Productivity and Inflation Dynamics *

Ludovica Ambrosino† Jenny Chan ‡ Silvana Tenreyro §

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Abstract

How does higher productivity affect inflation? Productivity shifts both supply and demand (through real incomes), and its effect on inflation depends on their relative magnitude and timing, and on how monetary policy responds. We distinguish between a one-off level shock that temporarily increases productivity above trend and a persistent increase in productivity growth. A one-off shock lowers marginal costs and lifts potential output, generating downward pressure on the price level. However, this is a level effect and once prices adjust, inflation returns to target. In contrast, higher productivity growth raises expected permanent income, which stimulates investment and consumption, pushing up on the natural real rate. Absent a tightening of monetary policy, this can generate inflationary pressures. Anticipation effects are central: if demand rises ahead of realised productivity gains, inflation can rise even as productive capacity expands. In an open economy, the impact on inflation also depends on whether productivity gains occur in the tradable or the non-tradable sector. Altogether, the inflationary consequences of higher productivity are a priori ambiguous and depend on the balance and timing of demand and supply responses, the composition of consumption across tradables and non-tradables, and importantly, the monetary policy response.

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JEL classification: E3, E4, E5, F4, O4

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†London Business School. Contact: lambrosino@london.edu

‡Bank of England; CFM, CEPR. Contact: jenny.chan@bankofengland.co.uk

§London School of Economics, UK; CFM, CEPR. Contact: Tenreyro: Department of Economics and Centre for Macroeconomics, London School of Economics, Houghton Street, London, WC2A 2AE, United Kingdom; s.tenreyro@lse.ac.uk

1 Introduction

Recent technological breakthroughs raise an important question for policymakers: will higher productivity lead to inflationary or disinflationary pressures? The answer depends on the balance between supply and demand. Higher productivity expands productive capacity, placing downward pressure on inflation, while higher real incomes stimulate aggregate demand, leading to upward pressure on inflation. The relative strength and timing of these effects determine the overall inflationary impact. Since monetary policy operates primarily through aggregate demand, the key question for central banks is: how policy should respond to productivity shocks to keep inflation at target?

We analyse the demand and supply channels in a small open economy New Keynesian model. To reconcile competing views on the effect of productivity, we distinguish between a level shock and a growth shock. A one-time increase in productivity raises potential output and lowers marginal costs. The initial effect is disinflationary, as lower marginal costs place downward pressure on prices. However, this is a level effect, not a permanent change in inflation. Once prices adjust, inflation returns to target.

By contrast, an increase in productivity *growth* implies a persistent increase in the level of productivity and potential output. Therefore, as productivity rises over time, the expected returns to investment and the marginal product of labour increase, leading to higher expected permanent income. In anticipation of higher future returns and incomes, aggregate demand increases as firms raise current investment and households raise current consumption. This increase in demand leads to a higher natural rate of interest. If monetary policy does not adjust in line with the equilibrium rate of interest, inflationary pressures may emerge.[1,2]

In either case, whether the increase in productivity is one-off or sustained, inflation depends on the relative speed of supply and demand adjustment. On the supply side, higher productivity raises potential output and lowers marginal costs, capturing the partial equilibrium intuition that higher productivity is disinflationary. Our model embeds this mechanism within a general equilibrium setting in which demand responds as well. Higher expected permanent income raises consumption, while stronger expected returns increase investment. The effect on inflation therefore depends on the balance between expanding productive capacity and the magnitude and timing of the demand response.

The anticipation margin is critical. If households and firms revise expectations upward ahead of realised productivity gains, demand may rise before supply adjusts. In this case,

[1] This distinction clarifies why historical analogies, such as the late 1990s IT boom, are often misinterpreted. Although higher productivity growth initially coincided with subdued inflation, it did not eliminate risks of inflation. Core PCE rose increased steadily until the 2001 recession. Stronger productivity growth raised the natural rate of interest, prompting monetary tightening shortly thereafter, and policy would likely have remained tighter for longer absent the 2001 recession (Furman (2026)).

[2] For a distinction between a shock to levels versus growth, see also Christiano, Trabandt, and Walentin (2011).

inflationary pressures can emerge despite higher productive capacity. To illustrate the importance of the relative timing and magnitude of the demand and supply response, we consider two scenarios (see also Ambrosino, Chan, and Tenreyro (2026)). In a front-loaded scenario, where productivity gains are realised immediately, marginal costs fall, generating disinflationary pressures. Although higher real incomes also stimulate demand and higher returns to capital stimulate investment, the rapid expansion of supply dominates and inflation falls.[3] Next, we consider a scenario where productivity gains materialise slowly over time.[4] Households and firms increase consumption and investment in anticipation of higher future income and returns, raising demand before supply fully adjusts. The natural rate increases and monetary policy must tighten in order to prevent excess demand.[5] In the absence of such a response, inflationary pressures would build. Therefore, whether higher productivity is inflationary or disinflationary depends critically on whether demand precedes the supply expansion or whether the supply expansion keeps pace with demand.

The impact of higher productivity on inflation will also depend on its sectoral incidence. A positive productivity shock is therefore not mechanically disinflationary, as its aggregate effect will depend on how relative prices adjust and on the degree of price flexibility in the sectors where gains occur. In a small open economy, a productivity increase in the tradables sector does not reduce domestic tradable prices, which are anchored to the world price. Higher real incomes can instead raise demand for non-tradables, whose supply is typically less elastic. The rise in non-tradables prices can potentially increase CPI inflation, absent countervailing changes in tradables prices. By contrast, a productivity increase in the non-tradables sector expands supply in a sector which faces downward-sloping domestic demand. Although higher real incomes increase demand for tradables, their prices are dependent on global prices. The decline in non-tradables prices may therefore dominate, lowering CPI inflation. To summarise, the composition of consumption also determines whether aggregate price pressures rise or fall following a positive productivity shock.

[3] Other scenarios may also be disinflationary. For example, demand could weaken if uncertainty dampens investment, if labour-displacing innovation increases precautionary savings, or if frictions lead to slow or uneven reallocation (Ferrante, Graves, and Iacoviello 2023; Guerrieri, Lorenzoni, Straub, and Werning 2021). A sequence of one-sided unanticipated, positive productivity shocks may also be disinflationary if demand consistently fails to adjust upwards.

[4] This scenario is plausible if innovation diffuses gradually. Historical experience with general-purpose technologies, such as electricity and the internet, show that productivity gains materialise gradually as applications develop and firms reorganise production. Consistent with this view, Jones and Tonetti (2026) use growth-based accounting to suggest that even when automation accelerates, aggregate growth may only increase gradually due to bottlenecks and low substitution across tasks (”weak links”).

[5] Higher productivity does not automatically imply higher r^* either. Higher productivity may raise desired savings more than desired consumption if it increases uncertainty (for example, through higher unemployment). In this case, precautionary savings may increase, and if the increase in desired savings outweighs the increase in desired investment, aggregate demand may weaken and the natural real rate may fall. Similarly, if productivity gains disproportionally accrue to capital owners with high savings propensity than labour income, r^* could fall. Disinflationary pressures would emerge if the policy response were insufficiently expansionary.

Across all scenarios, the natural rate of interest provides a guide for the appropriate stance of monetary policy. Higher productivity growth raises expected income and investment demand, increasing the natural real rate. Stabilising inflation requires monetary policy to track this shift. The key message is that the inflationary consequences of higher productivity are *a priori* ambiguous. They depend on whether the shock affects levels or growth, on the timing of supply and demand responses, on the sectoral incidence, and, ultimately, on the monetary policy response.

Related Literature A large literature studies productivity shocks as drivers of business-cycle fluctuations. Early real business cycle models such as Long and Plosser (1983) emphasise technology shocks as primary sources of aggregate fluctuations, with positive shocks leading to an expansionary effect on output and hours. New Keynesian frameworks refined this analysis, showing how short-run effects of productivity shocks depend on nominal rigidities and the policy response (Galí 1999). Under flexible prices, a positive productivity shock lowers marginal costs and inflation while raising output. However, the presence of nominal rigidities and endogenous policy can attenuate or even lead to a contractionary response of hours and output, depending on how policy responds to changes in the natural rate and expected marginal costs. Christiano, Eichenbaum, and Vigfusson (2004) and Basu, Fernald, and Kimball (2006) offer a different view, showing that the contractionary effects identified in some studies are sensitive to identification assumptions and measurement. Alternative specifications show that technology shocks increase hours and output, consistent with RBC predictions. Whether or not productivity shocks are expansionary or contractionary, Smets and Wouters (2007) show that their contribution to inflation and output volatility may be limited. Finally, Christiano, Trabandt, and Walentin (2011) also study the distinction between level and growth-rate productivity shocks. We contribute to this literature by showing that the impact of productivity on inflation is ambiguous and crucially depends on how demand adjusts, which is shaped by the policy response and by the sector and time profile of the productivity process. Specifically, we go beyond the distinction between level and growth-rate TFP shocks by distinguishing productivity gains in the tradable versus the non-tradable sector, and by differentiating a temporary level shock from a permanent one and, within the latter, a front-loaded increase from a gradual one.

The information technology (IT) boom of the 1990s renewed interest in the macroeconomic consequences of technological change. Oliner and Sichel (2000), Stiroh (2002), and Jorgenson, Ho, and Stiroh (2008) document the contribution of IT capital deepening to the increase in U.S, labour productivity and growth. At the time, however, there was also considerable debate about whether this would lead to a permanent shift in trend growth. Gordon (2000) argued at the time that the gains were concentrated in a small number of sectors and therefore unlikely to generate sustained, broad-based effects comparable to earlier general purpose technologies. In retrospect, the evidence suggests a nuanced assessment. Productivity growth did increase in the 1990s and early 2000s. However, in line with the

”productivity J-curve” perspective, these gains required complementary intangible investments, sectoral reallocation, and time to diffuse across sectors. Productivity growth was not permanent, as U.S trend growth slowed again after the mid-2000s.

