Passive Investing and the Rise of Mega-Firms

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Abstract

We study how passive investing affects asset prices. Flows into passive funds disproportionately raise the stock prices of the economy's largest firms, and especially those large firms in high demand by noise traders. Because of this effect, the aggregate market can rise even when flows are entirely due to investors switching from active to passive strategies. Intuitively, passive flows increase the idiosyncratic risk of large firms in high demand, which discourages investors from correcting the flows' effects on prices. Consistent with our theory, prices and idiosyncratic volatilities of the largest S&P500 firms rise the most following flows into that index.

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1 Introduction

One of the most important capital-market developments of the past thirty years has been the growth of passive investing. Passive funds track market indices and charge lower fees than active funds. In 1993, passive funds invested in US stocks managed \$23 billion of assets. That was 3.7% of the combined assets managed by active and passive funds, and 0.44% of the US stock market. By 2021, passive assets had risen to \$8.4 trillion. That was 53% of combined active and passive, and 16% of the stock market.[1] The growth of passive investing has been estimated to be more than twice as high when accounting for the increasing tendency by active funds and other investors to stay close to their benchmark indices.[2]

The growth of passive investing has stimulated academic and policy interest in how it affects asset prices and the real economy. One effect that has been emphasized, drawing on the literature on rational expectations equilibria (REE) with asymmetric information (Grossman (1976), Grossman and Stiglitz (1980)), is that with fewer active funds, individual stocks become less liquid and their prices less informative. Another effect, drawing on the literature on index additions (Harris and Gurel (1986), Shleifer (1986)), is that the prices of the stocks included in the indices tracked by passive funds rise, while the prices of non-index stocks do not.

In this paper we show that the growth of passive investing disproportionately raises the stock prices of the economy's largest firms, and especially those large firms in high demand by noise traders. Passive investing thus reduces primarily the financing costs of the largest firms and makes the size distribution of firms more skewed. These effects are generated by a different mechanism than in the REE and index-addition literatures because information in our model is symmetric and the effects arise even when indices include all firms. We also show that the effects on the largest firms can be sufficiently strong to cause the aggregate market to rise even when the growth of passive comes entirely from investors switching from active (and not from new investors entering

[1]The data come from the 2022 Investment Company Institute (ICI) Factbook (Figure 2.9 and Tables 11 and 42), and from `https://data.worldbank.org/indicator/CM.MKT.LCAP.CD?locations=US`. We identify passive funds with index mutual funds and index exchange-traded funds (ETFs), and identify more generally passive investing with indexing throughout this paper.

[2]A measure of how far active funds stray from their benchmark indices is active share, defined in Cremers and Petajisto (2009). Petajisto (2013) finds that active share has been declining over time. Chinco and Sammon (2024) estimate that passive investing under its broader definition comprised 33.3% of the US stock market in 2021.

into stocks). Passive investing thus biases the stock market towards overvaluation. Consistent with our theory, we find that the prices of the largest firms in the S&P500 index rise the most following flows into that index.

While our results hold even in a CAPM world, the intuition is easier to convey when noise traders are present. Suppose that a large firm is in high demand by noise traders, and that active investors accommodate this demand by short-selling the firm in equilibrium. A switch by some investors from active to passive generates additional demand for the firm because passive investors hold the firm with its weight in the market index while active investors hold it with negative weight. Active investors can accommodate the additional demand by scaling up their short position. This renders them, however, more exposed to the firm's idiosyncratic risk, which is non-negligible because the firm is large. The firm's stock price must then rise to induce active investors to take on the additional risk. Crucially, because the stock price rises, the stock's idiosyncratic price movements become larger in absolute terms. This gives rise to an amplification loop: the short position of active investors becomes even riskier, causing the stock's price to rise even further, and the stock's idiosyncratic price movements to become even larger.

The amplification loop explains why passive flows have their largest effects on large firms in high demand by noise traders. It also explains why the effects of passive flows on these firms can be sufficiently strong so that a switch from active to passive causes the aggregate market to rise even though other firms might drop. It further explains why passive flows raise the idiosyncratic volatility of large firms more than of smaller firms, a result that we confirm empirically.

In our model, presented in Section 2, agents can invest in a constant riskless rate and in multiple stocks, over an infinite horizon. Each stock's dividend flow per share is the sum of a constant component and of a systematic and an idiosyncratic component that follow independent square-root processes. Some agents, the experts, can invest in all assets without constraints. They can be interpreted as investors who follow active strategies using stocks, mutual funds or hedge funds. Other agents, the non-experts, can only invest in the riskless asset and in a capitalization-weighted index. They can be interpreted as investors in passive funds. Experts and non-experts maximize a mean-variance objective over instantaneous changes in wealth. Noise traders can also be present, and hold a number of shares of each stock that is constant over time.

In the equilibrium of the model, derived in Section 3, the price of a stock is the sum of the present values of the constant, systematic and idiosyncratic dividends. The discount rate for idiosyncratic dividends increases in the stock's supply held by experts, and is approximately equal to the riskless rate for all but the largest stocks. The discount rate for systematic dividends increases in the aggregate supply of all stocks held by experts.

Section 4 shows analytically how stock prices respond to an increase in the measure of non-experts. When the measure of experts is held constant in this exercise, passive flows are due to entry by new investors into the stock market. When instead the measure of experts and non-experts is held constant, passive flows are due to a switch from active to passive. In a CAPM world where the index includes all stocks and noise traders are absent, a switch from active to passive leaves stock prices unchanged because experts and non-experts hold the same portfolio. When instead passive flows are not entirely due to a switch from active, their effect is an increasing function of CAPM beta for all but the largest stocks, and exceeds that function for the largest stocks. Intuitively, passive flows lower the market risk premium, and this lowers the discount rates for systematic and idiosyncratic dividends. The effect on the largest stocks is disproportionately large because of the changes to the idiosyncratic discount rate. Changes to that rate are approximately equal to zero for all but the largest stocks. Moreover, these changes have a larger effect on the present value of idiosyncratic dividends than equal changes to the systematic discount rate have on the present value of systematic dividends. This is because the idiosyncratic rate is lower than the systematic rate.

Section 5 calibrates the model using data on moments of asset returns and the size distribution of firms. The calibration assumes approximately 1,700 firms sorted into five size groups based on the aggregate dividends that firms pay to their shareholders. The assumed size distribution of firms conforms to a power law with exponent one, consistent with the empirical evidence (Axtell (2001)). We consider the case where the relative size of the systematic and idiosycratic components of dividends is the same for all stocks, and the case where the systematic component decreases with firm size in a way that generates the empirical negative relationship between size and CAPM beta (Fama and French (1992)). Consistent with the analytical results of Section 4, passive flows have disproportionately large effects on the firms in the largest size group. Furthermore, when noise traders are present, the effects of passive flows are strongest for those firms in the largest size group that are in high demand by noise traders.

Section 6 presents tests of our theory and relates our results to empirical findings in the literature. We take the index to be the S&P500 and flows to be into index mutual funds and index ETFs tracking it. Our flow data are quarterly, from 1996 to 2020. During quarters when index funds receive high inflows, the largest stocks in the index outperform the index. During the same quarters, index concentration as measured by, e.g., the Herfindahl index of index weights, increases. Following the same quarters, the idiosyncratic stock return volatility of large firms increases more than for smaller firms. We show additionally that the largest stocks outperform the index in the first week of each month. This is consistent with households investing at the beginning of each month a fraction of their monthly paychecks in passive funds through their retirement plans.

The effects of passive investing have mainly been analyzed within the framework proposed by Grossman and Stiglitz (1980, GS), in which informed and uninformed investors trade with noise traders. Informed and uninformed investors in GS can be interpreted as active and passive fund managers, respectively. A switch from active to passive lowers informational efficiency and can exacerbate the mispricing induced by noise traders.[3] The interpretation of GS investors as fund managers is developed in Garleanu and Pedersen (2018), in which investors search for informed managers, and the efficiency of the search market for managers affects the efficiency of the asset market. In Subrahmanyam (1991), the introduction of a market index facilitates passive investing and lowers liquidity for the assets that comprise the index. A switch from active to passive exacerbates noise-trader mispricing in our model as well. Our main results, however, concern how the effects of passive flows depend on stock size, and hold even in the absence of noise traders.[4]

[3] Pastor and Stambaugh (2012) and Stambaugh (2014) explain an increase in market efficiency, as reflected in a decline in active funds' expected returns, by the increase in the assets that active funds manage and by the decline in noise trading, respectively.

[4] Some papers show that the rise in passive investing can raise informational efficiency for individual stocks. In Bond and Garcia (2022), a decrease in the costs of index investing induces uninformed traders to switch to trading the index from trading individual stocks, and this lowers informational efficiency for the index but raises it for individual stocks. In Buss and Sundaresan (2023) passive investing can increase market efficiency when corporate investment responds to stock prices. On the empirical side, Ben-David, Franzoni, and Moussawi (2018) and Da and Shive (2018) find that the introduction of ETFs lowers informational efficiency for the underlying stocks because non-fundamental demand shocks spill over across stocks. Brogaard, Ringgenberg, and Sovich (2018) likewise find that the introduction of commodity indices results in worse production decisions by commodity firms. Bhojraj, Mohanram, and Zhang (2020) find instead that the introduction of sector ETFs renders stock prices more responsive to sector-level fundamental information. Glosten, Nallareddy, and Zou (2020) likewise find that ETFs render stock

A different literature studies how constraints or incentives of fund managers to not deviate from their benchmark indices affect asset prices. Brennan (1993), Kapur and Timmermann (2005), Cuoco and Kaniel (2011) and Basak and Pavlova (2013) show that compensating managers based on their performance relative to indices induces them to buy index assets, causing their prices to rise. Davies (2024) shows that passive flows have their strongest positive effects on the prices of stocks with high CAPM beta or in high demand by noise traders. Our model nests these results while also yielding the effects for the largest stocks. Chabakauri and Rytchkov (2021) show that passive flows cause market volatility to decrease when they are due to a switch from active to passive, and to increase when they are due to entry by new investors into the stock market. Our model is closest to Buffa, Vayanos, and Woolley (2022, BVW), who examine how constraints on managers' deviations from indices affect asset prices. We depart from BVW by introducing systematic risk and a size distribution of firms.

Our theory has implications for recent macroeconomic trends such as the rise in industry concentration and the decline in corporate investment. Autor, Dorn, Katz, Patterson, and van Reenen (2020) show that the rise of superstar firms can account for the rise in concentration (Grullon, Larkin, and Michaely (2019)) and the decline in the labor share (Elsby, Hobijn, and Sahin (2013), Karabarbounis and Neiman (2014)). Our theory suggests that the growth of passive investing can be one factor behind the rise of superstar firms, through the steeper decline of their financing costs. Alexander and Eberly (2018) and Crouzet and Eberly (2023) attribute the decline in corporate investment (Hall (2014), Fernald, Hall, Stock, and Watson (2017)) to intangible capital, while Gutiérrez and Philippon (2017) and Covarrubias, Gutiérrez, and Philippon (2019) show that the rise in concentration and changes in corporate governance are additional causes. Our theory suggests that the growth of passive investing may also have played a role because large overvalued firms experience the steepest decline in their financing costs but may not have the best investment projects.[5]

prices more responsive to economy-wide information, in the case of small stocks or stocks with low analyst coverage. Antoniou, Li, Liu, Subrahmanyam, and Sun (2022) find that ETFs cause firms' investment decisions to become more tightly linked to stock prices. Coles, Heath, and Ringgenberg (2022) and Koijen, Richmond, and Yogo (2024) find that the growth of passive investing does not have a significant impact on market efficiency. Haddad, Huebner, and Loualiche (2025) find that the growth of passive investing is associated with less price-elastic asset demand curves.

[5] Gutiérrez and Philippon (2017) find that firms with a large share of ownership by passive funds invest less. They emphasize governance-based explanations rather than valuation-based ones.

2 Model

Time t is continuous and goes from zero to infinity. The riskless rate is exogenous and equal to $r > 0$. There are N firms indexed by $n = 1, .., N$. The stock of firm n, also referred to as stock n, pays dividend flow D_{nt} per share and is in supply of $\eta_n > 0$ shares. The dividend flow of stock n is

$$D_{nt} = \bar{D}_n + b_n D_t^s + D_{nt}^i, \tag{2.1}$$

the sum of a constant component $\bar{D}_n \geq 0$, a systematic component $b_n D_t^s$ and an idiosyncratic component D_{nt}^i. The systematic component is the product of a systematic factor D_t^s times a factor loading $b_n \geq 0$. The systematic factor follows the square-root process

$$dD_t^s = \kappa^s \left(\bar{D}^s - D_t^s \right) dt + \sigma^s \sqrt{D_t^s} dB_t^s, \tag{2.2}$$

where $(\kappa^s, \bar{D}^s, \sigma^s)$ are positive constants and B_t^s is a Brownian motion. The idiosyncratic component follows the square-root process

$$dD_{nt}^i = \kappa_n^i \left(\bar{D}_n^i - D_{nt}^i \right) dt + \sigma_n^i \sqrt{D_{nt}^i} dB_{nt}^i, \tag{2.3}$$

where $\{\kappa_n^i, \bar{D}_n^i, \sigma_n^i\}_{n=1,..,N}$ are positive constants and $\{B_{nt}^i\}_{n=1,..,N}$ are Brownian motions that are mutually independent and independent of B_t^s. By possibly redefining factor loadings and the parameters of the square-root process (2.2), we set the long-run mean $\bar{D}^s$ of the systematic factor to one. By possibly redefining the supply η_n, the factor loading b_n and the parameters of the square-root process (2.3), we set the long-run mean $\bar{D}_n + b_n + \bar{D}_n^i$ of the dividend flow of stock n to one for all n.

Our specification (2.1)-(2.3) for dividends differs from typical specifications in the asset-pricing literature in two main respects. First, dividends are typically assumed to be non-stationary, while our specification yields stationarity because the systematic and idiosyncratic components of dividends mean-revert. Second, the volatility of dividends per share is typically assumed proportional to their level. That is the case, for example, when dividends follow a geometric Brownian motion. Under our specification instead, the volatility of the systematic and idiosyncratic components of dividends is proportional to the square root of their level.

Our model yields a non-stationary specification in the limit where the mean-reversion parameters $(\kappa^s, \{\kappa_n^i\}_{n=1,..,N})$ converge to zero. The analytical results shown in Section 4 carry through to that limit. Moreover, the calibration results shown in Section 5 remain similar across different values of $(\kappa^s, \{\kappa_n^i\}_{n=1,..,N})$. Thus, while stationarity yields a stochastic steady state in which we can compute unconditional moments of returns, it does not seem important for our results.

We assume that the volatility of dividends per share is proportional to the square-root of their level rather than to the level itself for tractability. The square-root specification preserves two important properties of typical specifications. First, dividends always remain positive. This is because when they converge to zero, their volatility converges to zero while their mean reversion pulls them towards their positive long-run mean. Second, the volatility of dividends increases with their level. This property is key for our results as we explain in subsequent sections. In Appendix C we show that the volatility of dividends per share of individual firms in the data increases with the level of dividends per share. Moreover, the increase appears to be concave rather than linear, consistent with a square-root specification. In Appendix D we show in a three-period model that results when the volatility of dividends per share is proportional to their level or to the square-root of their level are similar. We confine ourselves to three periods in Appendix D to ensure that the analysis of the level specification remains tractable, and we eliminate the mean-reversion so that the level specification becomes a geometric random walk.

Denoting by S_{nt} the price of stock n, the stock's return per share in excess of the riskless rate is

$$dR_{nt}^{sh} \equiv D_{nt}dt + dS_{nt} - rS_{nt}dt, \tag{2.4}$$

and the stock's return per dollar in excess of the riskless rate is

$$dR_{nt} \equiv \frac{dR_{nt}^{sh}}{S_{nt}} = \frac{D_{nt}dt + dS_{nt}}{S_{nt}} - rdt. \tag{2.5}$$

We refer to dR_t^{sh} as share return, omitting that it is in excess of the riskless rate. We refer to dR_t as return, omitting that it is per dollar and in excess of the riskless rate. All return moments that we compute in our calibration in Section 5 concern dR_t.

Agents are competitive and form overlapping generations living over infinitesimal time intervals. We assume infinitesimal lifespans for tractability because they yield simple mean-variance preferences as we show below. Each generation of agents includes experts and non-experts. Experts can invest in the riskless asset and in the stocks without constraints. These agents can be interpreted as investors who follow active strategies using stocks, mutual funds or hedge funds. Non-experts can invest in the riskless asset and in a stock portfolio that tracks an index. These agents can be interpreted as investors in passive funds.[6]

In addition to experts and non-experts, noise traders can be present. These agents generate an exogenous demand for each stock, which is smaller than the supply coming from the issuing firm. For tractability, we take the demand by noise traders to be constant over time when expressed in number of shares. A constant demand can capture slowly mean-reverting market sentiment. When noise traders are absent, or when their demand is proportional to the firm-issued supply in the cross-section of stocks, experts and non-experts hold the same portfolio of stocks in equilibrium.

The index includes all stocks or a subset of them. It is capitalization-weighted over the stocks that it includes, i.e., weights them proportionately to their market capitalization. We refer to the included and the non-included stocks as index and non-index stocks, respectively. We denote by $\mathcal{I}$ the subset of index stocks, by $\mathcal{I}^c$ its complement and by η'_n the number of shares of stock n included in the index. Since the index is capitalization-weighted over the stocks that it includes, η'_n for $n \in \mathcal{I}$ is proportional to the number of shares η_n issued by firm n. By possibly rescaling the index, we set $\eta'_n = \eta_n$ for $n \in \mathcal{I}$. For $n \in \mathcal{I}^c$, $\eta'_n = 0$.

