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THE DEMAND FOR WHEAT UNDER NON-LINEAR PRICING IN PAKISTAN*

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It is common for economic agents to face non-linear pricing schedules. This poses difficulties for the estimation of demand functions but also advantages in that we have price variation in a cross-section. We illustrate this with two examples from Pakistan: the urban rationing of wheat and the demand for wheat by rural households that produce and consume wheat. In the first case consumers have to pay higher prices for purchases above the ration, and in the second buying and selling prices differ. Demand elasticities are estimated using maximum likelihood methods. These estimates are considerably different from those obtained using standard LES assumptions.

1. Introduction

It is common in developing countries for households to face prices for commodities which depend on the amount purchased or sold. We examine here, for Pakistan, both the urban rationing of wheat and the demand or supply by rural households which are both producers and consumers. The data used are from the late 1970's. In the first case, consumers have to pay higher prices for purchases above the ration, and in the second, buying and selling prices may differ.

The existence of non-linear prices poses problems for estimation, yet it has advantages. The problems arise because marginal prices depend on decisions and the endogenous quantity purchased cannot be expressed simply as a function of the exogenous prices in the usual way. However, recently there have been many papers on methods for dealing with such problems, particularly in the context of labour supply [see, for example, Burtless and Hausman (1978) and Hausman (1979, 1985)]. The advantage is that the existence of more than one price allows for price variation in a cross-section. Usually one

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has to impose strong identifying restrictions such as the additive separability of utility functions (so that price responses are closely tied to income responses) in order to estimate price elasticities in cross-section data sets [see, e.g., Deaton and Muellbauer (1980)].

In this paper we focus on a single kink in the budget constraint, at the ration quantity for urban households, and at zero purchases and sales for rural households. This implies that in each case we have three regimes. In urban areas there are households that purchase less than the ration, those who purchase their exact ration, and those who enter the free market to purchase more than their ration. In rural areas, the three regimes correspond to net purchasers, net sellers, and those who neither sell nor purchase wheat. The model is described in section 2.

The estimates for Pakistan are based on a household survey conducted during 1976/77. We discuss the theory for both rural and urban households, but the estimates presented here relate to the latter. The data and its limitations are discussed in section 3. The estimation techniques and results are discussed in section 4. We briefly compare these with results from other forms of estimation and provide some concluding comments in section 5.

2. The model

2.1. Urban households

The difference between market price and ration price produces a kink in the budget constraint (in the absence of resale, see below). In the analysis of demand under such a budget constraint we follow the work by Chetty and Haliburn (1982) and Deaton (1981) on fair price shops in India. This, in turn, was influenced by the analysis of labour supply where taxes and social security benefits yield kinks in the budget constraint for consumption and leisure (see references above). Until recently in Pakistan, rationed wheat was provided to (mainly) urban households below the market price. The demand for wheat in this situation is depicted in fig. 1.

The consumption of wheat, Q_w, is measured along the vertical axis. On the horizontal axis we depict the composite consumption of non-wheat goods, Q_n. The ration available is given by OR, and amounts below this could be bought at the price p_r. Non-wheat goods are taken as the numeraire. Any extra wheat required would have to be purchased in the free market at price p_w, and one would expect that $p_w > p_r$. Thus, if resale is not permitted, the consumer is limited to the budget set ACD with a kink at C. Then CD represents *regime 1* where the consumer purchases less than the permitted quota; at C the exact quota is taken, representing *regime 2*; and along AC the consumer purchases more wheat then permissible under the quota and has to resort to open market purchases (*regime 3*).

