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OPTIMALLY UNIFORM COMMODITY TAXES, TASTE DIFFERENCES AND LUMP-SUM GRANTS

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Optimal commodity taxes for an economy with many households should be at a uniform proportional rate under certain conditions. These include (i) linear and parallel Engel curves with intercepts dependent on household composition, (ii) separability between goods and labour, and (iii) optimal demogrants.

1. Introduction

Atkinson and Stiglitz (1976) showed that with an optimal non-linear income tax, separability of goods from leisure implies the optimality of uniform proportional commodity taxes, while Deaton (1979) demonstrated that an optimal *linear* income tax is sufficient for the result provided the subutility function for goods is such as to guarantee linear Engel curves. Proofs of both of these results assumed that consumers have endowments only of time, have identical tastes, and differ only in their wage rates. Deaton's (1979) analysis apparently allows the intercepts of the Engel curves to vary over households, but his proof does not hold without further restriction on taste variation (see below).

In this paper, we work with the model used by Deaton (1979), in which goods are separable from leisure and there are linear Engel curves for goods in terms of total commodity expenditure, but we allow for taste variation by making the Engel curve intercepts functions of observable and unobservable household characteristics. We also allow the government to levy (or pay) a lump-sum tax (or grant) that varies with observable household characteristics. Formulae are derived that define the optimal structure of this scheme and we show that if the unobservable characteristics are of no interest to policy makers (in a sense defined below), then an optimal structure of the benefit or demogrant scheme implies the optimality of uniform commodity taxation.

Empirical work can tell us much about the effects of household structure on demand and many developed and developing countries operate lump-sum transfer schemes linked to household size and composition (e.g., child benefits and food rations). The results, therefore, allow the theory to be more closely linked to existing knowledge of demand and to practical policy. They also suggest a number of directions for applied research.

2. Analysis

We start with the specification of consumer preferences. There are $(n+1)$ goods, leisure q_0 and a vector of n commodities q, and the direct utility function for individual (household) h takes the form

$$u^h = v^h(q_0, q) = f^h\{q_0, \xi^h(q)\}, \tag{1}$$

so that goods q are separable from leisure q_0. If u_1^h is the maximum utility obtainable on goods, i.e., the maximum of $\xi^h(q)$, when goods prices are p and $p \cdot q = x^h$, then goods' Engel curves will be linear if u_1^h has the form

$$u_1^h = F^h\{(x^h - a^h(p))/b(p)\}, \tag{2}$$

where $F^h\{\ \}$ is monotone increasing. The functions $a^h(p)$ and $b(p)$ are linearly homogeneous in the vector of goods' prices p, and $b(p)$ is *not* indexed on h. This implies that the Engel curves for any commodity are parallel for different households, differing only in intercepts. Such a formulation is consistent with cross-sectional evidence of *non*-linear relationships between households linking demands to total expenditures, since nothing prevents a non-linear relationship between $a^h(p)$ and x^h. For $a^h(p)$ itself, we adopt a linear specification,

$$a^h(p) = a(p) + \sum_{j=1}^{J} \alpha^j(p) z^{jh} + \epsilon^h(p), \tag{3}$$

where z^{jh} is a measure of the jth *observable* characteristic for household h, $j = 1, \ldots, J$, for example, one may think of z^{jh} as the number of children of age j in household h, and $\epsilon^h(p)$ is the 'residual' or idiosyncratic taste variation. The functions $a(\ )$, $\alpha^j(\ )$ and $\epsilon^h(\ )$ are homogeneous degree one.

Taking (2) and (3) together, the sub-demand functions for goods are given by

$$q_i^h = \beta_i(p)x^h + \{a_i(p) - \beta_i(p)a(p)\} + \sum_{j=1}^{J} \{\alpha_i^j(p) - \beta_i(p)\alpha^j(p)\} z^{jh} + \{\epsilon_i^h(p) - \beta_i(p)\epsilon^h(p)\}, \tag{4}$$

where $\beta_i(p) = b_i(p)/b(p)$ is the marginal propensity to consume good i and $a_i(p)$, $b_i(p)$, $\alpha_i^j(p)$ and $\epsilon_i^h(p)$ are the ith partial derivatives of $a(p)$, $b(p)$, and $\alpha^j(p)$ and $\epsilon^h(p)$ respectively. One may think of (4) as an expenditure system with linear Engel curves where the marginal propensities may depend on p but are invariant across households and the minimum requirements (a_i^h) depend not only on prices but also on family structure and a term that is random across households.

The government's optimization problem is to maximize social welfare W given by

$$W = W(u^1, u^2, \ldots, u^h, \ldots, u^H) \tag{5}$$

over H households, subject to the government budget constraint and individual preferences. The government has a tax t_i on the consumption of good i and pays a (possibly negative) amount (demogrant) g^h to each household, where g^h is a linear function of the household characteristics vector z^h, i.e.,

$$g^h = \gamma_0 + \sum_{j=1}^{J} \gamma_j z^{jh}. \tag{6}$$

The instruments to be set are the n taxes and the $(J+1)$ γ's. Given (6), individual utilities are constrained by

$$c^h(u^h,\ p_0^h,\ p) = M^h + p_0^h T^h + \gamma_0 + \sum_{j=1}^{J} \gamma_j z^{jh}, \tag{7}$$

where $c^h(\ )$ is the cost function corresponding to the direct utility function (1), and T^h, the time endowment and M^h is exogenous lump-sum income. The government budget constraint is

$$\sum_h \sum_k t_k c_k^h(u^h,\ p_0^h,\ p) = R + \sum_h \left\{ \gamma_0 + \sum_{j=1}^{J} \gamma_j z^{jh} \right\}, \tag{8}$$

where $c_k^h(u^h,\ p_0^h,\ p)$ is the kth partial derivative of the cost function, and R is necessary revenue.