We denote by W_{1t} and W_{2t} the wealth of an expert and a non-expert, respectively, by z_{1nt} and z_{2nt} the number of shares of stock n that these agents hold, and by μ_1 and μ_2 these agents' measure. A non-expert thus holds $z_{2nt} = \lambda\eta'_n$ shares of stock n, where λ is a proportionality coefficient that the agent chooses optimally. We assume for tractability that non-experts choose λ once and for

[6]Investors' choice to invest in active or passive funds can result from trading off the superior returns on active funds with their higher fees, in the spirit of Grossman and Stiglitz (1980).

all at time zero and under the unconditional distribution of dividends. We denote by $u_n < \eta_n$ the number of shares of stock n held by noise traders. The special case where noise traders are absent corresponds to $u_n = 0$ for all n.

Experts and non-experts born at time t are endowed with wealth W. Their budget constraint is

$$dW_{it} = \left(W - \sum_{n=1}^{N} z_{int} S_{nt}\right) r dt + \sum_{n=1}^{N} z_{int}(D_{nt}dt + dS_{nt}) = W r dt + \sum_{n=1}^{N} z_{int} dR^{sh}_{nt}, \tag{2.6}$$

where dW_{it} is the infinitesimal change in wealth over their life, $i = 1$ for experts, and $i = 2$ for non-experts. They have mean-variance preferences over dW_{it}. Given infinitesimal lifespans, mean-variance preferences can be derived from any VNM utility u, using the second-order Taylor expansion

$$u(W + dW_{it}) = u(W) + u'(W)dW_{it} + \frac{1}{2}u''(W)dW_{it}^2 + o(dW_{it}^2). \tag{2.7}$$

Experts maximize the conditional expectation of (2.7). This is equivalent to maximizing

$$\mathbb{E}_t(dW_{1t}) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}_t(dW_{1t}) \tag{2.8}$$

with $\rho = -\frac{u''(W)}{u'(W)}$, because infinitesimal dW_{1t} implies that $\mathbb{E}_t(dW_{1t}^2)$ is equal to $\mathbb{V}\mathrm{ar}_t(dW_{1t})$ plus smaller-order terms. Non-experts maximize the unconditional expectation of (2.7). This is equivalent to maximizing

$$\mathbb{E}(dW_{2t}) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}(dW_{2t}), \tag{2.9}$$

because infinitesimal dW_{2t} implies that $\mathbb{E}(dW_{2t}^2)$ is equal to $\mathbb{V}\mathrm{ar}(dW_{2t})$ plus smaller-order terms.

3 Equilibrium

We look for an equilibrium where the price S_{nt} of stock n is

$$S_{nt} = \bar{S}_n + b_n S^s(D^s_t) + S^i_n(D^i_{nt}), \tag{3.1}$$

the sum of the present value $\bar{S}_n$ of dividends from the constant component, the present value $b_n S^s(D^s_t)$ of dividends from the systematic component, and the present value $S^i_n(D^i_{nt})$ of dividends from the idiosyncratic component. Assuming that the functions $(S^s(D^s_t), S^i_n(D^i_{nt}))$ are twice continuously differentiable, we can write the share return dR^{sh}_{nt} of stock n as

$$\begin{aligned} dR^{sh}_{nt} &= (\bar{D}_n + b_n D^s_t + D^i_{nt})dt + (b_n dS^s(D^s_t) + dS^i_n(D^i_{nt})) - r\left(\bar{S}_n + b_n S^s(D^s_t) + S^i_n(D^i_{nt})\right)dt \\ &= \mu_{nt}dt + b_n\sigma^s\sqrt{D^s_t}(S^s)'(D^s_t)dB^s_t + \sigma^i_n\sqrt{D^i_{nt}}(S^i_n)'(D^i_{nt})dB^i_{nt}, \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} \mu_{nt} \equiv \frac{\mathbb{E}_t(dR^{sh}_{nt})}{dt} &= \bar{D}_n - r\bar{S}_n \\ &+ b_n\left[D^s_t + \kappa^s(1 - D^s_t)(S^s)'(D^s_t) + \frac{1}{2}(\sigma^s)^2 D^s_t (S^s)''(D^s_t) - rS^s(D^s_t)\right] \\ &+ D^i_{nt} + \kappa^i_n(\bar{D}^i_n - D^i_{nt})(S^i_n)'(D^i_{nt}) + \frac{1}{2}(\sigma^i_n)^2 D^i_{nt}(S^i_n)''(D^i_{nt}) - rS^i_n(D^i_{nt}) \end{aligned} \tag{3.3}$$

is the instantaneous expected share return on stock n, and the second step in (3.2) follows from (2.2), (2.3) and Ito's lemma.

Using (2.6) and (3.2), we can write the objective (2.8) of experts as

$$\sum_{n=1}^N z_{1nt}\mu_{nt} - \frac{\rho}{2}\left[\left(\sum_{n=1}^N z_{1nt}b_n\right)^2 (\sigma^s)^2 D^s_t[(S^s)'(D^s_t)]^2 + \sum_{n=1}^N z^2_{1nt}(\sigma^i_n)^2 D^i_{nt}[(S^i_n)'(D^i_{nt})]^2\right]. \tag{3.4}$$

Using (2.6), (3.2) and $z_{2nt} = \lambda\eta'_n$, we can likewise write the objective (2.9) of non-experts as

$$\sum_{n=1}^N \lambda\eta'_n\mu_n - \frac{\rho}{2}\lambda^2\left[\left(\sum_{n=1}^N \eta'_n b_n\right)^2 (\sigma^s)^2\mathbb{E}\left[D^s_t[(S^s)'(D^s_t)]^2\right] + \sum_{n=1}^N \left(\eta'_n\right)^2(\sigma^i_n)^2\mathbb{E}\left[D^i_{nt}[(S^i_n)'(D^i_{nt})]^2\right]\right],$$

$$(3.5)$$

where $\mu_n \equiv \frac{\mathbb{E}(dR^{sh}_{nt})}{dt} = \mathbb{E}(\mu_{nt})$. Experts maximize (3.4) over positions $\{z_{1nt}\}_{n=1,..,N}$. Non-experts maximize (3.5) over λ. Taking the first-order condition in (3.4) and substituting $\{z_{1nt}\}_{n=1,..,N}$ from the market clearing equation

$$\mu_1 z_{1nt} + \mu_2 \lambda \eta'_n + u_n = \eta_n, \tag{3.6}$$

which requires that the demand of experts, non-experts and noise traders equals the supply coming from the issuing firm, we find

$$\mu_{nt} = \rho \left[b_n \left(\sum_{m=1}^{N} \frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} b_m \right) (\sigma^s)^2 D^s_t [(S^s)'(D^s_t)]^2 + \frac{\eta_n - \mu_2 \lambda \eta'_n - u_n}{\mu_1} (\sigma^i_n)^2 D^i_{nt} [(S^i_n)'(D^i_{nt})]^2 \right]. \tag{3.7}$$

We look for functions $(S^s(D^s_t), S^i_n(D^i_{nt}))$ that are affine in their arguments,

$$S^s(D^s_t) = a^s_0 + a^s_1 D^s_t, \tag{3.8}$$

$$S^i_n(D^i_{nt}) = a^i_{n0} + a^i_{n1} D^i_{nt}, \tag{3.9}$$

for positive constants $(a^s_0, a^s_1, \{a^i_{n0}, a^i_{n1}\}_{n=1,..,N})$. Substituting (3.3), (3.8) and (3.9) into (3.7), we can write (3.7) as

$$\begin{aligned} &\bar{D}_n - r\bar{S}_n + b_n \left[D^s_t + \kappa^s a^s_1 (1 - D^s_t) - r(a^s_0 + a^s_1 D^s_t) \right] + D^i_{nt} + \kappa^i_n a^i_{n1} (\bar{D}^i_n - D^i_{nt}) - r(a^i_{n0} + a^i_{n1} D^i_{nt}) \\ &= \rho \left[b_n \left(\sum_{m=1}^{N} \frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} b_m \right) (\sigma^s a^s_1)^2 D^s_t + \frac{\eta_n - \mu_2 \lambda \eta'_n - u_n}{\mu_1} (\sigma^i_n a^i_{n1})^2 D^i_{nt} \right]. \end{aligned} \tag{3.10}$$

Identifying terms in D^s_t yields a quadratic equation that determines a^s_1. Identifying terms in D^i_{nt} yields a quadratic equation that determines a^i_{n1}. Identifying the remaining terms yields $\bar{S}_n + b_n a^s_0 + a^i_{n0}$. Substituting $(a^s_1, \{a^i_{n1}\}_{n=1,..,N})$ into the first-order condition of non-experts yields an equation for λ, whose solution completes our characterization of the equilibrium. Proposition 3.1

characterizes the equilibrium. The proposition's proof is in Appendix A, where all proofs are gathered.

Proposition 3.1. *In equilibrium, the price of stock n is*

$$S_{nt} = \frac{\bar{D}_n}{r} + b_n a_1^s \left(\frac{\kappa^s}{r} + D_t^s \right) + a_{n1}^i \left(\frac{\kappa_n^i}{r} \bar{D}_n^i + D_{nt}^i \right), \tag{3.11}$$

where

$$a_1^s = \frac{2}{r + \kappa^s + \sqrt{(r+\kappa^s)^2 + 4\rho \left(\sum_{m=1}^N \frac{\eta_m - \mu_2 \lambda \eta_m' - u_m}{\mu_1} b_m \right) (\sigma^s)^2}}, \tag{3.12}$$

$$a_{n1}^i = \frac{2}{r + \kappa_n^i + \sqrt{(r+\kappa_n^i)^2 + 4\rho \frac{\eta_n - \mu_2 \lambda \eta_n' - u_n}{\mu_1} (\sigma_n^i)^2}}, \tag{3.13}$$

and $\lambda > 0$ solves

$$\begin{aligned} &\left(\sum_{m=1}^N \eta_m' b_m \right) \left(\sum_{m=1}^N (\eta_m - u_m) b_m \right) (\sigma^s a_1^s)^2 + \sum_{m=1}^N \eta_m' (\eta_m - u_m) (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \\ &= (\mu_1 + \mu_2) \lambda \left[\left(\sum_{m=1}^N \eta_m' b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^N \left(\eta_m' \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right]. \end{aligned} \tag{3.14}$$

The price depends on $(\mu_1, \mu_2, \sigma^s, \{b_m, \sigma_m^i, \eta_m, \eta_m', u_m\}_{m=1,..,M})$ only through $\left(\sum_{m=1}^N \frac{\eta_m - \mu_2 \lambda \eta_m' - u_m}{\mu_1} b_m \right) (\sigma^s)^2$ and $\frac{\eta_n - \mu_2 \lambda \eta_n' - u_n}{\mu_1} (\sigma_n^i)^2$, and is decreasing and convex in the latter two variables.

The present value $\bar{S}_n$ of the dividends of stock n that come from the constant component is $\frac{\bar{D}_n}{r}$. This is because constant dividends are discounted at the riskless rate r. The present value $b_n S^s(D_t^s)$ of dividends coming from the systematic component is $b_n a_1^s \left(\frac{\kappa^s}{r} + D_t^s \right)$, and the present value of dividends coming from the idiosyncratic component is $a_{n1}^i \left(\frac{\kappa_n^i \bar{D}_n^i}{r} + D_{nt}^i \right)$. The coefficients a_1^s and a_{n1}^i are inversely proportional to the discount rates. Indeed, when D_t^s is equal to its long-run mean of one and hence all future expected systematic dividends are also equal to one, the stream of these dividends is multiplied by $a_1^s \left(\frac{\kappa^s}{r} + 1 \right)$ and is thus discounted at the rate $\frac{r}{(\kappa^s + r) a_1^s}$. Likewise, when D_{nt}^i is equal to its long-run mean of $\bar{D}_n^i$, the stream of expected idiosyncratic dividends $\bar{D}_n^i$

is discounted at the rate $\frac{r}{(\kappa^i+r)a_{n1}^i}$.

Supply affects the discount rate for systematic dividends through $\left(\sum_{m=1}^N \frac{\eta_m-\mu_2\lambda\eta_m'-u_m}{\mu_1}b_m\right)(\sigma^s)^2$. This is a risk-adjusted measure of the aggregate supply of stocks that each expert holds in equilibrium, and we refer to it as systematic supply. Supply affects the discount rate for idiosyncratic dividends through $\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}(\sigma_n^i)^2$. This is a risk-adjusted measure of the supply of stock n held by each expert, and we refer to it as idiosyncratic supply. We calculate systematic and idiosyncratic supply as follows. The supply of stock n held by all experts combined is equal to the supply η_n coming from the issuing firm, minus the demand $\mu_2\lambda\eta_n'$ and u_n coming from non-experts and noise traders, respectively. We express it in per-expert terms by dividing by the measure μ_1 of experts. In the case of systematic supply, we risk-adjust by multiplying by the factor loading b_n of stock n and by the square of the diffusion parameter σ^s of the systematic factor, and we aggregate across all stocks. In the case of idiosyncratic supply, we multiply by the square of the diffusion parameter σ_n^i of the idiosyncratic component of the dividends of stock n.

A reduction in systematic or idiosyncratic supply lowers the discount rate of the corresponding component of dividends. The present value of dividends goes up and its movements become larger in absolute terms. Supply generates a positive relationship between price level and price volatility because discounting for risk works multiplicatively. When supply drops, a_1^s and a_{n1}^i rise, and so do price level and price volatility. Discounting is multiplicative in our model because the volatility of dividends per share is assumed to increase with the level of dividends per share. By contrast, in CARA-normal models, where the volatility of dividends per share is constant, discounting for risk works additively, by discounting expected dividends at the riskless rate and subtracting a term. A reduction in supply in those models raises the price level but does not affect price volatility.

4 Passive Flows and Stock Prices—Analytical Results

Passive flows in our model correspond to an increase in the measure μ_2 of non-experts. These flows can arise because of entry by new investors into the stock market, in which case the measure μ_1 of experts is not changing, or because of a switch by investors from active to passive, in which case μ_1 decreases. We nest the two cases by assuming that when μ_2 increases, μ_1 decreases by an amount equal to a fraction $\phi \in [0,1]$ of the increase in μ_2.

Proposition 4.1. *Suppose that μ_2 increases and μ_1 decreases by an amount equal to a fraction $\phi \in [0,1]$ of the increase in μ_2. The percentage change in the price of stock n is*

$$\frac{1}{S_{nt}}\frac{dS_{nt}}{d\mu_2} = \frac{\rho}{\mu_1 S_{nt}}\left[b_n\left(\sum_{m=1}^{N}\Delta_m b_m\right)(\sigma^s a_1^s)^2\left(\frac{\kappa^s}{r}+D_t^s\right)F^s + \Delta_n(\sigma_n^i a_{n1}^i)^2\left(\frac{\kappa_n^i}{r}\bar{D}_n^i + D_{nt}^i\right)F_n^i\right], \tag{4.1}$$

where

$$\Delta_n \equiv -\mu_1\frac{d}{d\mu_2}\left(\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}\right) = \frac{d(\mu_2\lambda)}{d\mu_2}\eta_n' + \phi\frac{\mu_2\lambda\eta_n'+u_n-\eta_n}{\mu_1}, \tag{4.2}$$

$$F^s \equiv \frac{1}{\sqrt{(r+\kappa^s)^2 + 4\rho\left(\sum_{m=1}^{N}\frac{\eta_m-\mu_2\lambda\eta_m'-u_m}{\mu_1}b_m\right)(\sigma^s)^2}}, \tag{4.3}$$

$$F_n^i \equiv \frac{1}{\sqrt{(r+\kappa_n^i)^2 + 4\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}(\sigma_n^i)^2}}. \tag{4.4}$$

To derive the implications of Proposition 4.1, we begin with a baseline case that corresponds to the CAPM. We assume that the index includes all stocks and is thus the market portfolio, i.e., $\mathcal{I} = \{1,..,N\}$. We also assume that noise traders hold the same fraction of shares of each stock, i.e., $\#\{\frac{u_m}{\eta_m} : m \in \{1,..,N\}\} = 1$, and denote that fraction by $\hat{u} \in [0,1)$. The latter assumption includes as a special case the absence of noise traders, i.e., $u_n = 0$ for all n. Since non-experts and noise traders hold the market portfolio, experts also hold the market portfolio and stocks are priced according to the CAPM.

Simple CAPM logic suggests that the effect of passive flows on a stock's price in the baseline case should increase in the stock's CAPM beta and should not depend on the stock's size. To present that logic and why it fails in our model, we begin with a simple example that is loosely connected to our model but has the advantage of illustrating the generality of the mechanism. We next show that the same mechanism operates within our model.

Consider a stock n that pays dividend flow D_{nt} per share, and suppose that the stock's expected dividend $\bar{D}_n$, the stock's CAPM beta β_n and the market risk premium MRP are constant. Under the CAPM, the stock's price is $S_n = \frac{\bar{D}_n}{r+\beta_n \mathrm{MRP}}$. Since passive flows generate demand for the market portfolio, they lower MRP. The percentage price change that they generate is thus proportional to

$$\frac{1}{S_n}\frac{\partial S_n}{\partial(-\mathrm{MRP})} = \frac{\bar{D}_n \beta_n}{S_n(r+\beta_n \mathrm{MRP})^2} = \frac{\beta_n}{r+\beta_n \mathrm{MRP}}$$

in the cross-section. It increases in β_n and it does not depend on the size of stock n. This is the simple CAPM logic.

