

# Web-based Technical Appendix for History and Industry Location: Evidence from German Airports

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## 1. Introduction

This web-based technical appendix develops in further detail the general equilibrium structure which underlies the model of air travel and hub creation outlined in Redding, Sturm and Wolf (2009).

### 1.1. *Endowments and Preferences*

We assume that each location (or city) supplies a differentiated non-traded service that can only be consumed at the point of production. To focus on the demand for air travel, we assume that air travel is the only means of consuming non-traded services in other cities. For a resident of a city to consume one unit of the non-traded service produced by another city requires one return flight. Consumers also derive utility from a homogeneous numeraire good which is assumed to be freely traded between cities.<sup>1</sup>

The representative consumer's preferences are Cobb-Douglas in a consumption index of non-traded services and in the homogeneous numeraire good. The modelling of the demand for non-traded services follows Anderson and van Wincoop (2003). The non-traded services consumption index is assumed to take the standard Constant Elasticity of Substitution (CES)

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<sup>1</sup>This formulation sweeps all economic activity that is traded through routes other than air travel into the homogeneous numeraire good, and allows us to focus on the demand for and supply of air travel.

form so that:

$$U_j = \left( \sum_{i=1}^N \beta_{ij}^{\frac{1-\sigma}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\left(\frac{\sigma}{\sigma-1}\right)\alpha} (q_j)^{1-\alpha}, \quad 0 < \alpha < 1, \sigma > 1 \quad (1)$$

where  $N = 3$  denotes the number of cities;  $\alpha$  is the share of expenditure on non-traded services;  $\sigma$  is the elasticity of substitution between the varieties of non-traded services;  $\beta_{ij}$  is an inverse measure of the weight allocated by consumers in city  $j$  to the non-traded services produced in city  $i$ ;  $c_{ij}$  denotes the consumption of non-traded services produced in city  $i$  by residents of city  $j$ ;  $q_j$  indicates the consumption of the homogeneous numeraire good.<sup>2</sup>

Cities are populated with a mass of  $L_i$  consumers who have identical preferences, have a fixed city of residence from which they may travel to consume non-traded services, and are endowed with one unit of labor that is supplied inelastically with zero disutility.

### 1.2. Technology and Market Structure

The numeraire good is produced under conditions of perfect competition and according to a constant returns to scale technology:  $y_i = l_i^y$ , where  $y_i$  and  $l_i^y$  denote output and labor used in production of the numeraire. We choose units in which to measure the numeraire good so that the unit labor requirement is equal to one. Since the numeraire good is freely traded, its price is equal to one in all cities:  $p_i^y = p^y = 1$ . In addition, we focus on parameter values for which all cities produce the numeraire good, which pins down the equilibrium wage as equal to one:  $w_i = w = 1$ .<sup>3</sup>

Non-traded services are produced under conditions of perfect competition and according to a constant returns to scale technology:<sup>4</sup>

$$x_i \equiv \sum_{j=1}^N x_{ij} = l_i^x \quad (2)$$

where  $x_i$  corresponds to total production of non-traded services in city  $i$ ,  $x_{ij}$  is the quantity of non-traded services produced in city  $i$  and sold to residents of city  $j$ , and  $l_i^x$  denotes total

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<sup>2</sup>Throughout the analysis, the first subscript corresponds to the point of production and the second subscript to the point of consumption. We use  $i$  to indicate the city of production and  $j$  to indicate the city of residence of consumers.

<sup>3</sup>Incomplete specialization can be ensured by an appropriate choice of values for the preference parameters  $\beta_{ij}$  and labor endowments for each city.

<sup>4</sup>Note that, from equation (1), non-traded services are differentiated by city of production (as in Armington 1969) but are homogeneous within cities. Allowing for differentiated varieties of non-traded services within cities is straightforward, but merely complicates the analysis without adding any additional insight.

employment of labor in non-traded services in city  $i$ . We also choose units in which to measure non-traded services so that the unit labor requirement for this sector is equal to one.

The differentiation of non-traded services by city of origin ensures that all cities produce non-traded services. With the equilibrium wage equal to one, perfect competition and the production technology (2) imply that the equilibrium price of non-traded services is equal to one:  $p_i^x = p^x = 1$ . Since consuming one unit of a non-traded service from another city requires one return flight, the number of passenger journeys ( $a_{ij}$ ) equals demand for non-traded services ( $c_{ij}$ ), that is  $a_{ij} = c_{ij}$  for  $i \neq j$ . As the source and destination cities are not necessarily symmetric, the total number of return flights between cities  $j$  and  $i$  is equal to  $a_{ij} + a_{ji}$ .

