Pigou, Taxation and Public Goods [1,2]

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I. INTRODUCTION

The results of Samuelson [5-7] in the theory of public goods have provided the basis for most subsequent discussion of the optimum provision of public goods. Samuelson showed that a necessary condition for Pareto Optimality (and hence for maximizing a social welfare function which responds positively to individual utilities) is that the sum of the marginal rates of substitution (Σ MRS) between a public good and a private good be equal to the marginal rate of transformation (MRT). The sole constraint is that production is in the aggregate production set. This optimum can be achieved as a competitive equilibrium with the government supplying the public good up to the point where Σ MRS = MRT and financing its production by lump-sum taxation. Lump-sum transfers may also be employed to achieve the appropriate income distribution.

The achievement of the " full " optimum described above depends on lump-sum taxes and transfers being feasible. If the taxation tools available exclude lump-sum taxation, then the optimization problem must be modified to include explicitly the means by which government revenue is raised. The importance of this point was clearly recognized by Pigou [3], who argued that the cost to consumers of the public good would be larger than just the necessary resources on account of the " indirect " damage caused by taxation:

> " Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen " ([3], p. 34).

This problem has recently received more formal attention from Diamond and Mirrlees [2] and Stiglitz and Dasgupta [8]. In particular, the latter authors have challenged Pigou's conclusion and have argued that in certain circumstances it may not be correct. The economics of their argument and the circumstances in which Pigou's reasoning breaks down was, however, obscure and they failed to distinguish between a number of different interpretations of Pigou's statement. The purpose of this paper is to clarify the meaning of the results obtained by Stiglitz and Dasgupta and to elucidate how Pigou's intuitively appealing (if informal) argument goes wrong.

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II. THE PROBLEM

In examining the effect of the means of finance on the optimum provision of public goods, it is important to distinguish two questions which have tended to be confused in the literature:

(i) the appropriate benefit measure for incremental output of the public good (e.g. for use in benefit-cost analysis),

(ii) the appropriate output level for public goods.

(i) *Benefit Measures and the Conventional Rule*

The first of these questions, which is the one to which we give most attention in this paper, may best be approached through consideration of the methods of benefit-cost analysis. This analysis proceeds by dividing the net effect of an increment in expenditure into costs and benefits. This division can in principle be carried out in many ways, but here we shall assume that the cost side is represented by the marginal rate of transformation and concentrate on the value to which this should be equated (or the *benefit measure*). The rationale for this procedure is that the cost-benefit analyst will usually work out the cost of producing a public good at some set of accounting prices and this will correspond to the marginal cost (or MRT). He will then want to know the benefit measure with which this cost should be compared. Traditional practice in cost-benefit analysis has been to use Σ MRS as the benefit measure (Prest and Turvey [4]): e.g. a road project where the benefits are measured by the sum of the value of possible savings to the users. In what follows the practice of using Σ MRS as a benefit measure will be referred to as the Conventional Rule.

The rationale for the Conventional Rule clearly derives from the first-order conditions for a full optimum, and where the expenditure has to be financed by distortionary taxation the benefit measure has to be modified.[1] The question with which we are concerned here is whether this modified benefit measure will be less than or greater than that indicated by the Conventional Rule. Pigou's intuition was that the Conventional Rule would overstate the true benefits because it ignored the " indirect damage " caused by raising revenue. Stiglitz and Dasgupta, on the other hand, argue that the condition for the Conventional Rule to over-state the benefits is that " the share of tax revenue from the ith commodity is . . . greater than the elasticity of tax revenue from an increase in the tax rate on the ith commodity: or equivalently, as the marginal revenue from raising the tax on the ith commodity by a unit is . . . less than C_i [consumption of good i] " ([8], p. 159). Unfortunately, no adequate interpretation of this result is given by Stiglitz-Dasgupta, and we have no means of telling from the above result whether the case where the Conventional Rule understates the benefits is a likely one or merely an abnormality. Moreover, Stiglitz and Dasgupta overlook the important consideration that their result depends on the choice of which goods are taxed. These questions are taken up in Section III.

