Bond Market Clienteles, the Yield Curve, and the Optimal Maturity Structure of Government Debt

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Abstract

We propose a clientele-based model of the yield curve and optimal maturity structure of government debt. Clienteles are generations of agents at different lifecycle stages in an overlapping-generations economy. An optimal maturity structure exists in the absence of distortionary taxes and induces efficient intergenerational risksharing. If agents are more risk-averse than log, then an increase in the long-horizon clientele raises the price and optimal supply of long-term bonds—effects that we also confirm empirically in a panel of OECD countries. Moreover, under the optimal maturity structure catering to clienteles is limited and long-term bonds earn negative expected excess returns.

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1 Introduction

The government-bond market involves many distinct investor clienteles. For example, pension funds invest typically in long maturities as a way to hedge their long-term liabilities, while banks' treasury departments hold shorter maturities. Clientele demands vary over time in response to demographic or regulatory changes. This time-variation can have important effects both on the yield curve and on the government's debt-issuance policy.

The UK pension reform provides a stark illustration of clientele effects. The Pensions Act of 2004 required pension funds to maintain their asset-liability ratio above a threshold. Moreover, liabilities had to be marked to market using the yields of long-term inflation-indexed bonds. To minimize variation in their marked-to-market asset-liability ratio, pension funds bought large quantities of those bonds. This caused a steep decline in long rates. For example, in January 2006 the inflation-indexed bond maturing in 2055 was yielding 0.48%, which was low relative to both the historical average of long real rates and the short rates prevailing at that time.[1] The steep decline in long rates induced the UK Treasury to tilt debt issuance towards long maturities. For example, bonds with maturities of fifteen years or longer constituted 58% of issuance during financial year 2006-7, compared to an average of 40% over the previous four years.[2]

While clientele considerations matter for the yield curve and the issuance of government debt, they are largely absent from theoretical models. Indeed most models of bond-price determination assume a representative agent, thus ruling out clienteles. Optimal issuance of government debt is also studied mostly in representative-agent models, which overcome the Ricardian equivalence result of Barro (1974) by assuming distortionary taxes.

In this paper we develop a model of the yield curve and optimal issuance of government debt that emphasizes the role of clienteles. We focus on demographic rather than regulatory clienteles, and model them as generations of agents at different stages of their life cycle in an overlapping-generations economy. Assuming that agents are more risk-averse than log, we show that an increase in the relative importance of the clientele with the longer investment horizon, i.e., the younger, has two related effects: it renders long-term bonds more expensive, and it increases their optimal supply by the government. Using a panel of OECD countries, we find empirical support for these links between demographics, the slope of the yield curve, and the average maturity of government debt.

Our model delivers a number of additional results. We show that an optimal maturity structure

[1] Dimson, Marsh and Staunton (2002) compute a historical average of 2.3% for long real rates in the UK over the 20th century, based on a maturity of approximately 20 years. The inflation-indexed bond maturing in 2009 was yielding 1.47% in January 2006.

[2] The issuance numbers are from the website of the UK Debt Management Office. For a more detailed account of the UK pension reform and its effects on yields and debt issuance, see Greenwood and Vayanos (2010). Another illustration of clientele-driven issuance is the French Treasury's first-time issuance of a 50-year bond in 2005, in response to strong demand by pension funds.

of government debt exists even in the absence of distortionary taxes, and is the tool through which the government induces efficient risksharing across generations. One consequence of efficient risksharing is that the government caters to clientele demand, e.g., increases issuance in a segment of the yield curve in response to increased clientele demand in that segment. Catering could also be a consequence of maximizing revenue from bond issuance. We show, however, that a welfare-maximizing government caters to clientele demand to a more limited extent than a revenue-maximizing one because catering affects the tax burden of future generations. A welfare-maximizing government also limits issuance of long-term bonds to a level where these earn negative expected excess returns relative to short-term bonds.[3]

Section 2 describes our overlapping-generations model, set in discrete time and infinite horizon. Generations live for three periods, receive an endowment in the first period of their lives, consume in the third period, and have CRRA preferences over consumption. They can invest their endowment in a linear production technology and in non-contingent government bonds. The government can levy non-distortionary taxes on agents' endowments. In our baseline model, studied in Sections 2-4, the technology yields a riskless return and the one-period interest rate, which is equal to that return because of no arbitrage, is stochastic only in one period. Without loss of generality, we take that period to be period 2, which is the first period when three generations co-exist since the first generation is born in period 0. We assume that the interest rate in period 2 can take two values, so one- and two-period bonds complete the market from the perspective of agents trading in period 1. We refer to one- and two-period bonds as short- and long-term, respectively. The assumptions of a riskless technology and uncertainty only in one period simplify our analysis and deliver the main insights. Section 5 relaxes these assumptions.

Section 3 examines how the term structure of interest rates in period 1 depends on clientele demand and bond supply. An increase in the relative importance of the long-horizon clientele, consisting of generation 1 born in period 1, relative to the short-horizon clientele, consisting of generation 0 born in period 0, renders long-term bonds more expensive if agents are more risk-averse than log. Indeed, under these preferences, agents with a long horizon value more highly assets whose returns increase when interest rates decrease, and long-term bonds have this property. Risk aversion larger than log is necessary for the intuitive clientele effects that we derive: the effects reverse if risk aversion is smaller than log and vanish for log agents.

Section 3 shows additionally that a lengthening of the maturity structure in period 1, defined as an increase in the supply of long-term bonds holding total debt value constant, renders long-term bonds cheaper. Ricardian equivalence thus does not hold in our model. This is because supply

[3] Long-term nominal bonds earn positive expected excess returns relative to their short-term counterparts (e.g., Cochrane (1999)). Our model, however, is real so our results concern real bonds. We return to the implications of our model for bond risk premia, and the link with the empirical evidence, after Proposition 4.6.

changes and the accompanying offsetting tax changes concern different generations: supply changes must be absorbed by generations 0 and 1, who are the only ones that can trade in period 1, but tax changes concern future generations.

Section 4 determines the optimal maturity structure of government debt in period 1. Maturity structure affects intergenerational risksharing. For example, if the government issues more long-term bonds in period 1, it helps generation 1 insure against the risk of reinvesting at a low interest rate in period 2. This insurance is provided by imposing more risk on future generations through a more uncertain tax burden. The government's optimal policy in period 1 is to issue the quantity of long-term bonds that generations 0 and 1 would buy from future generations if the latter were present in the market in period 1. Through this policy, the government induces complete participation by all generations in period 1 and efficient risksharing.

If agents are more risk-averse than log, then an increase in the size of the long-horizon clientele in period 1 (generation 1) increases the demand for insurance against a low interest rate in period 2. As a consequence, the optimal maturity structure involves more long-term debt. Note that in lengthening maturity structure in response to an increase in the size of the long-horizon clientele, the government is effectively catering to that clientele. Such catering, however, is limited, i.e., the issuance of long-term bonds is smaller than the increased demand. This is because the insurance provided to generation 1 imposes more risk on future generations. Note that when the interest rate is low, fewer aggregate resources are available. Therefore, efficient risksharing requires that generation 1 consumes less in that state. Since generation 1 is marginal in pricing bonds in period 1, long-term bonds earn negative expected excess returns relative to short-term bonds under the optimal maturity structure.

Section 5 extends our baseline model to a stochastic interest rate in all periods, a risky production technology, and a risky endowment of each generation. When stochastic shocks are small, the equilibrium can be characterized in closed form. Under a risky technology, clientele demand and bond supply affect the short-term interest rate, and this can reverse their effect on the prices of long-term bonds. The effect on expected excess returns remains the same, and so do the main properties of optimal maturity structure.

Section 6 tests our basic predictions on how clientele demand affects the yield curve and the maturity structure of government debt. We focus on OECD countries, for which data on the maturity structure of government debt are available. Our main demographic variable is the median age of the population: an increase in median age can be interpreted as a decrease in the relative importance of the long-horizon clientele. Since in our model generations live for three periods, we use 10- and 30-year government bonds as empirical proxies for the short- and long-term bond, respectively. Consistent with our model's predictions in the empirically plausible case where agents are more risk-averse than log, we find that median age is positively related to the slope of the yield curve, as measured by the 30-10 yield spread, and negatively related to the average maturity of government debt.

This paper is related to the literature on optimal public debt policy. The benchmark in that literature is the Ricardian equivalence result of Barro (1974): in a representative-agent model with non-distortionary taxes, the level and composition of government debt are irrelevant. Ricardian equivalence fails in the presence of distortionary taxes because debt can be used to smooth tax rates across time and states of nature. Distortionary taxes imply an optimal time path for the level of government debt, as shown in Barro (1979) and Aiyagari, Marcet, Sargent and Seppala (2002), and an optimal composition of the government debt portfolio. Lucas and Stokey (1983) derive the optimal portfolio in terms of Arrow-Debreu securities. Angeletos (2002) and Buera and Nicolini (2004) show how the optimal outcome can be implemented with non-contingent bonds of different maturities, provided that there are bonds of as many maturities as states of nature so that markets are complete. Nosbusch (2008) derives the optimal maturity structure under incomplete markets. Faraglia, Marcet and Scott (2010) extend the complete markets analysis of optimal maturity structure to a framework with capital accumulation.[4] Optimal maturity structure in our model is determined by clienteles and intergenerational risksharing rather than distortionary taxes. This yields insights that differ from and complement those of the distortionary-tax literature.

It is well known that Ricardian equivalence fails in models with overlapping generations. Government debt shifts taxes to future generations, as shown in Diamond (1965) and Blanchard (1985), and this can be Pareto-improving when the economy is dynamically inefficient. Government debt can also improve intergenerational risksharing, as shown in Fischer (1983) and Gale (1990). In both papers generations live for two periods, and the introduction of one-period non-contingent government bonds can be Pareto-improving. Gale shows additionally that two-period bonds can yield even larger improvements.[5] Our model differs because we allow for clienteles with different investment horizons, study how changes in clientele demand affect bond prices and the optimal maturity structure, and characterize the latter in terms of an equilibrium with complete participation by all generations.[6]

[4]A separate strand of the literature considers optimal debt policy when debt contracts are nominal and the government has control over inflation. The government can then use state-contingent inflation to smooth tax rates. Prominent examples of this approach include Lucas and Stokey (1983), Bohn (1988), Calvo and Guidotti (1990,1992), Barro (2002), Benigno and Woodford (2003), and Lustig, Sleet and Yeltekin (2008). Missale and Blanchard (1994) show that with nominal debt contracts, the optimal maturity can be decreasing in the total size of the debt.

[5]Blanchard and Weil (2001) provide general conditions under which the government can use bonds of different maturities to achieve Pareto improvements in a dynamically efficient economy. Weiss (1980) and Bhattacharya (1982) show that the government can use state-contingent inflation to improve intergenerational risksharing when debt contracts are nominal.

[6]Ball and Mankiw (2007) use the complete-participation equilibrium to characterize the optimal social security system in a two-period overlapping-generations model. They show that an optimal social security system implements the equilibrium that would prevail if all generations could trade risksharing contracts ex-ante. Social security and debt maturity structure can be viewed as alternative mechanisms to improve intergenerational risksharing. We return

The role of clienteles is emphasized in an early term-structure literature. According to the preferred-habitat hypothesis of Culbertson (1957) and Modigliani and Sutch (1966), there are investor clienteles with preferences for specific maturities and with limited ability to substitute across the yield curve. More recently, Vayanos and Vila (2009) develop a formal model of preferred habitat in which each maturity has its own clientele and substitution across maturities is performed by risk-averse arbitrageurs. Greenwood and Vayanos (2012) build on that model to explore how changes in debt supply affect bond yields and returns, and find empirically that an increase in maturity-weighted debt to GDP predicts high excess returns of long-term bonds. Unlike these papers, we model clienteles through CRRA preferences, dispense with arbitrageurs, and perform a normative analysis of maturity structure.

Finally, our paper is related to a recent literature studying how private firms make capital-structure decisions to cater to investor clienteles or sentiment (e.g., Stein (1996), Baker and Wurgler (2002), Greenwood, Hanson and Stein (2010)). The government in our model caters to clienteles, but to a limited extent because it internalizes the effect on future generations. Moreover, the government affects asset prices, while firms in the catering literature are price takers.

2 Model

Time t is discrete and goes from 0 to ∞. There is a single good. In each period a new generation is born and lives for three periods. Generation t, born in period t, is young in that period, middle-aged in period $t+1$, and old in period $t+2$. It receives an endowment α_t of the good when young and consumes c_{t+2} when old. Utility over consumption is CRRA

$$u(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}, \tag{2.1}$$

and the coefficient of relative risk aversion γ is equal across generations. Each generation derives its consumption by investing its net-of-tax endowment in a linear production technology and in government bonds.

The technology yields a return that is riskless but can vary stochastically across periods. Because of no-arbitrage, this return is equal to the one-period interest rate, which can thus be stochastic. We mainly focus on the case where the one-period interest rate is stochastic only in one period. This case is simple analytically and conceptually, and delivers the main insights. For simplicity we also assume that the endowment of each generation is riskless. Section 5 extends our analysis to a stochastic interest rate in all periods, a risky production technology, and a risky endowment of

to this issue in Section 4.2.

each generation.

The steady state of our model starts from period 2, which is the first period when three generations coexist. We assume that the period when the interest rate is stochastic belongs to the steady state, and without loss of generality we take it to be period 2. We set the gross one-period interest rate in all other periods to $R > 1$, i.e., one unit invested in period $t \neq 2$ yields R units in period $t + 1$. We denote the corresponding rate in period 2 by R_2, and assume that it takes the values $R_{2,h}$ and $R_{2,\ell} < R_{2,h}$ with probabilities p and $1 - p$, respectively. We use more generally the subscript $s = h, \ell$ to denote the value of a variable in state s. The value of R_2 is observed in period 2 but not in previous periods, and constitutes the only uncertainty in the model (until Section 5). The endowment α_t can depend on t. For simplicity we assume that α_t is bounded above, but our analysis can be extended to the case where α_t grows at an asymptotic gross rate smaller than R.

The government issues non-contingent bonds and collects income taxes. Since only young agents receive an income (endowment), taxes are only on them. For simplicity we set government spending to zero; assuming positive spending would not affect our results. Bond issuance and taxes can be negative, in which case they represent government investment in bonds and transfers towards the agents, respectively.

We assume that only one-period bonds are available in periods $t \neq 1$, while one- and two-period bonds are available in period 1. This entails no loss of generality relative to a more general maturity structure. Indeed, in periods $t \neq 1$, agents face no uncertainty one period ahead. Therefore, bonds of multiple maturities are redundant, and we can assume that only one-period bonds are available. In period 1, the uncertainty one period ahead is described by two states. Therefore, two maturities suffice to complete the market from the perspective of agents trading in period 1, and we take these to be one and two periods.

We denote by τ_t the tax rate in period t and by b_t the face value of one-period bonds available in the same period. One-period bonds included in b_t for $t \neq 2$ are only those issued by the government in period t. One-period bonds included in b_2 are both those issued by the government in period 2, and two-period bonds issued in period 1 that become one-period in period 2. We denote by B_1 the face value of two-period bonds issued by the government in period 1 and by L_1 the gross two-period interest rate in the same period.

The choice of maturity structure concerns the mix (b_1, B_1) of one- and two-period bonds issued by the government in period 1. The generations investing in bonds in that period are 0 and 1, and maturity structure affects how risk is shared between them and generations $t \geq 2$. We hence study the choice of maturity structure in conjunction with the taxes in periods $t \geq 2$, which can be made state-contingent and can concern only the endowments of generations $t \geq 2$. We treat taxes on generations $t \leq 1$ as exogenous, and for simplicity set them to zero. Since these taxes are zero, the face value b_0 of one-period bonds issued by the government in period 0 is also zero.

The government's budget constraint in period 1 is

$$\frac{b_1}{R} + \frac{B_1}{L_1^2} = 0. \tag{2.2}$$

Government revenue is the sum of taxes on generation 1, which are zero, plus the revenue b_1/R from issuing one-period bonds and B_1/L_1^2 from issuing two-period bonds. It is equal to the cost b_0 of repaying the one-period bonds issued in period 0, which is zero.

The government's budget constraint in period 2 is

$$\alpha_2\tau_2 + \frac{b_2 - B_1}{R_2} = b_1. \tag{2.3}$$

Government revenue is the sum of taxes $\alpha_2\tau_2$ on generation 2, plus the revenue $(b_2 - B_1)/R_2$ from issuing one-period bonds. It is equal to the cost b_1 of repaying the one-period bonds issued in period 1. The government's budget constraint in period $t > 2$ is similarly

$$\alpha_t\tau_t + \frac{b_t}{R} = b_{t-1}. \tag{2.4}$$

To eliminate Ponzi schemes by the government, we assume that debt must be bounded above, i.e.,

$$b_t \leq K \tag{2.5}$$

for a bound K and for all t.

3 Supply and Demand Effects

In this section we characterize equilibrium in period 1. We derive the equilibrium term structure and examine how it depends on bond supply and clientele demand.

3.1 Equilibrium Term Structure

In period 1, only generations 0 and 1 can trade bonds. We denote by $B_{1,0}$ and $B_{1,1}$, respectively, the face values of two-period bonds that they hold. (Bond holdings are expressed in absolute terms and not as fraction of endowment.) Generation 0's consumption in period 2 is

$$c_2 = R^2\alpha_0 + \left(\frac{L_1^2}{R_2} - R\right)\frac{B_{1,0}}{L_1^2}. \tag{3.1}$$

The first term in the right-hand side is the consumption achieved by investing the endowment α_0 in the linear production technology or in one-period bonds between periods 0 and 2. The second term is the additional return from investing in two-period bonds, which become available in period 1. It is equal to the excess return of two- relative to one-period bonds between periods 1 and 2, times the market value of two-period bonds held by generation 0. Generation 0 chooses its holdings $B_{1,0}$ of two-period bonds to maximize expected utility $\mathbb{E}[u(c_2)]$ subject to (3.1). The first-order condition is

$$\mathbb{E}\left[u'(c_2)\left(\frac{1}{R_2} - \frac{R}{L_1^2}\right)\right] = 0. \tag{3.2}$$

Since $u'(0) = \infty$, (3.2) has a solution $B_{1,0}$ for any two-period interest rate L_1 satisfying no-arbitrage, i.e., $L_1^2 \in (RR_{2,\ell}, RR_{2,h})$. Moreover, the solution is unique since utility is strictly concave.

Generation 1's consumption in period 3 is

$$c_3 = RR_2\alpha_1 + \left(L_1^2 - RR_2\right)\frac{B_{1,1}}{L_1^2}. \tag{3.3}$$

The first term in the right-hand side is the consumption achieved by investing the endowment α_1 in the linear production technology or in one-period bonds between periods 1 and 3. The second term is the additional return from investing in two-period bonds. It is equal to the excess return of two- relative to one-period bonds between periods 1 and 3, times the market value of two-period bonds held by generation 1. Note that the relevant excess return for generation 1 is over two periods because this generation consumes one period later than generation 0. The first-order condition is

$$\mathbb{E}\left[u'(c_3)\left(1 - \frac{RR_2}{L_1^2}\right)\right] = 0. \tag{3.4}$$

The same argument as for (3.2) implies that (3.4) has a unique solution $B_{1,1}$.

The two-period interest rate L_1 is determined from the market-clearing condition

$$B_{1,0} + B_{1,1} = B_1. \tag{3.5}$$

Proposition 3.1 shows that (3.5) has a unique solution L_1 because the demand functions $(B_{1,0}, B_{1,1})$ are increasing in L_1, converge to $-\infty$ when L_1^2 goes to $RR_{2,\ell}$, and converge to ∞ when L_1^2 goes to $RR_{2,h}$.