To explain why the above logic fails in our model, we next modify the example to account for different components of dividends. Suppose that the dividend flow D_{nt} of stock n is the sum of a constant component $\bar{D}_n$, a systematic component $b_n D_t^s$ and an idiosyncratic component D_{nt}^i. Suppose that the expected dividend $\bar{D}^s$ from the systematic factor D_t^s and the factor's CAPM beta β^s are constants, and normalize $\bar{D}^s$ to one. Suppose also that the stock's expected idiosyncratic dividend $\bar{D}_n^i$ and the CAPM beta β_n^i of idiosyncratic dividends are constants, and $\beta^s > \beta_n^i \geq 0$. Under the CAPM, the stock's price is $S_n = \frac{b_n}{r+\beta^s \mathrm{MRP}} + \frac{\bar{D}_n^i}{r+\beta_n^i \mathrm{MRP}}$. The percentage price change that passive flows generate is proportional to

$$\begin{aligned}\frac{1}{S_n}\frac{\partial S_n}{\partial(-\mathrm{MRP})} &= \frac{b_n \beta^s}{S_n(r+\beta^s \mathrm{MRP})^2} + \frac{\bar{D}_n^i \beta_n^i}{S_n(r+\beta_n^i \mathrm{MRP})^2} \\ &= w_n^s \frac{\beta^s}{r+\beta^s \mathrm{MRP}} + w_n^i \frac{\beta_n^i}{r+\beta_n^i \mathrm{MRP}},\end{aligned} \tag{4.5}$$

where $w_n^s \equiv \frac{b_n}{S_n(r+\beta^s \mathrm{MRP})}$ is the fraction of the price accounted by the systematic component and $w_n^i \equiv \frac{\bar{D}_n^i}{S_n(r+\beta_n^i \mathrm{MRP})}$ is the fraction accounted by the idiosyncratic component. Since the stock's CAPM beta is $\beta_n = w_n^s \beta^s + w_n^i \beta_n^i$, we can write (4.5) as

$$\frac{1}{S_n}\frac{\partial S_n}{\partial(-\mathrm{MRP})} = \frac{\beta_n}{r+\beta^s \mathrm{MRP}} + w_n^i \beta_n^i \left[\frac{1}{r+\beta_n^i \mathrm{MRP}} - \frac{1}{r+\beta^s \mathrm{MRP}}\right]. \tag{4.6}$$

For small stocks, β_n^i is negligible. Therefore, the second term in (4.6) is negligible and the price effect of passive flows increases in the stocks' CAPM beta β_n and does not depend on stock size. For large stocks, however, β_n^i is non-negligible because these stocks account for a non-negligible fraction of the market portfolio. Therefore, passive flows raise the prices of large stocks above and beyond their effect through the stocks' beta. Intuitively, the present value of idiosyncratic dividends is more sensitive to a drop in MRP, per unit of idiosyncratic beta β^i_n, than the present value of systematic dividends is, per unit of systematic beta β^s. This is because the discount rate for idiosyncratic dividends is lower than for systematic dividends ($\beta^i_n < \beta^s$).

The same mechanism as in the above example operates within our model. Setting $\eta'_n = \eta_n$ and $u_n = \hat{u}\eta_n$ for all n in (3.14), we find $\lambda = \frac{1-\hat{u}}{\mu_1+\mu_2}$. Setting $\eta'_n = \eta_n$ and $u_n = \hat{u}\eta_n$ for all n and $\lambda = \frac{1-\hat{u}}{\mu_1+\mu_2}$ in (4.2), we find $\Delta_n = \frac{(1-\phi)\mu_1(1-\hat{u})}{(\mu_1+\mu_2)^2}\eta_n$. Therefore, (4.1) implies that the percentage change in the price of stock n that passive flows generate is

$$\frac{1}{S_{nt}}\frac{dS_{nt}}{d\mu_2} = \frac{(1-\phi)\rho(1-\hat{u})}{(\mu_1+\mu_2)^2 S_{nt}}\left[b_n\left(\sum_{m=1}^N \eta_m b_m\right)(\sigma^s a^s_1)^2\left(\frac{\kappa^s}{r}+D^s_t\right)F^s + \eta_n(\sigma^i_n a^i_{n1})^2\left(\frac{\kappa^i_n}{r}\bar{D}^i_n + D^i_{nt}\right)F^i_n\right]. \tag{4.7}$$

We next write (4.7) in terms of the CAPM beta of stock n and of the difference in discount rates between systematic and idiosyncratic dividends. Using (3.2), (3.8) and (3.9), we find that the conditional covariance between the return on stock n and the share return on the market portfolio is

$$\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR^{sh}_{mt}\right) = \frac{1}{S_{nt}}\left[b_n\left(\sum_{m=1}^N \eta_m b_m\right)(\sigma^s a^s_1)^2 D^s_t + \eta_n(\sigma^i_n a^i_{n1})^2 D^i_{nt}\right]. \tag{4.8}$$

Using (4.8), we can write (4.7) as

$$\begin{aligned}\frac{1}{S_{nt}}\frac{dS_{nt}}{d\mu_2} = \frac{(1-\phi)\rho(1-\hat{u})}{(\mu_1+\mu_2)^2}&\left[\left(\frac{\kappa^s}{rD^s_t}+1\right)F^s\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR^{sh}_{mt}\right)\right.\\ &\left.+\frac{\eta_n(\sigma^i_n a^i_{n1})^2}{S_{nt}}\left(\left(\frac{\kappa^i_n}{r}\bar{D}^i_n + D^i_{nt}\right)F^i_n - \left(\frac{\kappa^s}{rD^s_t}+1\right)D^i_{nt}F^s\right)\right].\end{aligned} \tag{4.9}$$

Equation (4.9) is the counterpart within our model of (4.6), with the two terms inside the square bracket in the right-hand side of (4.9) corresponding to the two terms in the right-hand side of (4.6). Proposition 4.2 derives the implications of (4.9).

Proposition 4.2. *Suppose* $\mathcal{I} = \{1, .., N\}$ *and* $\#\{\frac{u_m}{\eta_m} : m \in \{1, .., N\}\} = 1$. *Suppose that* μ_2

increases and μ_1 decreases by an amount equal to a fraction $\phi \in [0,1]$ of the increase in μ_2. When $\phi = 1$, stock prices do not change. When $\phi < 1$, prices increase, with the following properties:

- *Consider two small stocks n and n' with $\eta_n, \eta_{n'} \approx 0$. Stock n experiences a larger percentage price increase than stock n' if $\beta_{nt} > \beta_{n't}$.*
- *Consider a large stock n with $\eta_n \not\approx 0$ and a small stock n' with $\eta_{n'} \approx 0$, and suppose $\beta_{nt} \geq \beta_{n't}$. Stock n experiences a larger percentage price increase than stock n' if*

$$\left(\frac{\kappa_n^i}{r}\bar{D}_n^i + D_{nt}^i\right) F_n^i > \left(\frac{\kappa^s}{rD_t^s} + 1\right) D_{nt}^i F^s. \tag{4.10}$$

When passive flows are due to a pure switch from active to passive ($\phi = 1$), stock prices do not change. This is because in the baseline case experts and non-experts hold the market portfolio. When instead passive flows are due, fully or partly, to entry by new investors into the stock market ($\phi < 1$), stock prices increase. For small stocks, the increase is fully described by CAPM beta and does not depend on stock size. This is because size can have an effect only through the covariance between the present value of idiosyncratic dividends and the return on the market portfolio, but that covariance is negligible for small stocks. For large stocks instead, the covariance is not negligible and passive flows raise their prices above and beyond their effect through beta provided that (4.10) holds. Condition (4.10) concerns the discount rates for systematic and idiosyncratic dividends. When (D_t^s, D_{nt}^i) are equal to their long-run means, (3.12), (3.13), (4.3) and (4.4) imply that (4.10) is equivalent to $a_{n1}^i\left(\frac{\kappa^s}{r}+1\right) > a_1^s\left(\frac{\kappa^s}{r}+1\right)$ and thus to the discount rate for idiosyncratic dividends being smaller than for systematic dividends.

We next turn to the case where the index does not include all stocks or where noise traders hold different fractions of shares across stocks. Using (4.8), we can write (4.1) as

$$\begin{aligned}\frac{1}{S_{nt}}\frac{dS_{nt}}{d\mu_2} = \frac{\rho}{\mu_1}\Bigg[&\frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m}\left(\frac{\kappa^s}{rD_t^s}+1\right)F^s \mathrm{Cov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right) \\ &+\frac{(\sigma_n^i a_{n1}^i)^2}{S_{nt}}\left(\Delta_n\left(\frac{\kappa_n^i}{r}\bar{D}_n^i + D_{nt}^i\right)F_n^i - \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m}\eta_n\left(\frac{\kappa^s}{rD_t^s}+1\right)D_{nt}^i F^s\right)\Bigg],\end{aligned} \tag{4.11}$$

which generalizes (4.9). Proposition 4.3 derives the implications of (4.11). One implication is derived in the special case where noise-trader demand is independent from stocks' other characteristics. Assumption 4.1 defines this independence case.

Assumption 4.1. *[**Independence**] The market consists of $N = GL$ stocks, which belong to G disjoint groups, each with cardinality L. The values of $(\bar{D}_n, b_n, \kappa_n^i, \bar{D}_n^i, \sigma_n^i, \eta_n, \eta_n')$ are the same across all stocks in any given group $g = 1, .., G$ and are denoted by $(\bar{D}_g, b_g, \kappa_g^i, \bar{D}_g^i, \sigma_g^i, \eta_g, \eta_g')$. The values of u_n differ across those stocks and are $\{\eta_g \hat{u}_\ell\}_{\ell=1,..,L}$, where $\{\hat{u}_\ell\}_{\ell=1,..,L}$ are the same across groups.*

In the independence case, all stock characteristics except noise-trader demand are the same across all stocks within each of a number of disjoint groups. The within-group distribution of the fraction of shares that are held by noise-traders is the same across groups. Our calibrations are made under Assumption 4.1, with the main difference between groups being stock size.

Proposition 4.3. *Suppose $\mathcal{I} \subsetneq \{1, .., N\}$ or $\#\{\frac{u_m}{\eta_m} : m \in \{1, .., N\}\} > 1$. Suppose that μ_2 increases and μ_1 decreases by an amount equal to a fraction $\phi \in [0, 1]$ of the increase in μ_2. The resulting stock price changes have the following properties:*

- *There exists a non-empty interval $[0, \phi_1) \subset [0, 1]$ such that for all $\phi \in [0, \phi_1)$, properties are the same as in Proposition 4.2, except that large stock n must also satisfy $n \in \mathcal{I}$.*
- *For all $\phi \in [0, 1]$, for any large stock n with $\eta_n \not\approx 0$, $n \in \mathcal{I}$ and $n \in \operatorname{argmax}_m \frac{u_m}{\eta_m}$, and for any small stock n' with $\eta_{n'} \approx 0$, stock n experiences a larger percentage price change than stock n' if $\beta_{nt} = \beta_{n't}$ and (4.10) holds.*
- *When Assumption 4.1 holds and $\mathcal{I} = \{1, .., N\}$, there exists a non-empty interval $(\phi_2, 1] \subset [0, 1]$ such that for all $\phi \in (\phi_2, 1]$ and for any two small stocks n and n' with $\eta_n, \eta_{n'} \approx 0$, their prices decrease and stock n experiences a larger percentage price decrease than stock n' if $\beta_{nt} > \beta_{n't}$.*

When passive flows are due, fully or mostly, to entry by new investors in the stock market

($\phi \in [0, \phi_1)$), the prices of all stocks increase. As in Proposition 4.2, the price increase for small stocks is fully described by CAPM beta and is larger for higher beta stocks. Unlike in Proposition 4.2, passive flows do not necessarily raise the prices of large stocks above and beyond their effect through beta. They do so, however, for those large stocks that are included in the index. Intuitively, passive flows affect index and non-index stocks differently because of their effect on the present value of idiosyncratic dividends. Only stocks that belong to the index experience an increase in that present value because passive flows lower their idiosyncratic supply and thus the discount rate for idiosyncratic dividends. The effect through idiosyncratic supply is negligible for small stocks but non-negligible for large stocks.

When passive flows are due, fully or mostly, to a switch from active to passive ($\phi \in (\phi_2, 1]$), stock prices can increase or decrease. The price change for small stocks is fully described by CAPM beta. Unlike in the case $\phi \in [0, \phi_1)$ and in Proposition 4.2, prices can decrease, in which case the decrease is larger for higher beta stocks. Moreover, passive flows do not raise the price of all large stocks above and beyond their effect through beta, but do so for those large stocks that are included in the index and are in high demand by noise traders. The prices of other large stocks can rise below the effect through beta and can even drop. Intuitively, a pure switch from active to passive raises the idiosyncratic supply for stocks that are not in the index because they are sold by experts but are not bought by non-experts. It also raises the idiosyncratic supply for stocks that are in low demand by noise traders since experts hold them with a weight larger than in the market portfolio while non-experts hold them with the market weight. As in the case where passive flows are due to entry, the effect through idiosyncratic supply is negligible for small stocks but non-negligible for large stocks.

Our calibrations indicate that the positive effect of passive flows on large stocks that are included in a large-stock index and are in high demand by noise traders overtakes any negative effects on other stocks. As a result, passive flows cause the aggregate market to rise even when they are purely due to a switch from active to passive. Proposition 4.3 does not examine how passive flows affect the aggregate market. The equations in this section provide an intuition, however. Since the discount rate for the idiosyncratic dividends of large stocks in high demand is small, the present value of those stocks' idiosyncratic dividends is highly sensitive to changes in the discount rate, generated by passive flows. The high sensitivity is reflected in the term $F_n^i = \frac{1}{\sqrt{(r+\kappa_n^i)^2+4\rho\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma_n^i)^2}}$ in (4.1) being large because η'_n is equal to η_n rather than to zero (stock n is included in the index) or because u_n is large (stock n is in high demand by noise traders). The large positive effect of passive flows on large stocks in high demand can be re-interpreted as the amplification effect described in the Introduction.[7]

5 Calibration

5.1 Parameter Values

The model parameters are the riskless rate r, the number N of stocks, the parameters $(\kappa^s, \bar{D}^s, \sigma^s)$ and $(b_n, \kappa_n^i, \bar{D}_n^i, \sigma_n^i)_{n=1,..,N}$ of the dividend processes, the supply parameters $(\eta_n, \eta'_n, u_n)_{n=1,..,N}$, the measures (μ_1, μ_2) of experts and non-experts, and the risk-aversion coefficient ρ.

We set the starting values of μ_1 and μ_2 so that their sum $\mu_1 + \mu_2$ is one. This is a normalization because we can redefine ρ. We set ρ to one. This is also a normalization because we can redefine the numeraire in the units of which wealth is expressed. Since the dividend flow is normalized by $\bar{D}_n + b_n + \bar{D}_n^i = 1$, redefining the numeraire amounts to rescaling the numbers of shares $(\eta_n, \eta'_n, u_n)_{n=1,..,N}$. We set the riskless rate r to 3%.

We set starting values $\mu_1 = 0.9$ and $\mu_2 = 0.1$, i.e., the measure of experts is nine times that of non-experts. We examine how stock prices change when μ_2 is raised to 0.6, i.e., the measure of non-experts rises six-fold. We consider two polar cases for the measure of experts. The first case is when flows into passive funds are entirely due to entry by new investors into the stock market ($\phi = 0$). In that case, the measure μ_1 of experts remains equal to 0.9. The second case is when flows into passive funds are entirely due to a switch by investors from active to passive ($\phi = 1$). In that case, the total measure $\mu_1 + \mu_2$ of experts and non-experts remains equal to one.

We calibrate the number N of stocks and the number η_n of shares of each stock based on the number and size distribution of publicly listed US firms. Axtell (2001) finds that the size distribu-

[7]The amplification effect can be seen formally through (3.10). Passive flows lower the idiosyncratic supply $\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}$ of a stock n in high demand. Holding constant a_{n1}^i in the right-hand side of (3.10), this raises a_{n1}^i in the left-hand side. Transposing the rise in a_{n1}^i to the right-hand side generates a further rise in a_{n1}^i in the left-hand side for a stock n that is in high demand and sold short by experts because $\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}$ is negative, and so on.

tion of all US firms, with size measured by sales or number of employees, is well approximated by a power law with exponent one.[8] Under that power law, if an interval $[x, \phi x]$ with $\phi > 1$ includes a fraction f of firms and their average size is s, then the adjacent interval $\left[\phi x, \phi^2 x\right]$ includes a fraction $\frac{f}{\phi}$ of firms and their average size is ϕs. Motivated by this scaling property, we set $\phi = 5$ and assume five size groups. Size group 5, the top group, includes six stocks, each of which is issued in $625 \times \eta$ shares. Size group 4 includes 30 ($= 5 \times 6$) stocks, each of which is issued in $125 \times \eta$ ($= \frac{1}{5} \times 625 \times \eta$) shares. Size group 3 includes 150 ($= 5 \times 30$) stocks, each of which is issued in $25 \times \eta$ ($= \frac{1}{5} \times 125 \times \eta$) shares. Size group 2 includes 750 ($= 5 \times 150$) stocks, each of which is issued in $5 \times \eta$ ($= \frac{1}{5} \times 25 \times \eta$) shares. Size group 1, the bottom group, includes 750 stocks, each of which is issued in η ($= \frac{1}{5} \times 5 \times \eta$) shares. We drop the scaling property for group 1 to better fit the data.

The five size groups in our calibration are defined based on the aggregate dividends that firms pay to their shareholders. Indeed, since the long-run mean of the dividend flow per share is normalized to one for each firm, the long-run mean of aggregate dividend flow for each firm is equal to the firm's number of shares. Market capitalization varies monotonically across size groups, with its ratio between two stocks in consecutive groups being close to five, as is the case for the number of shares. Constructing the market capitalization ratios in the data as in our calibration, we find values close to five as well.[9]

We consider three cases for index composition. The baseline is when the index includes all stocks and is thus the true market portfolio, i.e., $\eta'_n = \eta_n$ for all n. The second case is when the index includes only the stocks in our top three size groups, i.e., $\eta'_n = \eta_n$ for the 186 stocks in size groups 3, 4 and 5, and $\eta'_n = 0$ for the 1,500 stocks in size groups 1 and 2. That index can be

[8]For a survey on power laws and their relevance to Economics, see Gabaix (2016).