(ii) *Output of Public Goods and Over- and Under-supply*

The second question concerns the level of output of public goods. The statements by Stiglitz and Dasgupta that the Conventional Rule would lead to an " under- or oversupply " (p. 159) suggest that they have thrown light on this question, but they do not in fact specify what they mean by under- and over-supply. There are a number of possible interpretations which could be given. It could mean that the solution to the problem where public goods are financed by distortionary taxation but where Σ MRS = MRT is imposed as a constraint leads to output levels which are larger or smaller than the output levels where the constraint is removed. Alternatively, it could mean that the optimum output levels where public goods are financed by distortionary taxation are larger or smaller than

[1] With the procedure followed here (which is that adopted by Diamond and Mirrlees), the costs of raising revenue are subtracted from the benefit side. The same results could equally well be described as adding the costs of raising revenue to the cost side—as is done by Pigou and Stiglitz-Dasgupta.

the levels in the full optimum (financed by lump sum taxation). The latter interpretation [1] is perhaps the more interesting and it is the one on which we focus—see Section IV. The approach adopted is to examine the changes in optimum output levels as the availability of lump-sum finance increases (from zero to the level of the full optimum). This raises slightly more general questions of the welfare effects of changes from commodity taxation to lump-sum taxation. These questions are discussed further in the Appendix.

III. BENEFIT MEASURES AND COMMODITY TAXATION

A wide range of taxes could be used to finance expenditure on public goods, but in this paper it is assumed that *ad valorem* commodity and factor taxes are employed. In this we are following Diamond-Mirrlees and Stiglitz-Dasgupta, and the model described below is based on their work (the notation is that of Diamond-Mirrlees).

Assumptions

Households. There are assumed to be h identical households [2] maximizing utility functions $U(x, e)$ subject to $q.x = 0$, where x denotes the net consumption of the n private goods by the household (factors supplied being treated as negative demands), q denotes the prices faced by consumers and e denotes the supply of public goods. The indirect utility function is denoted by $V(q, e)$. It is assumed that the utility function is such that x is a well-defined function of q and e.

Production. The production constraint is written as $G(X, e) = 0$, where $X = hx$ is the total net consumption and it is assumed that there are constant returns to scale. Private producers are price-takers and the first-order conditions for profit maximization mean that G_k is proportional to p_k where p_k is the producer price of the kth good.[3] The tax rate on the kth good is $t_k = q_k - p_k$.

Government. The government is assumed to maximize a social welfare function $hV(q, e)$ subject to the production constraint. (The requirement of market clearing ensures that the government budget is balanced.) The controls at the government's disposal are the tax rates t_k and the expenditure on the public good e (for simplicity attention is focused on the case where there is a single public good, but the analysis can readily be extended). It may also be noted that we may assume without loss of generality that one good is not taxed (given that there are no lump-sum transfers, net consumer expenditure is zero, so that a proportional tax on all commodities raises no revenue). It is assumed that the untaxed good is good one, and that this is taken as the numeraire ($p_1 = q_1 = 1$). If we further assume that G is defined such that $G_1 = 1$, this implies that $G_k = p_k$.

The government's maximization problem may be formulated in terms of the Lagrangean: $\mathscr{L} = hV(q, e) - \lambda G[X(q, e), e]$. The first-order condition [4] for e is given by (where $G_i = \partial G/\partial X_i$, $G_e = \partial G/\partial e$)

$$h\frac{\partial V}{\partial e} - \lambda\left[\sum_{i=1}^{n} G_i \frac{\partial X_i}{\partial e} + G_e\right] = 0, \qquad \text{...(1)}$$

which may be written as

$$h\frac{\partial V/\partial e}{\alpha q_k} = \frac{p_k}{q_k}\frac{\lambda}{\alpha}\frac{G_e}{G_k} + \frac{\lambda}{\alpha q_k}\sum_{i=1}^{n} p_i \frac{\partial X_i}{\partial e}, \qquad \text{...(2)}$$

where α denotes the individual marginal utility of income.

1 The last sentence in the quotation from Pigou given above suggests that it was the comparison of output levels which he had in mind.

