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ON THE SWITCH FROM DIRECT TO INDIRECT TAXATION

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The paper is concerned with the inter-relationship between the theoretical specification of household behaviour and the application of econometric results to the analysis of public policy. The first part of the paper discusses the relationship between the optimum tax literature and econometric specification of joint labour supply and commodity demands. The second part introduces an 'activity model' of the allocation of money and time, which generalises the standard linear expenditure system, and describes the preliminary results of estimating the model for the United Kingdom Family Expenditure Survey data for 1973. The remainder of the paper uses the estimates to examine the effect of reducing income taxation and increasing value added tax on both labour supply and welfare.

1. Introduction

In a number of countries the balance between direct and indirect taxation is an active political question. In Britain it has been widely argued that it would be desirable to switch the burden of taxation away from income taxation to indirect taxation, and the Conservative Government has adopted the policy of reducing the basic rate of income tax and increasing the rate of value added tax (a selective indirect tax). Whilst this is not a structural reform of the tax system, involving only a change in the rates of tax, it does represent a significant departure from past trends.

The purpose of this paper is to use an empirically estimated labour supply/commodity demand system to assess some of the implications of such

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a switch in tax policy. The joint determination of labour supply and commodity demands is central to the analysis, and we pay particular attention to the specification of an appropriate model. The estimation of the model requires data on both consumption and labour supply, and we have for this purpose exploited the cross-section United Kingdom Family Expenditure Survey, a valuable source which has only recently become available in the form of micro-data.

A full assessment of the switch from direct to indirect taxation should take account of a wide range of considerations, such as the impact on savings, on portfolio choice, on perceived tax burdens, on the distribution of income, etc. Here, however, we concentrate on a single behavioural aspect — the implications for labour supply. The effect of the income tax on work effort has figured largely in the public debate, with the claim being made that the switch to indirect taxation would provide incentives and increase work effort. The precise status of this claim is open to debate. On one interpretation, it can be seen as saying that private decisions about work effort diverge from the socially desired outcome (presumably even in the absence of taxation). The supply of effort is akin to a merit good in that it should be encouraged. On a second interpretation, the concern may arise from the belief that the *tax system* has caused a reduction in work effort, and that taxation could be reformed to reduce the associated 'deadweight' loss. This position is implicitly based on the criterion of social welfare, with the effect on labour supply being an intermediate objective. In what follows, we consider both the absolute effect on labour supply and the implications for social welfare.

The first part of the paper discusses the relationship between tax theory and the choice of econometric specification (section 2) and introduces the particular model employed here (section 3). Preliminary results obtained from estimating the model using data for 1973 are described in section 4; and the remainder of the paper applies the results to the switch from income taxation to value added tax (sections 5 and 6).

2. Tax theory and econometric specification

The welfare implications of the switch from direct to indirect taxation have been discussed in the literature on optimum taxation [see Atkinson and Stiglitz (1980, ch. 14) for references]. This has typically been based on a model in which there are many individuals and where the government chooses differentiated taxes on commodities and an income tax schedule for wage income in order to maximise a social welfare function which depends on the utility levels of the individuals, whilst raising a given revenue.[1] The

[1] One can show that, if production is competitive and there are constant returns to scale, or if it is government controlled, the problem is equivalent to that where the constraint is on meeting consumer demands rather than on revenue.

individuals themselves choose labour supply and commodity demands to maximise utility given the post-tax prices and their budget constraints. It is clear that differences between individuals are essential to the problem, else the optimum would — in this model — be to specify the income tax schedule as a poll-tax, with neither marginal income taxation nor taxes on commodities.[2]

The results of the optimum tax literature are, in general, sensitive to the range of instruments assumed to be at the disposal of the government. One question of special importance is whether the government has full freedom to levy a *nonlinear* income tax. In this paper, we assume that this is not the case, and that the only income tax which is possible is linear, i.e. a uniform poll subsidy and a constant marginal tax rate. For administrative and other reasons, the income tax operated in Britain has a constant marginal rate over a wide range of incomes, and we focus on this case [the nonlinear tax is discussed in Atkinson and Stiglitz (1976)]. The choice treated here is therefore that of the appropriate balance between a linear income tax and a differentiated indirect tax system.

This problem has been analysed extensively for a world where the production sector is competitive. If individuals face the price vector $\boldsymbol{q}$, equal to the producer price vector $\boldsymbol{p}$ plus taxes $\boldsymbol{\tau}$, the demand for goods of the hth household is $\boldsymbol{x}^h(\boldsymbol{q}, w^h, M^h)$, where w^h is the wage and M^h the unearned income including the poll subsidy. The first order conditions for optimum indirect taxes [see Diamond (1975) and Atkinson and Stiglitz (1976)], when account is taken of the first order conditions for the choice of the poll subsidy, are:

$$\sum_i \tau_i \left(\sum_h s_{ik}^h / H \right) = \bar{x}_k \phi_k, \qquad k = 1, \ldots, n-1, \tag{1}$$

where there are $(n-1)$ goods, H is the number of households, s_{ik}^h is the Slutsky-compensated demand derivative for the hth household, $\bar{x}_k$ is the mean of x_k^h and

$$\phi_k = \operatorname{cov}[(x_k^h/\bar{x}_k), b^h], \tag{2}$$

where b^h is the net social marginal valuation of income for household h (one adjusts the usual social marginal valuation to allow for the propensity to pay tax and hence return income to the government). The term ϕ_k is closely related to the 'distributional characteristic' defined by Feldstein (1972). The condition for the optimum choice of the poll subsidy is that $\bar{b}$, the mean of b^h, be equal to unity.

It is clear from the conditions (1) and (2) that the optimum differentiated tax structure depends on the way in which demand patterns vary across

[2]This may need to be modified in the presence of uncertainty — see Eaton and Rosen (1980).

households. This naturally suggests that we should make use of empirically estimated demand functions. With an estimated demand relationship, and assumed values of b^h, one can calculate ϕ_k and hence examine the tax structures which satisfy the first-order conditions (1). The conclusions drawn may, however, depend crucially on the specification of the demand relationships. To illustrate the point we take the example of the linear expenditure system (LES) which has been widely used in empirical work.

Households are assumed to be identical in all respects apart from their wage, w^h, and M^h, and to maximise the Stone–Geary utility function:

$$U(x^h, l^h) = \sum_{i=1}^{n-1} \beta_k \log(x_k^h - \gamma_k) + \beta_n \log(T - l^h), \tag{3}$$

where T is the quantity of available hours, β_k, γ_k are parameters, and l^h is the supply of labour. Maximisation subject to the budget constraint

$$\sum_{k=1}^{n-1} q_k x_k^h = w^h l^h + M^h$$

yields the LES demands for the hth consumer:

$$x_k^h = \gamma_k + (\beta_k/q_k)(1/\lambda_h) = \gamma_k + (\beta_k/q_k)[M^h + w^h T - \boldsymbol{q}' \cdot \gamma], \qquad k = 1, \ldots, (n-1). \tag{4}$$

We can interpret γ_k as the minimum 'committed' consumption, β_k as the marginal propensity to consume good k, and λ^h as the private marginal utility of income. The Slutsky terms are given by:

$$\begin{aligned} s_{ik}^h &= \frac{\beta_i \beta_k}{\lambda^h q_i q_k}, \qquad \text{for } i \neq k, \\ s_{kk}^h &= \frac{\beta_k^2}{\lambda^h q_k^2} - \frac{\beta_k}{\lambda^h q_k^2}. \end{aligned} \tag{5}$$

Substituting into the left-hand side of (1), we obtain:

$$\frac{1}{H} \frac{\beta_k}{q_k} \sum_h \frac{1}{\lambda^h} \left(\beta^* - \frac{\tau_k}{q_k} \right), \tag{6}$$

where

$$\beta^* = \sum_i (\tau_i \beta_i / q_i).$$

On the other hand, the right-hand side of (1) is:

$$\bar{x}_k \phi_k = \operatorname{cov}[x_k^h - \bar{x}_k, b^h] = \frac{\beta_k}{q_k} \operatorname{cov}[1/\lambda^h, b^h]. \tag{7}$$

From (6) and (7) we see that τ_k/q_k is independent of k so that the only solutions to the first-order conditions must involve uniform taxation.[3] Thus the assumption of an LES specification would *impose* an answer to the question posed here. The empirical results themselves (e.g. estimates of γ and $\boldsymbol{\beta}$) would contribute nothing.