As discussed in the main text, we consider a monopoly airline that has the choice whether to operate direct connections between cities or to operate indirect connections via a hub. We assume that there is a fixed cost of  $F > 0$  units of labor of operating each direct connection and then a marginal cost in terms of labor for each return passenger. In addition, we assume that there is a sunk cost of  $H > 0$  units of labor of creating a hub. Since we focus on equilibria where specialization is incomplete, and so the wage in all cities is equal to one, the airline is indifferent as to where to source labor. The marginal cost is a function of the distance flown  $d_{ij}$ ,  $\psi(d_{ij})$ , where distance flown depends on whether a direct or indirect connection is operated between cities  $j$  and  $i$ . With a direct connection, the airline flies the shortest feasible distance between cities  $i$  and  $j$ ,  $\delta_{ij}$ , and so  $d_{ij} = \delta_{ij}$ . With an indirect connection, the airline flies the shortest feasible distance from city  $i$  to the hub in city  $k$  plus the shortest feasible distance from city  $k$  to city  $j$ , and so  $d_{ij} = \delta_{ik} + \delta_{kj} \geq \delta_{ij}$ . The total labor required for  $a_{ij}$  passenger journeys from city  $i$  to city  $j$  is therefore:

$$l_{ij}^a = \begin{cases} a_{ij}\psi(\delta_{ij}) + F & \text{if the connection is direct} \\ a_{ij}\psi(\delta_{ik} + \delta_{kj}) & \text{if the connection is indirect} \end{cases} \quad (3)$$

### 1.3. Airline Equilibrium Prices and Profits

Consumers are price-takers and take into account the full cost of consuming non-traded services, which equals their price at the point of production plus the cost of air-travel. Expenditure minimization yields the standard CES demand for non-traded services. Therefore city  $j$  residents' demand for the non-traded services produced in city  $i$ , and hence city  $j$  residents'

demand for air travel to city  $i$ , is:

$$c_{ij} = a_{ij} = \beta_{ij}^{1-\sigma} T_{ij}^{-\sigma} P_j^{\sigma-1} E_j^T \quad (4)$$

where  $\beta_{ij}$  is the inverse measure of the weight allocated by consumers in city  $j$  to the non-traded services produced in city  $i$ ;  $T_{ij} = p_i^x + p_{ij}^a$  is the composite cost of purchasing one unit of non-traded services at price  $p_i^x$  and one return air journey at price  $p_{ij}^a$ ;  $E_j^T = \alpha E_j = \alpha w L_j$  is expenditure on the composite good of non-traded services and air travel which equals a constant share of total expenditure which equals income;  $P_j$  is the CES price index summarizing the full cost of consuming non-traded services for residents in city  $j$ :

$$P_j = \left[ \sum_{i=1}^N (\beta_{ij} T_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (5)$$

Recall that we assume the airline is able to segment markets for travel between each pair of cities. Therefore profit maximization yields the standard result that the equilibrium price of a return trip between two cities is proportional to marginal cost:

$$p_{ij}^a = \begin{cases} \left( \frac{\varepsilon(a_{ij})}{\varepsilon(a_{ij})-1} \right) \psi(\delta_{ij}) & \text{if the connection is direct} \\ \left( \frac{\varepsilon(a_{ij})}{\varepsilon(a_{ij})-1} \right) \psi(\delta_{ik} + \delta_{kj}) & \text{if the connection is indirect} \end{cases} \quad (6)$$

where  $\varepsilon(a_{ij})$  denotes the elasticity of demand.

From the equilibrium pricing rule, variable profits from passenger journeys from city  $j$  to city  $i$  equal revenue divided by the elasticity of demand:  $\rho_{ij} = (p_{ij}^a a_{ij}) / \varepsilon(a_{ij})$ . Variable profits for the route as a whole equal the sum of variable profits on passenger journeys in each direction:  $\pi_{ij} = \rho_{ij} + \rho_{ji}$ . Variable profits will be lower if a route is served by an indirect rather than a direct connection for two reasons. First, marginal cost is higher if a route is served by an indirect connection, which increases prices. Since demand is elastic, the higher prices decrease revenues and so diminish variable profits. Second, one can allow for a disutility of changing planes on indirect connections (e.g. by assuming that  $\beta_{ij}$  is higher if a route is served by an indirect rather than a direct connection), which further reduces the demand for air travel on indirect connections, and so decreases revenue and variable profits.<sup>5</sup>

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<sup>5</sup> A richer model would be able to explain the co-existence of direct and indirect connections on routes and the empirically observed lower prices for indirect connections. While this would complicate the analysis, the decision to create a hub would still depend on the trade-off between profits on direct and indirect connections and the fixed costs of operating a direct connection.

#### 1.4. Bilateral Passenger Departures

The number of return passenger journeys from city  $j$  to city  $i$  is determined by equation (4). Since passenger journeys are round-trips, the total number of departing passengers from city  $j$  to city  $i$  is the sum of passengers travelling in each direction:

$$A_{ij} = a_{ij} + a_{ji} = \beta_{ij}^{1-\sigma} T_{ij}^{-\sigma} P_j^{\sigma-1} E_j^T + \beta_{ji}^{1-\sigma} T_{ji}^{-\sigma} P_i^{\sigma-1} E_i^T \quad (7)$$

Equation (7) implies that bilateral passenger departures depend on characteristics of the source city  $j$ , characteristics of the destination city  $i$ , and bilateral travel costs. Log-linearizing this relationship, collecting terms in source city characteristics in a fixed effect  $s_i$ , collecting terms in destination city characteristics in another fixed effect  $m_i$ , and allowing for a stochastic component to bilateral travel costs  $u_{ij}$ , we obtain the gravity equation for bilateral departures in the main text of the paper.

### References

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