2 This assumption is made for simplicity; for discussion of the case where consumers are not identical, see Diamond and Mirrlees [2].

3 The production side is discussed further in Diamond and Mirrlees. If non-constant returns permit positive profits the results carry through, in the case of identical consumers, if 100% profits taxes are allowed (see e.g. [8])

4 We assume in this paper that functions are differentiable to the relevant order.

The left-hand side of equation (2) is the sum of the marginal rates of substitution between the public good and the private good k. In order to proceed from this equation to discussion of the Conventional Rule (Σ MRS = MRT), we need to specify the good k used in the comparison. As usually presented (see, for example, Aaron and McGuire [1], p. 909) the Conventional Rule takes the marginal rate of substitution between the public good and the private good selected as numeraire, and in the present case this corresponds to the untaxed good ($k = 1$). Equation (2) may then be rewritten

$$\frac{G_e}{G_1} = \frac{\alpha}{\lambda}\frac{h\dfrac{\partial V}{\partial e}}{\alpha} - \sum_{i=1}^{n}(q_i - t_i)\frac{\partial X_i}{\partial e}$$

or ...(3)

$$\text{MRT} = \frac{\alpha}{\lambda}\Sigma\,\text{MRS} + \frac{\partial}{\partial e}\left[\sum_{i=1}^{n} t_i X_i\right],$$

where we have used the fact that $\sum_{i=1}^{n} q_i \partial X_i/\partial e = 0$ (from the budget constraint of the consumer).

Equation (3) allows us to see whether the benefit measure with commodity taxation is greater than or less than that with the Conventional Rule. Beginning with the second term on the right-hand side, we can see that this represents a factor overlooked by Pigou—the effect on tax revenue resulting from complementarity and substitutability between private and public goods. If increased government expenditure leads to a greater consumption of taxed private goods, this reduces the revenue which has to be raised and hence increases the benefit measure. An example of this would be if the provision of a further television channel increased demand for television sets and these were subject to an indirect tax. This point, which was brought out by Diamond and Mirrlees, is however a straightforward one and is not central to this paper, and in what follows it will be assumed that $\partial X_i/\partial e = 0$ for $i = 1, ..., n$.

If the second term vanishes, the right-hand side of equation (3) becomes $(\alpha/\lambda)\Sigma$ MRS and the departure of the benefit measure from the Conventional Rule depends simply on whether $\alpha \gtrless \lambda$. Since α is the marginal utility of income to the consumer and λ is the social marginal cost of raising revenue, Pigou's argument leads one to expect that where taxes are distortionary $\lambda > \alpha$ and hence that the true benefit is less than Σ MRS. In order to see where this breaks down, we need to look at the optimal tax structure. The first-order conditions are

$$h\frac{\partial V}{\partial q_k} = \lambda\left(\sum_{i=1}^{n} G_i \frac{\partial X_i}{\partial q_k}\right) = \lambda\frac{\partial}{\partial t_k}\left(\sum_{i=1}^{n} p_i X_i\right).$$

Since $V_k = -\alpha x_k$ and $\partial(\Sigma p_i X_i)/\partial t_k + \partial(\Sigma t_i X_i)/\partial t_k = 0$ (from differentiating the consumer budget constraint), we can write this as

$$\frac{\alpha}{\lambda} = \frac{\dfrac{\partial}{\partial t_k}\left(\sum_{i=1}^{n} t_i X_i\right)}{X_k} \qquad \text{...(4)}$$

on which the Stiglitz-Dasgupta condition is based (see p. 120). However, this condition is not framed in terms of readily recognizable parameters (neither of the demand functions nor of the utility function) and does not provide much insight into whether or not it is in fact likely that $\alpha > \lambda$.

Using the Slutsky relationship, equation (4) may be rewritten as

$$\frac{\alpha}{\lambda} = 1 - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I} + \sum_{i=1}^{n} t_i (S_{ik}/X_k), \qquad \text{...(5)}$$

where S_{ik} denotes the Slutsky term and $\partial X_i/\partial I$ denotes the income term

$$(\text{i.e. } \partial X_i/\partial q_k = S_{ik} - X_k(\partial X_i/\partial I)).$$

From this we can see that whether $\alpha \gtrless \lambda$ depends on two factors.

