# PRODUCTIVITY, WAGEA AND NUTRITION 

Part 1: The theory

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## 1. Introduction

No one can doubt that the low levels of consumption that pertain in many poor countries are not only the result of the low productivity of labour but are at the same time for various reasons one of the causes of low productivity. In some cases it may even be tiat the diet does not provide adequate calories to allow of a full day's hard work. This idea is one of those that people have in mind when referring to the 'vicious circle of poverty'. Low consumption leads to low productivity which in turn leads to low consumption. As Myrdal (1968) puts it: ${ }^{\prime}$

The main cause of undernourishment and malnutrition in South Asia is, of course, poverty and, in particular, the low productivity of man and land in agriculture. The remedy is development, but the way will not be easy, partly because the dietary deficiencies themselves have reduced people's ability to work. On the other hand, as the nutritional deficiencies tend to lower labour input and efficiency and to decrease vitality in general, the; themselves constitute one of the obstacles standing in the way of development, particularly in agriculture.

The claim that there is a connection berween productivity and consumption and that it is an important connection is not likely to be disputed.

[^0]But one can go further and claim that the link between productivity and consumption exerts an important intluence on wages. According to this line of argument competition will not press wages down beyond a certain point because a lower level of wages would not provide workers with enough consumption to enable them to work effectively. It seems that the first person to explore the theoretical implications of this idea in detail was Leibenstein who reached the following conclusion: ${ }^{2}$

What all this implies is that at very low wages there may be a labour deficit because the units of work produced per man are so few. But at higher wages the units of work per man increase so rapidly that a labour surplus is created. For underdeveloped areas this may mean that the allegedly observed manpower surpluses in agriculture do not really exist when wages are very low, but that they do indeed become a fact when wages rise sufficiently.

Leibenstein's theoretical treatment was far from complete but he did atternpt an assessment of the likely empirical importance of his model by comparing levels of calorie intake commonly observed in poor countries with estimated calorie requirements for certain types of work.

## 2. The Mirrlees-Stiglitz model

More recently the influence of the productivity-consumption link on wages has been the subject of more thorough theoretical investigation in papers by Mirrlees (1976) and by Stiglitz (1976). Both these writers range widely in their discussions and consider questions cutside the scope of the present paper, such as optimum allocation and shadow wages for a system in which the productivity-consumption link is important. Our present concern however, is the positive theory of wages. ${ }^{3}$

Assume that the working day is of a given length in terms of hours. We shall distinguish between 'clock hours' and 'sfficiency hours'. The former are the usual units of time while the latter are the measure of the productivity of the labourer's effort. A more productive worker will prodace a higher number of efficiency hours of labour in a given number of clock hours. The number of efficiency hours produced per clock hour worked depends upon the worker's consumption level $c$ and this relation will be denoted $h(c)$. The

[^1]assumption that the length of the working day is fixed allows us to use the number of clock hours and the number of workers interchangeably.

All workers will be employed for the same number of clock hours and will receive the same wage $w$. The wage is all consumed and for the time being the worker will be assumed to have no other source of consumption. The number of clock hours worked is $l$, which is proportional to the number of men employed, and the number of efficiency hours produced is $i \cdot h(w)$. Output depends upon the number of efficiency hours as

$$
\begin{equation*}
y=f[\ln (w)] . \tag{1}
\end{equation*}
$$

Note the distinction between the 'daily wage', which is the wage, $w$, received by a man for a day's work, and the 'wage per efficiency hour', here $w / h(w)$, which is the cost to the employer of buying an efficiency hour of labour. Suppose the labour is freely available to an employer at any wage not less than $\bar{w}$. It might seem that no rational employer would pay more thar $\bar{w}$ but this is not the case. Since different wages buy labour of different efficiencies, an employer facing an unlimited supply of labour will choose to pay that wage which minimizes the average cost of one efficiency hour of labour. Thus $w$ will be chosen to minimize $w / h(w)$ regardless of th. level of $l$, provided only that the optimum cost-minimizing level of $w$ exceeds $\bar{w}$. Having selected his wage the employer will then employ sufficiently many workers to produce the output he requires.

Formally, the problem that the employer solves is

$$
\min _{w, l} w \cdot l,
$$

subject to

$$
f[\ln (w)] \geqq \bar{y}, \quad w \geqq \bar{w} .
$$

That is the employer minimizes his wage bill $w \cdot l$ subject to producing at least an output $\bar{y}$.

For this problem, assuming that the second constraint does not bind, the Lagrangean form to be maximized is

$$
\begin{equation*}
-w l+\theta[f[\ln (w)]-\bar{y}] . \tag{3}
\end{equation*}
$$

For an intetior solution (i.e. $l$ and $u$ both positive) the necessary conditions for a maximum are

$$
-w+\vartheta f^{\prime} h(w)=0,
$$

and

$$
\begin{equation*}
-l+\theta f^{\prime} l h^{\prime}(w)=0 \tag{5}
\end{equation*}
$$

where a prime ' denotes a first derivative. From (4) and (5) we ótain

$$
\begin{equation*}
\frac{w}{l}=\frac{h(w)}{l^{\prime}(w)} \tag{6}
\end{equation*}
$$

or, re-arranging,

$$
\begin{equation*}
\frac{w}{h(w)}=\frac{1}{h^{\prime}(w)} \tag{7}
\end{equation*}
$$

We shall sometimes refer to the wage rate that solves (2) as the efficiency wage' and to the theory that workers will receive this wage as 'the efficiency wage theory'. Eq. (7) says that the average cost of an efficiency hour is, at the optimura, equal to the marginal cost $\left(1 / h^{\prime}(w)\right)$. This is as it should be sinc. we are minimizing average cost. The efficiency wage $w^{*}$ is given by the tangent from the origin to the $h($ ) curve-see fig. 1 .

We have drawn the wage-productivity curve as starting at a positive wage, then rising at an increasing rate and later rising at a declining rate. ${ }^{4}$ The supposition is that a certain amount of consumption is required to enable some one to undertake any work as opposed to merely existing. Once that


Fig. 1

[^2]basic consumption has been provided there are increasing returns to consumption and then later diminishing returns. We will return later to the question of how to interpret the origin. There are two possibilities: either workers at the origin survive at a very low level of existence or they leave the economy under consideration.

These then are the main outlines of the model that will be the material for our investigations in part $I$ and which we shall attempt to evaluate empirically in part II. In section 3 we shall extend this model to consider the case in which some labourers have a source of consumption apart from wage income. We shall think in particular of labourers who own a small piece of land. The following cases will each be discussed in turn. In the first we compare the wages that an employer will pay land-owning labour which is freely available at a given wage, to the wage $w^{*}$ that he would pay if the labour owned no land. In the second case we consider a monopsonistic employer who employs both kinds of labour at once (because there is not enough labour with an alternative source of consumption to meet his requirements). Again we compare the wages paid to the two types of labour, both when the alternative source of consumption is constant and when it varies with the wage offered, because the labourer will work on his own land if his marginal return there is higher. Finally we consider the case of pure competition between employers for the two types of labour.

In section 4 we consider whether the functional form for the relationship assumed in (1) is plausible. We ask in section 5 how strong the relationship between productivity and consumption is likely to be as observed by an employer.

Sections 6 to 8 are concerned with making exolicit the links between the efficiency wage theory and more orthodox general equilibrium theory and theories of supply and demand for labour. In particular we consider how it comes about that there can be a discontinuity in the supply of labour. The conclusions of this investigation are summarized in section 9 .

## 3. Further implications

The fact that the Mirrlees-Stiglitz model arrives at the conclusion that there could be unemployment with no tendency for wages to decline is not decisive evidence that it is correct. Even if such unemploymeni is important in reality, there could be other explanations for it. We should go on therefore to examine some other conclusions of the model. So far we have allowed only for consumption coming from the wage paid. But suppose that workers have other sources of income, say from cultivation of their own land. We compare the wages chosen by two employers, each of whom can pay any wage he chooses, provided that it is not less than $\bar{w}$. One employer has only landless labourers tc hire; the other hires only labourers who have some
land of their own. Note that we are here comparing the separate choices of distinct employers in distinct labour markets-one interpretation would be the comparison of wages in different villages with different land distributions. Later we shall briefly consider the question of what an employer will choose to pay to the two different types of wonker if both are available for him to hire and he can choose a wage for each type.

