

## HOW SHOULD IGNORANCE OF RESPONSE TO INCENTIVES AFFECT TAX RATES? \*

Nicholas STERN

*London School of Economics, London WC2A 2AE, UK*

Received 27 July 1989

Accepted 17 August 1989

Our knowledge of labour supply response is limited. How should this affect tax rates? We provide an example in which greater uncertainty about the responsiveness of labour supply to the wage would have no effect on taxes.

### 1. Introduction

Models of optimum taxation [see e.g. Mirrlees (1971)] contain as a central ingredient a labour supply function which embodies the disincentive effects of taxation on labour supply. In such models, these disincentive effects are traded off against the redistributive benefits to find the optimum tax rates. The optimum rates which emerge from these models are quite sensitive to the elasticity of labour supply which is assumed [see e.g. Stern (1976)]. Unfortunately, our knowledge of labour supply elasticities in the relevant sense, notwithstanding worthy research on this subject, is somewhat limited. The notion of labour supply embodied in the models is long-term and includes effort, initiative, skill acquisition and so on as well as hours of work. It is on this last aspect that empirical research has been concentrated. Whilst informational difficulties with other aspects of labour make this concentration readily understandable we do remain in a state of considerable ignorance concerning the appropriate labour supply functions which should be used in our models.

The question we ask in this note is the following. What would be the effect of a reduction in the uncertainty concerning labour supply responses on the optimum tax rate? The answer to our question is likely to depend rather sensitively on the structure of the model and, no doubt, one could offer arguments why, both within and outside the analytical framework adopted here, less uncertainty about labour supply responses could shift judgements concerning the optimum income tax either up or down. The example we offer in this paper, using fairly standard ingredients, however, is one where a change in uncertainty has no effect whatsoever on the optimum tax schedule in a model of linear income taxation. In this sense ignorance, per se, tilts the argument neither for nor against high tax rates.

\* The example in this note was formulated in early 1985. Subsequent interest when I mentioned it to others has led me to make it more widely available. I am grateful to Tony Atkinson, Angus Deaton, Peter Diamond and Mervyn King for helpful comments. Support from the ESRC Programme Grant 'Taxation, Incentives and the Distribution of Income' is gratefully acknowledged.

## 2. The general problem

Suppose, in a model of linear income taxation with certainty, that  $V(t, \theta)$  is the level of social welfare as a function of the marginal tax rate and a measure of labour supply responsiveness  $\theta$ . We shall for the moment consider  $t$  and  $\theta$  as scalars. We shall think of a collection of individuals, with differing pre-tax wage rates,  $w$ , each choosing labour supply and consumption to maximise utility in the face of a post-tax budget constraint. There is a common utility function, or pattern of behaviour, which is characterised by  $\theta$ , a parameter of labour supply responsiveness. Social welfare depends on household welfare or utility. The government raises resources through marginal income taxation at a constant rate,  $t$ , to spend on a uniform transfer denoted by  $G$ . The government budget constraint will make the problem of optimum income taxation into a one-dimensional problem, which we think of as the choice of the constant marginal tax rate.

The first-order condition for the optimum tax rate in a world of certainty, where subscripts denote partial derivatives would be

$$V_t = 0. \quad (1)$$

This would yield the optimum tax rate as a function of  $\theta$ .

We do not, however, know  $\theta$  with certainty. Suppose that  $\theta$  is a random variable and that  $t$  has to be selected in advance of knowing  $\theta$ . We assume that the objective is the maximisation of the expectation of social welfare. The first-order condition becomes, where E denotes the expectation over  $\theta$ ,

$$EV_t = 0, \quad (2)$$

with a second-order condition

$$EV_{tt} < 0. \quad (3)$$

Suppose we now have an increase in riskiness of  $\theta$  [in the sense of Rothschild and Stiglitz (1970, 1971)] which holds its mean constant. Then, if  $V_t$  is a convex function of  $\theta$ , i.e.

$$V_{t\theta\theta} > 0, \quad (4)$$

the optimum tax rate will *rise*. This result is derived by Rothschild and Stiglitz (1971) and may be seen intuitively as follows. The convexity of  $V_t$  means that an increase in spread *raises*  $EV_t$ . From the second-order condition we have to *increase*  $t$  to bring  $EV_t$  back to zero. Similarly, if  $V_{t\theta\theta} < 0$ , then an increase in spread of the Rothschild–Stiglitz variety will reduce  $t$ . We think of less ignorance about incentives as a reduction in the spread of  $\theta$  and greater ignorance as an increase in spread.