(i) *A " distortionary effect "*, represented by the third term on the right-hand side. If we multiply by $t_k X_k$ and sum, we obtain

$$\left(\frac{\alpha}{\lambda} - 1 + \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}\right) = \left(\sum_{i=1}^{n} \sum_{k=1}^{n} t_i S_{ik} t_k\right) \Big/ R, \qquad \ldots(6)$$

where $R = \sum_{k=1}^{n} t_k X_k$, from which it can be deduced (following Diamond and Mirrlees) that where the revenue is positive (as it will be for positive government expenditure) then

$$\sum_{i=1}^{n} t_i (S_{ik}/X_k) \leqq 0 \qquad \ldots(7)$$

using the negative semi-definiteness of the Slutsky matrix. This term works in the direction of $\alpha < \lambda$, and may be interpreted as representing the distortionary effect with which Pigou was concerned—the excess burden (at the margin of tax revenue) associated with commodity as opposed to lump-sum taxation. (This may be demonstrated by examining the effect of allowing the government to use lump-sum taxation up to an amount T—see the Appendix.) To this extent Pigou's excess burden argument is, therefore, correct; however, he overlooked the second term in equation (5).

(ii) *A " revenue " effect*, represented by the second term on the right-hand side of (5) or the marginal rate of tax as income I increases. If this were always positive, then it would be guaranteed that $\alpha < \lambda$ and hence (where $\partial X_i/\partial e = 0$) that the benefit measure is less than Σ MRS. However, this condition cannot be guaranteed and the term may well be negative. Moreover, the sign of the term depends on the choice of which goods are taxed. This may be illustrated by the case where there is one private consumption good, and one factor supplied by the household. If public expenditure is positive, revenue must be positive, so that if the factor is untaxed the consumption good must be taxed at a positive rate. The marginal tax rate will then be positive where the consumption good is a normal good, which is clearly a rather weak requirement. If, on the other hand, the consumption good is untaxed, we have to subsidise leisure (a negative tax rate on leisure raising a positive revenue) and the marginal tax rate will only be positive where leisure is an inferior good.[1] The reason for the dependence of the sign on the choice of the taxed good is fairly clear: the " income " effect of taxation *reduces* the revenue from a consumption tax given normality but *increases* the revenue from a factor tax given normality of leisure.

In their paper, Stiglitz and Dasgupta give the impression that Pigou's intuitive argument was quite wrong. From the more detailed analysis given in this section it appears that —if interpreted carefully—the excess burden argument has some relevance; however, Pigou overlooked the other aspects of distortionary taxation described under (ii) which may reverse his conclusion that the Conventional Rule over-estimates the benefits.

IV. OUTPUT LEVELS AND DISTORTIONARY TAXATION— AN EXAMPLE

The second question outlined above—the relationship between the optimum level of public good output under distortionary taxation and that in the full optimum—may be approached by supposing that the government levies a lump sum tax T per household, with the balance of the revenue required to finance public expenditure being raised by

[1] In this case, the condition for $\alpha > \lambda$ (combining (i) and (ii)) is that the supply curve of labour be backward-bending, which is not implausible. This result is given by Stiglitz and Dasgupta (p. 159), but they do not appear to appreciate its dependence on the choice of untaxed good.

commodity taxation. We may then examine the relationship between T and the output of public goods, and in particular compare e_{CT} with e_{LS} where the subscript CT denotes the commodity tax optimum (where $T = 0$) and LS the lump-sum optimum (where $t = 0$). To keep things simple, we consider the special case where there is one private good and one factor, and where producer prices are taken as constant (at unity). It may be noted that the comparison of e_{CT} with e_{LS} is not affected by the choice of untaxed good, since the physical quantities in the two optima do not depend on the normalization. We assume for the details of our analysis that the factor (L) is taxed at rate t (so that the wage faced is $(1-t)$) and the commodity (X) is taken as the untaxed good. We also assume that the utility function is separable in public and private goods: $U(X, L)+H(e)$.