A rapidly growing literature studies the macroeconomic implications of artificial intelligence (AI) as a general purpose technology. Acemoglu (2024) develops a tractable framework in which AI affects task allocation and labour demand, showing how aggregate gains depend on whether AI complements or substitutes for labour.[6,7] Brynjolfsson, Rock, and Syverson (2021) show how intangible investments, such as organisational capital, skills, and complementary technologies can initially depress measured productivity before generating substantial gains. The transitional dynamics can entail reallocation and adjustment costs, complicating the assessment of productivity growth in real time. Recent contributions also highlight sectoral heterogeneity, intangible capital accumulation, AI's role in accelerating innovation itself (McElheran, Yang, Kroff, and Brynjolfsson 2024; Baily, Byrne, Kane, and Soto 2024; Filippucci, Gal, and Schief 2024; Bontadini, Corrado, Haskel, and Jona-Lasinio 2024). A common theme in this literature is that lags in diffusion and complementary investments affect the timing and magnitude of aggregate productivity effects. However, they do not consider the implications for inflation and the appropriate monetary policy response.

Our paper is also related to the literature on expectations and news shocks. Information about higher future productivity can lead to increases in consumption and investment prior to their realisation (Beaudry and Portier 2006; Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro 2024). In New Keynesian models, anticipated productivity growth raises the natural rate, generating short-run demand pressures even before supply expands (Woodford 2003; Laubach and Williams 2003).[8] These mechanisms are particularly relevant with investment dynamics, since forward-looking investment decisions may precede observable productivity gains. Relatedly, Ambrosino, Chan, and Tenreyro (2026) analyse various shocks related to trade fragmentation to show how the timing and relative magnitude of demand versus supply effects leads to different implications for inflation dynamics and policy.

Finally, we build on a rich literature on monetary policy in small open economies (SOEs), including the seminal work of Benigno and Benigno (2003), Gali and Monacelli (2005), and Corsetti, Dedola, and Leduc (2010). Other important contributions to this line of research include but are not limited to, Santacreu (2005) and De Paoli (2009), who study tradable

[6]The inflationary consequences also depend on how productivity affects the capital stock. If productivity gains are disembodied (raising the productivity of existing capital), marginal costs fall and investment demand may decline, leading to disinflationary pressures. However, if productivity is embodied in new capital, as in putty-clay models, firms must invest to realise gains as existing capital becomes obsolete (Gilchrist and Williams 2000; Martinez 2021; Proebsting 2024). In this case, investment demand rises, raising the natural rate. Without monetary tightening, these dynamics can become inflationary.

[7]See Yotzov et al. (2026) for survey-level evidence on AI adoption and its effects on jobs, productivity, and output.

[8]Holston, Laubach, and Williams (2017) provide empirical estimates linking shifts in trend productivity growth to movements in the equilibrium real interest rate.

and non-tradable sectors in SOEs, and Schmitt-Grohé and Uribe (2003), who introduce imperfect international risk sharing. Within this class of models, productivity shocks to the tradables sector affect relative prices, leading to real exchange rate appreciation and a reallocation of demand across sectors. Such shocks may increase non-tradables inflation despite falling marginal costs in tradables. Different price rigidities across sectors may also affect how changes in relative prices affect aggregate CPI inflation, with implications for optimal monetary policy (Aoki 2001). When frictions impede reallocation across sectors, asymmetric shocks can generate a tradeoff between stabilising inflation and the output gap (Guerrieri, Lorenzoni, Straub, and Werning 2023).

Outline Section 2 sets out the theoretical framework. Section 3 describes the model calibration while Section 4 considers several scenarios that entail higher productivity in the non-tradables sector. Section 5 considers the case where productivity gains instead occur in the tradables sector. Finally, Section 6 presents concluding remarks and potential directions for future research.

2 Baseline Model

The goal of this section is to deliver qualitative insights into the macroeconomic effects of productivity shocks. We present a small open economy New Keynesian model with two production sectors (tradable and non-tradable), sector-specific capital accumulation, imported intermediate inputs, and nominal rigidities, building on Drechsel, McLeay, and Tenreyro (2019), Ambrosino, Chan, and Tenreyro (2026), and Ferrero and Seneca (2019). Within this setting, we study the channels through which higher productivity in the nontradables sector affects inflationary pressures across different scenarios.

2.1 Households

There is a continuum of households with identical preferences at any given point in time t. Each household consumes C_t and supplies labour N_t, at wage W_t, leading to expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_\ell \frac{N_t^{1+\phi}}{1+\phi} \right\}$$

The parameters β, σ, and ϕ capture the discount factor, the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity, respectively, while κ_ℓ is the disutility weight placed on labour.

The household trades domestic and foreign bonds and accumulates an aggregate capital stock, K_t. The period budget constraint of these households is given by:

$$
\begin{aligned}
P_tC_t + B_t + P_tI_t + \mathcal{E}_tB_t^* = & B_{t-1}(1+i_{t-1}) + (1+i_{t-1}^k)K_{t-1} + \mathcal{E}_tB_{t-1}^*(1+i_{t-1}^*) \\
& + W_tN_t + P_t\Psi_t - \frac{\chi}{2}\mathcal{E}_tP_t^*\left(\frac{B_t^*}{P_t^*} - b^*\right)^2.
\end{aligned} \tag{1}
$$

where P_t is the aggregate price level and B_t denotes the holdings of a risk-free one-period nominal bond in domestic currency, which pays the nominal interest rate i_t. The risk-free one-period nominal bond in foreign currency is denoted by B_t^*. The foreign interest rate is denoted by i_t^* and the nominal exchange rate (expressed in terms of domestic currency relative to foreign currency) is denoted by $\mathcal{E}_t$. The real profits from firms in both tradable and non-tradable sectors (Ψ_t) are rebated to the agent. K_{t-1} represents the stock of capital the agent starts the period with, i_t^k is the return on capital and I_t is investment.

Aggregate investment across sectors yields the following law of motion for capital

$$
K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]I_t. \tag{2}
$$

Sectoral capital stocks $(K_{N,t}, K_{H,t})$ are determined within the period by firms through their factor demand decisions.

Following Schmitt-Grohé and Uribe (2003), we assume that there is a quadratic cost of changing the real bond position relative to a real steady-state value (b^*) when trading in the foreign bond market. This cost is a common feature of small open economy models and ensures that the economy returns to a unique steady-state net foreign asset position following a transitory shock. The cost (in units of the consumption index) is denoted by a non-negative parameter, χ, while P_t^* is the aggregate price level in the foreign country.

Households maximise their expected lifetime utility by choosing $\{C_t, N_t, K_t, I_t, b_t, b_t^*\}_{t=0}^{\infty}$ subject to the series of (real) budget constraints (1) and law of motion of capital (2), where $b_t = \frac{B_t}{P_t}, b_t^* = \frac{B_t^*}{P_t^*}$.

Consequently, the optimality conditions are as follows,

$$
\kappa_\ell(N_t)^\phi = (C_t)^{-\sigma}\frac{W_t}{P_t} \tag{3}
$$

$$
\frac{1}{(1+i_t)} = \beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{1+\pi_{t+1}}\right] \tag{4}
$$

$$
q_t = \beta\,\mathbb{E}_t\left[q_{t+1}(1-\delta) + C_{t+1}^{-\sigma}\frac{1+i_t^k}{1+\pi_{t+1}}\right] \tag{5}
$$

$$
[1+\chi(b_t^* - b^*)] = \beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1+i_t^*}{1+\pi_{t+1}^*}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\right] \tag{6}
$$

$$C_t^{-\sigma} = q_t\left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \phi_I\left(\frac{I_t}{I_{t-1}} - 1\right)\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\,\mathbb{E}_t\left[q_{t+1}\phi_I\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{7}$$

where $\Pi_{t+1} = (1 + \pi_{t+1}) = \frac{P_{t+1}}{P_t}$ is the gross inflation rate, $\Pi^*_{t+1} = (1 + \pi^*_{t+1}) = \frac{P^*_{t+1}}{P^*_t}$ is the foreign gross inflation rate, $\mathcal{S}_t = \frac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate, and $(1 + r^k_{t+1}) = \frac{(1+i^k_{t+1})}{(1+\pi_t)}$. We define $\Lambda_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$ as the relevant stochastic discount factor, since only the unconstrained households have access to financial markets. The household's optimality condition for labour yields the labour supply relation (3). The Euler equation (4) follows from the first order condition for b_t. Finally, households' choices of foreign and domestic bonds give rise to an uncovered interest rate parity condition, which links the expected change in the exchange rate to the differential between the domestic and foreign interest rates. The conditions on b_t and b^*_t imply equation (6), the deviation from the uncovered interest-rate parity (UIP). According to this equation, international risk sharing will generally be imperfect, and aggregate demand across countries will fluctuate inefficiently,

$$\chi(b^*_t - b^*) = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{(1 + i^*_t)}{(1 + \pi^*_{t+1})}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} - \frac{1 + i_t}{1 + \pi_{t+1}}\right)\right]$$

As in Santacreu (2005), aggregate consumption is a CES aggregate of both *tradable* (T) and *non-tradable* (N) goods,

$$C_t \equiv \left[(1 - \varsigma)^{\frac{1}{\iota}} C_{T,t}^{\frac{\iota-1}{\iota}} + \varsigma^{\frac{1}{\iota}} C_{N,t}^{\frac{\iota-1}{\iota}}\right]^{\frac{\iota}{\iota-1}}$$

where ς is the share of non-tradable goods in domestic consumption and ι is the elasticity of substitution between tradable and non-tradable goods. $C_{T,t}$ is a CES aggregate of tradable goods produced in the domestic and foreign economy,

$$C_{T,t} = \left[(1 - \theta)^{\frac{1}{\mu}} C_{H,t}^{\frac{\mu-1}{\mu}} + \theta^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{8}$$

where $1 - \theta$ captures the home bias: smaller values of θ imply that the economy consumes less foreign goods. The elasticity of substitution between home-produced tradable and foreign tradable goods is given by μ.