If resale is permitted, then the budget constraint changes. Suppose a consumer buys a quantity Q_r of the rationed wheat, $(Q_w - Q_r)$ on the open

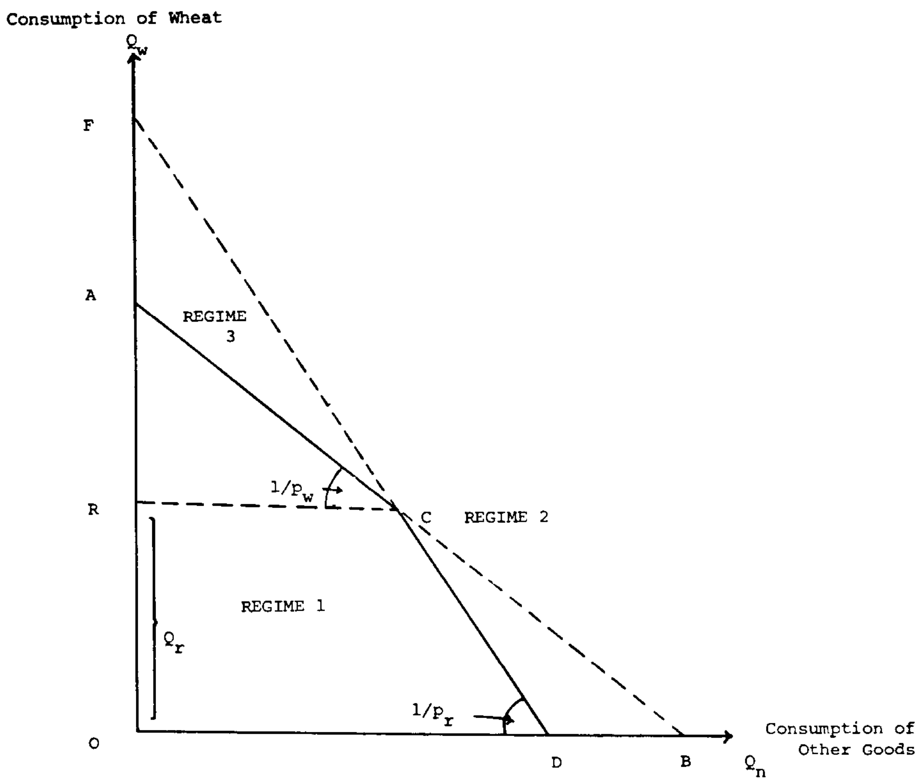

Fig. 1. Urban households – the budget constraint for wheat.

market (where $Q_w - Q_r$ is positive for free market purchases and negative for sales), and Q_n of other goods. If M is the exogenously given expenditure level, the consumer's budget constraint with resale becomes

$$Q_n + p_w(Q_w - Q_r) + p_r Q_r = M, \tag{1}$$

which can be rewritten

$$p_w Q_w + Q_n = M + (p_w - p_r) Q_r. \tag{2}$$

Thus we can now think of the ration recipient as having an expenditure of $M + (p_w - p_r)Q_r$, rather than the exogenously given sum M, and the second term on the r.h.s. of (2) acts like an income transfer. In terms of fig. 1, the budget constraint with resale is given by AB. In practice such resale involves transaction costs, and given the amounts that are allowed under the ration, may not be particularly widespread. If unlimited purchases were allowed at the ration price, the budget constraint would become DF.

We shall assume in this paper that resale is not allowed and that there is a fixed quota. This accords with the prevailing rule, and largely it seems the practice at the time. The budget set is then ACD. The consumer's utility function is $U(Q_w, Q_n)$ over wheat and other goods. Maximising with respect to a linear budget constraint $pQ_w + Q_n = Y$, for income Y and wheat price p, would yield the ordinary Marshallian demand function $F(\cdot)$, where

$$Q_w = F(p, Y). \tag{3}$$

We may take the demand function simply as defining the preferences rather than incorporating an assumption that the budget constraint is in fact linear. It is straightforward to find a utility function associated with $F(p, Y)$ provided that the standard conditions on the Slutsky derivatives apply [see, e.g. Hausman (1981) and Stern (1986)].

If Q_1 and Q_2 are the demands along the hypothetical constraints FD and AB, the demand for wheat in the two regimes is given by

$$Q_1 = F(p_r, M), \tag{4}$$

$$Q_2 = F[p_w, M + (p_w - p_r)Q_r]. \tag{5}$$

If $Q_1 < Q_r$, the consumer will buy less than the quota and will be in regime 1. Also, if $Q_2 > Q_r$, the consumer will purchase in the open market and will be in regime 3. Moreover, if both $Q_1 > Q_r$ and $Q_2 < Q_r$, then exactly Q_r will be purchased and the person will be in regime 2.