[9]As of 2 April 2024, average market capitalization was \$2.175tn for the top six publicly listed US firms (Microsoft, Apple, NVIDIA, Alphabet, Amazon, Meta), \$379.2bn for the next 30 firms, \$94.14bn for the next 150 firms, \$16.06bn for the next 750 firms and \$2.887bn for the next 750 firms. The combined market capitalization of all 3,605 publicly listed US firms was \$53.54tn. The combined market capitalization of the 1,686 (=6+30+150+750+750) firms in our size groups 1, 2, 3, 4 and 5 was \$52.76tn. The market capitalization ratios are 5.74 ($= \frac{2175}{379.2}$) between size groups 5 and 4, 4.03 ($= \frac{379.2}{94.14}$) between size groups 4 and 3, 5.86 ($= \frac{94.14}{16.06}$) between size groups 3 and 2, and 5.56 ($= \frac{16.06}{2.887}$) between size groups 2 and 1. The counterparts of these ratios generated by our model are 4.51, 4.86, 4.97 and 4.99 in the baseline of the constant-b_n calibration, and 5.01, 5.30, 5.46 and 5.54 in the baseline of the varying-b_n calibration. If size group 1 is enlarged to include the remaining 1,919 (=3,605-1,686) publicly listed US firms, then the ratio between size groups 2 and 1 in the data jumps up to 14.53. The stocks in our size groups 1, 2, 3, 4 and 5 account for more than 98.5% of the market capitalization of all publicly listed US firms. All market capitalization data come from `https://companiesmarketcap.com/usa/largest-companies-in-the-usa-by-market-cap/`.

interpreted as a large-stock index such as the Russell 200 or the S&P500.[10] The third case is when the index includes only the stocks in our bottom three size groups, i.e., $\eta'_n = \eta_n$ for the 1,650 firms in size groups 1, 2 and 3, and $\eta'_n = 0$ for the 36 firms in size groups 4 and 5.

We consider two cases for noise-trader demand u_n. The baseline is when u_n is equal to zero for all stocks and thus there are no noise traders. The second case is when u_n is equal to zero for one-half of the stocks in each size group, and to 30% of the shares issued for the remaining half ($u_n = 30\% \times \eta_n$). The former stocks are the low-demand ones and the latter stocks are the high-demand ones.

We set the mean-reversion parameters κ^s and $\{\kappa^i_n\}_{n=1,..,N}$ to a common value κ. We set the long-run means $\{\bar{D}^i_n\}_{n=1,..,N}$ and diffusion parameters $\{\sigma^i_n\}_{n=1,..,N}$ of the idiosyncratic components to common values $\bar{D}^i$ and σ^i, respectively. The stationary distribution of D^i_{nt} generated by the square-root process (2.3) is gamma with support $(0, \infty)$ and density

$$f(D^i_{nt}) = \frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i - 1} e^{-\beta^i D^i_{nt}}, \tag{5.1}$$

where $\alpha^i \equiv \frac{2\kappa\bar{D}^i}{(\sigma^i)^2}$, $\beta^i \equiv \frac{2\kappa}{(\sigma^i)^2}$ and Γ is the Gamma function. The stationary distribution of D^s_t generated by the square-root process (2.2) is also gamma, with density given by (5.1) in which D^i_{nt} is replaced by D^s_t, α^i by $\alpha^s \equiv \frac{2\kappa\bar{D}^s}{(\sigma^s)^2} = \frac{2\kappa}{(\sigma^s)^2}$, and β^i by $\beta^s \equiv \frac{2\kappa}{(\sigma^s)^2}$. We set $\frac{\sigma^i}{\sqrt{\bar{D}^i}} = \frac{\sigma^s}{\sqrt{\bar{D}^s}} = \sigma^s$. This ensures that the distributions of D^s_t and D^i_{nt} are the same when scaled by their long-run means: $\frac{D^i_{nt}}{\bar{D}^i}$ has the same distribution as $\frac{D^s_t}{\bar{D}^s} = D^s_t$.

We allow for correlation between size and the loading b_n of dividends on the systematic factor. We assume that for stocks in size group $m = 1, .., 5$, $b_n = \bar{b} - (m-3)\Delta b \geq 0$. Varying Δb changes the relationship between size and CAPM beta.

The parameters left to calibrate are $(\kappa, \bar{D}^i, \bar{b}, \Delta b, \sigma^s, \eta)$. We calibrate them based on stocks' unconditional expected returns, return variances, CAPM betas and CAPM R-squareds. We use as calibration targets the values of these moments for the starting measures $(\mu_1, \mu_2) = (0.9, 0.1)$ and for the baseline where the index includes all stocks and there are no noise traders. The formulas

[10] While the S&P500 accounts for a larger fraction of market capitalization than an index made of the 186 stocks in our size groups 3, 4 and 5, it leaves out a non-negligible fraction. As of 2 April 2024, the S&P500 accounted for 81.5% of the combined market capitalization of all publicly listed US firms. Our size groups 3, 4 and 5 accounted for 72.0%.

for the moments are in Appendix B and the values of the moments for $(\mu_1, \mu_2) = (0.9, 0.1)$ and for the baseline are in Table 5.1.

The effects of changing κ on return moments and other numerical results are similar to those of changing the other parameters. We set $\kappa = 4\%$. The values of $(\bar{D}^i, \bar{b}, \Delta b)$ must satisfy $\bar{b} + (m - 3)\Delta b + \bar{D}^i \leq 1$ for all $m = 1, .., 5$ because of $\bar{D}_n \geq 0$ and the normalization $\bar{D}_n + b_n + \bar{D}^i = 1$. Inequality $\bar{b} + (m-3)\Delta b + \bar{D}^i \leq 1$ for all $m = 1, .., 5$ is equivalent to $\bar{b} + 2|\Delta b| + \bar{D}^i \leq 1$. We assume that the latter inequality holds as an equality. This minimizes the constant component $\bar{D}_n \geq 0$, which becomes zero for the largest-b_n stocks. Minimizing $\bar{D}_n$ maximizes return variances, bringing them closer to their empirical counterparts as we explain below.

We consider two cases for Δb. The first case is when $\Delta b = 0$ and thus the loading b_n of dividends on the systematic factor is the same for all stocks. In this constant-b_n case, CAPM beta increases monotonically with size because the contribution of idiosyncratic dividends to beta is larger for larger stocks. While a positive relationship between size and beta is counterfactual, as the empirical relationship is negative (Fama and French (1992)), the constant-b_n case serves as a useful benchmark. Our assumption that the constant component of dividends is zero for the largest-b_n stocks implies that it is zero for all stocks in the constant-b_n case, and thus plays no role. The second case is when Δb takes the positive value $\Delta b = 0.04$ that generates a negative relationship between size and beta approximating the empirical one. In this varying-b_n case, beta is 1.38 for the stocks in size group 1 and 0.96 for the stocks in size group 5. Constructing the same size groups in the data as in our model, we find that average beta is 1.26 for size group 1 and 0.93 for size group 5 when stocks within groups are weighed according to their market capitalization.[11]

We calibrate the relative size of $\bar{b}$ and $\bar{D}^i$ based on CAPM R-squared. CAPM R-squared in the data averages to 29.71% across the stocks in all size groups when they are weighted according to their market capitalization.[12] In the constant-b_n case, this R-squared is achieved by setting $\bar{b} = 0.75$ and $\bar{D}^i = 0.25$. In the varying-b_n case, this is achieved for $\bar{b} = 0.725$ and $\bar{D}^i = 0.195$.

[11]In each quarter during the sample period of our empirical exercise in Section 6, we sort the 1,686 largest stocks into five size groups as in our model. We regress the quarterly value-weighted excess returns on the resulting five portfolios on the excess return on the market (CRSP index) to compute CAPM betas.

[12]We construct the five size groups as in the CAPM beta exercise, compute R-squared for each stock from a CAPM regression with monthly returns and a five-year lookback window, and average across stocks using market-capitalization weights.

We calibrate the supply parameter η based on stocks' expected returns. We target expected returns (in excess of the riskless rate) to average 4% across the stocks in all size groups when they are weighted according to their market capitalization. In the constant-b_n case, this is achieved for $\eta = 0.00004$. In the varying-b_n case, this is achieved for $\eta = 0.00007$.

We calibrate the diffusion parameter σ^s based on stocks' return variances. Raising σ^s (and σ^i through $\frac{\sigma^i}{\sqrt{\bar{D}^i}} = \sigma^s$) has a non-monotone effect on variances. For given values of D_t^s and $\{D_{nt}^i\}_{n=1,..,N}$, variances rise. At the same time, the stationary distributions of D_t^s and $\{D_{nt}^i\}_{n=1,..,N}$ shift weight towards very small or very large values, for which variances are low under the square-root specification. The maximum return variances that our model generates are lower than in the data because of the low variances at the extremes. Given the discrepancy, one approach is to set σ^s to the value that maximizes return variances. That value, however, yields prices that are overly low relative to the calibrated expected returns. Another approach is to use a lower value for σ^s, further undershooting return variances, but obtaining prices more in line with expected returns. The two approaches yield similar results on the effects of passive flows. We follow the former approach in Appendix E.1, setting $s^s = 2.2$, which is the value in the varying-b_n case that maximizes the average return variance across stocks in all size groups when they are weighted according to their market capitalization. We follow the latter approach in the rest of this section, setting $s^s = 0.5$.[13]

Table 5.1 shows the unconditional average of the price and the unconditional return moments for $(\mu_1, \mu_2) = (0.9, 0.1)$ and for the baseline, in the constant-b_n case (Panel A) and the varying-b_n case (Panel B). When moving from the smallest to the largest size group, expected return and CAPM beta rise in the constant-b_n case but decline in the varying-b_n case. Stocks in size group 5 have the largest CAPM R-squared, even in the varying-b_n case where their CAPM beta is the smallest. This is because Proposition 3.1 implies that stock prices are less sensitive to idiosyncratic dividend shocks when idiosyncratic supply is large.

[13]For $s^s = 2.2$, the average price across the stocks in all size groups when they are weighted according to the number of shares issued is 4.29 in the constant-b_n case and 5.16 in the varying-b_n case. In comparison, discounting expected dividends of one at the sum of the riskless rate of 3% plus the average expected return (in excess of the riskless rate) of 4% yields $\frac{1}{7\%} = 14.29$. The discrepancy arises because of the expected returns' time-variation. For $\sigma^s = 2.2$, the stationary distributions of D_t^s and $\{D_{nt}^i\}_{n=1,..,N}$ give high weight to extreme values. Moreover, expected returns are close to zero for small values of D_t^s and $\{D_{nt}^i\}_{n=1,..,N}$, but increase significantly away from zero, and average prices are primarily determined by expected returns away from zero. The discrepancy is significantly smaller for $s^s = 0.5$ because the average price is 12.39 in the constant-b_n case and 12.61 in the varying-b_n case.

Panel A: Constant-b_n case

Size Group	**Price**	**Expected Return (%)**	**Return Volatility (%)**	**CAPM Beta**	**CAPM R^2 (%)**
1 (Smallest)	12.93	3.89	12.56	1.01	26.18
2	12.91	3.90	12.56	1.01	26.28
3	12.83	3.93	12.56	1.02	26.81
4	12.46	4.07	12.53	1.07	29.32
5 (Largest)	11.24	4.62	12.45	1.24	39.78

Panel B: Varying-b_n case

Size Group	**Price**	**Expected Return (%)**	**Return Volatility (%)**	**CAPM Beta**	**CAPM R^2 (%)**
1 (Smallest)	10.42	5.34	12.51	1.38	28.82
2	11.54	4.65	11.09	1.19	27.46
3	12.59	4.11	9.93	1.04	26.41
4	13.35	3.76	8.88	0.96	27.52
5 (Largest)	13.37	3.72	7.64	0.96	37.04

Table 5.1: Price and Return Moments for $(\mu_1, \mu_2) = (0.9, 0.1)$ and the Baseline.

5.2 Passive Flows and Stock Prices—Calibration Results

5.2.1 Baseline

Table 5.2 shows how flows into passive funds affect stock prices in the baseline. We compute the percentage change of the unconditional average of the price. Computing instead the unconditional average of the percentage change yields similar results. Since the price is linear in D_t^s and D_{nt}^i, we can compute its unconditional average by setting the systematic component D_t^s and the idiosyncratic component D_{nt}^i of dividends to their long-run means, $\bar{D}^s = 1$ and $\bar{D}_n^i$.

The second and third columns of Table 5.2 report the percentage price change when μ_2 is raised to 0.6 and μ_1 is held equal to 0.9. Passive flows in these columns are due entirely to entry by new investors into the stock market ($\phi = 0$). The second column corresponds to the constant-b_n case and the third column to the varying-b_n case. The fourth and fifth columns are counterparts of the second and third columns when μ_2 is raised to 0.6 and μ_1 is lowered to 0.4. Passive flows in these columns are due entirely to a switch by investors from active to passive ($\phi = 1$).

Size Group	Entry into the Stock Market		Switch from Active to Passive	
	Constant-b_n	Varying-b_n	Constant-b_n	Varying-b_n
1 (Smallest)	7.03	7.67	0	0
2	7.08	6.63	0	0
3	7.34	5.99	0	0
4	8.45	6.17	0	0
5 (Largest)	11.78	7.53	0	0

Table 5.2: Percentage Price Change Caused by Passive Flows in the Baseline.

Consistent with Proposition 4.1, passive flows do not affect stock prices when they are due to a switch by investors from active to passive, and raise prices when they are due to entry by new investors into the stock market. When passive flows are due to entry, the percentage price increase that they generate is larger for larger stocks in the constant-b_n case, and is a J-shaped function of stock size in the varying-b_n case. These results as well are consistent with Proposition 4.1. Indeed, the proposition shows that when $\phi < 1$, the effect of passive flows is increasing in CAPM beta and is larger for large stocks holding beta constant. In the constant-b_n case, beta increases with size, so the two effects work in the same direction causing the effect of passive flows to increase with size. In the variable-b_n case, beta decreases with size. Therefore, the effect of passive flows decreases with size for small stocks but can increase for large stocks. Table 5.2 shows that the effect of size dominates that of beta for size groups 4 and especially 5.

To illustrate why the effect of size can dominate that of beta, we return to (4.6) and (4.9). When $\kappa^s = \kappa^i_n = \kappa$ and (D^s_t, D^i_{nt}) are equal to their long-run means, we can write (4.9) as

$$\frac{1}{S_{nt}}\frac{dS_{nt}}{d\mu_2} = \frac{(1-\phi)\rho(1-\hat{u})(\kappa+r)F^s}{(\mu_1+\mu_2)^2 r}\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^{N}\eta_m dR^{sh}_{mt}\right)\left(1+\gamma^i_n\frac{F^i_n - F^s}{F^s}\right), \tag{5.2}$$

where

$$\gamma^i_n \equiv \frac{\frac{\eta_n(\sigma^i_n a^i_{n1})^2\bar{D}^i_n}{S_{nt}}}{\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^{N}\eta_m dR^{sh}_{mt}\right)}$$

is the fraction of stock n's CAPM beta that is driven by stock n's idiosyncratic dividends. The percentage price rise for stock n exceeds that for stock n' if

$$\frac{1+\gamma_n^i \frac{F_n^i - F^s}{F^s}}{1+\gamma_{n'}^i \frac{F_{n'}^i - F^s}{F^s}} > \frac{\mathbb{C}\mathrm{ov}_t\left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)}{\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)}. \tag{5.3}$$

When stock n is larger than stock n' and has smaller beta, the effect of size dominates that of beta if the left-hand side of (5.3) exceeds the right-hand side, which exceeds one. In the varying-b_n case, the fraction γ_n^i of beta that is driven by idiosyncratic dividends is 8.04% for stocks in size group 5 and 2.43%, 0.53%, 0.10% and 0.02% for stocks in size groups 4, 3, 2 and 1, respectively. The ratio $\frac{a_{n1}^i}{a_1^s}$ of the discount rate for systematic dividends to that for idiosyncratic dividends takes the values 5.15, 6.33, 6.72, 6.81 and 6.83 for stocks in size groups 5, 4, 3, 2 and 1, respectively. The corresponding ratio $\frac{F_n^i - F^s}{F^s}$ takes the values 6.66, 9.92, 11.26, 11.59 and 11.66. Therefore, the term $\gamma_n^i \frac{F_n^i - F^s}{F^s}$ is 53.52% (=8.04% $\times$ 6.66) for stocks in size group 5 and 24.06%, 5.94%, 1.20% and 0.23% for stocks in size groups 4, 3, 2 and 1, respectively. The left-hand side of (5.3) for a stock n in size group 5 and a stock n' in size group 3 is thus 1.45 ($=\frac{1+53.52\%}{1+5.94\%}$) and exceeds the ratio of betas of stock n' to stock n in the right-hand side.

The effect of passive flows on the aggregate market in Table 5.2 translates into a demand elasticity higher than in the literature. Suppose that the measure μ_2 of non-experts increases from 0.1 to 0.6, holding the measure μ_1 of experts equal to 0.9. In the constant-b_n case, the aggregate market rises by 8.48% and non-experts' holdings (equal to $\mu_2\lambda$ times the value of the market) increase by 33.39% of the market's initial value. The resulting elasticity is 3.93 ($=\frac{33.39}{8.48}$). In the varying-b_n case, the elasticity is 4.92. By contrast, Gabaix and Koijen (2021) estimate an elasticity of 0.2 for the aggregate market, while the literature on index additions estimates elasticities ranging from 0.4 to 4 for individual firms. Our model might be generating a high elasticity for two reasons. First, the fraction of truly active investors might be smaller than in our calibration because many active funds in practice have constraints limiting their deviations from benchmark indices. Second, the elasticity estimates in the literature mostly concern short-run elasticities, while the elasticities in our model are long-run.

