Chapter 14

THE THEORY OF COST–BENEFIT ANALYSIS

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1. Basic principles

1.1. Introduction

Cost–benefit analysis is very widely used and it is therefore important that its methods be properly understood. In this chapter we try to contribute to the understanding by giving a formal description of the subject and examining the theoretical basis for some of the techniques which have become accepted tools of decision-making around the world.

The purpose of cost–benefit analysis is to provide a consistent procedure for evaluating decisions in terms of their consequences. This might appear as an obvious and sensible way to proceed, but it is by no means the only one (examples of alternative procedures are majority voting, collective bargaining, the exercise of power, or the assertion of rights). So described, cost–benefit analysis clearly embraces an enormous field. To keep our subject-matter manageable, we confine most of our attention in this chapter to its best-known and most important application: the evaluation of public sector projects. Nevertheless, the

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theory we develop also offers clear guidelines for the evaluation of government decisions in such varied fields as tax, trade or incomes policies, the provision of public goods, the distribution of rationed commodities, or the licensing of private investment.

We shall concentrate on theory. Furthermore, we shall not attempt to present a survey or summary of the vast literature on the theoretical aspects of the subject. Rather, we shall put forward a view of how cost–benefit analysis should proceed, give a fairly unified account of the most salient results of the theoretical literature, show how the framework encompasses a number of approaches to the definition and formulation of cost–benefit problems, and then discuss implications for a number of practical issues.

Accordingly, the contents of the paper are as follows. In Section 1 we introduce the basic concepts of cost–benefit analysis for project evaluation. In particular we show how and when shadow prices can be used to construct cost–benefit tests which evaluate projects in terms of their net effect on social welfare. For this to be the case, the shadow price of a commodity must be defined as the total impact on social welfare of a unit increase in the net supply of that commodity from the public sector. In order for this definition to be operational, it must be possible to predict all the repercussions of a project. We shall embody this idea in the notion of a "policy" and emphasise the close relationship between shadow prices and the choice of policies. We attempt, in Section 2, to draw together some important results selected from the theoretical literature, by analysing a single model and following the principles outlined in Section 1. In Section 3 we review a number of more practical issues at the centre of the literature on applied project evaluation (the treatment of traded and non-traded goods; the discount rate; the shadow wage, and so on) in the light of the previous results. Section 4 contains concluding comments.

1.2. Project evaluation, cost–benefit analysis, and shadow prices

In this subsection we introduce some basic concepts which will be used throughout. They are given formal structure, discussed and developed in the rest of the paper.

By a *project*, we shall understand a change in the net supplies of commodities from the public sector. The term "public sector" is interpreted here in a somewhat restricted sense, which will be clarified below; however, the theory we develop also provides precise guidelines for the evaluation of projects in the private sector. The analysis will be conducted from the point of view of a *planner*, who has to assess projects and who has preferences over states of the economy, embodied in a well-defined objective function or "social welfare" function. The interpretation, specification and necessity of the objective function will be discussed in detail below.

The process of judging whether or not a project should be accepted is called *project evaluation*. *Cost–benefit analysis* is the examination of a decision in terms of its consequences or costs and benefits. The *shadow price* of a good measures the net impact on social welfare of a unit increase in the supply of that good by the public sector.

In the context of project evaluation a *cost–benefit test* is a simple decision rule which consists of accepting only those projects which make a positive profit at shadow prices. As we show below, our definition of shadow prices ensures that shadow profits are precisely a (first-order) measure of the net effect of a project on social welfare, so that cost–benefit tests succeed in identifying welfare-improving projects.

In order to evaluate a project from the point of view of its consequences, it is crucial to have a model which predicts the total effect on the state of the economy of undertaking a particular project. This total effect involves a comparison of the economy "with" the project and the economy "without" it. Formally, we embody the relation between a project and its consequences in the notion of a "policy", i.e. a rule which associates a state of the economy with each public production plan. It is a recurring theme of this chapter that different policies correspond to different rules for shadow pricing. To the extent that several policies are genuinely available, we argue that the policy and project should be selected with respect to the same criterion, the level of social welfare. We also examine closely the special case where there is no real choice and only one policy is available to the planner.

The two basic ingredients of the approach to cost–benefit analysis which is adopted in this chapter are therefore the ability to predict consequences (a model) and the willingness to evaluate them (an objective function).

A major purpose of using the techniques of cost–benefit analysis, and particularly shadow prices, is to allow decisions at the level of the enterprise in the public sector. In principle one could imagine a planner who is endowed with information on the working of the entire economy and well informed about possible projects, who could calculate the level of social welfare associated with any possible course of action. Formally this is how most optimising models appear. We know, however, that it is generally impossible for one office or bureau to be fully acquainted with possibilities and difficulties at each enterprise and household. Thus, we seek to leave many decisions at a level closer to the individual enterprise but to provide information by which individual decisions may be co-ordinated. With this information each enterprise can take decisions whilst exploiting its own detailed knowledge of its own circumstances. Thus, our approach does not assume full knowledge of production possibilities but is simply concerned with providing information to public enterprises for the appraisal of their own projects.

1.3. The basic theory of shadow prices

We develop in this subsection a model which formalises the concepts introduced above. It will be given further structure in Sections 2 and 3.

Our economy consists of "private agents" and a "planner". The theory as such allows the identity of the planner to be interpreted in various ways, e.g. at one extreme the planner could represent a powerful government agency controlling many policy instruments, and at another it could designate an analyst solely concerned with the evaluation of a single project. For purposes of interpretation, we shall also speak of the planner as "the government", bearing in mind, however, that when governments are not "monolithic", all the agencies not under the control of the planner should formally be included in the set of private agents.

Private agents behave systematically, in response to a vector $s = (s_k)$ of *signals* summarising all the relevant variables affecting their behaviour (prices, taxes, quantity constraints, etc.). Thus, given the vector s, called the *environment*, one knows exactly how a private agent will respond, e.g. his net demands or supplies and his level of utility or profit. In particular the vector E of aggregate net demands for commodities from private agents is assumed to be a well-defined function of s. This is not restrictive provided the vector of signals is defined comprehensively (e.g. it could include scale factors for constant returns to scale industries – see Section 2.3.5). Commodities are indexed by i, taking into account, if necessary, the time of their delivery and the state of the world. Problems raised by time and uncertainty will be discussed separately in Section 3. The net supply of the public sector, or *public production plan*, is represented by the vector z with components z_i (where $z_i = 0$ if the ith commodity is neither used nor produced in the public sector). The *public sector* is identified with the set of firms directly under the control of the planner; in particular the planner should have full control over both the production plan and profits of these firms – the notion is further discussed in Section 2.3.1.

Two types of constraints restrict the set of environments which may realistically be considered by the planner as feasible. The *scarcity constraints* require the matching of net supplies and demands. In addition, *side constraints* describe any further limitations on the selection of s by the planner – e.g. permissible tax rates may be restricted, or quotas which he cannot influence may apply. Formally, these constraints are respectively written as

$$E(s) - z = 0 \tag{1.1}$$

and

$$s \in S, \tag{1.2}$$

where $E(s)$ is the vector of net demands from the private sector, and S is the *opportunity set* of the planner.

We write (1.1) with strict equality since otherwise the use of some of the net public supply would not be described. To write it as an inequality constraint would involve the unnecessary assumption that free disposal is possible. Free disposal is an aspect of production possibilities and it may or may not be a property of the public sector production set (denoted by Z). We shall not always assume that the public production set is known to the planner (indeed, as we have argued above, the use of cost–benefit techniques may aim partly at avoiding the difficulties associated with centralising such knowledge). Rather, we shall consider an arbitrary initial value of z, and represent a *small project* as an infinitesimal perturbation $\mathrm{d}z$ of this public production plan. A small project is feasible if $(z + \mathrm{d}z) \in Z$. We shall not be concerned with assessing the feasibility of projects, but rather with appraising the desirability of a priori specified, and presumably feasible, projects. The concentration on small projects motivates our use of differential techniques; all the functions appearing in this paper are assumed to be once continuously differentiable [the reader who wishes to pursue non-differentiability, corner solutions and the like should see Guesnerie (1979)].

The normative element of the model consists of an *objective function* which reflects the planner's preferences between different environments:

$$V: s \to V(s) \tag{1.3}$$

(recall that the behaviour of private agents can always be inferred from s). As before we also speak of $V(\cdot)$ as the *social welfare function*. Together (1.1), (1.2) and (1.3) constitute the *model of the planner*.

For future reference we should point out what is involved in writing the constraints in the above manner. First, note that writing demands and preferences as functions of s in (1.1) and (1.3) (rather than of s and z) involves no loss of generality since s is defined to include all the variables relevant to the behaviour of private agents. Secondly, it is not restrictive in (1.2) to regard S as independent of z, since one can always substitute for z using (1.1). Thirdly, we could not use this last procedure where there is a constraint linking s with the production plan of individual firms in the public sector. Examples where this might arise are externalities from a specific public firm to consumers or private firms, or a firm-specific budget constraint (of the Boiteux type) applying to a public firm. At this stage such problems are precluded by the notion of full control (by the planner) over the public sector; however, they will be explicitly examined in Section 2.

By a (feasible) *policy*, we shall understand a function, denoted by $\phi(\cdot)$, which associates with each public production plan z an environment s such that (s, z) satisfies (1.1) and (1.2). We assume that at least one feasible policy exists. This is not very restrictive since it amounts to saying that any public production plan is compatible with at least one environment (at least around the initial value of z). Once a policy ϕ is specified there is a unique environment associated with each production plan [the value of s_k being given by $\phi_k(z)$]. We can then associate with each production plan a level of social welfare, $V(\phi(z))$.

We now consider an arbitrary project, $\mathrm{d}z$. With the policy ϕ this yields a change in social welfare, $\mathrm{d}V$, where

$$\mathrm{d}V = \frac{\partial V}{\partial s}\frac{\partial \phi}{\partial z}\,\mathrm{d}z, \tag{1.4}$$

where $\partial V/\partial s$ is the row vector with kth entry $\partial V/\partial s_k$ and $\partial \phi/\partial z$ is a matrix with kith entry $\partial \phi_k/\partial z_i$.

The sign of $\mathrm{d}V$ provides the natural project evaluation criterion: if $\mathrm{d}V > 0$ the project yields a welfare improvement.

Thus, if the shadow price vector ν is defined as

$$\nu \equiv \frac{\partial V}{\partial s}\frac{\partial \phi}{\partial z}, \tag{1.5}$$

then the *cost–benefit test*,

$$\text{accept the project } \mathrm{d}z \text{ if and only if } \Pi \equiv \nu\,\mathrm{d}z > 0 \tag{1.6}$$

correctly identifies welfare improving projects; we call Π the *shadow profits* of the project. This property motivates the definition of shadow prices we have given in Section 1.2. To see that (1.5) and our previous definition are equivalent simply consider a project with $\mathrm{d}z_j = 1$ and $\mathrm{d}z_i = 0$, for $i \neq j$.

The above expression for shadow prices applies to any well-defined policy ϕ. In the important special case where the planner's opportunity set is so restricted that there is only one feasible s for any given z, the constraints (1.1) and (1.2) define a unique feasible policy and we say that the model is *fully determined*.

More generally, however, several feasible policies will exist. Given that $V(\cdot)$ captures the planner's objectives and provided that all the relevant constraints have been correctly described, a consistent pursuit of these objectives requires choosing the best available policy, i.e. the policy ϕ which solves the problem

$$\text{(P)} \qquad \max_s V(s) \quad \text{s.t.} \begin{cases} E(s) - z = 0, \\ s \in S. \end{cases}$$

For simplicity we shall assume that a unique such policy exists (and is differentiable). We shall call (P) *the planner's problem* and denote its solution by s^*.

When the policy is chosen in this fashion, the shadow prices in (1.5) are also given by

$$\nu = \frac{\partial V^*}{\partial z}, \tag{1.7}$$

where $V^*(z)$ is the maximum of (P).

We shall call $V^*(z)$ the *social value of the public production plan* z. Formula (1.7) thus enables us to interpret shadow prices very naturally as *marginal rates of substitution* in a social utility function $V^*(\cdot)$ which is well-defined on the commodity space. Under certain regularity conditions, ν as given by (1.7) will be equal to the Lagrange multipliers on the scarcity constraints (1.1) in the maximisation problem (P) – see Section 2.1.

Notice that when the model is fully determined, the unique feasible policy ϕ is also trivially a solution of (P). Thus, provided that policies are chosen as we have described whenever a choice arises, the definitions (1.5) and (1.7) are equivalent; we shall use them interchangeably.

Multiplying (1.5) and (1.7) by $\mathrm{d}z$ shows the equivalence of the "primal" and the "dual" approaches to project evaluation: a project can be evaluated either by valuing its inputs and outputs at shadow prices (sometimes called the "dual" method) or by tracing all its general equilibrium effects, and then comparing the world with and without the project from the point of view of social welfare (the "primal" method). The equivalence is an immediate consequence of our definition of shadow prices.

Our approach to the definition of shadow prices may appear unnecessarily abstract or convoluted. However, much of the literature on cost–benefit analysis reveals the sensitivity of shadow prices to "policies", and it is therefore quite important to be clear about how much one can legitimately "assume" about the optimality of policies. We do not propose "assuming" that, in the real world, governments at large follow optimal policies; but we recommend that planners who wish to promote a consistent use of cost–benefit techniques should consistently pursue their objectives *within their area of control*. This area of control can, in our framework, be arbitrarily small, the lowest degree of freedom being represented by the "fully determined model" in which a single policy is feasible; we shall keep this special case firmly in mind throughout the chapter. And, clearly, any irreversible commitment to particular decisions should be embodied in the constraints circumscribing the planner's choice.

Our exclusive concentration on policies which solve (P) should not, therefore, be considered as restrictive. Note also that when a welfare improvement is possible *without* a project, it is very difficult to give rigorous meaning to the notion of "welfare-improving" project. For instance, the reader may easily verify that, if s^0 is an arbitrary initial environment, then, unless $s^0 = s^*$, *every* $\mathrm{d}z$ will satisfy the following:

$$\exists \varepsilon > 0, \mathrm{d}s$$

$$\text{s.t. } (\mathrm{d}s, \varepsilon \cdot \mathrm{d}z) \text{ is feasible and } \frac{\partial V}{\partial s}\mathrm{d}s > 0. \tag{1.8}$$

This follows from a continuity assumption and observing that this criterion is satisfied by the zero project $\mathrm{d}z = 0$. In other words, any sufficiently small project can "look favourable" by combining it with a suitable change in s. This possibility has also been emphasised by Bertrand (1974) and Diewert (1983).

To summarise, we wish to emphasise two points. First, shadow prices cannot be properly defined without specifying a policy, and, as we shall see, they can be quite sensitive to the policy chosen. Secondly, when several policies are genuinely feasible, the planner should consistently pursue his objectives (within his area of control), and therefore choose the best available.

1.4. Shadow prices and optimal public production

Up to this point we have considered (small) changes in the public production plan from an arbitrary initial point. This is natural since project evaluation techniques are of particular interest when the public production set is imperfectly known, or new opportunities arise in public production. If the public production set is known, however, a consistent pursuit of objectives involves choosing z to maximise $V^*(z)$ subject to the relevant technological constraints:

$$\max_z V^*(z)$$

(Q)

$$\text{s.t. } z \in Z. \tag{1.9}$$

When the public production set Z is convex, an optimal production plan z^* has maximum shadow profits in Z, i.e.

$$\nu^* z^* = \max_{z \in Z} \nu^* z \tag{1.10}$$

where

$$\nu^* = \frac{\partial V^*}{\partial z} \quad \text{at } z^*.$$

Indeed, suppose there existed an alternative production plan $\tilde{z} \in Z$ such that $\nu^*\tilde{z} > \nu^* z^*$. The project

$$\mathrm{d}z = \varepsilon(\tilde{z} - z^*)$$

for $\varepsilon > 0$ would then have positive shadow profits; moreover, it would be feasible since Z is convex and $z^*, \tilde{z} \in Z$. This would contradict the optimality of z^* since, for small ε, shadow profits are an accurate measure of welfare change.

This result has several important implications. First, if Z allows free disposal, all the components of ν^* must be non-negative. This is not a property of shadow prices in general: for arbitrary z, negative shadow prices are not ruled out, and imply that locally it is socially beneficial to waste some outputs in the public sector, or to use more inputs than are necessary.

Secondly, a sufficient condition for optimal public production to occur on the boundary of Z is that the associated shadow prices should not all be zero. Where shadow prices are also non-negative (e.g. there is free disposal) optimal production will be weakly efficient (in the sense that it is impossible to have strictly more of every good). Diamond and Mirrlees (1971) elucidated the conditions under which weak production efficiency of the public sector is a feature of the optimum. They have argued that these conditions are not very restrictive (intuitively, they require that a simultaneous increase in the availability of every good has some socially beneficial use), and we shall henceforth assume them to be fulfilled.

Thirdly, recalling the standard first-order conditions for profit maximisation, we find that, when public production is optimal, shadow prices are proportional to marginal rates of transformation in the public sector. In fact, the latter result holds irrespective of the convexity of Z [as the reader may verify by deriving the first-order conditions for a maximisation of V^* under the technological constraint $F(z) \le 0$].

Let us now briefly consider production decisions at the level of individual public enterprises. If there are no externalities arising within the public sector then individual public enterprises should each use the same shadow prices ν. This should be obvious since without such externalities a project in one public firm would not interfere with the production plan of other public firms.

As a corollary, if there are no externalities and an individual public enterprise, l, knows its production set Z^l an argument analogous to the previous one establishes that this enterprise should maximise shadow profits over Z^l, if Z^l is convex, irrespective of the production decisions of other public firms. Furthermore, with no externalities shadow profits can be maximised over the aggregate public production set Z only if each public firm maximises its shadow profits.

To summarise, where there are no externalities within the public sector:

(i) Projects which display positive shadow profits are welfare improving. Since this is true wherever the project arises in the public sector, the same shadow prices should be used throughout the public sector.

(ii) Shadow prices will coincide with the marginal rates of transformation in any optimally managed public firm. For a firm with a convex production set this implies maximising shadow profits.

(iii) An optimal aggregate public production plan has maximum shadow profits in the aggregate public production set (assuming it is convex); shadow prices then coincide with marginal rates of transformation throughout the public sector.

Of course, the shadow prices associated with an optimal production plan and those appropriate for the evaluation of local changes around an initial production plan away from the optimum are calculated at different points and thus will not have the same *values*. But the *definition* of shadow prices in terms of the rate of change of social welfare with respect to public production ($\nu = \partial V^*/\partial z$) is the same in both cases.

1.5. Cost–benefit analysis and the theory of reform

So far we have concentrated on the application of cost–benefit techniques to the project evaluation problem. The scope of these techniques, however, goes far beyond project evaluation and the principles we have outlined in the context of project evaluation apply to more general problems of decision-making. More precisely, if ω denotes any collection of "parameters", and if $V^*(z;\omega)$ denotes the corresponding maximum value function of (P), then the gradient of V^* defines both a vector of shadow prices for commodities ($\partial V^*/\partial z$), and a vector of "marginal social values" for parameters ($\partial V^*/\partial \omega$), which allow the application of cost–benefit tests to any marginal change $(\mathrm{d}z, \mathrm{d}\omega)$. We shall call $\mathrm{d}\omega$ a *reform*.

Some care has to be exercised, however, in the specification of the collection of "parameters". Three interpretations are possible. According to the first, these parameters indicate variables which are genuinely outside the control of the planner and can legitimately be considered as "exogenous" to the model. An example would be world prices for a small open economy. From this viewpoint, a marginal change $\mathrm{d}\omega$ represents an exogenous *perturbation* of the economy – e.g. a shift in world prices.

According to the second interpretation, a marginal change $\mathrm{d}\omega$ reflects a *new opportunity* not previously available to the planner – e.g. the introduction of (small) lump-sum taxes. A "new opportunity" of this kind is a straightforward generalisation of the notion of a "project".

A third interpretation, however, is to consider ω as a collection of variables which are, strictly speaking, within the planner's area of control, but which he *chooses* to *treat* as parametric in order to evaluate possible "reforms" in that space. Suppose, for instance, that the planner can control a vector t of indirect taxes. One could argue that instead of choosing the optimal value, t^*, of t (subject to all the relevant constraints), the planner could also, less ambitiously, consider an arbitrary initial value t^0, and from there examine the vector of "marginal social values" $\partial V^*/\partial t$ to discover the set of desirable "tax reforms" $\mathrm{d}t$ (i.e. the set of all $\mathrm{d}t$'s satisfying $(\partial V^*/\partial t)\,\mathrm{d}t > 0$).

This approach is particularly attractive when the planner "inherits" an initial environment s^0 which is not optimum but lacks the *information* necessary to locate the optimum s^*. If the collection of parameters is then specified in such a way that the model is "fully determined" (i.e. (P) has a unique solution for *given* z and ω), then the calculation of shadow prices and marginal social values involves the use of *local* information only, i.e. information on marginal responses around s^0. This method is familiar in the theory of reform, which has mostly been concerned with examining the set of desirable directions of change $(\mathrm{d}z, \mathrm{d}\omega)$ starting from an arbitrary initial environment, on the basis of local information within a fully determined model [see Guesnerie (1977), Ahmad and Stern (1984) and Newbery and Stern (1987)].