In the case of regime 1, $Q_1 < Q_r$, an individual faced with a budget constraint FD would choose a point on CD. Since the ration has not been taken up fully, other points along CD are available along with Q_1 and faced with the market price (and budget constraint AB), Q_1 is revealed preferred to all points on CA. Thus faced with the constraint AB only a point on BC would be chosen, and $Q_1 < Q_r$ implies $Q_2 < Q_r$. Similarly, $Q_2 > Q_r$ implies $Q_1 > Q_r$ in regime 3. Thus $Q_1 < Q_r$ and $Q_2 > Q_r$ are mutually exclusive and no individual will both purchase less than the quota and buy from the open market at a higher price. Exceptions encountered in reality include external constraints to taking up the ration such as distance or availability or discrimination in the award of quotas, and also quality differences reflecting inedible rationed wheat for example. Inferior quality wheat is often reputed to be supplied through ration shops. It is suggested that this arises either through corruption or as a deliberate targeting device; for the purpose of this paper we do not take into account quality differences explicitly.

The regimes may be formally written as

$$\begin{aligned} &\text{Regime 1} \quad F(p_r, M) < Q_r, \\ &\text{Regime 2} \quad F[p_w, M + (p_w - p_r)Q_r] < Q_r < F(p_r, M), \\ &\text{Regime 3} \quad F[p_w, M + (p_w - p_r)Q_r] > Q_r. \end{aligned} \tag{6}$$

If we now assume that the demand functions $F(\cdot)$ have a specific functional form and contain a random term known to the household but which cannot be observed directly, then we have a stochastic model whose likelihood can be written down fairly easily – see section 4 below.

2.2. Rural households

A similar analysis can be used to describe the choice of the rural household facing different buying and selling prices for wheat (p_w and p_r, respectively). This is illustrated in fig. 2. We first consider a household that does not buy or sell wheat. If its lump-sum income is M, and the cost of production of a quantity Q_w of wheat is $C(Q_w, q)$, where input prices are q, then it can spend on other goods an amount $M - C(Q_w, q)$. The frontier describing its consumption possibilities is given by the convex curve EE' in fig. 2 (the convexity follows from increasing marginal costs of production, which we shall assume).

