi^*(s) < 1$ for $s < T - Y$ or $\phi^*(s) < \bar{\phi} + \delta(T-s)$ for $T - Y \leq t < T$. If this is the case then using lemma 7 $d^*(t) = 0$ for $t \in [s,T]$ and there is an immediate contradiction as successful individuals will strictly prefer to implement at s than wait until T. □

These lemmas provide an upper and lower bound on the values of $\phi^*(t)$ in equilibrium. As before, the proof for uniqueness now proceeds by showing that the only equilibrium strategies which support values of ϕ between these bounds are the ones given in the propositions.

Proof. **i) Case 1:** $T < X$

Strategy of successful player: $d^*(t) = 0$ for all t. Strategy of unsuccessful player: $d^*(t) = 0$ for all t and $e^*(t) = e_{\max}$ for all t. Beliefs: Beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for all t.

This follows immediately from Lemma 13.

ii) Case 2: $X < T < Y$

Strategy of successful player: $d^*(t) = 0$ for all t. Strategy of unsuccessful player: $d^*(t) = 0$ for all t and $e^*(t)$ satisfies

$$\exp\left(-\lambda \int_0^t e^*(s)\, ds\right) \geq \bar{\phi} + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_0^T e^*(s)\, ds\right) = \bar{\phi}.$$

Beliefs: Beliefs evolve according to $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for all t.

Lemma 7 and $\phi^*(T) = \bar{\phi}$ (as shown above from Lemmas 13 and 14) imply that $d^*(t) = 0$ for all t. The restriction on $e^*(t)$ comes from Lemma 15. Beliefs are given by Bayesian updating.

iii) Case 3: $T > \frac{1}{\delta}(1-\bar{\phi})\alpha_2$

Strategy of successful player: $d^*(t) = \textit{implement}$ for $t < T - Y$ and $d(t) = 0$ for $t \geq T - Y$

Strategy of unsuccessful player: $d^*(t) = 0$ for all t and $e^*(t) = \frac{\delta}{c}$ for $t < T - Y$. $e(t)$ satisfies

$$\exp\left(-\lambda \int_{T-Y}^t e^*(s)\, ds\right) \geq \bar{\phi} + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_{T-Y}^T e^*(s)\, ds\right) = \bar{\phi}.$$

for $t \geq T - Y$. Beliefs: $\phi(t) = 1$ for $t < T - Y$ and $\phi^*(t) = \exp\left(-\lambda \int_0^t e^*(s)\, ds\right)$ for $t \geq T - Y$.

The proof for Case 2 encompasses the subgames for $t \geq T - Y$. The uniqueness for $t < T - Y$ is completely analogous to the proof in case 4 for large incentives of the uniqueness of equilibrium strategies for $t < T - Y_d - Z$. □

B.4 Modes of Communication

In this section, we provide an explicit characterization of the setup and the results when relaxing our assumptions about the value of breakthroughs and the modes of communication as discussed in Section 6.3.

B.4.1 Setup

We assume that an individual is prepared to exert effort for a second breakthrough, $\alpha_2 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$. We also assume that an individual is prepared to delay implementation if the other player is exerting maximum effort to produce a third breakthrough, $\lambda \alpha_3 e_{\max} > \delta$, and that this is no longer true for a fourth breakthrough, $\lambda \alpha_4 e_{\max} < \delta$.

From a modeling standpoint it is also reasonable to draw a distinction between implementing the project and communicating that a breakthrough has been produced. Hence we allow agents to reveal at each instance that they have produced a breakthrough without implementing the project. We denote this action by $w_i(t, n_i) : [0,T] \times \{0,1,2\} \to \{0,1,...,n_i\}$ where n_i denotes the breakthrough an agent has produced. We restrict this to be non-decreasing such that an agent may only reveal that a breakthrough was produced but cannot subsequently conceal this having previously revealed it. To simplify notation we also assume that if the other agent has revealed a breakthrough, $w_{-i} = 1$ then the action w_i is no longer available. Common knowledge about the production of 2 breakthroughs will lead to a decision in a similar way to common knowledge of the production of a single breakthrough in the earlier model would also lead to a decision. Hence in the event that the other agent has previously announced the production of a breakthrough then announcing and deciding to implement are equivalent for an agent.