Proposition 3.1. *There exists a unique equilibrium two-period interest rate L_1 in period 1.*

The equilibrium term structure in period 1 consists of the one-period interest rate R and the two-period interest rate L_1. Since supply and demand do not affect R, their effects on L_1, on the slope of the term structure, and on the expected excess return of two- relative to one-period bonds are all in the same direction.

3.2 Bond Supply

Bond supply in period 1 can be measured by the face values b_1 and B_1 of one- and two-period bonds, respectively. We hold total supply constant in market value terms, and examine how a shift towards two-period bonds, i.e., a lengthening of the maturity structure, affects the two-period interest rate L_1. This amounts to increasing B_1 holding $b_1/R + B_1/L_1^2$ constant.

Proposition 3.2. *A lengthening of the maturity structure in period 1 raises the two-period interest rate L_1.*

Intuitively, when B_1 increases, generations 0 and 1 must absorb more two-period bonds, and this gives them larger exposure to the risk that the one-period interest rate will increase in period 2. They are compensated for that risk through a decrease in the price of two-period bonds. The two-period interest rate thus increases.

The effect of supply derived in Proposition 3.2 stands in contrast to the Ricardian equivalence result of Barro (1974), derived in a representative-agent model with non-distortionary taxes. In that model, supply changes can affect consumption and bond demand, but the effects are exactly offset by those of the accompanying tax changes.[7] In our model, however, supply and tax changes concern different agents: supply changes must be absorbed by generations 0 and 1, who are the only ones that can trade in period 1, but tax changes concern future generations.

[7] Supply effects can arise in representative-agent models with distortionary taxes. The mechanism driving these effects, however, is different than in our model, and the effects themselves are typically in the opposite direction. In particular, an increase in the supply of two-period bonds can increase their price, holding the one-period interest rate constant. Indeed, when supply increases, the government must collect more taxes in states where the interest rate is low because more resources are required in these states to eventually repay the two-period bonds. Because taxes are distortionary, consumption is reduced in the low-interest rate states. As a consequence, two-period bonds, which are more valuable in these states, become more attractive to the representative agent and their price increases.

3.3 Clientele Demand

Clienteles in our model correspond to generations that are alive in any given period and differ in their investment horizons. For example, the clienteles in period 1 are the young generation 1, with a two-period investment horizon, and the middle-aged generation 0, with a one-period horizon. We refer to them as the long- and short-horizon clientele, respectively. To examine how the mix of these clienteles affects the term structure, we change the relative size of their endowments holding the value of their aggregate endowment measured as of period 1 constant. Thus, an increase in the size of the long-horizon clientele amounts to increasing α_1 holding $R\alpha_0 + \alpha_1$ constant.

Proposition 3.3. *If $\gamma > 1$, then an increase in the size of the long-horizon clientele in period 1 lowers the two-period interest rate L_1. The result is reversed if $\gamma < 1$.*

According to the preferred-habitat hypothesis of Culbertson (1957) and Modigliani and Sutch (1966), short-term bonds are demanded mainly by short-horizon investors, while long-term bonds are demanded by long-horizon investors. Therefore, when generation 1 commands more resources, two-period bonds should be in higher demand and thus more expensive. Proposition 3.3 confirms this intuition when agents' coefficient of relative risk aversion γ is larger than one. When $\gamma < 1$, however, the effect is reversed, and when $\gamma = 1$ (logarithmic utility) the clientele mix has no effect. Intuitively, when utility is logarithmic, agents behave myopically and their portfolio choice is independent of the time when they need to consume. When instead $\gamma \neq 1$, generation 1 has a hedging demand which depends on assets' covariance with the interest rate in period 2. In the case $\gamma > 1$, the hedging demand favors assets whose returns increase when the interest rate decreases. Two-period bonds have this property, and hence generation 1 invests a larger share of its wealth in these bonds than generation 0. In summary, our model can generate intuitive clientele effects, while also highlighting that such effects can arise only when $\gamma > 1$. Clientele effects do not arise, in particular, under logarithmic utility, an assumption commonly used in term-structure models.

4 Optimal Maturity Structure

The properties derived in Section 3 concern a general maturity structure, i.e., hold for any quantities of one- and two-period bonds that the government issues in period 1. In this section we show that there exists an optimal maturity structure and derive its properties.

4.1 Complete Participation

To set the stage for our analysis, we start with the benchmark case where all generations are available to trade one- and two-period bonds in period 1. We also remove the government, setting bond supply and taxes to zero. This yields an equilibrium with complete participation and efficient risksharing across generations. We then show that the government can use maturity structure and taxes to induce this allocation when only generations 0 and 1 are available to trade in period 1.

The optimization problems of generations 0 and 1 are as in Section 3.1, so we only need to consider those of generations $t \geq 2$. Generation 2's consumption in period 4 is

$$c_4 = RR_2\alpha_2 + R\left(L_1^2 - RR_2\right)\frac{B_{1,2}}{L_1^2}, \tag{4.1}$$

and generation $t > 2$'s consumption in period $t+2$ is

$$c_{t+2} = R^2\alpha_t + R^{t-1}\left(L_1^2 - RR_2\right)\frac{B_{1,t}}{L_1^2}, \tag{4.2}$$

where $B_{1,t}$ denotes the face value of two-period bonds held by generation $t \geq 2$. (As in Section 3.1, bond holdings are expressed in absolute terms and not as fraction of endowment.) The interpretation of (4.1) and (4.2) is analogous to that of (3.1) and (3.3): the first term in the right-hand side is the consumption achieved by investing the endowment in the linear production technology or in one-period bonds, and the second term is the additional return from investing in two-period bonds. The excess return of two- relative to one-period bonds is computed between periods 1 and 3, and brought forward to period t at the rate R. The first-order condition of generation $t \geq 2$ is

$$\mathbb{E}\left[u'(c_{t+2})\left(1 - \frac{RR_2}{L_1^2}\right)\right] = 0. \tag{4.3}$$

The same argument as in Section 3.1 implies that (4.3) has a unique solution $B_{1,t}$.

The two-period interest rate L_1 under complete participation is determined by the market-clearing condition

$$\sum_{t=0}^{\infty} B_{1,t} = 0. \tag{4.4}$$

Eq. (4.4) differs from its counterpart (3.5) in Section 3.1 because the summation is over all generations, and the supply of two-period bonds is zero. Proposition 4.1 shows that (4.4) has a solution L_1.[8]

Proposition 4.1. *Under complete participation, there exists an equilibrium two-period interest rate*

[8] We discuss uniqueness of the solution in Section 4.4.

L_1 in period 1.

Combining the first-order conditions (3.2), (3.4) and (4.3), and noting that there are two states (h, ℓ), we find

$$\frac{u'(c_{2,h})}{u'(c_{2,\ell})} = \frac{R_{2,h}}{R_{2,\ell}} \frac{u'(c_{t+2,h})}{u'(c_{t+2,\ell})} \tag{4.5}$$

for all $t \geq 1$, where the subscript $s = h, \ell$ denotes consumption in state s. Eq. (4.5) characterizes intergenerational risksharing under complete participation. Risk is shared optimally, and the condition for optimality is that the ratio across states of the marginal benefit from receiving the good in period 2 is identical for all generations. Since generation 0 consumes in period 2, its marginal benefit coincides with its marginal utility of consumption. The marginal benefit of generation $t \geq 1$ is equal to the marginal utility of consumption times the return on investment from period 2 to period $t + 2$, which is when that generation consumes. This return is $R^{t-1}R_2$, and its ratio across states h and ℓ is $R_{2,h}/R_{2,\ell}$. Since this ratio is identical for all generations $t \geq 1$, so is the ratio across states of the marginal utility of consumption. Thus, if one of generations $t \geq 1$ consumes more in state h than in state ℓ, so do all of them.

Since the one-period interest rate is higher in state h than in state ℓ, more aggregate resources are available in that state. As a consequence, the consumption of all generations $t \geq 1$ is higher in state h. The consumption of generation 0, however, can be lower. The reason why this is consistent with efficient risksharing is that while the marginal utility of generations $t \geq 1$ is lower in state h, their return from investment is higher. As a consequence, their marginal benefit from receiving the good in period 2 can be higher in state h, in which case efficient risksharing requires that the marginal utility of consumption of generation 0 is also higher. Put differently, it can be socially efficient that generation 0 foregoes consumption in state h since investment in that state has high return for future generations.

Whether or not generation 0 consumes more in state h than in state ℓ depends on the coefficient of relative risk aversion γ. If γ is high, then the marginal benefit of generations $t \geq 1$ from receiving the good in period 2 is driven more by their marginal utility of consumption than by their return on investment. Since their marginal utility is lower in state h, so is that of generation 0, meaning that generation 0 consumes more in state h. The comparison is reversed for lower values of γ.

Proposition 4.2. *Under complete participation, generations $t \geq 1$ consume more in state h than in state ℓ. Generation 0 consumes more in state h than in state ℓ if γ exceeds a threshold $\bar{\gamma} > 1$, but the comparison is reversed if $\gamma \leq 1$.*

Figure 1 plots the consumption of all generations in states h and ℓ. We consider both the case of complete participation and that of autarky, where each generation invests all its endowment in the linear production technology. We assume that endowments are constant ($\alpha_t = 1$ for all t), the two states are equally likely ($p = 0.5$), and $\gamma = 4$. We interpret one period to last for 15 years and set the gross return R of the technology to $(1 + 2.1\%)^{15}$. The annual rate 2.1% is the historical average of the long real interest rate in the US in the 20th century (Dimson, Marsh, and Staunton (2002)). We assume that the return of the technology is 10% above its mean in state h and 10% below in state ℓ. We plot consumption normalized by its expected value, R^2. Under autarky, only generations 1 and 2 are exposed to the return shock. Under complete participation, by contrast, all generations share the risk. Consistent with Proposition 4.2, the consumption c_{t+2} of generations $t \geq 1$ is higher in state h. Moreover, the parameter value $\gamma = 4$ is in the region where the consumption c_2 of generation 0 is also higher in state h.

We next determine equilibrium bond holdings. Under complete participation, generations 1 and 2 seek to hedge against the risk that their return on investment in period 2 will be low (state ℓ). Generations $t > 2$, who have no exposure to that risk, provide them with insurance by selling them two-period bonds. Since initial positions are zero, generations $t > 2$ short-sell two-period bonds and use the proceeds to invest in one-period bonds. The investment in one-period bonds is the reason why the consumption of these generations is low in state ℓ (Proposition 4.2).

Generation 0 could be an additional seller of two-period bonds because it has no exposure to return risk. Generation 0, however, differs from generations 1 and 2 not only in risk exposure, but also in investment horizon: that of generation 0 is one period, while that of generations 1 and 2 is two periods. If $\gamma > 1$, then a longer horizon implies larger investment in two-period bonds (Proposition 3.3). Therefore, generation 0 invests less in two-period bonds than generations 1 and 2, which implies that generations 1 and 2 have to be buyers of these bonds. Generation 0 can be a buyer or a short-seller; it is a short-seller if γ exceeds the threshold $\bar{\gamma} > 1$ since its consumption is higher in state h than in state ℓ (Proposition 4.2). If instead $\gamma \leq 1$, then a longer horizon implies smaller investment in two-period bonds. Therefore, generation 0 invests more in two-period bonds than generations 1 and 2, which implies that generation 0 has to be a buyer of these bonds and consume more in state ℓ (Proposition 4.2). Generations 1 and 2 can be buyers or short-sellers of two-period bonds.

Corollary 4.1. *Under complete participation,*

- *Generations $t > 2$ short-sell two-period bonds.*
- *Generations 1 and 2 buy two-period bonds if $\gamma \geq 1$.*
- *Generation 0 short-sells two-period bonds if $\gamma \geq \bar{\gamma}$, and buys them if $\gamma \leq 1$.*

4.2 Inducing Complete Participation through Debt Maturity Structure

We now return to the case where only generations 0 and 1 can trade in period 1. We show that although generations $t \geq 2$ cannot trade in that period, there exists a choice of maturity structure and taxes that results in efficient risksharing between them and generations 0 and 1. Through this choice, the government can effectively induce complete participation.

Suppose that the government issues the aggregate quantity B_1^* of two-period bonds that generations 0 and 1 buy under complete participation. Then, the two-period interest rate coincides with that under complete participation. The same is true for the quantity of two-period bonds bought by each of generations 0 and 1, and for these generations' consumption. Generations $t \geq 2$ can also consume the same as under complete participation through a choice of state-contingent taxes that meet the government's budget constraint.

Proposition 4.3. *Suppose that only generations 0 and 1 can trade in period 1. Then, the government can achieve the same outcome as under complete participation by issuing the aggregate quantity B_1^* of two-period bonds that generations 0 and 1 buy under complete participation, and levying appropriate state-contingent taxes $\{\tau_t^*\}_{t\geq 2}$ on generations $t \geq 2$.*

Since the government issues the same quantity of two-period bonds as that sold by generations $t \geq 2$ under complete participation, it is effectively trading on behalf of these generations. Thus, the government induces efficient risksharing by replicating the trades of private agents not present in the market. Note that the government raises welfare by supplying the right quantity of two-period bonds rather than by introducing these bonds into the market. Two-period bonds can exist even in the government's absence; the government's role is to trade the right quantity of them.

To illustrate the government's optimal policy, we return to the example used in Figure 1 of Section 4.1. Under complete participation, generations 1 and 2 share their risk with generations 0 and $t > 2$ by buying two-period bonds from them. Moreover, generations 0 and 1 buy two-period bonds in the aggregate, and the quantity bought is 0.7. Therefore, the government can induce complete participation by issuing a quantity $B_1^* = 0.7$ of two-period bonds.

Figure 2 depicts government debt and taxes in states h and ℓ for $t \geq 2$. In state h, the government makes a capital gain on its bond portfolio since it has issued two-period bonds. It also taxes generation 2 ($\tau_{2,h} > 0$), which buys two-period bonds under complete participation and consumes less in state h than under autarky. The government then invests its trading profits and tax proceeds in one-period bonds, rolls them over, and pays out the interest that it earns as a transfer to generations $t > 2$. Thus, transfers to generations $t > 2$ are constant over time ($\tau_{t,h} \equiv \tau_h < 0$ for $t > 2$). Constant transfers are optimal because generations $t > 2$ have the same initial endowment, CRRA preferences and investment opportunity set, and hence the same consumption profile under complete participation. Conversely, in state ℓ, the government makes a capital loss on its bond portfolio, and makes a transfer to generation 2 ($\tau_{2,\ell} < 0$). It then finances its trading losses and transfer payments by issuing one-period bonds, rolls them over, and pays the interest by levying a constant tax on generations $t > 2$ ($\tau_{t,\ell} \equiv \tau_\ell > 0$ for $t > 2$).[9]

4.3 Uniqueness of Optimal Maturity Structure

Section 4.2 derives a maturity structure that induces complete participation and efficient risksharing. We next study more formally the optimization over maturity structures, and show that a maturity structure is optimal if and only if it is as in Section 4.2.

A maturity structure is fully characterized by the quantity B_1 of two-period bonds issued in period 1; the quantity b_1 of one-period bonds issued in that period is determined by the government's budget constraint (2.2). We study the choice of B_1 in conjunction with the state-contingent taxes $\{\tau_t\}_{t\geq 2}$. The policy $(B_1, \{\tau_t\}_{t\geq 2})$ must satisfy the government's budget constraints (2.2)-(2.4) and the bound (2.5). We refer to such a policy as admissible.

We define optimality of a maturity structure by aggregating across agents gains and losses from a policy change. To ensure that gains and losses are comparable across agents, we measure them in monetary terms as of period 1. Consider a policy change from $(B_1, \{\tau_t\}_{t\geq 2})$ to $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$, and denote by c_{t+2} and $\hat{c}_{t+2}$ the consumption of generation t in the corresponding equilibria. Define the gain T_t of generation t as the dollar amount that if invested in one-period bonds from period 1 to $t+2$ and subtracted from $\hat{c}_{t+2}$ would leave generation t with the same expected utility as under c_{t+2}. The gain of generation 0 is given by

$$\mathbb{E}u(c_2) \equiv \mathbb{E}u(\hat{c}_2 - RT_0) \tag{4.6}$$

since investing T_0 in one-period bonds from period 1 to 2 yields RT_0. Similarly, the gain of generation $t \geq 1$ is given by

$$\mathbb{E}u(c_{t+2}) \equiv \mathbb{E}u(\hat{c}_{t+2} - R^t R_2 T_t) \tag{4.7}$$

since investing T_t in one-period bonds from period 1 to $t+2$ yields $R^t R_2 T_t$. The gain T_t is analogous

[9] Proposition 4.3 shows that the government can achieve the complete-participation allocation through a combination of debt maturity structure in period 1 and taxes on future generations $t \geq 2$. An alternative mechanism that achieves the same allocation is a social security system, provided that taxes in period 2 can also be imposed on generations 0 and 1. Since, however, taxes are only on income in our model, and agents receive income only in the first period of their lives, generations 0 and 1 cannot be taxed in period 2. Even under more general tax schemes and income profiles, debt maturity structure has the advantage over social security to not require taxes on generations 0 and 1: the revenue is raised instead through the capital losses that these generations realize on their investments in government bonds. Taxes could have the drawback of being distortionary (a consideration which is outside our model).

to the concept of compensating variation (e.g., Varian (1992)).

Definition 1. *A maturity structure* B_1 *is optimal if it is associated to an admissible policy* $(B_1, \{\tau_t\}_{t\geq 2})$ *such that the sum* $\sum_{t=0}^{\infty} T_t$ *of gains from switching to any other admissible policy* $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$ *is non-positive.*

Optimality in definition 1 is a stronger version of Pareto optimality: the maturity structure B_1 is not optimal if the allocation corresponding to $(B_1, \{\tau_t\}_{t\geq 2})$ is Pareto-dominated by the allocation corresponding to $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$ combined with a set of transfers. The policy $(B_1^*, \{\tau_t^*\}_{t\geq 2})$ of Proposition 4.3 corresponds to a standard competitive equilibrium with complete participation, and hence to a Pareto optimal allocation among all feasible allocations, i.e., those satisfying the aggregate resource constraint. Since the allocation corresponding to $(B_1^*, \{\tau_t^*\}_{t\geq 2})$ is not Pareto-dominated by any other allocation, B_1^* is optimal according to Definition 1. Proposition 4.4 shows that the converse is true as well, i.e., any optimal maturity structure must correspond to an equilibrium with complete participation.

Proposition 4.4. *A maturity structure is optimal if and only if it is equal to the aggregate quantity* B_1^* *of two-period bonds that generations 0 and 1 buy in a complete-participation equilibrium.*

4.4 Properties of Optimal Maturity Structure

We next derive two key properties of the optimal maturity structure in our model. The first concerns the effect of changes in the clientele mix. The second concerns the expected excess return of long- relative to short-term bonds.

4.4.1 Clientele Effects

We consider a change in the clientele mix in period 1. As in Proposition 3.3, we change the relative size of clienteles' endowments holding the value of their aggregate endowment measured as of period 1 constant. Thus, an increase in the size of the long-horizon clientele, generation 1, relative to the short-horizon clientele, generation 0, amounts to increasing α_1 holding $R\alpha_0 + \alpha_1$ constant. We make the following technical assumption:

Assumption 1. *The demand functions of generations* $t > 2$ *for two-period bonds under complete participation are increasing in the two-period interest rate* L_1*.*

Assumption 1 ensures that the aggregate demand function for two-period bonds in the equilibrium with complete participation is a standard downward-sloping demand function: it is increasing in L_1 and hence is decreasing in the bond price. This is because the demand functions of generations 0, 1 and 2 for two-period bonds are increasing in L_1, as shown in Propositions 3.1 and 4.1. A downward-sloping demand function ensures that the equilibrium with complete participation is unique. Assumption 1 holds under the sufficient condition $\gamma \leq 1$, and hence holds also for values of γ larger than one.[10]

Proposition 4.5. *Suppose that Assumption 1 holds. If $\gamma > 1$, then an increase in the size of the long-horizon clientele in period 1:*

- *Raises the optimal supply B_1^* of two-period bonds.*
- *Lowers the equilibrium two-period interest rate L_1 that prevails when two-period bonds are in supply B_1^*.*

The results are reversed if $\gamma < 1$.