This example illustrates that the information required in optimum tax calculations may go considerably beyond that typically estimated (the same applies to calculations as to whether one is moving in a welfare-improving direction), and that the specification may be crucial. As noted by Deaton, 'it is likely that empirically calculated tax rates, based on econometric estimates of parameters, will be determined in structure, not by the measurements actually made, but by arbitrary, untested (and even unconscious) hypotheses chosen by the econometrician for practical convenience' [Deaton (1978, p. 1)].

Similar considerations apply when we consider the absolute effect of the switch in taxation on labour supply. In the LES case, the labour supply function for household h may be written:

$$w^h l^h = w^h T - \beta_n [M^h + w^h T - \boldsymbol{q}' \cdot \gamma]. \tag{8}$$

If we now ask how labour supply is affected by indirect taxation, we can see that q_i influences l^h only through the term $\boldsymbol{q}' \cdot \gamma$. The same is true of the effect of q_i on the demand for good $k \neq i$ (see eq. (4)). Thus there is a general notion of gross complementarity/substitutability captured in the single parameter γ_i. It does not allow for the possibility that there may be a particular degree of complementarity or substitutability between good i and leisure (as opposed to other goods). This arises in large part because of the straitjacket imposed by the assumption of additive separability, an assumption which is commonly made in demand studies. As Deaton (1974) has emphasised, the way in which the income and substitution responses are tied together by this specification limits severely the flexibility of the estimated equations. By using a functional form which makes this assumption, we are again running the risk of imposing an answer on the data rather than using the data in an informative way.

[3]For more details, see Atkinson (1977). More generally, Deaton (1979) shows that, with a linear income tax, a sufficient condition is that there be weak separability between goods and leisure, coupled with linear Engel curves for goods. Where a nonlinear income tax may be levied, weak separability alone is sufficient [Atkinson and Stiglitz (1980, ch. 14)].

These considerations lead one to ask whether the specification can be generalised in a way which does not have strong implications of the kind described but at the same time does not lose the practical convenience of forms such as the LES.

One route out of this difficulty is to make use of the so-called 'flexible functional forms' for consumer demand functions. This has been followed in the case of labour supply by Wales and Woodland (1979) and Ashworth and Ulph (1977). This approach has the advantage of introducing the needed flexibility but is not without disadvantages. In particular many of the more flexible forms, such as the generalised linear function Diewert (1971), do not -- for an arbitrary set of parameters -- satisfy the conditions on the utility function of global monotonicity and/or quasi-concavity. The imposition of conditions sufficient to ensure that these properties hold globally reduces the flexibility of the form. For some purposes local properties may well be sufficient, so that this problem is not too worrying. However, this is not necessarily true for the simulation of tax changes, which -- if some of the proponents are to believed -- could take us far from the present position.

In view of this we have tried a different tack, considering the generalisations which are suggested by two related aspects of labour supply and commodity demand: first, that goods are usually purchased for use in particular activities, and secondly that these activities involve the use of time.

3. Activity model of labour supply and commodity demands

The analysis presented here is based on the household production model developed by Becker (1965).[4] This model assumes that the ultimate utility of the household is derived from activities (an m-dimensional vector $\boldsymbol{c}$) and leisure, and that these activities require the input of goods (of which there are n) and time, these being in fixed proportions. The cost to a consumer of an activity is the payment for the goods and the value of the time required, e.g. the cost to a businessman of 'playing golf' depends on the price of golf clubs, balls, etc. and on the opportunity cost of his time.

The model, and its special assumptions (such as fixed input coefficients), are open to criticism; they do however allow us to explore the relationship between labour supply and the consumption of different goods in a relatively simple framework. Even with an LES specification for the utility derived from activities, which we later adopt, the welfare implications of a switch from direct to indirect taxation are no longer implied merely by the choice of functional form. Moreover, the notion of an 'activity' is one which does seem

[4]This section of the paper draws on Atkinson and Stern (1980), where we discuss -- among other aspects -- the relationship of the model to the characteristics approach of Gorman (1956) and Lancaster (1971).

to capture some of the ways in which tax changes have been discussed, e.g. the notion of taxing goods involved in leisure pursuits.

The model may be set out formally as follows (where we drop the superscript h). The output of activity j is

$$c_j = \min\left[a_{1j}^{-1} x_{1j}, a_{2j}^{-1} x_{2j}, \ldots, a_{nj}^{-1} x_{nj}, t_j^{-1} T_j\right], \tag{9}$$

where x_{ij} is the quantity of good i allocated to activity j and T_j is the time allocated to the activity. Assuming that all goods have strictly positive prices, the purchases of goods are, where A is the $n \times m$ matrix with elements $[a_{ij}]$,

$$\boldsymbol{x} = A\boldsymbol{c}, \tag{10}$$

and the shadow prices of activities in terms of goods inputs are given by the vector $\boldsymbol{r}$, where

$$\boldsymbol{r}' = \boldsymbol{p}'A. \tag{11}$$

The time requirement is given by

$$\boldsymbol{t}' \cdot \boldsymbol{c}. \tag{12}$$

Suppose that a household is endowed with M units of unearned income and T units of time in excess of that required for 'essential' tasks such as sleeping. If hours of work are l and the wage rate per hour, w, and if the household behaves as a single utility-maximising entity, then the consumer choice problem may be written as

$$\max U(\boldsymbol{c}, l)$$

subject to

$$\begin{aligned} \boldsymbol{p}'A\boldsymbol{c} &= M + wl, \\ \boldsymbol{t}' \cdot \boldsymbol{c} &= T - l, \\ l, c_1, \ldots, c_m &\geqq 0. \end{aligned} \tag{13}$$

We have assumed both budget and time constraints hold with equality.[5] As noted by Becker (1965), where l does not enter the utility function, the problem can be reduced to a single constraint [the more general multiple-constraint case is discussed in Atkinson and Stern (1980)]. Making this

[5] This will be true where l does not enter the utility function (which we assume below), the household is not satiated and at least one of the r_i is strictly positive.

assumption, and eliminating l from the constraints, we have

$$\boldsymbol{q}'\cdot\boldsymbol{c}\equiv(\boldsymbol{r}+w\boldsymbol{t})'\cdot\boldsymbol{c}=M+wT, \tag{14}$$

where $q_j=\sum_i p_i a_{ij}+wt_j$ denotes the 'total price' of activity j, which we assume to be positive (although t_j may be positive or negative).

