Heterogeneity, Demand for Insurance and Adverse Selection
WEB APPENDIX

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Web Appendix A: Extending Propositions 1 and 2 for General Discrete Distributions

In this web appendix, I extend the results analyzed in Section 2 for general distributions. In particular, I consider a finite population $\vartheta = \{\zeta_1, \zeta_2, ..., \zeta_N\}$ and analyze two variations of the joint distribution of revealed and true values that affect the correlation and the relative dispersion respectively. I show how Propositions 1 and 2 generalize if the introduction of demand frictions can be related to these two variations.

The first variation captures a reduction in the correlation between revealed and true values by assigning to a type who has a higher true value than another type a revealed value that is lower than for the other type.

Definition 1 *$\hat{v}$ is a confound of v if for some pairs of individuals characterized by ζ_x, ζ_y with $v(\zeta_x) < v(\zeta_y)$: $\hat{v}(\zeta_x) = v(\zeta_x) + \varepsilon$ and $\hat{v}(\zeta_y) = v(\zeta_y) - \varepsilon$ with $v(\zeta_x) + \varepsilon > v(\zeta_y) - \varepsilon$. For all other $\zeta : v(\zeta) = \hat{v}(\zeta)$.*

A natural example of a confound is when each type of a pair perceives to be the other type. This keeps the marginal distribution of the true and revealed values identical, but reduces the correlation.

The second variation increases the spread of the revealed values relative to the true values.

Definition 2 *$\hat{v}$ is an exaggeration of v if for some pairs of individuals characterized by ζ_x, ζ_y with $v(\zeta_x) \geq v(\zeta_y)$: $\hat{v}(\zeta_x) = v(\zeta_x) + \varepsilon$ and $\hat{v}(\zeta_y) = v(\zeta_y) - \varepsilon$ for some $\varepsilon > 0$. For all other $\zeta : v(\zeta) = \hat{v}(\zeta)$.*

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Notice that an exaggeration coincides with a mean-preserving spread when starting from $v_x = v_y$, in which case it corresponds to the introduction of random noise. Both confounds and exaggerations make types for whom the revealed value exceeds the true value overrepresented among the insured.

Proposition 1 *If the revealed values are the result of a sequence of exaggerations and confounds of the true values, the demand curve overestimates the insurance value for the insured and underestimates the insurance value for the uninsured. That is,*

$$E\left(\varepsilon|\hat{v} \geq p\right) \geq 0 \geq E\left(\varepsilon|\hat{v} < p\right) \text{ for any } p.$$

Proof. *Assume $\hat{v}$ is a confound or exaggeration of v. Consider the set of individuals buying insurance at a price p, $\{\zeta \in \vartheta|\hat{v}(\zeta) \geq p\}$. If there is an individual $\zeta_y \in \vartheta$ buying insurance for whom $\hat{v}\left(\zeta_y\right) < v\left(\zeta_y\right)$, then there is also a individual $\zeta_x \in \vartheta$ buying insurance for whom $\hat{v}(\zeta_x) > v(\zeta_x)$, with $\hat{v}(\zeta_x) - v(\zeta_x) = v\left(\zeta_y\right) - \hat{v}\left(\zeta_y\right)$. However, the opposite is not true. That is, for any individual $\zeta_y \in \vartheta$ for whom $\hat{v}\left(\zeta_y\right) < v\left(\zeta_y\right)$, there is a price $p \in \left(\hat{v}\left(\zeta_y\right), \hat{v}(\zeta_x)\right]$ at which an individual $\zeta_x \in \vartheta$ for whom $\hat{v}(\zeta_x) > v(\zeta_x)$ buys while individual ζ_y does not. Hence, $E_\zeta\left(v|\hat{v} \geq p\right) \geq E_\zeta\left(\hat{v}|\hat{v} \geq p\right)$.* ■

We can adjust the notion of confounds and exaggerations for general discrete distributions such that the sign of the wedge between the demand and value curve only depends on the market coverage q. For this to be the case, it is sufficient that all confounds and exaggerations are centered around some value $\bar{v}$ in the following sense.

Definition 3 *$\hat{v}$ is a $\bar{v}$- centered confound of v if for some pairs of individuals characterized by ζ_x, ζ_y with $v(\zeta_x) \leq v\left(\zeta_y\right) : \hat{v}(\zeta_x) = v(\zeta_x) + \varepsilon$ and $\hat{v}\left(\zeta_y\right) = v\left(\zeta_y\right) - \varepsilon$ with $v(\zeta_x) + \varepsilon \geq \bar{v} \geq v\left(\zeta_y\right) - \varepsilon$. For all other $\zeta : v(\zeta) = \hat{v}(\zeta)$.*

Definition 4 *$\hat{v}$ is a $\bar{v}$- centered exaggeration of v if for some pairs of individuals characterized by ζ_x, ζ_y with $v(\zeta_x) \geq \bar{v} \geq v\left(\zeta_y\right) : \hat{v}(\zeta_x) = v(\zeta_x) + \varepsilon$ and $\hat{v}\left(\zeta_y\right) = v\left(\zeta_y\right) - \varepsilon$ for some $\varepsilon > 0$. For all other $\zeta : v(\zeta) = \hat{v}(\zeta)$.*

Proposition 2 *If the revealed value is the result of a sequence of $\bar{v}$- centered exaggerations and confounds of the true value, the demand function underestimates (overestimates) the value of insurance for the marginal buyer if market coverage is sufficiently high (low). That is,*

$$E\left(\varepsilon|\hat{v} = p\right) < (>)\, 0 \text{ for } p < (>) \bar{v}.$$

Proof. *Assume $\hat{v}$ is a $\bar{v}$- centered confound or exaggeration of v. Consider the set of marginal buyers at a price p, $\{\zeta \in \vartheta|\hat{v}(\zeta) = p\}$. If p is above $\bar{v}$, for any marginal buyer $\hat{v}(\zeta) \geq v(\zeta)$. If p is below $\bar{v}$, for any marginal buyer $\hat{v}(\zeta) \leq v(\zeta)$. Hence, $E_\zeta\left(v|\hat{v} = p\right) \geq E_\zeta\left(\hat{v}|\hat{v} \geq p\right)$ for $p \leq \bar{v}$ and vice versa.* ■

The Proposition implies that the demand and value curve intersect only once, at price $p = \bar{v}$. However, the difference between the two curves is not necessarily monotone in the price as in Proposition 2 in the main text.