Proposition 4.5 is closely related to Proposition 3.3, which examines how an increase in the size of the long-horizon clientele affects the equilibrium two-period interest rate L_1. Proposition 3.3 takes the supply of two-period bonds as fixed, while Proposition 4.5 examines both how the optimal supply changes and how the equilibrium two-period interest rate changes taking into account the change in supply. The effects are closely related. For example, Proposition 3.3 shows that if $\gamma > 1$, then an increase in the size of the long-horizon clientele raises the aggregate demand of generations 0 and 1 for two-period bonds. Due to the higher demand, these generations buy a larger quantity of two-period bonds in the equilibrium with complete participation, and the two-period interest rate in that equilibrium decreases. This, in turn, implies Proposition 4.5: the optimal supply B_1^* of two-period bonds in the equilibrium with incomplete participation increases, and the two-period interest rate prevailing under B_1^* decreases.

Propositions 3.3 and 4.5 reveal a similarity between welfare maximization and revenue maximization. Suppose that $\gamma > 1$. Holding maturity structure constant, an increase in the size of the long-horizon clientele lowers the two-period interest rate L_1 (Proposition 3.3). This prompts the government to change the maturity structure by increasing the supply B_1^* of two-period bonds (Proposition 4.5). Hence, a welfare-maximizing government responds to demand shocks in a way that appears consistent with maximizing revenue from bond issuance. The same is true when $\gamma < 1$ since the effects of demand shocks on L_1 and B_1^* are both in the opposite direction than when $\gamma > 1$.

Revenue maximization, suitably extended to account for the variance of revenue in addition to the mean, is viewed as a relevant objective by many practitioners (Bernaschi, Missale and Vergni (2009)). Our analysis identifies a similarity between revenue maximization and welfare

[10]The equilibrium with complete participation is unique in the example used in Figures 1 and 2, where $\gamma = 4$.

maximization, pointed out in the previous paragraph, but also an important difference. A revenue-maximizing government in our model would respond to an increase in the demand for two-period bonds by increasing supply up to the point where the two-period interest rate remains unchanged. This is because it equates the revenue from issuing two-period bonds to the expected revenue from issuing one-period bonds and rolling over, and the latter does not depend on the demand shock because the one-period interest rate is equal to the return of the production technology. By contrast, a welfare-maximizing government lets the two-period interest rate decrease (Proposition 4.5). Thus, it does not fully accommodate the increase in demand, catering to clienteles to a more limited extent than a revenue-maximizing government. This is because catering affects future generations. Indeed, by increasing the supply of long-term bonds, the government helps the long-horizon clientele insure against the risk of reinvesting at a low interest rate. This insurance, however, is provided by imposing more risk on future generations through a more uncertain tax burden: high taxes when the interest rate is low and low taxes when it is high.

4.4.2 Expected Excess Returns of Long-Term Bonds

We consider the expected excess return of two- relative to one-period bonds in period 1 over both a one- and a two-period horizon. The two-period return is determined by the first-order condition (3.4) of generation 1, which is investing over two periods. Generation 1 consumes less in state ℓ than in state h under complete participation (Proposition 4.2). This is because productivity and the one-period interest rate are lower in state ℓ, and hence fewer aggregate resources are available in that state. Since two-period bonds dominate one-period bonds in state ℓ, they are a valuable hedge and offer negative expected excess return over two periods. The one-period return is determined by the first-order condition (3.2) of generation 0, which is investing over one period. If γ exceeds the threshold $\bar{\gamma} > 1$, then generation 0 consumes less in state ℓ than in state h under complete participation, and two-period bonds offer negative expected excess return over one period as well.

Proposition 4.6. *Under the optimal maturity structure, the expected excess return of two- relative to one-period bonds in period 1 is negative over a two-period horizon. The corresponding return over a one-period horizon is negative if $\gamma \geq \bar{\gamma}$, but positive if $\gamma \leq 1$.*

It has been well documented that long-term nominal bonds earn positive expected excess returns relative to their short-term counterparts.[11] Our model, however, is real so our results concern real bonds. Ang, Bekaert and Wei (2008) find that long-term real bonds in the US offer no significant excess returns. In Section 6 we provide evidence consistent with their results: we find that the average slope of the real term structure across the UK and the US is slightly negative.

[11] See, for example, Cochrane (1999). Campbell, Sunderam and Viceira (2013) find, however, that the expected excess returns of nominal bonds have turned negative over the past 15 years.

Hence, the result of Proposition 4.6 that long-term bonds earn negative expected excess returns is not at odds with the data.

Proposition 4.6 identifies a difference between welfare maximization and revenue maximization, which is related to the one in Section 4.4.1. Since the expected return of two-period bonds over a two-period horizon is lower than of one-period bonds, a government could raise expected revenue by issuing more long-term bonds. This raises the tax burden of future generations when interest rates are low, which is also when these generations' consumption is low. A welfare-maximizing government internalizes the risk imposed on future generations and limits the supply of long-term bonds below the level where their expected returns are equalized to those of short-term bonds.

5 Extensions

Our analysis so far assumes that the return of the linear production technology is riskless and that the one-period interest rate, to which this return is equal because of no-arbitrage, is stochastic only in one period. In this section we relax both assumptions. We also allow the endowment of each generation to be risky.

5.1 Stochastic Interest Rate in All Periods

We first maintain a riskless return and endowment, but allow the one-period interest rate to be stochastic in all periods $t \geq 2$, i.e., in the steady state when three generations coexist. We denote by R_t the gross one-period interest rate in period $t \geq 2$, and assume that it takes the values $R_{t,h}$ and $R_{t,\ell} < R_{t,h}$ with probabilities p_t and $1 - p_t$, respectively, where $R_{t,\ell}$ exceeds a lower bound $\underline{R} > 1$. For simplicity, we take R_t to be independent across periods. We assume that the interest rate in periods 0 and 1 is equal to R, and set $R_0 = R_1 \equiv R$.

Without loss of generality, we assume that only one-period bonds are available in period 0, while one- and two-period bonds are available in periods $t \geq 1$. We denote by τ_t the tax rate in period t and by b_t the face value of one-period bonds available in the same period. One-period bonds included in b_{t+1} are both those issued by the government in period $t + 1$, and two-period bonds issued in period t that become one-period in period $t + 1$. We denote by B_t the face value of two-period bonds issued by the government in period $t \geq 1$, and by L_t the gross two-period interest rate in the same period. As in Section 2, we set $\tau_0 = \tau_1 = 0$, which imply $b_0 = 0$.

The government's budget constraint in period $t \geq 1$ is

$$\alpha_t \tau_t + \frac{b_t - B_{t-1}}{R_t} + \frac{B_t}{L_t^2} = b_{t-1}. \tag{5.1}$$

Government revenue is the sum of taxes $\alpha_t \tau_t$ on generation t, plus the revenue $(b_t - B_{t-1})/R_t$ from issuing one-period bonds and B_t/L_t^2 from issuing two-period bonds. It is equal to the cost b_{t-1} of repaying the bonds that mature in period $t-1$. We eliminate Ponzi schemes by the government by assuming that debt must be bounded above, i.e.,

$$\frac{b_t}{R_t} + \frac{B_t}{L_t^2} \leq K \tag{5.2}$$

for a bound K and for all t.

As in Section 4, we can link optimal maturity structure to equilibrium with complete participation. Complete participation requires that in any given period $t \geq 1$ all generations $t' \geq t-1$ are available to trade one- and two-period bonds. This includes the unborn generations $t' > t$, but not the old generations $t' < t-1$ that consume in periods up to t and cannot share risk concerning a later period. As in Section 4.2, the government can effectively induce complete participation through a choice of maturity structure and taxes. In particular, the government should issue in period t the aggregate quantity B_t^* of bonds that generations $t-1$ and t buy under complete participation.

Proposition 5.1. *Suppose that only generations $t-1$ and t can trade in period t for all $t \geq 1$. Then, the government can achieve the same outcome as under complete participation by issuing for $t \geq 1$ the aggregate quantity B_t^* of two-period bonds that generations $t-1$ and t buy under complete participation, and levying appropriate state-contingent taxes $\{\tau_t^*\}_{t\geq 2}$ on generations $t \geq 2$.*

A maturity structure policy by the government is characterized by the quantity $\{B_t\}_{t\geq 1}$ of two-period bonds issued in periods $t \geq 1$ and the state-contingent taxes $\{\tau_t\}_{t\geq 2}$ on generations $t \geq 2$. The policy $(\{B_t\}_{t\geq 1}, \{\tau_t\}_{t\geq 2})$ must satisfy the government's budget constraint (5.1) and the bound (5.2). We refer to such a policy as admissible. The definition of optimality generalizes Definition 1 in Section 4.3. Proceeding as in that section, we can show that a maturity structure $\{B_t^*\}_{t\geq 1}$ that induces complete participation is optimal.

Proposition 5.2. *A maturity structure is optimal if and only if it equal to the aggregate quantity $\{B_t^*\}_{t\geq 1}$ of two-period bonds that generations 0 and 1 buy in a complete-participation equilibrium.*

Solving for the equilibrium when the interest rate is stochastic in all periods can only be done numerically. We can, however, derive closed-form solutions when the stochastic variation is small. We define the mean-zero random variable η_t by $\eta_t \equiv R_t - \mathbb{E}(R_t)$, and denote its values (corresponding to the two values of R_t) by $\eta_{t,h}$ and $\eta_{t,\ell} < \eta_{t,h}$. We assume that $\{\eta_{t,h}, \eta_{t,\ell}\}_{t\geq 2}$ are proportional to a parameter $\bar{\eta} > 0$. For small $\bar{\eta}$, we can compute the equilibrium under both complete and incomplete participation in closed-form, and show that the properties derived in Sections 3 and 4.4 continue to hold.

Proposition 5.3. *Suppose that interest-rate risk is small. Then, Propositions 3.2 and 3.3 (supply and clientele effects under a general maturity structure) and Propositions 4.5 and 4.6 (clientele effects and expected excess returns under an optimal maturity structure) hold.*

5.2 Risky Production Technology and Demographic Uncertainty

We next relax the assumptions that the return of the linear production technology and the endowment of each generation are riskless. Since the endowment of a generation can be interpreted as the generation's size, a risky endowment corresponds to demographic uncertainty. We set $\zeta_t \equiv \alpha_t - \mathbb{E}(\alpha_t)$, where α_t is the endowment of generation t and ζ_t is a mean-zero random variable observed in period t. We denote the gross return of the technology in period t by R_t, i.e., one unit invested in the technology in period t yields R_t units in period $t+1$, and use R_t^f to denote the gross one-period interest rate. We assume that $R_t = \mathbb{E}(R_t) + \eta_t + \theta_{t+1}$, where the random variables η_t and θ_{t+1} are mean-zero and observed in periods t and $t+1$, respectively. Variables observed in different periods are independent of each other, but variables observed in the same period can be correlated. The expected return of the technology conditional on information available at the time of investment is $\mathbb{E}(R_t) + \eta_t$. When $\{\zeta_t, \eta_t, \theta_{t+1}\}_{t\geq 1}$ are small, in the sense of Section 5.1, we can solve for the equilibrium under incomplete participation in closed form.

Proposition 5.4. *Suppose that endowment and productivity risk are small. Then, a lengthening of the maturity structure in period t raises the expected excess return of two- relative to one-period bonds, over both a one- and a two-period horizon. It raises the two-period interest rate L_t if*

$$\mathbb{V}ar(\eta_{t+1}) + \frac{\mathbb{E}(R_{t+1})}{\mathbb{E}(R_t)}\mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) > 0, \tag{5.3}$$

and the one-period interest rate R_t^f if

$$\mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) > 0. \tag{5.4}$$

A decrease in the size of the long-horizon clientele in period t has the same effects if $\gamma > 1$, and the opposite effects if $\gamma < 1$.

A lengthening of the maturity structure in period t raises the expected excess return of two- relative to one-period bonds for the same reasons as in Proposition 3.2. The two-period interest rate, however, does not always increase because there can be a countervailing decrease in the one-period rate (an effect not present with a riskless technology, where no-arbitrage pins the one-period rate to the technology's return). The one-period rate decreases when the technology offers unexpectedly low returns during times of high interest rates, i.e., θ_{t+1} is negatively correlated with η_{t+1}. Indeed, two-period bonds then correlate positively with an investment in the technology, and an increase in their supply exposes agents to more risk. This induces a greater preference to invest in riskless one-period bonds than in the technology, causing the one-period interest rate to decrease.

If $\gamma > 1$, then a decrease in the size of the long-horizon clientele in period t has the same effects as in the previous paragraph. The effect on the expected excess return of two- relative to one-period bonds arises for the same reasons as in Proposition 3.3. The one-period interest rate decreases when θ_{t+1} is negatively correlated with η_{t+1} because agents with a longer horizon are then less averse to investing in the technology. A decrease in the relative size of these agents can thus result in a greater aggregate preference to invest in riskless one-period bonds than in the technology.

With a risky production technology or demographic uncertainty, the link between optimal maturity structure and equilibrium with complete participation becomes more complicated. The government can generally achieve less intergenerational risksharing relative to a complete-participation equilibrium in which all generations can invest at all times in the technology and can trade assets that pay off contingent on endowment shocks. This is because the government's risksharing tools are limited, e.g., bonds are non-contingent. The optimal maturity structure can be characterized through complete-participation equilibrium, as in Section 4, when the shock η_{t+1} can only take two values and is perfectly correlated with α_{t+1} and θ_{t+1}. In that case, all uncertainty is spanned by shocks to the one-period interest rate. When, in addition, stochastic shocks are small, the clientele effects derived in Proposition 4.5 continue to hold for the optimal supply of two-period bonds and their expected excess return.

A final extension of our analysis is to assume that assets can be issued not only by the government but also by private firms. Holding constant the asset side of the corporate sector's balance sheet, changes to the maturity structure of corporate debt, and to the mix of corporate liabilities more generally, would not matter in our model because of the Modigliani-Miller theorem. Changes to the mix of government liabilities matter, by contrast, because they are accompanied by changes in taxes on future generations. Thus, only the maturity structure of government debt and not that of corporate debt can be used to improve intergenerational risksharing.

6 Empirical Tests

In this section we test the basic predictions of our model. In the empirically plausible case where the coefficient of relative risk aversion γ is larger than one, our model predicts that an increase in the size of the long-horizon clientele lowers long rates relative to short rates. This is shown in Proposition 3.3 under a general maturity structure and in Proposition 4.5 under the optimal maturity structure. Our model also predicts that an increase in the size of the long-horizon clientele induces a welfare-maximizing government to raise the average maturity of its debt structure, as shown in Proposition 4.5. These results hold in our baseline model and in the extensions studied in Section 5.

Clienteles in our model are defined by demographic characteristics, i.e., young agents are the long-horizon clientele and old agents the short-horizon one. Hence, a natural way to test our basic predictions is to run the following two regressions for a panel of countries:

$$Slope_{it} = a + bDem_{it} + u_i + e_{it},$$

$$Maturity_{it} = c + dDem_{it} + v_i + f_{it},$$

where i indexes countries and t time periods, $Slope_{it}$ is the slope of the yield curve, i.e., the difference in yields between long- and short-term government bonds, Dem_{it} is a demographic variable capturing the relative size of the long- and short-horizon clienteles, $Maturity_{it}$ is a variable capturing the average maturity of government debt, and u_i and v_i are country effects. An appealing feature of these regressions is that the demographic variable can plausibly be assumed exogenous.

6.1 Data Description

Our empirical analysis focuses on OECD countries, for which data on the maturity structure of government debt are available. For a subset of these countries, we also observe yields on government bonds. We sample the data annually because demographic variables are slow-moving.

We start with the sample of 30 countries that were members of the OECD at the beginning of 2010. We drop country-year observations where the country's Standard and Poor's credit rating is below AA-. We do this to abstract from the credit component of bond yields, since our model is not about sovereign default risk. Our filter eliminates the Czech Republic, Greece, Hungary, Korea, Mexico, Poland, the Slovak Republic and Turkey from the sample. It also drops some country-year observations for Iceland (2008-10), Ireland (2010), Italy (2006-10), and Portugal (2009-10).

Our main demographic variable is the median age of the population from the United Nations Population Division for the period 1960-2010. Since in our model the young generation is the long-horizon clientele, an increase in the median age can be interpreted as a decrease in that clientele's relative importance. We also use OECD Health Data for life expectancy at age 40 to control for differences in life expectancy.[12]

[12]The importance of a generation in our model is measured by income, which is the product of population size

Our measure of the maturity structure of government debt comes from two sources. For the period 1980-2010, we use the weighted average maturity of marketable government debt series from the OECD Central Government Debt Statistical Yearbook. In order to obtain the longest possible time series we merge these data with comparable data for the period 1960-1979 from Missale (1999). Data on maturity structure are available for all countries in our reduced sample, except for Switzerland and Portugal. This leaves us with 20 countries.

In our model generations live for three periods, and so a period should be thought of as quite long. If working life starts at around age 25, it would be natural to think of a period as lasting around 15 to 20 years.[13] Our empirical proxy for the long-term bond is the 30-year benchmark government bond, and for the short-term bond is the 10-year benchmark government bond. We thus measure slope using the 30-10 bond yield spread. The specific maturity choices are driven by data availability: 30-year bonds is the longest maturity for which there are relatively consistent data across OECD countries, and the benchmark status of the 10-year bond ensures more and better price data than for the 15-year bond. We use data on nominal government bond yields from Global Financial Data. Out of the sample of all OECD countries, only 10 countries have data on 30-year yields. In the case of the UK, we substitute the 30-year yield with the consol yield which is available for a much longer time period. (Our results do not depend on this choice.)

Table 1 provides summary statistics for the entire sample, and Table 2 breaks them down by countries. In both tables, Panel A refers to the sample of 10 countries used in the regression of slope on median age, and Panel B to the sample of 20 countries used in the regression of maturity on median age. Table 1 shows that there is substantial variation in our variables when we consider the entire sample. Table 2 shows that there is also substantial cross-country variation.

While our model is real, our empirical measure of slope is constructed using nominal yields. This choice is driven by data availability: only two countries in our sample, the UK and the US, have direct measures of long-term real bond yields, and even for these countries the measures are available only for recent years.

As a check for the validity of using nominal slope as a proxy for real slope, we examine the period 1998-2010 for the UK and the US. This is the longest period for which both nominal and real long-term yields are available. For the UK we use nominal and real spot rates published by the Bank of England. The longest available maturity is 25 years, and we calculate nominal and

times income per capita. Our median-age measure is based on population size, and it would be desirable to also account for income per capita. Unfortunately data on how wealth differs across generational cohorts are not available for most OECD countries. Lack of data availability is also the reason why we do not include additional measures of bond demand, such as foreign holdings, and measures of bond supply in our regressions.

[13]In our model only the young generation receives an income (endowment), while the evidence is that income peaks at middle age, around age 50. We assume this income profile to draw a sharp distinction between the role of debt maturity structure and social security in improving intergenerational risksharing. Introducing income for the middle-aged generation would blur this distinction. We expect, however, that the main predictions of our model would carry through provided that there is a cost of raising taxes (e.g., distortions). See Footnote 9.

real slope using the 10- and 25-year maturities. For the US we use the 10-year and 30-year nominal and real bond yields from Global Financial Data.