There is a difficulty, however, in combining this third interpretation with decentralised project evaluation procedures. The problem is that an element of arbitrariness is introduced here in the specification of the collection of parameters. There will, in general, be many possible specifications, each implying a different shadow price vector, and no single specification obviously commends itself in the context of public production decisions. Indeed it is very difficult, with this third interpretation, to formulate appealing criteria for the specification of parameters – a treatment of this issue would require giving the model a richer structure, e.g. incorporating explicitly the informational constraints which prevent the location of an optimum in the first place.

The problem alluded to here has been aptly summarised by Bell and Devarajan:

> For purposes of social cost–benefit analysis, a project may be viewed as a disturbance to the economy, displacing it from some initial equilibrium to a new one. But the new configuration will depend on which particular variables adjust to restore equilibrium. Since there may be more than one admissible form of adjustment, it is natural to ask how – if at all – the corresponding shadow prices for project evaluation depend on the nature of adjustment. Now, the manner in which the economy equilibrates depends on how the government responds to the disturbance that the project generates. ...

> These equilibrating mechanisms are specified as different rules for "closing" a general equilibrium system which is initially specified so as not to be fully determined. By taking different combinations of variables to be fixed exogenously, one arrives at different formulations of the way in which the economy adjusts to the introduction of a project. It is then possible to solve for the different variables which enter into the social welfare function, the gradient of which yields the vector of shadow prices for the economy... It is worth remarking also that... apparently minor changes in the choice of endogenous variables can lead to quite different results. [Bell and Devarajan (1983) pp. 457–8]

A number of other authors have been concerned with precisely the same problem e.g. Blitzer, Dasgupta and Stiglitz (1981), Diewert (1983), and particularly Hammond (1985).

It is obviously outside the scope of this paper to solve the difficulties raised by this third interpretation. In the formal model of Section 2, we shall include a vector of parameters without explicitly addressing the question of how it is specified.

2. Lessons from second-best theory

Our insistence on the simultaneous and consistent choice of policies and shadow prices underlines the link between cost–benefit analysis and the theory of the second-best, embodied in the work of authors such as Meade (1955), Diamond and Mirrlees (1971), Lesourne (1975) and Guesnerie (1979). In this section we shall attempt to make this link clearer by bringing together, in an example of the planner's model, a number of important results belonging to both traditions.

Section 2.1 contains a description of the model which is based on Drèze (1982) and owes much to the work of Guesnerie (1979). The framework can accommodate a wide range of restrictions on policy tools and market imperfections, such as price rigidities, quantity rationing (including unemployment), trade quotas, exogenous taxes and transfers and so on. In Section 2.2 we draw attention to some elementary properties of the model which will be important in interpreting and understanding what follows. We examine, in Section 2.3, the validity of certain simple rules for the determination of shadow prices in distorted economies and in particular their relation to producer prices, consumer prices and world prices. Whilst such rules are suggestive we shall argue that their range of application is limited and that in general we have to investigate the structure of the model and policies in more detail. That is the task of Section 2.4.

2.1. The model

The model we shall consider is an example of the general model of Section 1. All the assumptions of Section 1 are retained including differentiability and the existence and uniqueness of a solution to the planner's problem (P) – see Section 1.3.

As before, commodities are indexed by i, which runs from 1 to I. Inputs for firms are treated as negative outputs, and household supplies are treated as negative demands. The model allows for many periods and uncertainty insofar as these can be taken into account by simply indexing commodities according to their date of delivery and the state of the world. Problems associated with time and uncertainty will be discussed in greater detail in Section 3.

2.1.1. Private agents

Private agents consist of consumers and producers. Consumers are indexed by $h = 1, \ldots, H$. The hth consumer, confronted with a price vector q, a money income m^h, and a vector of rations or "quotas" $\bar{x}^h = (\bar{x}_-^h, \bar{x}_+^h)$ indicating lower and upper bounds on h's consumption (net of endowments) of each commodity, chooses a consumption plan $x^h(q, \bar{x}^h, m^h)$ solving

$$\max_{x^h} U^h(x^h)$$

$$\text{s.t.} \begin{cases} qx^h = m^h, \\ \bar{x}_-^h \le x^h \le \bar{x}_+^h \end{cases} \tag{2.1}$$

[for simplicity we assume that the solution of (2.1) is always in the interior of h's consumption set].

The aggregate net consumption vector is denoted by $x \equiv \sum_h x^h$.

Similarly, the net supply vector $y^g(p, \bar{y}^g)$ of the gth producer (where g runs from 1 to G) solves

$$\max_{y^g} \Pi^g \equiv py^g$$

$$\text{s.t.} \begin{cases} y^g \in Y^g = \{ y^g : F^g(y^g) \le 0 \}, \\ \bar{y}_-^g \le y^g \le \bar{y}_+^g, \end{cases} \tag{2.2}$$

where p is the vector of producer prices and Y^g is a convex production set (F^g concave). The aggregate net supply vector of private producers is denoted by $y \equiv \sum_g y^g$.

Subscripts to U^h and F^g denote partial derivatives. Equality constraints are special cases of inequality constraints, arising when the lower bound is identical to the upper bound. In every case the binding bound (if any) is simply denoted by $\bar{x}_i^h$, $\bar{y}_i^g$. We shall assume some familiarity with the constrained demand and supply functions $x^h(\cdot)$, $y^g(\cdot)$, as well as with the constrained indirect utility functions $V^h(q, \bar{x}^h, m^h)$ and the constrained profit functions $\Pi^g(p, \bar{y}^g)$ – for a more detailed treatment see e.g. Neary and Roberts (1980), and Fuss and McFadden (1978). Note that when quotas are allowed it is not very restrictive to assume that demands and supplies for consumers and firms are single valued (in particular constant returns industries are permitted, see Section 2.3.5). Externalities are discussed separately in Section 3.8. The unit of account is discussed in Section 2.2.4.

The vector of indirect taxes is denoted by $t \equiv q - p$. Non-linear taxes and taxes on intermediate goods are excluded. The lump-sum income m^h of consumer h consists of the sum of a lump-sum transfer r^h from the government and profit income:

$$m^h \equiv r^h + \sum_g \theta^{gh} \Pi^g, \tag{2.3}$$

where θ^{gh} is h's share in g's profits. The government's share in g's profits is thus $1 - \sum_h \theta^{gh} \equiv \zeta^g$. Profit taxes are taken into account in the calculation of profit shares and there is no need to introduce them separately; for instance, a small increase in the gth profit tax can be treated as a small proportional reduction of the relevant private shares.

This formulation allows for the existence of an arbitrary number of publicly-, privately- and jointly-owned firms. For any firm, government interference with production decisions can take the form of a set of quantity restrictions, or administered prices, observance of which leaves the firm free to choose a profit-maximising production plan.

There are two ways of representing farm households (or the family firm). First such households can be included within the set of consumers provided that the demand functions are interpreted as net trade functions. Such functions possess all the necessary properties for the results in this chapter. Secondly one could regard a farm household as a firm wholly owned by a single household. The distinction turns mainly on the prices at which the household is considered to transact. In this respect the first formulation will usually be more natural.

To simplify the presentation, foreign trade will be treated as follows. Foreign exchange is considered as a separate commodity, indexed by the subscript f. Thus, p_f is the exchange rate, i.e. the price of a unit of foreign exchange. We shall regard the vector of net import levels n as a vector of rations applying with equality to a specific firm, indexed by the superscript f – this firm is a sort of

State Trading Corporation (STC). Thus $\bar{y}_i^f \equiv n_i$ $(i \neq f)$. Given a vector of net imports n, the foreign exchange earnings from trade are given by $y_f^f(\bar{y}^f)$, where

$$-\frac{\partial y_f^f}{\partial \bar{y}_i^f} \equiv p_i^f \tag{2.4}$$

is the marginal cost in terms of foreign exchange of commodity i on the world market. Thus, under the standard "small country" assumption, p_i^f is simply the (fixed) world price of commodity i. Since consumers and producers are interested in commodities, and not in foreign exchange as such, $x_f^h \equiv y_f^g \equiv 0$ for all h, $g(\neq f)$.

The STC transacts at prices p on the domestic market. Hence, its profits,

$$\Pi^f \equiv p \cdot y^f \equiv p_f y_f^f + \sum_{i \neq f} p_i n_i, \tag{2.5}$$

are naturally interpreted as the (net) tariff revenue (the difference between the cost of net imports on the world market and their value at producer prices). However, the STC has no scope for profit maximisation since its net supplies are determined by rations for every commodity except f. Since tariff revenue accrues to the government, $\theta^{fh} = 0$ for all h, i.e. $\zeta^f = 1$. The balance of payments is simply the fth scarcity constraint, $-y_f^f(\cdot) = z_f$, where z_f can be interpreted as an endowment of foreign exchange. Notice that, so far, the model allows for any trade regime since we have not yet specified the relation between domestic and world prices nor the way imports are determined. Thus the STC is simply a notational device for summarising foreign trade transactions and its use does not involve special restrictions on the treatment of foreign trade.

2.1.2. The planner

In the example of the planner's model studied here, the elements of the vector s of "signals" form a subset of dimension K of the following variables:

$$(p_i),\quad (t_i),\quad (r^h),\quad (\bar{x}_i^h),\quad (\bar{y}_i^g),\quad (\theta^{gh}). \tag{2.6}$$

The remaining variables will be considered as exogenous characteristics of the economy, called "parameters" or "predetermined variables", and denoted by ω. The interpretation of ω was discussed in Section 1.5. The components of s will be called "control variables" (interchangeably with "signals"), to distinguish them more sharply from predetermined variables. The distinction between predetermined variables and control variables is not formally necessary since any signal can be constrained, by a suitable specification of S, to take a fixed predetermined value (marginal social values of parameters would then become

Lagrange multipliers on those constraints); however, it will much facilitate the exposition of this section.

In most of our work we shall regard the value of ω as fixed and exogenous, and neglect mentioning ω explicitly, the main exception arising when we consider shifts in ω of the comparative static kind (in Section 2.2.2).

The net excess demand function of the private sector here reduces to

$$E(s;\omega) \equiv \sum_h x^h(p+t,\bar{x}^h,m^h) - \sum_g y^g(p,\bar{y}^g), \tag{2.7}$$

with m^h as in (2.3).

The planner's objectives are embodied in a Bergson–Samuelson social welfare function (its interpretation is discussed in Section 2.2.5):

$$V(s;\omega) = W(\dots,V^h(p+t,\bar{x}^h,m^h),\dots) \tag{2.8}$$

with m^h as in (2.3) again.

There remains to discuss the specification of the opportunity set S. In some of our work we shall assume that there are no constraints further to the scarcity constraints and in that case speak of the "model without side constraints". In the model without side constraints the latitude of the planner's choice is essentially described by specifying the variables to be considered as "predetermined".

The assumptions underlying the model without side constraints are somewhat simplistic. In particular, they imply that the decisions of all the public agencies not under the control of the planner (e.g. the fiscal authorities, or the import licensing authority) are exogenous to the planner's model. While this short cut is a commonly adopted one in second-best theory, a more realistic treatment of the endogenous behaviour of government agencies requires the introduction of side constraints. Similarly, a thorough study of such phenomena as monopolistic behaviour, endogenous rationing schemes or collective bargaining would involve a detailed specification and analysis of the appropriate side constraints (e.g. the requirement that, for a monopolistic firm, marginal cost equals marginal revenue). While we shall usually avoid an explicit consideration of the complexities introduced by side constraints, many of our results allow S to have a very general structure.

In the model with side constraints, where S has an arbitrary structure, we shall make extensive use of the notion of "locally unrestricted control variable" to designate a control variable which, near the optimum s^*, does not enter the side constraints. Formally, the kth control variable s_k is locally unrestricted, or "*unrestricted*" for short, if there exists a neighbourhood N of s^* and a set S_{-k} in R^{K-1} such that, within that neighbourhood:

$$s \in S \Leftrightarrow s_{-k} \in S_{-k}, \tag{2.9}$$

where s_{-k} is the vector s without its kth component. When all control variables are unrestricted, s^* can be shown to be in the interior of S; for our purposes this special case is formally equivalent to that of the model without side constraints.

As before, z denotes the public production plan, and Z the public production set. The planner's problem (P) of Section 1 now reads:

$$
\text{(P)} \quad \max_{s} W\left(\ldots, V^h\left(p+t, \bar{x}^h, r^h + \sum_g \theta^{gh} \Pi^g(\cdot)\right), \ldots\right)
$$

$$
\text{s.t.} \begin{cases} \sum_h x^h(\cdot) - \sum_g y^g(\cdot) - z = 0, \\ s \in S, \end{cases} \tag{2.10}
$$

and the shadow price vector v is the gradient of the maximum value function $V^*(z;\omega)$ of (P).

2.1.3. On the interpretation of the model

We conclude our exposition of the model with some comments on its interpretation. In the model we have just presented the dimensionality of the vector of control variables does not represent the dimensionality of the planner's choice: intuitively, with K signals and, say, J binding constraints, the planner really has only $K-J$ "instruments" (or "degrees of freedom"), the remaining J signals being endogenously determined by the constraints. It is tempting, then, to introduce a distinction, within the set of control variables, between "directly controlled" variables (or policy instruments) and "implicitly controlled" (or endogenous) variables. This distinction is formally unnecessary for our purposes since the whole set of K controls is chosen simultaneously subject to the J constraints. However, we shall occasionally use it for purposes of interpretation.

A simple example may help to clarify the issue. Consider an economy where the ith market clears through foreign trade, i.e. the amount $\bar{y}_i^f$, of net imports of the ith commodity is determined by the excess of demand for the ith commodity over domestic supply, or, equivalently, "solves" the ith excess demand equation. In such circumstances, $\bar{y}_i^f$ would be an endogenous variable in our model, and hence, a "control variable". Now consider a different economy, where the ith commodity is traded under a quota. If the planner can control this quota, $\bar{y}_i^f$ will again be considered as a control variable. The distinction between these two different sets of circumstances is irrelevant to the formal structure of the model. When it can help its interpretation we could call imports in the former case implicitly controlled and in the latter directly controlled.

A related point of interpretation is the following. In much of the remainder of this section, we shall study the first-order conditions of (P) which, together with the constraints, determine optimal policies and shadow prices. Usually a given first-order condition (or subset of first-order conditions) can alternatively be interpreted as a "shadow pricing rule" (i.e. an expression satisfied by the shadow price system) or as an "optimal policy rule" (helping to determine the optimal values of policy instruments). Here again, the distinction can be helpful for purposes of interpretation, but it is formally unnecessary since shadow prices and policies are determined simultaneously. For instance, we shall see (in Sections 2.4 and 3.3) that, when net imports are "locally unrestricted", relative shadow prices for traded goods coincide under fairly general conditions with their relative border prices. Whether this result should be interpreted as a shadow pricing rule or as a criterion for choosing optimal quotas is a point of interpretation and not of substance.

As a corollary of this observation, the informational burden of calculating shadow prices and optimal values of instruments should always be assessed jointly. For instance, in a model with optimal commodity taxes where shadow prices coincide with producer prices (see Section 2.3), it is tempting to claim that shadow prices are very easy to calculate. In interpreting this statement, however, one should remember that using these shadow prices is conditional upon implementing optimal taxes, the choice of which will require, for instance, information on consumer preferences, and interpersonal comparisons of welfare (see Sections 3 and 3.2).

Finally we should note that we have chosen to write (P) using indirect utility functions and uncompensated demand functions. It is possible to rewrite the problem using expenditure functions and compensated demand functions, or using direct utility functions. Different authors adopt different formulations and this may affect the appearance of formulae or the ease of derivation of certain results; and occasionally it takes some effort to establish the relationships between results which are differently expressed. However, there are generally no real differences of substance and we have chosen a formulation which is convenient for the broad range of results we shall discuss.

2.2. General properties of the model

2.2.1. Shadow prices and Lagrange multipliers

Shadow prices and Lagrange multipliers are independent concepts, even though their values coincide under some circumstances. We shall here try to clarify the relationship between these two analytical tools.

For simplicity, consider first the model without side constraints. Under a simple rank condition (namely, that the $I \times K$ Jacobian matrix $\partial E/\partial s$ has full row rank, i.e. the constraints are locally linearly independent) the necessary first-order conditions for a maximum of (P) (2.10) are

$$\frac{\partial V}{\partial s} - \lambda \frac{\partial E}{\partial s} = 0, \tag{2.11}$$

where λ is a vector of Lagrange multipliers associated with the scarcity constraints, see Dixit (1976a, p. 7). Apostol (1957, p. 153) provides a formal statement and proof of the theorem on the existence of Lagrange multipliers for the case $I < K$. For $I = K$ the proof is trivial since when the rank condition is satisfied the rows of $\partial E/\partial s$ span the vector space in which $\partial V/\partial s$ lies.

Now consider a small project $\mathrm{d}z$. Any compatible small change $\mathrm{d}s$ (in particular that associated with the planner's policy) must satisfy

$$\frac{\partial E}{\partial s}\,\mathrm{d}s = \mathrm{d}z. \tag{2.12}$$

The consequent change in social welfare is

$$\mathrm{d}V = \frac{\partial V}{\partial s}\,\mathrm{d}s = \lambda \frac{\partial E}{\partial s}\,\mathrm{d}s = \lambda\,\mathrm{d}z. \tag{2.13}$$

Since this holds for arbitrary $\mathrm{d}z$, λ must actually coincide with the shadow price vector ν. In this model, therefore, shadow prices are identical to the Lagrange multipliers associated with the scarcity constraints. It should be emphasised, however, that shadow prices are certainly not *defined* as Lagrange multipliers, and the coincidence obtained here is contingent upon the particular format of the model. More precisely, there are in general several ways of expressing a given set of constraints, and while shadow prices are invariant to the format adopted, Lagrange multipliers are not (for an illustration, see Section 2.2.3). The coincidence of shadow prices and Lagrange multipliers is often analytically convenient, however, and this fact underlies the construction of our own model.

The crucial assumptions underlying the above reasoning are, first, the differentiability of V^* (which ensures the existence and uniqueness of a shadow price vector ν), and second, the rank condition [guaranteeing the existence of a vector λ satisfying (2.11), and hence, coinciding with ν].

The case where there are exactly as many signals as there are constraints deserves explicit consideration, since, intuitively, it will be often associated with the model being "fully determined". Differentiating the identity $E(\phi(z)) \equiv z$ we obtain:

$$\frac{\partial E}{\partial s}\frac{\partial \phi}{\partial z} = I,$$

where I here denotes the identity matrix. Therefore, when the rank condition is satisfied:

$$\frac{\partial \phi}{\partial z} = \left[\frac{\partial E}{\partial s}\right]^{-1}, \tag{2.14}$$

and hence

$$\nu = \frac{\partial V^*}{\partial z} = \frac{\partial V}{\partial s}\frac{\partial \phi}{\partial z} = \frac{\partial V}{\partial s}\left[\frac{\partial E}{\partial s}\right]^{-1}, \tag{2.15}$$

so that (2.11) holds and has ν as a unique solution for λ.

This example illustrates a very important point: under mild regularity conditions, all the theory developed in this chapter applies without alteration to the "fully determined" model. This observation is crucial because the fully determined case where no policy choice is available is often considered as significant by cost–benefit theorists, and is also central to the theory of reform, as we argued in Section 1.5.

It is useful to restate (2.11) as

$$\frac{\partial \mathscr{L}}{\partial s} = 0,$$

where

$$\mathscr{L}(s; \omega) \equiv V(\cdot) - \nu[E(\cdot) - z] \tag{2.17}$$

is the Lagrangian of the planner's problem for the model without side constraints.

The principles outlined above generally apply with more complicated side constraints, though care has to be taken with conditions ensuring the existence of Lagrange multipliers – see Abadie (1967), Apostol (1957), Dixit (1976a) or Gale (1967). Given the structure of the model, shadow prices still coincide with the Lagrange multipliers of the scarcity constraints, while other multipliers are associated with side constraints; for instance, if the side constraints can be written as $G(s; \omega) \leq 0$, the Lagrangian of the planner's problem becomes

$$\mathscr{L}(\cdot) \equiv V(\cdot) - \nu[E(\cdot) - z] - \mu G(\cdot), \tag{2.18}$$

where μ is a vector of Lagrange multipliers associated with the side constraints. The first-order conditions for a maximum require a zero gradient of $\mathscr{L}$ with respect to s at s^*. Also, if s_k is unrestricted, the side constraints can be considered to be (locally) independent of s_k. The first-order conditions imply

$$\frac{\partial V}{\partial s_k} - \nu \frac{\partial E}{\partial s_k} = 0, \quad \text{for each } s_k \text{ unrestricted.} \tag{2.19}$$

We shall make extensive use of this result.