The history of the game at a given point in time is given by (μ, ν). $\mu \in \{0, 1, 2\}$ indicates whether no breakthrough has been announced, $\mu = 0$, or if one has been announced which player announced it, $\mu = 1, 2$. $\nu \in [0, t]$ indicates at what time the player announced successful production of a breakthrough $\nu \in (0, t]$. If no breakthrough has been announced, $\nu = 0$.

B.4.2 Short Deadline

We first consider a short deadline. The equilibrium exhibits no revelation of private information about successful breakthroughs, $w_i(t, n_i) = 0$, no decisions on the equilibrium path prior to the deadline, maximum effort by agents who have only produced one or two breakthroughs and zero effort by those who have produced two breakthroughs.

We define $\tilde{X}$ as

$$\lambda \left[\begin{array}{l} \exp(-\lambda e_{\max}\tilde{X})\alpha_2 + \lambda e_{\max}\tilde{X}\exp(-\lambda e_{\max}\tilde{X})\alpha_3 \\ + \left(1 - \exp(-\lambda e_{\max}\tilde{X}) - \lambda e_{\max}\tilde{X}\exp(-\lambda e_{\max}\tilde{X})\right)\alpha_4 \end{array} \right] = c.$$

Note that

$$\exp(-\lambda e_{\max}\tilde{X})\alpha_2 + \left(1 - \exp(-\lambda e_{\max}\tilde{X})\right)\alpha_3 > \frac{c}{\lambda}.$$

Also, we assume parameter values such that $\tilde{X} < \tilde{X}_d$, where $\tilde{X}_d$ is given by

$$\lambda e_{\max}\left[\exp(-\lambda e_{\max}\tilde{X}_d)\alpha_3 + \lambda e_{\max}\tilde{X}_d\exp(-\lambda e_{\max}\tilde{X}_d)\alpha_4\right] = \delta.$$

Proposition 9. *$\exists T > 0$ such that a symmetric Perfect Bayesian Equilibrium is*

$$\begin{aligned}
w_i^*(t, n_i) &= 0 \\
d_i^*(t, n_i, \mu, \nu) &= \begin{cases} \textit{implement if } \mu \in \{1, 2\} \\ 0 \textit{ otherwise} \end{cases} \\
e_i^*(t, n_i, \mu, \nu) &= \begin{cases} 0 \textit{ if } \mu \in \{1, 2\} \textit{ or } n_i \geq 2 \\ e_{\max} \textit{ otherwise} \end{cases} \\
\phi_i^*(t, \mu, \nu) &= \begin{cases} \{0, 0, 1\} \textit{ if } \mu \in \{1, 2\} \\ \{\exp(-\lambda e_{\max}t), \lambda e_{\max}t\exp(-\lambda e_{\max}t), 1 - (\lambda e_{\max}t + 1)\exp(-\lambda e_{\max}t)\} \textit{ otherwise.} \end{cases}
\end{aligned}$$

Proof. Define $V_i(t, n_i, \mu, \nu)$ as the continuation value for an agent at time t depending on the breakthrough they have produced and the history of announcements of breakthroughs by themselves and the other agent.
Define $\chi(n_i)$ as

$$\chi(n_i) = V_0 + \sum_{j=1}^{n_i}\alpha_j + (1 - \exp(-\lambda e_{\max}T))\alpha_{n_i+1} + (1 - \exp(-\lambda e_{\max}T) - \lambda e_{\max}T\exp(-\lambda e_{\max}T))\alpha_{n_i+2}.$$