The government's maximization problem may be written in terms of the Lagrangean

$$hV(t, T, e)+\lambda[hT+thL-e]$$

and the first-order conditions are

$$V_t+\lambda\left[L+t\frac{\partial L}{\partial t}\right]=0 \qquad \ldots(8)$$

$$hV_e-\lambda = 0 \qquad \ldots(9)$$

$$hT+htL-e = 0. \qquad \ldots(10)$$

In the maximization described above, T is taken as a parameter; we are interested in the way in which t and e change as T is allowed to vary. Differentiating the first-order conditions with respect to T,[1]

$$\begin{bmatrix} V_{tt}+\lambda\left(2\frac{\partial L}{\partial t}+t\frac{\partial^2 L}{\partial t^2}\right) & 0 & L+t\frac{\partial L}{\partial t} \\ 0 & hV_{ee} & -1 \\ h\left(L+t\frac{\partial L}{\partial t}\right) & -1 & 0 \end{bmatrix}\begin{bmatrix} \frac{dt}{dT} \\ \frac{de}{dT} \\ \frac{d\lambda}{dT} \end{bmatrix} = -\begin{bmatrix} V_{tT}+\lambda\left(\frac{\partial L}{\partial T}+t\frac{\partial^2 L}{\partial t\partial T}\right) \\ 0 \\ h\left(1+t\frac{\partial L}{\partial T}\right) \end{bmatrix}. \qquad (11)$$

Evaluating at the full optimum ($t = 0$, $\alpha = \lambda$), and denoting the matrix of coefficients by A, we obtain

$$\frac{1}{h}\mid A\mid\frac{de}{dT}=\left(V_{tT}+\alpha\frac{\partial L}{\partial T}\right)L-V_{tt}-2\frac{\alpha\partial L}{\partial t}.$$

Since

$$V_{tT}=-\alpha_t,\ V_{tt}=-\alpha_t L-\alpha\frac{\partial L}{\partial t}$$

this gives

$$\frac{1}{h}\mid A\mid\frac{de}{dT}=\alpha L\frac{\partial L}{\partial T}-\alpha\frac{\partial L}{\partial t}=\alpha S,$$

where S denotes the compensated change in L in response to a rise in the wage rate. The second-order conditions require $\mid A\mid \geqq 0$, and since $S>0$, it follows that a small reduction in the possibilities for lump-sum taxation from the full optimum would lead to a fall in the optimum quantity of the public good.

The establishment of global results is more difficult, but the points at issue are brought out in Figure 1. From equation (9) we can draw the " demand " schedule for public goods

[1] It should be noted that the assumption of constant producer prices is important at this point.

$\lambda = hV_e$; and we can use equations (8) and (10) to define a social " supply " curve for the commodity tax optimum. From (8), and using the fact that $V_t = -\alpha L$,

$$\lambda = \alpha \bigg/ \left(1 + \frac{t}{L}\frac{\partial L}{\partial t}\right), \qquad \text{...(12)}$$

where t is a function of e from (10). Similarly, we may derive a " supply " curve $\lambda = \alpha$ for the case where the revenue is raised solely by lump-sum taxation. It should be noted