5.2.2 Partial Index

Table 5.3 shows how flows into passive funds affect stock prices when the index includes only the stocks in the top three size groups (Panel A) and when it includes only the stocks in the bottom three size groups (Panel B). The columns are as in Table 5.2.

Panel A: Index Includes Only Top Three Size Groups

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	6.90	7.50	-0.42	-0.56
2	6.91	6.44	-0.60	-0.69
3	7.32	6.00	0.04	0.13
4	8.80	6.59	1.50	1.67
5 (Largest)	13.43	9.03	5.98	5.32

Panel B: Index Includes Only Bottom Three Size Groups

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	7.02	7.66	-0.02	-0.01
2	7.11	6.67	0.15	0.17
3	7.56	6.21	0.97	0.93
4	7.29	5.09	-4.12	-3.64
5 (Largest)	8.13	4.80	-9.54	-6.58

Table 5.3: Percentage Price Change Caused by Passive Flows into a Partial Index.

When passive flows are due to entry by new investors into the stock market, their effect on small stocks is approximately independent of index composition and same as in the baseline. The effect on large stocks, by contrast, depends significantly on index composition. Stocks in size group 4 and especially 5 rise significantly more when the index includes only size groups 3, 4 and 5 than when it includes only size groups 1, 2 and 3, with the baseline effect being in-between. The comparisons to the baseline carry through to the case where passive flows are due to a switch from active to passive. The effect on small stocks is approximately independent of index composition and equal to zero, as in the baseline. By contrast, stocks in size group 4 and especially 5 rise significantly when the index includes only size groups 3, 4 and 5, and drop significantly when it includes only size groups 1, 2 and 3. These results reflect the result of Proposition 4.3 that the effects of passive flows depend on index inclusion for large stocks but not for small stocks.

Table 5.3 yields two additional implications. First, passive flows have a disproportionately large effect (relative to simple CAPM logic) on the largest stocks in an index only when these stocks are also the largest in the economy. Indeed, the rising part of the J-shape shown in Table 5.2 in the varying-b_n case does not arise among index stocks when the index includes only size groups 1, 2 and 3, but arises when the index includes only size groups 3, 4 and 5.

The second implication of Table 5.3 is that passive flows into an index that does not include all stocks affect the valuation of the aggregate market even when the flows are a pure switch from active to passive. A pure switch from active to the passive large-stock index raises the aggregate market by 1.49% in the constant-b_n case and 1.63% in the varying-b_n case. A pure switch from active to the passive small-stock index lowers the aggregate market by 2.75% in the constant-b_n case and 2.31% in the varying-b_n case. These movements arise because the switch has negligible effects on small stocks but significant effects on large stocks.

5.2.3 Noise Traders

Table 5.4 shows how flows into passive funds affect stock prices when stocks differ in noise-trader demand. There are ten groups of stocks: five size groups and two demand subgroups within each size group. The columns are as in Table 5.2 with the addition of a column that indicates whether a stock is in high or low demand.

The effects of passive flows are approximately independent of noise-trader demand for small stocks. For large stocks instead, especially in size group 5, passive flows have larger effects on high-demand stocks. These results reflect the result of Proposition 4.3 that the effects of passive flows depend on noise-trader demand for large stocks but not for small stocks.

Table 5.4 implies additionally that passive flows in the presence of noise traders raise the aggregate market even when they are a pure switch from active to passive. The positive effects on large high-demand stocks thus exceed the negative effects on large low-demand stocks. Intuitively, because the discount rate for idiosyncratic dividends is lower for the high-demand stocks, the present

Size Group	Noise-Trader Demand	Entry into the Stock Market		Switch from Active to Passive	
		Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	Low	7.27	7.95	-0.02	-0.02
	High	7.27	7.95	-0.00	-0.01
2	Low	7.32	6.89	-0.04	-0.05
	High	7.31	6.89	0.02	0.03
3	Low	7.54	6.20	-0.17	-0.18
	High	7.52	6.18	0.15	0.16
4	Low	8.49	6.22	-0.68	-0.63
	High	8.47	6.25	0.73	0.70
5 (Largest)	Low	11.31	7.24	-1.90	-1.38
	High	11.71	7.78	2.59	2.03

Table 5.4: Percentage Price Change Caused by Passive Flows with Noise Traders.

value of their idiosyncratic dividends is more sensitive to changes in the discount rate. The effects of the aggregate market are smaller than in the case of a partial index: a pure switch from active to the passive raises the market by 0.09% in both the constant-b_n and the varying-b_n case.

The effects of passive flows on large high-demand stocks become particularly large and asymmetric when passive flows are into a large-stock index. This is shown in Table 5.5, in which the index is assumed to include only the stocks in the top three size groups. The columns are as in Table 5.4.

In both the constant-b_n and the varying-b_n case, a pure switch from active to passive has large positive effects on high-demand stocks in size group 5, and significantly smaller effects on all other stocks. Because of the asymmetrically large effects on the large high-demand stocks, the aggregate market rises, by 1.47% in the constant-b_n and 1.68% in the varying-b_n case.

5.2.4 Return Volatility

Since passive flows raise the present value of the idiosyncratic component of dividends of large stocks in high demand, they cause movements to that component to become larger. Those movements also become larger relative to the stocks' price provided that the price does not increase by as much. When passive flows are due to a switch from active to passive, the change in the market

Size Group	Noise-Trader Demand	Entry into the Stock Market		Switch from Active to Passive	
		Constant-b_n	Varying-b_n	Constant-b_n	Varying-b_n
1 (Smallest)	Low	7.15	7.80	-0.43	-0.59
	High	7.15	7.79	-0.42	-0.57
2	Low	7.16	6.72	-0.60	-0.71
	High	7.16	6.71	-0.54	-0.64
3	Low	7.51	6.19	-0.21	-0.16
	High	7.49	6.18	0.13	0.21
4	Low	8.76	6.55	0.40	0.57
	High	8.77	6.61	2.08	2.28
5 (Largest)	Low	12.56	8.37	2.05	1.99
	High	13.37	9.39	10.03	9.58

Table 5.5: Percentage Price Change Caused by Passive Flows into a Partial Index with Noise Traders.

risk premium is small and so is the change in the present value of the systematic component of dividends. Therefore, the idiosyncratic volatility of large stocks in high demand rises. When instead passive flows are due to entry by new investors in the stock market, the present value of the systematic component of dividends rises significantly, and idiosyncratic volatility can fall. In both cases, however, the idiosyncratic volatility of large stocks in high demand rises more (or falls less) than for small stocks because passive flows do not affect the present value of the idiosyncratic component of small stocks' dividends.

Table 5.6 shows the effect of passive flows on idiosyncratic volatility when flows are due to a switch from active to passive. The table confirms that the idiosyncratic volatility of large stocks in high demand rises more than for other stocks, and especially so when the index is a large-stock one. Idiosyncratic volatility averaged across size groups also rises more for large stocks. Table E.6 is the counterpart of Table 5.6 when flows are due to entry by new investors in the stock market. Idiosyncratic volatility can rise or fall, but rises more (or falls less) for large stocks.

Size Group	Noise-Trader Demand	All-Stock Index		Large-Stock Index	
		Constant-b_n	Varying-b_n	Constant-b_n	Varying-b_n
1 (Smallest)	Low	0.00	0.00	0.06	0.09
	High	0.00	0.00	0.06	0.09
2	Low	0.00	0.00	0.05	0.06
	High	0.00	0.00	0.05	0.06
3	Low	0.00	-0.01	0.07	0.07
	High	0.00	0.01	0.08	0.10
4	Low	-0.03	-0.05	0.10	0.12
	High	0.03	0.05	0.16	0.24
5 (Largest)	Low	-0.08	-0.12	0.18	0.24
	High	0.11	0.16	0.43	0.73

Table 5.6: Change in Idiosyncratic Volatility Caused by Passive Flows from Active.

6 Empirical Evidence

In this section we present tests of our theory and relate our results to empirical findings in the literature. We take the index to be the S&P500 index, and passive flows to be the flows in or out of US listed index mutual funds and index ETFs tracking it. The S&P500 index accounts for the bulk of passive investing in US stocks: index mutual funds tracking the S&P500 index account for 47% to 87% of assets of all index mutual funds invested in US stocks in our sample. We refer to index mutual funds and index ETFs tracking the S&P500 index as S&P500 index funds.

6.1 Data and Descriptive Statistics

Our data on stock returns, market capitalization, and the composition of the S&P500 index come from the Center for Research in Security Prices (CRSP). Our data on net assets of S&P500 index mutual funds come from the Investment Company Institute (ICI). Our data on net assets of S&P500 index ETFs come from CRSP. We include in our analysis only plain-vanilla ETFs, excluding alternative ETFs such as leveraged ETFs, inverse ETFs and buffered ETFs. Our ETF sample consists of the SPDR S&P500 ETF Trust, the iShares Core S&P500 ETF, and the Vanguard S&P500 Index Fund ETF, which collectively account for almost all of the plain-vanilla S&P500 ETF market. Our sample begins in the second quarter of 1996 and ends in the fourth quarter of 2020.

Table 6.1 reports descriptive statistics. The descriptive statistics in Panel A concern aggregate variables, measured at a quarterly frequency. The descriptive statistics in Panel B concern firm-level variables pertaining to all S&P500 firms, measured at a daily or a quarterly frequency. All the variables except VIX and $\log(Vol_{Idio})$ are multiplied by 100.

	Mean	Std Dev	25th Pctl	50th Pctl	75th Pctl	Skewness	Exc Kurt	N
Panel A: Aggregate Variables								
$R_{Top\ 10}$	-0.02	3.86	-2.04	-0.45	2.59	-0.04	0.39	99
$R_{Top\ 50\ EW}$	-0.15	1.56	-1.07	-0.39	0.72	0.51	0.95	99
$R_{Top\ 50}$	-0.17	1.86	-1.27	-0.20	1.09	-0.16	0.50	99
$R_{Top\ 100}$	-0.17	1.28	-0.91	-0.23	0.60	-0.61	2.39	99
$R_{Top\ 150}$	-0.19	0.95	-0.69	-0.19	0.42	-0.49	1.76	99
$R_{Top\ 200}$	-0.17	0.78	-0.61	-0.19	0.37	-0.47	1.28	99
R_{Index}	2.68	8.50	-0.77	3.41	7.60	-0.56	0.55	99
$Passive\ Flow$	0.05	0.09	0.01	0.05	0.10	0.33	3.62	99
$\Delta\log(w_{Top\ 10})$	0.51	3.87	-1.98	0.41	3.17	0.20	0.69	99
$\Delta\log(Dispersion)$	0.51	3.58	-1.90	0.45	2.37	0.41	1.23	99
$\Delta\log(HHI)$	0.81	5.61	-2.85	0.58	3.55	0.52	1.57	99
VIX	20.36	7.59	14.57	19.31	24.92	1.80	6.03	99
Panel B: Firm-Level Variables for All Firms								
R_{Daily}	0.05	2.45	-0.96	0.02	1.04	0.47	41.66	2,974,228
$\log(Vol_{Idio})$	-4.28	0.50	-4.64	-4.31	-3.95	0.34	0.40	45,737

Table 6.1: Descriptive Statistics.

The first six rows in Panel A concern quarterly returns on large-stock portfolios in excess of the index return, and the seventh row concerns the return on the index. We compute the excess return on the portfolio of the top 10 firms in the index according to market capitalization, the top 50 firms, the top 100 firms, the top 150 firms and the top 200 firms. We measure market capitalization at the end of the previous quarter. We compute value-weighted returns except in the case of the top 50 firms where we also compute equally-weighted returns as a robustness check.

The eighth row concerns passive flows. We measure flows into S&P500 index funds in any given quarter by the ratio of S&P500 index fund net assets to index market capitalization (i.e., combined capitalization of all S&P500 stocks) minus the same ratio in the previous quarter:

$$PassiveFlow_t = \frac{\$SP500IndexAssets_t}{\$SP500IndexCap_t} - \frac{\$SP500IndexAssets_{t-1}}{\$SP500IndexCap_{t-1}}.$$

The mean of passive flow is 0.05% quarterly. Cumulating over the 99 quarters of our sample, we find that an extra 5% of market capitalization is held by S&P500 index funds at the end of our sample period relative to the beginning.

The ninth, tenth and eleventh rows concern the first difference of the natural logarithm of three measures of index concentration: the combined portfolio weight of the stocks of the top 10 firms in the index, denoted by w_{Top10}, the standard deviation of index weights across all S&P500 firms, denoted by $Dispersion$, and the Herfindahl index of index weights across all S&P500 firms, denoted by HHI. Index concentration has been growing during our sample period, by rates ranging from 0.51% to 0.81% per quarter. The twelfth row concerns VIX, the CBOE volatility index.

The first and second rows in Panel B concern daily returns and the natural logarithm of idiosyncratic volatility, respectively, for all index firms. Idiosyncratic volatility is the quarterly standard deviation of daily residual stock returns from the Fama-French three-factor model.

6.2 Tests

Table 6.2 reports the results from regressing excess returns on large-stock portfolios on passive flows. Results in Panel A concern the top 50 firms, value- or equally-weighted, with and without controls. Results in Panel B concern all large-stock portfolios, value-weighted, with controls. Controls are the S&P500 return, the one-quarter lagged S&P500 return and VIX.

For ease of interpretation, we standardize $PassiveFlow$ to have a mean of zero and a standard deviation of one. We denote the resulting variable $\widehat{Passive\,Flow}$. We use the same notation for VIX and for the three measures of S&P500 index concentration. The t-statistics, in parentheses, are based on Newey-West heteroskedasticity- and autocorrelation-consistent standard errors with three lags. Our findings are robust to increasing the number of lags.

Consistent with our model, the relationship between passive flows and excess returns on large firms is positive and significant economically and statistically. Panel A shows that an one-standard-deviation increase in $Passive\,Flow$ is associated with an increase in the quarterly excess return on the top-50 firm portfolio by an amount ranging from 0.528% to 0.557%, depending on whether returns are value- or equally-weighted and controls are added or not. This is approximately one-third of the quarterly standard deviation of excess return in Table 6.1. The t-statistic ranges from 3.65 to 4.19. Panel B shows that the effect of $Passive\,Flow$ becomes strongest when limiting the

Panel A: Top 50 Firms

Variables	$R_{Top\,50\,EW}$	$R_{Top\,50}$	$R_{Top\,50\,EW}$	$R_{Top\,50}$
$\widehat{Passive\,Flow}$	0.00557	0.00553	0.00531	0.00528
	(3.65)	(3.69)	(4.19)	(3.66)
Constant	-0.00150	-0.00168	-0.000253	-0.00137
	(-0.92)	(-0.80)	(-0.12)	(-0.54)
Observations	99	99	99	99
Controls	N	N	Y	Y
Adjusted R-squared	0.127	0.088	0.210	0.125

Panel B: All Large-Stock Portfolios

Variables	$R_{Top\,10}$	$R_{Top\,50}$	$R_{Top\,100}$	$R_{Top\,150}$	$R_{Top\,200}$
$\widehat{Passive\,Flow}$	0.00687	0.00528	0.00303	0.00208	0.00145
	(2.46)	(3.66)	(3.02)	(2.28)	(1.64)
Constant	-0.00156	-0.00137	-0.00177	-0.00189	-0.00149
	(-0.32)	(-0.54)	(-1.00)	(-1.40)	(-1.34)
Observations	99	99	99	99	99
Controls	Y	Y	Y	Y	Y
Adjusted R^2	0.049	0.125	0.105	0.124	0.125

Table 6.2: Passive Flows and Excess Returns on Large Stocks.

large-firm portfolio to only the largest firms. A one-standard-deviation increase in $Passive\,Flow$ is associated with an increase in the quarterly excess return on the value-weighted portfolio of top 200 firms by 0.145%, of top 150 firms by 0.208%, of top 100 firms by 0.303%, of top 50 firms by 0.528% and of top 10 firms by 0.687%.

Converting our quarterly estimates to cumulative estimates over the length of our sample yields large effects. Recall from Table 6.1 that the mean and standard deviation of $Passive\,Flow$ are 0.05% and 0.09%, respectively. Since our sample comprises 99 quarters, the cumulative effect of $Passive\,Flow$ on the excess return on the value-weighted top-50 firm portfolio is $0.528\% \times \frac{0.05\%}{0.09\%} \times 99 = 29.04\%$. According to this estimate, the rise in passive investing over the past 25 years caused a firm that was in the top 50 of the S&P500 index during the entire period to rise by 29% more than the index.

The estimated 29% effect of passive flows in Table 6.2 is larger than in our calibration. For example, the difference between the return on size group 5 and the average return on size groups 3, 4 and 5 in Tables 5.2-5.5 ranges from zero (Table 5.2, switch from active to passive, all firms in index) to 4% (Table 5.5, switch from active to passive, size groups 3-5 in index). The discrepancy might be arising for the same two reasons mentioned in the context of elasticities in Section 5.2. First, the fraction of truly active investors might be smaller than in our calibration. Second, the 30% estimate concerns a contemporaneous effect of passive flows, which can partly mean-revert.

The finding in Table 6.2 that passive flows raise the stock prices of the largest firms the most are consistent with other findings in the literature. Ben-David, Franzoni, and Moussawi (2018) find that increases in a firm's ownership by ETFs have significantly larger effects on the firms in the S&P500 than on the smaller firms in the Russell 3000 (Table IV). Haddad, Huebner, and Loualiche (2025) find that demand elasticities are smaller for large firms than for smaller firms (Figure 3), implying that an increase in demand proportional to firms' market capitalization causes the stocks of large firms to rise the most.