Formally, the model is as follows. Workers of type 0 are landless; workers of type 1 all own the same amount of land. If a type 1 worker applies $h_{1}$ efficiency units of labour to his land he obtains an output $g\left(h_{1}\right)$, where $g(\cdot)$ is a concave function and $g(0)=0$. We are here assuming diminishing returns to the application of labour to land. We consider two formulations of the problem which we shall immediately show to be identical in effect. Assume first that in the case of type 1 workers the employer make; an offer to the worker of a wage per hour for a specified number of hours which the worker is not frte to vary if he accepts the offer. Whatever time remains to him the type I worker will devote to work on his own land. Thus we retain the assumption that the total clock hours worked in the day are fixed. As with type 0 workers, the wage $w$ will be interpreted as the wage for all the hours available to a worker. Now however the worker will work a proportion of his time $\lambda$ on his own land and a proportion ( $1-\lambda$ ) for the employer. The employer chooses $\lambda$, that is the feature of our present formulation. Let $l$ be the total number of men hired. These men consume an amount $c$ and can provide $h(c)$ efficiency hours of labour. Thus if an employer hires a worker who consumes $c$ for a fraction $(1-\lambda)$ of his time he obtains ( $1-\lambda$ )h(c) efficiency units of labour. We assume throughout this section that we are dealing with workers in families with just one member. Thus problems of allocation within the family are ignored.

The analogue of problem (2) now is seen to be:

$$
\min _{w, \lambda, l, c} w \cdot(1-\lambda) \cdot l,
$$

subject to

$$
\begin{align*}
& (1-\lambda) h(c) l \geqq 1,{ }^{5} \\
& c \leqq(1-\lambda) w+g[\lambda h(c)], \\
& w \geqq \bar{w} . \tag{9}
\end{align*}
$$

[^3]For this problem the Lagrangean is

$$
\begin{align*}
-w \cdot(1-\lambda) \cdot l & +\mu_{1}[(1-\lambda) \cdot h(c) \cdot l-1] \\
& +\mu_{2}[(1-\lambda) w+g[/ h(c)]-c] \tag{10}
\end{align*}
$$

assuming the constraint $w \geqq \bar{w}$ not be binding.
Since the first constraint of (9) will be binding if the problem is to be interesting we shall have

$$
\begin{equation*}
w(1-\lambda) l=\frac{w(1-\lambda) l}{(1-\lambda) h(c) l}=\frac{w}{h(c)} . \tag{11}
\end{equation*}
$$

Hence an equivalent formulation so (9) requires that $w$ ' $(c)$ be minimized.
Assuming an interior solution for all variables the derivatives of the Lagrangean (10) with respect to all choice variables ifll vanish. Hence we obtain

$$
\begin{align*}
& w: \quad-(1-\lambda) l+(1-\lambda) \mu_{2}=0 \quad \text { or } \quad \mu_{2}=l,  \tag{12}\\
& \lambda: \quad w l-\mu_{1} h(c) l-\mu_{2}\left[w^{\prime}-g^{\prime} h(c)\right]=0  \tag{13}\\
& l: \quad-w(1-\lambda)+\mu_{1}(1-\lambda) h(c)=0 \tag{14}
\end{align*}
$$

(It is easier to discuss conditions for an interior solution for $\lambda$ when we have established fig. 2.)

Substituting for $\mu_{2}$ from (12) and cancelling yields

$$
\begin{equation*}
\mu_{1}=g^{\prime}[\lambda k(c)] \tag{15}
\end{equation*}
$$

Using (15) one may reduce (14) to

$$
\begin{equation*}
g^{\prime}[\lambda h(c)]=\frac{w}{h(c)} \tag{16}
\end{equation*}
$$

The above has an impcrtant interpretation which ve will explore at once. The left-hand side of (16) is the marginal product if efficiency hours of iahn'r on the worker's own land. The right-hand sid is the wage that he is paid per efficiency hour of labour provided. If the wo er were free to divide his time as he wished bet'veen his own and the em lioyer's land he would equate the marginal product of efficiency hours or: his own land to the opportunity cost of efficiency hours, which is $w / h(c)$. Now (16) tells us that the employer will choose to divide the worker's tin: between the worker's
lend and the employer's land just as the worker would choose. In other words, it makes no difference whether we assume that the employer constrains the workers to work the hours that the employer prefers or leaves the werker free to choose. The outcome is the same in either case. The reason is that, given $w$, the employee and the employer have a mutual interest in maximiziag consumption, one because he values it, the other because it increases the labourer's productivity. This gives rise to the problem:

$$
\max _{i} c,
$$

subject to

$$
\begin{equation*}
c=(1-\lambda) w+g[\lambda h(c)] \tag{17}
\end{equation*}
$$

and $w$ given.
Which in turn gives us

$$
\begin{equation*}
\frac{\mathrm{d} c}{\mathrm{~d} \lambda}=-w+g^{\prime}[\quad]\left[h(c)+\lambda h^{\prime}(c) \frac{\mathrm{d} c}{\mathrm{~d} \lambda}\right]=0 \tag{18}
\end{equation*}
$$

But $\mathrm{d} c / \mathrm{d} \lambda$ must be zero if $c$ is maximized, hence (16) follows.
Returning to the conditions derived from the differentiation of the Lagrangean (10), we have finally:

$$
\begin{equation*}
c: \quad \mu_{1}(1-\lambda) h^{\prime}(c) l+\mu_{2}\left[g^{\prime} \lambda h^{\prime}-1\right]=0, \tag{19}
\end{equation*}
$$

which, taking into account earlier results, reduces to

$$
\begin{equation*}
g^{\prime}[\lambda h(c)]=\frac{1}{h^{\prime}(c)} \tag{20}
\end{equation*}
$$

This last condition is illustrated in fig. 2.
The production function $g(\cdot)$ is 'turned over' relative to iss position in economics textbooks because the independent variable, efficiency hours, here appears on the vertical axis, so that the horizontal and wertical axes have been inter changed. Taking that into account it will be seen to be an ordinary production function.

Once again the employer wants to move the worker along a line of the steepest possibie slope, that is to make the cost of buying an efficiency unit of labour $\varepsilon: 5$ small as possible. Now, however, he can start not just from the origin but from any point in the shaded area $\mathscr{P}$. It is easily confirmed that the steepest feasible line is the common tangent to the shaded areas $\mathscr{B}^{3}$ and $\mathscr{H}$.


Fig. 2
The tangent to $\mathscr{H}$ has slope $h^{\prime}(c)$. Also the slope of the production function, remembering that it is drawn with horizontal and vertical axes interchanged, is $1 / g^{\prime}[h(c)]$. Hence (20).

Fig. 2 illustrates the conditions for an interior solution $0<\lambda<1$. 'Where $\mathrm{A}^{\prime}$ and $\mathbf{B}^{\prime}$ coincide $\lambda=1$ and the employer hires an infinite number of workers for an infinitesimal number of hours. The credibility of the assumption of an infinitely elastic supply is strained. There are many possible wajs corner solutions can arise- $\mathrm{A}^{\prime}$ to the right of $3^{\prime}, g$ and $h$ ir tersecting, no double tangent to the curve $h(c)$. Thus the worker with land receives a lower wage

From fig. 2 we may derive at once a conclusion concerning the comparison of wages paid to landless labourers and to laburers with land. The slope of the line $A^{\prime} B^{\prime}$ is steeper than the slope of a lire drawn from 0 and tangent to the curve $h(c)$. Thus the worker with land receives a lower wage than the worker without land. This is easily seen from fig. 2 because the wage is EB. Moreover he consumes at a lower level since the tangent to $h(c)$ from the origin must meet $h(c)$, which is concave in that region, to the right of $\mathrm{B}^{\prime}{ }^{6}$ The total cost to the employer who has only landle s labourers to

[^4]hire is greater than for the employer who has labour with land and thus, ceteris paricus, profits are higher in the latter case. The comparison between the optima for the two cares can be undersiood as follows. If the average cost per efficiency hour is h.jiner in the landless case, then so is the marginal cost (being equal to the average cost at the optimum) and hence consumptior in that case must be higher (recall that the marginal cost of an efficiency hour is $1 / h^{\prime}$ ).