The aggregate CPI price index (P_t) and the tradable good price index ($P_{T,t}$) are respectively given by

$$P_t \equiv \left[(1 - \varsigma)P_{T,t}^{1-\iota} + \varsigma P_{N,t}^{1-\iota}\right]^{\frac{1}{1-\iota}}$$

$$P_{T,t} \equiv \left[(1 - \theta)P_{H,t}^{1-\mu} + \theta P_{F,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}$$

Aggregate prices are a function of the prices for both tradable and non-tradable goods. In turn, tradable prices are a composite of prices for domestically-produced tradable goods and foreign-produced tradable goods, with weights reflecting the degree of home bias. The non-tradable goods and price index are given, respectively, by

$$C_{N,t} \equiv \left(\int_0^1 C_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad P_{N,t} \equiv \left(\int_0^1 P_{N,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}},$$

where ϵ captures the substitutability across different non-tradable consumption varieties i.

2.2 Firms

Households supply labour to both the tradable and non-tradable sectors, such that

$$N_t = N_{H,t} + N_{N,t} \tag{9}$$

Labour is completely mobile across sectors, therefore there is only one wage rate in equilibrium. In a similar manner, capital K_t and foreign input $M_{F,t}$ are also supplied to both the tradable and non-tradable sectors, such that

$$K_t = K_{H,t} + K_{N,t} \tag{10}$$

$$M_{F,t} = M_{H,t} + M_{N,t} \tag{11}$$

2.2.1 Non-tradable goods sector

Final Goods Producers Competitive final goods producers assemble intermediate goods. We denote by $Y_{N,t}(i)$ the quantity demanded of each variety i, and by $P_{N,t}(i)$ the price charged by the individual firm producing variety i. The final good producer's optimisation problem is to choose $\{Y_{N,t}(i)\}, i \in [0,1]$ to maximise profits

$$\max_{Y_{N,t}(i)} \left\{ P_{N,t} Y_{N,t} - \int_0^1 P_{N,t}(i) Y_{N,t}(i) di \right\}$$

subject to an aggregation technology with constant elasticity of substitution,

$$Y_{N,t} = \left(\int_0^1 Y_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Profit maximization, taking as given the final goods price $P_{N,t}$ and the prices for the intermediate goods $P_{N,t}(i)$, yields the set of demand schedules

$$Y_{N,t}(i) = \left[\frac{P_{N,t}(i)}{P_{N,t}} \right]^{-\epsilon} Y_{N,t} \tag{12}$$

Intermediate Goods Producers Intermediate goods firms use labour and an *intermediate imported input* $M_{F,t}$ in production,

$$Y_{N,t} = A_{N,t} M_{N,t}^{\kappa_1} N_{N,t}^{\kappa_2} K_{N,t-1}^{1-\kappa_1-\kappa_2} \tag{13}$$

where $A_{N,t}$ is the exogenous sector-specific productivity. $M_{N,t}$ is the foreign country's final good used in the production of non-tradable goods, capturing intermediate input utilisation in the production function.

Imported intermediate inputs used in production are purchased at price $P_{F,t}$, which is fully determined by the law of one price,

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*, \tag{14}$$

where $P_{F,t}^*$ denotes the foreign-currency price of imported goods and $\mathcal{E}_t$ is the nominal exchange rate. Imported input prices therefore affect production costs in both sectors through the firms' optimal input demand conditions.

Firms are monopolistically competitive and adjust prices according to Rotemberg (1982), incurring an adjustment cost each time,

$$AC_t(i) = \frac{\xi}{2}\left(\frac{P_{N,t}(i)}{P_{N,t-1}(i)} - \Pi_N\right)^2 Y_{N,t} P_{N,t}$$

where ξ summarises the degree of nominal rigidity in the economy and Π_N denote steady state non-tradable gross inflation. Finally, we obtain the Phillips Curve for non-tradable goods,

$$\Pi_{N,t}\left(\Pi_{N,t} - \Pi_N\right) = \mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_{N,t+1}\left(\Pi_{N,t+1} - \Pi_N\right)\frac{Y_{N,t+1}P_{N,t+1}}{Y_{N,t}P_{N,t}}\right] + \frac{\epsilon}{\xi}\left(\frac{MC_t}{P_{N,t}} - \frac{\epsilon-1}{\epsilon}\right) \tag{15}$$

where MC_t is the marginal cost obtained from differentiating the total cost function.[9]

Using the demand relation and the labour market clearing condition $N_{N,t} = \int_0^1 N_{N,t}(i)di$, we can write the aggregate production function as

$$Y_{N,t}\Delta_t = A_{N,t} M_{F,t}^{\kappa_1} N_{N,t}^{\kappa_2} K_{t-1}^{\kappa_3}$$

where $\Delta_t = \left(1 - \frac{\xi}{2}(\Pi_{N,t} - \Pi_N)^2\right)$ captures the output loss caused by the costly adjustment of prices.

2.2.2 Tradable goods sector

The tradable sector is internationally competitive. Domestic producers are perfectly competitive and take the home-tradable price $P_{H,t}$ as given, with no markup over marginal cost.

[9]See Appendix A for derivations.

Although firms are price takers, the domestic-currency price of home tradables does not adjust instantaneously to the law of one price. Following Krugman (1987) and Gopinath (2015), we allow for incomplete exchange-rate pass-through into $P_{H,t}$ itself,

$$\log P_{H,t} = \rho_{P_H} \log P_{H,t-1} + (1 - \rho_{P_H}) \log \left(\mathcal{E}_t P^*_{H,t} \right), \tag{16}$$

where $\rho_{P_H} \in (0, 1)$ governs the speed at which the domestic price of home tradables converges to the level implied by the international relative price $\mathcal{E}_t P^*_{H,t}$.[10] We assume that foreign price dynamics ($P^*_{H,t}$) are driven by developments in world markets and therefore exogenous from the perspective of our small open economy.

Tradable sector production is given by

$$Y_{H,t} = A_{H,t} M_{H,t}^{\zeta_1} N_{H,t}^{\zeta_2} K_{H,t-1}^{1-\zeta_1-\zeta_2} \tag{17}$$

where $A_{H,t}$ is the exogenous sector-specific productivity. The problem of a firm in the tradable sector is to maximise profits

$$\max_{N_{H,t}, M_{H,t}, K_{H,t-1}} P_{H,t} Y_{H,t} - W_t N_{H,t} - P_{F,t} M_{H,t} - (1 + i^k_t) K_{t-1}$$

subject to the production technology (17). This yields

$$W_t N_{H,t} = \zeta_2 Y_{H,t} P_{H,t} \tag{18}$$

$$P_{F,t} M_{H,t} = \zeta_1 Y_{H,t} P_{H,t} \tag{19}$$

$$(1 + i^k_t) K_{H,t-1} = (1 - \zeta_1 - \zeta_2) Y_{H,t} P_{H,t} \tag{20}$$

2.3 Monetary Policy

The monetary policy authority sets the interest rate according to the following Taylor rule,

$$\frac{I_t}{I} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \tag{21}$$

where $I_t = (1 + i_t)$ is the gross nominal interest rate and I, Π, Y are the steady state level of nominal interest rate, inflation and output, respectively.[11]

2.4 Equilibrium

Given the tradable and foreign import prices $P^*_{T,t}, P^*_{F,t}$, the monetary policy rule determining i_t, foreign output Y^*_t, inflation Π^*_t and and interest rates i^*_t, and an initial condition on price

[10] This is consistent with pricing-to-market and dominant-currency-pricing behaviour, whereby exporters set prices with reference to the destination market's currency rather than adjusting one-for-one with the exchange rate.

[11] We chose a Taylor rule that targets the inflation rate, rather than unobservable variables such as the natural rate or the output gap. This type of rule approximates the remit of most central banks that follow a price stability mandate, and is in line with the literature (Galí (2015)).

dispersion, the equilibrium is given by a sequence of quantities $\{C_{H,t}, C_{T,t}, C_{N,t}, C_{F,t}, C_t, N_t, N_{H,t}, N_{N,t}, B^H_{t+1}, B^*_{t+1}, Y_{N,t}, Y_{H,t}, M_{N,t}, M_{H,t}, K_{H,t}, K_{N,t}\Psi_t\}_{t=0}^{\infty}$ and prices $\{\Lambda_{t,t+1}, \Pi_{H,t}, \Pi_{N,t}, \Pi_{T,t}, \Pi_{F,t}, \Pi_t, W_t, \mathcal{T}_t, \mathcal{S}_t, \mathcal{E}_t, r^k_t, \Delta_t\}_{t=0}^{\infty}$ such that firms and households maximise their objectives, and the goods, labour and financial markets clear,

$$\begin{aligned}
Y_t &= C_{H,t} + C^*_{H,t} + C_{N,t} \\
Y_{H,t} &= C_{H,t} + C^*_{H,t} \\
Y_{N,t} &= C_{N,t} \\
B_t &= 0 \\
B^*_t &= B^* \\
N_t &= N_{H,t} + N_{N,t} \\
M_{F,t} &= M_{H,t} + M_{N,t} \\
K_{t-1} &= K_{H,t-1} + K_{N,t-1} \\
Y^*_t &= C^*_t
\end{aligned}$$

2.5 Natural Real Interest Rate and Natural Level of Output

To understand the impact of our shocks and to determine the appropriate real policy interest rate, we calculate the natural level of output (Y^n_t) and the natural real interest rate (r^n_t).