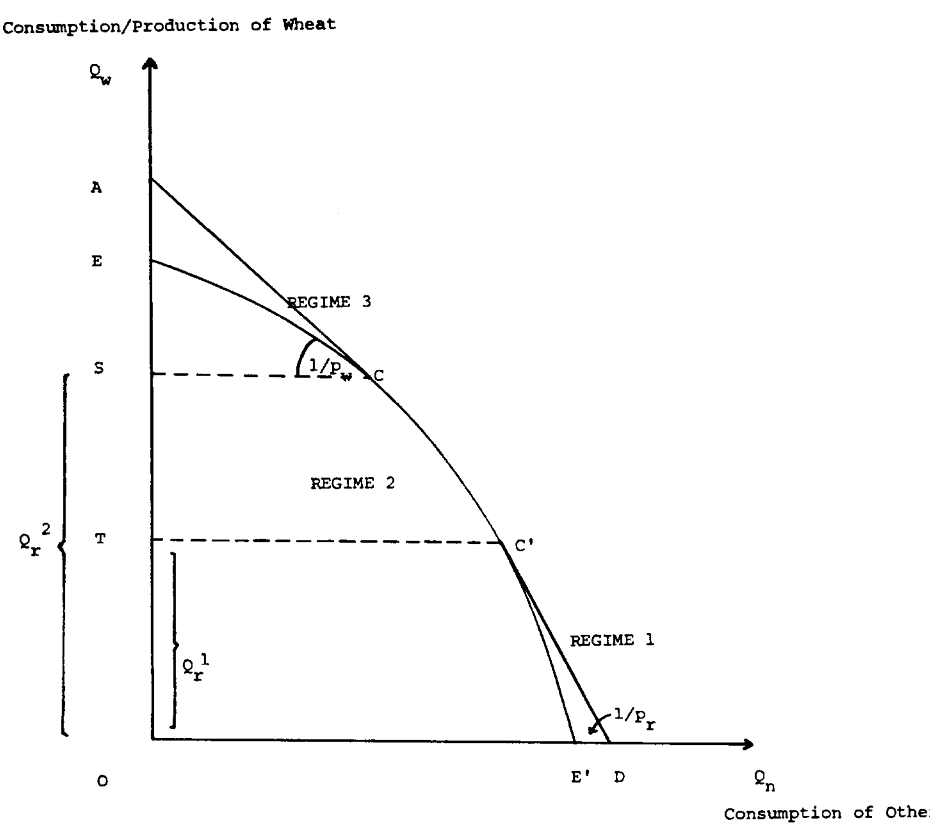

Fig. 2. Rural households – the budget constraint for wheat.

Suppose now that the household can buy at p_w and sell at p_r: the consumption possibility frontier becomes $ACC'D$ (see fig. 2). The region AC involves buying wheat on the open market with production OS, or Q_r^2. The point C is defined by the gradient of EE' being $-1/p_w$. We call AC regime 3. The region $C'E'$ we call regime 1 and it corresponds to production of an amount Q_r^1, equal to OT, and selling of wheat at price p_r. At the point C' the gradient of EE' is $-1/p_r$. Households which neither buy nor sell are on CC', or regime 2.

We can now bring in preferences and demand as before. The convexity of the consumption possibility frontier $ACC'D$ implies, using the same type of revealed preference argument as for the urban case, that the three regimes are mutually exclusive as possible consumption choices. As before we describe the preferences through the demand function $F(p, Y)$ and the three regimes may be written formally, analogously to (6),

$$\begin{aligned}
&\text{Regime 1} \quad F\{p_r, M+\Pi(p_r, q)\} < Q_r^1, \\
&\text{Regime 2} \quad \begin{array}{l} F\{p_w, M+\Pi(p_w, q)\} < Q_r^1, \\ F\{p_r, M+\Pi(p_r, q)\} > Q_r^2, \end{array} \\
&\text{Regime 3} \quad F\{p_w, M+\Pi(p_w, q)\} > Q_r^2,
\end{aligned} \tag{7}$$

where $\Pi(p, q)$ is the maximum profit associated with wheat price p and input prices q, and M is other lump-sum income. The profit function will be restricted if certain factor inputs are fixed.

If we now take specific functional forms for $F(\cdot)$ and the cost function $C(\cdot)$ (and hence Π) and include a random term (known to the household but not the observer) in, say, F, we have a stochastic model whose likelihood may be written down (for an example using the urban model, see section 4). Note that given $C(\ )$, p_r, and p_w, we can calculate Q_r^1 and Q_r^2. The data set (see below) did not contain sufficient information on input prices, so we were not able to estimate the model directly. We will, however, present cost functions based on an alternative data set for rural households in a subsequent paper.

3. The data

The non-linear pricing models described above can be conveniently examined using data from the 1977 *Micro-Nutrient Survey* (or *MNS*) conducted under the aegis of the Pakistan Planning Commission. This provides information on the consumption of wheat, with a clear distinction between rationed supplies and open market purchases. Also there are data on production, consumption, sales and purchases of wheat by rural households. However, there is only scanty information on inputs into the production process, and for the estimates of the cost and profit function required by eqs. (7) we will make use of the 1977 Water and Power Development Authority's (WAPDA) *Agricultural Economics Survey of the Indus Basin* (or the *Indus Basin Survey*). The rural estimates will be presented in a subsequent paper.

3.1. Urban households and rationing

The *MNS* data show that on average the ration price for wheat was Rs 0.90 per kg across the urban clusters sampled. While there was some variation in the price across clusters, within-cluster variance of the ration price was zero and the range of the ration price was between Rs 0.883 and Rs 1.01. The market price of wheat was Rs 1.20 on average across the whole sample, ranging from a low of Rs 1.00 to a high of Rs 2.75, with some within-cluster variation as well. At the time the ration allotment was 1.75 kg per adult per week, with children between the ages of 2 and 14 being entitled to half the allotment.