Stone–Geary case

At this stage we specialise by taking the case where the utility function is of the Stone–Geary form:

$$U(\boldsymbol{c})=\sum_{j=1}^{m}\beta_j\log(c_j-\gamma_j), \tag{15}$$

where $\beta_j\geqq 0$ and $\sum_{j=1}^{m}\beta_j=1$. Where $\boldsymbol{c}>0$, $l>0$, the first-order conditions (compare with (4)) are then

$$q_jc_j=q_j\gamma_j+\beta_j/\lambda,\qquad j=1,\dots,m, \tag{16}$$

where λ is the Lagrange multiplier associated with the budget constraint (the private marginal utility of income). Hence

$$1/\lambda=M+wT-\boldsymbol{q}'\cdot\gamma \tag{16a}$$

and

$$q_jc_j=q_j\gamma_j+\beta_j[M+wT-\boldsymbol{q}'\cdot\gamma]. \tag{17}$$

The implied demands for goods and labour supply are:

$$x_i=\sum_{j=1}^{m}a_{ij}\gamma_j+\left(\sum_{j=1}^{m}(a_{ij}\beta_j/q_j)\right)[M+wT-q'\cdot\gamma],$$

$$i=1,\dots,n, \tag{18}$$

$$l=T-\boldsymbol{t}\cdot\gamma-\left(\sum_{j=1}^{m}(t_j\beta_j/q_j)\right)(M+wT-\boldsymbol{q}'\cdot\gamma). \tag{19}$$

The implications depend on the specification of the activity matrix, A, and the vector, $\boldsymbol{t}$, which enters both directly and via the total price $\boldsymbol{q}$. Where we have pure consumption of $n-1$ goods (with $a_{ii}=1$, $t_i=0$, for $i=1,\dots,n-1$, and $a_{ij}=0$, for $i\neq j$) and pure leisure ($a_{in}=a_{nj}=0$, for $i,j=1,\dots,n$, and $t_n=1$)

it reduces to the standard LES model, with

$$\begin{aligned} q_j &= p_j \qquad \text{for} \quad j=1,\ldots,n-1, \\ q_n &= w, \end{aligned} \tag{20}$$

and demand functions, $i=1,2,\ldots,n-1$,

$$p_i x_i = p_i \gamma_i + \beta_i \left[M + wT' - \sum_{1}^{n-1} p_j \gamma_j \right], \tag{21}$$

where $T' = T - \gamma_n$, and labour supply function

$$wl = wT' - \beta_n \left[M + wT' - \sum_{1}^{n-1} p_j \gamma_j \right]. \tag{22}$$

This is similar to the form discussed earlier (eq. (8)) and it has the restrictive implications brought out there.

There are several ways in which the strong assumptions of the standard LES model may be relaxed using this activity framework. That followed here is perhaps the simplest. We retain the assumptions about A, but allow for time inputs into activities, i.e. we allow $t_j \neq 0$ for $j=1,\ldots,n-1$. This yields, for $i=1,\ldots,n-1$,

$$p_i x_i = p_i \gamma_i + \frac{\beta_i p_i}{p_i + t_i w} \left[M + wT'' - \sum_{j=1}^{n-1} p_j \gamma_j \right], \tag{23}$$

where

$$T'' = T - \gamma_n - \sum_{j=1}^{n-1} t_j \gamma_j \tag{24}$$

with a labour supply function

$$wl = wT'' - \left(\sum_{j=1}^{n-1} \underline{\frac{t_j \beta_j w}{p_j + t_j w}} + \beta_n \right) \left[M + wT'' - \sum_{j=1}^{n-1} p_j \gamma_j \right]. \tag{25}$$

Using $q_i = p_i + w_i t_i$, $i=1,2,\ldots,n-1$, $t_n = 1$ and $q_n = w$, eq. (25) follows immediately from (19). The demand for leisure follows from the time constraint or, alternatively, eq. (18) for $i=n$.

The difference from the standard LES labour supply function is brought out by the underlined term[6]: this induces an additional effect of the price of

[6] The standard LES supply function (22) is obtained from (25) by setting $t_j = 0$ for $j=1,\ldots,n-1$. It is then the case that $T'' = T'$.

goods on labour supply (in addition to the term in $\sum p_j\gamma_j$). This new effect can be seen to depend on β_j and on t_j. Compare, for example, an increase in taxation on a good used for an activity with a large time requirement and an increase in the tax on a good used for an activity with a low time requirement. There are two factors at work. First, a rise in price which reduces the consumption of the time-intensive activity leads to a greater availability of time for work; second, for this activity a rise in the cost of the good has a smaller proportionate effect on the 'total' price. In discussion of this question it is the first effect which tends to be given most weight, but one must not forget the second factor. Thus, the suggestion that taxing golf clubs rather than chocolate will cause businessmen to spend more time at work overlooks the fact that the price of clubs may be a very small fraction of the total cost to him. In the same way, we can see that, comparing people with different wages, the effect on total price is proportionately less for those with high wages. The golf-club tax may drive manual workers back to the shopfloor but not the businessman back to his desk.

The effect of the terms t_j has been discussed in terms of activities requiring time, but it is possible that some activities *save* time. If the purchase of a good or service k reduces the amount of time required for essential tasks, increasing the effective time endowment, then t_k can be negative. This means that the household 'buys time' at the price of $(-p_k/t_k)$. If this price is less than the wage, the model breaks down since it becomes attractive to buy an indefinite amount of the good and sell the 'acquired' time at w. Thus (25) is valid only for $w<(-p_k^*/t_k^*)$, where k^* is the value of k for which $(-p_k/t_k)$ is a minimum amongst those for which $t_k<0$. Where some t_k are negative, then it is assumed that attention is confined to the range for which (25) is valid, and we discuss this further when presenting the empirical results.

The data used in the next section to estimate the model are obtained from a cross-section survey and it is assumed that there is no variation in commodity prices. The effect of indirect taxes is therefore inferred from the response to the total price, which varies with the wage rate. As such, the evidence on responses to prices and taxes is 'once removed'. Moreover, the LES assumption is still restrictive at the level of activities, and the diagonality assumption about the activity matrix rules out much of the richness of the approach. We think however that the model does offer a significant first step towards extending the standard LES, and other additively separable systems, with respect to the relationship between goods and labour (if not between one good and another).[7]

[7]As was pointed out to us by Angus Deaton, the expenditure function has the form

$$E(\mathbf{p},w,U)=\sum(p_k+t_kw)\gamma_k+U\prod(p_k+t_kw)^{\beta_k}.$$

From this it may be seen that the condition of weak-separability between leisure and goods is not satisfied [see Deaton (1979, eq. (17))] where $t_k\neq 0$.

4. Estimation of the model

In this section we describe the estimation of the activity model of eqs. (23) and (25), and — for comparison — the standard LES. It should be emphasised that these estimates are preliminary and that they need to be refined in several respects. We assume that each equation contains an additive stochastic term. In view of the budget constraint, for a given observation, the stochastic terms in the expenditure equations must sum to the stochastic term in the earnings equation. It follows that the variance–covariance matrix of the error terms for the full expenditure and earnings system is singular. The procedure we have followed is to delete the earnings equation from the system, i.e. eq. (25), or (22) in the case of the LES. The assumption made concerning the remaining $(n-1)$ equations is that ε_i is normally distributed with variance σ_i^2, and that the covariance $(\varepsilon_i, \varepsilon_j)=0$ for $i \neq j$. These assumptions imply that the errors are uncorrelated across expenditure equations but positively correlated with the error in earnings. They are not necessarily satisfactory, and we may expect the covariance matrix to be nondiagonal where the specification has not adequately captured the extent of complementarity/substitutability. The assumptions about the stochastic specification should therefore be regarded only as a first step.

The data used to estimate the model are drawn from the Family Expenditure Survey (FES) for April–December 1973. The FES is a continuous survey which covers a nationally representative sample of all private households in the United Kingdom and is carried out by the Office of Population Censuses and Surveys on behalf of the Department of Employment. A sample of around 10,400 private households is selected each year and of these some 70% agree to cooperate fully. The main purpose of the survey is to collect information for the annual adjustment of the weights used in the Retail Price Index, but the survey contains a great deal of other data. (For a description, see Kemsley (1969) and Stark (1978).)