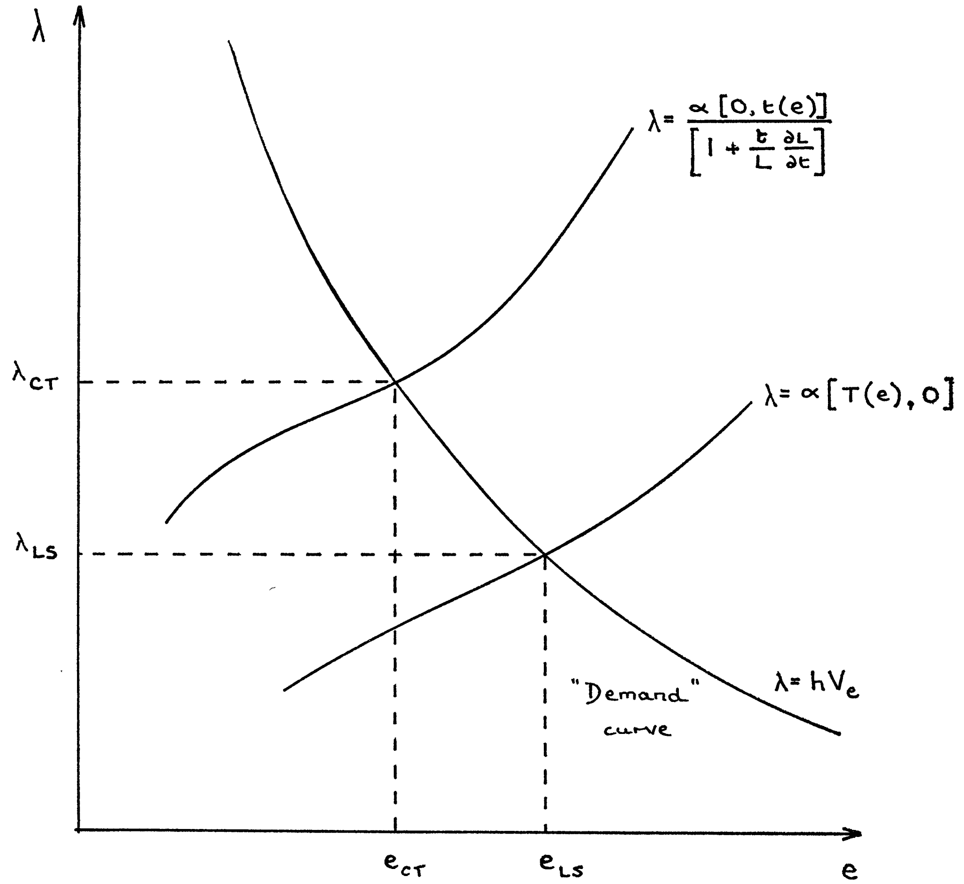

FIGURE 1

Note: It is assumed that $H_{ee}<0$ so that the demand curve is downward sloping, and the second-order conditions ensure that the supply curve cuts the demand curve from below.

that α is a function of T and t, so that in the first case we may write $\alpha[0, t(e)]$ and in the second $\alpha[T(e), 0]$. For $e_{CT}<e_{LS}$ (as shown in the diagram), we require

$$\lambda_{CT}>\lambda_{LS}(=\alpha_{LS}).$$

The condition given by Stiglitz-Dasgupta (in this case that the supply curve of labour be upward sloping) ensures that

$$\lambda_{CT}>\alpha_{CT},$$

but it is clear that this condition is not by itself sufficient for $e_{CT}<e_{LS}$. It has still to be demonstrated that $\alpha_{CT}>\alpha_{LS}$.

The case of the Cobb-Douglas utility function

$$U(X, L, e) = a \log x + (1-a) \log (1-L) + H(e)$$

(where $0<a<1$) illustrates the inadequacy of the Stiglitz-Dasgupta condition since the condition is exactly satisfied at the commodity tax optimum,[1] and according to their analysis we should expect the Conventional Rule to give the right answer. However, it is clear (see Figure 1) that

$$e_{CT} \gtrless e_{LS} \text{ as } \alpha_{CT} \lesseqgtr \alpha_{LS}.$$

From the first-order conditions and the individual budget constraint,

$$1/\alpha = (1-t) - T.$$

From the government budget constraint

$$hT + ht\left(a + \frac{(1-a)T}{1-t}\right) = e$$

so that

$$1/\alpha_{CT} = (1 - e_{CT}/ah)$$

$$1/\alpha_{LS} = 1 - e_{LS}/ah$$

Since $e_{CT} > e_{LS}$ and $\alpha_{CT} < \alpha_{LS}$ is inconsistent with this pair of equations, it follows that the level of public good provision is lower in the commodity tax optimum than in the lump-sum optimum.