We may categorise the three regimes in terms of households that consumed ration wheat in urban areas. There were 22 in regime 1 consuming less than the quota, 119 in regime 3 consuming both rationed wheat and market wheat, and 105 households on the kink (regime 2). There were 483 urban households in the full urban sample. The 170 excluded households involve those with missing values for wheat consumption or rations and those not consuming rationed wheat. Of the households not consuming rationed wheat almost half cited quality as the reason for not taking up the ration entitlement, and the remainder either had home grown stocks, gifts or were prevented from taking up the ration for a variety of reasons, including *inter alia* the non-availability of ration shops, or stocks in the ration shop, or access to a ration card. We assume that these households have different preferences or opportunities and thus cannot be represented as coming from the same model (for further discussion see below). We therefore exclude them from the sample.

4. Estimation

We use a linear demand function for wheat

$$Q_{\mathrm{w}} = F(p, M) = \beta_0 + \beta_1 p + \beta_2 M, \tag{8}$$

where Q_{w} and M are per capita wheat demand and per capita expenditures, respectively. As emphasised, this is simply a way of representing preferences and does not involve either the assumption that the budget constraint is linear or that the demand function can be estimated by linear regression. For the utility function corresponding to this demand function, see Hausman (1981) and Stern (1986). We have also experimented with two other functional forms: log-linear and a modification of (8) above with a quadratic price term, and the elasticities estimated have been remarkably stable. A more detailed treatment of family composition has not been attempted given the limitations of the data. With an additive stochastic term, we have

$$Q^i_{\mathrm{w}} = \beta_0 + \beta_1 p^i + \beta_2 M^i + \varepsilon^i, \tag{9}$$

where the superscript denotes household i. The stochastic term ε^i may be interpreted as an unobservable component of taste (known to the household), which is assumed to be independently and normally distributed,

$$\varepsilon^i \sim \mathrm{N}(0, \sigma^2). \tag{10}$$

Given the characterisation of the three regimes from the previous section, we may develop the likelihood function in the following manner. Denote the probability density at a consumption point Q_{w} of a household in regime 1 by $\varphi_1(Q_{\mathrm{w}})$ and that of a regime 3 household by $\varphi_3(Q_{\mathrm{w}})$. Thus

$$\varphi^i_1(Q^i_{\mathrm{w}}) = (2\pi\sigma^2)^{-1/2}\exp\left\{-\frac{1}{2\sigma^2}\left(Q^i_{\mathrm{w}} - \beta_0 - \beta_1 p^i_{\mathrm{r}} - \beta_2 M^i\right)\right\}, \tag{11}$$

and

$$\varphi^i_3(Q^i_{\mathrm{w}}) = (2\pi\sigma^2)^{-1/2}\exp\left\{-\frac{1}{2\sigma^2}\left[Q^i_{\mathrm{w}} - \beta_0 - \beta_1 p^i_{\mathrm{w}} - \beta_2\left(M^i + \left(p^i_{\mathrm{w}} - p^i_{\mathrm{r}}\right)Q^i_{\mathrm{r}}\right)\right]\right\}. \tag{12}$$

From (6) and (9) above it can be seen that a household belongs to regime 1 if

$$\varepsilon^i < Q_{\mathrm{r}} - \beta_0 - \beta_1 p_{\mathrm{r}} - \beta_2 M^i, \tag{13}$$

which has a probability Φ^i_1, given by (14) where $\Phi(\cdot)$ is the standard normal distribution function,

$$\Phi^i_1 = \Phi\left[\frac{Q_{\mathrm{r}} - \beta_0 - \beta_1 p^i_{\mathrm{r}} - \beta_2 M^i}{\sigma}\right]. \tag{14}$$

Similarly a household belongs to regime 3 if

$$\varepsilon^i > Q_r - \beta_0 - \beta_1 p^i_{\mathrm{w}} - \beta_2\left[M^i + \left(p^i_{\mathrm{w}} - p^i_{\mathrm{r}}\right)Q_{\mathrm{r}}\right], \tag{15}$$

which has a probability

$$\Phi_3^i = \Phi\left[\frac{\beta_0 + \beta_1 p_w^i + \beta_2\{M^i + (p_w^i - p_r^i)Q_r\} - Q_r}{\sigma}\right], \tag{16}$$

and the probability of a household being in regime 2 is thus

$$\Phi_2^i = 1 - \Phi_1^i - \Phi_3^i. \tag{17}$$

The log-likelihood is then, where N_j is the set of households in regime j,

$$\begin{aligned}\log L(\beta_0, \beta_1, \beta_2, \sigma) = &\sum_{i \in N_1} \log \varphi_1^i + \sum_{i \in N_3} \log \varphi_3^i \\ &+ \sum_{i \in N_2} \log \Phi_2^i.\end{aligned} \tag{18}$$