It is important to underline the point that we have not so far arrived at any conclusion concerning the wages that will be paid to landless and landowning workers by a hirer of labour where both types sell their labour in the same labour market. In that case the total cost of buying an efficiency unit of labour is the sum of the costs for each type so that the objective of the employer is no longer to minimize the cost for one type taken alone.

The analysis of the case of discriminating monopsony with two types of land ownership can become quite complicated, so we start with a simple case. Suppose that type 1 workers obtain a fixed amount of consumption $\bar{c}_{0}$ $>0$ from :heir land by applying a given amount of labour but this cannot be augmented by further work. There will be no loss of generality if we assume the given amount of labour is zero. This is the case of a production fanction with a right angle corner, thus $\Gamma$, if drawn with the axes arranged as usual, or thus 7 on our diagram.

The employer's problem now is to choose $l_{0}$ and $l_{1}$, the numbers of mandays of the two types of labour hired, and the wages $w_{0}$ and $w_{1}$ paid to them, to solve:

$$
\begin{equation*}
\min _{w_{0}, l_{0}, w_{1}, l} w_{0} \cdot l_{0}+w_{1} \cdot l_{1} \tag{21}
\end{equation*}
$$

subject to

$$
\begin{aligned}
& l_{0} h\left(w_{0}\right)+l_{1} h\left(\bar{c}+w_{1}\right) \geqq 1, \\
& w_{0} \geqq \bar{w}, \quad w_{1} \geqq \bar{w}, \quad l_{1} \leqq \bar{l}_{1} .
\end{aligned}
$$

The wage at which labour is freely available could be different for the two types of worker without affecting the subsequent analysis, provided the wage constraints do not bind. We have added now a new constraint concerning the availauility of type 1 labour, for unless there is some such limit it is obvious that the employer will hire only landed labour and will obtain all the labour that he requires at a lower cost for an efficiency hour of labour than he would with landless laborr, as was shown above. Indeed it is immediate that the employer will hire labour with land up to the maximum available before he hires any landless labour. For suppose not. Then he could substitute for a landless labourer a abourer with land, paying him the same wage, and this would increase production at no extra cost. Thus the
solutions for $l_{0}$ and $l_{1}$ in (21) are clear: $l_{1}=\bar{l}_{1}$ and $l_{0}$ will follow from the first inequality of (21), which of course will be satisfied exacily, once $w_{0}$ and $w_{1}$ are known.

The Lagrangean of (21), assuming that the constraints $w_{0} \geqq \bar{w}$ and $w_{1} \geqq \bar{w}$ do not biad, is:

$$
-w_{0} \cdot l_{0}-w_{1} \cdot \bar{l}_{1}+\mu\left[l_{0} h\left(w_{0}\right)+\bar{l}_{1} h\left(\bar{c}+w_{1}\right)-1\right] .
$$

From the derivatives with respect to $l_{0}, w_{0}$ and $w_{1}$, we obtain

$$
\begin{array}{cl}
l_{0}:-w_{0}+\mu h\left(w_{0}\right)=0, \\
w_{0}: & -l_{0}+\mu l_{0} h^{\prime}\left(w_{0}\right)=0,  \tag{22}\\
w_{1}: & -\bar{l}_{1}+\mu \bar{l}_{1} h^{\prime}\left(\bar{c}+w_{1}\right)=0 .
\end{array}
$$

It follows that

$$
\begin{equation*}
w_{0}=w^{*} . \tag{23}
\end{equation*}
$$

Since we are assuming throughout that type 0 workers are in infinitely elastic supply at a wage less than $w^{*}$ we shall always have the result that they will be paid and consume $w^{*}$. Any other outcome would involve an employer, competitive or otherwise, in extra cost. We have also from (22) that

$$
\begin{equation*}
h^{\prime}\left(w_{0}\right)=h^{\prime}\left(\bar{c}+w_{1}\right) . \tag{24}
\end{equation*}
$$

Hence

$$
\begin{equation*}
w_{0}=\bar{c}+w_{1} .{ }^{7} \tag{25}
\end{equation*}
$$

This result has a straightforward economic interpretation. What it says is that both types of labour will receive the same level of consumption. The wages of the 'better off' land-owning labourers will be lower by just enough to put them in the same position as the landless labourers. That this must be the case is easily seen by considering the economic interpretation of (24), which says that the marginal increase in efficiency from increasing consumption must be the same for each type of worker. Were that not so it would pay the employer to shift some consumption from one type of worker to another, increasing productivity at no extra cost. If we compare the conclusion oit this analysis with that of a comparison of separate labour markets we see that once again the land-owning labourers receive iower

[^5]wages but now they no longer consume less than employed landless labourers. ${ }^{8}$ Of course they do gain the advantage that they all get employed.

These results on the relative wages and consumption for the two types of worker raise natural questions in game theory and in welfare economics. For example, if it is a disadvantage to hold land, the landed have an incentive to conceal ownership or dispense with land. But note that the advantage of holding land is not simply measured by the wage that will be received by a land-owning labourer, because land-owning labourers are employed in preference to landless labourers. These are interesting questions but we shall not pursue them here. We return to our analysis of the situation where type 0 and type 1 workers coexist in the same labour market.

The case of the labourer's land with an invariant product was presented because it is easy to follow and serves to fix ideas. However it is a special case and in some ways a misleading one. The point is that the supply of tr e 1 labour in clock hours, $\bar{l}_{1}$, is completely inelastic when we assume irs marginal productivity on cwn land to be zero. Hence the employer has no incentive to pay a higher wage to get more of it than would otherwise be available. When the supply is elastic, and given our assumption of fixed total clock hours per worker, it is upward sloping when we assume a more usual production function, we certainly will not arrive at (24). In fact we must have in this case:

$$
\begin{equation*}
h^{\prime}\left(c_{0}\right)>h^{\prime}\left(c_{1}\right), \tag{26}
\end{equation*}
$$

and

$$
\begin{equation*}
c_{0}<c_{1} . \tag{27}
\end{equation*}
$$

For otherwise it would pay the employer to shift wage payments to type 1 workers and increase their supply at no cost to productivity. Hence in every case land-owning labourers will consume at a higher level. But that does not tell us whether they will receive a higher or a lower wage. We have not been able to find an argument to rule out either possibility.

We shall not pursue further the discussion of the monopsonistic employer choosing between landless and land-owning labourers, and wages for each type, except to note one important point. In considering the case in which the employer hires land-owning labour alone, there being 10 landless labour

[^6]availablc, we observed that the amployer will not choose to exercise his power to compel the labourer to work a specified number of hours. This is because it will be optimum for the employer to choose to split the hours of the iabourer between the labourer's own land and the employer's land so that the marginal product of an efficiency of labour on the labourer's land will equal the wage cost to the emplcyer of an efficiency hour of labour.

In the present case this conclusion does not hold. If the employer can make a take-it-or-leave-it offer of a wage for a specified number of hours to the land-owning labourers 'type 1) then he will do so and his wage will not be equal to the marginal product of labour on the labourer's own land. In this case the land-owning worker will definitely receive a lower wage for one efficiency hour of labour and, because he will consume at the same level as the landless labourer (thus giving equal efficiency hours per clock hour), a lower wage for one hour's work than the landless labouer.

The problem for the landlord is now the following:

$$
\min _{l_{0}, c, w, \lambda} w^{*} \cdot l_{0}+(1-\lambda) w_{1} T_{1},
$$

subject to

$$
\begin{align*}
& h\left(w^{*}\right) l_{0}+(1-\lambda) h(c) \bar{l}_{1} \geqq 1, \\
& c \leqq(1-\lambda) w_{1}+g[\lambda h(c)], \tag{28}
\end{align*}
$$

where the minimization is achieved by choice of $l_{0}, c, w$ and $\lambda$. Here we have already set $l_{1}-T_{1}$, the maximum numbe of men of type 1 available, since it is clearly optimum for the employer to employ all of these before employing any landless labour if, as he indeed can, he can get one efficiency hour of labour from them more cheaply than he can get it from landless labourers. The wage for landless labourers has been set equal to $w^{*}$, the efficiency wage for type 0 labour, because it is optimum to pay hat wage to landless labourers in excess supply regardless of the fact that $t$ lere is another kind of labour available for hire.