2.2.2. Shadow prices and the marginal social value of parameters

In the model without side constraints, consider a small exogenous perturbation $\mathrm{d}\omega$ of the values of predetermined variables. In order to satisfy the scarcity constraints, a corresponding change $\mathrm{d}s$ must take place such that

$$\frac{\partial E}{\partial \omega}\mathrm{d}\omega + \frac{\partial E}{\partial s}\mathrm{d}s = 0. \tag{2.20}$$

Using the first-order conditions (2.11) with $\lambda = \nu$ and (2.20) the net effect on social welfare is

$$\begin{aligned} \mathrm{d}V &\equiv \frac{\partial V}{\partial \omega}\mathrm{d}\omega + \frac{\partial V}{\partial s}\mathrm{d}s \\ &= \frac{\partial V}{\partial \omega}\mathrm{d}\omega + \nu \frac{\partial E}{\partial s}\mathrm{d}s \\ &= \frac{\partial V}{\partial \omega}\mathrm{d}\omega - \nu \frac{\partial E}{\partial \omega}\mathrm{d}\omega \\ &= \frac{\partial \mathscr{L}}{\partial \omega}\mathrm{d}\omega. \end{aligned} \tag{2.21}$$

In other words, the net effect on social welfare of a small shift of any parameter is indicated by the gradient of $\mathscr{L}$. For any parameter ω_k, we shall call

$$\frac{\partial V^*}{\partial \omega_k} \equiv MSV_{\omega_k} = \frac{\partial \mathscr{L}}{\partial \omega_k} = \left[\frac{\partial V}{\partial \omega_k}\right] - \left[\nu \frac{\partial E}{\partial \omega_k}\right] \tag{2.22}$$

the *marginal social value* of ω_k. The value of a parameter is optimal from the point of view of the planner when its marginal social value is zero. By analogy, we shall identify the gradient of $\mathscr{L}$ with respect to s as the marginal social value of control variables. The first-order conditions (2.11) state that at s^* the marginal social value of control variables is zero.

To illustrate (2.22), consider the marginal social value of a lump-sum transfer to consumer h, denoted b^h:

$$b^h \equiv MSV_{r^h} = \frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial m^h} - \nu \frac{\partial x^h}{\partial m^h} \equiv \beta^h - \nu \frac{\partial x^h}{\partial m^h}. \tag{2.23}$$

The marginal social value of a lump-sum transfer to consumer h is therefore the difference between the social marginal utility of h's income (β^h), usually called his *welfare weight*, and the social cost $\nu(\partial x^h/\partial m^h)$ of h's additional consumption.

This line of interpretation is very general. In the model without side constraints, the marginal social value of any parameter can always be regarded as the difference between its direct impact on social welfare (holding all other signals constant) and the social cost of the net additional commodity demands it generates – see (2.22); and the same applies for a control variable.

When side constraints are present, additional terms have to be taken into account, insofar as the parameters considered interact with these constraints. For instance, if the side constraints can be written as $G(s; \omega) \le 0$, then, under appropriate regularity conditions, the marginal social values of parameters and control variables will still be well defined as the gradient of the Lagrangian (2.18).

2.2.3. *The government budget constraint, Walras' law, and shadow revenue*

The accounts of the government in our model can be presented in many equivalent ways, according to the prices at which public firms are considered to transact. For instance, when a consumer buys from a public firm at prices q, the breakdown of q between indirect taxes levied by the government and the prices charged by the public firm is arbitrary and has no consequences for the theory.

To be specific, let us consider that the public sector transacts at prices p. The net public revenue R_p (the subscript indicates our accounting convention) becomes the sum of profits in the public sector (pz), indirect taxes (tx), the government's share of private profits ($\sum_g \zeta^g \Pi^g$) inclusive of tariff revenue, and lump-sum taxes ($-\sum_h r^h$):

$$R_p \equiv pz + tx + \sum_g \zeta^g \Pi^g - \sum_h r^h. \tag{2.24}$$

A few manipulations using the accounting identities and budget constraints of the system imply (*irrespective* of the specification of S):

$$R_p \equiv p(z + y - x). \tag{2.25}$$

A corollary of this identity is Walras' law:

$$R_p = 0 \Leftrightarrow p(z + y - x) = 0. \tag{2.25a}$$

Notice that (2.25a) does not involve the assumption that $x - y = z$. But of course it implies that if the scarcity constraints are met (including that applying to foreign exchange) then the government budget is balanced. This property of the model justifies the omission of a separate public budget constraint in (P).

Three remarks are in order. First, dispensing with the public budget constraint by invoking Walras' law does not at all free the analyst from enquiring about the process through which the government meets its budget constraint. This process is implicitly reflected here in the specification of the control variables and the planner's opportunity set. Several recent papers in the cost–benefit literature, e.g. Johansson (1982), Bell and Devarajan (1983), Blitzer, Dasgupta and Stiglitz (1981), and Diewert (1983), have stressed the possible sensitivity of shadow prices to alternative assumptions concerning the source (or destination) of a marginal unit of public revenue. In this model, the problem is best viewed as a particularly important example of the need to identify clearly the opportunity set of the planner.

Secondly, we can use (2.25a) to illustrate our remark in Section 2.2.3 that Lagrange multipliers will coincide with shadow prices only when the constraints are written appropriately. Indeed in view of (2.25a) it would be legitimate to replace any scarcity constraint, say the Ith, by the government budget balance $R_p = 0$. Then the ith shadow price will as usual be the rate of change of social welfare with respect to z_i and thus equal to the partial derivative with respect to z_i of the Lagrangian. We can see, however, that this derivative is $\lambda_i + \alpha p_i$, for $i \neq I$, where λ_i is the Lagrange multiplier on the ith constraint and α that for the government budget constraint. Hence when the problem is written in this way ν no longer coincides with λ.

Thirdly, a word should be said about the financing of projects. It is often suggested that projects should be distinguished according to the way they are financed. When projects are financed out of general revenue, however (i.e. the government faces the single budget constraint $R_p = 0$), it is not necessary to consider separately how individual projects are financed. What matters is just their shadow profits; how government revenue is raised will be reflected in the "policy", and hence in the shadow prices. On the other hand, if budgetary constraints apply to particular parts of public activity, as when some tax revenues are earmarked, or where government firms have financial targets, Walras' law can obviously not be invoked to ignore them, and they should be included explicitly in the model as side constraints. Their implications for shadow pricing will be briefly considered in Section 3.6.

The generalisation of the above formulae is straightforward. Suppose we do the government accounts with respect to a price vector ρ and define the government revenue as

$$R_\rho \equiv \rho z + (q - \rho)x + (\rho - p)y + \sum_g \zeta^g \Pi^g - \sum_h r^h, \tag{2.26}$$

where $(q-\rho)$ and $(\rho-p)$ may be interpreted as "taxes" called *shadow taxes* when $\rho=\nu$. Then, analogously to (2.25),

$$R_\rho \equiv \rho(z+y-x), \tag{2.27}$$

and again $R_\rho=0$ when the scarcity constraints are satisfied.

The notion of *shadow revenue* (R_ν) provides an alternative and useful way of interpreting the marginal social value of parameters discussed above. Indeed, in the model without side constraints, (2.27) allows us to write:

$$\mathscr{L}(\cdot)\equiv V(\cdot)+R_\nu(\cdot) \tag{2.28}$$

and

$$MSV_{\omega_k}=\frac{\partial V}{\partial \omega_k}+\frac{\partial R_\nu}{\partial \omega_k}. \tag{2.29}$$

The marginal social value of a parameter (or control variable) is now the sum of its direct effect on social welfare, and its effect on the "shadow revenue" of the government. For instance, according to this approach, the marginal social value of a lump-sum transfer to the hth consumer reduces to

$$b^h=\beta^h+\frac{\partial R_\nu}{\partial r^h}=\beta^h-1+(q-\nu)\frac{\partial x^h}{\partial m^h}, \tag{2.30}$$

which is easily verified to be equivalent to (2.23). Equation (2.30) is a familiar expression in the literature on optimal taxation, especially in the case where $\nu=p$ [see, for example, Atkinson and Stiglitz (1980, Lecture 14)].

2.2.4. *Normalisation rules and the numeraire*

2.2.4.1. Market prices. Our social welfare function, and the scarcity constraints, are easily verified to be homogeneous of degree 0 in (p, t, r). Whenever the side constraints also have this property, it will not be restrictive to impose that at least one of these variables should be predetermined (i.e. take a fixed exogenous value), provided that the value of the chosen variable has a definite, known sign at a solution of (P) (2.10). A simple example of such a normalisation rule is $p_{i_0}\equiv 1$ for some i_0.

When choosing a normalisation rule, care should be taken not to introduce unintended restrictions in the model. For instance, the above normalisation rule would not be permissible a priori in a model where wages are fixed in terms of a non-homogeneous price index (because the latter restriction would itself destroy the homogeneity of the model). Notice also that it is not permissible, in general, to impose a priori restrictions of the type $t_{i_0} \equiv 0$, *unless* there is a guarantee that an optimum with this feature exists. Examples where such a guarantee would exist are (i) where the model is separately homogeneous in p and in (q, r), as in Diamond and Mirrlees (1971), and (ii) where the good i_0 cannot be taxed.

2.2.4.2. Shadow prices. For purposes of project evaluation, only relative shadow prices matter (since $\nu\, \mathrm{d}z > 0 \Leftrightarrow \alpha \cdot \nu\, \mathrm{d}z > 0$ for all $\alpha > 0$). It is permissible, then, to scale the social welfare function so as to satisfy a particular normalisation rule, e.g. $\nu_{i_0} = 1$ for some i_0 such that $\nu_{i_0} > 0$, or $\beta^{h_0} = 1$ for some h_0.

The choice of normalisation rule is a matter of convenience. It does not alter the *sign* of the shadow profits of a project, but only their magnitude and interpretation. For instance, with the normalisation rule $\nu_f = 1$, a project yielding a social profit Π has exactly the same social value as a gift of Π units of foreign exchange.

2.2.5. *The social welfare function and Meade's formula*

In Section 1 we introduced $V(s)$ as the *objective function* of the planner. We also call it the *social welfare function* (SWF). There is no suggestion that this welfare function provides some kind of scientific measure of society's "well-being"; it simply represents the preferences of the planner (however determined, see Section 3.2) over environments. When there are no externalities in preferences, the choice of a social welfare function of the Bergson–Samuelson type amounts to making two important assumptions regarding the planner's preferences.

First, we restrict the arguments of these preferences to consumption profiles. This ensures the existence of a function Ψ such that

$$V(s) = \Psi(\ldots, x^h(s), \ldots). \tag{2.31}$$

Second, we assume that the marginal rate of substitution between any two commodities going to a particular individual is the same according to the planner's preferences and according to those of the concerned individual himself. When this is the case, the SWF is said to be *individualistic*. If the SWF is individualistic, then, for an arbitrary cardinalisation of each individual utility function U^h, and for an arbitrary cardinalisation of V, the SWF, there exists a function W such that (2.31) can be written as

$$V(s) = W(\ldots, U^h(x^h(s)), \ldots).$$

This result is a straightforward application of separarable preferences – see, for example, Deaton and Muellbauer (1980, p. 127).

These two assumptions may or may not appeal, though we would argue that for many practical purposes they are not exceedingly far-fetched. Whether these two assumptions are regarded as acceptable will depend on the type of problem being studied (e.g. one would not necessarily want to assume an individualistic SWF in considering, say, preferences for heroin). In any case, we do not suggest that the welfare weights β^h introduced above involve objective measures of individual "happiness"; rather, they reflect subjective judgements which have a straightforward intuitive interpretation. Their measurement will be discussed further in Section 3.2.

Notice that when the SWF takes the Bergson–Samuelson form, the effect on social welfare of any small change $\mathrm{d}s$ of the environment can be written as

$$\mathrm{d}V = \sum_h \beta^h \left[\frac{(\partial V^h/\partial s)\,\mathrm{d}s}{\partial V^h/\partial m^h} \right] \equiv \sum_h \beta^h \cdot MWP^h, \tag{2.32}$$

where MWP^h, h's marginal-willingness-to-pay for the change, measures the net worth of this change to the hth individual in money terms: $\mathrm{d}V^h = MWP^h(\partial V^h/\partial m^h)$. In particular, putting $\mathrm{d}s = (\partial\phi/\partial z)\,\mathrm{d}z$, we can see the net welfare gains of any project as a welfare-weighted sum of each individual's marginal-willingness-to-pay for the project; this is essentially Meade's formula [Meade (1955, Mathematical Supplement, ch. 2)].

2.2.6. *Ultimate and derived objectives*

Real-world planners are sometimes thought to reason not in terms of a single well-defined objective function, but rather in terms of a variety of objectives expressed in the more or less precise language of macro-economic aggregates such as national income, employment, growth, balance of payments deficits, inequality indices, etc. If such were to be the case these multiple objectives would have to be appropriately weighted before providing the foundation of operational project evaluation criteria. When such goals are interpreted as *derived* objectives, ultimately valued themselves from the point of view of intertemporal consumer welfare (or any other well-defined objective function), an explicit recognition of the underlying social welfare function will indicate not only how these objectives should each be quantified, but also which relative weights they should carry.

To illustrate this point, consider a very simple example of the model where only labour services (indexed by l) are rationed:

$$V = W\big(\ldots, V^h(q, \bar{x}_l^h, m^h)\ldots\big). \tag{2.33}$$

Differentiating (2.33) we obtain:

$$\mathrm{d}V=\sum_h \beta^h\left[\mathrm{d}m^h - x^h\,\mathrm{d}q + \left(\rho_l^h - q_l\right)\mathrm{d}\bar{x}_l^h\right] \tag{2.34}$$

$$=\sum_h \beta^h\left[\mathrm{d}e^h - \tilde{x}^h\,\mathrm{d}\tilde{q} + \rho_l^h\,\mathrm{d}\bar{x}_l^h\right] \tag{2.35}$$

$$=\mathrm{d}e + \sum_h\left(\beta^h - 1\right)\mathrm{d}e^h - \mathrm{d}\tilde{q}\Big(\sum_h \beta^h\tilde{x}^h\Big) + \sum_h \beta^h\rho_l^h\,\mathrm{d}\bar{x}_l^h, \tag{2.36}$$

where e^h is h's expenditure ($e^h \equiv m^h - q_l\bar{x}_l^h$), $e \equiv \sum_h e^h$, $\tilde{x}^h$ and $\tilde{q}$ are the vectors x^h and q^h without their lth component, and ρ_l^h is h's "reservation wage". In this simple illustration, social welfare becomes the weighted sum of four terms which represent familiar concepts such as, respectively, national income, an index of inequality, a consumer price index and a measure of unemployment. There are, however, also other interpretations of such objectives, e.g. in terms of a right to work, which do not rely on a Bergson–Samuelson social welfare function but which could still (sometimes) be included in a more general objective function.

Though we would not deny the potential usefulness of derived objectives in interpreting the model we shall work directly with ultimate objectives and, further, confine our analysis to a SWF of the Bergson–Samuelson type.

2.2.7. *The first best*

Let z^0 be an arbitrary public production plan and s^0 an environment such that $E(s^0) = z^0$, and consider the corresponding allocation $[(x^{h0}),(y^{g0})] = [(x^h(s^0)),(y^g(s^0))]$ of z^0. This allocation is said to be *first best* if it maximises social welfare subject to the irreducible technological constraints of production, i.e. if it solves

$$\max_{(x^h,\,y^g)} W\left(\ldots,U^h(x^h),\ldots\right)$$

(Q)

$$\text{s.t.}\begin{cases}\sum_h x^h - \sum_g y^g - z = 0,\\ y^g \in Y^g\end{cases} \tag{2.37}$$

(for simplicity we still ignore consumption sets).

By *a first-best policy* we shall mean a policy which achieves a first-best allocation. Of course such a policy may not be feasible, i.e. compatible with the planner's opportunity set.

Now when W is increasing in each argument, a first-best allocation must be a Pareto-efficient allocation. In the usual way the marginal rates of substitution between commodities must be the same for each consumer and producer (we denote the vector of common marginal rates by ρ). At a first-best allocation, moreover, the marginal social utility of a commodity going to any two individuals must be the same – otherwise social welfare could be improved by reallocating commodities between consumers. It follows that the relative social value of commodities is the same in every use, and coincides with their private marginal rates of substitution. Clearly (suitably normalised) ρ will provide the correct shadow price vector associated with any first-best policy.

A formal proof that shadow prices are collinear with ρ is as follows.

Consider the maximum value function of (Q), say $V^*(z)$, and let $\nu \equiv \partial V^*/\partial z$. Suppose that ν failed to be collinear with ρ. Then we could find h and $\mathrm{d}x^h$ s.t.

$$\frac{\partial U^h}{\partial x^h}\mathrm{d}x^h > 0, \quad \nu\,\mathrm{d}x^h < 0. \tag{2.38}$$

Putting $z_\varepsilon \equiv z^0 + \varepsilon\,\mathrm{d}x^h$:

$$V^*(z_\varepsilon) \geq W\big(U^1(x^{10}),\dots,U^{h-1}(x^{h-1,0}),U^h(x^{h0}+\varepsilon\,\mathrm{d}x^h),\dots,U^H(x^{H0})\big) \tag{2.39}$$

$$= W\big(U^1(x^{10}),\dots,U^h(x^{h0}),\dots,U^H(x^{H0})\big)$$

$$+\varepsilon\frac{\partial W}{\partial U^h}\frac{\partial U^h}{\partial x^h}\mathrm{d}x^h + o(\varepsilon^2) \tag{2.40}$$

$$> V^*(z^0), \quad \text{for small } \varepsilon > 0. \tag{2.41}$$

On the other hand,

$$V^*(z_\varepsilon) = V^*(z^0) + \nu\big(z_\varepsilon - z^0\big) + o(\varepsilon^2) \tag{2.42}$$

$$= V^*(z^0) + \varepsilon\nu\,\mathrm{d}x^h + o(\varepsilon^2) \tag{2.43}$$

$$< V^*(z^0), \quad \text{for small } \varepsilon > 0. \tag{2.44}$$

This contradiction establishes the result.

Notice that this result is not concerned with how first-best allocations are actually achieved; in particular, it says nothing about market prices. However, we know that under certain conditions, a first-best allocation may be achieved as a competitive equilibrium with optimal lump-sum transfers; in such a case, (relative) shadow prices coincide with (relative) market prices, both being equal to the common marginal rates of substitution. The reader may verify that, where producer prices and lump-sum transfers are unrestricted control variables in problem (P) (2.10), while taxes and quotas are absent, and the economy is closed, a first-best allocation is achieved, shadow prices coincide with market prices, and welfare weights are equal for all consumers.

2.3. *Some important second-best theorems*

Some of the early contributions to second-best theory were pessimistic regarding the possibility of defining operational rules for public decision-making outside the perfectly competitive model; they stressed, for instance, that distortions occurring anywhere in the economy would lead to a disruption of the standard conditions for Pareto efficiency everywhere else [Lipsey and Lancaster (1956–57)].

Since then much work has been directed at establishing more positive results, and exhibiting the conditions under which reasonably simple and operational rules for decision-making (e.g. the desirability of aggregate production efficiency) exist in spite of the failure to achieve a first-best allocation. We shall present some of the more important results in our framework. A more formal synthesis can be found in Guesnerie (1979).

2.3.1. *Production efficiency of the public sector*

So far we have considered the "public sector" to cover all firms whose production plan is directly under the control of the planner, and whose profits accrue entirely to the government. It may happen, however, that the production plan of a private firm can effectively be manipulated by the planner (e.g. via fiscal instruments or quantity constraints), without affecting other private agents (except, of course, indirectly, via the scarcity constraints). If, in addition, the profits of such a firm either accrue entirely to the government, or can be taxed optimally, we call it *fully controlled*; and it is intuitively clear that the planner should evaluate the production plan of such a firm according to the same criteria as those applying to other public firms. As a corollary, weak production efficiency (in the sense of Section 1.4) is clearly desirable for the subset of fully controlled firms.

As a simple example, consider a firm, say the gth, for which each commodity is rationed – except one, say the first, whose net output will therefore be chosen to satisfy the technological constraint $F^g(y^g)=0$. Suppose, furthermore, that the profits of this firm are all taxed: $\zeta^g=1$ (an optimal profit tax for this firm would be an equally adequate assumption). If $(\bar{y}_i^g)_{i\neq 1}$ are unrestricted signals, the first-order conditions (2.19) imply

$$\nu_i = -\nu_1 \frac{\partial y_1^g}{\partial \bar{y}_i^g} = \nu_1 \frac{F_i^g}{F_1^g}. \tag{2.45}$$

Hence, when $\nu_1 \neq 0$:

$$\frac{\nu_i}{\nu_j} = \frac{F_i^g}{F_j^g}, \quad \text{for all } i,\ j, \tag{2.46}$$

which illustrates the desirability of joint production efficiency for the set of fully-controlled firms. Notice that when g's production set is strictly convex, "full control" can equally be achieved by a set of (unrestricted) firm-specific taxes – which would bring the relative prices faced by the firm into equality with relative shadow prices.

Examples of firms which are not fully controlled in the above sense are: (i) firms whose profits are not fully taxed (these are dealt with in Section 3.6); (ii) public firms facing a budget constraint of the Boiteux type, e.g. $py^g=\overline{\Pi}^g$ (also in Section 3.6); and (iii) firms whose net output of some commodities only can be controlled by the planner (see Section 2.4.3 below).