The function (18) is then maximised with respect to β_0, β_1, β_2, and σ. The results from the urban sample can be summarised as in table 1. The figures in parentheses are t-ratios. We observe that the estimated parameters have the right signs and that these are significant at the 5% level. The estimated elasticities at the sample mean for ration consuming households are an income elasticity of 0.18 and uncompensated and compensated price elasticities of 0.84 and -0.63, respectively. These estimates are considerably different from those obtained with standard ELES methods [see Ahmad, Ludlow and Stern (1987) for the ELES estimates]. However, Strauss (1986) obtained rural price elasticities for rice consumption in Sierra Leone that are comparable (-1.26 for the poorer rural income group) to those of this paper.

Table 1

Estimates of the demand for wheat by urban households.[a]

Ration users
$n_1 = 22$, $n_2 = 105$, $n_3 = 119$
$\beta_0 = 24.6$ (21.13)
$\beta_1 = 8.20$ (3.39)
$\beta_2 = 0.02$ (2.41)
Log-likelihood $= -465.34$
Income elasticity $= 0.18$
Uncompensated price elasticity $= -0.84$
Compensated price elasticity $= -0.63$

[a] t-ratios in parentheses; Q_w in seers per month; prices in Rs per seer; n_j is the number of households in regime j.

The likelihood for the rural model can be constructed in a similar way, using (7). From the cost function we can calculate Π and Q_r^1 and Q_r^2. Thus we can write Φ_i^1 and Φ_i^3, the densities for regimes 1 and 3, in a similar manner to (14) and (16) and then the probability of regime 2 as in (17).

5. Concluding remarks

In this paper we have modelled the demand for wheat where budget constraints are non-linear. Examples in Pakistan which include rationing for urban households, and the joint decision to produce and consume by farm households, are quite common. One can exploit the price differentials in the different regimes in each case to estimate price response in a cross section. These estimates differ from the now standard and somewhat restricted methods of measuring price response in cross-section models in the absence of explicit price data [see, for instance, the Extended Linear Expenditure System (ELES) of Lluch et al. (1973), Barnum and Squire (1979), or Ahmad, Ludlow and Stern (1987) who experimented with a variant of the ELES with the MNS data for Pakistan]. The own-price and income elasticities for the demand for wheat by ration consuming urban households derived in this paper are -0.84 and 0.18. These differ markedly from ELES estimates with the same data set from Ahmad, Ludlow and Stern (1987) of -0.28 and 0.30, respectively. The price elasticities estimated with the model of this paper are much larger in absolute magnitude than the ELES estimates and the income elasticities much lower. However, additively separable systems of which the ELES is one, impose strong restrictions on demand elasticities – indeed we have the approximation in such systems, known as Pigou's Law, that $e_{kk} \approx -\varphi e_k$, where e_{kk} is the uncompensated price elasticity for good k, e_k the income elasticity and φ a scalar independent of k [see Deaton and Muellbauer (1980)]. At the level of aggregation of most demand studies a value of φ less than one is common. Hence, given that the income elasticity for wheat is likely to be low, the price elasticities of the order quoted from the ELES studies are hardly surprising. Our method does remove the straight-jacket of additive separability. However, since the sample size in the MNS is small and the treatment of a number of aspects (particularly quality, and family composition) is unsatisfactory, the results in this paper should be considered only as illustrative of the method. Nonetheless the estimates from the techniques adopted were of significance, according to the conventional criteria, and of the 'right' sign. Their substantial difference from those arising from standard methods for cross-section data should add to our circumspection concerning the latter. We hope to illustrate the method described for rural households in subsequent work.

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