The Lagrangean for (28) is

$$
\begin{align*}
-w^{*} \cdot l_{0}-(1-\lambda) \cdot w_{1} \cdot \bar{l}_{1} & +\mu_{1}\left[h\left(w^{*}\right) \cdot l_{0}+(1-\hat{\lambda}) \cdot h(c) \cdot \bar{l}_{1}-1\right] \\
& \left.+\mu_{2}\left[(1-\lambda) \cdot w_{1}+g_{1} \dot{\lambda} h(c)\right\}-c\right] . \tag{29}
\end{align*}
$$

From which, assuming $0<\lambda<1$ [see the discussion of this condition for problem (9) and fig. 2], we obtain

$$
\begin{array}{ll}
w_{1}:-(1-\lambda) \bar{l}_{1}+\mu_{2}(1-\lambda)=0 & \text { or } \mu_{2}=\bar{l}_{1} \\
l_{0}:-w^{*}+\mu_{1} h\left(w^{*}\right)=0 & \text { or } \mu_{1}=w^{*} / h\left(w^{*}\right) . \tag{31}
\end{array}
$$

$$
\begin{array}{ll}
\lambda: & w_{1} \bar{l}_{1}-\mu_{1}^{\prime}\left(1(c) \cdot \bar{l}_{1}-\mu_{2}\left[w_{1}-g^{\prime}\{\lambda h(c)\} h(c)\right]=0,\right. \\
c: & \mu_{1}(1-\lambda) h^{\prime}(c) \bar{l}_{1}-\mu_{2}\left[1-g^{\prime}\{\lambda h(c)\} \lambda h^{\prime}(c)\right]=0 . \tag{33}
\end{array}
$$

Substituting (30) and (31) into (32), and simplifying, yields

$$
\begin{equation*}
\frac{w^{*}}{h\left(w^{*}\right)}-g^{\prime}\{\lambda h(c)\}=0 . \tag{34}
\end{equation*}
$$

This has a straightforward economic interpretation. The first term, $w^{*} / h\left(w^{*}\right)$, is the wage paid for one efficiency unit of labour by the employer when he hires extra workers of type 0 ; it measures the marginal cost of an efficiency unit of labour to the employer. The other term, to which it must be equal, is the marginal product of efficiency units of labour on the workers' own land.

Eq. (34) has an intuitive explanation as follows. The employer always has the option to substitute type 0 labour for type 1 labour while holding the consumption of type 1 labour constant. He would then choose a smaller value of $i$ so that he would have one less efficiency hour of labour from type 1 workers. They would then produce the marginal product of one efficiency hour on their own land and this would be a gain to the employer since he would have that much less to contribute to their consumption. The extra cost would be the cost of one efficiency hour of labour from type 0 workers. At the optimum the gain must equal the extra cost. Compare this conclusion to (20) and its interpretation above. However, here the wage to which the marginal product will be equal is not now the wage for type 1 labour but the wage per efficiency hour for type 0 labour, and that wage per efficiency hour must be higher than the wage for type 1 labour. Only if type 1 labour provides efficiency hours of labour more cheaply will it be optimum to employ it all first and it is clearly feasible for type 1 labour to provide efficiency units of labour more cheaply. Hence the employer having the power to constrain the labourer to work a specified number of hours will use it. One further conclusion follows from previous analysis: type 0 and type 1 labour must consume at the same level. This is so because once the employer has fixed the efficiency hours of work on the labourer's own land it musi be impossible to increase production by paying more wages to one type of labour and less to the other type; we have a 'problem within a problem' of the type of (21). The problem is not strictly identical to (21) because the type 1 workers do not start on the horizontal axis, however this is immaterial as we can see by referring forward to fig. 3 (which is explained on p. 346). The type 1 labourer moves from $E^{\prime}$ to $E$ in selling his labour. Hence the slope of $\mathrm{E}^{\prime} \mathrm{E}$ is $h(c) / w_{1}$. But the triangle EE'D is similar to $\mathrm{EDO}^{\prime}$ and $\mathrm{ED}=h(c)$. Hence $\mathrm{O}^{\prime} \mathrm{D}=w_{1}$ and we have the analogy with (21).

Formally from (2i), taking into account (30), (31) and (34), one obtains

$$
\begin{equation*}
g^{\prime}[\lambda h(c)]=\frac{w^{*}}{h\left(w^{*}\right)}=\frac{1}{h^{\prime}(c)} \tag{35}
\end{equation*}
$$

And, from (6),

$$
\begin{equation*}
\frac{w^{*}}{h\left(w^{*}\right)} h^{\prime}\left(w^{*}\right)=1, \tag{36}
\end{equation*}
$$

so that

$$
h^{\prime}\left(w^{*}\right)=h^{\prime}(c) .
$$

From this it follows that $w^{*}$ is equal to $c$, as required.
We can conclude (see below) that the wage paid to land-owning labour for an efficiency hour of labour will be less than the rate paid to landless labourers. However, since both will consume at the same rate, each type will provide the same number of efficiency hours of labour in one clock hour's work. Hence the hourly wage rate will be less foi land-owning labour. Fig. 3


Fig. 3
illustrates. The line $O E$, the tangent tc $h()$ through the origin has slope $h\left(w^{*}\right) / w^{*}$. The line $\mathrm{O}^{\prime} \mathrm{E}^{\prime}$ parallel to OE and tangent to g has the same slope. The wage paid to land-owning labour is measured by the slope of E'E, which is steeper than OE. Hence $h(c) / w_{1}$ is greater than $h\left(w^{*}\right) / w^{*}$, so that the wage rate for land-owning labourers is lower.

The double tangent is included in lig. 3 for comparison with fig 2 . If there were several different types of so:ker with different quantities of land [less land shifting the $g()$ function to the left] then the price per efficiency hour for the marginal type that is emp:oyed is given by the double tangent for the $g()$ function for that type. (Th is result of double tangency for the marginal type will hold however competitive the market is.) Where : ne marginal type is landless we have the sitation illustrated in fig. 3.

In devoting so much space to variou; cases of pure monopsony we have arguably given that type of case more emphasis than it deserves. Rural labour markets are notorious for imperfections of competition; nevertheless there is always some competition and scmetimes quite a lot. Moreover, there is something to be said for examining polar cases. In the case of pure competition many of our foregoing conclusions no longer stand or are exactly reversed. However one conclusion always stancis and it is on this one that we would like to lay the emphasis because it will be a major plank for empirical testing of the theory when we come to consider that in part II. Even under a market as competitive as one cares to imagine, in the sense that there is a very large number of errployers between which workers can choose and a very large number of workers between which employers may choose, no employer will pay other then the efinciency wage to a landless labourer with no other consumption so long as the labour of these workers is in excess supply. This conclusion is he firm and enduring result of the efficiency wage theory.

More conclusions from a competitive model may be derived quite simply. Take the case of pure competition betveen employers as a reference point and assume worktrs cannot have more than one employer. It is obvious that the cost of buying an efficiency hour of labour must be the same for all labourers who get employed. In that case land-owning labourers will benefit from the economic rent implicit in their having an alternative source of consumption and will be paid more per day and consume more. In this case, in sontrast to that of monopsony, amongst those types of worker who get employed all have an equal chance, but some receive lower wages than others, those lower wages being exactly compensated, from the employers' point of view, by their productivity. Even under competition there may be labourers who have no chance of beng empluyed (they cannot provide efficiency hours of labour at the market rate with the consumption which follows from the market wage). In the case where the marginal worker, to gain employment, must have some consumption background families will
concentrate consumption on the potential wage earner in order to give him a chance of being employed. Poorer families will be able to 'efford' less wage earners [see Mirrlees (1976)].

