Economics Letters 3 (1979) 119–123

A NOTE ON THE ALLOCATION OF TIME *

A.B. ATKINSON
University College London, London WC1E 6BT, UK

N.H. STERN
University of Warwick, Coventry, UK

Received 19 September 1979

A multi-commodity model in which time as well as nominal income is necessary in the process of consumption is analyzed. Duality theory is used to provide a counter example to the 'Linder Theorem' that a rise in real wages will decrease consumption of some good if it is more time intensive than the weighted average of the time intensity of all other commodities.

The theory of the consumer has for the most part dealt with models which concentrate on the constraint on the purchase of goods which results from the money budget of the household, and have ignored the constraints on consumption activities imposed by the limitations of time. Becker (1965), however, presented a model which included the constraint on time and showed how it could be used to understand many observed market phenomena. Linder (1970) developed similar ideas and used them to explain attitudes to time at different stages of economic development and, in particular, the apparent failure of productivity increases to bring with them a reduction in the intensity of effort and pace of life. Baumol (1973, p. 629) expanded on Linder's analysis arguing that 'the substitution effect of a rise in real wages will decrease consumption of some good or service if the time needed to consume one dollar's worth of the item is greater than the average time used to consume a dollar's worth of all other commodities (in the proportion he consumes them)'.

The main purpose of this paper is to show that Baumol's claim to have established the above result is incorrect. He is correct for the two-activity model he was analysing but the result does not generalise to his assertion in the above quotation. A secondary purpose is to illustrate the power of the expenditure function approach to this problem. Finally we show how the model allows an alternative explanation

* This research was supported by a Social Science Research Council programme grant on Taxation, Incentives and the Distribution of Income.

for some empirical results on the value of time spent travelling to that advanced recently by Glejser (1978).

The model is as follows. Activity j involves goods with a money cost p_j per unit of the activity. We suppose that the activity requires a bundle of goods in fixed proportions so that the money cost is independent of the level of the activity (or any other activities). The activity has a time cost of t_j hours per unit and it is impossible to indulge in two activities simultaneously. If $(c_0, c_1, ..., c_m)$ is the vector of activities the consumer's problem is to

$$\begin{aligned} &\text{maximise} \quad u(c_0, c_1, ..., c_m), \\ &\text{subject to} \quad \sum_{j=0}^{m} p_j c_j \leqslant M, \\ &\qquad\qquad \sum_{j=0}^{m} t_j c_j \leqslant T, \\ &\qquad\qquad c_0, c_1, ..., c_m \geqslant 0, \end{aligned} \tag{1}$$

where M and T are the total available money and time respectively. The formulation is fairly flexible. An activity may involve earning money (p_j could be negative) or travelling to work. It may be 'pure' leisure ($p_j = 0$), and one can allow time-saving activities ($t_j < 0$).

In our analysis of the Baumol claim we consider the case where the activity zero is work and we shall write $c_0 = l$ and $p_0 = -w$. We at first suppose that c_0 does not enter the utility function so that problem (1) becomes:

$$\begin{aligned} &\text{maximise} \quad u(c_1, ..., c_m), \\ &\text{subject to} \quad \sum_{j=1}^{m} p_j c_j \leqslant M + wl, \\ &\qquad\qquad \sum_{j=1}^{m} t_j c_j \leqslant T - l, \\ &\qquad\qquad l, c_1, ..., c_m \geqslant 0. \end{aligned} \tag{2}$$

The time and money constraints bind at the optimum if the consumer is not satiated and at least one of the p_j, $j = 1, ..., m$, is positive. We assume $l > 0$ at the optimum. The problem can then be written

$$\begin{aligned} &\text{maximise} \quad u(c_1, ..., c_m), \\ &\text{subject to} \quad \sum_{j=1}^{m} q_j c_j = M + wT, \\ &\qquad\qquad c_1, ..., c_m \geqslant 0, \end{aligned} \tag{3}$$

where $q_j = p_j + wt_j$. The price q_j may be considered as the total price involving both goods and time.

Problem (3) is the utility maximisation model in standard form and we can use the expenditure function to analyse it. We write $E(\boldsymbol{q}, u)$ as the expenditure function for problem (3) where $\boldsymbol{q}$ is the m-vector of prices q_j. The standard properties of E apply: E is concave in $\boldsymbol{q}$ and the compensated demand for the jth activity, $c_j(\boldsymbol{q}, u)$ is equal to the partial derivative of E with respect to q_j, which we write E_j. Note that the formulation of the problem in (3) implies that it does not matter whether compensation is through the time constraint or the money constraint.

Baumol considers the case $m = 2$ and asks what will happen to compensated consumption of the two activities if the wage changes. From the definition of q_k we have $\partial q_k/\partial w = t_k$. Writing $\partial c_j/\partial w$ for the compensated effect of the wage change we have

$$\frac{\partial c_j}{\partial w} = \sum_{k=1}^{m} t_k E_{jk} \,. \tag{4}$$

From the homogeneity of E_j in $\boldsymbol{q}$ we have

$$0 = \sum_{k=1}^{m} q_k E_{jk} \,. \tag{5}$$

And from (4) and (5) we have for the two good case

$$\frac{\partial c_1}{\partial w} = q_1 E_{11}\left(\frac{t_1}{q_1} - \frac{t_2}{q_2}\right) . \tag{6}$$

Hence, using $E_{11} < 0$ (concavity of E) and $q_j = p_j + wt_j$, we have

$$\operatorname{sign} \frac{\partial c_1}{\partial w} = \operatorname{sign}\left(\frac{t_2}{p_2} - \frac{t_1}{p_1}\right) . \tag{7}$$

Eq. (7) is the Linder theorem for the two good case: if good 1 is more time-intensive ($t_1/p_1 > t_2/p_2$), the substitution effect of an increase in w is against good 1.