Y^n_t is the level of output that would arise under flexible prices. To derive it, we need to determine the profit-maximizing flexible price for the domestic non-tradable good firms, which is the only sector facing nominal rigidities. Profit-maximising firms set the flexible optimal price in order to equalise marginal cost and marginal revenue. This is equivalent to setting the real marginal cost to the inverse of the desired markup,

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}.$$

This yields

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \left[C^{\sigma}_t N^{\phi}_t \frac{1}{\kappa_2} \right]^{\kappa_2} \left[\frac{P_{F,t}}{P_t} \frac{1}{\kappa_1} \right]^{\kappa_1} \left[\frac{(1 + i^k_t)}{P_t} \frac{1}{1 - \kappa_1 - \kappa_2} \right]^{1-\kappa_1-\kappa_2}. \tag{22}$$

Using the UIP condition as well as the equilibrium condition for output, we can express the natural real interest rate as the risk-free real interest rate consistent with the Euler equation when output is at its natural level at all times,

$$\begin{aligned}
(C^n_{t+1})^{\sigma} &= \beta(1 + r^n_t)(C^n_t)^{\sigma} \\
(1 + r^n_t) &= \frac{1}{\beta} \left[\left(\frac{Y^n_{t+1}}{Y^n_t} \right) \frac{\Sigma_{\varsigma\theta,t}}{\Sigma_{\varsigma\theta,t+1}} \right]^{\sigma}
\end{aligned}$$

where $C_t = \frac{1}{\Sigma_{\varsigma\theta,t}} Y_t$ and $\Sigma_{\varsigma\theta,t} \equiv \left[\varsigma \left(\frac{P_t}{P_{N,t}} \right)^{\iota} + (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}} \right)^{\mu} \left(\frac{P_t}{P_{T,t}} \right)^{\iota} + \theta \left(\frac{P_t}{P_{H,t}} \right)^{\mu} \frac{\mathcal{S}^{\mu-1}_t}{\mathcal{D}^{\frac{1}{\sigma}}_t} \right]$.[12]

[12]See Appendix A for a full derivation.

3 Calibration

We calibrate the model at a quarterly frequency. Our aggregate baseline calibration is standard, as several preference and technology parameters are shared with the standard New Keynesian literature. We assume a discount factor $\beta = 0.9877$ which implies an annual nominal rate of 5% in steady state. We set the elasticity of intertemporal substitution $\sigma = 1$. For simplicity, we consider low elasticities of substitution between foreign and domestic tradable goods (μ), and between tradable and non-tradable goods (ι), and the Frisch elasticity. In the baseline model, θ equals 0.60, which corresponds to a 60 percent import share. We set κ_1 and κ_2, the income share of foreign primitive input in the production of tradable and non-tradable goods respectively, close to 0 in the baseline. Finally, we calibrate ς, the share of non-tradable goods in the consumption basket to be 0.7.

Parameter	Definition	Value	Source / Target
β	Household discount factor	0.9877	Annual net nominal rate $r_{ss} \approx 5\%$
σ	Household risk aversion	2	Corsetti, Dedola, and Leduc (2009)
κ_ℓ	Labour disutility	≈ 5	Literature
ϕ	Inverse Frisch elasticity	3	Gali and Monacelli (2005)
θ	Share of foreign tradables	0.60	Harrison and Oomen (2010)
ς	Share of non-tradables	0.7	Literature
μ	Elasticity of substitution between home and foreign goods	1	Literature
ι	Elasticity of substitution between tradables and non-tradables	1	Literature
ϵ	Elasticity of substitution between non-tradable varieties	11	10% gross final markup
ϕ_π	Interest sensitivity to inflation	1.5	Literature
ϕ_y	Interest sensitivity to output	0	Literature
ξ	Rotemberg adjustment cost	19.8	Avg lifetime of prices 2Q
ζ_2	Labour share in tradables	0.7	Literature
ζ_1	Foreign input share in tradables	≈ 0	Literature
κ_2	Labour share in non-tradables	0.80	Literature
κ_1	Foreign input share in non-tradables	≈ 0	Literature
ρ_s	Persistence coefficient	0.9	$s \in \{A_N, A_H\}$
ρ_{P_H}	Home-tradable price pass-through persistence	0.7	Literature
χ	Portfolio adjustment cost	0.001	Literature

Table 1: This table presents the baseline quarterly calibration.

4 Results

To establish some basic intuition for our results, we study three scenarios in our baseline calibration, in which a positive productivity shock affects the non-tradables sector.[13] This section begins with a textbook example, a temporary productivity shock. The motivation is to illustrate the standard disinflationary mechanism associated with higher productivity in New Keynesian models.

[13] See section 5 for the case where a positive productivity shock affects the tradable sector.

Next, we explore two scenarios to emphasise how the timing and magnitude of the relative supply- and demand-side effects matter for the inflation response. In particular, we compare a front-loaded permanent increase in productivity with a gradual, permanent increase. Although both scenarios lead to the same higher long-run level of productivity, we highlight how the timing of demand versus supply effects lead to different inflationary outcomes. On the supply side, higher productivity raises potential output and lowers marginal costs, capturing the partial equilibrium intuition that higher productivity is disinflationary. Considering the demand response provides a more nuanced perspective. Higher expected permanent income raises consumption while higher expected returns increase investment, creating inflationary pressures. Inflation therefore depends on whether expanding supply capacity outpaces the increase in demand or if demand keeps pace.

Temporary increase in productivity Figure 1 plots the impulse response functions to a temporary increase in non-tradables productivity ($A_{N,t}$), which rises by 10% on impact and gradually returns to its steady-state level. Higher productivity increases the marginal product of labour in the non-tradables sector, leading to higher real wages and a reallocation of labour away from the non-tradables sector. Despite higher real wages, marginal costs in the non-tradables sector fall as higher productivity more than offsets the increase in labour costs. Output in the non-tradables sector rises due to higher productivity. The expansion in non-tradables supply leads to a decline in non-tradables prices relative to tradables prices. Since the shock is temporary, the increase in investment is short-lived, leading to a moderate increase in the capital stock. Aggregate output rises, reflecting the expansion in the non-tradables sector. Consumption rises only temporarily as the increase in the real interest rate dampens the rise in consumption. Moreover, the wealth effect from a temporary productivity shock is limited. As demand does not keep pace with the expansion in supply capacity, this scenario is disinflationary. Following an initial decline, aggregate CPI inflation gradually returns to its steady state level. The limited demand response is reflected in a lower natural rate of interest, consistent with the disinflationary pressures generated by this scenario.

Front-loaded and permanent increase in productivity Figure 2 shows an immediate and permanent increase in non-tradables productivity. On the supply-side, dynamics are similar to the case of a temporary increase in TFP. Higher productivity leads to a substantial increase in output and a fall in labour in the non-tradables sector. Relative to the case of a temporary productivity shock, the increase in investment is similar in magnitude but more gradual. While the initial effects on supply are similar to the case of a temporary increase in productivity, a permanent increase in TFP generates a much stronger wealth effect: households revise their expectations of permanent income upward, and consumption rises by more and remains persistently elevated, relative to the temporary case. This stronger demand response raises the non-tradables sector's use of labour and other inputs, pushing up

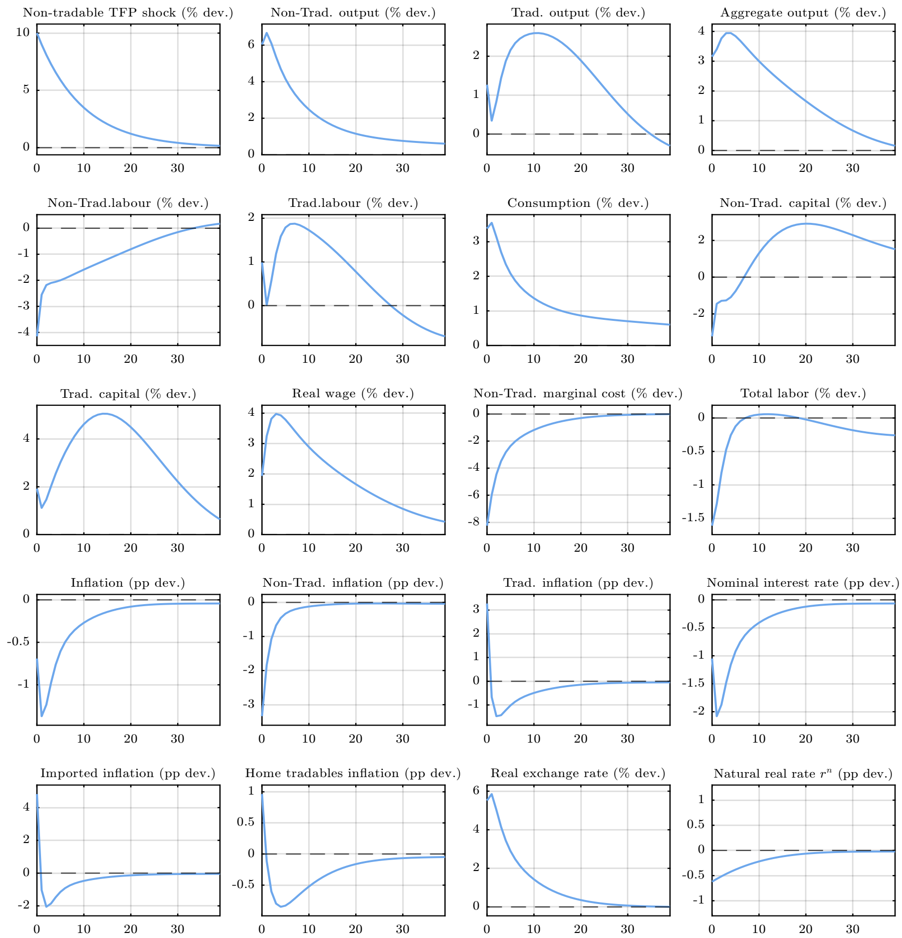

Figure 1: IRFs to a 10% Temporary Increase in Non-Tradable TFP.

the economy-wide wage and partly offsetting the fall in marginal costs, so non-tradables inflation falls by less than in the temporary case. The same wealth effect raises demand for tradables, adding a further inflationary contribution from tradables prices. As a result, the decline in inflation is smaller than the temporary TFP scenario. Due to the front-loaded nature of this shock, the expansion in supply capacity outpaces the increase in demand. Overall, the permanent productivity shock leads to a much weaker disinflationary response relative to the temporary shock. Reflecting the offsetting effects of higher supply capacity and a stronger demand response, the natural rate of interest is unchanged in this scenario.