## 4. Aggregation

The theory discussed so far is a rather particular version in that it specifies the production function for employers as depending on labour and consumption as

$$
\begin{equation*}
y=f[k, l \cdot h(c)], \tag{37}
\end{equation*}
$$

1. ler than the fully general form

$$
\begin{equation*}
y=f[k, l, c], \tag{38}
\end{equation*}
$$

where $\boldsymbol{k}$ is the vector of other inputs, such as land and capital. To assume (37) as opposed to (38) is to assume that it is labour power that matters for production, what we have called efficiency units of labour, and that the effect of consumption is to augment the labour power of men in proportion to their numbers and independently of other inputs. Strictly this specification cannot be correct. Imagine for example that capital inputs are so large that production is highly automated. It is incredible that labour time and consumption could then be substituted to form a sub-aggregate labour power in exactly the same way as if production methods were very primitive. However, confining our attention to relatively backward agriculture it may well be that something like labour power is what matters.

## 5. The strength of the productivity-consumption link from the employer's point of view

An empirical assessment of the Mirrlees-Stiglitz model depends upon some important questions of interpretation. One of the intercsting and suggestive features of the model is the emergence of an efficiency wage, a wage which all employers will choose to pay, independently of variations in the supply of labour, to all workers with no alternative source of consumption. But we have to decide whether this wage will be the same wage in different areas, different years, at different times of year and with different techniques of production. This will be sc if the shape of the $h(\cdot)$ function is invariant. We shall pay particular attention to a version of the model, called here the 'strong' version in which it is supposed that the shape of the $h(\cdot)$ function is invariant over a wide area of regions, techniques and seasons. The strong version natural'y has the advantage of leading to strong conclusions. Also some who have
thought of the influence of nutrition on wages have supposed that this factor would make for wage stability, say from season to season. ${ }^{9}$

It is tempting to refuse such an extreme assumption. However, it is clear that the theory can easily be put beyond the reach of refatation unless a strong version is admitted. If one is not willing to cornmit oneself to supposing that the $h(\cdot)$ function is invariant, or at least insensitive, to a wide range of conditions then the theory is almost devoid of implications.

On a priori grounds the most plausitle reason why the function will vary is with the physical demands that the work involves. A strong link between productivity and consumption is more likely to emerge with heavy manual labour than with light work. Consider, for example, the case of a man employed as a night-watchman. Perhaps he will be more alert and watchful if well-fed. But this is not an effect to be compared to that which wou. 1 operate in the case of a man shifting earth to build a dam.

Some iurther questions take on particular importance when we come to consider evidence relevant to assessing the validity of the theory in part II. So far in our theoretical arguments we have treated income as being identical to consumption for the worker. This cannot be strictly accurate for more than one reason. Given income, the size of consumption is reduced by saving and whenever it happens that someone other than the wage-earner himself consumes out of his income. Saving out of agricultural wages does occur, but the possibility that a large slice of an addition to the wage would be consumed by the wage-earner's family is a more serious consideration. If extra income were to accrue to a worker by way of a simple wage increase, then it is hard to believe that a good deal of the extra consumption would not normally be enjoyed by members of the worker's family. Against this it might be said that if there is a link between productivity and consumption and if the wage is importantly influenced by the productivity of the worker, then it might be in the interest of both worker and employer to concentrate the consumption on the worker himself.

It depends on the arrangements and institutions how, and particularly when, a higher consumption is reflected in productivity. Suppose that the employer knows that if his workers eat more he gets more or better work from them. Then he has an incentive so to arrange things that his workers will eat. An obvious way of trying to achieve this is feeding the worker on the job as happens where a meal is provided during a 'lunch break'. However the worker who is fed on the job could eat less at home, so as a method of enforcing a dietary level it may not be very effective.

The argument in this section has, so far, examined possible determinants of the strength of the relation $h(\cdot)$ as seen by the employer. This anproach, however, can be misleading. Where income levels are sufficiently high the

[^7]employer might suppose that any desire to eat more consequent upon exird work would lead to the worker spending a higher proportion of income on food for himself of his own volition and that there is no need for the employer to take this into account in fixing the wage. This presumably is what happens with some heavy manual work in rich countries. The question whether the same might be true in poor countries is one to which we shall return in part II.

The strength of the productivity-consumption lirk as perceived by the employer depends importantly on the time period under considcration. If workers are hired on a day-to-day basis then it is worth the while of the employer to make a point of feeding his workers well only if the effects of a good meal are reflected in the work of that same day. To the extent that an employer makes a worker more productive on subsequent days by feeding him more he confers an external economy on future employers.

Some of the links between productivity and consumption manifest themselves only in weeks or months raiher than days. One such link is the effect of nutrition in building up skeletal muscle. Where longer-term links are important we might expect to observe long-term employment contracts which would enable employers to take advantage of such links. The institution of permanent labour provides for that possibility. Indeed it is one of the implications of the theory that we would expect to see a prevalence of long-term employment contracts or arrangements, for these would enable an employer to 'capture' to the fullest possible extent the gains to productivity from paying higher wages. This concerns models of the Mirrlees-Stiglitz type, that is those in which it is the employer who takes into account the link between productivity and consumption. This is clear because the employer chooses the wage. But even if the institutions of the employment contract do not allow the employer to benefit from taking the relation into account it may nevertheless be accounted by the peasant household and this is discussed in some detail by both Mirrlees and Stiglitz.

## 6. A further look at the Mirrlees-Stiglitz model

We started our investigations a little uneasy with the Mi riees-Stiglitz approach to the case of the cost-minimizing producer who chocses the wage in that much of the emphasis of the analysis is thrown on to the cost side rather than on to the worker who supplies the effort and consumes the wages. Any preferences the worker may have between more effective work together with more consumption and less effective work toge' her with less consumption do not appear explicitly in the analysis. It could be argued that the worker is at the limit of what is feasible for him, or to use the languase of modern formal theory of the consumer, on the boundary of his consumption set. From this point of view we need only discuss orderings on the
boundary of the consumplition set and we might further suppose that the boundary was ordered in such a way as to give utility increasing with consumption (even when the extra work is taken into account). This is precisely the assumption made by both Mirrlees and Stiglitz in their discussions of optimum allocation within the family.

The reference to the consumption set above reminds us that the relation of the model to standard consumer theory remains unclarified. One difference is obvious, the consumption set defined by the area under the curve $h(c)$ is not convex as we have drawn our figures above. However, we shall soon see that this is not an essential difference; for much of what went before we could have as well made $h(c)$ a concave function provided that we were happy not to include the origin.

In the standard theory [cf. Debreu (1959)] different types of labour are treated as different commodities. Thus one of the jobs of the consumption set is to display the ability or inability of certain individuals to provide certain kinds of labour service at all (e.g. the inability of the authors of this paper to perform as trapeze artists) or to provide certain kinds of labour service unless consuming in a specified manner. This last feature is precisely what is involved in the consumption-productivity relation. Now, considering the $h(c)$ function as specifying a Debreu consumption set let us examine once again the 'disequilibrium' in the labour market in the form of willing workers being unable to find jobs which we seemed to show to be an 'equilibrium' position in the sense that it would tend to pers st.

We think of the state of affairs in which there are workers, who will work at any wage not less than $\bar{w}$, failing to find employment when the wage is above $\bar{w}$, as a disequilibrium. In such a situation one might imagine that the unemplojed workers could undercut he employed workers by agreeing to work for less, but it is important to understand that strictly the unemployed workers cannot undercut the employed workers and, of course, if they could do so a rational employer would be only too glac to accept their offers. This point becomes clear the moment ws conside: the type of labour to be performed to be specified exactly as it would be in Debreu's model. In that case a worker is hired not just to work a day but rather to provide one day's labour of a specified efficiency. Equivaliently we may suppose that the work is specified in terms of 'tasks" ( (e.g. ploughing a specified area) so that what the employer buys is not undifferentiated 1 bour time but efficiency units of labor as such. In this case an unemployed worker cannot offer more efficiency units of labour for the same wage sinaply because the employed worker is on the boundary of his consumption set and hence offering the employer the maximum number of efficiency units of labour consistent with the wage that he receives. However from this it des not follow that the unemployed worker would not prefer to be employed, in which case there would be an excess supply of labour but no possibility of undercutting the employed
workers. To see whether there would indeed be an excess supply in this sense we need to examine the ordering over the consumption set. It could be argued that we need only discuss orderings on the boundary of the consumption set and we might further suppose (following Mirrlees and Stiglitz) that the boundary was ordered in such a way as to give utility increasing with consumption (even when the extra number of tasks performed is taken into account).