With three activities

$$\frac{\partial c_1}{\partial w} = t_1 E_{11} + t_2 E_{12} + t_3 E_{13} \,, \tag{8}$$

$$0 = q_1 E_{11} + q_2 E_{12} + q_3 E_{13} \,. \tag{9}$$

From (8) and 9 we have

$$\frac{\partial c_1}{\partial w} = E_{11} q_1 \left(\frac{t_1}{q_1} - \frac{t_2}{q_2}\right) + E_{13} q_3 \left(-\frac{t_2}{q_2} + \frac{t_3}{q_3}\right) . \tag{10}$$

It is clear from (10) that even if $t_1/p_1 > t_2/p_2 > t_3/p_3$ we can have $\partial c_1/\partial w > 0$.

Intuitively, if activities 1 and 3 are strong complements then taken together their relative time cost might not exceed that for activity 2. Thus care must be taken with the patterns of substitutes and complements before one is justified in claiming that (compensated) levels of the time-intensive activities decline if the wage rate goes up. An example where a money-intensive and a time-intensive activity may be complements is a large, expensive meal followed by a long walk. Hence Baumol's claim quoted at the beginning of this paper is incorrect. [1]

It is straightforward to derive in the above model the result that the substitution effect of an increase in the wage is in favour of labour [see Atkinson and Stern (forthcoming, eqs. (11)–(13)) – one differentiates the equation $l = T - \Sigma_{j=1}^{m} t_j c_j$ with respect to w]. This result is, however, more general than the case considered above, i.e., that where the utility function is independent of c_0. Returning to problem (1) we have a special case of the general problem of rationing – we have two constraints rather than several. Diamond and Yaari (1972) have shown that a change in price in a given constraint, together with compensation to keep utility constant by adjusting the budget in the same constraint, leads to own price effects which are symmetric [their equation (15)] and non-positive [their equation (16)]. In our case this implies $\partial l/\partial w \geqslant 0$ where compensation is by an income change [recall $p_0 = -w$ in our interpretation of problem (1)]. James Mirrlees has pointed out to us that the result can also be derived by a straightforward extension of the standard expenditure function approach to incorporate extra constraints. Thus one defines E as the minimum income which is required subject to achieving a given utility level and satisfying the other constraints on the problem (here $\Sigma_{j=0}^{m} t_j c_j \leqslant T$). The standard proofs of the concavity of the expenditure function and the result that its first derivatives are compensated demands go through with only minor modifications and we immediately have the result that the own price effects are symmetric and non-positive. Note that one can define an expenditure function corresponding to any one of the constraints – for example, one could use a function which is defined as the minimum time expenditure required to achieve a given utility level whilst satisfying the income constraint.

Baumol did consider the case where c_0 enters the utility function. This case provides a further example of the way in which his analysis fails to generalise where we have a third activity (c_3). We know that the substitution effect of a wage increase is in favour of labour (c_0). Suppose that the time intensive activity is strongly complementary with labour (for example, resting), then the substitution effect of a wage increase will be in favour of, not against, the time intensive activity. [2]

[1] One cannot amalgamate all other goods into a single composite commodity because their relative price varies with the wage rate.

[2] Note that this example is invalid where we have only two activities. Suppose we take the case most favourable to the argument – where c_0 and c_1 are perfect complements – we have then essentially only the activity c_2 and the joint activity zero and one. With two constraints (time and money) and only two variables the consumer's choice will, if both constraints bind, be independent of the utility function – thus the substitution effect will be zero.

Finally we use the model to comment on the opportunity cost of time. We take Lagrange multipliers λ_M and λ_T for the money and time constraints respectively. From the first order condition for choice of labour supply (recalling our convention that $c_0 \equiv l$) we have, where we have set the time required for a unit of work (t_0) equal to 1,

$$\frac{\lambda_T}{\lambda_M} = w + \frac{1}{\lambda_M}\frac{\partial u}{\partial l}. \tag{11}$$

The left-hand side is the value of time in terms of money, which we shall call $\hat{w}$, and we see that

$$\frac{\hat{w}}{w} = 1 + \frac{1}{w\lambda_M}\frac{\partial u}{\partial l} \tag{12}$$

[results similar to (12) have been derived in the travel-time literature – see, for example Harrison and Quarmby (1972)]. If time spent at work has positive (negative) marginal utility the value of time will be greater than (less than) the wage rate. Note that the standard model of choice, with the income constraint being the only one of relevance, prevents us from considering the case where, at the optimum, time at work provides pleasure on the margin since, in that model, an increase in work would increase utility without violating the budget constraint.

It is commonly found in studies of the valuation of travel time that the value is less than the gross-of-tax wage but that the value as a proportion of the wage tends to rise with income. An ingenious explanation for this is advanced by Glejser (1978); a more prosaic alternative is that those with lower wages dislike work more intensely relative to their payment.

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