This is the expected value of the project at the deadline conditional on an agent having n_i breakthroughs in equilibrium. Note that for $n_i = 1, 2$ and $T < \tilde{X}$,

$$\begin{aligned}
\chi(n_i) - \chi(n_i - 1) &= \exp(-\lambda e_{\max}T)\alpha_{n_i} + \lambda e_{\max}T\exp(-\lambda e_{\max}T)\alpha_{n_i+1} \\
&\quad + (1 - \exp(-\lambda e_{\max}T) - \lambda e_{\max}T\exp(-\lambda e_{\max}T))\alpha_{n_i+2} \\
&> \frac{c}{\lambda}.
\end{aligned}$$

The potential histories in the game can be organized into the following three categories:

$$\begin{aligned}(1) &\ : \ (t,0,0,0)\,;(t,1,0,0)\,;(t,2,0,0)\\ (2) &\ : \ (t,0,-i,v)\,;(t,1,-i,v)\,;(t,2,-i,v)\\ (3) &\ : \ (t,1,i,\nu)\,;(t,2,i,\nu)\,.\end{aligned}$$

The first category contains the histories on the equilibrium path. The continuation values for the first category are:

$$\begin{aligned}V_i(t,2,0,0) &= \chi(2)-\delta(T-t)\\ V_i(t,1,0,0) &= (1-\exp(-\lambda e_{\max}(T-t)))\,\chi(2)+\exp(-\lambda e_{\max}(T-t))\chi(1)\\ &\quad -\frac{c}{\lambda}(1-\exp(-\lambda e_{\max}(T-t)))-\delta(T-t)\\ V_i(t,0,0,0) &= \exp(-\lambda e_{\max}(T-t))\chi(0)+\lambda e_{\max}(T-t)\exp(-\lambda e_{\max}(T-t))\chi(1)\\ &\quad +(1-\exp(-\lambda e_{\max}(T-t))-\lambda e_{\max}(T-t)\exp(-\lambda e_{\max}(T-t)))\,\chi(2)\\ &\quad -\delta(T-t)-\frac{2c}{\lambda}\left(1-\exp\left[-\lambda e_{\max}(T-t)\right]-\frac{\lambda e_{\max}(T-t)}{2}\exp\left[-\lambda e_{\max}(T-t)\right]\right).\end{aligned}$$

The off-equilibrium-path continuation values for the second category are given by:

$$\begin{aligned}V_i(t,0,-i,v) &= V_0+\alpha_1+\alpha_2\\ V_i(t,1,-i,v) &= V_0+\alpha_1+\alpha_2+\alpha_3\\ V_i(t,2,-i,v) &= V_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4\end{aligned}$$

The off-equilibrium-path continuation values for the third category are given by:

$$\begin{aligned}V_i(t,1,i,v) &= \begin{cases} V_0+\alpha_1+(1-\exp(-\lambda e_{\max}t))\,\alpha_2+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_3 \text{ for } \nu=t\\ V_0+\alpha_1+\alpha_2+\alpha_3 \text{ for } \nu<t\end{cases}\\ V_i(t,2,i,v) &= \begin{cases} V_0+\alpha_1+\alpha_2+(1-\exp(-\lambda e_{\max}t))\,\alpha_3+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_4 \text{ for } \nu=t\\ V_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4 \text{ for } \nu<t.\end{cases}\end{aligned}$$

Note that $\frac{c}{\lambda}>\alpha_3$ immediately implies that an effort intensity of 0 is optimal when an agent believes that at least two breakthroughs have been produced. This is the case when an agent herself has produced 2 breakthroughs or the agent is at a history where both agents have announced that they have produced a breakthrough. In the remaining cases effort is $e_{\max}$ which is optimal provided that the continuation values satisfy:

$$\begin{aligned}V_i(t,2,0,0)-V_i(t,1,0,0) &> \frac{c}{\lambda}\\ V_i(t,1,0,0)-V_i(t,0,0,0) &> \frac{c}{\lambda}\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}V_i(t,2,0,0)-V_i(t,1,0,0) &= \frac{c}{\lambda}+\exp(-\lambda e_{\max}(T-t))\left[\chi(2)-\chi(1)-\frac{c}{\lambda}\right]\\ &> \frac{c}{\lambda}\\ V_i(t,1,0,0)-V_i(t,0,0,0) &= \lambda e_{\max}(T-t)\exp(-\lambda e_{\max}(T-t))\,(\chi(2)-\chi(1))\\ &\quad +\exp(-\lambda e_{\max}(T-t))\,(\chi(1)-\chi(0))\\ &\quad \frac{c}{\lambda}(1-\exp(-\lambda e_{\max}(T-t)))+ce_{\max}(T-t)\exp(-\lambda e_{\max}(T-t))\\ &= \frac{c}{\lambda}+\lambda e_{\max}(T-t)\exp(-\lambda e_{\max}(T-t))\left(\chi(2)-\chi(1)-\frac{c}{\lambda}\right)\\ &\quad +\exp(-\lambda e_{\max}(T-t))\left(\chi(1)-\chi(0)-\frac{c}{\lambda}\right)\\ &> \frac{c}{\lambda}.\end{aligned}$$

The implementation decision strategy is optimal provided that

$$\begin{aligned}
V_i(t,2,0,0) &\geq V_0+\alpha_1+\alpha_2+(1-\exp(-\lambda e_{\max}t))\,\alpha_3+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_4\\
V_i(t,1,0,0) &\geq V_0+\alpha_1+(1-\exp(-\lambda e_{\max}t))\,\alpha_2+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_3\\
V_i(t,0,0,0) &\geq V_0+(1-\exp(-\lambda e_{\max}t))\,\alpha_1+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_2
\end{aligned}$$

Taking the first constraint,

$$\chi(2)-\delta(T-t)\geq V_0+\alpha_1+\alpha_2+(1-\exp(-\lambda e_{\max}t))\,\alpha_3+(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))\,\alpha_4$$

$\Leftrightarrow$

$$\begin{aligned}
\delta(T-t) \quad \leq \quad & [(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)-(\lambda e_{\max}T+1)\exp(-\lambda e_{\max}T)]\,\alpha_4\\
&+\exp(-\lambda e_{\max}t)\,(1-\exp(-\lambda e_{\max}(T-t)))\,\alpha_3.
\end{aligned}$$

At $t=T$ the inequality is satisfied. The derivative of the LHS wrt t is $-\delta$ the derivative of the RHS is

$$-\lambda e_{\max}\left[\exp(-\lambda e_{\max}t)\alpha_3+\lambda e_{\max}t\exp(-\lambda e_{\max}t)\alpha_4\right].$$

which for $0\leq t<\tilde{X}_d$ is strictly less than $-\delta$ hence the inequality is satisfied for $0\leq t\leq T\left(\leq\tilde{X}\right)$. We may apply the same argument for the other two constraints after noting:

$$\begin{aligned}
V_i(t,1,0,0) \quad &\geq \quad (1-\exp(-\lambda e_{\max}T))\,\alpha_2+(1-(\lambda e_{\max}T+1)\exp(-\lambda e_{\max}T))\,\alpha_3-\delta(T-t)\\
V_i(t,0,0,0) \quad &\geq \quad (1-\exp(-\lambda e_{\max}T))\,\alpha_1+(1-(\lambda e_{\max}T+1)\exp(-\lambda e_{\max}T))\,\alpha_2-\delta(T-t).
\end{aligned}$$

This implies that the announcement strategy is optimal as well,

$$\begin{aligned}
V_i(t,2,0,0) \quad &\geq \quad V_i(t,2,i,t)\\
V_i(t,1,0,0) \quad &\geq \quad V_i(t,1,i,t)
\end{aligned}$$

These conditions are identical to the conditions for the implementation decision strategy earlier.
Finally, for the optimality of implementation decisions after an announcement, we require:

$$\begin{aligned}
V_i(t,2,-i,v) \quad &\geq \quad V_i(s,2,-i,v)-\delta(s-t) \text{ for all } s\in(t,T]\\
V_i(t,1,-i,v) \quad &\geq \quad V_i(s,1,-i,v)-\delta(s-t) \text{ for all } s\in(t,T]\\
V_i(t,0,-i,v) \quad &\geq \quad V_i(s,0,-i,v)-\delta(s-t) \text{ for all } s\in(t,T]\\
V_i(t,2,i,v) \quad &\geq \quad V_i(s,2,i,v)-\delta(s-t) \text{ for all } s\in(t,T]\\
V_i(t,1,i,v) \quad &\geq \quad V_i(s,1,i,v)-\delta(s-t) \text{ for all } s\in(t,T]
\end{aligned}$$