The correlation coefficient between nominal and real slope is 0.68. Excluding the crisis year 2008, which appears to be an outlier for the UK, the coefficient rises to 0.89. Figure 3 shows the high correlation. A univariate regression of real slope on nominal slope produces a coefficient of 0.53 (t-statistic of 4.45), an intercept of -0.19 (t-statistic of -2.99), and an R-squared of 0.46. Excluding 2008, the respective numbers are 0.60 (8.81), -0.17 (-4.64), and 0.79. The high correlation between nominal and real slope suggests that nominal slope is a good proxy for real slope.[14]

The average real slope across the UK and the US, i.e., the average y-coordinate of the points in Figure 3, is -0.06%. The average nominal slope, i.e., the average x-coordinate, is 0.25%. Inflation risk premia could be the reason why the average nominal slope is positive despite the average real slope being negative.

6.2 Results

Our regression results are in Table 3, Panels A and B. The first column in each panel reports results from a pooled OLS regression; the second column from a between regression, in which the data for each country are averaged over time and regressed across countries; the third column from a fixed-effects regression, derived by including country dummy variables; and the fourth column from a random-effects regression. Standard errors in the OLS, fixed-effects, and random-effects regressions are clustered by country.

Consider first Panel A, in which the slope of the yield curve is regressed on median age. Since an increase in median age corresponds to a decrease in the relative importance of the long-horizon clientele (the young generation), our model implies a positive regression coefficient. The coefficient is indeed positive for all four regressions. It is significant at the 1% level for OLS and random effects, and at the 5% level for the between regression. Figure 4 plots the country means of slope and median age, and confirms visually the statistical significance of the between regression.

The fixed- and random-effects regressions produce coefficients of similar magnitude. This suggests that the country-specific component of the error term is uncorrelated with the independent variable, as assumed in the random-effects model. A Hausman specification test yields a p-value of 42.3%, confirming this impression and supporting random effects.

In addition to being statistically significant, our results are economically significant. Using, for example, the point estimate from the random-effects regression, we find that a one-standard-

[14] A high correlation between nominal and real slope is also consistent with many of the leading inflation forecasting models, e.g., the unobserved components model with stochastic volatility in Stock and Watson (2007), which generate nearly identical inflation forecasts at the 10- and 30-year horizons. Thus, changes in inflation expectations should have a small influence on the nominal 30-10 slope.

deviation increase in median age is associated with an increase in slope of 19 basis points. This corresponds to approximately one-third of the in-sample standard deviation of the slope variable.

Consider next Panel B, in which the average maturity of government debt is regressed on median age. Since an increase in median age corresponds to a decrease in the relative importance of the long-horizon clientele, our model implies a negative regression coefficient. The coefficient is indeed negative in all four regressions. It is significant at the 5% level for OLS, fixed effects and random effects, but insignificant for the between regression (p-value of 24%). Figure 5 plots the country means of maturity and median age, and shows that the lack of statistical significance of the between regression is driven by the UK. The results in Panel B are not affected if we restrict the analysis to the sub-sample of countries used in Panel A.

The fixed- and random effects regressions produce coefficients of similar magnitude. A Hausman specification test yields a p-value of 85%, supporting the random-effects model. Using the point estimate from the random-effects regression, we find that a one-standard-deviation increase in median age is associated with an increase in maturity of 0.59 years. This corresponds to approximately one-fourth of the in-sample standard deviation of the maturity variable. The results are thus economically significant.

One possible concern with using median age as a measure of the relative size of the short-horizon clientele is that differences in median age could partly reflect differences in life expectancy. That is, high life expectancy could cause median age to be high without lowering the relative size of the long-horizon clientele. As a robustness check, we replace median age in our regressions by median age divided by life expectancy at age 40. The coefficients in Table 3 maintain their predicted sign, with somewhat lower statistical significance.

7 Conclusion

We propose a clientele-based model of the yield curve and optimal maturity structure of government debt. Clienteles are generations of agents at different stages of their lifecycle in an overlapping-generations economy. An optimal maturity structure exists in the absence of distortionary taxes and induces efficient intergenerational risksharing. If agents are more risk-averse than log, then an increase in the long-horizon clientele raises the price and optimal supply of long-term bonds—effects that we also confirm empirically in a panel of OECD countries. Moreover, under the optimal maturity structure catering to clienteles is limited and long-term bonds earn negative expected excess returns.

Our model emphasizes a new and empirically relevant determinant of maturity structure: changes in demand in different segments of the yield curve, generated by clienteles. The literature has emphasized that an additional important determinant is the government's desire to manage its balance-sheet risk and smooth tax rates. Combining the two determinants, by introducing distortionary taxes in our model, could yield a richer theory of optimal maturity structure and be an interesting extension of our work. It could also be interesting to develop a calibrated version of our model and give precise quantitative prescriptions on how maturity structure should respond to changes in clientele demand. This might require extending the lives of generations to more than three periods. Finally, while our model concerns demographic clienteles, it could be interesting to also study regulatory clienteles. This would require modeling financial institutions, such as pension funds, and the agency problems that give rise to regulation.

APPENDIX

Proof of Proposition 3.1: We first show that the demand functions $(B_{1,0}, B_{1,1})$ are increasing in L_1. Setting $(\omega_1, y_2, y_{2,h}, y_{2,\ell}) \equiv (R/L_1^2, 1/R_2, 1/R_{2,h}, 1/R_{2,\ell})$ and using (3.1), we can write (3.2) as

$$\mathbb{E}\left\{u'\left[R^2\alpha_0 + (y_2 - \omega_1)B_{1,0}\right](y_2 - \omega_1)\right\} = 0$$

$$\Leftrightarrow p\left[R^2\alpha_0 + (y_{2,h} - \omega_1)B_{1,0}\right]^{-\gamma}(y_{2,h} - \omega_1) + (1-p)\left[R^2\alpha_0 + (y_{2,\ell} - \omega_1)B_{1,0}\right]^{-\gamma}(y_{2,\ell} - \omega_1) = 0, \tag{A.1}$$

where the second step follows because utility is CRRA. Solving (A.1), we find

$$B_{1,0} = R^2\alpha_0 \frac{1 - Y}{Y(y_{2,\ell} - \omega_1) + \omega_1 - y_{2,h}}, \tag{A.2}$$

where

$$Y \equiv \left[\frac{p(\omega_1 - y_{2,h})}{(1-p)(y_{2,\ell} - \omega_1)}\right]^{\frac{1}{\gamma}}.$$

Differentiating (A.2) with respect to ω_1, we find

$$\frac{\partial B_{1,0}}{\partial \omega_1} = R^2\alpha_0 \frac{(y_{2,h} - y_{2,\ell})\frac{\partial Y}{\partial \omega_1} - (1-Y)^2}{\left[Y(y_{2,\ell} - \omega_1) + \omega_1 - y_{2,h}\right]^2}. \tag{A.3}$$

Since $\partial Y/\partial \omega_1 > 0$, (A.3) implies that $\partial B_{1,0}/\partial \omega_1 < 0$. Therefore, $B_{1,0}$ is increasing in L_1. When L_1^2 converges to $RR_{2,\ell}$, and so ω_1 converges to $y_{2,\ell}$, (A.2) implies that $B_{1,0}$ converges to $-\infty$. When L_1^2 converges to $RR_{2,h}$, and so ω_1 converges to $y_{2,h}$, (A.2) implies that $B_{1,0}$ converges to ∞.

Using (3.3) and (3.4) we can similarly compute $B_{1,1}$:

$$B_{1,1} = R\alpha_1 \frac{R_{2,h} - ZR_{2,\ell}}{Z(1 - \omega_1 R_{2,\ell}) + \omega_1 R_{2,h} - 1}, \tag{A.4}$$

where

$$Z \equiv \left[\frac{p(\omega_1 R_{2,h} - 1)}{(1-p)(1 - \omega_1 R_{2,\ell})}\right]^{\frac{1}{\gamma}}.$$

Differentiating (A.4) with respect to ω_1, we find

$$\frac{\partial B_{1,1}}{\partial \omega_1} = R\alpha_1 \frac{(R_{2,\ell} - R_{2,h})\frac{\partial Z}{\partial \omega_1} - (R_{2,h} - ZR_{2,\ell})^2}{\left[Z(1-\omega_1 R_{2,\ell}) + \omega_1 R_{2,h} - 1\right]^2}. \tag{A.5}$$

Since $\partial Z/\partial \omega_1 > 0$, (A.5) implies that $\partial B_{1,1}/\partial \omega_1 < 0$. Therefore, $B_{1,1}$ is increasing in L_1. When L_1^2 converges to $RR_{2,\ell}$, and so $\omega_1 R_{2,\ell}$ converges to one, (A.4) implies that $B_{1,1}$ converges to $-\infty$. When L_1^2 converges to $RR_{2,h}$, and so $\omega_1 R_{2,h}$ converges to one, (A.4) implies that $B_{1,1}$ converges to ∞. Since $B_{1,0} + B_{1,1}$ is increasing in L_1, and takes values from $-\infty$ to ∞, (3.5) has a unique solution L_1. ∎

Proof of Proposition 3.2: An increase in B_1 holding $b_1/R + B_1/L_1^2$ constant has no effect on tax rates. Therefore, (3.1)-(3.4) imply that the only effect on $(B_{1,0}, B_{1,1})$ is through L_1. Since $B_{1,0} + B_{1,1}$ is increasing in L_1, (3.5) implies that L_1 is increasing in B_1. ∎

Proof of Proposition 3.3: We first show that an increase in the size of the long-horizon clientele, i.e., an increase in α_1 holding $R\alpha_0 + \alpha_1$ constant, raises the aggregate demand for two-period bonds $B_{1,0} + B_{1,1}$ if $\gamma > 1$, and lowers it if $\gamma < 1$. Because of CRRA utility, the quantities

$$\phi_{1,0} \equiv \frac{B_{1,0}/L_1^2}{R\alpha_0},$$

$$\phi_{1,1} \equiv \frac{B_{1,1}/L_1^2}{\alpha_1},$$

characterizing the fraction of wealth that generations 0 and 1 invest in two-period bonds in period 1, are independent of α_0 and α_1. Therefore, an increase in α_1 holding $R\alpha_0 + \alpha_1$ constant raises $B_{1,0} + B_{1,1}$ if and only if $\phi_{1,1} > \phi_{1,0}$. To compare $\phi_{1,1}$ and $\phi_{1,0}$, we substitute (3.1) and (3.3) into (4.5), written for $t = 1$. (Eq. (4.5) holds for generations 0 and 1 in Section 3, and for all generations in Section 4.1.) Because of CRRA utility,

$$\frac{1 + \phi_{1,0}(\frac{L_1^2}{RR_{2,h}} - 1)}{1 + \phi_{1,0}(\frac{L_1^2}{RR_{2,\ell}} - 1)} = \left(\frac{R_{2,h}}{R_{2,\ell}}\right)^{1-\frac{1}{\gamma}} \frac{1 + \phi_{1,1}(\frac{L_1^2}{RR_{2,h}} - 1)}{1 + \phi_{1,1}(\frac{L_1^2}{RR_{2,\ell}} - 1)}.$$

Therefore, the quantity

$$\frac{1 + \phi_{1,0}(\frac{L_1^2}{RR_{2,h}} - 1)}{1 + \phi_{1,0}(\frac{L_1^2}{RR_{2,\ell}} - 1)} \frac{1 + \phi_{1,1}(\frac{L_1^2}{RR_{2,\ell}} - 1)}{1 + \phi_{1,1}(\frac{L_1^2}{RR_{2,h}} - 1)} - 1 = \frac{(\phi_{1,1} - \phi_{1,0})\left(\frac{L_1^2}{RR_{2,\ell}} - \frac{L_1^2}{RR_{2,h}}\right)}{\left[1 + \phi_{1,0}(\frac{L_1^2}{RR_{2,\ell}} - 1)\right]\left[1 + \phi_{1,1}(\frac{L_1^2}{RR_{2,h}} - 1)\right]}$$

has the same sign as

$$\left(\frac{R_{2,h}}{R_{2,\ell}}\right)^{1-\frac{1}{\gamma}} - 1.$$

Since $R_{2,h} > R_{2,\ell}$, the sign of $\phi_{1,1} - \phi_{1,0}$ is the same as of $\gamma - 1$. Since $B_{1,0} + B_{1,1}$ is increasing in L_1, (3.5) implies that an increase in α_1 holding $R\alpha_0 + \alpha_1$ constant lowers L_1 if and only if it raises $B_{1,0} + B_{1,1}$, i.e., if and only if $\gamma > 1$. ∎

Proof of Proposition 4.1: Eqs. (3.3), (3.4), (4.1), (4.3) for $t = 2$, and CRRA utility imply that

$$\frac{B_{1,2}}{\alpha_2} = \frac{B_{1,1}}{R\alpha_1}. \tag{A.6}$$

Therefore, $B_{1,2}$ has the properties of $B_{1,1}$ shown in the proof of Proposition 3.1: it is increasing in L_1, converges to $-\infty$ when L_1^2 converges to $RR_{2,\ell}$, and to ∞ when L_1^2 converges to $RR_{2,h}$. Using (4.2) and (4.3), and proceeding as in the proof of Proposition 3.1, we can compute $B_{1,t}$ for $t > 2$:

$$B_{1,t} = R^{-(t-3)}\alpha_t \frac{1 - Z}{Z(1 - \omega_1 R_{2,\ell}) + \omega_1 R_{2,h} - 1}, \tag{A.7}$$

where Z is defined in the proof of Proposition 3.1. The demand function $B_{1,t}$ is not always increasing in L_1, but converges to $-\infty$ when L_1^2 converges to $RR_{2,\ell}$, and to ∞ when L_1^2 converges to $RR_{2,h}$. Since $\sum_{t=0}^{\infty} B_t$ takes values from $-\infty$ to ∞, (4.4) has a solution L_1. ∎

Proof of Proposition 4.2: Since utility is CRRA, we can write (4.5) as

$$\frac{c_{2,h}}{c_{2,\ell}} = \left(\frac{R_{2,h}}{R_{2,\ell}}\right)^{-\frac{1}{\gamma}} \frac{c_{t+2,h}}{c_{t+2,\ell}}. \tag{A.8}$$

Eq. (3.1) implies that

$$\frac{c_{2,h}}{c_{2,\ell}} = \frac{R^2\alpha_0 + \left(\frac{1}{R_{2,h}} - \frac{R}{L_1^2}\right) B_{1,0}}{R^2\alpha_0 + \left(\frac{1}{R_{2,\ell}} - \frac{R}{L_1^2}\right) B_{1,0}}. \tag{A.9}$$

Moreover,

$$\frac{c_{t+2,h}}{c_{t+2,\ell}} = \frac{\sum_{t=1}^{\infty} \frac{c_{t+2,h}}{R^{t-3}}}{\sum_{t=1}^{\infty} \frac{c_{t+2,\ell}}{R^{t-3}}}$$

$$= \frac{RR_{2,h}\alpha_1 + R_{2,h}\alpha_2 + \sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-1}} - \left(1 - \frac{RR_{2,h}}{L_1^2}\right) B_{1,0}}{RR_{2,\ell}\alpha_1 + R_{2,\ell}\alpha_2 + \sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-1}} - \left(1 - \frac{RR_{2,\ell}}{L_1^2}\right) B_{1,0}}, \tag{A.10}$$

where the first step follows because $\frac{c_{t+2,h}}{c_{t+2,\ell}}$ is identical for all $t \geq 1$, and the second from (3.3), (4.1), (4.2) and (4.4).

If $B_{1,0} \geq 0$, then (A.10) implies that $\frac{c_{t+2,h}}{c_{t+2,\ell}} > 1$ for $t \geq 1$. If $B_{1,0} < 0$, then (A.9) implies that $\frac{c_{2,h}}{c_{2,\ell}} > 1$, and hence (A.8) implies that $\frac{c_{t+2,h}}{c_{t+2,\ell}} > 1$ for $t \geq 1$.

Since $\frac{c_{t+2,h}}{c_{t+2,\ell}} > 1$ for $t \geq 1$, (A.8) implies that $\frac{c_{2,h}}{c_{2,\ell}} > 1$ for γ exceeding a threshold $\bar{\gamma} > 1$. Suppose next that $\gamma \leq 1$. If $B_{1,0} > 0$, then (A.9) implies that $\frac{c_{2,h}}{c_{2,\ell}} < 1$. If $B_{1,0} \leq 0$, then we can use (A.10) to write the right-hand side of (A.8) as

$$\left(\frac{R_{2,h}}{R_{2,\ell}}\right)^{1-\frac{1}{\gamma}} \frac{\frac{c_{t+2,h}}{R_{2,h}}}{\frac{c_{t+2,\ell}}{R_{2,\ell}}} = \left(\frac{R_{2,h}}{R_{2,\ell}}\right)^{1-\frac{1}{\gamma}} \frac{R\alpha_1 + \alpha_2 + \frac{1}{R_{2,h}} \sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-1}} - \left(\frac{1}{R_{2,h}} - \frac{R}{L_1^2}\right) B_{1,0}}{R\alpha_1 + \alpha_2 + \frac{1}{R_{2,\ell}} \sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-1}} - \left(\frac{1}{R_{2,\ell}} - \frac{R}{L_1^2}\right) B_{1,0}}.$$

Since this is smaller than one, (A.8) implies that $\frac{c_{2,h}}{c_{2,\ell}} < 1$. ∎

Proof of Corollary 4.1: Eqs. (4.2) and $c_{t+2,h} > c_{t+2,\ell}$ imply that $B_{1,t} < 0$ for $t > 2$. If $\gamma \geq \bar{\gamma}$, then (3.1) and $c_{2,h} > c_{2,\ell}$ imply that $B_{1,0} < 0$. If $\gamma \leq 1$, then (3.1) and $c_{2,h} < c_{2,\ell}$ imply that $B_{1,0} > 0$. Suppose finally that $\gamma \geq 1$ and one of $(B_{1,1}, B_{1,2})$ is non-positive. Eq. (A.6) implies that both of $(B_{1,1}, B_{1,2})$ are non-positive. Since $\gamma \geq 1$, generation 0 invests a smaller fraction of its wealth in two-period bonds than generation 1 (proof of Proposition 3.3), and hence $B_{1,0} \leq 0$. The non-positivity of $(B_{1,0}, B_{1,1}, B_{1,2})$, together with the negativity of $B_{1,t}$ for all $t > 2$ is inconsistent with (4.4), a contradiction. Therefore, both of $(B_{1,1}, B_{1,2})$ are positive. ∎

Proof of Proposition 4.3: Suppose that the government issues a quantity B_1^* of two-period bonds. Since the demand functions of generations 0 and 1 are the same as under complete participation, the two-period interest rate is also the same. As a consequence, each of generations 0 and 1 has the same investment in two-period bonds and the same consumption as under complete participation. Generations $t \geq 2$ can also have the same consumption as under complete participation through an appropriate choice of state-contingent taxes that we denote by $\{\tau_t^*\}_{t\geq 2}$. Since the budget constraint of generation 2 under incomplete participation is

$$c_4 = RR_2\alpha_2(1-\tau_2), \tag{A.11}$$

(4.1) implies that the taxes that leave that generation with the same consumption as under complete participation are given by

$$\begin{aligned} RR_2\alpha_2 + R\left(L_1^2 - RR_2\right)\frac{B_{1,2}}{L_1^2} &= RR_2\alpha_2(1-\tau_2^*) \\ \Leftrightarrow \alpha_2\tau_2^* &= \left(\frac{R}{L_1^2} - \frac{1}{R_2}\right)B_{1,2}. \end{aligned} \tag{A.12}$$