Evidence on expenditure is collected partly by interview and partly by records kept by individual members of the household (participants maintain a detailed 'diary' of all expenditure during a 14 day period). The data used are the total household expenditures grouped into nine broad commodity categories: (1) food, (2) alcoholic drink, (3) tobacco, (4) clothing and footwear, (5) durable household goods, (6) other goods, (7) transport and vehicles, (8) services, and (9) 'composite'.[8] The final category is defined as the residual of income after deducting the sum of expenditures on the other eight categories [for details of the construction, see Atkinson and Stern (1980)]. It

[8]The data differ from those used in Atkinson and Stern (1980), in that they include in addition to diary entries expenditure recorded elsewhere in the enquiry (e.g. items purchased on credit).

includes savings and items of expenditure which we have not tried to explain directly (e.g. on housing).

Evidence on income from different sources and hours of work are collected by interview, relating both to the most recent pay period and to 'normal' income and hours. The wage variable used is normal hourly wage of the head of household, calculated by dividing normal earnings per week by normal working hours.[9] Other household income is calculated as the net income of the household excluding the earnings of the head. Thus this net income includes all unearned income, social security benefits and the earnings of other family members. The family decision making process is assumed to be such that the head makes all the expenditure decisions and decides how much to work taking the earnings of other household members as fixed.[10] The FES also contains a great deal of information on demographic and other characteristics which seem likely to affect the pattern of expenditure; these are assumed to influence the 'subsistence' parameters (γ_i).

The FES is a long-established survey, which has been in continuous operation since 1957, and the data appear in general to be of high quality. There are however certain problems which should be noted. First, expenditure on certain items, such as alcoholic drink and tobacco, is thought to be substantially under-reported. The second problem is that there are a number of zero entries. For some expenditure categories, such as tobacco, this may be the correct normal expenditure; in other cases it may reflect either failure to report expenditure or the short period over which diary records are kept. At this stage, we have simply estimated equations for those values where there is strictly positive expenditure.[11] The third problem is that of errors in the recording of the income and wage data. Comparison with other sources, such as the national accounts, suggests under-reporting of investment income, self-employment income (partly because this is obtained on a retrospective basis) and part-time earnings. This is likely to mean that the variable M is particularly subject to measurement error. It should also be noted that it is a measure of current rather than permanent income. The data on earnings are thought to be much more reliable; on the other hand, the procedure used to calculate wage rates introduces further econometric problems.

[9]This is not fully satisfactory for several reasons, including the fact that it does not allow for nonproportional wage schedules. It will be affected by both overtime premia and by unpaid overtime (if included in normal hours). This point has been explored in the work of Brown, Levin and Ulph (1976).

[10]It has been suggested to us that a more reasonable assumption would be the opposite case of secondary workers making their decisions conditional on those of the household head. In future work, we plan to explore this and intermediate cases.

[11]This clearly introduces problems of sample selection bias: see, for example, Hausman and Wise (1977) and Heckman (1979).

The focus of the paper is on households with a male head (aged 18–64) who is employed (i.e. excluding the self-employed) full-time. A household may face a quite complicated budget constraint when account is taken of income tax, National Insurance (NI) contributions, and income-related social security benefits. This aspect has been examined by Burtless and Hausman (1978), Wales and Woodland (1979), Ashworth and Ulph (1977); and we plan to explore it with the aid of the FES data. For the present we decided to concentrate on a range of earnings where the budget constraint is relatively straightforward: the 1,617 households where the man's hourly wage (in 1973) fell in the range of £0.85–£3.00. We hope by this process to exclude most households receiving income-related benefits and most higher rate tax payers.[12] In April 1973, the start of the sample period, the mean gross hourly earnings for adult male workers was £0.94 (New Earnings Survey, 1973, table I). Most households faced a marginal tax rate of either 34.75% (i.e. basic rate of income tax plus NI contributions of 4.75% (up to 30th September 1973), or 35% (basic rate and 5% NI contributions after 1st October 1973), or 30% (basic rate only).[13] For simplicity, we averaged, taking a figure of 32.5%. We used this tax rate to calculate the marginal net wage. The fixed component of income, M, was taken as after tax. We are supposing, therefore, that there is a linear budget constraint which gives a disposable income equal to M, plus 0.675 times gross earnings, plus the cash value of tax allowances. We have simplified at this stage by not calculating the value of the tax allowances for each household, which will thus be reflected in the constant in the expression for full income (see (27) below) and in the coefficients on household characteristics. We have also excluded it from the expenditure on the 'composite' category, p_9x_9, which is then

$$p_9x_9=(1-\tau)wl+M-\sum_1^8 p_ix_i, \tag{26}$$

where w is the gross-of-tax wage, and $\tau=0.325$.

The introduction of the linear income tax system, and of household characteristics, means that the demand system (23) becomes

$$p_ix_i=p_i\gamma_i+\frac{\beta_ip_i}{p_i+w(1-\tau)t_i}\left[M+G^*+w(1-\tau)T''-\sum_j p_j\gamma_j\right] \tag{27}$$

[12]This approach is not without difficulties. It avoids the problems associated with truncation on the dependent variable, but one should, in principle, allow for the fact that the budget constraint outside the range chosen is likely to lead to rather different behaviour. This applies particularly to the problem of nonparticipation. Moreover, it might be more appropriate to treat the wage as an endogenous variable.

[13]The personal tax allowances were such that nearly all families in the subsample would have been liable for tax: for a couple with 4 children (2 aged 11–16) they were some £32 per week. The NI contribution was payable up to £48 a week (£54 after 1 October 1973).

and

$$p_i\gamma_i = g_{0i} + \sum_k g_{ki} Z_k, \tag{28}$$

where G^* denotes the value of the tax allowances, and Z_k is the vector of household characteristics. The latter was taken to be, after some initial experimentation:[14]

OWN: equal to 1 if the household is an owner-occupier, zero otherwise;
NEARN: the number of earners in the household;
NCH: the number of children.

In the cross-section data used, we assume that prices are identical for all households. The equations to be estimated for the expenditure, E_i, on good i may therefore be written

$$E_i = g_{0i} + \sum_k g_{ki} Z_k + h_{1i}\left(\frac{M + \alpha_1 w + \alpha_2}{1 + h_{2i} w}\right). \tag{29}$$

Comparing (29) and (27) we have:

$$\beta_i = h_{1i}, \tag{30a}$$

$$t_i/p_i = h_{2i}/(1-\tau), \tag{30b}$$

$$T'' = \alpha_1/(1-\tau), \tag{30c}$$

$$G^* = \alpha_2 + \sum_j p_j\gamma_j \tag{30d}$$

$$= \alpha_2 + \sum_j \left\{ g_{0j} + \sum_k g_{kj} Z_k \right\}, \tag{30e}$$

where we have used (28). From this we can see that we can identify the parameters necessary to calculate the effects of changes in indirect taxes and income tax on labour supply, commodity demands and welfare. That one can estimate price elasticities where there is no price variation is also a feature of the standard LES model. It should however be emphasised that it reflects in that case the tight specification imposed by the Stone–Geary utility function, whereas our demand system is not the LES and we do have variation in the *total* price of activities from variation in the wage rate.

The estimation of the equations is by maximum likelihood and posed a number of difficulties. In particular the estimation of α_2, and the constraints

[14]Although it may be reasonable to treat *NCH* as exogenous, this is clearly questionable for *OWN* and *NEARN*. The choice of household tenure is discussed in King (1980); the joint determination of participation of household members will be the subject of further work.

implied by eq. (30e), caused problems. Our procedure at this preliminary stage was to omit the constraints and to set $\alpha_2=0$.[15] (The estimation is described in Atkinson and Stern (1980).) For this, and other reasons indicated above, the results should be treated with caution.