V. CONCLUSIONS

The analysis of this paper supports the criticism made by Stiglitz and Dasgupta of Pigou in the sense that the Conventional Rule as defined above may be an over- or under-estimate of the incremental benefits of a public good. The correct benefit measure with distortionary taxation may exceed the sum of the marginal rates of substitution:

(*a*) where the public good is complementary with taxed private goods,

(*b*) where a rise in exogenous income would lead to a fall in the net tax paid

$$\partial(\Sigma t_i X_i)/\partial I < 0).$$

This is likely to occur where taxed goods are inferior or normally supplied factors are subsidised. In such cases, substitution away from the taxed good is not as large as it would otherwise be, and the change in taxation needed to raise an extra £1 is smaller.

On the other hand, Stiglitz and Dasgupta failed to point out that whether the Conventional Rule provides an under- or over-estimate depends on the choice of taxed goods (as is clear from (*b*) above). We have also seen that the question of the appropriate output level for public goods is a rather different one—a point not made clear by Stiglitz-Dasgupta—and that their analysis throws no light on whether output levels in the full optimum are greater or less than the optimum output levels when expenditure is financed by distortionary taxation.

APPENDIX

In the text (p. 123) it was asserted that the term $\sum_{i=1}^{n} t_i(S_{ik}/X_k)$ corresponded to the excess burden associated with the use of (optimum) commodity taxes as opposed to lump-sum taxes. This may be demonstrated by examining the effect of allowing the government to use lump-sum taxation at level T. The government's maximization problem may then be formulated in terms of the Lagrangean (where $h = 1$)

$$\mathscr{L} = V(q, e, T) - \lambda G[X(q, e, T), e] \qquad \text{...(A.1)}$$

[1] The supply curve of labour is given by $L = a + T(1-a)/(1-t)$, so that $\partial L/\partial t = 0$ at $T = 0$.

giving the same first-order conditions for the choice of t_k as before (equation (5))

$$\sum_{i=1}^{n} t_i(S_{ik}/X_k) = \alpha/\lambda - 1 + \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}. \qquad \ldots(A.2)$$

Let us examine the effect of allowing the government to make a small increase dT in the lump-sum tax (moving away from the optimum described above), where adjustments are made in the commodity tax rates to hold e constant and ensure that the production constraint holds. Since $\partial\mathscr{L}/\partial t = 0$ it follows that $d\mathscr{L}/dt = \partial\mathscr{L}/\partial T$. Moreover, given that the production constraint continues to hold $d\mathscr{L} = dV$, which is the change in welfare brought about by the adjustment.[1] From (*A.1*),

$$\frac{\partial\mathscr{L}}{\partial T} = -\alpha - \lambda \sum_{i=1}^{n} G_i \frac{\partial X_i}{\partial T} = -\alpha - \lambda \sum_{i=1}^{n} (q_i - t_i) \frac{\partial X_i}{\partial T}$$

(since $V_T = -\alpha$). From the consumer budget constraint we may note that

$$\sum_{i=1}^{n} q_i \frac{\partial X_i}{\partial T} = -1.$$

Moreover, $\partial X_i/\partial T = -\partial X_i/\partial I$, so that

$$\frac{\partial\mathscr{L}}{\partial T} = -\alpha + \lambda\left(1 - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}\right)$$

which from (*A.2*)

$$= -\lambda \sum_{i=1}^{n} t_i(S_{ik}/X_k). \qquad \ldots(A.3)$$

In the text it has been shown that the right-hand side of this expression is positive, so that there is a welfare gain to the increased use of lump-sum taxes. It should, however, be emphasized that this result only holds where *optimum* commodity taxes are employed. In the case where the tax rates are selected arbitrarily, there is no guarantee that a switch to lump-sum taxation will raise the level of welfare.

A similar argument shows that there is a " route " from the commodity tax optimum to the lump-sum optimum such that welfare increases all the way. For each level of T choose the optimum $e(T)$ and $t(T)$, so that along the path $\partial\mathscr{L}/\partial e = \partial\mathscr{L}/\partial t = 0$. Then at each point on the path we have $dV/dT = d\mathscr{L}/dT = \partial\mathscr{L}/\partial T > 0$.

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