Likewise, since the budget constraint of generation $t > 2$ under incomplete participation is

$$c_{t+2} = R^2\alpha_t(1-\tau_t), \tag{A.13}$$

(4.2) implies that the taxes that leave that generation with the same consumption as under complete participation are given by

$$\begin{aligned} R^2\alpha_t + R^{t-1}\left(L_1^2 - RR_2\right)\frac{B_{1,t}}{L_1^2} &= R^2\alpha_t(1-\tau_t^*) \\ \Leftrightarrow \alpha_t\tau_t^* &= R^{t-3}R_2\left(\frac{R}{L_1^2} - \frac{1}{R_2}\right)B_{1,t}. \end{aligned} \tag{A.14}$$

To determine whether the policy $(B_1^*, \{\tau_t^*\}_{t\geq 2})$ satisfies (2.5), we compute b_t using the government's budget constraints (2.2)-(2.4) and the initial condition $b_0 = 0$. For $t \geq 2$,

$$b_t = R^{t-2}\left(1 - \frac{RR_2}{L_1^2}\right)B_1^* - R^{t-2}R_2\alpha_2\tau_2^* - \sum_{t'=3}^{t} R^{t+1-t'}\alpha_{t'}\tau_{t'}^*. \tag{A.15}$$

Using $\sum_{t'=2}^{\infty} B_{1,t'} = -B_1^*$, we can write (A.15) as

$$\begin{aligned} b_t &= -R^{t-2}\left(1 - \frac{RR_2}{L_1^2}\right)\sum_{t'=2}^{\infty} B_{1,t'} - R^{t-2}R_2\alpha_2\tau_2^* - \sum_{t'=3}^{t} R^{t+1-t'}\alpha_{t'}\tau_{t'}^* \\ &= -R^{t-2}\left(1 - \frac{RR_2}{L_1^2}\right)\sum_{t'=t+1}^{\infty} B_{1,t'} \\ &= -\left(1 - \frac{RR_2}{L_1^2}\right)\frac{1-Z}{Z(1-\omega_1 R_{2,\ell}) + \omega_1 R_{2,h} - 1}\sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{R^{t'-t-1}}, \end{aligned} \tag{A.16}$$

where the second step follows from (A.12) and (A.14), and the third from (A.7). Eq. (A.16) implies that b_t is bounded. ∎

Proof of Proposition 4.4: We first derive an intertemporal budget constraint for the government and a resource constraint. Writing (A.15) for an admissible policy $(B_1, \{\tau_t\}_{t\geq 2})$ and dividing by $R^{t-2}R_2$, we find

$$\frac{b_t}{R^{t-2}R_2} = \left(\frac{1}{R_2} - \frac{R}{L_1^2}\right) B_1 - \alpha_2\tau_2 - \frac{1}{R_2}\sum_{t'=3}^{t} \frac{\alpha_{t'}\tau_{t'}}{R^{t'-3}}. \tag{A.17}$$

Summing (3.1), (3.3) divided by R_2, (A.11) divided by RR_2, and (A.13) divided by $R^{t+1}R_2$ for all $t' = 3, .., t$, we find

$$c_2 + \frac{1}{R_2}\sum_{t'=1}^{t} \frac{c_{t'+2}}{R^{t'-1}} = R^2\alpha_0 + R\alpha_1 + \alpha_2(1-\tau_2) + \frac{1}{R_2}\sum_{t'=3}^{t} \frac{\alpha_{t'}(1-\tau_{t'})}{R^{t'-3}} + \left(\frac{1}{R_2} - \frac{R}{L_1^2}\right)(B_{1,0} + B_{1,1}). \tag{A.18}$$

Combining (A.17) and (A.18), and using (3.5), we find

$$c_2 + \frac{1}{R_2}\sum_{t'=1}^{t} \frac{c_{t'+2}}{R^{t'-1}} = R^2\alpha_0 + R\alpha_1 + \alpha_2 + \frac{1}{R_2}\sum_{t=3}^{t} \frac{\alpha_{t'}}{R^{t'-3}} + \frac{b_t}{R^{t-2}R_2}. \tag{A.19}$$

Taking the limit when $t \to \infty$, and noting that the limit of the last term in the right-hand side is non-positive because of (2.5), yields the resource constraint

$$c_2 + \frac{1}{R_2}\sum_{t=1}^{\infty} \frac{c_{t+2}}{R^{t-1}} \leq R^2\alpha_0 + R\alpha_1 + \alpha_2 + \frac{1}{R_2}\sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-3}}. \tag{A.20}$$

Taking the limit when $t \to \infty$ in (A.17) yields likewise the government's intertemporal budget constraint

$$\alpha_2\tau_2 + \frac{1}{R_2}\sum_{t=3}^{\infty} \frac{\alpha_t\tau_t}{R^{t-3}} - \left(\frac{1}{R_2} - \frac{R}{L_1^2}\right) B_1 \geq 0. \tag{A.21}$$

We next show that B_1^* is an optimal maturity structure. We proceed by contradiction, and suppose that there exists an admissible policy $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$ to which a switch from $(B_1^*, \{\tau_t^*\}_{t\geq 2})$ generates a positive sum $\sum_{t=0}^{\infty} T_t$ of gains. Denote by $\hat{c}_{t+2}$ and c_{t+2}^* the consumption of generation

t in the corresponding equilibria. The allocation $\{\check{c}_{t+2}\}_{t\geq 0}$ defined by

$$\check{c}_2 \equiv \hat{c}_2 + R\sum_{t=1}^{\infty} T_t, \tag{A.22}$$

$$\check{c}_{t+2} \equiv \hat{c}_{t+2} - R^t R_2 T_t \quad \text{for} \quad t \geq 1, \tag{A.23}$$

yields the same utility as $\{c^*_{t+2}\}_{t\geq 0}$ for generations $t \geq 1$ because of (4.7), and higher utility for generation 0 because of (4.6) and $\sum_{t=0}^{\infty} T_t > 0$. This is a contradiction because $\{c^*_{t+2}\}_{t\geq 0}$ is Pareto optimal among all feasible allocations. To show Pareto optimality and the contradiction, we denote the risk-neutral probabilities for states $s = h, \ell$ under complete participation by

$$\pi_{1,h} \equiv \frac{\frac{L_1^2}{R_{2,\ell}} - R}{\frac{L_1^2}{R_{2,\ell}} - \frac{L_1^2}{R_{2,h}}},$$

$$\pi_{1,\ell} \equiv 1 - \pi_{1,h},$$

respectively. We also use (3.1), (3.3), (4.1) and (4.2) to write the optimization problem of generation $t \geq 0$ under complete participation as maximizing utility within the budget set $\mathcal{B}_t$, where

$$\mathcal{B}_0 \equiv \left\{c_2 : \pi_{1,h} c_{2,h} + \pi_{1,\ell} c_{2,\ell} \leq R^2 \alpha_0\right\},$$

$$\mathcal{B}_1 \equiv \left\{c_3 : \pi_{1,h} \frac{c_{3,h}}{R_{2,h}} + \pi_{1,\ell} \frac{c_{3,\ell}}{R_{2,\ell}} \leq R\alpha_1\right\},$$

$$\mathcal{B}_2 \equiv \left\{c_4 : \pi_{1,h} \frac{c_{4,h}}{RR_{2,h}} + \pi_{1,\ell} \frac{c_{4,\ell}}{RR_{2,\ell}} \leq \alpha_2\right\},$$

$$\mathcal{B}_t \equiv \left\{c_{t+2} : \pi_{1,h} \frac{c_{t+2,h}}{R^2 R_{2,h}} + \pi_{1,\ell} \frac{c_{t+2,\ell}}{R^2 R_{2,\ell}} \leq \alpha_t \left(\frac{\pi_{1,h}}{R_{2,h}} + \frac{\pi_{1,\ell}}{R_{2,\ell}}\right)\right\} \quad \text{for} \quad t > 2.$$

Since c^*_2 maximizes the utility of generation 0 within $\mathcal{B}_0$, $\check{c}_2$, which yields higher utility, is outside $\mathcal{B}_0$. Likewise, since c^*_{t+2} maximizes the expected utility of generation $t \geq 1$ within $\mathcal{B}_t$, $\check{c}_{t+2}$, which yields the same utility, is weakly outside $\mathcal{B}_t$. Combining the corresponding inequalities, we find

$$\sum_{s=h,\ell} \pi_{1,s} \left(\check{c}_{2,s} + \frac{1}{R_{2,s}} \sum_{t=1}^{\infty} \frac{\check{c}_{t+2,s}}{R^{t-1}}\right) > \sum_{s=h,\ell} \pi_{1,s} \left(R^2\alpha_0 + R\alpha_1 + \alpha_2 + \frac{1}{R_{2,s}} \sum_{t=3}^{\infty} \frac{\alpha_t}{R^{t-3}}\right). \tag{A.24}$$

Eq. (A.24) contradicts the resource constraint (A.20). Indeed, (A.20) holds for $\{\hat{c}_{t+2}\}_{t\geq 0}$, since this allocation corresponds to the admissible policy $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$. Moreover, because of (A.22) and

(A.23), (A.20) holds also for $\{\check{c}_{t+2}\}_{t\geq 0}$.

We finally show that an optimal maturity structure must equal B_1^*. Consider an optimal maturity structure B_1 and an associated admissible policy $(B_1, \{\tau_t\}_{t\geq 2})$. The ratio across states of marginal utilities must be equal across all generations $t \geq 2$; otherwise, it would be possible to generate a positive sum of gains by reallocating taxes across two generations so that one is kept equally well off and the other becomes better off. Moreover, the government's intertemporal budget constraint (A.21) holds with equality; otherwise the government would be accumulating a growing surplus that it could instead distribute to the agents and raise their utility, contradicting the optimality of B_1. Consider next an infinitesimal change $(dB_1, \{d\tau_t\}_{t\geq 2})$ that preserves admissibility and the equality in (A.21). Eqs. (4.6) and (4.7) imply that

$$dT_0 = \frac{\mathbb{E}\left[u'(c_2)dc_2\right]}{R\mathbb{E}u'(c_2)}, \tag{A.25}$$

$$dT_t = \frac{\mathbb{E}\left[u'(c_{t+2})dc_{t+2}\right]}{R^t\mathbb{E}[u'(c_{t+2})R_2]} \quad \text{for} \quad t \geq 1. \tag{A.26}$$

Using (3.1) and (A.25), and recalling the definition $\omega_1 \equiv R/L_1^2$, we find

$$dT_0 = \frac{\mathbb{E}\left\{u'(c_2)\left[\left(\frac{1}{R_2} - \omega_1\right) dB_{1,0} - d\omega_1 B_{1,0}\right]\right\}}{R\mathbb{E}u'(c_2)} = -\frac{d\omega_1 B_{1,0}}{R}, \tag{A.27}$$

where the second step follows from (3.2). Using (3.3) and (A.26) for $t = 1$, we find

$$dT_1 = \frac{\mathbb{E}\left\{u'(c_3)\left[(1 - \omega_1 R_2)\, dB_{1,1} - d\omega_1 R_2 B_{1,1}\right]\right\}}{R\mathbb{E}[u'(c_3)R_2]} = -\frac{d\omega_1 B_{1,1}}{R}, \tag{A.28}$$

where the second step follows from (3.4). Using (A.11) and (A.26) for $t = 2$, we find

$$dT_2 = -\frac{\mathbb{E}\left[u'(c_4)\alpha_2 R_2 d\tau_2\right]}{R\mathbb{E}[u'(c_4)R_2]}, \tag{A.29}$$

and using (A.13) and (A.26) for $t > j$, we find

$$dT_t = -\frac{\mathbb{E}\left[u'(c_{t+2})\alpha_t d\tau_t\right]}{R^{t-2}\mathbb{E}[u'(c_{t+2})R_2]}. \tag{A.30}$$

Since B_1 is an optimal maturity structure, the sum $\sum_{t=0}^{\infty} dT_t$ should be zero; if it were negative, the opposite infinitesimal change would make it positive. Therefore, (A.27)-(A.30) imply that

$$
\begin{aligned}
& -\frac{d\omega_1(B_{1,0}+B_{1,1})}{R} - \frac{\mathbb{E}\left[u'(c_4)\alpha_2 R_2 d\tau_2\right]}{R\mathbb{E}[u'(c_4)R_2]} - \sum_{t=3}^{\infty} \frac{\mathbb{E}\left[u'(c_{t+2})\alpha_t d\tau_t\right]}{R^{t-2}\mathbb{E}[u'(c_{t+2})R_2]} = 0 \\
\Leftrightarrow & -\frac{d\omega_1 B_1}{R} - \frac{\mathbb{E}\left[u'(c_{t+2})R_2\left[\alpha_2 d\tau_2 + \frac{1}{R_2}\sum_{t=3}^{\infty}\frac{\alpha_t d\tau_t}{R^{t-3}}\right]\right]}{R\mathbb{E}[u'(c_{t+2})R_2]} = 0,
\end{aligned}
\tag{A.31}
$$

for all $t \geq 2$, where the second step follows from (3.5) and because the ratio across states of marginal utilities is equal across all generations $t \geq 2$. Differentiating (A.21), we find

$$
\alpha_2 d\tau_2 + \frac{1}{R_2}\sum_{t=3}^{\infty}\frac{\alpha_t d\tau_t}{R^{t-3}} = \left(\frac{1}{R_2} - \omega_1\right) dB_1 - \omega_1 dB_1. \tag{A.32}
$$

Substituting (A.32) into (A.31), we find

$$
\mathbb{E}\left[u'(c_{t+2})(1-\omega_1 R_2)\right] = 0 \tag{A.33}
$$

for all $t \geq 2$. Eq. (A.33) can be interpreted as a first-order condition for generations $t \geq 2$ under complete participation. To make this interpretation precise, we consider a representative agent who has CRRA utility with coefficient of relative risk aversion γ, and consumes in period 4 the present value of the consumption stream of generations $t \geq 2$:

$$
C = \sum_{t\geq 2}\frac{c_{t+2}}{R^{t-2}} = RR_2\left(\alpha_2 + \frac{1}{R_2}\sum_{t=3}^{\infty}\frac{\alpha_t}{R^{t-3}}\right) - R(1-\omega_1 R_2)B_1, \tag{A.34}
$$

where the second step in (A.34) follows from (A.11), (A.13) and (A.21). Because of CRRA utility, the common ratio across states of marginal utilities of generations $t \geq 2$ is equal to the ratio of marginal utilities of the representative agent. Therefore, the first-order condition (A.33) holds for the representative agent as

$$
\mathbb{E}\left[u'(C)(1-\omega_1 R_2)\right] = 0,
$$

and implies that if the representative agent were to invest in one- and two-period bonds in period 1 subject to the budget constraint

$$
C = RR_2\left(\alpha_2 + \frac{1}{R_2}\sum_{t=3}^{\infty}\frac{\alpha_t}{R^{t-3}}\right) + R(1-\omega_1 R_2)\hat{B}_1, \tag{A.35}
$$

then he would choose a quantity $\hat{B}_1 = -B_1$ of two-period bonds. As a consequence, the two-period interest rate L_1 clears the market with complete participation by all generations, i.e., in the equilibrium of Section 4.1. Indeed, (4.3) implies (A.33), and (4.1) and (4.2) imply that (A.34) holds with $\sum_{t=2}^{\infty} B_{1,t}$ instead of $-B_1$. Since the representative agent's demand for two-period bonds is uniquely determined, $\sum_{t=2}^{\infty} B_{1,t} = -B_1 \equiv -(B_{1,0} + B_{1,1})$. Therefore, (4.4) is met, and $B_1 = B_1^*$. ∎

Proof of Proposition 4.5: We consider a change from α_1 to $\alpha_1 + \epsilon$ holding $R\alpha_0 + \alpha_1$ constant. Differentiating (4.4) with respect to ϵ and treating L_1 as a function of ϵ, we find

$$\frac{\partial}{\partial \epsilon}(B_{1,0} + B_{1,1}) + \frac{dL_1}{d\epsilon}\frac{\partial}{\partial L_1}\sum_{t=0}^{\infty} B_{1,t} = 0$$

$$\Rightarrow \frac{dL_1}{d\epsilon} = -\frac{\frac{\partial}{\partial \epsilon}(B_{1,0} + B_{1,1})}{\frac{\partial}{\partial L_1}\sum_{t=0}^{\infty} B_{1,t}}, \tag{A.36}$$

where the first step follows because ϵ does not affect $B_{1,t}$ for $t \geq 2$. The numerator in (A.36) has the same sign as $\gamma - 1$, as shown in the proof of Proposition 3.3. The denominator is positive because the demand functions $B_{1,t}$ for $t \geq 0$ are increasing in L_1: for $t \in \{0, 1\}$ this is shown in the proof of Proposition 3.1, for $t = 2$ it is shown in the proof of Proposition 4.1, and for $t > 2$ it follows from Assumption 1. Therefore, $\frac{dL_1}{d\epsilon}$ has the same sign as $1 - \gamma$.

Differentiating $B_{1,0} + B_{1,1}$ with respect to ϵ and treating L_1 as a function of ϵ, we find

$$\begin{aligned}\frac{d}{d\epsilon}(B_{1,0} + B_{1,1}) &= \frac{\partial}{\partial \epsilon}(B_{1,0} + B_{1,1}) + \frac{dL_1}{d\epsilon}\frac{\partial}{\partial L_1}(B_{1,0} + B_{1,1}) \\ &= \frac{\partial}{\partial \epsilon}(B_{1,0} + B_{1,1})\frac{\frac{\partial}{\partial L_1}\sum_{t=2}^{\infty} B_{1,t}}{\frac{\partial}{\partial L_1}\sum_{t=0}^{\infty} B_{1,t}},\end{aligned} \tag{A.37}$$

where the second step follows from (A.36). The numerator and denominator in (A.37) are positive because the demand functions B_t for $t \geq 0$ are increasing in L_1. Therefore, $\frac{d}{d\epsilon}(B_{1,0} + B_{1,1})$ has the same sign as $\frac{\partial}{\partial \epsilon}(B_{1,0} + B_{1,1})$ and hence as $\gamma - 1$. The comparative statics in the proposition follow from $B_1^* = B_{1,0} + B_{1,1}$, and because the equilibrium two-period interest rate L_1 under incomplete participation and B_1^* coincides with that under complete participation. ∎

Proof of Proposition 4.6: Eqs. (3.4) and $c_{3,h} > c_{3,\ell}$, which follows from Proposition 4.2 for the complete-participation equilibrium and from Propositions 4.3 and 4.4 for the optimal maturity structure, imply that $L_1^2 < R\mathbb{E}(R_2)$. If $\gamma \geq \bar{\gamma}$, then (3.2) and $c_{2,h} > c_{2,\ell}$ imply that $\mathbb{E}\left(\frac{L_1^2}{R_2}\right) < R$. If $\gamma \leq 1$, then (3.2) and $c_{2,h} < c_{2,\ell}$ imply that $\mathbb{E}\left(\frac{L_1^2}{R_2}\right) > R$. ∎

Proof of Proposition 5.1: To determine the consumption of generation t in period $t+2$ under complete participation, we denote by $w_{t',t}$ generation t's wealth in period $t' = 0, .., t+2$ and by $B_{t',t}$ the face value of two-period bonds held by generation t in period $t' = 1, .., t+1$. The dynamics of $w_{t',t}$ are

$$w_{t',t} = \alpha_t 1_{\{t'=t\}} + R_{t'-1} w_{t'-1,t} + \left(\frac{L_{t'-1}^2}{R_{t'}} - R_{t'-1} \right) \frac{B_{t'-1,t}}{L_{t'-1}^2}, \tag{A.38}$$

where $t' = 0, .., t+2$, $w_{-1,t} = B_{-1,t} = B_{0,t} \equiv 0$, and $1_{\mathcal{S}}$ is equal to one if condition $\mathcal{S}$ is satisfied and zero otherwise. Solving (A.38) between 0 and $t+2$, we find

$$c_{t+2} = w_{t+2,t} = R_t R_{t+1} \alpha_t + \sum_{t'=1}^{t} \left(\prod_{t''=t'+2}^{t+1} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',t} + \left(\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} \right) B_{t+1,t}. \tag{A.39}$$

The first-order condition of generation t with respect to $B_{t',t}$ is

$$\mathbb{E}_{t'} \left[u'(c_{t+2}) \left(\prod_{t''=t'+2}^{t+1} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) \right] = 0 \tag{A.40}$$

for $t' = 1, .., t$, and

$$\mathbb{E}_{t+1} \left[u'(c_{t+2}) \left(\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} \right) \right] = 0 \tag{A.41}$$

for $t' = t+1$, where $\mathbb{E}_{t'}$ denotes expectation conditional on information available in period t'. A complete-participation equilibrium consists of two-period interest rates $\{L_t\}_{t \geq 1}$ and holdings $\{B_{t',t}\}_{t'=1,..,t+1}$ of two-period bonds for each generation $t \geq 0$ such that the market for two-period bonds clears in each period, i.e.,

$$\sum_{t=t'-1}^{\infty} B_{t',t} = 0 \tag{A.42}$$

for all $t' \geq 1$.