We hope that these examples help to indicate which type of firm can legitimately be regarded as belonging to the "public sector". For our purposes, no distinction is needed between the public sector and the set of fully controlled firms.

2.3.2. Aggregate production efficiency

The reasoning we have applied for a single firm can be applied to the private production sector as a whole. Thus, if the planner's opportunity set is such as to give him an *effective ability to induce any (feasible) aggregate private production plan*, without directly affecting consumer's demands and utilities, *aggregate production efficiency* (defined as weak production efficiency for all firms taken together) will clearly be desirable; and, more generally, private marginal rates of transformation will provide a suitable shadow price vector for evaluating marginal changes in the public production plan.

Once again this result says nothing about producer prices as such. With convex production sets, however, it implies that the private production plan y^* associiated with the solution of (P) (2.10) can be supported by the price vector ν:

$$\nu y^* = \max_{y \in Y} \nu y, \tag{2.50}$$

where Y is the aggregate production set for the private sector. As a corollary, if such a support vector is unique and there are no quotas, *shadow prices coincide with producer prices*. However, the general relationship between the desirability of aggregate production efficiency on the one hand, and the equality of shadow prices and producer prices on the other is quite complicated when quotas apply and we shall not investigate it further here.

Several sets of assumptions can be invoked to ensure that the tools available to the planner are powerful enough to enable him to "control" the private sector. Most of them are restrictive from a practical point of view. The best known of these sets of assumptions are those of the *Diamond–Mirrlees* model of optimal taxation and public production [see Diamond and Mirrlees (1971)]: competitive private producers, unrestricted commodity taxation and full taxation of private profits. When these conditions are satisfied:

> The government can induce private firms to produce any efficient net output bundle by suitable choice of producer prices p. ... The choice of p does not affect consumer demands or welfare, since pure profit arising from decreasing returns to scale go to the government, and since, any commodity taxes being possible, q can be chosen independently of p [Diamond and Mirrlees (1971, p. 17)].

It is interesting to compare these statements with the following (optimistic) quote from the Technical Note to the Sixth Five-Year Plan of India [Government of India (1981, p. 4)]:

> As for the private sector, the relevant macro plans are formulated by the Planning Commission, in complete tune with the general development strategy of the country and of the public sector. This part of the plan is regarded as indicative and subsequently appropriate measures are undertaken by different Ministries through fiscal, monetary and income policies to ensure their fulfilment.

We can now provide a formal proof of the Diamond–Mirrlees result for the case where all private firms have strictly decreasing returns of scale (for the case of constant returns see Section 2.3.5). Let $\zeta^g = 1$, for all g, and (p_i), (t_i) all be unrestricted signals. Then the first-order conditions (2.19) imply

$$\nu \frac{\partial y}{\partial p} = 0. \tag{2.51}$$

But also, by homogeneity of degree zero of private net supply functions:

$$p\frac{\partial y}{\partial p} = 0. \tag{2.52}$$

If the matrix $\partial y/\partial p$ has rank $(I-1)$, then ν must be collinear with p. As a corollary, if private firms face no quantity constraints, aggregate production efficiency is desirable. It is clear that these results extend to the case where the profits of each firm are optimally taxed since then the social value of any effect of price changes operating through profits is zero and (2.51) applies [see Dasgupta and Stiglitz (1972)].

The required rank condition is mild when producers face no quantity constraints. For a producer facing quantity constraints the matrix $\partial y^g/\partial p$ has at least as many zero columns as there are binding quotas; but the above rank condition will remain weak provided that for each good there is at least one unrationed producer. For further references on aggregate production efficiency see Stiglitz and Dasgupta (1971), Mirrlees (1972) and Roberts (1978).

2.3.3. *Efficiency of public production and consumption*

By an analysis similar to that of the preceding sections, one can examine the conditions of efficiency between the public sector and private consumers. The details are left to the reader, who is also referred to Guesnerie (1979). The following results should be intuitively clear from our previous discussion.

(1) If the planner can directly control the consumption plan of a consumer, he should choose it so as to equalise the ratios of the marginal utilities of different commodities for that consumer to shadow price ratios ($U_i^h/U_j^h = \nu_i/\nu_j$, for all i, j).

(2) When the planner has an effective ability (e.g. using taxes and lump-sum transfers) to manipulate private consumption plans without directly affecting producers' behaviour, shadow price ratios will coincide with marginal rates of substitution for each consumer; as a corollary, optimal public production will involve equating marginal rates of transformation in the public sector with the common marginal rates of substitution for consumers.

The latter property has been called "*C-C* efficiency" by Guesnerie (1979) – see also Hammond (1980). As a simple example, let $(r^h), (t_i)$ all be unrestricted signals. Equation (2.19) then implies:

$$\beta^h - \nu\frac{\partial x^h}{\partial m^h} = 0, \quad \text{for all } h, \tag{2.53}$$

$$-\sum_h \beta^h x_i^h - \nu\frac{\partial x}{\partial q_i} = 0, \quad \text{for all } i. \tag{2.54}$$

Substituting the Slutsky identity in (2.54) and using (2.53) we obtain:

$$\nu \frac{\partial \hat{x}}{\partial q} = 0, \tag{2.55}$$

where $\hat{x} = \sum_h \hat{x}^h$ and $\hat{x}^h(q, \bar{x}^h, u^h)$ is h's compensated demand vector. Moreover, the homogeneity of compensated demands implies:

$$q \frac{\partial \hat{x}}{\partial q} = 0. \tag{2.56}$$

Under a mild rank condition (see the discussion in Section 2.3.2) and a suitable normalisation rule, (2.55) and (2.56) imply $\nu = q$. As a corollary, if consumers face no quantity constraints, *C-C* efficiency is desirable. Notice, incidentally, that when $\nu = q$, $\nu(\partial x^h/\partial m^h) = 1$ for all h, and so from (2.53), $\beta^h = 1$: all consumers have identical welfare weights at s^*.

In this illustration the optimality of individual lump-sum transfers plays a crucial role. However, when restrictions are placed on the structure of preferences the same results can be derived under much weaker assumptions concerning the set of available transfers. This principle is familiar from the literature on optimal taxation. For example if all households have the same linear Engel curves, and differ only in their earning power (and labour is separable from other goods) then the optimality of a poll tax or transfer will imply here that consumer prices are proportional to shadow prices. The proof follows from (2.53) and (2.54) using an argument analogous to Deaton (1979). Similarly if Engel curves for different households are linear and parallel but their intercepts depend only on observable household characteristics, or unobservable characteristics uncorrelated with welfare weights, then optimal transfers based on observable household characteristics will imply that consumer prices are proportional to shadow prices [this is an extension of Deaton and Stern (1986)].

It should be noted that all the results of this section hold irrespective of non-convexities in private production, and this considerably enhances their usefulness.

2.3.4. *Efficiency of public production and trade*

If the conditions under which aggregate production efficiency is desirable are in general rather restrictive, it will often be much less far-fetched to assume that the planner has substantial control over foreign trade. When $n_i \equiv \bar{y}_i^f$ is unrestricted [implying that $\bar{y}_i^f$ enters the Lagrangian (2.18) only through $y^f(\cdot)$] the first-order conditions (2.19) include [recalling (2.4)]:

$$\nu_i \frac{\partial y_i^f}{\partial \bar{y}_i^f} + \nu_f \frac{\partial y_f^f}{\partial \bar{y}_i^f} = 0 \tag{2.57}$$

or

$$\nu_i = \nu_f p_i^f, \tag{2.58}$$

i.e. the shadow price of the ith commodity is simply its world price (or marginal cost) multiplied by the marginal social value of foreign exchange (ν_f); we shall call this the "border marginal cost rule".

Notice that this result holds irrespective of the form of the side constraints, provided these do not involve n_i so that this control variable remains unrestricted. It is also easy to see that ν_i will remain proportional to p_i^f *even* if the side constraints involve n_i, *provided that* the effects they capture operate only through the balance of payments y_f^f.

There are two important ways of interpreting the appearance of n_i as a control variable of the planner's problem (in the language of Section 2.1.3 we can think of n_i as being either directly controlled or implicitly controlled). The first is where the ith commodity is traded under a quota, and this quota can be chosen optimally. In this case, (2.58) is perhaps more naturally interpreted as an optimality rule for choosing the appropriate quota (see Section 2.1.3). Secondly, when the ith market clears by trade in the sense that the amount traded is endogenously determined from the scarcity constraints, the variable n_i can again be regarded as a control variable. The border marginal cost rule can then be interpreted as follows: since the effect of a small increase in the net public supply of commodity i is a corresponding reduction in its net imports, the social value of that commodity is its marginal cost (or revenue) in world trade, multiplied by the marginal social value of foreign exchange.

In both these examples, the result requires, in addition, that n_i should be unrestricted, i.e. it should not interfere with the side constraints. A simple counterexample would occur when an exogenous tariff t_i^f applies and the world price of commodity i depends on the quantity traded, implying a side constraint linking p_i, t_i^f and n_i.

2.3.5. *Constant returns to scale*

Constant returns to scale is a characteristic of the technology, and by itself it has no immediate implication for the values of shadow prices, which involve not only the fixed characteristics of the economy, but also policies and the behaviour of agents. It has been suggested, however, that under some simple assumptions (e.g. competition among private producers), the existence of constant returns in the private sector implies a close relationship between shadow prices and producer prices [see Diamond and Mirrlees (1976)]. We shall examine this idea in our model.

Consider a private firm, say g, operating under constant returns to scale (i.e. Y^g is a cone). If this firm faces no quantity constraint, its profit-maximising production plan will be either zero or indeterminate. When strictly positive profits are possible it would want to expand production without limit. If profits are negative at any positive scale it would want zero output. If maximum profits are zero, as they are in the standard characterisation of competitive equilibrium with constant returns to scale, any scalar multiple of a profit-maximising production plan is also profit maximising. The possible indeterminacy of the production plan will be resolved, however, if the firm faces a binding quantity constraint, say $\bar{y}_1^g$, which may be called its *scale factor*.

Now suppose that this scale factor is a control variable. This would be the case, for instance, if firm g is the sole producer of good 1, and adjusts its output to the demand it faces (i.e. $\bar{y}_1^g$ is an endogenous variable); or if the planner can directly manipulate the gth firm's output of good 1. If, in addition, $\bar{y}_1^g$ is unrestricted, then the first-order conditions (2.19) imply:

$$v\frac{\partial y^g}{\partial \bar{y}_1^g} = -b^g\left(p\frac{\partial y^g}{\partial \bar{y}_1^g}\right),$$

where $b^g \equiv \sum_h \theta^{gh} b^h$ and b^h is as in (2.23). If firm g faces no quantity constraint other than the scale factor then this can be rewritten as

$$vy^g = -b^g(py^g). \tag{2.59}$$

Moreover, if firm g makes zero profit then

$$vy^g = 0. \tag{2.60}$$

This expresses the well-known Diamond and Mirrlees (1976) result according to which the production plan of a firm operating under constant returns breaks even at shadow prices. This result also holds if firm g makes positive profits provided that these profits are fully or optimally taxed; this follows immediately from (2.59) with $b^g = 0$.

As we have seen it is common in models where some firms have constant returns to scale to impose the condition that profits of such firms are zero (the "no pure-profit condition"). It is important to note that side constraints of the kind $\Pi^g(\cdot) = 0$ do not prevent the scale factors of the corresponding firms from being unrestricted since these constraints can always be stated in terms of *unit* profits.

The Diamond–Mirrlees result is trivial when the firm under consideration operates at zero scale. In its non-trivial form, the result appears quite general, but its validity relies on the following conditions being satisfied. First, the firm under consideration must operate at a non-zero scale at the equilibrium s^*. Second, the scale factor for this firm should be a control variable; this may require the planner being able to control the distribution of output between firms within industries, particularly if (2.60) is considered to hold for many firms. Finally, the firm should not face any quantity constraints other than that imposed by its scale factor.

As a corollary of the above result, if (2.60) holds for $(I-1)$ firms with linearly independent production plans and zero profits, ν is proportional to p [Diamond and Mirrlees (1976)]. The circumstances under which this assumption is likely to be satisfied, however, have not been thoroughly elucidated, and the discussion above should make clear that they may be quite restrictive. The simplest example of an economy where the required conditions are likely to be met is one where the assumptions of the non-substitution theorem (constant returns, no joint production, and a single scarce factor) are satisfied, and the private sector is competitive. The details are left to the reader.

As a final remark, notice that the equality of producer prices and shadow prices can also be brought about by a suitable collection of optimal scale factors for constant returns industries and optimal taxes. If firm g operates under constant returns with zero profits and its scale factor is unrestricted, we have, as before (2.60):

$$\nu y^g = 0.$$

On the other hand, if both p_i and t_i are unrestricted, and profits are fully (or optimally) taxed, we obtain, as in (2.51):

$$\nu \frac{\partial y}{\partial p_i} = 0. \tag{2.61}$$

Both these equations are also satisfied by the producer price vector. Thus if $(I-1)$ of them hold (and are linearly independent), ν will be proportional to p. It should be noted that the assumption of unrestricted p_i is somewhat implausible in this context, since the "no pure-profit" condition for constant returns firms will usually amount to side constraints involving prices. It can be shown, however, that the result holds if there are no *further* side constraints and, in addition, *each* p_i and t_i is a control variable. To see this, rewrite (2.61) as

$$\nu \frac{\partial y}{\partial p_i} + \sum_{g \in C} \alpha^g y_i^g = 0$$

where C is the set of constant returns firms and α^g is the Lagrange multiplier associated with the no pure-profit condition applying to firm g. Post-multiplying by ν and using $\nu y^g = 0$ ($g \in C$) we obtain

$$\nu \frac{\partial y}{\partial p} \nu = 0.$$

When the matrix $\partial y/\partial p$ has rank $(I-1)$, the latter implies that ν is collinear to p since $\partial y/\partial p$ is a positive semi-definite matrix.

2.3.6. Conclusion

Two elementary lessons seem to emerge from the results of this sub-section. First, there is little point in arguing in a vacuum about the relative weights one should give to producer prices, consumer prices or world prices in shadow pricing formulae; a useful discussion of such issues should involve an understanding of the functioning of the economy and an explicit recognition of the constraints and objectives relevant to the decisions of the planner. Second, the conditions under which shadow pricing rules of uniform simplicity are valid appear to be rather restrictive for practical purposes. In the next subsection we shall study the more general shadow pricing rules applicable in our model; their implications will be pursued in Section 3.

2.4. Shadow prices in distorted economies

In this section we study more explicitly the features of shadow prices and optimal policies in our model under alternative specifications of the vector of control variables. Our first task is to find the marginal social value of each of these variables (see Section 2.2.2), by differentiation of the Lagrangian of (P) (2.10):

$$MSV_{r^h} = \beta^h - \nu \frac{\partial x^h}{\partial m^h} \equiv b^h, \tag{2.62}$$

$$MSV_{p_i} = -\sum_h \beta^h x_i^h - \nu \left[\frac{\partial x}{\partial q_i} - \frac{\partial y}{\partial p_i} \right] + \sum_g \sum_h \theta^{gh} b^h \frac{\partial \Pi^g}{\partial p_i}, \tag{2.63}$$

$$MSV_{t_i} = -\sum_h \beta^h x_i^h - \nu \frac{\partial x}{\partial q_i}, \tag{2.64}$$

$$MSV_{\bar{y}_i^g} = \nu \frac{\partial y^g}{\partial \bar{y}_i^g} + \sum_h \theta^{gh} b^h \frac{\partial \Pi^g}{\partial \bar{y}_i^g}, \tag{2.65}$$

$$MSV_{n_i} = \nu_i - \nu_f p_i^f, \tag{2.66}$$

$$MSV_{\bar{x}_i^h} = \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial \bar{x}_i^h} - \nu \frac{\partial x^h}{\partial \bar{x}_i^h}. \tag{2.67}$$

Each of these expressions holds with arbitrary side constraints, as long as the control variable (s_k) being considered is unrestricted. The same expressions hold for the marginal social value of each predetermined variable (ω_k) which does not (locally) affect the side constraints.

By setting the marginal social value of control variables to zero, we can study shadow pricing rules as well as optimal policy rules. By studying the marginal social value of predetermined variables, we could also study directions of welfare-improving reforms (see Section 1.5) – this, however, is not our purpose here.

In the remainder of Section 2.4 we investigate optimal policies and shadow prices by examining the first-order conditions that marginal social values for control variables should be zero.

2.4.1. *Unrestricted producer prices: The Ramsey–Boiteux rule*

If the ith producer price is an unrestricted control variable its marginal social value (2.63) should be zero. As explained in Section 2.1.3, the notion that a price is a control variable can be seen either as an assumption that it can be set optimally or that the price adjusts to clear the market (for example we may consider a traded good with an optimal tariff or a non-traded good with a market-clearing price). Accordingly, the first-order conditions may be interpreted either as optimality rules or as expressions to be satisfied by shadow prices.

If the marginal social value of p_i in (2.63) is zero we have

$$-\sum_h \beta^h x_i^h - \nu\left[\frac{\partial x}{\partial q_i} - \frac{\partial y}{\partial p_i}\right] + \sum_g b^g y_i^g = 0, \tag{2.68}$$

where, as before, $b^g \equiv \sum_h \theta^{gh} b^h$, and b^h is given in (2.62). Note that b^g can be interpreted as the marginal social value of a lump-sum tax on the profits of the gth firm. The first term on the l.h.s. of equation (2.68) embodies the direct effect on consumers of the price increase, the second the social cost of meeting the extra net demands induced and the third the social value of the extra profits generated. Using the decomposition of consumer demands into income and substitution effects one can easily derive

$$\nu\left[\frac{\partial y}{\partial p_i} - \frac{\partial \hat{x}}{\partial q_i}\right] = d_i, \tag{2.69}$$

where $d_i = \sum_h b^h x_i^h - \sum_g b^g y_i^g$ and may be viewed as a "net distributional characteristic" of the ith commodity. This net distributional characteristic captures the marginal social value of all the income effects associated with a marginal change in the ith price. With optimal lump-sum transfers for consumers, each b^h is zero so that d_i is zero for each i. Note that, if a commodity is rationed for every private agent, the left-hand side of (2.69) vanishes; in other words, the rule for optimal pricing of a rationed commodity is simply $d_i = 0$ (this is because the price charged on a rationed commodity affects its users in the same manner as a lump-sum tax). More generally, d_i will be higher, the more it is consumed by "transfer-deserving" household (with $b^h > 0$) and produced by firms whose profits should be more heavily taxed ($b^g < 0$).

Viewed as a shadow pricing rule, (2.69) suggests (intuitively speaking) that if the ith market clears by price adjustment, then, other things being equal, the shadow price of the ith good will be greater: (i) the greater is the distributional term d_i, and (ii) the more (less) socially valuable are its substitutes (complements). More precisely, (2.69) provides a formula for evaluating the marginal social value of the general equilibrium effects of a small change in the ith price; these effects are "broken down" into compensated changes in net commodity demands and changes in real income.

Equation (2.69) is sometimes written as

$$\tau^c \frac{\partial \hat{x}_i}{\partial q} + \tau^p \frac{\partial y_i}{\partial p} = d_i, \tag{2.70}$$

using the symmetry of the Slutsky matrix and the net supply derivatives, the homogeneity of $\hat{x}_i(\cdot)$ and $y_i(\cdot)$, and the definition of τ^c and τ^p as shadow taxes $(q - \nu)$ and $(\nu - p)$, respectively. A number of variants of this formula have been investigated in the second-best literature on optimal production and pricing and synthesised under the name of Ramsey–Boiteux rules by Guesnerie (1980). To bring out further the implications of Ramsey–Boiteux rules viewed as shadow pricing rules, let us first ignore the distributional effects by assuming that $d_i = 0$. Equation (2.69) can then be re-written as

$$\nu_i = \alpha_i MSC_i + (1 - \alpha_i)\widehat{MSC}_i \tag{2.71}$$

with

$$MSC_i \equiv - \sum_{k \neq i} \nu_k \frac{\partial y_k}{\partial p_i} \Big/ \frac{\partial y_i}{\partial p_i} \tag{2.71a}$$

$$\widehat{MSC}_i \equiv - \sum_{k \neq i} \nu_k \frac{\partial \hat{x}_k}{\partial p_i} \Big/ \frac{\partial \hat{x}_i}{\partial p_i} \tag{2.71b}$$

and

$$\alpha_i = \frac{\partial y_i}{\partial p_i} \left[\frac{\partial y_i}{\partial p_i} - \frac{\partial \hat{x}_i}{\partial p_i} \right]^{-1} \tag{2.71c}$$

(note that α_i lies between 0 and 1). We call MSC_i the *marginal social cost* of the ith commodity: it is the cost at shadow prices of the inputs involved in the production of an extra unit of the ith commodity brought about by a change in price. Symmetrically if the ith good is an input we can think of MSC_i as its *marginal social product*. The definition of $\widetilde{MSC}_i$ is analogous: it measures the social value of the commodities involved in compensating consumers for a unit loss of the ith good. Equation (2.71) then expresses the ith shadow price as a weighted average of MSC_i and $\widetilde{MSC}_i$, the weights being given by the proportion of the extra unit coming from (or going to) each source (destination) as determined by the price elasticities.