The above argument, however, begs certain important questions. Is the employer in a position to force the worker back to the boundary of the consumption set? If so, how is this boundary ordered by the worker? If not, how is the analysis extended to include preferences inside in the boundary? What do we mean by the limit of what is feasible for the worker? Does this limit have the form described in the Mirrlees-Stiglitz analysis? What is the relation between the long-run preference and the boundary and short-run preferences and the boundary? Given an assumption about the answers to some of the above questions what is the character of any equilibrium in such a model? In the next section we try to answer some of these questions. In other words we examine the labourer-consumer side of the model in more detail.

## 7. Prcferences over the consumption set

We shall suppose below the existence of an aggregate 'labour-power', discussed above. We shall think of labour power as measured by the number of tasks performed in the day and use the notation $n$ for this number. We are using the 'day' as a time unit. A great many wage contracts are for one day, ${ }^{10}$ and for the purposes of the analysis a 'day' is to be interpreted as the prevailing contract period. We shall allude again, briefly, to the contract period in part II. We shall consider, then, the consumption set and preferences defined in the space of pairs $(c, n)$ of consumption per day and tasks per day. In doing so we have already suppressed one interesting aspect of the problem, namely, the manner in which the $n$ tasks are performed. Are these performed at high intensity in a small number of hours or more leisurely over a longer working day? We could write $n=r(t, t)$ where $r$ is a function of the number of hours $\ell$ and a measure of work intensity $t$. The problem of course, is that, although $n$ and $\ell$ may be observable $r()$ and $t$ are nct. We can interpret our preference ordering over ( $:, n$ ) as derived from one over ( $c, t, t$ ) in one of two ways. We can suppose the length of the working day as fixed at $\bar{f}$ so that $u(c, n)=u^{0}\left(c, \overline{t^{7}}, t_{n}\right)$ where $u$ is our utility

[^8]function over ( $c, n$ ) space, $u^{0}$ over $(c, \ell, t)$ space and $t_{n}$ satisfies $n=r\left(\bar{\ell}, t_{n}\right)$. Alternatively we can suppose that the worker is free to perform the number of tasks $n$ in the day as he wishes 50 that
\[

$$
\begin{equation*}
u(c, n)=\max _{\ell, t: n=n(f, t)} u^{0}(c, \ell, t) \tag{39}
\end{equation*}
$$

\]

In either case the concavity of $u^{0}$ and $r$ imply she concavity of $u$.
We shall suppose then that each worker has $a$ convex consumption set $\zeta$ defined in ( $c, n$ ) space and a concave utility fanction $u(\quad)$ defined over $\zeta$. We suppose that there is a roinimum consuntiontion per worker per day $\stackrel{C}{0}_{0}$ necessary for survival so that the lower boundary of the consumption set passes through the point $X \equiv\left(C_{0}, 0\right)$. We post oone for the moment any statement as to the long. versus short-run character of $\zeta$. The above assumptions are illustrated in fig. 4(i).

In fig. 4(ii) we diaw the Mirrlées-Stiglitz relation for comparison where we choose the special form which intersects the horizontal axis at point $Y$ and is concave over ( $C_{0}, \infty$ ). Mirrlees (1976) and Stigliz (1976) both worked with a relation which intersected the horizontal axis at the origin. We have been interpreting the number of efficiency hours $h$ as the number of tasks $n$, and we suppose, at the moment, that all wages are zonsumed. We see that fig. $4(i)$ is merely a rotation through $90^{\circ}$ of fig. 4(ii) where the point $Y$ becomes the point $X$ when we move 4(ii) to (i). There ae, however, two important differences from the Mirrlees-Stiglitz approach. First we have indifference curves in 4(i) where we do not in 4 (ii) and second the boundary of the consumption set intersects the $c$-axis at: $X$, which is $C_{0}$ above 0 .

We shall below pay more attention to the former of these differences but note in passing that the second point: is of considerable importance to the


Fig. 4

Mirrlees-Stiglitz result that there will be two different consumption groups in the family of peasant farmers. They suppose, as noted above, that the $h(\cdot)$ relation in 4(ii) goes through the origin rather than stopping at Y. It is the non-cenvexity thereby introduced which is at the root of the advantage to be gained from two consumption levels inside the family. Without the nonconvexity, the family which maximizes the sum of utilities $u(c, n)$ a cross the identical family members for given $\sum c=C$ and $\sum n=N$ will choose, if an optimum for its problem exists, an equal allocation $(c, n)$ for eack member of the family if $u(c, n)$ is strictly concave (and there will still be an optimum with equality if we drop the strictness assumption).

We return now to the first of the differences mentioned above, between the approaches described in the two diagrams of fig. 4. We recall that a rocation through $90^{\circ}$ of fig. 4(ii) (the Mirrlees-Stiglitz approach gives the consumption set $\zeta$ of fig. 4(i) and that the Mirrlees-Stiglitz approach considers only consumption-work pairs on the frontier of $\zeta$ so that the interior of $\zeta$ is irrelevant and therefore not ordered. Along the frontier, Mirrlees and Stiglitz assume that utility increases with consumption, notwithstanding the corresponding increase in work. We represent in fig. 5(i)-(iv) examples of four ways of ordering the consumption set which give very different orderings of the frontier. For each of these four orderings we can consider a function $v(c)$ which describes how utility changes as we move round the boundary from

$\left(C_{0}, 0\right)$. The corresponding of • functions are sketched in (ig. 6(i)-(iv). The second case in figs. 5 and 6 represents that of Mirrlees and Stiglitz. We suggest that the most plausible case is the third, although, of course all four are conceivable and they do not ewhaust the possibilities.


Fig. 6

The claim that 6 (iii) is the most plausible case is not something that lends itseif to proof but one can get 2 feeling for what is likely by posing one or two questions. The frontier describes the combinations of consumption per day and tasks per day which constiute the limit of the individual's capabilities. We shall discuss the meaning of 'limit' in more detail in part II but for the moment we can ask ourselves whether we should prefer total idleness together with the minimun pessible consumption $C_{0}$, to extreme hard work at much higher levels of consumption. It is most unlikely that we should be indifferent between all such consumption-work pairs. We suppose that a little work together with the mininnum extra consumption needed to stay in the consumption set would be preferred to total idleness, and that at extreme levels of work some relaxation of work effort, together with the appropriate reduction in consumption, would be welcomed. We presume furthe: that there are work-consumpticn pairs on the boundary involving such intensive activity that there are dermed to be worse than total idieness on minimum possible consurnption. Thus $p\left(C_{0}\right)$ in fig. 6 (iii) is not the minimum utility level. Indeed, the atili y which we suppose is attached to
leisure leads us to guess that the first case $5(i)$ and $6(i)$ is 10 less likely than the second - the Mirrlees--Stiglitz configuration. We suggest then, that the most common case is where there is some medium level bf work combined with a medium consumption level, $C_{I}$, that is rank. 1 the highest amongst points on the frontier and that we have an indifference mat of the third type.

We have been supposing, until now, that the relevant houndary for the employer is the physical limit for the individual. There may, however, be a bound to the offers an employer can make successfully which is set by opportunities, or expected opportunities, which are availaple elsewhere. The rural worker may have some estimation of the chances of pbtaining work in a town and if forced to sufficiently low utility levels (lower than some $\bar{u}$ say) might actually migrate. Alternatively he may have relations of friends on farms elsewhere who might provide him with consumption and work yielding $\bar{u}$. He may be able to make a living in the village doing an ad hoc collection of menial tasks in his own or other households. Such alte, natives might not exist but we suspect, on the other hand, that they may be common. In this case the boundary for the monopolist would be those poitts on the frontier, if any, of $\zeta$ providing utility levels higher than $\bar{u}$ tpgether with the indifference curve corresponding to $\bar{u}$.