Gradual and permanent increase in productivity ($A_{N,t}$) Next, consider a more gradual expansion in supply capacity. Figure 3 shows the effects of a gradual and permanent increase in non-tradables productivity. Although both the gradual and front-loaded scenarios converge to the same higher long-run level of productivity, the inflationary consequences differ due to the timing of the supply and demand effects. The supply-side dynamics are similar to the previous scenarios: higher $A_{N,t}$ raises non-tradables output relative to the labour input required. As productivity rises over time, marginal costs decline, placing downward pressure on non-tradables inflation. Since the improvement in productivity takes place gradu-

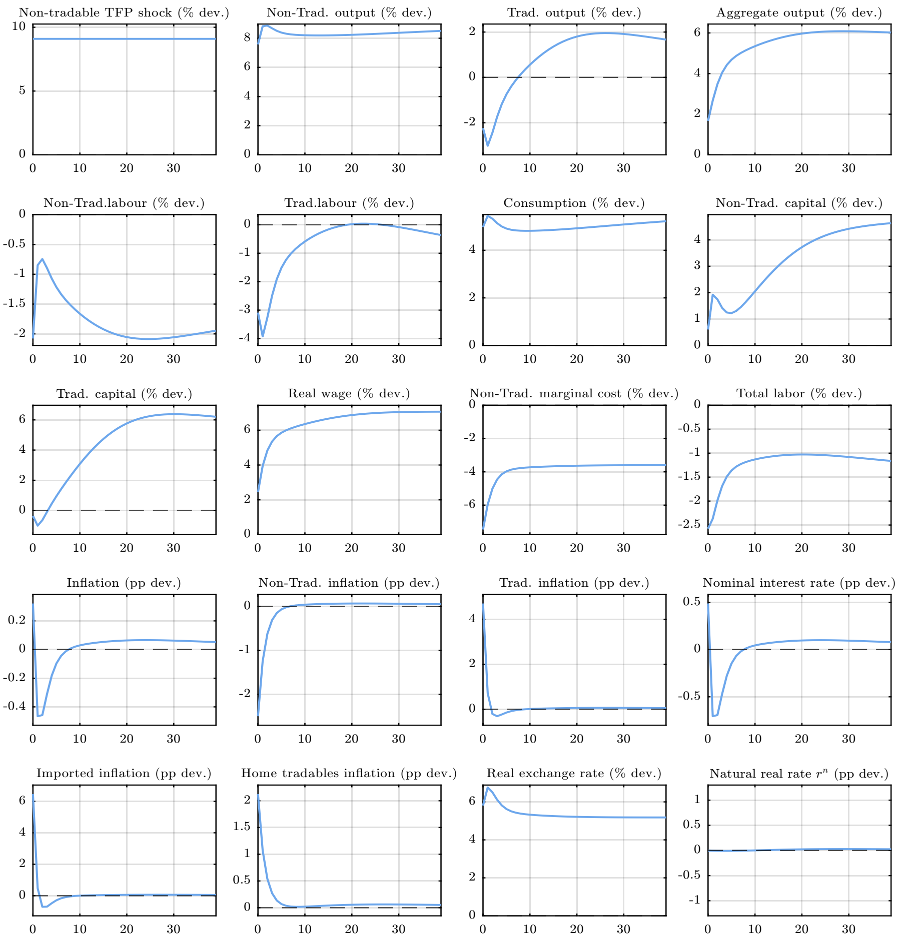

Figure 2: IRFs to a 10% Frontloaded and Permanent Increase in Non-Tradable TFP.

ally, the decline in marginal costs is also gradual.

Meanwhile, expectations of higher future productivity raise expected permanent income, leading to strong wealth effects. Higher real wages support higher consumption, while wealth effects also lead to a fall in labour supply. This increase in aggregate demand partly offsets the disinflationary impact of higher productivity in the non-tradables sector. Higher income also raises demand for tradables, contributing to an increase in tradables inflation. Gradual productivity gains also increase the expected return to investment. As a result, forward-looking firms expand their capital stock over time. As consumption and investment rise before the full increase in productive capacity materialises, demand temporarily grows faster than supply. As a result, aggregate CPI inflation rises despite the fall in non-tradables inflation and the steady decline in non-tradables marginal costs. The increase in aggregate demand is reflected in a higher natural rate of interest, which requires a tightening of monetary policy in order to contain inflationary pressures.

These scenarios highlight how the inflationary impact of higher productivity depend on the timing and magnitude of the relative supply and demand responses. When the increase in productivity is temporary, the increase in supply dominates the relatively limited demand response, leading to a moderate fall in inflation as marginal costs decline. The frontloaded

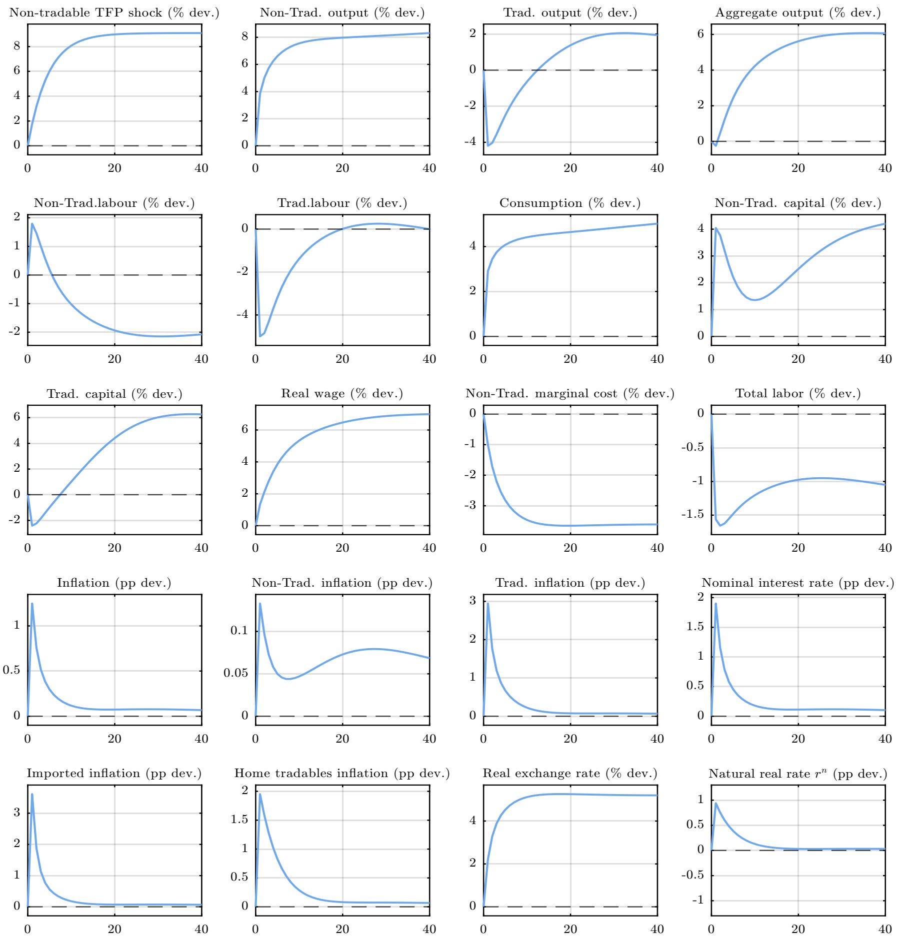

Figure 3: IRFs to a 10% Gradual and Permanent Increase in Non-Tradable TFP.

and permanent increase in productivity features a similar mechanism: since productivity increases immediately, the supply effect outweighs the increase in demand, resulting in disinflation. By contrast, when productivity increases gradually and permanently, the actions of forward-looking households and firms in anticipation of future productivity gains may lead to demand strengthening ahead of supply. In this case, the natural rate of interest increases and monetary policy must tighten to prevent excess demand. These results show that the inflationary impact of productivity depends not only on their persistence, but also on whether supply or demand adjusts more rapidly.

5 Productivity Gains in the Tradables Sector

In this section, we show how the effect of productivity gains on aggregate CPI inflation depends on the sector in which they occur. Following the classic Balassa-Samuelson logic, this section considers a productivity shock in the tradables sector. In a small open economy, as tradables prices are largely pinned down by world prices, the adjustment takes place through higher income and wages, which raise demand for non-tradables. With upward pressure on non-tradables prices, aggregate CPI inflation may increase although productivity has risen. A productivity shock in the tradables sector can therefore be inflationary.

This contrasts with the baseline case, in which productivity gains occur in the non-tradables sector. In that case, the effects on inflation operate more directly through domestic supply. With downward-sloping demand for non-tradables, an increase in supply capacity lowers their relative price and reduces non-tradables inflation. Although higher income also raises demand for tradables, their prices remain anchored to world prices. As a result, relative to the case where productivity gains occur in tradables, a supply expansion in non-tradables leads to lower domestic inflation, as the decline in non-tradables inflation reduces CPI inflation overall.

We begin this section with a benchmark scenario featuring a temporary productivity shock in the tradables sector. We then consider alternative scenarios to highlight how the balance between the timing and magnitude of supply and demand responses affect inflationary pressures.

Temporary increase in tradables productivity ($A_{H,t}$) Figure 4 shows the effect of a temporary increase in tradables productivity ($A_{H,t}$). A positive productivity shock in this sector raises the return to both labour and capital in tradables production, drawing both factors away from the non-tradables sector. Because the increase in productivity is short-lived, reallocation takes place sharply and on impact. Firms and workers shift resources into the tradables sector while it remains temporarily more profitable. With less labour and capital available to the non-tradables sector, marginal costs rise, placing upward pressure on non-tradables inflation. Tradables prices fall to some extent as production expands, but the rise in non-tradables inflation outweighs this decline, leading to an increase in aggregate CPI inflation.

Front-loaded increase in tradables productivity ($A_{H,t}$) Figure 5 shows the effect of a permanent and immediate increase in tradables productivity. Output in the tradables sector rises sharply and remains persistently higher. Labour initially rises, but subsequently falls below its starting level as production requires fewer workers per unit of output. Capital similarly dips before rising to a permanently higher level, reflecting the cost of adjusting investment quickly. Real wages and consumption increase as well.

Alongside these supply-side dynamics, tradables inflation falls as production expands. The increase in non-tradables inflation reflects stronger demand for non-tradables and higher marginal costs there. Overall, the increase in non-tradables inflation outweighs the fall in tradables inflation, leading to higher aggregate CPI inflation. In contrast to its counterpart in the non-tradables case, an immediate and permanent increase in tradables productivity leads to an increase in aggregate CPI inflation.