Whilst (2.71) provides an interesting interpretation of ν_i, taken by itself it does not allow us to calculate ν_i since the right-hand side includes other shadow prices – as usual all shadow prices are determined simultaneously. The greater the number of goods for which (2.71) applies the more we can deduce from this formula about the vector of shadow prices. Thus, consider the case where (2.70) holds for each i: this would apply for instance in a competitive economy where prices clear markets but taxes and transfers are exogenous. Let us assume for simplicity that at least one commodity – say the first – is untaxed ($q_1 = p_1$) at the optimum, and use the normalisation $\nu_1 = p_1$. A few manipulations of (2.70) then give

$$\tilde{\nu} = \tilde{p}\Gamma + \tilde{q}(I - \Gamma), \tag{2.72}$$

with

$$\Gamma \equiv Y_p(Y_p - X_q)^{-1} \equiv I + X_q(Y_p - X_q)^{-1},$$

where a tilde above a vector indicates the deletion of its first element, and X_q and Y_p denote, respectively, the matrices $\partial\hat{x}/\partial q$ and $\partial y/\partial p$ deprived of their first row and column (the invertibility of $(Y_p - X_q)$ involves a mild rank condition; without the truncation the corresponding $I \times I$ matrix could not be inverted – this follows from (2.69) with $d_i = 0$ for each i).

In equation (2.72) the shadow price vector appears as a generalised weighted average of producer and consumer prices, where the weights are computed simultaneously from all price elasticities. Notice that Γ is the product of two positive semi-definite matrices, and so is $I - \Gamma$ (e.g. weights are non-negative in the two-commodity case).

We have seen in (2.71) and (2.72) two weighted average rules, the former averaging social costs on the production and consumption side for a single commodity and the latter producer and consumer price vectors. Weighted average rules have been popular in both the theoretical and applied literature and we have shown that some such rules can be given a general equilibrium foundation. The general equilibirum rules are not simple, however, and care has to be exercised in deriving short cuts or "approximations" for empirical applications.

Moreover the weighted average rules explicitly omit the distributional terms d_i. The importance of paying attention to the distributional aspects can be seen from the following example. Where p_i adjusts to clear each market it is immediate from (2.69) that if $p = q$ and d_i is different from zero for some i then $\nu \neq p$. Thus even in a perfectly competitive economy without indirect taxes market prices will generally be inadequate as shadow prices because they ignore distributional considerations.

In deriving the second "generalised weighted average" rule, we have assumed producer prices to be "unrestricted" for *each* commodity. This may appear as a stringent assumption when some commodities are traded and their domestic price depends on their world price. In Section 3.3 we shall see that under certain conditions the Ramsey–Boiteux rule, applied to *non-traded* goods exclusively, has a particular solution of the form

$$\nu_i = p_i + \frac{\mathrm{d}T}{\mathrm{d}z_i} \quad \text{(each } i \text{ non-traded)}$$

where $\mathrm{d}T/\mathrm{d}z_i$ is the *general equilibrium* effect on tax and tariff revenue of a small change in the net supply of good i in the public sector.

Whilst Ramsey–Boiteux rules have attracted a great deal of attention in the second-best literature, project evaluation manuals, on the other hand, often lack full rigour in their treatment of prices, e.g. by assuming fixed producer prices or relying on partial equilibrium analyses of the effects of price changes. A better integration between these two lines of research is an important step towards the understanding of shadow pricing rules in economies where some important markets (such as food grains) are fairly competitive and may be considered to clear by price adjustment.

2.4.2. Unrestricted commodity taxes: The generalised Ramsey rule

When the ith tax is unrestricted, the equation $MSV_{t_i} = 0$ holds, implying, using the same derivation as for (2.70):

$$\tau^c \frac{\partial \hat{x}_i}{\partial q} = \sum_h b^h x_i^h, \tag{2.73}$$

which may be interpreted along the same lines. As an illustration, if shadow prices are proportional to producer prices (as in the Diamond–Mirrlees model mentioned in Section 2.3.2), then (2.73) immediately reduces to the well-known "many-person Ramsey rule" for optimal taxation [see Diamond (1975)], namely

$$t\frac{\partial \hat{x}_i}{\partial q} = \sum_h b^h x_i^h. \tag{2.74}$$

Its generalisation in (2.73) is of striking simplicity, in spite of the much greater variety of models in which it is applicable. This illustrates the power of shadow prices in summarising much of the information contained in an economic model: when markets clear according to rules different from those applying in the Diamond–Mirrlees economy, shadow prices will deviate from producer prices, but, in a large class of models, the optimal taxation formula (2.73) will remain valid. Accordingly, (2.73) perhaps deserves to be called the *generalised Ramsey rule*.

Since the generalised Ramsey rule is in effect a special case of the Ramsey–Boiteux rule (the two are equivalent when all private firms are fully controlled), we shall not analyse it separately. To conclude our outline of optimal pricing rules, however, let us consider briefly the case where both p_i and t_i are unrestricted. Letting $MSV_{p_i} = MSV_{t_i} = 0$ gives (2.73) and, in addition:

$$v\frac{\partial y}{\partial p_i} = -\sum_g b^g y_i^g. \tag{2.75}$$

To interpret this, define the *marginal cost* (or marginal product, if $y_i^g < 0$) of commodity i in firm g as

$$MC_i^g \equiv -\sum_{j\neq i} p_j \frac{\partial y_j^g}{\partial \bar{y}_i^g}, \qquad \text{if } i \text{ is rationed for } g, \tag{2.76}$$

$$MC_i^g \equiv -\sum_{j\neq i} p_j \frac{\partial y_j^g/\partial p_i}{\partial y_i^g/\partial p_i}, \quad \text{otherwise.} \tag{2.77}$$

The *marginal social cost* (or marginal social product) of good i in firm g, denoted by MSC_i^g (or MSP_i^g), is defined similarly, by weighting the derivatives on the r.h.s. of (2.76) and (2.77) by shadow prices rather than by producer prices. Using (2.77), (2.75) becomes

$$\sum_g (v_i - MSC_i^g)\frac{\partial y_i^g}{\partial p_i} = -\sum_g b^g y_i^g \tag{2.78}$$

or

$$v_i = \sum_g \frac{\partial y_i^g/\partial p_i}{\partial y_i/\partial p_i} MSC_i^g - \frac{\sum_g b^g y_i^g}{\partial y_i/\partial p_i}. \tag{2.79}$$

where the first term on the r.h.s. of (2.79) is precisely the marginal social cost of good i as defined in (2.71a).

Thus, in this model, when both p_i and t_i are control variables (e.g. the ith commodity is optimally taxed, and exchanged at a market-clearing price) and unrestricted, the shadow price of the ith good reduces to its marginal social cost (suitably averaged over firms), corrected by a distributional term; the latter follows a simple inverse-elasticity rule, and vanishes if profits are fully or optimally taxed.

A similar marginal social cost rule applies when a producer ration is unrestricted. This brings us to (2.65).

2.4.3. *Unrestricted producer rations: The marginal social cost rule*

When $\bar{y}_i^g$ is unrestricted, $MSV_{\bar{y}_i^g} = 0$, and so, from (2.65):

$$0 = v\frac{\partial y^g}{\partial \bar{y}_i^g} + b^g\frac{\partial \Pi^g}{\partial \bar{y}_i^g} = v_i - MSC_i^g + b^g(p_i - MC_i^g) \tag{2.80}$$

or

$$v_i = MSC_i^g - b^g(p_i - MC_i^g), \tag{2.81}$$

which is another variety of marginal social cost rule. The shadow price of good i is its marginal social cost less the social value of the extra profits. Recall that b^g will be negative where the government would like to tax profits more heavily.

When $\bar{y}_i^g$ is considered as a policy instrument, (2.81) is perhaps best interpreted as a decision rule for choosing an optimal quota applying to firm g. On the other hand, if $\bar{y}_i^g$ is regarded as an endogenous variable (e.g. in a model with price rigidities and quantity rationing), (2.81), viewed as a shadow pricing rule, reflects the fact that a small increase in the net public supply of good i leads to a corresponding reduction of its net output in firm g. An important application of this arises with firms in "Keynesian equilibrium"; for such a firm output adjusts to demand and $\bar{y}_i^g$ (where i indexes the output of the firm) is an endogenous variable in the model – hence (2.81) applies. In such an equilibrium, is it correct to say that a commodity in excess supply ($p_i > MC_i^g$) should be devalued in the shadow price system relative to the market price system? This notion is intuitively appealing and not completely devoid of content, but as usual general equilibrium effects can invalidate it. Thus when $b^g = 0$ we can write (2.81) as

$$\frac{v_i}{p_i} = \left(\sum_{j\neq 1}\alpha_j\frac{v_j}{p_j}\right)\left(\frac{MC_i^g}{p_i}\right) < \sum_{j\neq i}\alpha_j\frac{v_j}{p_j}$$

where the α_j's ($j \neq i$) add to one and represent the shares of each input in the marginal cost of good i. Thus the "accounting ratio" (ν_i/p_i) of good i is *lower* than a weighted average of the accounting ratios of its inputs by a fraction measuring the discrepancy between marginal cost and price. In particular if the production of good i involves a single input (say k), the price of i relative to k will be lower in the shadow price system than in the producer price system. Further results of this kind have been explored in the literature on cost–benefit analysis in "general disequilibrium" (see references below).

The analogy between (2.79) and (2.81) brings out the potential equivalence between price and quantity instruments (here taxes and quotas) as well as its limits: on the one hand, taxes and quotas have different distributional implications; on the other hand, they may have differing degrees of effectiveness, e.g. according to the extent to which they can be individualised.

2.4.4. *Unrestricted consumer rations and the marginal willingness-to-pay*

Quantity constraints in consumption are not quite so easy to analyse as quantity constraints in production. To interpret (2.67), let us define the "pure substitution effect" of a small change in h's ration of good i on his net demand for good j as

$$\sigma_{ji}^h \equiv \frac{\partial x_j^h}{\partial \bar{x}_i^h} + q_i \frac{\partial x_j^h}{\partial m^h}. \tag{2.82}$$

This measures the effect on h's demand for good j of a small gift of the ith good, and it can be shown that, if h's utility is weakly separable between good i and other goods, then $\sigma_{ji}^h = 0$, for all $j \neq i$. Using (2.82), the equation $MSV_{\bar{x}_i^h} = 0$ becomes

$$0 = \beta^h(\rho_i^h - q_i) - \nu_i - \sum_{j \neq i} \nu_j \left(\frac{\partial x_j^h}{\partial \bar{x}_i^h} \right) \tag{2.83}$$

$$= -\nu_i + \beta^h(\rho_i^h - q_i) + \sum_{j \neq i} \nu_j \left(\sigma_{ji}^h - q_i \frac{\partial x_j^h}{\partial m^h} \right) \tag{2.84}$$

or (using $\sum_{j \neq i} q_j \sigma_{ji}^h = 0$)

$$\nu_i = \beta^h \rho_i^h - b^h q_i + \sum_{j \neq i} \tau_j^c \sigma_{ji}^h, \tag{2.85}$$

where ρ_i^h is h's marginal willingness-to-pay for good i:

$$\rho_i^h \equiv \frac{\partial U^h / \partial x_i^h}{\partial V^h / \partial m^h}.$$

In (2.85) the ith shadow price appears as the sum of three terms which clearly reflect the various effects of a marginal change in h's quota. The first is the hth consumer's welfare-weighted marginal willingness-to-pay for good i; the second is the net social value of the payment made by consumer h in return for an extra "ration ticket" (notice that this payment is precisely equivalent to a lump-sum tax); the third measures the net social cost of the change in h's consumption pattern through pure substitution effects, and vanishes when good i is weakly separable from other goods in h's utility.

An interesting and important application of (2.85) concerns non-marketed goods. Consider, for instance, a pure public good, produced exclusively in the public sector, and available uniformly to all consumers. This situation may be formalised by putting $q_i = 0$ and $\bar{x}_i^h \equiv \bar{x}_i$, for all h for that commodity. Setting $MSV_{\bar{x}_i} = 0$, we obtain, much as in (2.85):

$$\nu_i = \sum_h \beta^h \rho_i^h + \sum_{j \neq i} \tau_j^c \sigma_{ji}. \tag{2.86}$$

where $\sigma_{ji} \equiv \Sigma_h \sigma_{ji}^h$. In this model, then, the shadow price of a pure public good simply consists of a welfare-weighted sum of consumers' marginal willingness-to-pay, plus the induced contribution to shadow tax revenue through pure substitution effects; the latter term is zero if separability holds between public and private goods – see also Diamond and Mirrlees (1971, p. 271).

An application of the theory of cost–benefit analysis with consumer and producer rationing which is of particular significance is that developed in recent models of macro-economic disequilibrium. Some examples include Dixit (1976b), Drèze (1984), Johansson (1982), Marchand, Mintz and Pestieau (1983), and Roberts (1982).

2.5. Conclusion

The shadow pricing rules discussed in this section have no special claim at universality. In particular situations there is no alternative, in principle, to identifying carefully the functioning of the economy, the planner's objectives, his instruments and the constraints circumscribing their use, and then deriving the appropriate shadow pricing formulae. It is useful, however, to investigate the extent to which one can derive reasonable rules-of-thumb or guidelines based on characteristics showed by the economies in which cost–benefit techniques are to be applied. This enquiry has motivated some influential writings in the project evaluation literature [e.g. the Little–Mirrlees method (1974), and the UNIDO Guidelines – Dasgupta, Marglin and Sen (1972)]; the remainder of this chapter is devoted to a more informal discussion of some selected aspects of this area of research in the light of the theoretical results obtained above.

3. Selected applications of the theory

3.1. Introduction

In the previous section we have set out the theory of shadow prices in a fairly general model. The purpose of the present section is to show how this theory can throw light on a number of issues facing the cost–benefit analyst in practice. In Section 3.2 we discuss the role of the distribution of welfare or income. We argue that its explicit recognition should not be avoided and further illustrate how one could carry out a discussion of the value judgements necessary to evaluate changes in welfare for different households.

The treatment of shadow prices for traded and non-traded goods has been a central issue in discussions of cost–benefit analysis and planning in developing countries in the 1970s [e.g. Little and Mirrlees (1974)]. In Section 3.3 we apply the theory of Section 2 to characterise the circumstances which could justify using the widely debated rule that shadow prices for traded goods should be world or border prices. We examine also the common suggestion that shadow prices for non-traded goods should be equal to their marginal cost of production at shadow prices. And we discuss briefly the distinction between traded and non-traded goods.

Another major topic in the literature is shadow wages. Much of that discussion has used very aggregated models and we show in Section 3.4 that many of the basic ideas can fit into a general equilibrium framework of the kind we have been using.

In Section 3.5 we examine numeraires, discount rates, and foreign exchange. These topics are treated together and we shall emphasise their interrelatedness. Thus, as we shall see, the discount rate cannot be defined without the specification of the numeraire. And the value of particular premia (e.g. on foreign exchange) will depend on the numeraire chosen for the evaluation. Furthermore, the choice of discount rate is closely related to the issue of how such premia move over time. We also see how simple rules for deciding on the discount rate (for example linking it to a particular interest rate or rate or return) correspond to different assumptions about the opportunities and constraints faced by the planner.

We show in Section 3.6 how the analysis of shadow prices for public sector firms may be extended in a direct and natural way to private sector firms or public firms which are not fully controlled.

We examine in Section 3.7 how time and uncertainty can be treated more explicitly in our model. From the formal point of view we can use much the same general framework although the choice of the particular structure of the model for any application would often, particularly in the case of uncertainty, raise quite severe difficulties.

The final subsection of Section 3 groups together a number of topics which are important but which have not received special attention here such as public goods, externalities, and large projects.

The notation in this section is consistent with Section 2 but we shall also introduce further symbols as necessary for particular models.

3.2. The distribution of welfare

We argued in Section 2 that the social welfare function, interpreted as the objective function of the planner, should play a central role in the theory of shadow prices. In the expressions involving shadow prices the particular aspect of the welfare function which was prominent was the collection of social marginal utilities of income or welfare weights (β^h). In this subsection we first examine (Section 3.2.1) ways which have been suggested of avoiding the use of the social welfare function, and then see (Section 3.2.2) how one might discuss and specify a social welfare function or set of weights.

In practice the analyst should present results involving a selection of possible social welfare functions. This would be necessary not only because individuals or policy-makers will differ in, or be unsure of, their values but also because one would wish to know the sensitivity of shadow prices with respect to values. This sensitivity would indicate how important it would be to specify values precisely or to resolve differences between those who do not seem to agree. For some shadow prices welfare weights will not matter, for example where shadow prices are world prices; but for others the welfare weights may be very important indeed – e.g. see below on the shadow wage.

3.2.1. "Avoiding" value judgements

One can imagine four ways of apparently avoiding the use in cost–benefit analysis of judgements concerning distribution across households. First, one can attempt to identify actual Pareto improvements. Secondly, one can seek potential Pareto improvements. Thirdly, one can use criteria which are not based on household welfare. Fourthly, one can argue that distribution can be ignored in cost–benefit analysis because, if it is a problem, it should be dealt with using policy instruments other than public projects. An alternative and rather more fierce version of this fourth case is that the distribution of income is not an appropriate concern for the government. In practice this fourth view is translated as the assumption that money benefits or costs accruing to households can simply be added without weights. The third and fourth positions do not really dispense with the objective function but involve particular assumptions on its form. We examine these four viewpoints in turn.

The first approach is to seek to identify projects which yield Pareto improvements. Accordingly, for a given policy ϕ, a project evaluation criterion of the following kind might be proposed: accept $\mathrm{d}z$ if, for each h,

(A)

$$\frac{\partial V^h}{\partial s}\,\mathrm{d}s > 0,$$

where

$$\mathrm{d}s = \frac{\partial \phi}{\partial z}\,\mathrm{d}z.$$

This in effect amounts to requiring a project to pass a series of cost–benefit tests of the kind we have described in Section 1. We can think of a social welfare function which consists simply of the utility of household h and consider the associated shadow prices ν^h. The project increases the welfare of the hth household if $\nu^h \mathrm{d}z > 0$. Then a project yields a (strict) Pareto improvement if $\nu^h \mathrm{d}z > 0$ for each h. An immediate problem with this approach is that the collection of shadow prices which is needed to guide production decisions now contains as many elements as there are consumers, which is inconvenient for decentralisation. Furthermore, it could be that very few projects pass the test, thus effectively paralysing investment decisions.

The second approach consists of seeking potential Pareto improvements from an arbitrary initial situation s^0, z^0. Thus, a project $\mathrm{d}z$ would be deemed acceptable if there exists some compatible change $\mathrm{d}s$ for which (A) is satisfied. Compensation criteria of the Hicks–Kaldor type are familiar examples of this kind of criterion, which have also been explored by a number of other authors, e.g. Allais (1943), (1977), Debreu (1951), Diewert (1984) and Farrell (1957). On compensation criteria see, for example, the discussions in Graaff (1957), Boadway (1974), Bruce and Harris (1982) and Chipman and Moore (1979).

A few remarks may help to clarify the nature of this criterion. First we should note that if the initial environment s^0 is not Pareto efficient [given z^0 and the constraints in problem (P)], then *all* sufficiently small projects will pass the test. This should be obvious since the zero project ($\mathrm{d}z = 0$) passes the test and thus, with continuity, so will all small enough projects. Secondly, provided the initial environment is Pareto efficient no problems of circularity arise (i.e. if the move from z^0 to z^1 is compatible with a Pareto improvement, then the move from z^1 to z^0 cannot be); hence, the objections to the Hicks–Kaldor compensation criteria on this score are avoided. Thirdly, designing cost–benefit tests for this criterion raises similar difficulties to those indicated for actual Pareto improvements – except in rather special circumstances a single shadow price vector cannot provide such a test. An example of a model where a single cost–benefit test provides a sufficient condition for a potential Pareto improvement is Diewert (1983) – see also Dixit and Norman (1980, ch. 3).

An important and obvious ambiguity with the proposed criterion is that it remains vague on whether Pareto improvements will be actually implemented. If no such guarantee exists, then the criterion is certainly unacceptable.

In any case, a fundamental shortcoming of evaluation criteria based on Pareto improvements, whether actual or potential, is that, unless they are taken to imply that Pareto-improving changes are the only acceptable ones (a view which we regard as extremely unappealing and which attaches undue weight to the status quo), they provide no decision criterion for projects which cannot lead to Pareto improvements. It is difficult to overcome this problem without accepting the need to specify a social welfare function which embodies more definite judgements.

A third way of seemingly avoiding value judgements in project evaluation is to rely on objective functions which do not involve interpersonal comparisons of welfare. As we saw in Section 2.2.7, criteria based on statistics such as national income, indices of inequality, price indices, and unemployment rates are often most sensibly interpreted as deriving from the social welfare function. It is likely that other objectives which are often proposed, such as self-sufficiency, could also be derived in a similar framework but with a richer structure. For example, self-sufficiency may be seen as a derived objective when the external world subjects the economy to random shocks which are costly or difficult to control. Where the objectives are derived then they simply represent convenient means for summarising the criteria embodied in the social welfare function. Or the derived objectives may provide a useful means of discourse with those who are uncomfortable or unfamiliar with the notion of a social welfare function.