The maximization of (5) subject to (7) and (8) is the standard many-person optimal tax problem with only the γ's non-standard. As usual for such models, it is useful to define θ^h, the social marginal utility of money to h [the welfare weight à la Meade $= (\partial W/\partial u^h)/(\partial c^h/\partial u^h)$], and the quantity λ^h,

$$\lambda^h = \theta^h + \bar{\theta} r^h/(1-\bar{r}), \tag{9}$$

where r^h is the marginal propensity of h to spend on commodity taxes, and overbars denote (arithmetic) means across households. One can think of λ^h as the welfare weight adjusted for the marginal propensity to pay taxes and λ^h is readily shown to be the Lagrange multiplier associated with the preference constraint (7) given that γ_0 is optimally set [the optimality of γ_0 implies that the Lagrange multiplier on (8) is $\bar{\theta}/(1-\bar{r})$] i.e., $\bar{\lambda}$. The optimisation with respect to the demogrant parameters γ_j gives, for all j,

$$\sum_h \lambda^h z^{jh} = \bar{\lambda} \sum_h z^{jh}, \tag{10}$$

so that demogrants are set so as to exploit any correlation between observable characteristics and the (adjusted) social value of money given to h. Optimality with respect to the taxes t_i gives the familiar condition,

$$\sum_{k=1}^{n} \bar{s}_{ik} t_k = -(\bar{q}_i - q_i^*), \tag{11}$$

where $\bar{s}_{ik}$ is the mean of individual Slutsky terms and the star denotes a weighted average using the $\lambda^h/H\bar{\lambda}$ as weights, i.e.,

$$q_i^* = \sum_h \frac{\lambda^h}{H\bar{\lambda}} q_i^h. \tag{12}$$

In Deaton (1979), conditions are explored under which (11) permits a uniform tax solution, i.e., $t_k = \tau p_k$. Substituting into the left-hand side of (11) (and using homogeneity of the compensated demand derivatives), the separability between goods and leisure which we have assumed implies that the individual (and hence aggregate) Slutsky terms S_{i0}^h are equal to the common income effect $\beta_i(p)$ in the Engel curves scaled by a factor that does not depend on i. Thus the left-hand side of (11) becomes $\tau\beta_i(p)$ times a factor of proportionality. Hence, if the right-hand side of (11) is also proportional to $\beta_i(p)$, i.e., equal to $\beta_i(p)(\bar{x} - x^*)$, then a uniform tax solution is possible. In general, taste differences will imply that $(\bar{q}_i - q_i^*)$ is not proportional to $\beta_i(p)$, so that Deaton's original result, contrary to the statement he gives, requires identical tastes [see in particular Deaton (1979, eq. (16))]. In the present case however, use (12) and the Engel curves (4) to give

$$(\bar{q}_i - q_i^*) = \beta_i(p)\{\bar{x} - x^*\} + \sum_{j=1}^{J} \{\alpha_i^j(p) - \beta_i(p)\alpha^j(p)\}(\bar{z}^j - z^{*j})$$

$$+ \{(\bar{\epsilon}_i(p) - \epsilon_i^*(p)) - \beta_i(p)(\bar{\epsilon}(p) - \epsilon^*(p))\}. \qquad (13)$$

The demogrant optimality conditions (10) guarantee that the second term on the right-hand side of (13) is zero, so that uniform taxation will be optimal provided the last term, i.e., that involving idiosyncratic taste differences, is zero. This will be so, if the covariance of the derivatives of $\epsilon^h(p)$ with the weights λ^h is zero. This may or may not be the case. Referring back to (9) for the definition of λ^h, it is clear that λ^h varies with h only in so far as θ^h, the social marginal utility of money, varies, since the parallel linear Engel curve assumption guarantees that r^h is independent of h. Hence, if the pattern of idiosyncratic commodity demands over individuals is independent of social marginal utilities of money, and given the other assumptions, optimal commodity taxes should be uniform. This will be the case if society sees no reason to discriminate among equally well-off individuals according to the idiosyncratic components of their tastes. Exceptions would occur when the demand for a commodity reveals a characteristic that is not directly observable but would be taxed if it were. Goods are thereby taxed (subsidized) not so much to discourage (encourage) their consumption but because the people who consume them are thought to be particularly undeserving (deserving) and cannot be reached in any more direct way. For example, a relentlessly conformist government, though unable to tax the unorthodox directly because they will not openly reveal their predilections, can attempt to get at them by taxing implements or aids used in their activities. Though such examples can certainly be constructed, it seems to us that the indexation of the demogrant on observable characteristics will, given the other assumptions, normally be sufficient to guarantee the uniformity result.

Given a set of value judgements it should be relatively straightforward in practice to check the optimality of the demogrant. The results suggest that in applied work on policy it may be misleading to examine or propose indirect taxes in isolation from transfers to households.

References

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