In fig. 6(iii) for example the frontier of $\zeta$ would be relevant only for consumption levels between $C_{2}$ and $C_{3}$, and outside this range the appropriate boundary would be given by the indifference curve. This is illustrated in fig. 7. There are corresponding boundaries for the other three cases when we impose the constraint $u \geqq \bar{u}$.


Fig. 7

If we use a constraint $u \geqq \bar{u}$ then we have a model withou long-term rural unemployment. ${ }^{11}$ Tae constraint $u \geqq \bar{u}$ may, of course, represent expected opportunities in the town, and if urban wages are sufficiendly high workers will migrate on the probability of a jot attached to being present in the town. Thus urban unemployment is not ruled out, although long-term rural unemployment is. However long-tierm, as opposed to seasonal unemployment, is not a feature of all rural areas. Indeed, we should suggest that for less developed countries, models which predict long-term rural unemployment do not have ary special claim to attention.

We attach some importance to the case where there are alternatives yielding $\bar{u}$ and we introduce the following terminology in order to retain distinctions in the subsequent discussion. We call $\bar{u}$ the reservation utilit , level. That part of the consumption set lying on or above the reservation indifference curve will be called $R$, the relevant set. The tangent from the origin to $R$ may meet $R$ either at a point on tae reservation indifference curve or at a point, with utility higher than $\bar{u}$, where the frontier of $R$ coincides with the frontier of the consumption set $\zeta$. We shall indicate in the diagrams to follow whether $\zeta$ or $R$ is intended.

Having discussed the possible orderings of the consumption set in some detail we now turn to the difference the existence of orderings in the interior of the consumption would make to the consumption-work reactions perceived by the wage-setting employer. From this viewpoint the MirrleesStiglitz employer of section 2 seems now to be something rather more than a wage-setter. The employer insists, or supposes that, after setting the wage, the response of the worker is on the frontier of the consumption set. On the other hand, if the worker assumes he can provide as many completed tasks per day ( $n$ ) as he wishes at the given piece-rate per task $(c / n$ ) then the relevant set of responses is described by the offer curve where it lies in R. Let us look in more detail at the form of the offer curve. The situation is illustrated in fig. 8. A given piece-rate per task, $p$, is represented by the line OW. If as indifference curve is tangent to $O W$, at the point $Y$ say, then the number of tasks offered is $n_{\mathrm{y}}$. We can then draw the supply curve for the worker $n(p)$ and an example is drawn in figg. 9. The point T, where the tangent from 0 to the frontizr of $\zeta$ meeis the frontier, lies on the offer curve since for that price per task $p_{\mathrm{T}}, \mathrm{T}$ is the only feasible option giving at least $\bar{u}$. Thus $p_{\mathrm{T}}$ is the minimum price at which the employer can obtain labour.
We have drawn, in fig. 9 , the supply curve as downwari sloping for much of its range. It might of course, be either upward or downward sloping for the entire range or its slope may change as shown (or in the opposite direction). This particular feature of the supply curve is not our special

[^9]

Fig. 8


Fig. 9
concern. We want to concentrate on the relationship between the offer curve in fig. 8 (and hence the supply curve in fig. 9) and the frontiers of $\zeta$ and $R$.

It is clearly possible for the offer curve to lie along the frontier of $\zeta$ for part of its length. This possibility is illustrated in fig. 10, where we suppose that OT is tangent to $R$ at a boundary yoint of $\zeta$. Consider a constant piecerate line OUV meeting at the frontier of $\zeta$ (coinciding locally with the frontier of $R$ ) at $U$ and $V$, where $U$ has lower consumption than $V$. If the indifference curve through $U$ is steeper than OUV (as drawa in fig. 10) then maximum utility occurs at U [under our assumption of concave $u(c, n)$ ]. If the indifference curve through V is flatter than OUV the optimum occurs at $V$. If neither of these two eventualities nccur the optimum for the individual lies between $U$ and $V$ at the point of tangency of an indifference curve with OUV. The offer curve will depart from the frontier of $\zeta$ at a point, say $Z \mathrm{n}$ fig. 10 , where the indifference curve through $Z$ is tangential to $O Z$. In the example of fig. 10 the offe: curve lies along the frontier from $T$ to $Z$ and $\mathrm{il}_{\mathrm{i} \cdot \mathrm{n}}$

moves into the interior. Similar, ${ }^{3}$ y one can construct an example where the offer curve follows the frontier upwards from $\mathbf{T}$ before it leaves the frontier for the interior.

It is clear that, in cases such as 5(iii), if the gradient of an indifference curve changes continuously round ine frontier of $\zeta$ (which we still suppose coincides with the frontier of $R$ ), that $Z$ will lie on the frontier between $\overline{\boldsymbol{l}}$ and $I$, where $I$ is the best point on the frontier. An offer curve for case 5 (iii) might therefore look as TZF in fig. 11(i) with a corresponding suppiy curve as in fig. 11(ii).

A model of a price-setting employer who has to foilow the offer curve then coincides with the Mirrlees-Stiglitz approach (where movement is always around the frontier) only for price per task between $p_{T}$ and $p_{\mathrm{Z}}$ and consumption levels between $c_{\mathrm{T}}$ and $c_{\mathrm{Z}}$.

The preceding analysis of the offer curve has been for the case where the tangent from the origin to $R$ meets $R$, at $T_{k}$ say, where $T_{R}$ lies on the frontier of $\zeta$ strictly above the reservation indifference curve (hence $T_{R}=T$ ). We call this case the Mirrlees case since the minimum piece rate is $p_{\mathrm{T}}$ as in the Mirrlees-Stiglitz solution in section 2.

We distinguish two further cases. $\mathrm{T}_{\mathrm{R}}$ may lie on the reservation indifference curve strictly above the frontier of $\zeta$. We call this second case the interior case. The analysis in terms of $Z$ becomes irrelevant -the offer curve leaves the frontier of $R$ at $T_{R}$. The cost-minimizing employer chooses the price line $\mathrm{OT}_{R}$.

The third case can be illustrated in fig. 10 . If NU were the reservation


Fig. 11
indifference curve then $U$ would coincide with $T_{R}$ which lics both on the frontier of $\zeta$ and the reservation indifference curve. The zost-minimizing employer will choose the price line OU.

Returning to the Mirrlees case where $\mathrm{T}_{R}$ lies above the reservation utility level we considered movements around the frontier of $\zeta$ betwe n T and Z for the case where a best point on the boundary exists [see fig. 5 (iii)]. It is clear that $Z$ must also lie on the frontiers of both $R$ and $\zeta$. Take the case of fig. 10 for example. At the point $Z$ the indifference curve must be steeper than the boundary of $\zeta$ whereas at the point $J$, see fig. 7 , below $T$ where the boundary of $R$ becomes the reservation indifference curve, this indifference curve must be flatter than the boundary. It is clear $t_{1.2}^{+} c_{\mathrm{J}}<c_{1}$. In the case of fig. 10 we have therefor $c_{\mathrm{T}}>c_{\mathrm{Z}}>c_{\mathrm{I}}>c_{\mathrm{J}}$. We have assuned that $c_{\mathrm{T}}$ and $c_{\mathrm{Z}}$ are different points. They will be distinct unless $T=I$ when the tanger ay point $T$ is the best along the frontier. Where $c_{\mathrm{I}}$ is greater than $c_{\mathrm{T}}$ we have $c_{\mathrm{J}}>c_{\mathrm{I}}>c_{\mathrm{Z}}>c_{\mathrm{T}}$, (where $c_{\mathbf{y}}$, is from the upper point where the reservation indifference curve meets the frontier of $\zeta$-see fig. $7, c_{\mathrm{J}} \equiv C_{2}$ and $c_{\mathrm{J}} \equiv C_{3}$ ).

We have seen, then, that in the Mirrlees case the solution for the pricesetting employer who is trying to minimise his cost per task remains at the point $T$ with a cost per task equal to $p_{T}$, the gradient of OT, when we switch to the offer curve as a constraint rather than the frontier of $\zeta$. In this case those who are employed enioy a utility level higher than the reservation level $\bar{u}$, and those who are not employed exist at $\bar{u}$.