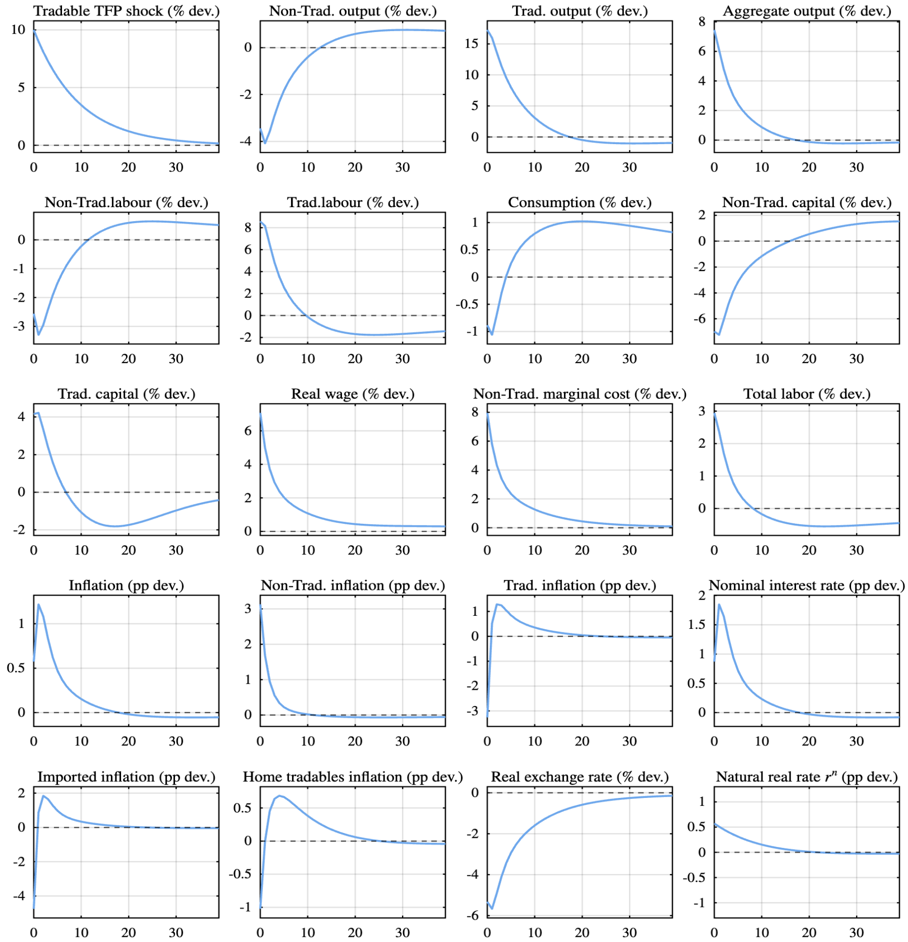

Figure 4: IRFs to a 10% Temporary Increase in Tradable TFP.

Gradual increase in tradables productivity ($A_{H,t}$) Finally, consider a permanent and gradual increase in productivity in the tradables sector (Figure 6). As expected, higher tradables sector productivity leads to a large increase in output in this sector relative to the non-tradables sector.[14] Labour initially falls sharply as production requires fewer workers per unit of output, and remains below its initial level even as the sector expands. The capital stock also declines initially, reflecting the cost of adjusting investment quickly, before rising gradually to a permanently higher level. Both real wages and consumption increase considerably, leading to an increase in aggregate demand. Consumption rises gradually to a permanently higher level. Because the improvement in tradables productivity unfolds gradually, resources move out of the non-tradables sector only slowly, so non-tradables marginal costs and inflation rise only modestly. By contrast, the nominal exchange rate, which reflects the entire anticipated path of future productivity gains, appreciates quickly, placing immediate downward pressure on tradables and imported inflation (Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro (2024)).[15] As the appreciation-driven fall in tradables in-

[14]Frictions in capital and labour reallocation would dampen the magnitude of this response.

[15]This is closely related to the mechanism in Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro (2024), who interpret the Brexit referendum as news of a persistent slowdown in UK tradable-sector productivity growth. They show that sterling depreciates and the relative price of non-tradables falls immediately upon the

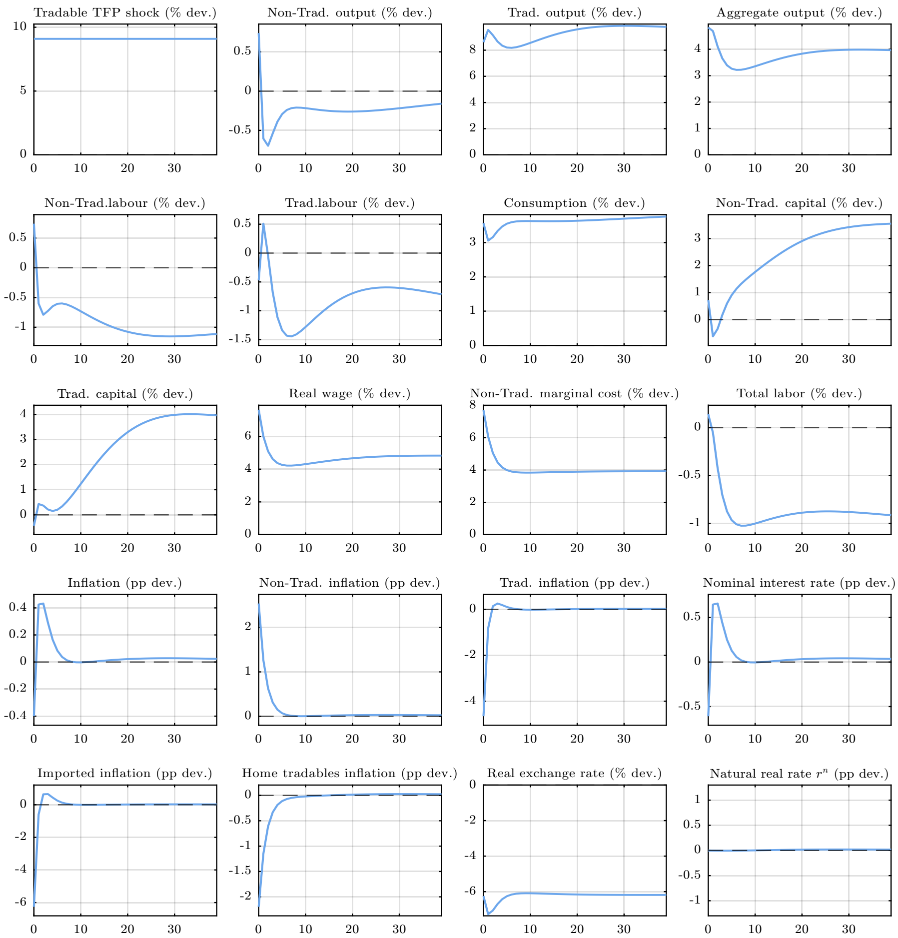

Figure 5: IRFs to a 10% Front-loaded and Permanent Increase in Tradable TFP.

flation outweighs the modest rise in non-tradables inflation, aggregate CPI inflation falls, in contrast to the front-loaded scenario above.

In summary, as tradables prices are anchored to world prices, the adjustment in response to a productivity shock in the tradables sector occurs largely through wages, output, employment and non-tradables inflation, rather than through tradables prices themselves. Because of this, the sectoral origin of a shock can reverse its inflationary implications: a front-loaded productivity gain is disinflationary when it occurs in the non-tradables sector, but inflationary when it occurs in tradables, since it operates through demand-driven non-tradables inflation rather than a fall in the shocked sector's own marginal cost. Conversely, a gradual productivity gain is inflationary in the non-tradables sector but disinflationary in tradables, as the exchange rate appreciates in anticipation of the future productivity gain more quickly than domestic resources can be reallocated. The impact of a productivity shock on inflation therefore depends not only on whether supply or demand adjusts more rapidly, but also on the sector in which the shock originates.

news but before the anticipated slowdown is actually realised.

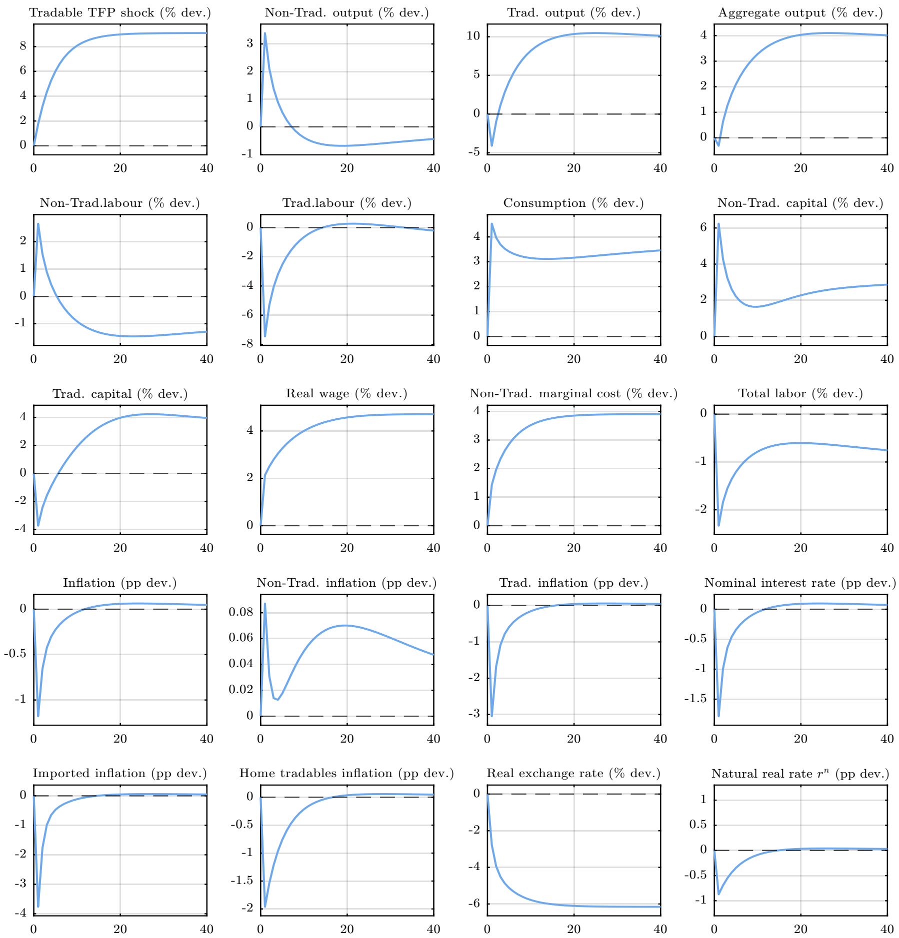

Figure 6: IRFs to a 10% Gradual and Permanent Increase in Tradable TFP.