The problem becomes one of the sign of  $V_{t\theta\theta}$ . This is a problem in pure theory, given the functional forms assumed, although we may not in general, from only the assumed functional forms be able to establish that  $V_{t\theta\theta}$  has a single sign. We may then have to appeal to applied work to help us with formulating restrictions on the location and shape of distributions to be considered if we are to obtain a single sign. Generally we are likely to find that the function  $V_t$  is sufficiently complicated that we have to turn to the computer to establish the effects of a change in spread on the optimum tax rate. In the next section we provide an example where we can get a simple and striking conclusion analytically.

We may also consider an alternative interpretation of  $\theta$  which is not as a random variable whose distribution reflects our ignorance of the labour supply parameter (common to everyone), but as a parameter which varies across households and is thus distributed within the population. The expectation  $E$  would then correspond to a sum across the population, and an increase in the spread of  $\theta$  to a more uneven distribution within the population of the parameter characterising the responsiveness of labour supply. See also Ordover (1987) for a discussion of the effect of changes in the spread of the distribution of wages using the Rothschild–Stiglitz approach.

### 3. An example

We suppose that the labour supply,  $\ell$ , is a linear function of post-tax wages,  $w(1 - t)$ , and of the lump-sum grant  $G$ . There is a distribution of pre-tax wages with density function  $f(w)$ . We assume, to keep things simple, that the range of the distribution and the lump-sum grant are such that there is no voluntary unemployment in the sense that for no individual the optimum labour supply is zero. We allow, however, for the possibility that some fraction  $(1 - \mu)$  of the population may, for reasons of sickness or whatever, be unable to work. Those who are unable to work must live on  $G$ . We suppose that those who have to subsist on  $G$  have the lowest welfare. An over-riding concern for the worst off would lead to an objective which would be the maximisation of  $G$ . The effect of uncertainty about labour response parameters on the optimum tax rate may be derived by examining the derivative  $G_{t\theta\theta}$ . If  $G_{t\theta\theta} > 0$  an increase in the spread will increase the optimum tax, and will reduce it if  $G_{t\theta\theta} < 0$ . We write the labour supply function as

$$\ell = \alpha w(1 - t) + \beta G + \gamma, \tag{5}$$

where we shall consider distributions of the parameters  $\alpha, \beta, \gamma$ . It is straightforward [see e.g. Stern (1986)] to calculate utility functions which correspond to this linear labour supply function. We may then calculate the lump-sum grant (and the level of social welfare)  $G$  as

$$G = \mu t \int w \ell f(w) \, dw, \tag{6}$$

where we have normalised total population to be unity. Therefore, where  $\bar{w}$  and  $\sigma_w^2$  are the mean and variance of the  $w$ -distribution, using (5) we have

$$G = \mu t(1 - t)\alpha(\sigma_w^2 + \bar{w}^2) + \mu t(\gamma + \beta G)\bar{w}, \tag{7}$$

or

$$G = (\mu\alpha t(1 - t)(\sigma_w^2 + \bar{w}^2) + \mu\gamma t\bar{w}) / (1 - \mu\beta t\bar{w}), \tag{8}$$

where we assume that the parameters are such that this value of  $G$  is positive. The effect of an increase in uncertainty concerning  $\alpha$ , for example, on the optimum tax rate is given by the sign of  $G_{t\alpha\alpha}$ . But we see immediately from (8) that  $G_{\alpha\alpha}$  is zero (and the same is true of  $G_{\gamma\gamma}$ ). Hence neither an increase nor a decrease in uncertainty concerning the response of labour supply to the wage will have any effect whatsoever on tax policy.

**References**

- Mirrlees, J.A., 1971, An exploration in the theory of optimum income taxation, *Review of Economic Studies* 38, no. 114, April, 175–208.
- Ordoover, J.A., 1987, Redistributing income: Ex rate or ex post, *Economic Inquiry* 19, April, 333–349.
- Rothschild, M. and J.E. Stiglitz, 1970, Increasing risk, Part I, A definition, *Journal of Economic Theory* 2, 225–243.
- Rothschild, M. and J.E. Stiglitz, 1971, Increasing risk, Part II, Its economic consequences, *Journal of Economic Theory* 3, 66–84.
- Stern, N.H., 1976, On the specification of models of optimum income taxation, *Journal of Public Economics* 6, nos. 1–2, July–Aug., 123–162.
- Stern, N.H., 1986, On the specification of labour supply functions, in: R. Blundell and I. Walker, eds., *Unemployment, search and labour supply* (Cambridge University Press, Cambridge) Ch. 9.