Results

The standard LES, with variable labour supply, is given by the special case of (29) where $h_{2i}=0$ for all i. We estimate both this special case (table 1) and the more general form where consumption involves time (table 2). The results suggest to us that the special case should be rejected in favour of the more general specification. First, there is the argument based on the comparison of the log-likelihood values. It is standard [see Berndt et al. (1974)] to argue that twice the difference in log-likelihood values is distributed as chi-square with degrees of freedom equal to the number of restrictions relaxed (here there are 9).[16] The log-likelihood value for the more general model[17] is -25400.07 and for the LES is -25422.80 (see table 1). The chi-square value is therefore 45.6 and this is to be compared with the 1% significance level for a chi-square variate with 9 d.o.f. of 21.7. On this basis, one can confidently reject the null hypothesis that the LES restrictions $h_{2i}=0$ are correct.

The second argument for rejecting the LES results of table 1 is based on the properties of the derived labour supply function. Using the budget constraint $M+0.675wl=\sum_{i=1}^{n-1} p_i x_i$, this may be written as

$$\begin{aligned} 0.675wl = & -M+5.825+1.041(M+24.199w)-0.036OWN \\ & -0.918NEARN+0.446NCH, \end{aligned} \tag{31}$$

where the coefficients on the r.h.s. of (31) are the corresponding column sums in table 1. Note that $\sum_{i=1}^{n-1}\beta_i$ is 1.041, implying that β_n is -0.041 (see eqs. (8) and (22)), so that there is a (small) negative valuation of leisure. This is inconsistent with our assumption that the β_i's are nonnegative (see eq. (15)); if β_n were negative, then the utility maximising decision would be $l=T'$. Note

[15]Since the 'composite' excluded the value of the tax allowances, the assumption that the tax allowance is independent of the characteristics would imply that the omitted constraints are:

$$\sum_j g_{kj}=0, \qquad \text{for all } k,$$

$$\sum_j g_{0j}=0.$$

[16]For a fuller treatment of this argument, see Holly (1978), and for the application to our model see Gomulka and Pemberton (1980).

[17]This is not the model of table 2 since that also incorporates the restriction $\Sigma_1^9\beta_i=1$.

Table 1

Standard linear expenditure system: Maximum likelihood estimates.[a]

Commodity group (expenditure in £/week)	No. of nonzero cases	Constant	h_{1i}	*OWN*	*NEARN*	*NCH*	R^2
1. Food	1,617	4.368	0.085	0.041	1.566	1.559	0.2756
		(0.415)	(0.008)	(0.241)	(0.232)	(0.093)	
2. Alcoholic drink	1,408	0.081	0.038	−0.973	1.152	0.097	0.1546
		(0.306)	(0.006)	(0.189)	(0.173)	(0.075)	
3. Tobacco	1,136	1.442	0.0078	−0.747	0.665	0.080	0.1543
		(0.159)	(0.0034)	(0.101)	(0.094)	(0.040)	
4. Clothing and footwear	1,509	−0.028	0.078	0.022	0.987	0.392	0.0931
		(0.553)	(0.011)	(0.333)	(0.309)	(0.128)	
5. Durable household goods	1,576	1.920	0.052	1.077	−0.166	−0.183	0.0104
		(1.054)	(0.020)	(0.656)	(0.589)	(0.251)	
6. Other goods	1,617	0.950	0.053	0.250	0.141	0.186	0.0756
		(0.340)	(0.007)	(0.204)	(0.191)	(0.079)	
7. Transport and vehicles	1,608	2.021	0.131	1.228	0.099	−0.462	0.0983
		(0.779)	(0.015)	(0.464)	(0.434)	(0.178)	
8. Services	1,617	0.817	0.143	0.277	−1.003	−0.390	0.0622
		(0.906)	(0.017)	(0.538)	(0.494)	(0.206)	
9. Composite	1,617	−5.746	0.453	−1.211	−4.359	−0.833	0.1072
		(2.054)	(0.041)	(1.172)	(1.137)	(0.451)	

[a] $\alpha_1 = 24.199$ (2.394); log-likelihood value $= -25422.797$.

Notes:
(1) Data base: Family Expenditure Survey, 1973.
(2) The variable to be examined in each equation is the expenditure (£/week for the commodity group indicated).
(3) Cases included are those households interviewed after 5 April where the head is in full-time employment at a wage between 0.85p and £3.00 per hour. For each commodity group, cases involving zero expenditure have been omitted. Maximum number of cases is 1,617.
(4) Asymptotic standard errors appear in parentheses.
(5) *NCH* number of children, *NEARN* number of earners in household, *OWN* 1 if owner-occupier, 0 otherwise.
(6) h_{1i} is the marginal propensity (out of full income) to spend on the commodity.
(7) α_1 is $0.675T'$. Recall that the wage here is gross of tax and $T' = T - \gamma_n$, where T is total time (hours/week) available and γ_n is the minimum requirement of leisure.
(8) The log-likelihood value is for comparison with that of the less restricted model where $h_{2i} \neq 0$, i.e. -25400.07.
(9) R^2 is calculated 'as if' the equations were independent.

that there would be little point in imposing the constraint $\beta_n = 0$ for then we should again have the trivial labour supply function $l = T'$.

We also found a negative valuation of leisure when we estimated (29) without the constraint that the h_{2i} are zero; $\sum_1^{n-1} \beta_i$ was in that case estimated to be 1.217, which again violates our assumption that the β_i's are nonnegative. Accordingly we imposed the constraint that $\sum_{i=1}^{n-1} \beta_i = 1$ so that pure leisure has no value. Unlike the standard LES case this does not pose a problem to the derivation of a labour supply curve since other activities use time. Indeed one can argue [see Atkinson and Stern (1980, §2.2)] that pure leisure, using no complementary inputs, is not likely to be an activity of importance. The constraint $\sum_{i=1}^{n-1} \beta_i = 1$ may be tested using the values of the maximised likelihood with and without the constraint. Twice the difference of the logarithm of the likelihoods is 1.89, which is to be compared with a 5% significance level for a chi-square variate with 1 d.o.f. of 3.84 (and a 10% level of 2.71). Thus we should accept the null hypothesis that the constraint is correct at the 5% level (and also at the 10% level).

The results from estimating the system (29) incorporating the constraint $\sum_{i=1}^{n-1} \beta_i = 1$ are presented in table 2. The results are not discussed in detail, but we may note some of the more interesting features. The significant coefficients for the characteristics variables are: *OWN* alcohol, tobacco (negative) and transport and vehicles (positive); *NEARN* food, alcohol, tobacco and clothing (positive) and composite (negative); *NCH* food, clothing, and other goods (positive) and transport and vehicles (negative). The coefficients h_{2i} allow us to estimate the time required in consumption: $t_i = p_i h_{2i}/0.675$. Hence the implied time required to consume a pint of beer costing say 20p (in 1973) is $(0.2 \times 0.517)/0.675 = 0.153$ hours or 9.2 minutes. The coefficient for services is significantly negative, i.e. the consumption of services reduces the time that would otherwise have been taken for certain activities. It may be checked that the total price is positive for services (the commodity for which $-h_{2i}$ is a maximum) for all wage rates in the range considered.

5. The switch from income tax to VAT and the effects on labour supply

Our labour supply equation is derived from[18]

$$0.675wl = \sum_{i=1}^{9} E_i - M, \tag{32}$$

where the E_i are calculated using (29) and the estimates in table 2. The effects of varying the wage, presented in table 3, are for the case where there are no

[18] This is not directly comparable with eq. (25) because we have not imposed the constraints (30e).