6 Conclusion

Higher productivity is neither inherently inflationary or disinflationary. The inflationary impact depends on how supply and demand adjust and whether monetary policy responds appropriately. Our model provides a framework for understanding the various channels through which higher productivity may affect inflationary pressures, with an emphasis on the following aspects: the distinction between level and growth effects, the relative speed of supply and demand adjustment, the sectoral incidence of the shock, and the stance of monetary policy relative to the natural real interest rate.

We highlight the different implications of a one-off increase in the level of productivity versus a permanent increase in the growth rate of productivity, and show that these implications also depend on the sector in which productivity rises. In the non-tradables sector, a one-time productivity shock lowers marginal costs and raises potential output, placing downward pressure on the price level; this is a relative price and level effect, rather than a sustained change in inflationary dynamics. Even if inflation falls temporarily, lower marginal costs do not imply persistently lower inflation. In the tradables sector, by contrast, the same one-off shock instead raises the price level, since tradables prices are anchored to world markets and the adjustment operates instead through stronger demand and higher marginal costs in the non-tradables sector.

A persistent increase in productivity growth has fundamentally different implications. Higher productivity growth raises output growth, and consumption increases through higher expected permanent income. When productivity gains occur in the non-tradables sector, this pushes up the natural rate of interest, and an inflation-targeting central bank must adjust policy in line with the higher natural rate to stabilise demand. Maintaining price stability may therefore require tighter, not looser, policy. When the same gradual productivity gains occur in the tradables sector instead, the natural rate of interest falls, as the exchange rate appreciates in anticipation of future productivity gains more quickly than resources can be reallocated out of non-tradables. In this case, maintaining price stability may instead call for looser policy.

More generally, productivity shifts both aggregate supply and aggregate demand. On the supply side, while higher productivity may raise real wages, lower marginal costs exert downward pressure on prices. On the demand side, higher expected permanent income leads to higher consumption, while higher expected returns increase investment. The impact on inflation is therefore a priori ambiguous, as it depends not only on the expansion of productive capacity, but also on the magnitude of the adjustment in demand. Timing is also critical. If demand, through higher consumption or investment, outpaces realised gains in supply, inflationary pressures may emerge. Conversely, if supply expands rapidly or demand remains contained, the impact on inflation may be subdued or even disinflationary.

These considerations suggest that policy should respond when productivity-induced demand pressures risk exceeding supply, with changes in the natural rate of interest as a guide. Conversely, one-off relative price adjustments that leave medium-term inflation dynamics unchanged do not warrant a policy response, provided inflation expectations remain anchored. The key challenge for policymakers is to assess in real-time, whether productivity gains are changing trend growth and the natural rate of interest, or simply generating temporary level effects on prices.

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A Model

A.1 Demand Side

Unconstrained households maximise their expected lifetime utility by choosing a sequence $\{C_t, N_t, b_t, b_t^*\}_{t=0}^{\infty}$ subject to the series of budget constraints (1). For simplicity, we can rewrite the budget constraint in real terms as

$$\begin{aligned} C_t + b_t + \mathcal{S}_t b_t^* + I_t = b_{t-1}\frac{(1+i_{t-1})}{(1+\pi_t)} + \mathcal{S}_t b_{t-1}^* \frac{(1+i_{t-1}^*)}{(1+\pi_t^*)} + \frac{(1+i_{t-1}^k)}{(1+\pi_t)} k_{t-1} \\ + w_t N_t + \Psi_t - \frac{\chi}{2}\mathcal{S}_t \left(b_t^* - b^*\right)^2 \end{aligned} \tag{A.1}$$

where $b_t = \frac{B_t}{P_t}, b_t^* = \frac{B_t^*}{P_t^*}, w_t = \frac{W_t}{P_t}, \mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate, and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the net inflation rate.

The first-order conditions with respect to $C_t, N_t, b_t, b_t^*, k_t, I_t$ are respectively given by:

$$\begin{aligned} (C_t)^{-\sigma} &= \lambda_t \\ \kappa_\ell (N_t)^\phi &= \lambda_t \frac{W_t}{P_t} \\ \lambda_t &= \beta \mathbb{E}_t \left[\frac{(1+i_t)}{(1+\pi_{t+1})}\lambda_{t+1}\right] \\ \lambda_t[\mathcal{S}_t + \mathcal{S}_t \chi (b_t^* - b^*)] &= \beta \mathbb{E}_t \left[\frac{(1+i_t^*)}{(1+\pi_{t+1}^*)}\mathcal{S}_{t+1}\lambda_{t+1}\right] \\ \lambda_t q_t &= \beta \mathbb{E}_t \left[\lambda_{t+1}\frac{(1+i_t^k)}{(1+\pi_{t+1})} + (1-\delta) q_{t+1}\right] \\ q_t &= \lambda_t \end{aligned}$$

where δ_t is the Lagrange multiplier on the budget constraint, and q_t is the Lagrange multiplier on the law of motion of capital. Total consumption expenditure by households is given by the sum of expenditures on domestic and foreign goods,

$$P_t C_t = P_{T,t} C_{T,t} + P_{N,t} C_{N,t} = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + P_{N,t} C_{N,t}$$

The system of demand functions is given by

$$C_{H,t} = (1-\theta)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\mu} C_{T,t}$$
$$C_{F,t} = \theta\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\mu} C_{T,t}$$
$$C_{N,t} = \varsigma\left(\frac{P_{N,t}}{P_{t}}\right)^{-\iota} C_{t}$$
$$C_{T,t} = (1-\varsigma)\left(\frac{P_{T,t}}{P_{t}}\right)^{-\iota} C_{t}$$
$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\epsilon} C_{N,t}$$

The terms of trade are defined as the price of imports in terms of the price of domestic goods,

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{A.2}$$

A.2 Non-Tradable Firms

Firm i in the non-tradable sector maximises profits

$$\max_{M^F_{N,t}(i),N_{N,t}(i),P_{N,t}(i),k_{N,t-1}(i)} P_{N,t}(i)Y_{N,t}(i) - \left(P_{F,t}M^F_{N,t}(i) + W_t N_{N,t}(i) + (1+i^k_t)k_{N,t-1}\right) - AC_t(i)$$

subject to its production technology (13) and the demand function for its product (12). This problem yields the following first-order conditions

$$W_t = MC_t(i)\kappa_2 \frac{Y_{N,t}(i)}{N_{N,t}(i)} \tag{A.3}$$
$$P_{F,t} = MC_t(i)\kappa_1 \frac{Y_{N,t}(i)}{M^F_{N,t}(i)} \tag{A.4}$$
$$(1+i^k_t) = MC_t(i)(1-\kappa_1-\kappa_2)\frac{Y_{N,t}(i)}{k_{N,t-1}(i)} \tag{A.5}$$

where $MC_t(i)$ is the Lagrange multiplier on the technology constraint. We can interpret this multiplier as the shadow cost of producing an additional unit of good $Y_{N,t}(i)$, that is, the marginal cost.

Substituting equations (A.3), (A.4), (A.5) into the production function, we obtain the marginal cost function:

$$MC_{N,t}(i) = \frac{1}{A_{N,t}}\left[\frac{W_t}{\kappa_2}\right]^{\kappa_2}\left[\frac{P_{F,t}}{\kappa_1}\right]^{\kappa_1}\left[\frac{(1+i^k_t)}{1-\kappa_1-\kappa_2}\right]^{1-\kappa_1-\kappa_2} \tag{A.6}$$

Since equation (A.6) is solely a function of factor prices and productivity, $MC_{N,t}(i) = MC_{N,t}\ \forall i$. We can rewrite nominal marginal cost in real terms as follows,

$$mc_{N,t}(i) = \frac{MC_{N,t}}{P_t} = \frac{1}{A_{N,t}}\left[\frac{W_t}{P_t}\frac{1}{\kappa_2}\right]^{\kappa_2}\left[\frac{P_{F,t}}{P_t}\frac{1}{\kappa_1}\right]^{\kappa_1}\left[\frac{(1+i^k_t)}{P_t}\frac{1}{1-\kappa_1-\kappa_2}\right]^{1-\kappa_1-\kappa_2} \tag{A.7}$$

A.3 Tradable Firms

Tradable sector production is given by

$$Y_{H,t} = A_{H,t} M_{H,t}^{\zeta_1} N_{H,t}^{\zeta_2} K_{H,t-1}^{1-\zeta_1-\zeta_2} \tag{A.8}$$

where $A_{H,t} = (A_{H,t}^{ss})^{1-\rho_h} A_{H,t-1}^{\rho_h} \epsilon_{H,t}$, and $\rho_h \in (0,1]$. Therefore, the problem of the tradable firm is to maximise profits

$$\max_{N_{H,t}, M_{H,t} K_{H,t-1}} \quad P_{H,t} Y_{H,t} - W_t N_{H,t} - P_{F,t} M_{H,t} - (1+i_t^k) K_{H,t-1}$$

subject to its production technology (17). This yields

$$\frac{W_t}{P_{H,t}} = \zeta_2 A_{H,t} N_{H,t}^{\zeta_2 - 1} M_{H,t}^{\zeta_1} K_{H,t-1}^{1-\zeta_1-\zeta_2} \implies W_t N_{H,t} = \zeta_2 Y_{H,t} P_{H,t} \tag{A.9}$$

$$\frac{P_{F,t}}{P_{H,t}} = \zeta_1 A_{H,t} M_{H,t}^{\zeta_1 - 1} N_{H,t}^{\zeta_2} K_{H,t-1}^{1-\zeta_1-\zeta_2} \implies P_{F,t} M_{H,t} = \zeta_1 Y_{H,t} P_{H,t} \tag{A.10}$$

$$\frac{(1+i_t^k)}{P_{H,t}} = (1-\zeta_1-\zeta_2) A_{H,t} N_{H,t}^{\zeta_2} M_{H,t}^{\zeta_1} K_{H,t-1}^{\zeta_1-\zeta_2} \implies (1+i_t^k) K_{H,t-1} = (1-\zeta_1-\zeta_2) Y_{H,t} P_{H,t} \tag{A.11}$$