We corroborate the findings in Table 6.2 through two additional tests, reported in Appendix E.3. First, we regress changes in index concentration on passive flows. Consistent with our model, the relationship between passive flows and changes in concentration is positive and significant economically and statistically. Second, we use a beginning-of-month dummy as a proxy to capture exogenous variation in passive flows. Since many US households invest a fraction of their monthly paychecks (together with the contributions from their employers) in passive funds through retirement plans such as 401(K), passive flows increase at the beginning of each month. Consistent with our model, returns on large stocks rise more than the index at the beginning of each month, with the effect becoming strongest when limiting the large-firm portfolio to only the largest firms.[14]

Our model predicts additionally that passive flows should raise the idiosyncratic return volatility of the largest firms more than that of smaller firms. To test for this prediction, we perform panel regressions of idiosyncratic volatility on one-quarter lagged passive flow interacted with a large firm

[14] Beginning-of-month passive flows can generate return predictability even though they themselves are predictable. Vayanos and Woolley (2013) show theoretically that rational prices do not fully reflect expected future flows if flows are uncertain. Hartzmark and Solomon (2024) find empirical evidence consistent with predictable flows generating return predictability.

indicator. The indicator is equal to one if the firm belongs to the top 50 and to zero otherwise. As additional variables in the regressions we include the two constituents of the interaction term, the one-quarter lagged index return, the logarithm of one-quarter lagged idiosyncratic volatility and firm fixed effects. Alternatively, we introduce time fixed effects to absorb the time-series variation, and drop lagged passive flow and index return. We conservatively double-cluster standard errors by firm and time. The regression results are in Table 6.3.

	$\log(Vol_{Idio})$	$\log(Vol_{Idio})$
L.$Passive\,Flow \times Large$	19.32	18.46
	(2.53)	(2.45)
L.$Passive\,Flow$	20.63	
	(1.21)	
L.$Large$	-0.0477	-0.0671
	(-2.86)	(-4.83)
Observations	45,737	45,737
Controls	Y	Y
Firm fixed effects	Y	Y
Time fixed effects	N	Y
Adjusted R^2	0.601	0.712

Table 6.3: Passive Flows and Idiosyncratic Return Volatility

Consistent with our model, passive flows impact more strongly the idiosyncratic return volatility of the largest firms, and this effect is significant economically and statistically. An one-standard-deviation increase in $PassiveFlow$ is associated with an increase in idiosyncratic volatility by 1.86% (=20.63 $\times$ 0.09%) for firms outside the top 50, and this effect approximately doubles to 3.60% $(=(19.32 + 20.63) \times 0.09\%)$ for firms in the top 50. Moreover, the incremental effect for large firms is statistically significant while the effect for other firms is not.

7 Conclusion

The growth of passive investing over the past thirty years and its effects on asset prices and the real economy have attracted attention by academics and policy-makers. In this paper we show that flows into passive funds disproportionately raise the prices of the economy's largest firms. Large firms are thus less liquid than small firms, in the sense that an increase in demand proportional to firms' market capitalization causes large firms' stock prices to rise the most. The effects of passive flows that we show in our model arise even when the indices tracked by passive funds include all firms, and can be sufficiently strong to cause the aggregate market to rise even when flows are entirely due to investors switching from active to passive. Our model implies additionally that passive flows raise the idiosyncratic return volatility of large firms more than of smaller firms. Consistent with our theory, we find that the prices and idiosyncratic volatilities of the largest firms in the S&P500 index rise the most following flows into that index.

Our theory implies that passive investing reduces primarily the financing costs of the largest firms in the economy and makes the size distribution of firms more skewed. Quantifying these effects is a natural extension of our research. A quantification exercise would also determine the contribution of the rise in passive investing to recent macroeconomic trends such as the rise in industry concentration and the decline in corporate investment. Some papers quantifying these trends emphasize heterogeneity in financing costs, which they often model through borrowing constraints. Our theory links this heterogeneity to stock-market distortions, which can be a more relevant channel for large firms.

An additional extension of our research concerns the design of indices. Passive funds in our model track capitalization-weighted indices. While such indices are the most common in practice, other types of indices, such as price-weighted or equal-weighted, also exist. It would be interesting to determine how indices should be designed to achieve welfare objectives. If the growth of passive funds reduces primarily the financing costs of the largest or overvalued firms, and this leads to welfare-reducing industry concentration or capital misallocation, then should capitalization-weighting be moderated? Should upper bounds be imposed on weights, as is the case for some sovereign-bond indices? Is capitalization-weighting the best solution despite its drawbacks?

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Appendix – For Online Publication

A Proofs

Proof of Proposition 3.1. The quadratic equation derived from (3.10) by identifying terms in D_t^s is

$$\rho\left(\sum_{m=1}^{N}\frac{\eta_m-\mu_2\lambda\eta_m'-u_m}{\mu_1}b_m\right)(\sigma^s a_1^s)^2+(r+\kappa^s)a_1^s-1=0. \tag{A.1}$$

The quadratic equation derived by identifying terms in D_{nt}^i is

$$\rho\frac{\eta_n-\mu_2\lambda\eta_n'-u_n}{\mu_1}(\sigma_n^i a_{n1}^i)^2+(r+\kappa_n^i)a_{n1}^i-1=0. \tag{A.2}$$

The equation derived by identifying the remaining terms is

$$\bar{D}_n-r\bar{S}_n+b_n\left(\kappa^s a_1^s-ra_0^s\right)+\kappa_n^i a_{n1}^i\bar{D}_n^i-ra_{n0}^i=0. \tag{A.3}$$

When $\sum_{m=1}^{N}(\eta_m-\mu_2\lambda\eta_m'-u_m)b_m\geq 0$, the left-hand side of (A.1) is increasing for positive values of a_1^s, and (A.1) has a unique positive solution, given by (3.12). When $\sum_{n=1}^{N}(\eta_m-\mu_2\lambda\eta_m'-u_m)b_n<0$, the left-hand side of (A.1) is hump-shaped for positive values of a_1^s, and (A.1) has either two positive solutions (including one double positive solution) or no solution. When two solutions exist, (3.12) gives the smaller of them, which is the continuous extension of the unique positive solution when $\sum_{m=1}^{N}(\eta_m-\mu_2\lambda\eta_m'-u_m)b_m>0$. Equation (3.13) gives the analogous solution of (A.2). Equation (A.3) yields

$$\bar{S}_n+b_n a_0^s+a_{n0}^i=\frac{\bar{D}_n+b_n\kappa^s a_1^s+\kappa_n^i a_{n1}^i\bar{D}_n^i}{r}. \tag{A.4}$$

Substituting (3.8), (3.9) and (A.4) into (3.1), we find (3.11).

Substituting $\mu_n = \mathbb{E}(\mu_{nt})$ and (3.7)-(3.9) into the first-order condition

$$\sum_{n=1}^{N} \eta'_n \mu_n = \rho\lambda \left[\left(\sum_{n=1}^{N} \eta'_n b_n \right)^2 (\sigma^s)^2 \mathbb{E}\left[D_t^s [(S^s)'(D_t^s)]^2 \right] + \sum_{n=1}^{N} \left(\eta'_n \right)^2 (\sigma_n^i)^2 \mathbb{E}\left[D_{nt}^i [(S_n^i)'(D_{nt}^i)]^2 \right] \right] \tag{A.5}$$

of non-experts, we find

$$\begin{aligned}
&\left(\sum_{m=1}^{N} \eta'_m b_m \right) \left(\sum_{m=1}^{N} \frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} b_m \right) (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \eta'_m \left(\frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} \right) (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \\
&= \lambda \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right], \qquad \text{(A.6)}
\end{aligned}$$

which we can rewrite as (3.14). Since $\eta_m > u_m$ for all m, (3.14) implies $\lambda > 0$.

Equations (3.11)-(3.13) imply that the price depends on $(\mu_1, \mu_2, \sigma^s, \{b_m, \sigma_m^i, \eta_m, \eta'_m, u_m\}_{m=1,..,M})$ only through $\left(\sum_{m=1}^{N} \frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} b_m \right) (\sigma^s)^2$ and $\frac{\eta_n - \mu_2 \lambda \eta'_n - u_n}{\mu_1} (\sigma_n^i)^2$. The price is decreasing and convex in the latter two variables if a_1^s is decreasing and convex in $\left(\sum_{m=1}^{N} \frac{\eta_m - \mu_2 \lambda \eta'_m - u_m}{\mu_1} b_m \right) (\sigma^s)^2$, and a_{n1}^i is decreasing and convex in $\frac{\eta_n - \mu_2 \lambda \eta'_n - u_n}{\mu_1} (\sigma_n^i)^2$. These properties hold if the function

$$\Psi(z) \equiv \frac{1}{A + \sqrt{B + Cz}}$$

is decreasing and convex for $z \geq -\frac{B}{C}$, where (A, B, C) are positive constants. The function $\Psi(z)$ is decreasing because its derivative

$$\Psi'(z) = -\frac{C}{2\sqrt{B + Cz}} \frac{1}{\left(A + \sqrt{B + Cz} \right)^2}$$

is negative. Since, in addition, $\Psi'(z)$ is increasing, $\Psi(z)$ is convex.

An equilibrium exists if (A.6), in which a_1^s and $\{a_{n1}^i\}_{n=1,..,N}$ are implicit functions of λ defined by (3.12) and (3.13), respectively, has a solution. For all non-positive values of λ, both sides of (A.6) are well-defined because the non-negativity of $\sum_{m=1}^{N} (\eta_m - \mu_2 \lambda \eta'_m - u_m) b_m$ and $\eta_n - \mu_2 \lambda \eta'_n - u_n$ ensures that (3.12) and (3.13) have a solution for a_1^s and $\{a_{n1}^i\}_{n=1,..,N}$, respectively. Moreover, the left-hand side of (A.6) is positive, and exceeds the right-hand side which is non-positive. An equilibrium exists if both sides of (A.6) remain well-defined for a sufficiently large positive value of λ that renders them equal.

If an equilibrium exists, then it is unique. Indeed, since the function $\Psi(z)$ is decreasing, (3.12) and (3.13) imply that the right-hand side of (A.6) is increasing in λ for positive values of λ. Equations (3.12) and (3.13) also imply that the left-hand side of (A.6) is decreasing in λ if the function

$$\Phi(z) \equiv \frac{z}{\left(A+\sqrt{B+Cz}\right)^2}$$

is increasing for $z \geq -\frac{B}{C}$, where (A, B, C) are positive constants. Showing that $\Phi(z)$ is increasing is equivalent to showing that

$$\hat{\Phi}(y) \equiv \frac{y^2 - B}{(A+y)^2}$$

is increasing for $y \equiv \sqrt{B+Cz} \geq 0$. The latter property follows because the functions $\hat{\Phi}_1(y) \equiv \frac{y}{A+y}$ and $\hat{\Phi}_2(y) \equiv -\frac{B}{(A+y)^2}$ are increasing for $y \geq 0$. Since the left-hand side of (A.6) is decreasing in λ and the right-hand side is increasing, a positive solution λ of (A.6) is unique. $\square$

Proof of Proposition 4.1. We first derive the second equality in (4.2). Differentiating $\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}$ with respect to μ_2 and using $\frac{d\mu_1}{d\mu_2} = -\phi$, we find

$$\begin{aligned}
&-\mu_1 \frac{d}{d\mu_2}\left(\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right) \\
&= -\mu_1 \left[\frac{d(\mu_2\lambda)}{d\mu_2}\frac{\partial}{\partial(\mu_2\lambda)}\left(\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right) - \phi\frac{\partial}{\partial\mu_1}\left(\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right)\right] \\
&= \frac{d(\mu_2\lambda)}{d\mu_2}\eta'_n + \phi\frac{\mu_2\lambda\eta'_n + u_n - \eta_n}{\mu_1}.
\end{aligned}$$

We next derive (4.1). Differentiating (3.12) with respect to μ_2, we find

$$\frac{da_1^s}{d\mu_2} = \frac{d}{d\mu_2}\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta'_m - u_m}{\mu_1} b_m\right) \frac{\partial a_1^s}{\partial\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta'_m - u_m}{\mu_1} b_m\right)}$$

$$
\begin{aligned}
&= -\left(\sum_{m=1}^{N} \frac{d}{d\mu_2}\left(\frac{\eta_m - \mu_2\lambda\eta'_m - u_m}{\mu_1}\right) b_m\right) \rho(\sigma^s a_1^s)^2 F^s \\
&= \frac{\rho}{\mu_1}\left(\sum_{m=1}^{N} \Delta_m b_m\right)(\sigma^s a_1^s)^2 F^s, \qquad \text{(A.7)}
\end{aligned}
$$

where the second step follows from (3.12) and (4.3), and the third step follows from (4.2). Differentiating (3.13) with respect to μ_2, we likewise find

$$
\begin{aligned}
\frac{da_{n1}^i}{d\mu_2} &= \frac{d}{d\mu_2}\left(\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right) \frac{\partial a_{n1}^i}{\partial\left(\frac{\eta_n - \mu_2\lambda\eta'_n - u_n}{\mu_1}\right)} \\
&= \frac{\rho}{\mu_1}\Delta_n(\sigma_n^i a_{n1}^i)^2 F_n^i, \qquad \text{(A.8)}
\end{aligned}
$$

where the second step follows from (3.12), (4.2) and (4.4). Differentiating (3.11) with respect to μ_2 and using (A.7) and (A.8), we find (4.1). $\square$

Proof of Proposition 4.2. We first derive $\Delta_n = \frac{(1-\phi)\mu_1(1-\hat{u})}{(\mu_1+\mu_2)^2}\eta_n$, which is used in the text before the proposition's statement to derive (4.7) from (4.1). When $\eta'_n = \eta_n$ and $u_n = \hat{u}\eta_n$ for all n, (3.14) implies $\lambda = \frac{1-\hat{u}}{\mu_1+\mu_2}$. Setting $\eta'_n = \eta_n$ and $u_n = \hat{u}\eta_n$ for all n and $\lambda = \frac{1-\hat{u}}{\mu_1+\mu_2}$ in (4.2), we find

$$
\begin{aligned}
\Delta_n &= \frac{d\left(\frac{\mu_2(1-\hat{u})}{\mu_1+\mu_2}\right)}{d\mu_2}\eta_n + \phi\frac{\frac{\mu_2(1-\hat{u})}{\mu_1+\mu_2}\eta_n + \hat{u}\eta_n - \eta_n}{\mu_1} \\
&= \left[\frac{\partial\left(\frac{\mu_2}{\mu_1+\mu_2}\right)}{\partial\mu_2} - \phi\frac{\partial\left(\frac{\mu_2}{\mu_1+\mu_2}\right)}{\partial\mu_1}\right](1-\hat{u})\eta_n - \frac{\phi}{\mu_1+\mu_2}(1-\hat{u})\eta_n \\
&= \frac{\mu_1+\phi\mu_2}{(\mu_1+\mu_2)^2}(1-\hat{u})\eta_n - \frac{\phi(\mu_1+\mu_2)}{(\mu_1+\mu_2)^2}(1-\hat{u})\eta_n \\
&= \frac{(1-\phi)\mu_1(1-\hat{u})}{(\mu_1+\mu_2)^2}\eta_n.
\end{aligned}
$$

The results in the proposition follow from (4.9). When $\phi = 1$, the right-hand side of (4.9) is zero and thus prices of all stocks do not change. When instead $\phi < 1$, the right-hand side of (4.9) is positive and thus prices of all stocks increase. Consider next small stocks n and n' with $\eta_n, \eta_{n'} \approx 0$. Since the second term inside the square bracket in the right-hand side of (4.9) is negligible, stock n experiences a larger percentage price increase than stock n' if $\mathbb{C}\mathrm{ov}_t\left(dR_{nt}, \sum_{m=1}^{N}\eta_m dR_{mt}^{sh}\right) >$

$\mathbb{C}\text{ov}_t\left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)$, which is equivalent to $\beta_{nt} > \beta_{n't}$. Consider next a large stock n with $\eta_n \not\approx 0$ and a small stock n' with $\eta_{n'} \approx 0$. The second term inside the square bracket in the right-hand side of (4.9) is negligible for stock n' but non-negligible for stock n. Moreover, it is positive for stock n if (4.10) holds. Therefore, if $\mathbb{C}\text{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right) \geq \mathbb{C}\text{ov}_t\left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)$, which is equivalent to $\beta_{nt} \geq \beta_{n't}$, then stock n experiences a larger percentage price increase than stock n'. $\square$

Proof of Proposition 4.3. We first compute $\frac{d\lambda}{d\mu_2}$. Differentiating (3.14) with respect to μ_2 and using $\frac{d\mu_1}{d\mu_2} = -\phi$, (A.7) and (A.8), we find

$$\begin{aligned}
&\frac{2\rho}{\mu_1}\left[\left(\sum_{m=1}^N \eta'_m b_m\right)\left(\sum_{m=1}^N (\eta_m - u_m) b_m\right)\left(\sum_{m=1}^N \Delta_m b_m\right)(\sigma^s)^4 (a_1^s)^3 F^s\right.\\
&\left.+\sum_{m=1}^N \eta'_m (\eta_m - u_m)\Delta_m (\sigma_m^i)^4 (a_{n1}^i)^3 \bar{D}_n^i F_n^i\right]\\
&= (\mu_1+\mu_2)\lambda\frac{2\rho}{\mu_1}\left[\left(\sum_{m=1}^N \eta'_m b_m\right)^2\left(\sum_{m=1}^N \Delta_m b_m\right)(\sigma^s)^4 (a_1^s)^3 F^s + \sum_{m=1}^N (\eta'_m)^2 \Delta_m (\sigma_m^i)^4 (a_{n1}^i)^3 \bar{D}_n^i F_n^i\right]\\
&\quad+\left[\frac{d\lambda}{d\mu_2}(\mu_1+\mu_2) + (1-\phi)\lambda\right]\left[\left(\sum_{m=1}^N \eta'_m b_m\right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^N \left(\eta'_m\right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i\right].
\end{aligned}\tag{A.9}$$