Table 2

Nonlinear 'LES' where consumption involves time: Maximum likelihood estimates.[a]

Commodity group (expenditure in £/week)	Constant	h_{1i}	h_{2i}	*OWN*	*NEARN*	*NCH*	R^2
1. Food	4.106	0.105	0.103	0.056	1.402	1.556	0.2763
	(0.605)	(0.021)	(0.108)	(0.241)	(0.276)	(0.093)	
2. Alcoholic drink	−0.279	0.090	0.517	−0.926	0.797	0.091	0.1597
	(0.420)	(0.026)	(0.274)	(0.190)	(0.211)	(0.074)	
3. Tobacco	1.240	0.064	2.120	−0.721	0.458	0.076	0.1625
	(0.192)	(0.048)	(2.158)	(0.101)	(0.113)	(0.040)	
4. Clothing and footwear	−0.234	0.091	0.084	0.039	0.883	0.390	0.0934
	(0.689)	(0.028)	(0.140)	(0.334)	(0.377)	(0.128)	
5. Durable household goods	2.061	0.033	−0.118	1.088	0.091	−0.182	0.0104
	(1.098)	(0.033)	(0.241)	(0.656)	(0.696)	(0.252)	
6. Other goods	0.839	0.058	0.053	0.254	0.102	0.185	0.0733
	(0.432)	(0.016)	(0.124)	(0.205)	(0.234)	(0.079)	
7. Transport and vehicles	2.036	0.111	−0.045	1.224	0.360	−0.460	0.0983
	(0.932)	(0.029)	(0.090)	(0.464)	(0.517)	(0.178)	
8. Services	1.398	0.063	−0.226	0.193	0.105	−0.368	0.0749
	(0.910)	(0.017)	(0.037)	(0.534)	(0.524)	(0.205)	
9. Composite	−5.646	0.385	−0.039	−1.169	−3.425	−0.830	0.1066
	(2.525)	(0.058)	(0.058)	(1.172)	(1.209)	(0.451)	

[a] $\alpha_1 = 25.591$ (4.54); log-likelihood value $= -25401.019$.

Notes: (1) See notes (1)–(5) and (9) in table 1.

(2) h_{1i} is the modified marginal propensity to spend on the commodity concerned. We have imposed the constraint $\Sigma_1^9 h_{1i} = 1$.

(3) h_{2i} is $(1-\tau)t_i/p_i$. Thus $(0.675wt_i)/p_i$ is the time cost (in money terms) divided by the money cost. The average wage for the sample is £1.235 per hour.

(4) α_1 is $0.675T''$. Recall that the wage here is gross of tax and T'' is total time available less that associated with the minimum requirement of each commodity (including leisure).

(5) The log-likelihood value may be compared with that which occurs when the constraint $\Sigma_1^9 h_{1i} = 1$ is not imposed, i.e. -25400.07.

Table 3

Expendíture (£/week) and labour supply with no tax changes.[a]

Gross wage (£/hour)	Food (1)	Alcoholic drink (2)	Tobacco (3)	Clothing and footwear (4)	Durable household goods (5)	Other goods (6)	Transport and vehicles (7)	Services (8)	Composite (9)	wl	Labour supply (hours/week)
1.0	11.86	2.80	2.35	4.97	4.25	3.66	7.62	4.61	4.37	45.46	45.46
1.5	12.84	3.09	2.33	5.87	4.88	4.29	9.25	6.41	9.94	63.88	42.59
2.0	13.75	3.30	2.32	6.71	5.61	4.90	10.97	8.95	15.74	83.65	41.83
2.5	14.57	3.47	2.31	7.49	6.46	5.47	12.77	12.81	21.80	105.76	42.30
3.0	15.34	3.61	2.30	8.22	7.47	6.02	14.67	19.39	28.13	132.40	44.13
1.235	12.33	2.94	2.34	5.40	4.54	3.96	8.38	5.39	6.96	53.99	43.72

[a]*Notes:* (1) Calculations are for characteristics *OWN*, *NEARN*, *NCH* and *M* set to their average levels for the sample: 0.653, 1.409, 1.161, and £15.78 per week, respectively.
(2) The sample was selected so that gross wages were between 0.85p and £3.00 per hour. The average wage in the sample is £1.235.
(3) Expenditures and labour supply are calculated using the results presented in table 2.

tax changes, providing a base line for the subsequent simulations. Lump-sum income and household characteristics are held constant at their average levels for the whole sample and w is varied as we move down the rows. The results are presented graphically in fig. 1. The labour supply first decreases with the wage but then starts to increase. The minimum is around $w = £2.0$ per hour. At the mean of the sample the uncompensated wage elasticity is -0.16, which is within the range one commonly finds reported from studies of the response of hours worked by adult males [see, for example, Ashenfelter and Heckman (1973) and Stern (1976)]; however at higher wage rates labour supply increases with the wage. It is interesting that where nonmonotonicity has been found in U.S. studies, it is usually of the opposite kind — labour supply at first increases with the wage and then decreases at higher wage rates [see Hall (1973)] — whereas the study by Brown, Levin and Ulph (1976) for the U.K. finds a nonmonotonicity of a similar kind to that shown in fig. 1.[19]

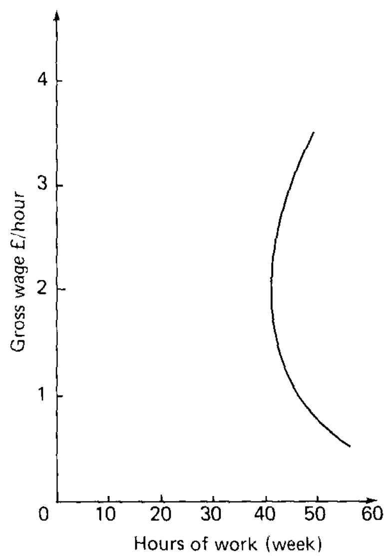

Fig. 1.

The (-shape of the labour supply curve may of course be an artefact of the functional form of the labour supply equation which we have chosen — indeed we warned in §2 that the choice of special forms may impose certain answers. As a partial check whether the danger exists for our case, we plotted labour supply against gross wages for our sample. It must be

[19]It has been brought to our attention by Leif Johansen that this possibility was discussed by Ragnar Frisch (1932): see, for example, his fig. 17.

remembered that such a procedure takes no account of the effects of other variables and cannot, therefore, be directly compared with the labour supply results of table 3. However it does seem fair to say that the form of the nonmonotonicity found in the simulation of table 3 does not produce a relation which is wildly at variance with the simple plot of labour supply against the wage. In addition, the simple regression of labour supply against w and w^2 gives coefficients of -14.47 (2.31) and 3.60 (0.71), where standard errors are in brackets; this gives a minimum at $w=£2.01$ per hour.

In our discussion of the labour supply equation (25) we noted that if t_k is negative for some k (here $h_{2k}<0$) then the household can buy time by purchasing good k and can, for example, increase its labour supply. The higher the wage the more attractive this option becomes. We noted in our discussion of table 2 that the time required for services is negative and this would provide a reason for the (-shape; it may be that higher wage households buy more services thus allowing extra time for work.

5.1. *Reduction in rate of income tax*

We now examine a reduction in the basic rate of income tax, and to make the discussion concrete we consider a reduction from the rate of $32\frac{1}{2}\%$ assumed in the estimation procedure to a rate of 25%. The Chancellor of the Exchequer has himself talked of a 25% basic rate as an eventual target[20] so that a reduction of this magnitude has some political interest. The effects of changes in VAT are discussed after those of the income tax and we then examine revenue considerations.

The income tax change would affect not only the marginal tax rate but also lump-sum income. The latter effect would arise both because of the change in the value of the allowances (the intercept of the tax function) and because of the effect on the tax paid on the income of other members of the household. The combined impact on lump-sum income is quite complex, and will vary from household to household. In the simulations we represent the effect by assuming that M is transformed to $mM+\mu$. The effect of the marginal tax rate change may be seen as multiplying w by $0.75/0.675=1.11$. It is assumed that the gross wage rates, and gross other income, are unaffected by the tax change.