Suppose next that participation is incomplete, i.e., only generations $t-1$ and t can trade in period t for all $t \geq 1$. Suppose that the government issues a quantity B_t^* of two-period bonds in period $t \geq 1$ and levies a tax

$$\alpha_t \tau_t^* = \alpha_t - w_{t,t} \tag{A.43}$$

on generation $t \geq 2$. The tax (A.43) leaves generation t with wealth

$$\alpha_t - \alpha_t \tau_t^* = w_{t,t},$$

in period t, which is the same as under complete participation. Hence, the demand function of generation t for two-period bonds is also the same. Since the supply of two-period bonds in period t is the same as the quantity that generations $t-1$ and t buy under complete participation, the two-period interest rate is also the same. As a consequence, each generation has the same investment in two-period bonds and the same consumption as under complete participation.

To determine whether the policy $(\{B_t^*\}_{t\geq 1}, \{\tau_t^*\}_{t\geq 2})$ satisfies (5.2), we compute b_t using the government's budget constraint (5.1) and the initial condition $b_0 = 0$. For $t \geq 2$,

$$b_t + \frac{R_t B_t^*}{L_t^2} = \sum_{t'=1}^{t-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t'}^* - \sum_{t'=2}^{t} \left(\prod_{t''=t'}^{t} R_{t''} \right) \alpha_{t'} \tau_{t'}^*. \tag{A.44}$$

Using $\sum_{u=t'+1}^{\infty} B_{t',u} = -B_{t'}^*$, we can write (A.15) as

$$\begin{aligned}
b_t + \frac{R_t B_t^*}{L_t^2} &= -\sum_{t'=1}^{t-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) \sum_{u=t'+1}^{\infty} B_{t',u} - \sum_{t'=2}^{t} \left(\prod_{t''=t'}^{t} R_{t''} \right) \alpha_{t'} \tau_{t'}^* \\
&= -\sum_{u=2}^{\infty} \sum_{t'=1}^{\min\{t-1,u-1\}} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} - \sum_{t'=2}^{t} \left(\prod_{t''=t'}^{t} R_{t''} \right) \alpha_{t'} \tau_{t'}^* \\
&= -\sum_{u=2}^{t} \sum_{t'=1}^{u-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} - \sum_{t'=2}^{t} \left(\prod_{t''=t'}^{t} R_{t''} \right) \alpha_{t'} \tau_{t'}^* \\
&\quad - \sum_{u=t+1}^{\infty} \sum_{t'=1}^{t-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u}.
\end{aligned} \tag{A.45}$$

Replacing t in (A.38) by u, and solving between 0 and u, we find

$$
\begin{aligned}
w_{u,u} &= \alpha_u + \sum_{t'=1}^{u-2} \left(\prod_{t''=t'+2}^{u-1} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} + \left(\frac{1}{R_u} - \frac{R_{u-1}}{L_{u-1}^2} \right) B_{u-1,u} \\
&= \alpha_u + \frac{1}{R_u} \sum_{t'=1}^{u-1} \left(\prod_{t''=t'+2}^{u} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u}. \qquad \text{(A.46)}
\end{aligned}
$$

Eqs. (A.43) and (A.46) imply that

$$
\alpha_u \tau_u^* = -\frac{1}{R_u} \sum_{t'=1}^{u-1} \left(\prod_{t''=t'+2}^{u} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u}. \qquad \text{(A.47)}
$$

Eq. (A.47) implies that for $u = 2, .., t$,

$$
\begin{aligned}
-\sum_{t'=1}^{u-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} &= -\sum_{t'=1}^{u-1} \left(\prod_{t''=t'+2}^{u} R_{t''} \prod_{t''=u+1}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} \\
&= \left(\prod_{t''=u}^{t} R_{t''} \right) \alpha_u \tau_u^*. \qquad \text{(A.48)}
\end{aligned}
$$

Eq. (A.47) also implies that for $u > t$,

$$
\begin{aligned}
\mathbb{E}_t^Q \left[\frac{\alpha_u \tau_u^*}{\prod_{t''=t+1}^{u-1} R_{t''}} \right] &= -\sum_{t'=1}^{t-1} \left(\prod_{t''=t'+2}^{t} R_{t''} \right) \left(1 - \frac{R_{t'} R_{t'+1}}{L_{t'}^2} \right) B_{t',u} \\
&\quad - \mathbb{E}_t^Q \left[\frac{1}{\prod_{t''=t+1}^{t'} R_{t''}} \sum_{t'=t}^{u-1} \left(\frac{1}{R_{t'+1}} - \frac{R_{t'}}{L_{t'}^2} \right) B_{t',u} \right], \qquad \text{(A.49)}
\end{aligned}
$$

where $\mathbb{E}^Q$ denotes expectation under the risk-neutral probabilities under complete participation. (As in the proof of Proposition 4.3, the risk-neutral probabilities conditional on information available in period t for states $s = h, \ell$ in period $t+1$ are

$$
\pi_{t,h} \equiv \frac{\frac{L_t^2}{R_{t+1,\ell}} - R_t}{\frac{L_t^2}{R_{t+1,\ell}} - \frac{L_t^2}{R_{t+1,h}}},
$$

$$
\pi_{t,\ell} \equiv 1 - \pi_{t,h},
$$

respectively.) Noting that

$$\mathbb{E}_t^Q\left[\frac{1}{\prod_{t''=t+1}^{t'} R_{t''}}\sum_{t'=t}^{u-1}\left(\frac{1}{R_{t'+1}}-\frac{R_{t'}}{L_{t'}^2}\right)B_{t',u}\right]=\mathbb{E}_t^Q\left[\frac{1}{\prod_{t''=t+1}^{t'} R_{t''}}\sum_{t'=t}^{u-1}\left[\mathbb{E}_{t'}^Q\left(\frac{1}{R_{t'+1}}-\frac{R_{t'}}{L_{t'}^2}\right)\right]B_{t',u}\right]=0$$

and using (A.48) and (A.49), we can write (A.45) as

$$b_t+\frac{R_tB_t^*}{L_t^2}=\sum_{u=t+1}^{\infty}\mathbb{E}_t^Q\left[\frac{\alpha_u\tau_u^*}{\prod_{t''=t+1}^{u-1}R_{t''}}\right]. \tag{A.50}$$

Because of CRRA utility, consumption must be non-negative, and hence $w_{t,t}$ is also non-negative. Therefore, (A.43) implies that $\alpha_t\tau_t^*\leq\alpha_t$ and (A.50) implies that

$$b_t+\frac{R_tB_t^*}{L_t^2}\leq\sum_{u=t+1}^{\infty}\mathbb{E}_t^Q\left[\frac{\alpha_u}{\prod_{t''=t+1}^{u-1}R_{t''}}\right]. \tag{A.51}$$

The right-hand side of (A.51) is bounded because α_t is bounded and R_t exceeds a lower bound larger than one. Therefore, (5.2) holds. ∎

Proof of Proposition 5.2: We first derive a resource constraint analogous to (A.20). Writing (A.44) for an admissible policy $(\{B_t\}_{t\geq1},\{\tau_t\}_{t\geq2})$ and dividing by $\prod_{t''=0}^{t}R_{t''}$, we find

$$\frac{b_t+\frac{R_tB_t}{L_t^2}}{\prod_{t''=0}^{t}R_{t''}}=\sum_{t'=1}^{t-1}\frac{1}{\prod_{t''=0}^{t'+1}R_{t''}}\left(1-\frac{R_{t'}R_{t'+1}}{L_{t'}^2}\right)B_{t'}-\sum_{t'=2}^{t}\frac{\alpha_{t'}\tau_{t'}}{\prod_{t''=0}^{t'-1}R_{t''}}. \tag{A.52}$$

The consumption of generation t in the equilibrium with incomplete participation is

$$c_{t+2}=R_tR_{t+1}\alpha_t(1-\tau_t)+\left(1-\frac{R_tR_{t+1}}{L_t^2}\right)B_{t,t}+\left(\frac{1}{R_{t+2}}-\frac{R_{t+1}}{L_{t+1}^2}\right)B_{t+1,t}. \tag{A.53}$$

(Eq. (A.53) can be derived from (A.39) by setting $B_{t',t}=0$ for all $t'<t$ and replacing α_t by $\alpha_t(1-\tau_t)$.) Dividing (A.53) by $\prod_{t''=0}^{t+1}R_{t''}$, summing over $t'=0,..,t$, and using $B_t=B_{t,t-1}+B_{t,t}$

for $t \geq 1$ (market clearing for two-period bonds) and $B_{0,0} = 0$, we find

$$\sum_{t'=0}^{t} \frac{c_{t'+2}}{\prod_{t''=0}^{t'+1} R_{t''}} = \sum_{t'=0}^{t} \frac{\alpha_{t'}(1-\tau_{t'})}{\prod_{t''=0}^{t'-1} R_{t''}} + \sum_{t'=1}^{t} \left(\frac{1}{\prod_{t''=0}^{t'} R_{t''}} \right) \left(\frac{1}{R_{t'+1}} - \frac{R_{t'}}{L_{t'}^2} \right) B_{t'} + \frac{1}{\prod_{t''=0}^{t+1} R_{t''}} \left(\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} \right) B_{t+1,t}. \tag{A.54}$$

Writing (A.52) for $t+2$ instead of t, subtracting it from (A.54), and using $\tau_0 = \tau_1 = 0$, we find

$$\sum_{t'=0}^{t} \frac{c_{t'+2}}{\prod_{t''=0}^{t'+1} R_{t''}} = \sum_{t'=0}^{t} \frac{\alpha_{t'}}{\prod_{t''=0}^{t'-1} R_{t''}} + \sum_{t'=t+1}^{t+2} \frac{\alpha_{t'}\tau_{t'}}{\prod_{t''=0}^{t'-1} R_{t''}} - \frac{1}{\prod_{t''=0}^{t+1} R_{t''}} \left(\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} \right) B_{t+1,t+1} + \frac{b_{t+2} + \frac{R_{t+2}B_{t+2}}{L_{t+2}^2}}{\prod_{t''=0}^{t+2} R_{t''}}. \tag{A.55}$$

We next take the limit of (A.55) when $t \to \infty$. Because of CRRA utility, consumption must be non-negative, and hence $\alpha_t \tau_t \leq \alpha_t$. Therefore, the limit of the second term in the right-hand side of (A.55) is non-positive. Moreover, the term

$$\left(\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} \right) B_{t+1,t+1}$$

which is the capital gain of generation $t+1$ on its investment in two-period bonds in period $t+1$ must be bounded below, otherwise the generation's consumption would be negative. Therefore, the limit of the third term in the right-hand side of (A.55) is non-positive. Since, in addition, the limit of the fourth term is non-positive because of (5.2), we find the resource constraint

$$\sum_{t=0}^{\infty} \frac{c_{t+2}}{\prod_{t'=0}^{t+1} R_{t'}} \leq \sum_{t=0}^{\infty} \frac{\alpha_t}{\prod_{t'=0}^{t-1} R_{t'}}. \tag{A.56}$$

We next show that $\{B_t^*\}_{t \geq 1}$ is an optimal maturity structure. Our definition of optimality generalizes Definition 1 by replacing $(B_1, \hat{B}_1)$ by $(\{B_t\}_{t\geq 1}, \{\hat{B}_t\}_{t \geq 1})$ and computing the gain T_t as

$$\mathbb{E}u(c_{t+2}) \equiv \mathbb{E}u \left(\hat{c}_{t+2} - \prod_{t'=1}^{t+1} R_{t'} T_t \right). \tag{A.57}$$

To show that $\{B_t^*\}_{t\geq 1}$ is optimal, we proceed by contradiction, and suppose that there exists an admissible policy $(\{\hat{B}_t\}_{t\geq 1}, \{\hat{\tau}_t\}_{t\geq 2})$ to which a switch from $(\{B_t^*\}_{t\geq 1}, \{\tau_t^*\}_{t\geq 2})$ generates a positive sum $\sum_{t=0}^{\infty} T_t$ of gains. Denote by $\hat{c}_{t+2}$ and c_{t+2}^* the consumption of generation t in the corresponding equilibria. The allocation $\{\check{c}_{t+2}\}_{t\geq 0}$ defined by

$$\check{c}_2 \equiv \hat{c}_2 + R_1 \sum_{t=1}^{\infty} T_t, \tag{A.58}$$

$$\check{c}_{t+2} \equiv \hat{c}_{t+2} - \left(\prod_{t'=1}^{t+1} R_{t'} \right) T_t \quad \text{for} \quad t \geq 1, \tag{A.59}$$

yields the same utility as $\{c_{t+2}^*\}_{t\geq 0}$ for generations $t \geq 1$ because of (A.57), and higher utility for generation 0 because of (A.57) and $\sum_{t=0}^{\infty} T_t > 0$. This is a contradiction because $\{c_{t+2}^*\}_{t\geq 0}$ is Pareto optimal among all feasible allocations. To show Pareto optimality and the contradiction, we write the optimization problem of generation t under complete participation as maximizing utility within the budget set $\mathcal{B}_t$, where

$$\mathcal{B}_t \equiv \left\{ c_{t+2} : \mathbb{E}^Q \left(\frac{c_{t+2}}{\prod_{t'=0}^{t+1} R_{t'}} \right) \leq \mathbb{E}^Q \left(\frac{\alpha_{t+2}}{\prod_{t'=0}^{t-1} R_{t'}} \right) \right\}.$$

Since c_2^* maximizes the utility of generation 0 within $\mathcal{B}_0$, $\check{c}_2$, which yields higher utility, is outside $\mathcal{B}_0$. Likewise, since c_{t+2}^* maximizes the expected utility of generation $t \geq 1$ within $\mathcal{B}_t$, $\check{c}_{t+2}$, which yields the same utility, is weakly outside $\mathcal{B}_t$. Combining the corresponding inequalities, we find

$$\mathbb{E}^Q \left(\sum_{t=0}^{\infty} \frac{\check{c}_{t+2}}{\prod_{t'=0}^{t+1} R_{t'}} \right) > \mathbb{E}^Q \left(\sum_{t=0}^{\infty} \frac{\alpha_t}{\prod_{t'=0}^{t-1} R_{t'}} \right). \tag{A.60}$$

Eq. (A.60) contradicts the resource constraint (A.56). Indeed, (A.56) holds for $\{\hat{c}_{t+2}\}_{t\geq 0}$, since this allocation corresponds to the admissible policy $(\hat{B}_1, \{\hat{\tau}_t\}_{t\geq 2})$. Moreover, because of (A.58) and (A.59), (A.56) holds also for $\{\check{c}_{t+2}\}_{t\geq 0}$. ∎

Proof of Proposition 5.3: We first study the equilibrium under incomplete participation. We look for an equilibrium in which the two-period interest rate in period $t \geq 1$ is given by

$$L_t^2 = R_t \mathbb{E}(R_{t+1}) + \mu_t \bar{\eta}^2 + o(\bar{\eta}^2), \tag{A.61}$$

where μ_t is a deterministic function of t, and $o(\epsilon)$ denotes small relative to ϵ. Using (A.61) and the definition of η_t, we find

$$1 - \frac{R_t R_{t+1}}{L_t^2} = 1 - \frac{R_t \left[\mathbb{E}(R_{t+1}) + \eta_{t+1}\right]}{R_t \mathbb{E}(R_{t+1}) + \mu_t \bar{\eta}^2} + o(\bar{\eta}^2)$$

$$= -\frac{\eta_{t+1}}{\mathbb{E}(R_{t+1})} + \frac{\mu_t \bar{\eta}^2}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} + o(\bar{\eta}^2), \tag{A.62}$$

and

$$\frac{1}{R_{t+2}} - \frac{R_{t+1}}{L_{t+1}^2} = \frac{1}{\mathbb{E}(R_{t+2}) + \eta_{t+2}} - \frac{R_{t+1}}{R_{t+1}\mathbb{E}(R_{t+2}) + \mu_{t+1}\bar{\eta}^2} + o(\bar{\eta}^2)$$

$$= \frac{1}{\mathbb{E}(R_{t+2})}\left[1 - \frac{\eta_{t+2}}{\mathbb{E}(R_{t+2})} + \frac{\eta_{t+2}^2}{\left[\mathbb{E}(R_{t+2})\right]^2}\right] - \frac{1}{\mathbb{E}(R_{t+2}) + \frac{\mu_{t+1}\bar{\eta}^2}{R_{t+1}}} + o(\bar{\eta}^2)$$

$$= -\frac{\eta_{t+2}}{\left[\mathbb{E}(R_{t+2})\right]^2} + \frac{\eta_{t+2}^2}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2} + o(\bar{\eta}^2). \tag{A.63}$$

Substituting (A.62) and (A.63) into (A.53), and using the definition of η_t, we find

$$c_{t+2} = R_t \left[\mathbb{E}(R_{t+1}) + \eta_{t+1}\right] \alpha_t (1 - \tau_t) + \left[-\frac{\eta_{t+1}}{\mathbb{E}(R_{t+1})} + \frac{\mu_t \bar{\eta}^2}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})}\right] B_{t,t}$$

$$+ \left[-\frac{\eta_{t+2}}{\left[\mathbb{E}(R_{t+2})\right]^2} + \frac{\eta_{t+2}^2}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2}\right] B_{t+1,t} + o(\bar{\eta}^2). \tag{A.64}$$

Using (A.64) and the fact that (η_{t+1}, η_{t+2}) are mean-zero, independent of each other, and independent of information available in period t, we find

$$\mathbb{E}\left[u(c_{t+2})\right] = u(c_{t+2}^0) + u'(c_{t+2}^0)\left\{\frac{\mu_t \bar{\eta}^2 B_{t,t}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} + \left[\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2}\right] B_{t+1,t}\right\}$$

$$+ \frac{1}{2}u''(c_{t+2}^0)\left\{\mathbb{V}ar[R_t(1-\tau_t)]\left[\mathbb{E}(R_{t+1})\right]^2 \alpha_t^2\right.$$

$$\left. + \mathbb{V}ar(\eta_{t+1})\left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}[R_t(1-\tau_t)]\alpha_t\right]^2 + \frac{\mathbb{V}ar(\eta_{t+2})B_{t+1,t}^2}{\left[\mathbb{E}(R_{t+2})\right]^4}\right\} + o(\bar{\eta}^2), \tag{A.65}$$

where

$$c_{t+2}^0 \equiv \mathbb{E}[R_t(1-\tau_t)]\mathbb{E}(R_{t+1})\alpha_t.$$

Maximizing the objective (A.65) with respect to $(B_{t,t}, B_{t+1,t})$, and using the fact that utility is

CRRA, we find the first-order conditions

$$\frac{\mu_t\bar{\eta}^2}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} = \frac{\gamma\mathbb{V}ar(\eta_{t+1})}{\mathbb{E}(R_t)\left[\mathbb{E}(R_{t+1})\right]^2\alpha_t\mathbb{E}(1-\tau_t)}\left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}(R_t)\alpha_t\mathbb{E}(1-\tau_t)\right] + o(\bar{\eta}^2), \tag{A.66}$$

$$\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2} = \frac{\gamma\mathbb{V}ar(\eta_{t+2})B_{t+1,t}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^4\alpha_t\mathbb{E}(1-\tau_t)} + o(\bar{\eta}^2). \tag{A.67}$$

Eqs. (A.66) and (A.67) yield bond demands, which we can substitute into the market-clearing condition $B_t = B_{t,t-1} + B_{t,t}$ and solve for μ_t. The demand $B_{t,t}$ can be computed from (A.66), and the demand $B_{t,t-1}$ from (A.67) written for $t-1$ instead of t. We find that

$$\mu_t = \frac{\gamma\mathbb{V}ar(\eta_{t+1})\left\{B_t - \mathbb{E}(R_t)\mathbb{E}(R_{t+1})\left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}\mathbb{E}(1-\tau_{t-1})}{\gamma} + \alpha_t\mathbb{E}(1-\tau_t)\right]\right\}}{\bar{\eta}^2\left[\mathbb{E}(R_{t+1})\right]^2\left[\mathbb{E}(R_{t-1})\alpha_{t-1}\mathbb{E}(1-\tau_{t-1}) + \alpha_t\mathbb{E}(1-\tau_t)\right]} + o(1). \tag{A.68}$$

Propositions 3.2 and 3.3 follow from (A.68). To show Proposition 3.2, we note that an increase in B_t holding $b_t/R_t + B_t/L_t^2$ constant (so that there is no effect on τ_t) raises μ_t and L_t because of (A.61) and (A.68). To show Proposition 3.3, we note that an increase in α_t holding $R_{t-1}\alpha_{t-1}(1-\tau_{t-1})+\alpha_t(1-\tau_t)$ constant, which in order zero amounts to holding $\mathbb{E}(R_{t-1})\alpha_{t-1}\mathbb{E}(1-\tau_{t-1}) + \alpha_t\mathbb{E}(1-\tau_t)$ constant, lowers the numerator in (A.68) if $\gamma > 1$ and raises it if $\gamma < 1$. Therefore, (A.61) and (A.68) imply that μ_t and L_t decrease if $\gamma > 1$ and increase if $\gamma < 1$.