A.4 Exogenous Processes

$$P_{F,t}^* = \left(P_{F,t}^{*ss}\right)^{1-\rho_F} \left(P_{F,t-1}^*\right)^{\rho_F} \exp(\nu_{F,t}) \quad \rho_F \in [0,1] \tag{A.12}$$

$$A_{N,t} = (A_N^{ss})^{1-\rho_N} A_{N,t-1}^{\rho_N} \exp(\nu_{N,t}), \quad \rho_N \in (0,1] \tag{A.13}$$

$$A_{H,t} = (A_{H,t}^{ss})^{1-\rho_H} A_{H,t-1}^{\rho_H} \exp(\nu_{H,t}), \quad \rho_H \in (0,1] \tag{A.14}$$

$$P_{H,t}^* = \left(P_{H,t}^{*ss}\right)^{1-\rho_{H^*}} \left(P_{H,t-1}^*\right)^{\rho_{H^*}} \exp(\nu_{H^*,t}) \quad \rho_{H^*} \in (0,1] \tag{A.15}$$

$$C_t^* = (C_t^{*ss})^{1-\rho_{C^*}} \left(C_{t-1}^*\right)^{\rho_{C^*}} \exp(\nu_{C^*,t}) \quad \rho_{C^*} \in (0,1] \tag{A.16}$$

$$r_t^* = (r_t^{*ss})^{1-\rho_{r^*}} \left(r_{t-1}^*\right)^{\rho_{r^*}} \exp(\nu_{r^*,t}) \quad \rho_{r^*} \in (0,1] \tag{A.17}$$

A.5 Equilibrium Conditions

We can derive demand functions for $Y_{H,t}$ and $Y_{N,t}$ in order to obtain the market clearing conditions. Tradables are by definition consumed both domestically and in the rest of the world,

$$C_{H,t} = (1-\theta) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} C_{T,t} = (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_t}{P_{T,t}}\right)^{\iota} C_t$$

$$C_{H,t}^* = \theta^* \left(\frac{P_t}{P_{H,t}}\right)^{\mu} \mathcal{S}_t^{\mu} C_t^*$$

$$Y_{H,t} = (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_t}{P_{T,t}}\right)^{\iota} C_t + \theta^* \left(\frac{P_t}{P_{H,t}}\right)^{\mu} \mathcal{S}_t^{\mu} C_t^*$$

where $C^*_{H,t}$, the foreign tradable demand, is derived by assuming symmetric preferences in the rest of the world. For non-tradables, instead, we calculate domestic demand as follows,

$$C_{N,t} = \varsigma \left(\frac{P_t}{P_{N,t}} \right)^{\iota} C_t \tag{A.18}$$

Therefore, we can express domestic aggregate output as follows,

$$Y_t = \varsigma \left(\frac{P_t}{P_{N,t}} \right)^{\iota} C_t + (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}} \right)^{\mu} \left(\frac{P_t}{P_{T,t}} \right)^{\iota} C_t + \theta^* \left(\frac{P_t}{P_{H,t}} \right)^{\mu} \mathcal{S}^{\mu}_t C^*_t. \tag{A.19}$$

A.6 Natural Level of Output

To derive Y^n_t, we set the real marginal cost to the inverse of the desired markup,

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}$$

Using the real marginal cost in equation (A.7), we obtain

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \left[\frac{W_t}{P_t} \frac{1}{\kappa_2} \right]^{\kappa_2} \left[\frac{P_{F,t}}{P_t} \frac{1}{\kappa_1} \right]^{\kappa_1} \left[\frac{(1+i^k_t)}{P_t} \frac{1}{1-\kappa_1-\kappa_2} \right]^{1-\kappa_1-\kappa_2} \tag{A.20}$$

Under perfectly competitive labour markets, we can write the labour supply condition as follows[16]

$$\frac{W_t}{P_t} = C^{\sigma}_t N^{\phi}_t$$

Therefore,

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \left[C^{\sigma}_t N^{\phi}_t \frac{1}{\kappa_2} \right]^{\kappa_2} \left[\frac{P_{F,t}}{P_t} \frac{1}{\kappa_1} \right]^{\kappa_1} \left[\frac{(1+i^k_t)}{P_t} \frac{1}{1-\kappa_1-\kappa_2} \right]^{1-\kappa_1-\kappa_2} \tag{A.21}$$

Next, we express C_t in terms of C^*_t. Using the household's first-order condition on foreign bond holdings, and assuming that the same conditions hold in the foreign economy,

$$\frac{1}{(1+i^*_t)} = \beta \mathbb{E}_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{1}{(1+\pi^*_{t+1})} \right]. \tag{A.22}$$

If $\Pi = \Pi^* = 1$ in steady state, then $r = r^* = \frac{1}{\beta}$. We can therefore write the international risk-sharing condition as follows

$$\mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \right] = \mathbb{E}_t \left[\left(\frac{C^*_t}{C^*_{t+1}} \right)^{\sigma} \frac{\mathcal{S}_t}{\mathcal{S}_{t+1}} \right] \left[1 + \chi(b^*_t - b^*) \right]. \tag{A.23}$$

[16] Galí, López-Salido, and Vallés (2007).

We can rewrite this as

$$\mathcal{D}_t^{\frac{1}{\sigma}} = \frac{C_t}{C_t^* \mathcal{S}_t} \qquad \mathcal{D}_t^{\frac{1}{\sigma}} = (1 + \chi(b_t^* - b^*)).$$

Using the goods market clearing equation and the assumption that international markets are incomplete, we can write

$$Y_t = \varsigma \left(\frac{P_t}{P_{N,t}}\right)^{\iota} C_t + (1-\theta)(1-\varsigma)\left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu}\left(\frac{P_t}{P_{T,t}}\right)^{\iota} C_t + \theta^* \left(\frac{P_t}{P_{H,t}}\right)^{\mu} \mathcal{S}_t^{\mu} C_t^*$$
$$Y_t = \left[\varsigma \left(\frac{P_t}{P_{N,t}}\right)^{\iota} + (1-\theta)(1-\varsigma)\left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu}\left(\frac{P_t}{P_{T,t}}\right)^{\iota} + \theta^* \left(\frac{P_t}{P_{H,t}}\right)^{\mu} \frac{\mathcal{S}_t^{\mu-1}}{\mathcal{D}_t^{\frac{1}{\sigma}}}\right] C_t$$
$$C_t = \frac{1}{\Sigma_{\varsigma\theta,t}} Y_t.$$

where $\Sigma_{\varsigma\theta,t} = \left[\varsigma \left(\frac{P_t}{P_{N,t}}\right)^{\iota} + (1-\theta)(1-\varsigma)\left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu}\left(\frac{P_t}{P_{T,t}}\right)^{\iota} + \theta^* \left(\frac{P_t}{P_{H,t}}\right)^{\mu} \frac{\mathcal{S}_t^{\mu-1}}{\mathcal{D}_t^{\frac{1}{\sigma}}}\right]$. The relationship between domestic consumption and aggregate domestic production is described by $\Sigma_{\varsigma\theta,t}$. In an open economy, these two variables do not move in lockstep; instead, their dynamics are influenced by the degree of openness to trade. Higher θ therefore widens the wedge between consumption and production, attributable to the increased share of tradable goods destined for export.

Since $N_t^n = N_{N,t}^n + N_{H,t}^n$, we can write:

$$1 = \frac{1}{A_{N,t}} \left[C_t^{\sigma} N_t^{\phi} \frac{1}{\kappa_2}\right]^{\kappa_2} \left[\frac{P_{F,t}}{P_t} \frac{1}{\kappa_1}\right]^{\kappa_1} \left[\frac{(1+i_t^k)}{P_t} \frac{1}{1-\kappa_1-\kappa_2}\right]^{1-\kappa_1-\kappa_2}$$
$$C_t^{\sigma} = \left[A_{N,t} \left[\frac{\kappa_1}{p_{F,t}}\right]^{\kappa_1} \left[\frac{1-\kappa_1-\kappa_2}{\tilde{i}_t^k}\right]^{1-\kappa_1-\kappa_2}\right]^{\frac{1}{\kappa_2}} \kappa_2 N_t^{-\phi}$$

$$Y_t^n = \left\{ \left[A_{N,t} \left[\frac{\kappa_1}{p_{F,t}}\right]^{\kappa_1} \left[\frac{1-\kappa_1-\kappa_2}{\tilde{i}_t^k}\right]^{1-\kappa_1-\kappa_2}\right]^{\frac{1}{\kappa_2}} \kappa_2 \left[N_{H,t}^n + N_{N,t}^n\right]^{-\phi} \right\}^{\frac{1}{\sigma}} \tag{A.24}$$

Equation (A.24) describes the natural level of output, which can vary with technology in both sectors, relative prices of input and foreign demand. Finally, the natural real interest rate is the risk-free real interest rate consistent with the Euler equation when output is at its natural level at all times,

$$(C_{t+1}^n)^{\sigma} = \beta(1 + r_t^n)(C_t^n)^{\sigma}$$
$$(1 + r_t^n) = \frac{1}{\beta}\left(\left(\frac{Y_{t+1}^n}{Y_t^n}\right) \frac{\Sigma_{\varsigma\theta,t}}{\Sigma_{\varsigma\theta,t+1}}\right)^{\sigma}$$

where we have used $C_t = \frac{1}{\Sigma_{\varsigma\theta,t}} [Y_t]$.