Where the objectives are not derived they must be justified in some other way and we suspect that often this would not be easy, at least in an ethically appealing manner. There may well be reasons for arguing, for example, that a society with high levels of unemployment is unsatisfactory. One would no doubt include the effects of unemployment on income distribution, the self-respect of the unemployed, or the availability of future skills; but most or all of these considerations operate through the level of welfare of current or future households. Nevertheless, it is quite possible that even after considerable reflection one would maintain that an objective was not derived in the sense we have described. For instance, one may wish to assert that every person has a right to gainful employment; a measure of the extent to which this right is satisfied, say the employment rate, may then be proposed as an objective. Even in this example interpersonal comparisons would seem to be involved since a measure such as the employment rate gives equal weight to the employment of different people. To summarise, whilst the idea of a social welfare function which dispenses with interpersonal comparisons of welfare is not, a priori, indefensible, in practice the range of situations in which it would be appealing is likely to be limited.

According to the fourth approach, we should ignore distribution either because it is not an appropriate concern for government or because distributional matters should be dealt with using other policy tools than public projects. These views are translated in practice into simply adding net money benefits, or willingness-to-pay, across households. The first version is, we hope, seen as clearly unattractive and irresponsible – at a minimum one would want to see governments accepting some obligation to the weak or disabled.

The second version represents a mistaken understanding of second-best welfare economics. It may be true that policy instruments exist – e.g. income taxes – which allow a more direct influence on the distribution of income than public projects; and the planner's model should include them. However, in general, redistributing income using these instruments will have social costs, and therefore the implications of projects for income distribution should not be ignored. For example the optimal non-linear income tax [see Mirrlees (1971), Stern (1976), or Christiansen (1981)] does not imply the equality of the social marginal utilities of income, β^h, across households: the disincentive aspects of the non-lump sum tax provide a reason for not fully equating them. Moreover, outside the fully competitive economy, even optimal lump-sum taxes ($b^h = 0$) do not equate β^h across households since the marginal social values of a transfer take into account the different social costs of different consumption patterns. It is noteworthy that whilst some non-economists, e.g. the lawyer Lord Roskill in chairing the enquiry into London's proposed third airport, refuse to accept weighting [see Roskill Commission (1971)], it has come to be accepted by some of those economists who had been its strongest opponents – see, for example, Harberger (1978) and Nwaneri (1970) and Layard, Squire and Harberger (1980).

3.2.2. The specification of value judgements

We are seeking here methods which can structure a discussion of value judgements so that we can clarify our ethical positions and translate them into a form which can be used directly in practical cost–benefit analysis. Value judgements, of course, have a subjective nature; but we can still, and should, discuss these judgements rationally and systematically, examine the use of evidence on the circumstances and feelings of individuals, and try to specify functions and parameters which capture our views. In this subsection we examine methods which involve inferring objectives from decisions. These decisions may be hypothetical or actual. In the former case we pose simple questions of the kind: "What would you do if asked to decide on policy in this simple problem?" An example would be whether or not to take an amount x from individual A in order to give an amount y to individual B. We can then use the values inferred in the simple case to characterise the social welfare function, or welfare weights, which will guide policy in more complicated cost–benefit models. In the case where actual decisions are used we have to model the problem perceived by the decision-maker so that we can infer his values from the decision actually taken. For example, the savings rate selected in a planned economy could be used to make inferences about judgements concerning distribution across generations; similarly, one might use the parameters of an income tax schedule to provide information on judgements across individuals. In the savings case one will have to model returns on investment, and in the income tax case responses to incentives, before one could use government decisions to infer the underlying values.

The procedure is familiar from demand analysis where individual preferences are inferred from consumption choices given a model of the budget constraint. The type of exercise we are proposing here is similar in spirit but involves the policy-maker rather than the individual. It is often called the "inverse optimum" problem. It might be argued that if the policy-maker is prepared to specify policies directly when given the structure of a model then it is unnecessary to introduce the optimising apparatus. This is to miss the point, however. We are treating simple cases where not only the formal problem can be solved easily but also one might expect to be able to make a direct judgement. This allows the inference of specific values which can be used in more complex problems.

The most straightforward way of thinking about welfare weights is to consider small imaginary transfers between different individuals. For instance, if a marginal transfer of α units to household h is judged to compensate exactly (from the point of view of social welfare, and all other signals being held constant) a loss of one unit of lump-sum income for household h', one may infer that

$$\alpha\beta^h - \beta^{h'} = 0. \tag{3.1}$$

Simple thought experiments of this kind can be used to generate the whole profile of welfare weights when the latter can be characterised by a relatively small number of parameters. Thus, consider a simple case where individuals are considered to differ significantly only in their lump-sum incomes (m^h) and supplies of a homogeneous type of labour indexed by l. If the SWF is anonymous, the welfare weights themselves would vary across individuals only according to these two factors, and we may write

$$\beta^h = \beta\left(m^h, x_l^h\right), \tag{3.2}$$

for some function β. Furthermore, if β can be approximated by a function of a simple form such as

$$\beta(\cdot) = \gamma\cdot\left(m^h\right)^{-\delta}\left(-x_l^h\right)^{-\varepsilon}, \tag{3.3}$$

then only two thought experiments of the above kind would yield all the relative welfare weights. Notice that in (3.3) the parameter δ is the familiar income elasticity of the social marginal utility of income – see Stern (1977).

Generally, of course, welfare weights will depend on the values taken by such signals as prices and rations – see, for example, Roberts (1980). As long as all consumers face the same signals, the above procedure remains valid for marginal changes around some initial environment. When discrete changes are considered, or when different individuals face different signals (e.g. different quantity constraints) or possess different characteristics (e.g. family structure), it will be necessary to take these additional factors into account as far as possible.

It is often helpful, in thinking about the specification of value judgements, to construct very simple models as "laboratories" for thought experiments. For example, one could pose a hypothetical optimal savings model where the government sets out to maximise

$$\int_0^\infty \frac{C^{1-\eta}}{1-\eta} e^{-\rho t} \mathrm{d}t \tag{3.4}$$

subject to a given $K(0)$ and $\dot{K} = rK - C$, where C is consumption, K is capital, r is a constant output–capital ratio and ρ a pure time discount rate. It is easy to show that the optimal savings ratio $(\dot{K}/rK)$ is $(1/\eta)$ $(1-\rho/r)$. Thus, if the government is prepared to judge the optimal savings rate given ρ and r, then we can immediately infer η. Depending on the interpretation of the model one can see η as reflecting distributional judgements across generations, or over time for given individuals. Under either view one can argue that distribution at a point in time embodies similar issues and regard η in an analogous manner to δ in (3.3).

The inverse optimum method can be applied to more complex problems, allowing, for instance, a greater number of parameters to be estimated. Its application to problems of optimal taxation and tax reform, in particular, has recently received close attention – see, for example, Guesnerie (1975), Ahmad and Stern (1984), Christiansen and Jansen (1978), and Stern (1977).

If the inverse optimum is to be used as a method for finding welfare weights it must be applied with considerable care. First, the calculated welfare weights may be sensitive to the model of the economy and to which tools are assumed optimally chosen. Secondly, the assumption that the government has optimised must be examined critically. One way of doing this would be to ask directly whether the calculated welfare weights correspond to plausible value judgements. One could go further and use inverse optimum calculations as part of a dialogue with the government concerning both its values and whether the current policies are optimum. Interpreted in this way, rather than as a mechanical device for deriving welfare weights, the inverse optimum exercise can be instructive.

In this section we have argued first that methods which avoid distributional value judgements are unsatisfactory in important respects, secondly, that such judgements can often be usefully embodied in welfare weights, and thirdly, that these welfare weights can be discussed and made explicit in rational and open ways.

3.3. Traded and non-traded goods

We examine in this subsection the implications of the results in Section 2 for the social valuation of traded and non-traded goods and for the classification between the two. These issues have been central in the discussion of cost–benefit methods, particularly those for developing countries – see, for example, Little and Mirrlees (1974), Dasgupta, Marglin and Sen (1972), and the *Bulletin of the Oxford University Institute of Economics and Statistics*, February 1972.

Our main result concerning the shadow price of traded goods was the border marginal cost rule (2.58), which says that the shadow price of the ith commodity is its world price or marginal cost multiplied by the marginal social value of foreign exchange:

$$v_i = v_f p_i^f. \tag{2.58}$$

This result holds whenever the variable $\bar{y}_i^f$ is an unrestricted control variable, i.e. the planner can (directly or implicitly) control the level of imports of the ith good and further $\bar{y}_i^f$ does not interfere with the side constraints.

As we have argued in Section 2, there are two kinds of circumstances where $\bar{y}_i^f$ will appear as a control variable in the planner's problem. In the first, $\bar{y}_i^f$ is interpreted as a market-clearing signal, i.e. trade adjusts endogenously to clear the ith market. In the second, the ith good is traded under a quota, but the planner can set the quota optimally. From a formal point of view there is no distinction between these two situations (see Section 2.1.3).

In both situations the validity of the result also depends, of course, on the variable $\bar{y}_i^f$ being "unrestricted". This does not necessarily preclude the existence of such things as tariffs and consumer or firm-specific quotas. What matters is that the rules by which these quotas and tariffs are set should not depend on $\bar{y}_i^f$. As a counter-example, suppose that an exogenous tariff $\bar{t}_i^f$ applies to good i, and that the world price of this good depends upon the quantity traded through the function $\psi(\cdot)$. The planner's problem would then involve the side constraint

$$p_i = p_f \cdot \psi(\bar{y}_i^f) + \bar{t}_i^f. \tag{3.5}$$

The first-order condition for $\bar{y}_i^f$ becomes, where μ is the Lagrange multiplier on (3.5):

$$\frac{\partial \mathscr{L}}{\partial \bar{y}_i^f} \equiv v_i + v_f \frac{\partial y_f^f}{\partial \bar{y}_i^f} + \mu p_f \frac{\partial \psi}{\partial \bar{y}_i^f} = 0. \tag{3.6}$$

We can readily interpret μ as the marginal social value of the tariff since $\mu \equiv \partial \mathscr{L}/\partial \bar{t}_i^f$. Equation (3.6) may be written as

$$\nu_i = \nu_f p_i^f - \mu p_f \frac{\partial \psi}{\partial \bar{y}_i^f}. \tag{3.7}$$

This differs from (2.58) unless either $\mu = 0$ (the tariff is optimal) or $\partial \psi / \partial \bar{y}_i^f = 0$ (the world price is fixed).

Intuitively, we can understand the border marginal cost rule as follows. The effect of an increase in the availability of an extra unit of the good in the economy can be treated simply as a reduction in net imports and an increase in foreign exchange earnings.

The conditions under which the border marginal cost rule is valid do not appear very restrictive where trade is market clearing and world prices are fixed. Where they are not fixed one requires in addition an optimal tariff or a flexible tariff which offsets domestically any change in the world price.

This rule has two great advantages. First, where world prices are fixed, it tells us quite a lot about shadow prices without needing to know a great deal about the structure of the economy, which may be very distorted. In this sense the rule is very robust. It is in part for this reason and because it is relatively easy to implement that this rule has been so popular as a basis for applied cost-benefit analysis. Secondly the rule holds commodity by commodity. When it holds for each traded commodity it can be seen as an example of the efficiency of the set of fully controlled firms (applied to the STC).

The border marginal cost rule obviously does not apply to goods which are traded under exogenous quotas, or to goods which are not traded at all. It is often suggested, for non-traded goods, that the shadow price should be based on the marginal social cost of production. We can examine this suggestion using the results of Section 2.4.

The analysis of Section 2.4 suggests two possible justifications for marginal social cost rules. The first relies on condition (2.79) for optimal (or endogenous) producer price:

$$\nu_i = \sum_g \frac{\partial y_i^g / \partial p_i}{\partial y_i / \partial p_i} MSC_i^g - \frac{\sum_g b^g y_i^g}{\partial y_i / \partial p_i}. \tag{2.79}$$

This result holds when p_i can be manipulated independently of q_i – e.g. if the tax t_i on commodity i can be chosen optimally, or if q_i is fixed exogenously. We may think of this rule intuitively as follows. Suppose an extra unit of commodity i is required and its price is increased to bring about its production. The first part of the cost is the marginal social cost of production, averaged appropriately, where the averaging is based on how much comes from each firm. The second part is an adjustment to take account of the social value of the change in profits brought about by the price increase – the price increase required to generate the extra unit is $(\partial y_i/\partial p_i)^{-1}$, the effect on profits of the gth firm of a unit price increase is y_i^g and the net social value of a unit increase in profits in firm g is b^g, which when combined and summed give the second term on the r.h.s. of (2.79).

The second potential justification for the marginal social cost rule arises when the gth producer's ration of good i is optimal (or endogenous, as when a firm produces according to demand). In that case, (2.81) applies:

$$v_i = MSC_i^g - b^g(p_i - MC_i^g). \tag{2.81}$$

The ith shadow price is then the marginal social cost of the extra resources required by firm g to increase the output of good i less the social benefit from increasing profits in firm g (by the excess of price over marginal cost).

The rule that shadow price equals marginal social cost for the ith good holds when the conditions for either (2.79) or (2.81) are satisfied and, in addition, the adjustment for changes in profits vanishes. This profit term is zero when profits are optimally or fully taxed ($b^g = 0$), or, in the case of (2.81), when price is equal to marginal cost. The application of (2.81) to the case of constant returns to scale has been discussed in Section 2.

These results suggest that the marginal social cost rule is not likely to be plausible for all non-traded goods. It is restrictive in its suggestion that the source of an extra unit of the ith good lies exclusively in extra production. When producer and consumer prices cannot be manipulated independently and price adjustments are an important element in market clearing, one should consider explicitly the effect on household demands and utilities of consumer price changes. The Ramsey–Boiteux rule (2.70) provides a means of weighting social costs arising on the production and consumption sides. It deserves to be given greater prominence in applied work on cost–benefit analysis.

An important alternative to the marginal social cost rule for non-traded goods is the following [an extension of Dinwiddy and Teal, (1987)]. Let us assume that (i) all consumers have identical welfare weights (though not necessarily identical preferences), normalised for convenience to unity; (ii) producers and consumers face no binding quantity constraints; (iii) taxes, tariffs and world prices are exogenous. Then we can write the total effect on social welfare of a project $\mathrm{d}z$ as

$$\begin{aligned}\mathrm{d}V &= q \cdot \mathrm{d}x \\ &= t \cdot \mathrm{d}x + p \cdot (\mathrm{d}y + \mathrm{d}z) \\ &= p \cdot \mathrm{d}z + t \cdot \mathrm{d}x + p \cdot \mathrm{d}y^f + \sum_{g \neq f} p \cdot \mathrm{d}y^g \\ &= p \cdot \mathrm{d}z + t \cdot \mathrm{d}x + p \cdot \mathrm{d}y^f\end{aligned}$$

using $p \cdot \mathrm{d}y^g = 0$ for $g \neq f$ from profit-maximisation. Hence, using $y^f \cdot \mathrm{d}p = 0$ (since the prices of traded goods are fixed):

$$\nu_i = p_i + \frac{\mathrm{d}T}{\mathrm{d}z_i}$$

where $T \equiv t \cdot x + p \cdot y^f$ is the sum of tax and tariff revenue (see (2.5)).

Interestingly the above rule seems applicable to traded goods as well as to commodities produced under constant returns, in which case it becomes *equivalent* to the border marginal cost rule (for traded goods) and the marginal social cost rule (for goods produced under constant returns)–whenever the latter rules are applicable. This equivalence, however, would not be easy to guess from the formula as it stands and this reminds us that the simplicity of this formula, and its intuitive appeal, are to some extent only apparent. In fact, the above rule embodies complicated general equilibrium effects and these are *important*.

The classification of commodities into traded and non-traded is a problem of substantial practical importance, especially in an intertemporal context; but we hope that our general discussion of the planner's model, as well as the more specific comments on the border marginal cost rule, make clear the theoretical principles which should underlie this classification. For instance, the implication of quotas for this classification depends crucially upon the control which the planner can exercise on them: if he can choose quotas unrestrictedly he should set them optimally along with adopting the border marginal cost rule; at another extreme, when quotas are exogenous from his point of view, he should consider the corresponding commodities as non-traded for the purposes of project evaluation. The classification can, of course, change over time, and in an intertemporal framework special difficulties will arise in specifying correctly future constraints and opportunities (e.g. movements in transport costs and quota restrictions); such difficulties, however, are a general feature of the planner's problem in its intertemporal interpretation. For further reading on the issues of this section see Little and Mirrlees (1974), Dasgupta, Marglin and Sen (1972), the symposium in the *Bulletin of the Oxford University Institute of Economics and Statistics*, February 1972, Bell and Devarajan (1983), and Kuyvenhoven (1978).

3.4. Shadow wages

The shadow price of a given type of labour is usually called a *shadow wage*. The theory of shadow wages therefore forms part of the general theory of shadow prices. However, shadow wages are often treated in either a partial equilibrium or a highly aggregated framework. In this subsection we provide some examples drawn from the model of Section 2 which will help indicate the important determinants of the shadow wage and relate our general results to some of the common notions appearing in simple models of shadow wage determination. We shall treat in turn the cases of a competitive labour market, involuntary unemployment with forced leisure and finally involuntary unemployment where there is residual absorption of labour in household activities.

Let l be the index for a particular type of labour service. If the market for this service is competitive, its price (wage), p_l, is a market-clearing variable, and (if p_l is also unrestricted) the Ramsey–Boiteux rule studied in Section 2.4.1 applies. For instance, if labour supplies are fixed (so that compensated consumer demands are inelastic with respect to q_l), we obtain from (2.69):

$$\nu_l = MSP_l + \frac{d_l}{\partial y_l/\partial p_l}, \tag{3.8}$$

where MSP_l is the marginal social product of labour in private enterprises [averaged over firms as in (2.79)], and, as before, $d_l = \sum_h b^h x_l^h - \sum_g b^g y_l^g$. The shadow wage here reduces to its marginal social product, corrected by a distributional term which follows a simple inverse-elasticity rule, and whose sign depends upon whether shareholders or labourers are relatively more "transfer-deserving". This is natural, since an increase in public sector employment resulting in an increase in wages will have both an allocative effect [reflected in the first term of (3.8)], and a distributional one (captured by the second term).

As a second example, consider a labour market with involuntary unemployment, which clears by rationing of supplies (the money wage being fixed, or in any case, related to signals other than the quantity of employment). For simplicity, assume that at the margin a single consumer, say h, is rationed in his supply, and that his utility is separable between leisure and consumption. Taking $\bar{x}_l^h$ as a market-clearing variable, we obtain at once from (2.85) (when $\bar{x}_l^h$ is unrestricted):

$$\nu_l = \beta^h \rho_l^h - b^h q_l, \tag{3.9}$$

where ρ_l^h is h's *reservation wage*, or disutility of labour in money terms (with $\rho_l^h < q_l$ since $\bar{x}_l^h$ is binding). This expression has a straightforward interpretation: in an economy of this type, where increased public employment affects leisure rather than employment elsewhere (by contrast with the first example), the shadow wage is equal to the (welfare-weighted) reservation wage, corrected by a distributional term which measures the marginal social value of the increase in income enjoyed by the extra labourer(s) hired (which here amounts to the wage rate). This simple shadow wage formula seems particularly relevant to some Western economies, where significant wage rigidities exist and a reduction in wage employment implies greater (forced) leisure rather than self-employment. The form of the enforced leisure postulated here is more relevant to an economy where labour is shared than to one where unemployment is a discrete phenomenon. Notice also that, with unemployment benefits, the second term in (3.9) is modified since the income increase from unemployment then amounts only to the difference between the wage and unemployment benefits.

As a final example, consider another type of unemployment situation where residual labour is absorbed in self-employment (e.g. on peasant farms), labour supplies being fixed. For the present purposes, we may consider a peasant farm absorbing residual labour as a particular firm, say g, owned by a single individual h (so that $\theta^{gh} = 1$), and facing a quota $\bar{y}_l^g$ on its employment of labour, $\bar{y}_l^g$ being determined endogenously from the solution of the lth scarcity constraint. When $\bar{y}_l^g$ is unrestricted (2.81) holds and so

$$\nu_l = MSP_l^g - b^h(p_l - MP_l^g), \tag{3.10}$$

where *MSP* and *MP* denote the marginal social product and marginal product of labour on the farm, respectively (defined as in Section 2). The r.h.s. of (3.10) is precisely the *Little–Mirrlees shadow wage* [see Little and Mirrlees (1974, pp. 270–271)]; and, once again, it has a straightforward interpretation: the first term represents the social value of the net loss of output caused by a withdrawal of one unit of labour from the peasant farm to the public sector; the second measures the marginal social value of the increase in income accruing to the peasant household.