These are all satisfied as $V_i(t,\cdot,-i,v)=V_i(s,\cdot,-i,v)$ and $V_i(t,\cdot,i,v)=V_i(s,\cdot,i,v)$. □

Note that in the proposed equilibrium, the off-equilibrium-path belief held by a player when her partner announces that she produced some number of breakthroughs, is that her partner produced two breakthroughs. This off-equilibrium-path belief is reasonable, because a player with one breakthrough prefers not to disclose that breakthrough even if she was believed to have one breakthrough. This is true because the best response of the other player would be (i) to implement the project if she had 2 breakthroughs, (ii) to stop exerting effort and to delay if she held one breakthrough, and (iii) to continue exerting effort until she produces one breakthrough and delay if she had 0 breakthroughs. The benefit of announcing successful breakthroughs for the player with only one breakthrough is that if the other player holds 2 breakthroughs then the project is implemented immediately. The cost is that the other player would no longer exert effort for a second breakthrough after having produced a first breakthrough. It is clear that for sufficiently short deadlines, the probability that the other player has produced 2 breakthroughs is too low for the benefit of announcing successful production to outweigh the cost of reducing the effort incentives of the other player. We show this more formally below.

The expected payoff from disclosing production of one breakthrough under this scenario equals

$$\zeta \equiv (1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))(V_0+\alpha_1+\alpha_2+\alpha_3)+(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)\hat{V},$$

where

$$\begin{aligned}\hat{V} &= V_0+\alpha_1+\exp(-\lambda e_{\max}(T-t))\left(1-\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\right)\alpha_2\\ &\quad+(1-\exp(-\lambda e_{\max}(T-t)))\left[\begin{array}{c}\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\alpha_2\\ +\left(1-\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\right)\alpha_3\end{array}\right]\\ &\quad-\frac{c}{\lambda}(1-\exp(-\lambda e_{\max}(T-t)))-\delta(T-t)\end{aligned}$$

is the payoff in the event that the announcement is not met with an implementation decision by the other player. In this case, the updated beliefs about the value of the project are

$$\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\alpha_2+\left(1-\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\right)\alpha_3,$$

where $\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}$ and $1-\frac{\exp(-\lambda e_{\max}T)}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}$ are the updated beliefs that the other player will have produced 0 and 1 breakthrough by the deadline given that they have not implemented the project upon receiving the announcement and hence do not have 2 breakthroughs.

Proposition 10. *$\exists 0<T<\tilde{X}: V_i(t,1,0,0)\geq\zeta$.*

Proof. Rearranging $V_i(t,1,0,0)\geq\zeta$ using the expressions above, we obtain

$$\begin{aligned}&\frac{\frac{\lambda e_{\max}t}{\lambda e_{\max}t+1}(1-\exp(-\lambda e_{\max}(T-t)))+\frac{1}{\lambda e_{\max}t+1}(1-(\lambda e_{\max}(T-t)+1)\exp(-\lambda e_{\max}(T-t)))}{1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\\ &\qquad\geq\frac{\delta(T-t)+\left(\frac{c}{\lambda}-\alpha_4\right)(1-\exp(-\lambda e_{\max}(T-t)))}{(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)\times[\exp(-\lambda e_{\max}(T-t))\alpha_3+(1-\exp(-\lambda e_{\max}(T-t)))\alpha_4]}\end{aligned}$$

This holds with equality for $t=T$ and holds strictly for $t=0$. It suffices to check that for any $t\in(0,T)$ the following holds,

$$\frac{\lambda e_{\max}t\exp(-\lambda e_{\max}t)}{1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\geq\frac{\delta}{\alpha_3}\frac{T-t}{1-\exp(-\lambda e_{\max}(T-t))}+\frac{\frac{c}{\lambda}-\alpha_4}{\alpha_3}.$$