Web Appendix B: Calibrations with Normally Distributed Heterogeneity

This appendix relaxes the linear demand assumption in the numerical examples and shows the robustness of the welfare results when assuming normal heterogeneity instead. In the original empirical analysis in EFC, a linear demand and cost curve are estimated. I use the slopes of these curves to estimate the covariance matrix of the normally distributed demand components. Note also that the expression for the welfare cost in (**??**) holds for normal heterogeneity, but relies on a linearization of the demand curve. Hence, this robustness exercise also allows to calculate the approximation error in this particular context, which is shown to be small.

TABLE APP1: THE COST OF ADVERSE SELECTION AS A FUNCTION OF THE NOISE RATIO.

Noise Ratio	**Cost of Adverse Selection**			
$\frac{cov\,(\varepsilon, \hat{v})}{cov\,(\varepsilon + r, \hat{v})}$	Γ (1)	Γ/S^* (2)	$\Gamma/\Gamma^{\text{RP}}$ (3)	$(\Gamma/\Gamma^{\text{RP}})_{\text{App.}}$ (4)
0	11.7	.04	1	1
.01	12.0	.04	1.02	1.02
.10	14.7	.05	1.24	1.25
.25	19.9	.07	1.69	1.76
.50	35.3	.14	2.49	3.33
1	102.2	.41	8.65	25.67$^{(1)}$

This Table repeats Table 1 in the main text, but assumes normal heterogeneity underlying the demand, value and cost curves. Column (4) shows the approximated bias in the estimated welfare cost, based on the formula in Proposition 4.

$^{(1)}$ This large value is due to the fact that the approximating formula in Corollay 2 does not account for the upperbound of 100% of insurance coverage, which is binding in this case when $cov(\varepsilon, \hat{v})/cov(\varepsilon + r, \hat{v}) = 1$.

Table App2: The welfare gain of subsidies and mandates

Noise Ratio	**Government Interventions**	
$\frac{cov(\varepsilon,\hat{v})}{cov(\varepsilon+r,\hat{v})}$	Price Subsidy $\Gamma-\Phi^S$ (1)	Universal Mandate $\Gamma-\Phi^M$ (2)
0	-41.2	-50.2
.01	-41.6	-48.9
.10	-48.5	-35.6
.25	-68.2	-12.4
.50	-126.9	26.1
1	$-654.0^{(1)}$	102.0

This Table repeats Table 2 in the main text, but assumes normal heterogeneity underlying the demand, value and cost curves. I assume that the marginal true value of insurance is greater than zero, which is relevant for the calculation of Φ^M.

(1)The large negative value is driven by the efficient price p^* being very negative. Because of the normal heterogeneity, the individuals for whom insurance is marginally efficient have a very negative revealed value of insurance.

Table App3: The Welfare Impact of Risk-Rating.

Risk Adj.	**No Noise**		**Scenario 1** *Independ.*		**Scenario 2** $var(\pi{+}\varepsilon)=var(\pi)$		**Scenario 3** $var(\pi{+}\varepsilon)=\frac{1}{2}var(\pi)$	
β	$\Delta S^c/S^c$ $(0a)$	Γ $(0b)$	$\Delta S^c/S^c$ $(1a)$	Γ $(1b)$	$\Delta S^c/S^c$ $(2a)$	Γ $(2b)$	$\Delta S^c/S^c$ $(3a)$	Γ $(3b)$
0	0	11.7	0	19.9	0	19.9	0	19.7
.10	.01	8.2	.01	15.4	.01	15.0	.00	13.0
.25	.03	4.3	.02	10.7	.02	9.5	.00	7.4
.50	.06	.9	.05	5.2	.02	4.9	$-.02$	5.1
.75	.08	.00	.06	2.6	.02	3.9	$-.05$	2.6
1	.09	0	.07	1.8	$.01^{(1)}$	5.2	$-.09$	1.9

This Table repeats Table 3 in the main text, but assumes normal heterogeneity underlying the demand, value and cost curves.

(1)The differences in column $(2a)$ are very small. They should be monotone in theory, but are not due to the simulation of the normal distribution using a finite number of draws.

Table App4: The Welfare Impact of Information Policies.

Noise Reduction	**Scenario 1** *Independence*		**Scenario 2** $corr\,(\pi, \pi + \varepsilon) \nearrow$		**Scenario 3** $corr\,(r, r + \varepsilon) \nearrow$	
$\Delta\sigma_\varepsilon^2/\sigma_\varepsilon^2$	$\Delta S^c/S^c$ (1a)	Γ (1b)	$\Delta S^c/S^c$ (2a)	Γ (2b)	$\Delta S^c/S^c$ (3a)	Γ (3b)
0	0	19.9	0	19.8	0	20.1
.10	.00	19.9	$-.00$	21.1	.01	19.1
.25	.01	19.1	.00	22.8	.03	17.6
.50	.02	18.6	.01	25.4	.06	15.3
1	.04	17.0	.01	32.5	.11	11.7

This Table repeats Table 4 in the main text, but assumes normal heterogeneity underlying the demand, value and cost curves.