Notice the importance played, in each of these examples, by the marginal social values of transfers $b^h = \beta^h - \nu(\partial x^h/\partial m^h)$. The values of b^h obviously depend crucially on the welfare weights; they are also closely related to what has become known in the literature as "savings premia". A plausible interpretation of the notion of a savings premium emerges in the intertemporal version of our model, where the marginal social value of a (first-period) transfer to consumer h becomes:

$$b^h = \beta^h - \sum_{i,\tau} \nu_{i\tau} \frac{\partial x_{i\tau}^h}{\partial m^h} = \beta^h - \sum_{i,\tau} \frac{\nu_{i\tau}}{q_{i\tau}} \frac{\partial (q_{i\tau} x_{i\tau}^h)}{\partial m^h}.$$

If first-period commodities are relatively more valuable socially than future-period commodities (perhaps because the planner has the opportunity to invest first-period commodities in very productive activities), then, other things being equal, marginal social values of transfers will be positively related to marginal propensities to save (i.e. to spend on future commodities). Consider, for instance, a simple two-period, one-commodity case. Dropping the commodity index, b^h becomes:

$$b^h = \beta^h - \frac{\nu_0}{q_0}(1 - MPS^h) - \frac{\nu_1}{q_1} MPS^h, \tag{3.11}$$

where MPS^h is h's marginal propensity to save in period 0. The marginal social value of a transfer from individual k to individual h then reduces to

$$b^h - b^k = (\beta^h - \beta^k) + \left(\frac{\nu_0}{q_0} - \frac{\nu_1}{q_1}\right)(MPS^h - MPS^k). \tag{3.12}$$

When a savings premium in the above sense exists ($\nu_0/q_0 > \nu_1/q_1$), then, the planner's inclination to redistribute on the grounds of unequal welfare weights may be modified by a recognition of differences in marginal propensities to save. As will be made clear in Section 3.5, the existence of a savings premium defined in this way is exactly equivalent to the shadow discount rate being higher than the consumer's rate of interest.

Each of the examples we have discussed relates specifically to a particular kind of situation, but we hope that together they provide a feel for the issues most commonly involved in calculating shadow wages. Particularly important elements of the planner's modelling exercise in this context are labour market institutions, migration behaviour, the organisation of peasant farms and the factors underlying marginal social values of transfers to different individuals. For further reading on the shadow wage, see, for example, Little (1961), Marglin (1976), Mazumdar (1976), Lal (1973), and Stiglitz (1981); on the savings premium see also Galenson and Leibenstein (1955), Diamond (1968) and Sen (1967).

3.5. Numeraires, discount rates, and foreign exchange

The discount rate allows us to compare the shadow values of goods which are used or produced in the future with those which are used or produced today. More precisely, the discount rate is defined as the rate of fall in the value of the numeraire against which goods are valued each year – this rate is used to convert shadow values in different years into common units this year or present values. One cannot discuss the value of the discount rate unless a numeraire has been defined and thus we have to treat the numeraire and the discount rate together. This is set out formally when we specify the definitions in Section 3.5.1. The determination and calculation of the discount rate are examined in Section 3.5.2. Finally, in Section 3.5.3 we discuss the shadow value of foreign exchange, and in particular the notion of a premium on foreign exchange, analogous to the premium on savings of the previous section.

3.5.1. Definitions

A good i which appears at time τ is obviously different from one which appears at time τ'. It is often useful, however, to recognise that these goods are physically the same and distinguished only by their date. In order to do this we shall use double subscripts so that $z_{i\tau}$ is the level of public supply of good i at time τ and

$\nu_{i\tau}$ is the shadow value of an extra unit of $z_{i\tau}$. The shadow value Π of a project can then be written as

$$\Pi \equiv \nu\, \mathrm{d}z \equiv \sum_{\tau} \nu_{\tau} \cdot \mathrm{d}z_{\tau} \equiv \sum_{\tau} \sum_{i} \nu_{i\tau}\, \mathrm{d}z_{i\tau}, \tag{3.13}$$

where ν_{τ} is the vector $(\ldots \nu_{i\tau} \ldots)$ and similarly for $\mathrm{d}z_{\tau}$.

We can now normalise the vector ν_{τ} in year τ by multiplying through by any scalar of our choice, for example to set one of its components to unity, forming the normalised vector $\bar{\nu}_{\tau}$. This gives us

$$\nu_{i\tau} \equiv \bar{\nu}_{i\tau} a_{\tau}, \tag{3.14}$$

where a_{τ} is called the *shadow discount factor*. If, for example, good i is the numeraire (with $\nu_{i\tau} > 0$ for all τ) then $\bar{\nu}_{i\tau} = 1$ for all τ, and $\nu_{i\tau} = a_{\tau}$. The shadow discount factor can then be seen as the marginal social value of a unit of numeraire accruing in year τ. From (3.13) and (3.14) we have

$$\Pi = \sum_{\tau} a_{\tau} \bar{\Pi}_{\tau}, \tag{3.15}$$

where

$$\bar{\Pi}_{\tau} = \bar{\nu}_{\tau}\, \mathrm{d}z_{\tau}. \tag{3.16}$$

Thus, we can interpret the social value of a project as a discounted stream of social benefits $\{\bar{\Pi}_{\tau}\}$ expressed in terms of a common numeraire. Hence the term "discount factor".

It should be emphasised that $\bar{\Pi}_{\tau}$ is not the marginal contribution of the project to "social welfare accruing in year τ" (supposing it were possible to identify such a contribution), but rather the contribution to intertemporal welfare of the net outputs in year τ of the project. For example, a capital good supplied by the public sector this year may generate benefits over a substantial future period and this will be embodied in its shadow value now.

The *shadow discount rate* is defined as the rate of fall of the discount factor

$$\rho_{\tau} \equiv \frac{a_{\tau} - a_{\tau+1}}{a_{\tau+1}}, \tag{3.17}$$

i.e. it is the rate of fall in the marginal social value of the numeraire. Where good i is the numeraire

$$\rho_{\tau} = \frac{\nu_{i\tau} - \nu_{i,\tau+1}}{\nu_{i,\tau+1}}. \tag{3.18}$$

From (3.17) we have, if $a_0 = 1$,

$$a_{\tau+1} = \frac{1}{(1+\rho_\tau)} \cdot \frac{1}{(1+\rho_{\tau-1})} \cdots \frac{1}{(1+\rho_0)}. \tag{3.19}$$

If the shadow discount rate were constant at r, then $a_{\tau+1}$ would be $1/(1+r)^{\tau+1}$, a familiar expression. We should emphasise, however, that there is no reason to expect ρ_τ to be constant, as should be clear from the next subsection.

Our definition obviously implies that the value of the shadow discount rate depends on the choice of numeraire. Suppose, for example, that a_τ and ρ_τ apply when good one is the numeraire (i.e. $\bar{\nu}_{1\tau} = 1$ for all τ) and a'_τ and ρ'_τ when good 2 is the numeraire ($\bar{\nu}_{2\tau} = 1$ for all τ). Then $a_\tau = (\nu_{1\tau}/\nu_{2\tau})a'_\tau$. Thus, using (3.17) and (3.18):

$$\rho_\tau = \rho'_\tau + \left[\frac{\dfrac{\nu_{1\tau}}{\nu_{2\tau}} - \dfrac{\nu_{1,\tau+1}}{\nu_{2,\tau+1}}}{\dfrac{\nu_{1,\tau+1}}{\nu_{2,\tau+1}}} \right] \frac{a'_\tau}{a'_{\tau+1}}.$$

Hence, $\rho_\tau = \rho'_\tau$ if and only if $\nu_{1\tau}/\nu_{2\tau} = \nu_{1,\tau+1}/\nu_{2,\tau+1}$, i.e. the relative shadow price of good 1 and good 2 does not change from τ to $\tau+1$. From now on we write the shadow discount rate when i is numeraire as $\rho^{(i)}$ and adopt a corresponding superscript notation when we wish to emphasise the dependence of a concept on the numeraire. Furthermore, for simplicity we shall assume in the remainder of Section 3.5 that there are only two periods (0 and 1).

3.5.2. The determination of the discount rate

Having defined the discount rate we now turn to its calculation. In particular we shall attempt to relate it to various rates of return in the economy.

To start with, let us consider a marginal project in the public sector as a small movement $\mathrm{d}y$ along the production frontier of a fully-controlled firm g. Thus,

$$\nabla F^g \mathrm{d}y = 0, \tag{3.20}$$

where ∇F^g is the vector of partial derivatives of g's production function. But we know from the result concerning efficiency among fully controlled firms that ∇F^g is proportional to ν [see, for example, (2.46)] and hence

$$\nu\, \mathrm{d}y = 0, \tag{3.21}$$

or, using time subscripts:

$$\nu_0 \,\mathrm{d} y_0 + \nu_1 \,\mathrm{d} y_1 = 0. \tag{3.22}$$

This allows us to define the *social rate of return* (SRR) in this firm, when good i is the numeraire, as

$$1 + SRR^{(i)} = -\frac{\bar{\nu}_1 \,\mathrm{d} y_1}{\bar{\nu}_0 \,\mathrm{d} y_0}. \tag{3.23}$$

Intuitively, the r.h.s. of (3.23) indicates the number of units of the numeraire that can be earned next year by investing a unit of numeraire this year in firm g. Note that (3.23) holds for any $\mathrm{d} y$ satisfying (3.20). Indeed from (3.14), (3.17), (3.22) and (3.23) we have

$$SRR^{(i)} = \frac{a_0 - a_1}{a_1} = \rho^{(i)}. \tag{3.24}$$

Thus, the shadow discount rate is equal to the social rate of return in every fully controlled firm. With our definition this is true irrespective of the choice of numeraire, though, of course, the value of SRR (or ρ) in (3.24) depends crucially on this choice.

It is often suggested that the shadow discount rate is equal to the social rate of return in a "marginal project", in the sense of a project which breaks even at shadow prices. The above result confirms the validity of this assertion. However, while the proposition is correct it is not directly useful since it really amounts to a restatement of the definition of "marginal project".

A useful application of (3.24) arises when we take foreign exchange as the numeraire in each year ($\nu_{f_\tau} = 1$ for each τ) and regard borrowing or lending abroad as a marginal project. If the STC can borrow and lend at fixed interest rates, then the social rate of return from having less foreign exchange this year (and more next year) is simply the interest rate it faces on the world capital market (the borrowing rate if it is borrowing at the margin and the lending rate if it is lending). If the STC can be legitimately regarded as a fully controlled firm, then the shadow discount rate is equal to the world interest rate.

It should be emphasised that the shadow discount rate here is the social rate of return only in a fully controlled firm. There is no reason, from what has been said, to suppose that this is the rate of return in the private sector. If private producers are unconstrained then under the conditions where relative producer prices are equal to relative shadow prices (see Section 2.3.2), the marginal rates of transformation in private and fully controlled firms will be the same. Then, using similar conventions for the numeraire for both shadow and producer prices we see that the shadow discount rate will be the private rate of return, i.e. the rate of return in terms of market prices (note that no distinction needs to be drawn here between pre and post-tax profits as long as profits taxes are proportional).

Intuitively under these conditions to each marginal public project there corresponds a marginal private project which is identical in all relevant respects so that the rates of return on the former and latter are the same. Alternatively, one could see the introduction of a public project as simply being accommodated in the general equilibrium by a displacement of an equivalent private project so that this private project can be seen as the opportunity cost of the public one.

Now consider a firm which is not fully controlled (for simplicity we shall omit the superscript g identifying this firm). As it stands, (3.23) does not unambiguously define the social rate of return in this firm, since the expression on the r.h.s. will no longer have the same value for any move dy on g's production frontier. Therefore we now define the social rate of return of good j in firm g as follows:

$$1 + SRR_j^{(i)} \equiv \frac{-\bar{\nu}_1 \dfrac{\partial y_1}{\partial \bar{y}_{j0}}}{\bar{\nu}_0 \dfrac{\partial y_0}{\partial \bar{y}_{j0}}} \qquad \text{if } j \text{ is rationed in firm } g \tag{3.25}$$

$$1 + SRR_j^{(i)} \equiv \frac{-\bar{\nu}_1 \dfrac{\partial y_1}{\partial p_{j0}}}{\bar{\nu}_0 \dfrac{\partial y_0}{\partial p_{j0}}} \qquad \text{otherwise} \tag{3.26}$$

where all derivatives are taken along the firm's supply function. The interpretation of these definitions is analogous to that of (3.23).

Using the same definition of "marginal social product" (MSP) as before (see Sections 2.4.2 or 3.4) it is then easy to show that

$$SRR_j^{(i)} = \rho^{(i)} \Leftrightarrow \nu_{j0} = MSP_{j0}. \tag{3.27}$$

In other words, the shadow discount rate is equal to the social rate of return of good j in firm g if and only if the shadow price of this good (period 0) is equal to its marginal social product in firm g. As we have seen in Sections 2.4 and 3.3, however, the latter is just the first-order condition for an optimal production level of good j (period 0) in firm g, provided that either price equals marginal cost for this good, or g's profits are fully (or optimally) taxed.

In summary, the equality between the shadow discount rate and social rates of return in the private sector hold under conditions which appear as neither straightforward nor general.

Let us now examine more closely the relationship between the shadow discount rate, social rates of return and private rates of return. The *private rate of return* (PRR) of good j in firm g is defined exactly as in (3.25) and (3.26), with producer prices replacing shadow prices (and using the numeraire of the producer price system). It is easy to verify that if firm g faces no binding constraint on its input of good j (period 0), then

$$PRR_j^{(i)} = \frac{p_{i0} - p_{i1}}{p_{i1}} = r^{(i)}, \tag{3.28}$$

where $r^{(i)}$ may be called the *producer rate of interest* using good i as numeraire (i.e. by selling one unit of good i this year, a producer can buy $1 + r^{(i)}$ units of it next year).

Private rates of return are obviously unlikely to coincide with social rates of return unless the vector of intertemporal shadow prices is itself collinear with the producer price vector. To see more directly the relationship between private rates of return and the shadow discount rate, observe that

$$r^{(i)} = \rho^{(i)} \Leftrightarrow \frac{v_{i0}}{p_{i0}} = \frac{v_{i1}}{p_{i1}} \tag{3.29}$$

(this follows immediately from the definitions). Thus, for a given numeraire, the shadow discount rate is equal to the producer rate of interest if and only if the ratio of shadow price to producer price for this numeraire remains constant over time.

One may define the *consumer rate of interest* for any commodity exactly as in (3.28) with producer prices replaced by consumer prices; (3.29) then has a straightforward counterpart. More generally one can analyse the relationship between the consumer rate of interest and the shadow rate of discount using the results of Section 2.4 on the general relationship between consumer prices and shadow prices much as we have done on the production side. Rather than doing this systematically we shall use these results to examine briefly some of the suggestions on the shadow discount rate found in the literature.

Diamond and Mirrlees (1971) have relative producer prices equal to relative shadow prices, see Section 2.3.2. Hence for a given numeraire the shadow discount rate, the producer rate of interest, and all private and social rates of return coincide, though they differ from the consumer rate of interest.

Arrow (1966) and Kay (1972) argue that it would often be reasonable to suppose that the ratio between shadow price and consumer price will be constant over time. They work at an aggregated level so the shadow price for each period is the Lagrange multiplier on the single scarcity constraint. Crudely speaking this would only change if the problems of financing public expenditure were expected to become more or less severe over time (recall the equivalence between the scarcity constraint and the government's budget constraint – see Section 2.2.3). As we have seen, if the ratio between shadow price and consumer price is constant for good i, then the shadow discount rate is equal to the consumer rate of interest for that good.

Little and Mirrlees (1974, pp. 291–305) argue explicitly that one would expect relative shadow prices and consumer prices to change over time as the government overcomes various constraints on prices and quantities (see also the characterisation of dynamic shadow prices in optimal growth models such as Stern, 1972).

Harberger (1973) claims that the shadow discount rate should be equal to the private rate of return on the grounds that private investment is a straightforward alternative to public investment. The validity of this argument, however, rests on the assumption that public projects are displacing private projects, which certainly involves a particular model of the economy [Bradford (1975)]. Moreover, as we have seen, in such a model it is the *social* rate of return in the private sector which would emerge as the shadow discount rate (and even this result would require some additional assumptions such as the full or optimal taxation of profits).

Sandmo and Drèze (1971) suggest a weighted average formula (averaging interest rates for consumers and producers) based on the degree to which funds for public investment come from private investment and private consumers – see also Baumol (1986), and Drèze (1974). This is precisely a special case of the Ramsey–Boiteux formula derived in Section 2.4.1 where shadow prices appear as a weighted average of consumer and producer prices.

Finally, let us mention a suggestion for finding the shadow discount rate which occurs quite commonly in the literature: namely, that the shadow discount rate should be set so that the total public investment budget is just exhausted. Thus, it is suggested that if we find ourselves accepting more projects than can be "afforded" with the existing discount rate then we should raise it, and conversely if too few projects pass the test of positive net present value. The shadow discount rate is seen as the price which clears the investment budget. Whilst this has some intuitive appeal, it is unsatisfactory in important respects. First, the argument runs in terms of "the" discount rate. There would, in general, be as many discount rates as periods and one could in principle adjust any or many of the discount rates to bring the amount of public investment to the desired level. In any case it is misleading to think of one single shadow price, or subset of shadow prices, as requiring adjustment. At shadow prices a marginal project should just break even and this is a condition on all the prices, not just on one or two of them. Thus, one can change the number of accepted projects by, for example, adjusting the shadow wage. Secondly, the nature and rationality of such a budget constraint should be carefully assessed in the first place (for the analysis of budget constraints in the public sector, the reader is referred to Sections 2.2.3 and 3.6). Thus, we would not recommend this approach to measuring the shadow discount rate.

Some further references on the topics of this subsection include Arrow and Kurz (1970), Marglin (1963a, 1963b, 1976), Pestieau (1975), and the collection of papers in Lind (1983).

3.5.3. *Premia on foreign exchange*

Foreign exchange is often seen as a particular problem in the planning of investment and growth. Some speak of "the foreign exchange constraint", others of "the premium on foreign exchange", "the shadow exchange rate" and so on. One might ask how such considerations fit into the framework we have set out.

The shadow value of foreign exchange has already appeared explicitly in our model: ν_f is the marginal social value of foreign exchange, since it measures the increase in social welfare resulting from a "gift" of one unit of foreign exchange. Since p_f is the exchange rate (the price of a unit of foreign exchange) it is tempting to call ν_f "a shadow exchange rate". Given that alternative interpretations have been placed on this concept it is not a terminology we shall adopt. Since we are free to normalise shadow prices we can set ν_f equal to p_f so that the market and shadow exchange rates are equal. This does not, however, allow us to avoid enquiring about the relative social value of foreign exchange and other commodities.

Intuitively the marginal social value of foreign exchange will depend on its source or destination at the margin or, in other words, on how the balance of payments is ensured. There are in principle many possibilities, such as changes in tariffs or quotas, adjustment of exchange rate, or fiscal adjustments; which of them are relevant will depend on the opportunities and constraints facing the planner. To give an explicit example which provides an illustration of the considerations involved, suppose that there is a quota system for the import of some commodities. The quota n_i of each of these commodities depends on an index (say α) of the amount of foreign exchange available. The rationing system is designed by a licensing authority beyond the control of the planner. The variable α adjusts endogenously to clear the balance of payments, and hence it is a control variable. We shall also assume it is unrestricted. We can then examine the first-order condition that the marginal social value of α should be zero.This gives [remembering $\bar{y}_i^f \equiv n$, (2.4) and (2.66)]:

$$\sum_i \frac{\partial n_i}{\partial \alpha}\left(\nu_i - \nu_f p_i^f\right) = 0, \tag{3.30}$$

where the summation is over the goods involved in the rationing scheme and p_i^f is the marginal cost of good i on the world market in terms of foreign exchange.

Rearranging we have:

$$v_f = \frac{\sum_i v_i \frac{\partial n_i}{\partial \alpha}}{\sum_i p_i^f \frac{\partial n_i}{\partial \alpha}}, \tag{3.31}$$

where again, and in what follows, the summations are over the goods involved in the rationing scheme. Thus, we can see the shadow value of foreign exchange as the shadow value of extra rations divided by the cost in terms of foreign exchange of obtaining them.

The premium on foreign exchange depends on the definition of the term and the opportunities and constraints for the use of foreign exchange. As an illustration suppose we decide to say that there is a premium on foreign exchange if the relative shadow price of foreign exchange and good 1 is higher than the relative market price for foreign exchange and good 1. Thus there is a premium on foreign exchange if

$$\frac{v_f}{p_f} - \frac{v_1}{p_1} > 0. \tag{3.32}$$

Assume that world prices are fixed for the goods involved in the rationing scheme ($p_i^f = \bar{p}_i^f$), that good 1 is amongst these goods, and that positive tariffs apply ($p_i = p_f p_i^f + t_i^f$, where t_i^f is the ith tariff). Suppose, further, that *relative* shadow prices for commodities do not differ from their relative market prices within the set of goods subject to rationing so that $v_i = (v_1/p_1)p_i$ in this group (note that the border marginal cost rule would not normally apply to these goods). Then (3.31) becomes:

$$\frac{v_f}{p_f} = \left[\frac{\sum_i \frac{\partial n_i}{\partial \alpha}\left(p_f p_i^f + t_i^f \right)}{\sum_i \frac{\partial n_i}{\partial \alpha} p_f p_i^f} \right] \frac{v_1}{p_1}. \tag{3.33}$$

If $\partial n_i/\partial \alpha \geq 0$ and $t_i^f \geq 0$ (with strict inequalities for at least one i), then the term in square brackets in (3.33) is greater than one so that (3.32) is satisfied and there is indeed a premium on foreign exchange.