Since $\frac{(1-\exp(-\lambda e_{\max}(T-t)))}{T-t}<1$, this condition is implied by

$$\frac{\exp(-\lambda e_{\max}t)-\exp(-\lambda e_{\max}T)}{T-t}\frac{\lambda e_{\max}t}{1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t)}\geq\frac{\delta}{\alpha_3}+\frac{\frac{c}{\lambda}-\alpha_4}{\alpha_3}. \tag{17}$$

Note that the RHS of (17) is clearly finite. Now consider the first term on the LHS of (17),

$$\frac{\exp(-\lambda e_{\max}t)-\exp(-\lambda e_{\max}T)}{T-t}.$$

This term is decreasing in t and thus has a finite positive lower bound of $\frac{1-\exp(-\lambda e_{\max}T)}{T}$ which in the limit $T\to 0$ approaches $\lambda e_{\max}$. Now consider the limit of the second term on the LHS of (17),

$$\lim_{t\to 0}\frac{\lambda e_{\max}t}{(1-(\lambda e_{\max}t+1)\exp(-\lambda e_{\max}t))}=\infty$$

Hence $\exists T>0$ such that (17) is satisfied for all $0<t\leq T$. $\square$

B.4.3 Infinite Deadline

To gain some intuition for what may occur a long way from the deadline we consider an equilibrium of the infinite horizon game. This is done to avoid the complications in a game with a finite horizon, of specifying the changes in behaviour for the subgames as we transition from behavior far away from the deadline to close to the deadline. The infinite horizon case allows one to focus on a setting where there is no effect of a future deadline. We show that the equilibrium strategies are similar to the equilibrium strategies in our earlier model far away from the deadline ($t < T - Y$). We find there is immediate revelation of private information about breakthroughs by each individual, $w_i^* = n_i$. Hence, the project is implemented whenever the combined number of breakthroughs reaches 2, $d_i^* = implement$ if $n_i + w_{-i} \geq 2$. This is similar to the earlier model whereby the agents immediately implement the project upon a breakthrough. Individuals also exert less than the maximum effort level $e_i^* = \frac{\delta}{c} < e_{\max}$ as in the earlier model which trades off freeriding incentives with incentives to bring forward the implementation of the project. We formalize this tradeoff in the following proposition.

Proposition 11. *A symmetric perfect Bayesian equilibrium of the infinite horizon game is*

$$\begin{aligned}
w_i^*(t, n_i) &= 1 \text{ if } n_i \geq 1 \\
d_i^*(t, n_i, \mu, \nu) &= \begin{cases} implement \text{ if } \begin{cases} n_i \geq 1 \text{ and } \mu = -i \\ n_i = 2 \end{cases} \\ 0 \text{ otherwise} \end{cases} \\
e_i^*(t, n_i, \mu, \nu) &= \begin{cases} 0 \text{ if } \begin{cases} n_i \geq 1 \text{ and } \mu = -i \\ n_i = 2 \end{cases} \\ \frac{\delta}{c} \text{ otherwise} \end{cases} \\
\phi_i^*(t, \mu, \nu) &= \begin{cases} \{0, 1, 0\} \text{ if } \mu = -i \\ 0 \text{ otherwise} \end{cases}
\end{aligned}$$

Proof. The continuation values are

$$\begin{aligned}
V_i(t, 2, i, \nu) &= V_0 + \alpha_1 + \alpha_2 \\
V_i(t, 2, -i, \nu) &= V_0 + \alpha_1 + \alpha_2 + \alpha_3 \\
V_i(t, 2, 0, 0) &= V_0 + \alpha_1 + \alpha_2 \\
V_i(t, 1, i, \nu) &= V_0 + \alpha_1 + \alpha_2 - \frac{c}{\lambda} \\
V_i(t, 1, -i, \nu) &= V_0 + \alpha_1 + \alpha_2 \\
V_i(t, 1, 0, 0) &= V_0 + \alpha_1 + \alpha_2 - \frac{c}{\lambda} \\
V_i(t, 0, -i, \nu) &= V_0 + \alpha_1 + \alpha_2 - \frac{c}{\lambda} \\
V_i(t, 0, 0, 0) &= V_0 + \alpha_1 + \alpha_2 - \frac{2c}{\lambda}.
\end{aligned}$$