The consequences of these changes for hours of work, as predicted by the estimates of table 2, are shown in table 4 for different values of the gross wage and for different combinations of m and μ. The predicted labour supply is obtained from (32), where w and M are transformed as described and the E_i are calculated as functions of w and M (both transformed) using the estimates in table 2. The largest increase in lump-sum income is that shown

[20]Although with the National Insurance contribution the rate would be higher.

in column 2 of table 4 where $\mu=0$ and $m=1.11$, i.e. the lump-sum income goes up by the same proportion as the marginal net wage (this may be regarded as an extreme case). On the other hand, it is possible that net income from other sources is unaffected by the change in the marginal rate (e.g. where the wife's earnings are below the threshold or where there are tax free social security benefits), but that μ is negative, reflecting the reduced value of tax allowances. This is illustrated by column 3 in table 4, where the value of μ is calculated as follows. The annual single person's allowance is

Table 4

Labour supply with income tax reduced to 25%.[a]

Gross wage/ hour	Before change (1)	Hours/week with income tax rate at 25% $m=1.11$ $\mu=0$ (2)	$m=1.0$ $\mu=-1.69$ (3)	$m=1.11$ $\mu=-1.69$ (4)
0.90	46.54	45.33	45.59	45.46
1.00	45.46	44.41	44.64	44.52
1.50	42.59	42.11	42.23	42.17
2.00	41.83	41.88	41.92	41.90
2.50	42.30	43.14	43.09	43.11
3.00	44.13	46.75	46.56	46.65
1.235 (mean)	43.72	42.96	43.14	43.05

[a]*Notes:* See notes to table 3.

£595. To this we added: 0.85 times the difference (£180) between the married and single person's allowance (assuming 85% of the heads of households were married); the average number of children (1.16) times the child allowance (£200); and 0.65 (65% of households in the sample are owner-occupiers) times a mortgage interest payment of £300 p.a. The total annual allowance comes to £1,175. The reduction in the marginal rate by $7\frac{1}{2}$ percentage points reduces the cash value of the tax allowance by £0.075 $\times$ 1,175 p.a., or £1.69 per week. Finally, column 4 shows the combined effect of μ and m.

The differences across columns 2, 3 and 4 in table 4 are small. For example at the average wage the largest difference amongst these columns (between 2 and 3) is only 10.8 minutes per week or 0.4 of 1% of hours per week. The difference in lump-sum income between columns 2 and 3 is 0.11 (15.78)+1.69 or £3.43 per week, at the average M in the sample of £15.78 per week; thus a percentage change in M of 22% yields a change in hours worked of only 0.4 of 1%. Hence the effects of changes in lump-sum income in this model are very small. Amongst columns 2–4 lump-sum income is highest in column 2 and lowest in column 3. We see, therefore, that the effect of increases in M is to reduce labour supply for gross wages up to £2.0 per hour but, for levels of the wage of £2.50 and above, increases in M increase labour supply. The reason the direction of the effect varies with the wage may be seen by examining the partial derivative of (32) with respect to M:

$$\frac{\partial}{\partial M}(0.675wl) = \sum_{i=1}^{9} \frac{h_{1i}}{(1+h_{2i}w)} - 1. \tag{33}$$

For w of £2.50 or £3.00 the right hand side of (33) is positive.

The effects of the 11% difference in the real wage between column 1 and the remaining columns are more substantial. Taking the difference between columns 1 and 4 at the average wage, we find that the reduction in income tax implies a reduction in labour supply of 40.2 minutes per week or 1.5%. On the other hand for wages above £2.0 per hour the effect of the reduction in income tax is to increase labour supply; the different direction is a reflection of the (-shape of the labour supply curve illustrated in fig. 1.

5.2. *Increase in rate of VAT*

If the effect of an increase in VAT is to raise the price of good i by a fraction τ_i, then this implies for our model (see eqs. (27) and (29)) the following:

Before VAT change *After VAT change*

$$g_{0i} + \sum_k g_{ki} Z_k \quad \rightarrow \quad (1+\tau_i)\Big[g_{0i} + \sum g_{ki} Z_k\Big], \tag{34}$$

$$h_{2i} \quad \rightarrow \quad h_{2i}/(1+\tau_i), \tag{35}$$

$$\alpha_2 \quad \rightarrow \quad \alpha_2 - \sum_i \tau_i \Big(g_{0i} + \sum_k g_{ki} Z_k\Big). \tag{36}$$

These price increases affect expenditure on each of the categories and hours worked (see (32)).

The pattern of relative price changes depends on the base of the VAT, on the rate structure, and on the responses of producers. We take as our starting point the 1973 VAT rates (0 and 10%), with the pattern broadly current at that date, and assume that any increase is fully reflected in consumer prices. The resulting price increase (τ_i) assumed to be associated with a ten percentage point increase in the VAT rate (i.e. from 10% to 20%) is shown in the first line of table 5. The changes in expenditure and labour supply, predicted using the estimated equations, after the ten percentage point increase in VAT, are illustrated in table 5. If we consider the household

Table 5

Changes in expenditure (£/week) and labour supply with a ten percentage point increase in VAT.[a]

Gross wage £/hour	Food	Alcoholic drink	Tobacco	Clothing and footwear	Durable household goods	Other goods	Transport and vehicles	Services	Composite	wl	Labour supply hours/week	Labour supply change
Percentage price increase (with 10 percentage point increase in VAT)												
	1.2	9.0	9.0	8.3	8.1	5.7	3.3	5.2	0.8	-		
Percentage change in expenditure												
1.0	0.1	1.8	7.2	1.6	4.2	1.2	0	−0.9	−9.2	45.63	45.63	+0.17
1.5	0.2	2.9	7.7	1.7	3.3	1.2	−0.1	−2.0	−4.1	63.97	42.65	+0.06
2.0	0.2	3.9	7.8	1.9	1.4	1.2	−0.2	−3.6	−2.6	83.48	41.74	−0.09
2.5	0.3	4.6	8.2	2.0	1.1	1.3	−0.2	−5.7	−2.0	104.95	41.98	−0.32
3.0	0.3	4.7	8.3	2.2	−0.1	1.3	−0.4	−9.2	−1.6	129.96	43.32	−0.81
1.235 (mean)	0.3	2.7	7.3	1.7	3.5	1.0	−0.1	−1.5	−5.7	54.14	43.84	+0.12

[a]*Notes:* (1) see notes to table 3.
(2) the assumed pattern of price increases is based broadly on calculations in the *National Institute Economic Review*, May 1972, table 10.

with the average wage, for the first six categories we have a rise in expenditure and for the last three there is a fall. The largest rise in expenditure is for tobacco where the increase is almost as large as the price increase (so that the fall in quantity is very small) and the biggest fall is for the composite. The fall in the real wage (approximately 5%) associated with the price increases goes with a reduction in the quantity of services of almost 7%. The total expenditure ($0.675wl+M$) is almost constant (rising 9p from £52.22 before the change). The pattern of expenditure responses varies with the wage — for example, at a wage of £3 per hour there is virtually no change in expenditure on durables, implying a fall of 8.1% in quantity, and a fall in expenditure on services of 9.2% implying a change in quantity of almost 15%. In contrast, at a wage of £1 per hour, while there is little change in expenditure on services, and there is a 4% increase on durables; there are at the same time bigger reductions in quantities for alcohol and for the composite.

The effect of different levels of VAT on labour supply are illustrated in table 6, which shows the response to overall rates of 10% (i.e. before change), 15%, 20% and 25%.

Table 6

Labour supply and increases in VAT.[a]

Gross wage £/hour	Before change (1)	Hours/week VAT increased to 15% (2)	20% (3)	25% (4)
0.90	46.54	46.64	46.73	46.81
1.00	45.46	45.55	45.63	45.70
1.50	42.59	42.62	42.65	42.68
2.00	41.83	41.78	41.74	41.71
2.50	42.30	42.13	41.98	41.85
3.00	44.13	43.69	43.32	43.00
1.235 (mean)	43.72	43.78	43.84	43.89

[a]*Note:* See notes to table 3.