If the premium on foreign exchange is defined as the l.h.s. of (3.32) divided by v_1/p_1, then it may be calculated from the value of a marginal bundle of commodities at domestic prices divided by its value at world prices. The content of the marginal bundle reflects the way in which an extra unit of foreign exchange will be allocated across commodities. Thus, in this example, and under this definition, the premium on foreign exchange is a measure of the extent to which domestic prices exceed world prices (more precisely, it can be expressed as a weighted average of tariff rates).

Formulae such as (3.33) are familiar in the cost–benefit literature. For instance, in terms of the above example, the Little–Mirrlees Standard Conversion Factor (SCF) for the group of goods involved (with the normalisation rule $\nu_f = p_f$) corresponds to ν_1/p_1 [see Little and Mirrlees, (1974, p. 218)] while the UNIDO Shadow Exchange Rate is analogous to one plus the premium on foreign exchange [see Dasgupta, Marglin and Sen (1972, ch. 16)].

As with the discount rate, many of the ideas on the marginal social value of foreign exchange found in the literature can be examined in terms of the general principles developed in Section 2. We shall not attempt this here but we hope that the above example illustrates convincingly the general idea that the marginal social value of foreign exchange depends upon the uses and sources of foreign exchange which are available to the planner. Some examples from the literature on the role of foreign exchange in cost–benefit analysis are: Bacha and Taylor (1971), Balassa (1974a and b), Bhagwati and Srinivasan (1981), Blitzer, Dasgupta and Stiglitz (1981), Scott (1974) and a collection of papers in *Oxford Economic Papers*, July 1974.

3.6. *Private projects and Boiteux firms*

To see how private projects should be evaluated, let us introduce, in the model of Section 2, a firm (indexed by 0) whose production plan y^0 is regarded as a vector of predetermined variables. A private project $\mathrm{d}y^0$ then induces a total change in social welfare $\mathrm{d}V$, where

$$\mathrm{d}V = \frac{\partial \mathscr{L}}{\partial y^0}\,\mathrm{d}y^0 = \nu\,\mathrm{d}y^0 + b^0(p\,\mathrm{d}y^0) \tag{3.34}$$

or

$$\mathrm{d}V = (\nu + b^0 p)\,\mathrm{d}y^0. \tag{3.35}$$

In other words, the price vector appropriate for the social evaluation of private projects is simply a linear combination of the shadow price vector and the producer price vector. This is not surprising, since in this context public and private projects differ only in respect of the distribution of profits, and these vary with net outputs at a rate indicated by producer prices.

A very similar result holds concerning the shadow prices appropriate for the evaluation of projects taking place in "Boiteux firms", or public firms facing a budget constraint [Boiteux (1956), or (1971)]. Indeed, it is easy to repeat the above analysis in the case where $\theta^{0h} = 0$ for all h and the side constraint $\rho y^0 = \overline{\Pi}$ applies for some price vector ρ; one then finds that the price vector appropriate for the evaluation of projects in Boiteux firms is a linear combination of the shadow price vector and the price vector defining the budget constraint:

$$\mathrm{d}V = \left(\nu + b^0\rho\right)\mathrm{d}y^0, \tag{3.36}$$

where now b^0 represents the Lagrange multiplier associated with the side constraint $\rho y^0 = \overline{\Pi}$. An instructive way of restating this result is the following: Boiteux firms should choose the production plan which has maximum profits at shadow prices (ν) among all those which satisfy its budget constraint. Guesnerie (1980) has shown that these decision rules for Boiteux firms are very general.

3.7. *Uncertainty and time*

Uncertainty and time have been relatively neglected in this paper and we comment very briefly in this subsection on their treatment in the models of Sections 1 and 2.

Projects are often undertaken in situations of considerable uncertainty and it is therefore important to consider how uncertainty should be treated in cost–benefit analysis. Formally uncertainty can be incorporated in the model of Section 1 by distinguishing commodities and signals according to the state of the world in which they occur [Debreu (1959)]. To proceed in this way is not to suppose that well-functioning markets for contingent commodities exist; it is only to recognise that, in every state of the world, every commodity has a source and destination and that private agents' demands for these commodities in each state of the world respond in a well-defined way to some collection of signals.

Pointing out that the general framework of Section 1 can encompass many models of economies with uncertainty is not to deny, of course, that its application to specific problems will often be greatly complicated by the presence of uncertainty. In particular, the specification of an objective function may involve difficult judgements, e.g. if private agents are thought to have mistaken expectations. Besides, the planner's problem (P) is now one of dynamic stochastic optimisation. It is important to recognise, however, that in spite of the intricate problems of modelling and application which it precipitates, cost–benefit analysis under uncertainty rests on similar theoretical foundations to the basic theory of shadow prices developed in Section 1. We shall here confine ourselves to illustrating this point with two simple examples.

As a first example, consider a very simple two-period economy with a single consumption good and labour. Private agents consist of a single peasant who consumes x_0 in period 0 (out of stocks), and supplies a fixed total amount of labour L, which is devoted both to employment on public works $(-z_l)$, and agricultural labour on the farm $(L+z_l)$. We assume that the peasant is always willing to take up extra public employment. The planner's objective function is simply the peasant's expected utility,

$$V = E_\sigma U(x_0, x_{1\sigma}) = \sum_\sigma \pi_\sigma U(x_0, x_{1\sigma}), \tag{3.37}$$

where σ is an index of the state of the world (e.g. a measure of rainfall), and π_σ is the perceived probability of occurrence of event σ.

Since public profits (losses) ultimately accrue to (are borne by) the peasant, we write, after appropriate substitutions:

$$V^*(z) \equiv E_\sigma U(x_0, F(L+z_l;\sigma) + z_{1\sigma} - x_0), \tag{3.38}$$

where $z_{1\sigma}$ is public production in period 1 and event σ, and F is the farm's production function. Letting $\nu \equiv \partial V^*/\partial z$ as usual, we find

$$\nu_{1\sigma} = \pi_\sigma \frac{\partial U(x_0, x_{1\sigma})}{\partial x_{1\sigma}} = \pi_\sigma U_{1\sigma}, \tag{3.39}$$

$$\nu_l = \sum_\sigma \pi_\sigma U_{1\sigma} F' = E(U_{1\sigma} F'). \tag{3.40}$$

The marginal social value for a sure delivery of a unit of the commodity in period 1 ($\mathrm{d}z_{1\sigma} = 1$, for all σ), say ν_1, is then

$$\nu_1 = \sum_\sigma \nu_{1\sigma} = E(U_{1\sigma}). \tag{3.41}$$

Combining (3.40) and (3.41) gives

$$\frac{\nu_l}{\nu_1} = \frac{E(U_{1\sigma}F')}{E(U_{1\sigma})} = E(F')(1+\alpha), \tag{3.42}$$

where α is the correlation coefficient between F' and $U_{1\sigma}$. The r.h.s. of (3.42) is the direct analogue in this model of the Little–Mirrlees shadow wage introduced in Section 3.4, and it differs from the latter in two respects. First, the distribution term here vanishes – this is natural since we now have a single consumer and no investment, and it has nothing to do with uncertainty as such. Secondly, and more importantly, the marginal product of labour is now replaced by its expected value, and is further corrected to take into account the correlation between marginal product on the farm and marginal utility of consumption. One would often expect this correlation to be negative (e.g. when bad weather means low marginal product and high marginal utility of income). In that case, the more risky is private production, the lower is the social cost of labour (in terms of ν_1); this makes intuitive sense.

Consider now a more general two-period model, where private agents' decisions in period 1 are taken with full knowledge of past and current signals, while those in period 0 have to be taken with imperfect knowledge of future signals. Considering a consumer's decision problem first, let $x_{1\sigma}(s_{1\sigma}; x_0)$ be his preferred consumption bundle in period 1 and event σ when x_0 has been chosen in period 0, and first-period signals take the value $s_{1\sigma}$. One can then appeal to the theory of decision-making under uncertainty to model the consumer's choice of x_0. Generally, this choice will depend upon the consumer's expectations about the values of future signals. These expectations in turn depend on the vector of current signals s_0. They may also depend, of course, on the quantities chosen or decisions taken by other agents in period 0, but since these decisions themselves are functions of current signals, ultimately the profile of demands in period 0 should depend upon s_0 only; and therefore, after fully taking into account the expectations process, one could still write $x_0 = x_0(s_0)$. After proceeding analogously for private producers, the planner's model becomes

$$\max_s V(s)$$

$$\text{s.t.} \begin{cases} \sum_h x_0^h(s_0) - \sum_g y_0^g(s_0) - z_0 = 0, \\ \sum_h x_{1\sigma}^h\left(s_{1\sigma}; x_0^h(s_0)\right) - \sum_g y_{1\sigma}^g\left(s_{1\sigma}; y_0^g(s_0)\right) - z_{1\sigma} = 0, \\ s \in S, \end{cases}$$

and the Lagrange multipliers associated with the scarcity constraints yield both a vector ν_0 of shadow prices for (certain) period 0 commodities and a collection of shadow price vectors $\nu_{1\sigma}$ for contingent commodities in period 1.

An uncertain project $\mathrm{d}z = (\mathrm{d}z_0, \dots \mathrm{d}z_{1\sigma} \dots)$ would be evaluated as before using the criterion: accept $\mathrm{d}z$ if $\nu\,\mathrm{d}z$ is positive.

Since an explicit analysis of uncertainty can quickly become quite involved, we shall not enter here into any substantial comment on the main issues of the literature. A major early contribution was that of Arrow and Lind (1970), who argued that under certain conditions uncertainty can be "ignored" in cost–benefit analysis in the sense that appropriate random variables can be replaced by their expected values. Roughly speaking, these conditions involve, first, that risks in public projects should be uncorrelated with private risks, and, secondly, that public risks should be borne by a large number of individuals. The latter condition can be regarded as a particular aspect of the planner's "policy" in the uncertainty context.

The limited validity of the Arrow–Lind result has prompted a good deal of subsequent research on the implications of uncertainty. Some valuable contributions include: Graham (1981) Henry (1974a, 1974b, 1981), Hirshleifer (1965), Sandmo (1972), Schmalensee (1976) and a discussion on the Arrow–Lind paper in the *American Economic Review* of March 1972. The issue of the correlation between project risks and the private sector is analogous to that examined using β-coefficients in the theory of finance (see the *Journal of Finance* for many contributions).

Very similar comments to the above apply to the idea of treating time in the general framework of Section 1 by dating commodities; indeed, the examples we have just given are intertemporal. Under appropriate *intertemporal separability* assumptions, however, the problem (P) can be reduced to a one-period format where private agents transact only in current commodities and the future is taken into account by imputing an appropriate social value to changes in the components of a suitably specified vector of stock variables (e.g. the capital stock of each firm, and the wealth of each individual). The correct social values to be assigned to these "investments" can of course ultimately be derived only by an explicit consideration of (P) in its full intertemporal version. A rigorous treatment of these issues is by no means trivial, and lies much beyond the scope of this chapter.

In the model of Section 2, we have effectively allowed for time by assuming perfect future information and either a complete set of forward markets, or perfect capital markets. These assumptions of course sound very stringent and should be relaxed whenever the simplifications involved are seriously misleading; we would argue, however, that the approach we have adopted permits an instructive discussion of a number of practical issues involving time, and we hope to have illustrated the point in Section 3, e.g. in the analysis of the discount rate.

3.8. Some neglected issues

We conclude this section by suggesting briefly how the framework we have developed can be applied or extended to deal with some issues which have not received here the attention they deserve, such as public goods, externalities, exhaustible resources and large projects.

From the formal point of view, public goods pose no acute problem. A public good is distinguished mainly by the special rules according to which it is allocated. For instance, in Section 2.4 we considered a public good for which $\bar{x}_i^h \equiv \bar{x}_i$ for all h and $q_i = 0$, and found its shadow price ν_i to satisfy (2.86):

$$\nu_i = \sum_h \beta^h \rho_i^h + \sum_{j \neq i} \tau_j^c \sigma_{ji}. \tag{3.43}$$

The first term on the r.h.s. is the welfare weighted marginal willingness-to-pay and the second is the induced effect on shadow tax revenue through pure substitution effects. The optimal level of public good provision is found, as for any other commodity, by equating its marginal rate of transformation with other goods in the public sector to the corresponding shadow price ratios. There may, of course, be important measurement problems in implementing this kind of rule, e.g. when the marginal-willingness-to-pay for public goods is not directly observable; but, as we have seen, analogous problems can arise for private goods as well (e.g. when the calculation of the shadow wage involves the reservation wage).

The measurement of the benefits from public activities has been a substantial topic in the literature on practical cost–benefit analysis. There are obvious examples arising in most of the major departments of government: a ministry of transport has to evaluate time saved by new roads; a ministry of health will be concerned with the appraisal of benefits from immunisation programmes; a water department will want to measure the benefits of more reliable water supply; educational allocations will require a view on the benefits of different expenditures and so on.

Only a few general comments are possible here. First, in many cases the willingness-to-pay of the consumer will be an important element and one can try to discover from questions, experiment and estimation how large this might be. An obvious example is the implicit valuation of time which might be revealed by an individual's choice of transport mode. The reader is referred to Chapter 9 in this Handbook on public goods for further discussion of these issues. Secondly, one must consider carefully whether it is appropriate to follow individual preferences. There are important problems of externalities (in, for example, immunisation), of information (in, for example, water supply), of identifying the consumer or his preferences (e.g. in education – both parent and child are involved). Thirdly, one must be humble and avoid giving the impression that very precise measurement is possible.

Public goods are discussed in more detail in Chapter 10 by Laffont, Chapter 9 by Oakland, and Chapter 11 by Rubinfeld in this Handbook. The literature on the economics of health, education or transport defies summary – the reader may consult journals specialising in such areas (e.g. *Journal of Health Economics*, *Journal of Human Resources*, and *Journal of Transport Economics and Policy*).

Externalities are well within the framework of Section 1, and can easily be introduced into the model of Section 2. For instance, the presence of externalities in private production no longer allows us to write the gth firm's supply function as $y^g(p, \bar{y}^g)$, since the production plans of other firms would become relevant signals for this firm's decisions. However, even after taking into account all the relevant interdependencies, the aggregate net supply function of the private sector should still be a well-defined function of $(p, \dots \bar{y}^g \dots)$, say $y^*(p, \dots \bar{y}^g \dots)$. Though this function would no longer satisfy all the usual properties of competitive supply functions, many of our results did not use these properties, while those which did can be restated appropriately by replacing the derivatives of $y(\cdot)$ by those of $y^*(\cdot)$. For instance, with full taxation of profits, the marginal social value of an unrestricted producer ration $\bar{y}_i^g$ now becomes:

$$MSV_{\bar{y}_i^g} = \nu \frac{\partial y^*}{\partial \bar{y}_i^g}. \tag{3.44}$$

The estimation of the functions $y^*(\cdot)$ may or may not be more difficult than that of supply functions involving no externalities. For example, if firms are unconstrained the informational requirements would be similar (a time series of prices and quantities) and in other circumstances one may be able to observe or find proxies for the rations. Externalities and public goods are particularly relevant in the context of environmental problems. These involve important and difficult applications of the theory. For an introduction see Johansson (1985) and Dasgupta (1982). The latter volume also includes an extensive discussion of cost–benefit analysis with exhaustible and renewable resources.

"Large projects" require a major extension of the framework of Section 1. For a large project $\Delta z \equiv z^1 - z^0$, one wishes to evaluate $V^*(z^1) - V^*(z^0)$. For this, simple cost–benefit tests based on a single shadow price vector are no longer available – though separate necessity and sufficiency tests can still be designed if V^* is quasi-concave [see, for example, Hammond (1980)]. The calculation of $V^*(z_1) - V^*(z^0)$ will usually involve much more demanding econometric exercises than that of shadow prices, e.g. the estimation of complete demand systems rather than of local elasticities. An interesting step forward in the analysis of large projects is provided by Hammond (1983), who develops *second-order* approximations of their welfare effects. Unfortunately, the theory of cost–benefit analysis for large projects is yet to make significant headway, though the extensive literature on consumer surplus is relevant in this context – for example Chipman and Moore (1980), Hammond (1983), Harris (1978), Hausman (1981), MacKenzie and Pearce (1982), and Willig (1976).

4. Summary and conclusions

We have tried to present a fairly general theory of cost–benefit analysis within which specific rules and procedures for setting shadow prices can be examined.

Throughout we have worked with one definition of the shadow price of a good: it is the effect on social welfare of a marginal increase in the supply of that good from the public sector. This definition is the natural one when we wish to ask of any (small) project whether it increases welfare: this will be the case if the value at shadow prices of the change in net supplies represented by the project is positive. The definition also reflects two central features of our approach. First, we are working in terms of social welfare; in other words we assume that there is a well-defined criterion against which the planner can evaluate outcomes. Secondly, there is just one environment corresponding to each public production plan so that the planner is able to work out the consequences of any particular change in public supplies. The welfare function can take many forms according to the values of the planner and we showed how it could be related to intermediate or derived objectives such as output, employment and income distribution. Similarly, whilst the appraisal of a project requires us to be specific about its consequences, the theory of cost–benefit analysis as we have presented it does not tie us to a particular model of the economy.

Where the planner has a genuine choice over the policies which will determine the consequences of a project we have argued that he should make these choices optimally. This essentially involves consistency in the sense that open choices will be settled using the same criterion as that applied in the appraisal of projects. Nevertheless, one can include in the model as many constraints on government policies as one wishes. And we retain as a special case throughout the "fully determined" model where constraints are so restrictive that the planner has no real degree of freedom in his choice, i.e. given a public production plan there is only one compatible environment. The issue of how policies are determined is important because the values of shadow prices can be very sensitive to alternative specifications of policy.

When the public production set is convex, shadow prices decentralise an optimal public production plan. The shadow prices associated with an optimal production plan may take different values from those which would be appropriate for evaluating local changes from an arbitrary production plan; but the underlying notion of shadow prices is the same.

We have also seen that, whether fully determined or not, the model can generally be written in such a way that shadow prices are equal to the Lagrange multipliers on the scarcity constraints in the planner's problem. This does not mean that unemployment of a resource implies zero shadow prices. For example, as shown in Section 3, extra public employment at some given wage would normally affect social welfare in an economy with unemployment of labour and thus the shadow wage would not be zero.

In Section 2 we have analysed a particular example of the planner's model. Firms and households trade as price-takers subject to quotas on their net trades. The model can incorporate an arbitrary degree of control by the planner over prices, indirect taxes, lump-sum transfers, quotas on households and firms, profits taxes and income taxes. On the other hand, such problems as oligopoly, migration or imperfect information would require extension of the model, although in principle they can be analysed by an appropriate specification of "side constraints".

We shall not attempt to summarise the various rules we have derived in Section 2 and discussed in Section 3. We merely recall some of the prominent issues. First, we have examined production efficiency and the relation between shadow prices and producer prices. Public production should take place on the boundary of the public production set (provided not all shadow prices are zero). Shadow prices should, generally, be the same throughout the public sector. Except in rather special circumstances, however, shadow prices will not be proportional to producer prices and aggregate production efficiency of public and private firms together will not usually be desirable given the opportunities available to the planner.

Secondly, we saw that, under simple conditions, relative shadow prices for traded commodities will coincide with relative world prices. This rule applies (commodity-wise) when either trade clears the market or an optimal quota can be set (in addition the amount traded does not affect producers and consumers except through the scarcity constraints). When world demand or supply curves are not perfectly elastic, one has to replace world prices by marginal costs or revenues.

Thirdly, for a non-traded good, when an optimal ration can be imposed on producers, or the producer price can be chosen independently of the consumer price, the shadow price is equal to the marginal social cost of production plus the shadow value of extra profits generated by extra production. As a special case of this rule we saw that, under certain conditions, competitive private sector firms operating under constant returns to scale will break even at shadow prices.

These rules are amongst the simplest and clearest of those we examined. Before they can be legitimately applied, however, the assumptions involved should be understood and assessed in context. In many situations it will be necessary to have recourse to the richer though more complicated set of rules investigated in Section 2.4 and Section 3.

In conclusion we should like to indicate some directions for further research which are suggested by the difficulties we have encountered in this chapter. First, the potentialities of the model of Section 2 have not been fully exploited. In our judgement, the general equilibrium approach which it illustrates should be pursued further to clarify many of the issues and methods appearing in the literature. Secondly, more vigorous efforts should be made to capture such phenomena as uncertainty and imperfect competition in a form which lends itself to similar comparative static methods. Thirdly, a more thorough integration between cost–benefit analysis and the theory of reform should prove particularly fruitful. Finally, an explicit treatment of informational problems would be of great value in bringing the difficulties of the practitioner to greater prominence in the theory.

In the meantime governments have to appraise projects. It is our judgement that the theory currently available is sufficiently rich and flexible to provide useful guidance for those decisions. We hope that the principles we have set out here will help to understand, assess and improve the theoretical foundations of the practical methods which have been advocated, and to provide guidance for their proper application.

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