We have also seen, however, that there is another case of importance-the intericr case - where the minimum piece-rate does not give a consumptionwork pair on the frontier of the consumption set. We have an equilibrium where the worker obtains his reservation utility and for higher wages the employer can move only along the usual offer curve. The employer offering insufficient wages obtains no labour.

But what would happen in the Mirrlees case if the employer, seeing an excess supply of labour wiliing to perform tasks $i_{T}$ per day for price $p_{\mathrm{T}}$, tried to lower the price? The excess supply would drop to zero since no-one would be able to supply any work at price per task below $p_{\mathrm{T}} .{ }^{12}$ This seems to us to indicate that such an equilibriurn is a rather implausible picture of an economy. On the other hand if there is a common utility level $\bar{u}$, as in the interior case, it is not surprising that a lower price from the monopolist would yield no offers. Rut in this case, there is no genuine excess supply of 'abour in the rural labour market.

## 8. A further look at the Mirrlees-Stiglita model: Conclusion

We have laid the emphasis in sections 6 and ${ }^{7}$ on the consumer side and have attempted to show how the consumption set, the boundary of which defines the relation, the preferences of the worker and alternative opportunities all play a part in determining the nature of the outcome. Our investigations have provided only limited support for the practice of confining attention to the frontier of the consumption set. An employer who makes 'all-or-nothing' offers to the worker would be able to force the worker on to the frontier of $R$ and would have an incentive to do so. The frontier of $R$ consists in part of the frontier of the consumption set and in part the reservation indifference curve and may be entirely one or the other. A pricesetting employer, however, is constrained to the offer curve of the workerthe consumption set frontier as such is unimportant. However the pricesetting employer will, as it happens, always take the worker to a point on his offer curve which is also a point on the frontier of $R$. These conclusions might seem to give the frontier of $\zeta$, as part of the frontier of $R$, an important role to play after all. But that depends critically upon the supposition that the outer limit of what a worker will do (frontier of $R$ ) is governed by his physical limits and not by alternative possib:lities providing for him a reservation utility level $\bar{u}$. Where this reservation level is the binding constraint we are back to a model with a familiar supply-derrand equilibrium. Urban unemployment is not rulec out. Long-term rural unemployment is ruled out although the alternative opportunities yielding $\bar{u}$ might be classified officially as unemployment.

## 9. Overall conclusions of the theory and ampirical implications

Having embarked on a thorough in restigation of the theoretical implications of the productivity-consumptio l-link hypothesis we have not been able to avoid a lengthy discussion. A su nmary of the conclusions of sections

[^10]6 and 7 , concerning the relation of the theory to standard supply and demand theory has been given in sextion 8. Section 4, concerning aggregation and section 5 on the strength of the relation as seen by the employer are short and do not need summaries. It would be a pity, however, if the reader were left at the end unable to see the important trees because of the forest around them. Despite the complexity of some parts of the analysis there is one basic and far-reaching conclusion which holds quite generally and it is on this corclusion that a good deal of the burden of testing the theory will fall. If the efficiency-wage theory as discussed in sections 2 and 3 is valid in its strong version then there will be one wage rate, the efficiency wage, which will be paid to labourers with no alternative source of consumption, whether there is competition or monopsony on the buyer's side of the market. That wage will be independent of sniall variations in the supply and demand for labour.

Lcoking forward to part II, which will be addressed to the empirical implications of the mc lel , it may be useful to gather together those conclusions from sections 2 and 3 of the analysis which are fairly amenable to empirical investigation.
(1) There is an efficiency wage which will always be paic to landless labourers whose labour is in excess supply (we might call these 'marginal' workers) regardless of the conditions of competition or monopoly and the supply and demand for latour. This wage should be simila in different regions and for different production conditions.
(2) Given that the productivity effect of consumption is likely to be different (and probably larger) in the long run than in the short run, we would be led by the theory $t$ ) expect that wages for workers employed under long-term centracts would be higher than for those employed ander shortterm contracts.
(3) Equally, on the same g ounds, we would expect long-term vontracts to 'offer advantages over short-term contracts from the employer's point of view, so that they might come to p redominate.
(4) The theory is not decisive in deciding whether landless labourers would be paid more or less per liour than labourers with land. In that regard conditions of competition o monopoly, and the exact quantitative significance of various factors do'matter for the outcome. However the theory throws up the possibility that wages for these two types of labour might be different in equilibrium and even that labourers wich land might receive a lower hourly wage rate.

All these conclusions, as is to be expected from a purely theoretical apr roach, are what might be called "qualitative". In part II we will devote
considerable attention to quantifyi g the relation postulated by the theory so as to test its quantitative significance. However we shall also compare the qualitative conclusions with some evidence to see whether the type of feature to which the theory gives rise is to be encountered in reality.

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[^0]:    *The original version of this paper was prepared for the Population and Human Resources Division of the I.B.R.D. and we are gratef il to T. King for advice and encouragement. The ideas were conceived and the first draft prepared when the authors were in India in the early part of 1975, working on a study of a village in West U.P. That stcy was rei concerned with nutrition and we shall be reporting on it elsewhere, see Bliss and Sterr (1979). We are geaterul for helpful comments to J.A. Mirrlees and P.K. Barchan. All opinions and errors are ours. The editor and referees made many constructive comments and we are gratefil to them.
    ${ }^{1}$ Myrdal (1968, vol. III, p. 1603).

[^1]:    ${ }^{2}$ See Leibenstein (1957, ch. 6, especially pp. 62-76).
    ${ }^{3}$ Mirrlees does not confine his discussion to agricultural labour only but addresses timself initially to the case of factory labour. Later he discusser allocation within a 'peasant' family. We are concerned only with agricultural labour. It might te thought that the model is more likely to apply to agriculture, where perhaps consumption levils are lower, but against this it could be argued that factory employers are more likely to exhibit the far-sighted rational caiculations that the model imputes to employers.

[^2]:    It is not necessary for the argument that the curve should both exhibit the region of strict convexity and that $C_{0}$ should lie to the right of $O$. Either of these properties will suffice. Note that the function $h(c)$ includes the horizontal axis to the left of $\mathcal{C}_{0}$. Hence it is not a concave fanction cven if $h(c)$ to the right of $C_{0}$ is concave.

[^3]:    ${ }^{5}$ With a suitable choice of units the constraint that the employer hire at least one efficiency unit of labour is equivalent to the requirement that he produce at least an output $\bar{y}$. We need only assume $f\left({ }^{\circ}\right)$ an increasing function.

[^4]:    ${ }^{6}$ We are indebted to J.A. Mirrlees for bringing this point to our attention. When this argument was put to M. Morishima he pointed out that the conclusion coule be different it the production function $g$ were to have a non-concave segment at the outsct ti.e. if there were to be increasing returns to the applicatior of efficiency hours of labour to land on the worker's own land for low levels of work).

[^5]:    ${ }^{7}$ It follows from (24) that the wages axe equal despite the fact that the $h$-function is not assumed to be concave. The solution will in each case always be on the concave segment of $h$.

[^6]:    ${ }^{8}$ In explaining our conclusions in the present case we have found that the following analogy sometimes helps to make it obvious. Prisoners of war are held in a camp and are fed with a view to getting work from them in the most economic manner. Some prisoners receive food parcels from relatives, while others to not. The conventions of war prohibit the confiscation of food parcels, otherwise the authorities would obviously do so. However they are able to discriminate between different prisoners according to the amount of food given. In that case it is most economic to give prisoners who receive food parcels precisely that much less food, hence in effect confiscating the parcels.

[^7]:    ${ }^{9}$ Cf. Rodgers (1975).

[^8]:    ${ }^{10}$ For the prevalence of daily contracts see the official publicanon Ag"icultural Wages in India. Ministry of Food and Agriculture (annual).

[^9]:    ${ }^{1}$ The worker who is achieving $\bar{u}$ in an unorganised manner may, of course, be classified as 'unemployed' by the collector of the official statistics, although this is not the usual practice [see Turnham (1971)].

[^10]:    ${ }^{13}$ Compare this conclusion with that of Leibeast in quoted on the second page of this paper.