As soon as an agent knows that a combined two breakthrough have been produced this results in zero effort as $\alpha_3 < \frac{c}{\lambda}$. This is true when $n_i = 2$ or when $n_i = 1$ and $\mu = -i$. Otherwise, this non-zero effort strategy is optimal provided that

$$\begin{aligned}
V_i(t, 2, i, \nu) - V_i(t, 1, i, \nu) &= \frac{c}{\lambda} \\
V_i(t, 2, 0, 0) - V_i(t, 1, 0, 0) &= \frac{c}{\lambda} \\
V_i(t, 1, 0, 0) - V_i(t, 0, 0, 0) &= \frac{c}{\lambda} \\
V_i(t, 1, -i, \nu) - V_i(t, 0, -i, \nu) &= \frac{c}{\lambda},
\end{aligned}$$

which is straightforward to verify from inspection of the continuation values. The decision not to implement the project is optimal provided that

$$\begin{aligned}
V_i(t,1,0,0) &\geq V_0+\alpha_1 \\
V_i(t,1,i,\nu) &\geq V_0+\alpha_1 \\
V_i(t,0,-i,\nu) &\geq V_0+\alpha_1 \\
V_i(t,0,0,0) &\geq V_0
\end{aligned}$$

which is straightforward to verify. For the history $(t,1,-i,\nu)$, the decision to implement is optimal provided that for all $s \geq t$

$$\begin{aligned}
V_i(t,1,-i,\nu) &\geq \int_t^s \left(V_0+\alpha_1+\alpha_2+\alpha_3-\delta(r-t)\right)\lambda\frac{\delta}{c}\exp\left(-\lambda\frac{\delta}{c}(r-t)\right)dr \\
&\quad +\exp\left(-\lambda\frac{\delta}{c}(s-t)\right)\left[V_i(s,1,-i,\nu)-\delta(s-t)\right] \\
&= \left[V_0+\alpha_1+\alpha_2+\alpha_3-\frac{c}{\lambda}\right]\left(1-\exp\left(-\lambda\frac{\delta}{c}(s-t)\right)\right)+\left(V_0+\alpha_1+\alpha_2\right)\exp\left(-\lambda\frac{\delta}{c}(s-t)\right),
\end{aligned}$$

which follows by $\frac{c}{\lambda} \geq \alpha_3$. An almost identical condition holds for $(t,2,i,\nu)$ and $(t,2,-i,\nu)$. A very similar condition is also derived for the history $(t,2,0,0)$:

$$\begin{aligned}
V_i(t,2,0,0) &\geq \int_t^s \left(V_i(r,2,-i,\nu)-\delta(r-t)\right)\lambda\frac{\delta}{c}\exp\left(-\lambda\frac{\delta}{c}(r-t)\right)dr \\
&\quad +\exp\left(-\lambda\frac{\delta}{c}(s-t)\right)\left[V_i(s,2,0,0)-\delta(s-t)\right] \\
&= \left[V_0+\alpha_1+\alpha_2+\alpha_3-\frac{c}{\lambda}\right]\left(1-\exp\left(-\lambda\frac{\delta}{c}(s-t)\right)\right)+\left(V_0+\alpha_1+\alpha_2\right)\exp\left(-\lambda\frac{\delta}{c}(s-t)\right),
\end{aligned}$$

which also follows by $\frac{c}{\lambda} \geq \alpha_3$. The announcement strategy is optimal provided that

$$\begin{aligned}
V_i(t,1,i,t) &\geq V_i(t,1,0,0) \\
V_i(t,2,i,t) &\geq V_i(t,2,0,0),
\end{aligned}$$

both of which hold with equality. □