Substituting Δ_n by its value in (4.2) and grouping all terms in $\frac{d\lambda}{d\mu_2}$ separately from the remaining terms, we find

$$\frac{d\lambda}{d\mu_2}\left(\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3\right) = \mathcal{N}_1 + \mathcal{N}_2 + \mathcal{N}_3, \tag{A.10}$$

where

$$\begin{aligned}
\mathcal{D}_1 &\equiv (\mu_1+\mu_2)\left[\left(\sum_{m=1}^N \eta'_m b_m\right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^N \left(\eta'_m\right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i\right],\\
\mathcal{D}_2 &\equiv \frac{2\mu_2\rho}{\mu_1}\left(\sum_{m=1}^N \eta'_m b_m\right)^2\left[(\mu_1+\mu_2)\lambda\left(\sum_{m=1}^N \eta'_m b_m\right) - \left(\sum_{m=1}^N (\eta_m - u_m) b_m\right)\right](\sigma^s)^4 (a_1^s)^3 F^s,\\
\mathcal{D}_3 &\equiv \frac{2\mu_2\rho}{\mu_1}\sum_{m=1}^N (\eta'_m)^2\left[(\mu_1+\mu_2)\lambda\eta'_m - (\eta_m - u_m)\right](\sigma_m^i)^4 (a_{n1}^i)^3 \bar{D}_n^i F_n^i,
\end{aligned}$$

$$\mathcal{N}_1 \equiv -(1-\phi)\lambda \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m\right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right],$$

$$\mathcal{N}_2 \equiv -\frac{2\rho}{\mu_1} \left(\sum_{m=1}^{N} \eta'_m b_m \right) \left[(\mu_1+\mu_2)\lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) - \left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) \right] \left(\sum_{m=1}^{N} \Delta'_m b_m \right) (\sigma^s)^4 (a_1^s)^3 F^s,$$

$$\mathcal{N}_3 \equiv -\frac{2\rho}{\mu_1} \sum_{m=1}^{N} \eta'_m \left[(\mu_1+\mu_2)\lambda \eta'_m - (\eta_m - u_m) \right] \Delta'_m (\sigma_m^i)^4 (a_{n1}^i)^3 \bar{D}_n^i F_n^i,$$

and

$$\Delta'_n \equiv \lambda \eta'_n + \phi \frac{\mu_2 \lambda \eta'_n + u_n - \eta_n}{\mu_1} = \Delta_n - \mu_2 \frac{d\lambda}{d\mu_2} \eta'_n. \tag{A.11}$$

We next show that $\sum_{m=1}^{N} \Delta_m b_m$ is positive in a non-empty interval $[0, \phi_1) \subset [0,1]$, and is negative in a non-empty interval $(\phi_2, 1] \subset [0,1]$ under Assumption 4.1 and $\mathcal{I} = \{1, .., N\}$. Using (A.10) and (A.11), we find

$$\begin{aligned}
\sum_{m=1}^{N} \Delta_m b_m &= \mu_2 \frac{d\lambda}{d\mu_2} \sum_{m=1}^{N} \eta'_m b_m + \sum_{m=1}^{N} \Delta'_m b_m \\
&= \mu_2 \frac{\mathcal{N}_1 + \mathcal{N}_2 + \mathcal{N}_3}{\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3} \sum_{m=1}^{N} \eta'_m b_m + \sum_{m=1}^{N} \Delta'_m b_m \\
&= \frac{\mu_2 \left(\mathcal{N}_1 + \mathcal{N}_2 + \mathcal{N}_3 \right) \left(\sum_{m=1}^{N} \eta'_m b_m \right) + \left(\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3 \right) \left(\sum_{m=1}^{N} \Delta'_m b_m \right)}{\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3} \\
&= \frac{\mu_2 \left(\mathcal{N}_1 + \mathcal{N}_3 \right) \left(\sum_{m=1}^{N} \eta'_m b_m \right) + \left(\mathcal{D}_1 + \mathcal{D}_3 \right) \left(\sum_{m=1}^{N} \Delta'_m b_m \right)}{\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3},
\end{aligned} \tag{A.12}$$

where the last step follows because the definitions of $\mathcal{D}_2$ and $\mathcal{N}_2$ imply

$$\mu_2 \mathcal{N}_2 \left(\sum_{m=1}^{N} \eta'_m b_m \right) + \mathcal{D}_2 \left(\sum_{m=1}^{N} \Delta'_m b_m \right) = 0.$$

In the proof of Proposition 3.1 we show that the right-hand side of (A.6) increases in λ and the left-hand side decreases in λ. Therefore, the difference between the right-hand and the left-hand side increases in λ. Multiplying that difference by μ_1 yields the difference between the right-hand and the left-hand side of (A.6), which thus also increases in λ. Therefore, the denominator in (A.12) is positive and $\sum_{m=1}^{N} \Delta_m b_m$ has the same sign as the numerator. Using the definitions of $\mathcal{D}_1$, $\mathcal{N}_1$ and Δ'_n, we find

$$
\begin{aligned}
&\mu_2 \mathcal{N}_1 \left(\sum_{m=1}^{N} \eta'_m b_m \right) + \mathcal{D}_1 \left(\sum_{m=1}^{N} \Delta'_m b_m \right) \\
&= \left[-(1-\phi)\mu_2 \lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) + (\mu_1 + \mu_2) \left(\sum_{m=1}^{N} \Delta'_m b_m \right) \right] \\
&\quad \times \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&= \left[\left[\mu_1 + \phi\mu_2 + \frac{\phi(\mu_1+\mu_2)\mu_2}{\mu_1} \right] \lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) - \frac{\phi(\mu_1+\mu_2)}{\mu_1} \left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) \right] \\
&\quad \times \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&= (1-\phi)\mu_1 \lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&\quad + \frac{\phi(\mu_1+\mu_2)}{\mu_1} \left[(\mu_1+\mu_2)\lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) - \left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) \right] \\
&\quad \times \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right].
\end{aligned} \tag{A.13}
$$

Since (3.14) implies

$$
\begin{aligned}
&\left[(\mu_1+\mu_2)\lambda \left(\sum_{m=1}^{N} \eta'_m b_m \right) - \left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) \right] \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&= \left(\sum_{m=1}^{N} \eta'_m b_m \right) \left[\left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \eta'_m (\eta_m - u_m) (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&\quad - \left(\sum_{m=1}^{N} (\eta_m - u_m) b_m \right) \left[\left(\sum_{m=1}^{N} \eta'_m b_m \right)^2 (\sigma^s a_1^s)^2 + \sum_{m=1}^{N} \left(\eta'_m \right)^2 (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i \right] \\
&= \sum_{m=1}^{N} \eta'_m \left[(\eta_m - u_m) \left(\sum_{\hat{m}=1}^{N} \eta'_{\hat{m}} b_{\hat{m}} \right) - \eta'_m \left(\sum_{\hat{m}=1}^{N} (\eta_{\hat{m}} - u_{\hat{m}}) b_{\hat{m}} \right) \right] (\sigma_m^i a_{m1}^i)^2 \bar{D}_m^i,
\end{aligned}
$$

we can write (A.13) as

$$
\begin{aligned}
&\mu_2\mathcal{N}_1\left(\sum_{m=1}^{N}\eta'_m b_m\right)+\mathcal{D}_1\left(\sum_{m=1}^{N}\Delta'_m b_m\right)\\
&=(1-\phi)\mu_1\lambda\left(\sum_{m=1}^{N}\eta'_m b_m\right)\left[\left(\sum_{m=1}^{N}\eta'_m b_m\right)^2(\sigma^s a_1^s)^2+\sum_{m=1}^{N}\left(\eta'_m\right)^2(\sigma_m^i a_{m1}^i)^2\bar{D}_m^i\right]\\
&\quad+\frac{\phi(\mu_1+\mu_2)}{\mu_1}\sum_{m=1}^{N}\eta'_m\left[(\eta_m-u_m)\left(\sum_{\hat{m}=1}^{N}\eta'_{\hat{m}}b_{\hat{m}}\right)-\eta'_m\left(\sum_{\hat{m}=1}^{N}(\eta_{\hat{m}}-u_{\hat{m}})b_{\hat{m}}\right)\right](\sigma_m^i a_{m1}^i)^2\bar{D}_m^i.
\end{aligned}
\tag{A.14}
$$

Using the definitions of $\mathcal{D}_3$, $\mathcal{N}_3$ and Δ'_n, we find

$$
\begin{aligned}
&\mu_2\mathcal{N}_3\left(\sum_{m=1}^{N}\eta'_m b_m\right)+\mathcal{D}_3\left(\sum_{m=1}^{N}\Delta'_m b_m\right)\\
&=\frac{2\mu_2\rho}{\mu_1}\sum_{m=1}^{N}\eta'_m\left[\Delta'_m\left(\sum_{\hat{m}=1}^{N}\eta'_{\hat{m}}b_{\hat{m}}\right)-\eta'_m\left(\sum_{\hat{m}=1}^{N}\Delta'_{\hat{m}}b_{\hat{m}}\right)\right]\left[\eta_m-u_m-(\mu_1+\mu_2)\lambda\eta'_m\right](\sigma_m^i)^4(a_{n1}^i)^3\bar{D}_n^i F_n^i\\
&=-\frac{2\phi\mu_2\rho}{\mu_1^2}\sum_{m=1}^{N}\eta'_m\left[(\eta_m-u_m)\left(\sum_{\hat{m}=1}^{N}\eta'_{\hat{m}}b_{\hat{m}}\right)-\eta'_m\left(\sum_{\hat{m}=1}^{N}(\eta_{\hat{m}}-u_{\hat{m}})b_{\hat{m}}\right)\right]\\
&\quad\times\left[\eta_m-u_m-(\mu_1+\mu_2)\lambda\eta'_m\right](\sigma_m^i)^4(a_{n1}^i)^3\bar{D}_n^i F_n^i.
\end{aligned}
\tag{A.15}
$$

Equations (A.14) and (A.15) imply that the numerator in (A.12) is equal to

$$
\begin{aligned}
&(1-\phi)\mu_1\lambda\left(\sum_{m=1}^{N}\eta'_m b_m\right)\left[\left(\sum_{m=1}^{N}\eta'_m b_m\right)^2(\sigma^s a_1^s)^2+\sum_{m=1}^{N}\left(\eta'_m\right)^2(\sigma_m^i a_{m1}^i)^2\bar{D}_m^i\right]\\
&+\frac{\phi}{\mu_1}\sum_{m=1}^{N}\eta'_m\left[(\eta_m-u_m)\left(\sum_{\hat{m}=1}^{N}\eta'_{\hat{m}}b_{\hat{m}}\right)-\eta'_m\left(\sum_{\hat{m}=1}^{N}(\eta_{\hat{m}}-u_{\hat{m}})b_{\hat{m}}\right)\right](\sigma_m^i a_{m1}^i)^2\bar{D}_m^i\\
&\times\left(\mu_1+\mu_2-\frac{2\mu_2\rho}{\mu_1}\left[\eta_m-u_m-(\mu_1+\mu_2)\lambda\eta'_m\right](\sigma_m^i)^2 a_{n1}^i F_n^i\right).
\end{aligned}
\tag{A.16}
$$

When $\phi=0$, (A.16) becomes

$$
\mu_1\lambda\left(\sum_{m=1}^{N}\eta'_m b_m\right)\left[\left(\sum_{m=1}^{N}\eta'_m b_m\right)^2(\sigma^s a_1^s)^2+\sum_{m=1}^{N}\left(\eta'_m\right)^2(\sigma_m^i a_{m1}^i)^2\bar{D}_m^i\right]
$$

and is positive. Therefore, $\sum_{m=1}^{N} \Delta_m b_m$ is positive and remains positive by continuity in a non-empty interval $[0, \phi_1) \subset [0, 1]$. When $\phi = 1$, (A.16) becomes

$$\frac{1}{\mu_1} \sum_{m=1}^{N} \eta'_m \left[(\eta_m - u_m) \left(\sum_{\hat{m}=1}^{N} \eta'_{\hat{m}} b_{\hat{m}} \right) - \eta'_m \left(\sum_{\hat{m}=1}^{N} (\eta_{\hat{m}} - u_{\hat{m}}) b_{\hat{m}} \right) \right] (\sigma^i_m a^i_{m1})^2 \bar{D}^i_m$$
$$\times \left(\mu_1 + \mu_2 - \frac{2\mu_2 \rho}{\mu_1} \left[\eta_m - u_m - (\mu_1 + \mu_2)\lambda \eta'_m \right] (\sigma^i_m)^2 a^i_{n1} F^i_n \right),$$

which we can write under Assumption 4.1 and using (3.13) and (4.4) as

$$\frac{1}{\mu_1} \sum_{g=1}^{G} \eta'_g \sum_{\ell=1}^{L} A_{g\ell} B_{g\ell}, \tag{A.17}$$

where

$$A_{g\ell} \equiv \eta_g (1 - \hat{u}_\ell) L \left(\sum_{\hat{g}=1}^{G} \eta'_{\hat{g}} b_{\hat{g}} \right) - \eta'_g \left(\sum_{\hat{g}=1}^{G} \eta_{\hat{g}} b_{\hat{g}} \sum_{\ell=1}^{L} (1 - \hat{u}_\ell) \right),$$
$$B_{g\ell} \equiv C^2_{g\ell} \bar{D}^i_g (\mu_1 + \mu_2 - F_{g\ell}),$$
$$C_{g\ell} \equiv \frac{2\sigma^i_g}{r + \kappa^i_g + \sqrt{(r + \kappa^i_g)^2 + 4\rho \frac{\eta_g (1 - \hat{u}_\ell) - \mu_2 \lambda \eta'_g}{\mu_1} (\sigma^i_g)^2}},$$
$$F_{g\ell} \equiv \frac{4\mu_2 \rho \frac{\eta_g (1 - \hat{u}_\ell) - \mu_2 \lambda \eta'_g}{\mu_1} (\sigma^i_g)^2}{r + \kappa^i_g + \sqrt{(r + \kappa^i_g)^2 + 4\rho \frac{\eta_g (1 - \hat{u}_\ell) - \mu_2 \lambda \eta'_g}{\mu_1} (\sigma^i_g)^2}} \frac{1}{\sqrt{(r + \kappa^i_g)^2 + 4\rho \frac{\eta_g (1 - \hat{u}_\ell) - \mu_2 \lambda \eta'_g}{\mu_1} (\sigma^i_g)^2}}.$$

To show that (A.17) is negative under Assumption 4.1 and $\mathcal{I} = \{1, .., N\}$, we show that $\sum_{\ell=1}^{L} A_{g\ell} B_{g\ell}$ is negative for all $g = 1, .., G$. This property follows if we show (i) $\sum_{\ell=1}^{L} A_{g\ell} = 0$, (ii) $A_{g\ell}$ decreases in $\hat{u}_\ell$, and (iii) $B_{g\ell}$ is positive and increases in $\hat{u}_\ell$. Indeed,

$$\sum_{\ell=1}^{L} A_{g\ell} B_{g\ell} = \sum_{\ell=1}^{L} \left(A_{g\ell} - \frac{\sum_{\ell'=1}^{L} A_{g\ell'}}{L} \right) B_{g\ell}$$
$$= \sum_{\ell=1}^{L} \left(A_{g\ell} - \frac{\sum_{\ell'=1}^{L} A_{g\ell'}}{L} \right) \left(B_{g\ell} - \frac{\sum_{\ell'=1}^{L} B_{g\ell'}}{L} \right) < 0,$$

where the first step follows from (i) and the last step follows from (ii), (iii) and $\#\{\frac{u_m}{\eta_m} : m \in$

$\{1, .., N\}\} > 1$. Property (i) follows because

$$\sum_{\ell=1}^{L} \left[\eta_g (1 - \hat{u}_\ell) L \left(\sum_{\hat{g}=1}^{G} \eta'_{\hat{g}} b_{\hat{g}} \right) - \eta'_g \left(\sum_{\hat{g}=1}^{G} \eta_{\hat{g}} b_{\hat{g}} \sum_{\ell=1}^{L} (1 - \hat{u}_\ell) \right) \right]$$
$$= L \left[\eta_g \left(\sum_{\hat{g}=1}^{G} \eta'_{\hat{g}} b_{\hat{g}} \right) - \eta'_g \left(\sum_{\hat{g}=1}^{G} \eta_{\hat{g}} b_{\hat{g}} \right) \right] \sum_{\ell=1}^{L} (1 - \hat{u}_\ell)$$
$$= L \left[\eta_g \left(\sum_{\hat{g}=1}^{G} \eta_{\hat{g}} b_{\hat{g}} \right) - \eta_g \left(\sum_{\hat{g}=1}^{G} \eta_{\hat{g}} b_{\hat{g}} \right) \right] \sum_{\ell=1}^{L} (1 - \hat{u}_\ell) = 0,$$

where the last step follows from $\mathcal{I} = \{1, .., N\}$. Property (ii) follows from the definition of $A_{g\ell}$. Property (iii) follows because $C_{g\ell}$ and $\mu_1 + \mu_2 - F_{g\ell}$ are positive and increasing in $\hat{u}_\ell$. (The term $\mu_1 + \mu_2 - F_{g\ell}$ is positive because $F_{g\ell} < \mu_2$ and is increasing in $\hat{u}_\ell$ because $F_{g\ell}$ is decreasing in $\hat{u}_\ell$.) Therefore, $\sum_{m=1}^{N} \Delta_m b_m$ is negative and remains negative by continuity in a non-empty interval $(\phi_2, 1] \subset [0, 1]$.