For the household with the average wage the increase in prices produces a small increase in labour supply (10 minutes per week or 0.4 of 1% of hours worked for a VAT-rate increase from 10% to 25%). At a gross wage of £3.00 per hour the VAT increase reduces labour supply (by 1 hour 8 minutes a week for a VAT rate increase to 25%).

The changes in income tax and in VAT have been analysed separately. The effect of making a reduction in income tax simultaneously with an increase in VAT may be judged by combining the two effects described above but one should remember that, strictly speaking, the effects are not additive. The magnitude of the increase in VAT required to finance a one percentage point reduction in the standard rate of income tax was, for 1973, around two percentage points [see, for example, Fiegehen and Lansley (1972)]. (In more recent years the base of VAT has been broadened.) Thus one may associate the cut from 32.5% to 25% in the marginal rate considered here for 1973 with an increase in the VAT rate from 10% to 25%. This would not, however, provide revenue balance for our sample; indeed one can expect a loss in revenue. For example, many households outside our sample where the head has wages below £0.85 per hour would benefit only slightly from reductions in the standard rate of tax, since they pay little tax, but bear the burden of increases in VAT.

6. The welfare implications

In Section 2 we discussed the optimum design of the tax structure subject to an overall revenue constraint. Here we consider the welfare implications of a cut in the marginal rate of income tax to 25%, with $m=1.11$ and $\mu=-£1.69$, and of different increases in the rate of VAT. Our sample is not intended to be representative of the population; the exercise is therefore a limited one. It may however serve to illustrate some features of the analysis.

The level of utility for an individual household which corresponds to a set of tax rates can be found by substituting the quantities purchased, as given by the demand function (29) (using the fact that $E_i=p_i c_i$) so forming the indirect utility function. This gives

$$v=\sum_{j=1}^{9}\beta_j\log\beta_j-\sum_{j=1}^{9}\beta_j\log(1+h_{2i}w)+\sum_{j=1}^{9}\beta_j\log(M+\alpha_1 w+\alpha_2)-\sum_{j=1}^{9}\beta_j\log p_j, \tag{37}$$

where we have used (28) and (30a). Recall that no utility is derived from the nth (here 10th) activity, pure leisure, since we have constrained $\sum_{j=1}^{9}\beta_j$ to be 1. The particular cardinal form of the utility function we have used will make no difference to our money measure of the change in welfare (equivalent variation — see below) for an individual but will make a difference to measures of the distribution of welfare in the population.

Using (37) we can compare, for a given individual, levels of utility before and after the tax change. The results of such a comparison might not be easily understood if expressed as a certain number of 'utils' and accordingly we use a money measure of changes in 'utils' defined as follows. We ask what lump sum (with no changes in price) we should have to give to the individual in order to produce the same change in welfare as that generated by the tax change. In other words we work with the equivalent variation, as has been used recently by Rosen (1978) who considered the dead-weight burden of taxation for the LES, extended to include labour supply, as estimated by Abbott and Ashenfelter (1976). We are interested in the distribution of welfare in our sample and we therefore present results for different wages and lump-sum incomes. It would, however, be unwieldy to present results for all 1,617 households in the sample. Therefore for specified ranges of wage rates we average the wage rate and lump-sum income to provide a 'representative individual' for that interval.[21] We then calculate the changes in welfare levels for the 16 representative individuals thereby produced. Characteristics are set at their mean level for the entire sample.

The implications of the switch from direct to indirect taxation are given in table 7, which shows the equivalent variation in two situations: A and B. A represents the situation where only the income tax is changed, B that where both the income tax is reduced and VAT is increased (with different rates of increase). At the average wage there is a welfare gain for VAT increases below 15 percentage points and a loss for increases above this magnitude. We suggested in the preceding section that for 1973 a 1 percentage point reduction in the income tax might be approximately financed, for the economy as a whole, by a 2 percentage point increase in VAT. Hence 15 percentage points may represent, approximately, constant revenue for the population (although for our sample it is likely there would be a loss in revenue). Those with high wages (above £2 per hour) gain for VAT increases up to 20 percentage points but those with the lowest wages are made worse off for VAT increases above 11 percentage points. The switch discriminates in favour of those with higher wages.

7. Concluding comments

We have argued in §2 that the specification of functional forms is crucial to the design of tax systems and the estimation of labour supply responses. Commonly used functional forms, in particular the linear expenditure system, impose strong restrictions on optimum tax schedules and labour supply responses. We provided in §3 an extension of the linear expenditure system to allow for the time used in different activities, and the estimation of this extended system for a subsample of the population was described in §4. The model and estimates were used to examine the effects of changes in income tax and VAT on labour supply in §5 and welfare in §6. We found that the

[21]The intervals are (lower bound in each case) 0.85–, 0.90–, 0.95–, 1.00–, 1.05–, 1.10–, 1.15–, 1.20–, 1.30–, 1.40–, 1.50–, 1.60–, 1.80–, 2.00–, 2.20–, 2.60–.

Table 7

Welfare effects of tax changes.[a]

Lower bound of interval for w	w (£/hour)	M (£/week)	Number of cases	A	B VAT increase of: 10%	15%	20%
0.85	0.876	13.760	192	2.124	0.204	−0.703	−1.577
0.90	0.924	13.680	168	2.245	0.289	−0.635	−1.526
0.95	0.975	15.698	175	2.594	0.528	−0.448	−1.389
1.00	1.024	14.974	158	2.651	0.570	−0.414	−1.362
1.05	1.074	14.948	132	2.784	0.662	−0.340	−1.306
1.10	1.122	15.873	85	3.011	0.819	−0.216	−1.214
1.15	1.177	14.388	86	3.005	0.818	−0.215	−1.210
1.20	1.246	16.668	152	3.438	1.115	0.019	−1.038
1.30	1.347	16.922	99	3.747	1.330	0.190	−0.910
1.40	1.441	18.040	79	4.133	1.597	0.400	−0.753
1.50	1.553	16.651	60	4.306	1.719	0.499	−0.676
1.60	1.681	16.911	72	4.712	1.999	0.720	−0.511
1.80	1.884	19.497	52	5.609	2.612	1.203	−0.153
2.00	2.092	17.315	31	6.040	2.903	1.431	0.016
2.20	2.369	19.459	53	7.239	3.703	2.050	0.466
2.60	2.792	20.326	23	9.057	4.859	2.920	1.073
Mean for sample	1.235	15.778	1617	3.314	1.030	−0.047	−1.087

[a] *Notes:* (1) w and M are the averages for the interval of w with lower bound given by the gross wage rate given in the column and upper bound given by the column below (the upper bound for the last case is £3.00).
(2) Characteristics have been set at their mean level for the sample.
(3) Column A shows the effects of an income tax cut of $7\frac{1}{2}$ percentage points with no change in VAT. Column B shows the effects of a 10, 15, 20 percentage-point increase in the basic rate of VAT together with the income tax cut.

model predicted for the subsample a labour supply response that was first decreasing with the wage, then increasing. For the average person in our sample, the effect of a reduction in the marginal rate of income tax from 32.5% to 25% would be to reduce weekly labour supply by 40 minutes. There would be an off-setting effect from a 15 percentage point increase in VAT of 10 minutes. There would be a net increase in labour supply by those with the highest wage rates, with the income tax cut increasing hours and the VAT change reducing them. Our analysis of welfare changes showed that, as predicted in less complex models, the benefits of a switch from income tax to VAT flow to those with higher wages.

Finally, we should emphasize that these empirical results are only preliminary, and are subject to a number of major qualifications. We have drawn attention to (among other aspects) the shortcomings of the basic data — including the measurement of wages, the special assumptions about the stochastic specification, the treatment of zero expenditures, and the problems encountered in estimation. The findings are valid only for the subsample of the population considered. There are many important questions which we have not considered, such as the effect of tax changes on the decisions of secondary workers and on effort. We hope to examine some of these limitations in subsequent work.

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