We next study the equilibrium under complete participation. Using (A.39) and the fact that the sequence $\{\eta_t\}_{t\geq 2}$ is independent and mean-zero, and proceeding as in the incomplete-participation case, we find

$$\begin{aligned}
&\mathbb{E}\left[u(c_{t+2})\right] = u(c^0_{t+2}) \\
&+ u'(c^0_{t+2})\left\{\sum_{t'=1}^{t}\left(\prod_{t''=t'+2}^{t+1}\mathbb{E}(R_{t''})\right)\frac{\mu_{t'}\bar{\eta}^2B_{t',t}}{\mathbb{E}(R_{t'})\mathbb{E}(R_{t'+1})} + \left[\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2}\right]B_{t+1,t}\right\} \\
&+ \frac{1}{2}u''(c^0_{t+2})\left\{\sum_{t'=1}^{t-2}\left(\prod_{t''=t'+2}^{t+1}\mathbb{E}(R_{t''})\right)^2\frac{\mathbb{V}ar(\eta_{t'+1})B^2_{t',t}}{\left[\mathbb{E}(R_{t'+1})\right]^2} + \mathbb{V}ar(\eta_t)\left[\frac{\mathbb{E}(R_{t+1})B_{t-1,t}}{\mathbb{E}(R_t)} - \mathbb{E}(R_{t+1})\alpha_t\right]^2\right. \\
&\left.+\mathbb{V}ar(\eta_{t+1})\left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}(R_t)\alpha_t\right]^2 + \frac{\mathbb{V}ar(\eta_{t+2})B^2_{t+1,t}}{\left[\mathbb{E}(R_{t+2})\right]^4}\right\} + o(\bar{\eta}^2),
\end{aligned} \tag{A.69}$$

where

$$c^0_{t+2} \equiv \mathbb{E}(R_t)\mathbb{E}(R_{t+1})\alpha_t.$$

Maximizing the objective (A.65) with respect to $\{B_{t',t}\}_{t'=1,..,t+1}$, and using the fact that utility is CRRA, we find the first-order conditions

$$\frac{\left(\prod_{t''=t'+2}^{t+1}\mathbb{E}(R_{t''})\right)\mu_{t'}\bar{\eta}^2}{\mathbb{E}(R_{t'})\mathbb{E}(R_{t'+1})} = \frac{\gamma\left(\prod_{t''=t'+2}^{t+1}\mathbb{E}(R_{t''})\right)^2\mathbb{V}ar(\eta_{t'+1})B_{t',t}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t'+1})\right]^2\alpha_t} + o(\bar{\eta}^2) \tag{A.70}$$

for $t' = 1, .., t-2$, and

$$\frac{\mathbb{E}(R_{t+1})\mu_{t-1}\bar{\eta}^2}{\mathbb{E}(R_{t-1})\mathbb{E}(R_t)} = \frac{\gamma\mathbb{V}ar(\eta_t)}{\left[\mathbb{E}(R_t)\right]^2\alpha_t}\left[\frac{\mathbb{E}(R_{t+1})B_{t-1,t}}{\mathbb{E}(R_t)} - \mathbb{E}(R_{t+1})\alpha_t\right] + o(\bar{\eta}^2), \tag{A.71}$$

$$\frac{\mu_t\bar{\eta}^2}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} = \frac{\gamma\mathbb{V}ar(\eta_{t+1})}{\mathbb{E}(R_t)\left[\mathbb{E}(R_{t+1})\right]^2\alpha_t}\left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}(R_t)\alpha_t\right] + o(\bar{\eta}^2), \tag{A.72}$$

$$\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2} = \frac{\gamma\mathbb{V}ar(\eta_{t+2})B_{t+1,t}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^4\alpha_t} + o(\bar{\eta}^2), \tag{A.73}$$

Eqs. (A.70)-(A.73) yield bond demands. We can substitute these demands into the market-clearing condition (A.42), which we can write as

$$\sum_{t'=t-1}^{\infty} B_{t,t'} = 0 \tag{A.74}$$

by inverting t and t', and solve for μ_t. The demand $B_{t,t}$ can be computed from (A.72), the demand $B_{t,t-1}$ from (A.73) written for $t-1$ instead of t, and the demand $B_{t,t'}$ for $t' > t$ from (A.70) and (A.71) after inverting t and $t' = 1, .., t-1$. We find

$$\mu_t = -\frac{\gamma\mathbb{V}ar(\eta_{t+1})\mathbb{E}(R_t)\left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t + \frac{\alpha_{t+1}}{\mathbb{E}(R_t)}\right]}{\bar{\eta}^2\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1}\mathbb{E}(R_{t''})}\right]} + o(1). \tag{A.75}$$

Propositions 4.5 and 4.6 follow from (A.75). To show Proposition 4.5, we note that

$$\begin{aligned}
B_t^* =& B_{t,t-1} + B_{t,t} \\
=& \frac{\mu_t \bar{\eta}^2 \left[\mathbb{E}(R_{t+1})\right]^2 \left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t\right]}{\gamma \mathbb{V}ar(\eta_{t+1})} + \mathbb{E}(R_t)\mathbb{E}(R_{t+1}) \left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t\right] + o(1) \\
=& \mathbb{E}(R_t)\mathbb{E}(R_{t+1}) \left[-\frac{\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t + \frac{\alpha_{t+1}}{\mathbb{E}(R_t)}}{\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty} \frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1} \mathbb{E}(R_{t''})}} \left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t\right] \right. \\
& \left. + \frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t \right] + o(1) \\
=& \frac{\mathbb{E}(R_t)\mathbb{E}(R_{t+1}) \left[-\frac{\alpha_{t+1}}{\mathbb{E}(R_t)} \left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t\right] + \left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t\right] \sum_{t'=t+1}^{\infty} \frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1} \mathbb{E}(R_{t''})} \right]}{\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty} \frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1} \mathbb{E}(R_{t''})}} + o(1),
\end{aligned} \tag{A.76}$$

where the second step follows from using (A.72) and (A.73), and the third from (A.75). The first statement of Proposition 4.5 follows from (A.76) and the second from (A.75).

To show Proposition 4.6, we note that (A.61) implies that the expected return of two- relative to one-period bonds over a two-period horizon is

$$L_t^2 - R_t \mathbb{E}(R_{t+1}) = \mu_t \bar{\eta}^2 + o(\bar{\eta}^2), \tag{A.77}$$

and over a one-period horizon is

$$\mathbb{E}\left[\frac{L_t^2}{R_{t+1}}\right] - R_t = \mathbb{E}\left[\frac{R_t \mathbb{E}(R_{t+1}) + \mu_t \bar{\eta}^2}{\mathbb{E}(R_{t+1}) + \eta_{t+1}}\right] - R_t + o(\bar{\eta}^2) = \frac{1}{\mathbb{E}(R_{t+1})} \left[\mu_t \bar{\eta}^2 + \frac{\mathbb{V}ar(\eta_{t+1}) R_t}{\mathbb{E}(R_{t+1})}\right] + o(\bar{\eta}^2). \tag{A.78}$$

Eq. (A.75) implies that $\mu_t < 0$ and

$$\mu_t\bar{\eta}^2 + \frac{\mathbb{V}ar(\eta_{t+1})R_t}{\mathbb{E}(R_{t+1})} = -\frac{\gamma\mathbb{V}ar(\eta_{t+1})\mathbb{E}(R_t)\left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t + \frac{\alpha_{t+1}}{\mathbb{E}(R_t)}\right]}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1}\mathbb{E}(R_{t''})}\right]} + \frac{\mathbb{V}ar(\eta_{t+1})R_t}{\mathbb{E}(R_{t+1})} + o(1)$$

$$= \frac{\mathbb{V}ar(\eta_{t+1})\mathbb{E}(R_t)}{\mathbb{E}(R_{t+1})}\left[-\frac{\gamma\left[\frac{\mathbb{E}(R_{t-1})\alpha_{t-1}}{\gamma} + \alpha_t + \frac{\alpha_{t+1}}{\mathbb{E}(R_t)}\right]}{\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1}\mathbb{E}(R_{t''})}} + 1\right] + o(1)$$

$$= \frac{\mathbb{V}ar(\eta_{t+1})\mathbb{E}(R_t)\left[(1-\gamma)\left[\alpha_t + \frac{\alpha_{t+1}}{\mathbb{E}(R_t)}\right] + \sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1}\mathbb{E}(R_{t''})}\right]}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t-1})\alpha_{t-1} + \alpha_t + \sum_{t'=t+1}^{\infty}\frac{\alpha_{t'}}{\prod_{t''=t}^{t'-1}\mathbb{E}(R_{t''})}\right]} + o(1).$$

Therefore, the sign of expected excess returns over a one- and a two-period horizon is as in Proposition 4.6. ∎

Proof of Proposition 5.4: We assume that the values of $\{\zeta_t, \theta_t\}_{t\geq 1}$ are proportional to $\bar{\eta}$, as are those of $\{\eta_t\}_{t\geq 1}$. We denote the investment of generation t in the technology in period t' by $x_{t',t}$. Generalizing (A.53), we find that the consumption of generation t in the equilibrium with incomplete participation is

$$\begin{aligned} c_{t+2} =& R_t^f R_{t+1}^f \alpha_t(1-\tau_t) + (R_t - R_t^f)R_{t+1}^f x_{t,t} + (R_{t+1} - R_{t+1}^f)x_{t+1,t} \\ &+ \left(1 - \frac{R_t^f R_{t+1}^f}{L_t^2}\right)B_{t,t} + \left(\frac{1}{R_{t+2}^f} - \frac{R_{t+1}^f}{L_{t+1}^2}\right)B_{t+1,t}. \end{aligned} \tag{A.79}$$

We look for an equilibrium in which the one- and two-period interest rates in period $t \geq 1$ are given by

$$R_t^f = \mathbb{E}(R_t) + \eta_t - \xi_t\bar{\eta}^2 + o(\bar{\eta}^2), \tag{A.80}$$

$$L_t^2 = R_t^f\mathbb{E}(R_{t+1}^f) + \mu_t\bar{\eta}^2 + o(\bar{\eta}^2), \tag{A.81}$$

where (ξ_t, μ_t) are deterministic functions of t. Using $R_t = \mathbb{E}(R_t) + \eta_t + \theta_{t+1}$, (A.62), (A.63), (A.79)-(A.81), the fact that $(\eta_{t+1}, \theta_{t+1})$ are independent of $(\eta_{t+2}, \theta_{t+2})$, and the fact that $(\eta_{t+1}, \theta_{t+1}, \eta_{t+2}, \theta_{t+2})$

are mean-zero and independent of information available in period t, we find

$$
\begin{aligned}
\mathbb{E}\left[u(c_{t+2})\right] =& u(c^0_{t+2}) + u'(c^0_{t+2}) \Bigg\{ \left[\xi_t \bar{\eta}^2 \mathbb{E}(R_{t+1}) + \mathbb{C}ov(\eta_{t+1}, \theta_{t+1})\right] x_{t,t} + \frac{\mu_t \bar{\eta}^2 B_{t,t}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} \\
& \xi_{t+1}\bar{\eta}^2 x_{t+1,t} + \left[\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2}\right] B_{t+1,t} \Bigg\} \\
& + \frac{1}{2} u''(c^0_{t+2}) \Bigg\{ \mathbb{V}ar[R^f_t \alpha_t (1-\tau_t)] \left[\mathbb{E}(R^f_{t+1})\right]^2 + \mathbb{V}ar(\theta_{t+1}) \left[\mathbb{E}(R_{t+1})\right]^2 x^2_{t,t} \\
& - 2\mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) \left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}[R_t \alpha_t (1-\tau_t)]\right] \mathbb{E}(R_{t+1}) x_{t,t} \\
& + \mathbb{V}ar(\eta_{t+1}) \left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}[R_t \alpha_t (1-\tau_t)]\right]^2 \\
& + \mathbb{V}ar(\theta_{t+2}) x^2_{t+1,t} - 2\mathbb{C}ov(\eta_{t+2}, \theta_{t+2}) \frac{x_{t+1,t} B_{t+1,t}}{\left[\mathbb{E}(R_{t+2})\right]^2} + \frac{\mathbb{V}ar(\eta_{t+2}) B^2_{t+1,t}}{\left[\mathbb{E}(R_{t+2})\right]^4} \Bigg\} + o(\bar{\eta}^2),
\end{aligned}
\tag{A.82}
$$

where

$$
c^0_{t+2} \equiv \mathbb{E}[R^f_t \alpha_t (1-\tau_t)] \mathbb{E}(R^f_{t+1}).
$$

Maximizing the objective (A.82) with respect to $(x_{t,t}, B_{t,t}, x_{t+1,t}, B_{t+1,t})$, and using the fact that utility is CRRA, we find the first-order conditions

$$
\begin{aligned}
& \xi_t \bar{\eta}^2 \mathbb{E}(R_{t+1}) + \mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) = \frac{\gamma}{\mathbb{E}(R_t)\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)} \\
& \times \left\{ \mathbb{V}ar(\theta_{t+1}) \mathbb{E}(R_{t+1}) x_{t,t} - \mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) \left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}(R_t)\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)\right]\right\} + o(\bar{\eta}^2),
\end{aligned}
\tag{A.83}
$$

$$
\begin{aligned}
& \frac{\mu_t \bar{\eta}^2}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})} = \frac{\gamma}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)} \\
& \times \left\{ -\mathbb{C}ov(\eta_{t+1}, \theta_{t+1}) x_{t,t} + \frac{\mathbb{V}ar(\eta_{t+1})}{\mathbb{E}(R_{t+1})} \left[\frac{B_{t,t}}{\mathbb{E}(R_{t+1})} - \mathbb{E}(R_t)\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)\right]\right\} + o(\bar{\eta}^2),
\end{aligned}
\tag{A.84}
$$

$$\xi_{t+1}\bar{\eta}^2 = \frac{\gamma}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)}\left\{\mathbb{V}ar(\theta_{t+2})x_{t+1,t} - \mathbb{C}ov(\eta_{t+2},\theta_{t+2})\frac{B_{t+1,t}}{\left[\mathbb{E}(R_{t+2})\right]^2}\right\} + o(\bar{\eta}^2), \tag{A.85}$$

$$\frac{\mathbb{V}ar(\eta_{t+2})}{\left[\mathbb{E}(R_{t+2})\right]^3} + \frac{\mu_{t+1}\bar{\eta}^2}{\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2} = \frac{\gamma}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})\left[\mathbb{E}(R_{t+2})\right]^2\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)}$$
$$\times\left\{-\mathbb{C}ov(\eta_{t+2},\theta_{t+2})x_{t+1,t} + \frac{\mathbb{V}ar(\eta_{t+2})B_{t+1,t}}{\left[\mathbb{E}(R_{t+2})\right]^2}\right\} + o(\bar{\eta}^2). \tag{A.86}$$

The market-clearing conditions are $B_t = B_{t,t-1} + B_{t,t}$ for two-period bonds and $b_t = b_{t,t-1} + b_{t,t}$ for one-period bonds, where $b_{t,t'}$ denotes the face value of one-period bonds held by generation t' in period t. The latter face values are given by

$$\alpha_t(1-\tau_t) = x_{t,t} + \frac{b_{t,t}}{R_t^f} + \frac{B_{t,t}}{L_t^2}, \tag{A.87}$$

$$\alpha_{t-1}(1-\tau_{t-1})R_{t-1}^f + x_{t-1,t-1}(R_{t-1} - R_{t-1}^f) + B_{t-1,t-1}\left(\frac{1}{R_t^f} - \frac{R_{t-1}^f}{L_t^2}\right) = x_{t,t-1} + \frac{b_{t,t-1}}{R_t^f} + \frac{B_{t,t-1}}{L_t^2}, \tag{A.88}$$

where the left-hand side of (A.87) and (A.88) is the income that generation t and $t-1$, respectively, enter period t with, and the right-hand side represents how this income is divided into an investment in the technology and in one- and two-period bonds. Using (A.80), (A.81), (A.87) and (A.88), we can write the market-clearing condition for one-period bonds as

$$b_t = \mathbb{E}(R_t)\left[\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t) + \mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1}) - (x_{t,t} + x_{t,t-1}) - \frac{B_{t,t} + B_{t,t-1}}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})}\right] + o(1).$$

Substituting $(x_{t,t}, B_{t,t}, x_{t,t-1}, B_{t,t-1})$ from (A.83)-(A.86) into the market-clearing conditions for one- and two-period bonds, we find

$$\xi_t = \gamma\frac{\mathbb{V}ar(\theta_{t+1})Z_{1,t} - \frac{\mathbb{C}ov(\eta_{t+1},\theta_{t+1})}{[\mathbb{E}(R_{t+1})]^2}\left[Z_{2,t} + \frac{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})}{\gamma}\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)\right]}{\bar{\eta}^2\mathbb{E}(R_t)\left[\mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1}) + \mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)\right]}, \tag{A.89}$$

$$\mu_t = \gamma\frac{\frac{\mathbb{V}ar(\eta_{t+1})}{[\mathbb{E}(R_{t+1})]^2}\left[Z_{2,t} - \frac{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})}{\gamma}\mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1})\right] - \mathbb{C}ov(\eta_{t+1},\theta_{t+1})Z_{1,t}}{\bar{\eta}^2\left[\mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1}) + \mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)\right]}, \tag{A.90}$$

where

$$Z_{1,t} \equiv \mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1}) + \mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t) - \frac{b_t}{\mathbb{E}(R_t)} - \frac{B_t}{\mathbb{E}(R_t)\mathbb{E}(R_{t+1})},$$

$$Z_{2,t} \equiv B_t - \mathbb{E}(R_t)\mathbb{E}(R_{t+1})\mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t).$$

Consider next the effect of increasing B_t holding $b_t/R_t^f + B_t/L_t^2$ constant, which in order zero amounts to holding $b_t/\mathbb{E}(R_t) + B_t/(\mathbb{E}(R_t)\mathbb{E}(R_{t+1}))$ constant. Consider also the effect of increasing α_t holding $R_{t-1}\alpha_{t-1}(1-\tau_{t-1}) + \alpha_t\mathbb{E}(1-\tau_t)$ constant, which in order zero amounts to increasing $\mathbb{E}(\alpha_t)$ holding $\mathbb{E}(R_{t-1})\mathbb{E}(\alpha_{t-1})\mathbb{E}(1-\tau_{t-1}) + \mathbb{E}(\alpha_t)\mathbb{E}(1-\tau_t)$ constant. Eqs. (A.77) and (A.78) imply that the effects on expected excess returns, over both a one- and a two-period horizon, are identical to those on μ_t. Eqs. (A.80) and (A.81) imply that the effects on the two-period interest rate L_t are identical to those on $-\xi_t\mathbb{E}(R_{t+1}) + \mu_t$. Eq. (A.80) implies that the effects on the one-period interest rate R_t^f are identical to those on $-\xi_t$. Eqs. (A.89) and (A.90) imply that these effects are as in the proposition. ∎

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Table 1: Summary statistics

Slope refers to the difference in nominal yields between 30-year and 10-year benchmark government bonds. Median age refers to the median age of the population. Maturity refers to the average maturity of government debt.

Panel A: Slope regression sample

	Slope (%)	Median age (years)
Mean	0.29	36.5
Median	0.35	37.2
St. Dev.	0.59	4.1
Min	-2.01	27.8
p5	-0.83	28.6
p95	1.04	42.5
Max	2.42	44.7

Number of observations: 235

Panel B: Maturity regression sample

	Maturity (years)	Median age (years)
Mean	6.0	34.6
Median	5.5	35.1
St. Dev.	2.6	4.0
Min	0.0	25.5
p5	2.5	27.6
p95	11.4	40.6
Max	14.5	44.1

Number of observations: 642

Table 2: Summary statistics by country

Panel A: Slope regression sample

Country	Country code	Number of observations	Slope in % (mean)	Slope in % (st.dev.)	Median age (mean)	Median age (st.dev.)
Belgium	BEL	13	0.48	0.21	40.0	0.9
Canada	CAN	35	0.35	0.24	34.0	3.8
France	FRA	13	0.51	0.24	38.7	0.9
Germany	DEU	23	0.38	0.39	40.2	2.2
Italy	ITA	13	0.45	0.26	39.9	1.2
Japan	JPN	12	0.85	0.18	42.9	1.2
Spain	ESP	11	0.54	0.30	38.3	0.9
Switzerland	CHE	13	0.83	0.58	40.0	1.2
United Kingdom	GBR	51	-0.13	0.88	36.0	1.8
USA	USA	51	0.17	0.45	31.8	2.9

Number of countries: 10

Panel B: Maturity regression sample

Country	Country code	Number of observations	Maturity (mean)	Maturity (st.dev.)	Median age (mean)	Median age (st.dev.)
Australia	AUS	36	5.3	1.3	32.7	3.0
Austria	AUT	29	6.2	0.9	37.3	2.1
Belgium	BEL	20	3.4	1.1	35.4	1.2
Canada	CAN	50	6.9	2.0	31.4	4.8
Denmark	DNK	32	4.5	1.2	37.4	1.9
Finland	FIN	19	3.7	0.4	38.3	2.5
France	FRA	20	6.3	0.4	37.5	1.5
Germany	DEU	43	5.1	0.8	37.9	2.7
Iceland	ISL	15	4.2	0.6	32.8	1.2
Ireland	IRL	48	8.3	2.2	29.2	2.6
Italy	ITA	46	5.2	2.7	35.8	3.1
Japan	JPN	11	5.4	0.4	42.4	1.1
Luxembourg	LUX	11	3.1	2.8	37.7	0.6
Netherlands	NLD	50	7.0	1.5	33.2	4.0
New Zealand	NZL	18	4.3	0.3	34.3	1.5
Norway	NOR	24	4.8	1.6	36.3	1.5
Spain	ESP	48	5.7	3.4	33.4	3.4
Sweden	SWE	24	4.0	1.4	38.0	1.5
United Kingdom	GBR	48	11.3	1.4	35.9	1.7
USA	USA	50	4.6	0.9	31.7	2.9

Number of countries: 20

Table 3: Regression results

Panel A reports results from panel regressions of the slope of the government bond yield curve (measured as the difference between the 30-year and 10-year yields) on the median age of the population (in years). Panel B reports results from panel regressions of the average maturity of government debt (in years) on the median age of the population. Groups refer to individual countries.

Panel A: Regressions of slope on median age

	(OLS)	(Between)	(Fixed effects)	(Random effects)
Median age	0.0469***	0.0611**	0.0389	0.0456***
	(3.94)	(2.80)	(1.63)	(2.64)
Constant	-1.421**	-1.890*	-1.129	-1.316*
	(2.95)	(2.26)	(1.29)	(1.93)
Number of observations	235		235	235
Number of groups		10		
R-squared	10.7%	49.5%	26.5%	
Hausman test: prob>chi2				42.3%

Panel B: Regressions of debt maturity on median age

	(OLS)	(Between)	(Fixed effects)	(Random effects)
Median age	-0.167**	-0.172	-0.146**	-0.147**
	(2.08)	(1.23)	(2.48)	(2.51)
Constant	11.74***	11.55**	11.02***	10.68***
	(4.29)	(2.32)	(5.39)	(4.98)
Number of observations	642		642	642
Number of groups		20		
R-squared	6.4%	7.7%	59.6%	
Hausman test: prob>chi2				85.0%

Absolute value of t- or z-statistics, computed with clustered standard errors, in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

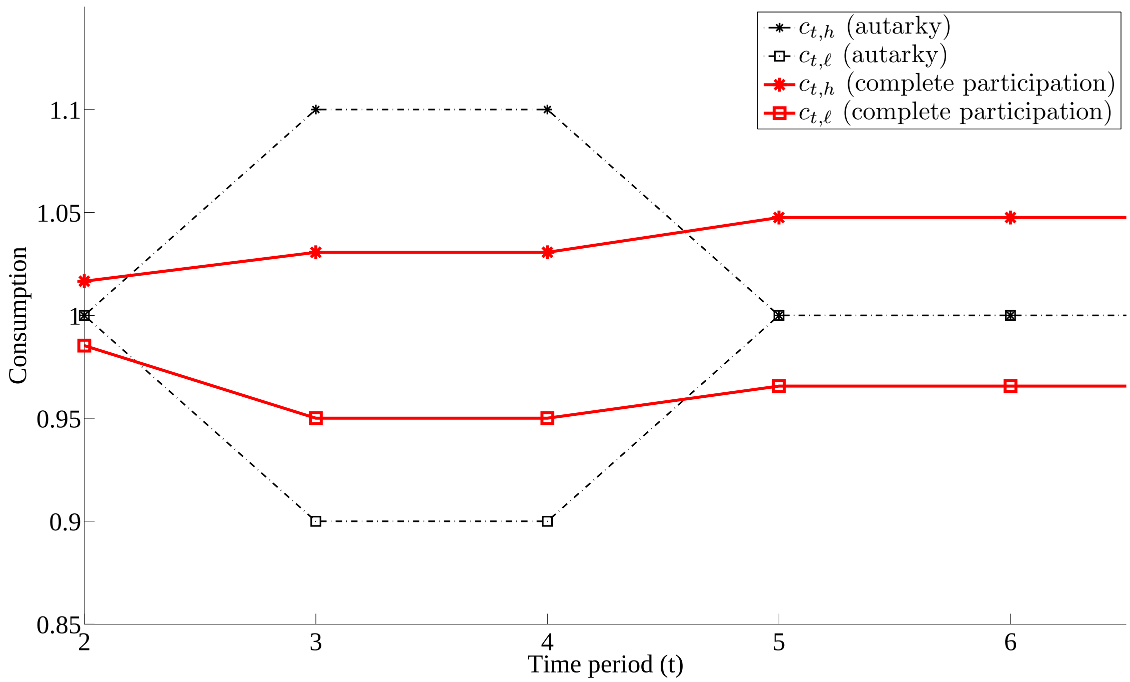

Figure 1: Complete participation vs. autarky. The figure is drawn for constant endowments ($\alpha_t = 1$ for all t), equally likely states ($p = 0.5$), coefficient of relative risk aversion $\gamma = 4$, average gross return of the linear production technology $R = (1 + 2.1\%)^{15}$, and realized return 10% above R in state h and 10% below in state ℓ. Consumption is normalized by its expected value, R^2.

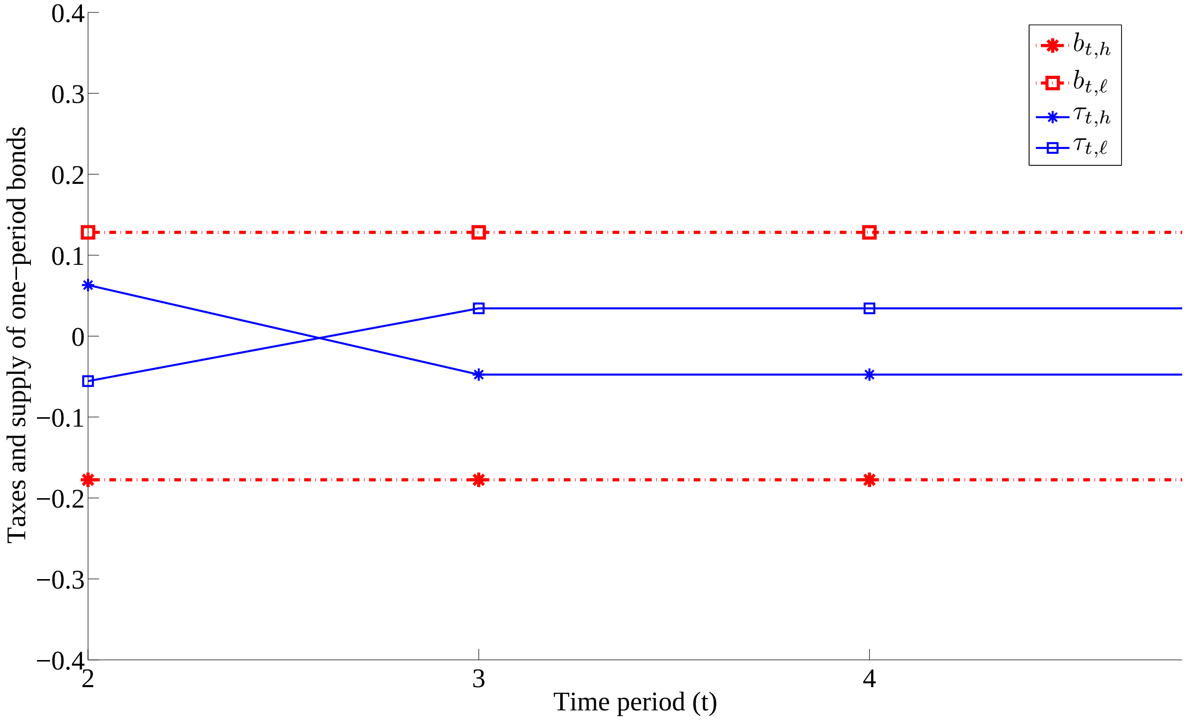

Figure 2: Taxes and supply of one-period bonds. The figure is drawn for the same parameter values as Figure 1.

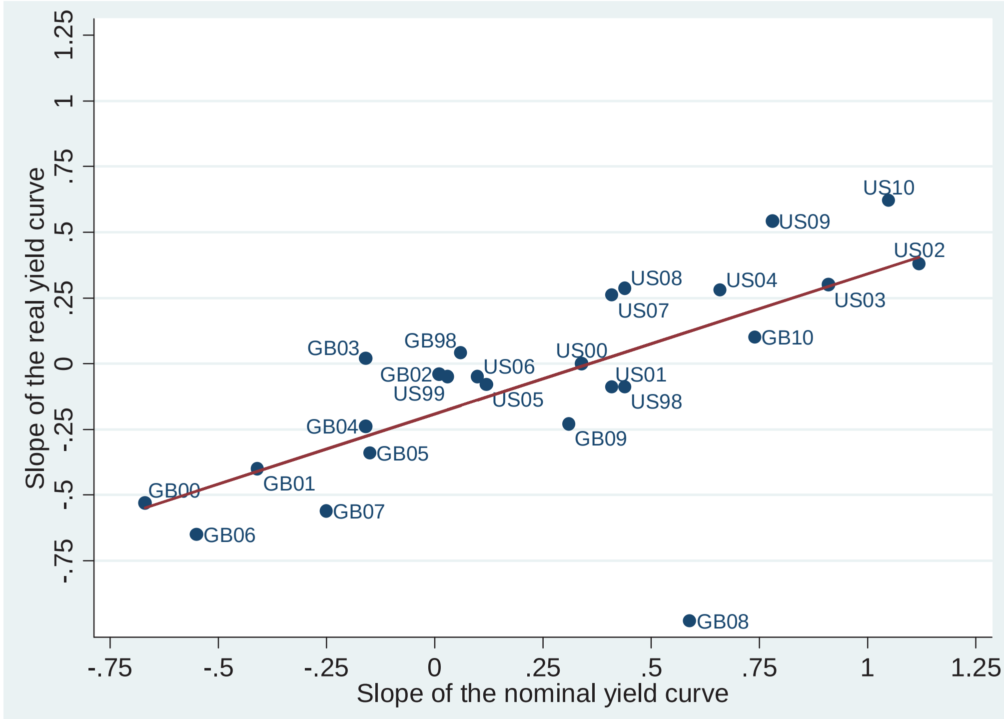

Figure 3: Real and nominal yield curve slopes. This graph plots country-year observations for the slope of the real yield curve for the United Kingdom (GB, from the Bank of England) and the United States (US, from Global Financial Data) against the slope of the nominal yield curve. The sample period is 1998-2010. The graph also includes a fitted regression line.

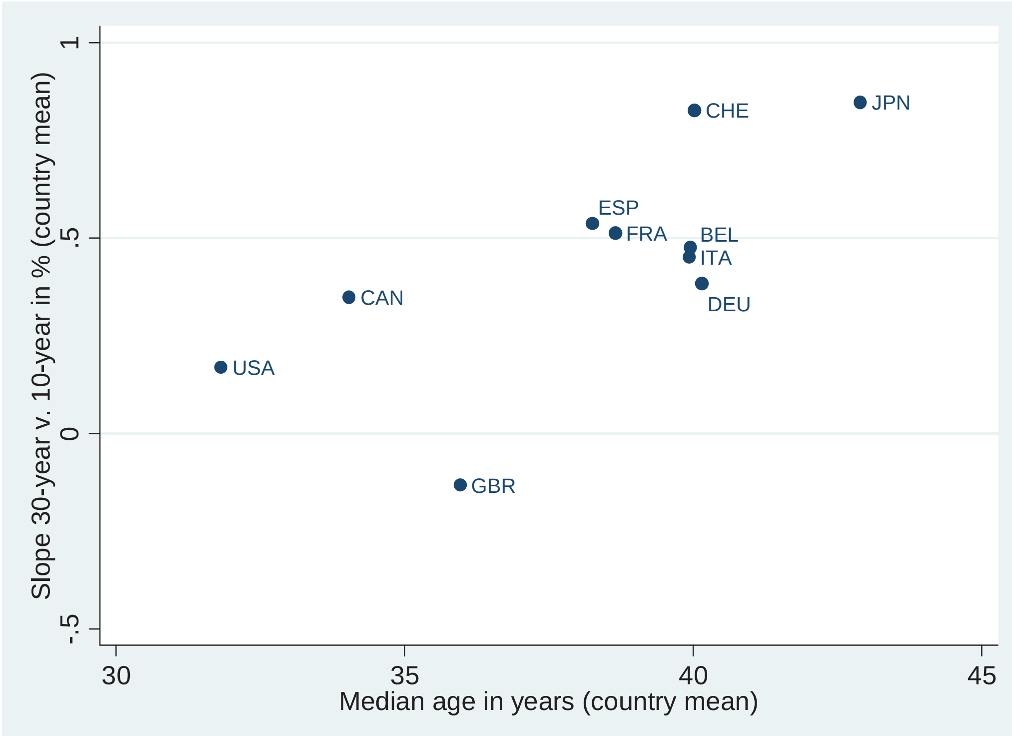

Figure 4: Slope of the yield curve and median age. This graph plots the sample means of the difference between 30-year and 10-year government bond yields for each country (from Global Financial Data) against the sample means of the median age for each country (from the United Nations Population Division).

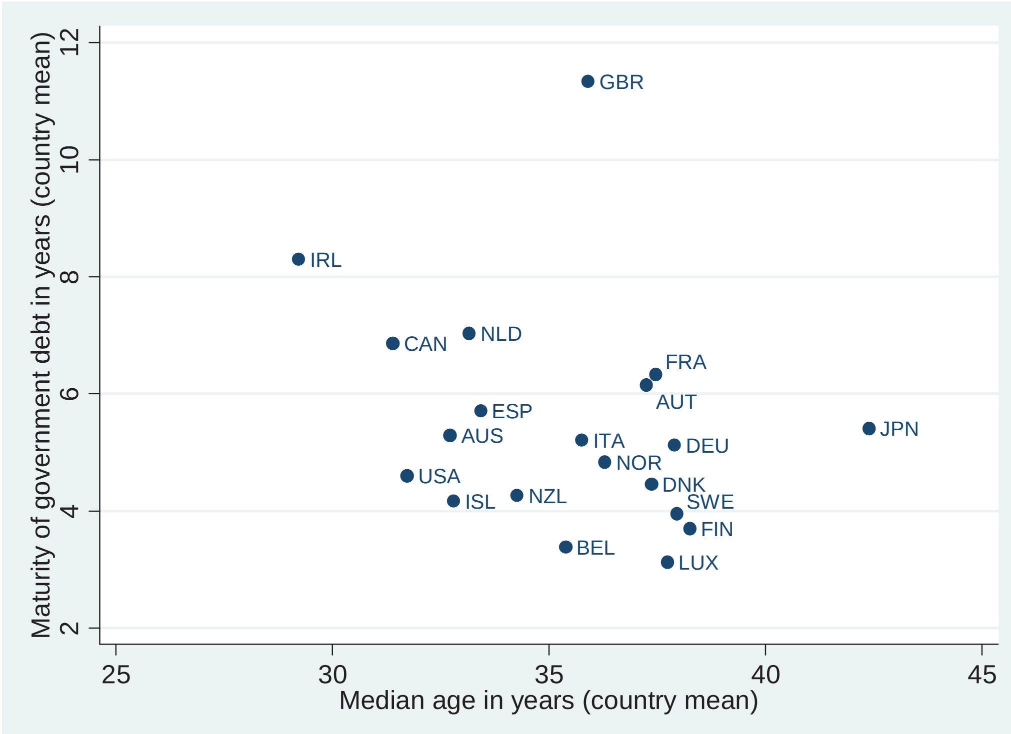

Figure 5: Maturity of government debt and median age. This graph plots the sample means of the maturity of government debt for each country (from the OECD and Missale (1999)) against the sample means of the median age for each country (from the United Nations Population Division).