We next show that

$$\hat{\Delta} \equiv \frac{d(\mu_2 \lambda)}{d\mu_2} + \phi \frac{\mu_2 \lambda + \max_m \frac{u_m}{\eta_m} - 1}{\mu_1} \tag{A.18}$$

is positive. For $\phi = 0$, $\hat{\Delta} = \frac{d(\mu_2\lambda)}{d\mu_2}$. This is positive because $\sum_{m=1}^{N} \Delta_m b_m$ is positive and (4.2) implies $\sum_{m=1}^{N} \Delta_m b_m = \frac{d(\mu_2\lambda)}{d\mu_2} \sum_{m=1}^{N} \eta'_m b_m$. To show that $\hat{\Delta}$ is positive for $\phi > 0$, we proceed by contradiction and assume $\hat{\Delta} \leq 0$. For $n \in \mathcal{I}$, (4.2) implies

$$\Delta_n = \left(\frac{d(\mu_2 \lambda)}{d\mu_2} + \phi \frac{\mu_2 \lambda + \frac{u_n}{\eta_n} - 1}{\mu_1} \right) \eta_n \leq \hat{\Delta} \eta_n \leq 0,$$

with the first inequality being strict for $n \notin \operatorname{argmax}_m \frac{u_m}{\eta_m}$. For $n \notin \mathcal{I}$, (4.2) implies

$$\Delta_n = \phi \frac{u_n - \eta_n}{\mu_1} < 0.$$

Therefore, $\Delta_n \leq 0$ for all n, with the inequality being strict for some n because $\mathcal{I} \subsetneq \{1, .., N\}$ or $\#\{\frac{u_m}{\eta_m} : m \in \{1, .., N\}\} > 1$. Combining this result with (4.2), (A.7), (A.8) and the function $\Phi(z)$ being increasing, we find that the derivative of the left-hand side of (A.6) with respect to

μ_2 is positive. Combining the same result with (4.2), (A.7), (A.8) and the function $\Psi(z)$ being decreasing, we find that the derivative of the term in square brackets in the right-hand side of (A.6) with respect to μ_2 is negative. Therefore, (A.6) implies $\frac{d\lambda}{d\mu_2} > 0$, in which case

$$\begin{aligned}
&\hat{\Delta} - \mu_2 \frac{d\lambda}{d\mu_2} < 0 \\
&\Rightarrow \lambda + \phi \frac{\mu_2 \lambda + \max_m \frac{u_m}{\eta_m} - 1}{\mu_1} < 0 \\
&\Rightarrow \phi\lambda + \phi \frac{\mu_2 \lambda + \max_m \frac{u_m}{\eta_m} - 1}{\mu_1} < 0 \\
&\Rightarrow (\mu_1 + \mu_2)\lambda + \max_m \frac{u_m}{\eta_m} - 1 < 0.
\end{aligned}$$

This yields a contradiction because (3.14) implies

$$(\mu_1 + \mu_2)\lambda \geq 1 - \max_m \frac{u_m}{\eta_m}.$$

Therefore, $\hat{\Delta} > 0$.

The results in the proposition follow from (4.11) and the signs of $\sum_{m=1}^N \Delta_m b_m$ and $\hat{\Delta}$. For $\phi = 0$, $\frac{d(\mu_2\lambda)}{d\mu_2} > 0$ implies $\Delta_n \geq 0$ for all n. Since, in addition, $\sum_{m=1}^N \Delta_m b_m > 0$ for all ϕ in a non-empty interval $[0, \phi_1)$, (4.1) implies that prices increase for all stocks for $\phi = 0$. The same is true by continuity for all ϕ in a non-empty interval $[0, \phi_1') \subset [0, 1]$. Redefining ϕ_1 as $\min\{\phi_1, \phi_1'\}$, prices increase for all stocks for all $\phi \in [0, \phi_1)$.

Consider next small stocks n and n' with $\eta_n, \eta_{n'} \approx 0$. Since $\sum_{m=1}^N \Delta_m b_m > 0$ for all $\phi \in [0, \phi_1)$ and since the second term inside the square bracket in the right-hand side of (4.11) is negligible, stock n experiences a larger percentage price increase than stock n' if $\mathbb{C}\text{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right) > \mathbb{C}\text{ov}_t\left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)$, which is equivalent to $\beta_{nt} > \beta_{n't}$. Conversely, since $\sum_{m=1}^N \Delta_m b_m < 0$ for all $\phi \in (\phi_2, 1]$ under Assumption 4.1 and $\mathcal{I} = \{1, .., N\}$, stocks n and n' experience a price decrease. Moreover, stock n experiences a larger percentage price decrease than stock n' if $\mathbb{C}\text{ov}_t\left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right) > \mathbb{C}\text{ov}_t\left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh}\right)$, which is equivalent to $\beta_{nt} > \beta_{n't}$.

Consider finally a large stock n with $\eta_n \not\approx 0$, $n \in \mathcal{I}$ and $n \in \text{argmax}_m \frac{u_m}{\eta_m}$, and a small stock n' with $\eta_{n'} \approx 0$. Since $n \in \mathcal{I}$ and $n \in \text{argmax}_m \frac{u_m}{\eta_m}$, (4.2) and (A.18) imply $\Delta_n = \hat{\Delta}\eta_n$. The second term inside the square bracket in the right-hand side of (4.11) is negligible for stock n' but non-negligible for stock n. It is positive for stock n if

$$\Delta_n \left(\frac{\kappa_n^i}{r} \bar{D}_n^i + D_{nt}^i \right) F_n^i - \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m} \eta_n \left(\frac{\kappa^s}{r D_t^s} + 1 \right) D_{nt}^i F^s > 0$$
$$\Leftrightarrow \hat{\Delta} \left(\frac{\kappa_n^i}{r} \bar{D}_n^i + D_{nt}^i \right) F_n^i - \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m} \left(\frac{\kappa^s}{r D_t^s} + 1 \right) D_{nt}^i F^s > 0. \tag{A.19}$$

Since $\hat{\Delta} > 0$, (4.10) implies (A.19) if

$$\hat{\Delta} \geq \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m} \Leftrightarrow \sum_{m=1}^N (\hat{\Delta}\eta_m - \Delta_m) b_m \geq 0. \tag{A.20}$$

Equation (A.20) holds because $\hat{\Delta}\eta_n \geq \Delta_n$ for $n \in \mathcal{I}$ and $\Delta_n < 0$ for $n \notin \mathcal{I}$. Therefore, if $\mathbb{C}\mathrm{ov}_t \left(dR_{nt}, \sum_{m=1}^N \eta_m dR_{mt}^{sh} \right) = \mathbb{C}\mathrm{ov}_t \left(dR_{n't}, \sum_{m=1}^N \eta_m dR_{mt}^{sh} \right)$, which is equivalent to $\beta_{nt} = \beta_{n't}$, then stock n experiences a larger percentage price change than stock n'. The same result holds when dropping the assumption $n \in \mathrm{argmax}_m \frac{u_m}{\eta_m}$, provided that ϕ is close to zero. Indeed, (A.19) is replaced by

$$\frac{\Delta_n}{\eta_n} \left(\frac{\kappa_n^i}{r} \bar{D}_n^i + D_{nt}^i \right) F_n^i - \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m} \left(\frac{\kappa^s}{r D_t^s} + 1 \right) D_{nt}^i F^s > 0. \tag{A.21}$$

For ϕ close to zero, $\frac{\Delta_n}{\eta_n}$ is positive because it is close to $\hat{\Delta}$. Therefore, (4.10) implies (A.21) if

$$\frac{\Delta_n}{\eta_n} \geq \frac{\sum_{m=1}^N \Delta_m b_m}{\sum_{m=1}^N \eta_m b_m} \tag{A.22}$$

and even if (A.22) holds in the reverse direction but its two sides are close (because (4.10) is a strict inegulity). For ϕ close to zero, (A.22) holds if $\mathcal{I} \subsetneq \{1, .., N\}$ and holds in the reverse direction with its two sides being close if $\mathcal{I} = \{1, .., N\}$ $\square$

B Return Moments

To compute conditional expected return, we divide the right-hand side of (3.10) by S_{nt}. Using (3.11), and dropping the subscript n from $(\kappa_n^i, \bar{D}_n^i, \sigma_n^i)$, we find

$$\frac{\mathbb{E}_t(dR_{nt})}{dt} = \rho r \frac{\left[b_n\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1} b_m\right)(\sigma^s a_1^s)^2 D_t^s + \frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}(\sigma_n^i a_{n1}^i)^2 D_{nt}^i\right]}{\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)}. \tag{B.1}$$

Unconditional expected return is the expectation of (B.1)

$$\frac{\mathbb{E}(dR_{nt})}{dt} = \rho r \mathbb{E}\left\{\frac{\left[b_n\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1} b_m\right)(\sigma^s a_1^s)^2 D_t^s + \frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}(\sigma^i a_{n1}^i)^2 D_{nt}^i\right]}{\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)}\right\}. \tag{B.2}$$

When the stationary distribution of (D_t^s, D_{nt}^i) is gamma, the expectation in (B.2) becomes

$$\begin{aligned}
&\int_{D_{nt}^i=0}^{\infty}\int_{D_t^s=0}^{\infty} \frac{\left[b_n\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1} b_m\right)(\sigma^s a_1^s)^2 D_t^s + \frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}(\sigma^i a_{n1}^i)^2 D_{nt}^i\right]}{\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)} \\
&\times \frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D_t^s)^{\alpha^s-1}e^{-\beta^s D_t^s}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D_{nt}^i)^{\alpha^i-1}e^{-\beta^i D_{nt}^i} dD_t^s dD_{nt}^i.
\end{aligned} \tag{B.3}$$

Because the functions $(D_t^s)^{\alpha^s-1}$ and $(D_{nt}^i)^{\alpha^i-1}$ go to ∞ when D_t^s and D_{nt}^i, respectively, go to zero, the numerical calculation of the double integral in (B.3) becomes slow and inaccurate if the lower bounds are close to zero. We instead use a fast and accurate method by writing the double integral as a sum of four terms. We fix a small $\epsilon > 0$ and a large M. The integration domain for the first term is $(D_t^s, D_{nt}^i) \in [\epsilon, M] \times [\epsilon, M\bar{D}^i]$, and we compute that term using Matlab's double integration routine. The integration domain for the second term is $(D_t^s, D_{nt}^i) \in [0, \epsilon] \times [\epsilon, M\bar{D}^i]$, and we compute that term as

$$\begin{aligned}
&\int_{D_{nt}^i=\epsilon}^{M\bar{D}^i}\int_{D_t^s=0}^{\epsilon} \frac{b_n\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1} b_m\right)(\sigma^s a_1^s)^2 D_t^s}{\bar{D}_n + b_n a_1^s\kappa^s + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D_t^s)^{\alpha^s-1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D_{nt}^i)^{\alpha^i-1}e^{-\beta^i D_{nt}^i} dD_t^s dD_{nt}^i \\
&+ \int_{D_{nt}^i=\epsilon}^{M\bar{D}^i}\int_{D_t^s=0}^{\epsilon} \frac{\frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}(\sigma^i a_{n1}^i)^2 D_{nt}^i}{\bar{D}_n + b_n a_1^s\kappa^s + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D_t^s)^{\alpha^s-1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D_{nt}^i)^{\alpha^i-1}e^{-\beta^i D_{nt}^i} dD_t^s dD_{nt}^i \\
&= \int_{D_{nt}^i=\epsilon}^{M\bar{D}^i} \frac{b_n\left(\sum_{m=1}^N \frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1} b_m\right)(\sigma^s a_1^s)^2}{\bar{D}_n + b_n a_1^s\kappa^s + a_{n1}^i(\kappa^i\bar{D}^i + rD_{nt}^i)}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}\frac{\epsilon^{\alpha^s+1}}{\alpha^s+1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D_{nt}^i)^{\alpha^i-1}e^{-\beta^i D_{nt}^i} dD_{nt}^i
\end{aligned}$$

$$+\int_{D^i_{nt}=\epsilon}^{M\bar{D}^i} \frac{\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma^i a^i_{n1})^2 D^i_{nt}}{\bar{D}_n+b_n a^s_1\kappa^s+a^i_{n1}(\kappa^i\bar{D}^i+rD^i_{nt})}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}\frac{\epsilon^{\alpha^s}}{\alpha^s}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i-1}e^{-\beta^i D^i_{nt}}dD^i_{nt}.$$

Thus, we approximate $\kappa^s+rD^s_t$ by κ^s and $e^{-\beta^s D^s_t}$ by one, then compute the exact integrals of $(D^s_t)^{\alpha^s}$ and $(D^s_t)^{\alpha^s-1}$ over $[0,\epsilon]$, and then use Matlab's integration routine to integrate with respect to D^i_{nt} over $[\epsilon, M\bar{D}_n]$. The integration domain for the third term is $(D^s_t, D^i_{nt})\in[\epsilon,M]\times[0,\epsilon]$, and we compute that term as

$$\begin{aligned}
&\int_{D^i_{nt}=0}^{\epsilon}\int_{D^s_t=\epsilon}^{M} \frac{b_n\left(\sum_{m=1}^N \frac{\eta_m-\mu_2\lambda\eta'_m-u_m}{\mu_1}b_m\right)(\sigma^s a^s_1)^2 D^s_t}{\bar{D}_n+b_n a^s_1(\kappa^s+rD^s_t)+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}e^{-\beta^s D^s_t}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i-1}dD^s_t dD^i_{nt}\\
&+\int_{D^i_{nt}=0}^{\epsilon}\int_{D^s_t=\epsilon}^{M} \frac{\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma^i a^i_{n1})^2 D^i_{nt}}{\bar{D}_n+b_n a^s_1(\kappa^s+rD^s_t)+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}e^{-\beta^s D^s_t}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i-1}dD^s_t dD^i_{nt}\\
&=\int_{D^s_t=\epsilon}^{M} \frac{b_n\left(\sum_{m=1}^N \frac{\eta_m-\mu_2\lambda\eta'_m-u_m}{\mu_1}b_m\right)(\sigma^s a^s_1)^2 D^s_t}{\bar{D}_n+b_n a^s_1(\kappa^s+rD^s_t)+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}e^{-\beta^s D^s_t}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}\frac{\epsilon^{\alpha^i}}{\alpha^i}dD^i_{nt}\\
&\quad+\int_{D^s_t=\epsilon}^{M} \frac{\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma^i a^i_{n1})^2}{\bar{D}_n+b_n a^s_1(\kappa^s+rD^s_t)+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}e^{-\beta^s D^s_t}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}\frac{\epsilon^{\alpha^i+1}}{\alpha^i+1}dD^i_{nt}.
\end{aligned}$$

Thus, we approximate $\kappa^i\bar{D}^i+rD^i_{nt}$ by $\kappa^i\bar{D}^i$ and $e^{-\beta^i D^i_{nt}}$ by one, then compute the exact integrals of $(D^i_{nt})^{\alpha^i}$ and $(D^i_{nt})^{\alpha^i-1}$ over $[0,\epsilon]$, and then use Matlab's integration routine to integrate with respect to D^s_t over $[\epsilon, M]$. The integration domain for the fourth term is $(D^s_t, D^i_{nt})\in[0,\epsilon]\times[0,\epsilon]$, and we compute that term as

$$\begin{aligned}
&\int_{D^i_{nt}=0}^{\epsilon}\int_{D^s_t=0}^{\epsilon} \frac{b_n\left(\sum_{m=1}^N \frac{\eta_m-\mu_2\lambda\eta'_m-u_m}{\mu_1}b_m\right)(\sigma^s a^s_1)^2 D^s_t}{\bar{D}_n+b_n a^s_1\kappa^s+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i-1}dD^s_t dD^i_{nt}\\
&+\int_{D^i_{nt}=0}^{\epsilon}\int_{D^s_t=0}^{\epsilon} \frac{\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma^i a^i_{n1})^2 D^i_{nt}}{\bar{D}_n+b_n a^s_1\kappa^s+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}(D^s_t)^{\alpha^s-1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}(D^i_{nt})^{\alpha^i-1}dD^s_t dD^i_{nt}\\
&=\frac{b_n\left(\sum_{m=1}^N \frac{\eta_m-\mu_2\lambda\eta'_m-u_m}{\mu_1}b_m\right)(\sigma^s a^s_1)^2}{\bar{D}_n+b_n a^s_1\kappa^s+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}\frac{\epsilon^{\alpha^s+1}}{\alpha^s+1}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}\frac{\epsilon^{\alpha^i}}{\alpha^i}dD^i_{nt}\\
&\quad+\frac{\frac{\eta_n-\mu_2\lambda\eta'_n-u_n}{\mu_1}(\sigma^i a^i_{n1})^2}{\bar{D}_n+b_n a^s_1\kappa^s+a^i_{n1}\kappa^i\bar{D}^i}\frac{(\beta_s)^{\alpha^s}}{\Gamma(\alpha^s)}\frac{\epsilon^{\alpha^s}}{\alpha^s}\frac{(\beta_i)^{\alpha^i}}{\Gamma(\alpha^i)}\frac{\epsilon^{\alpha^i+1}}{\alpha^i+1}dD^i_{nt}.
\end{aligned}$$

Thus, we approximate $\kappa^s+rD^s_t$ by κ^s, $\kappa^i\bar{D}^i+rD^i_{nt}$ by $\kappa^i\bar{D}^i$, and $e^{-\beta^s D^s_t}$ and $e^{-\beta^i D^i_{nt}}$ by one, and then compute the exact integrals of $(D^s_t)^{\alpha^s}$, $(D^s_t)^{\alpha^s-1}$, $(D^i_{nt})^{\alpha^i}$ and $(D^i_{nt})^{\alpha^i-1}$ over $[0,\epsilon]$. The sum of the four terms is independent of ϵ for ϵ ranging from 0.00001 to 0.01. For larger values of ϵ the approximations become inaccurate, and for smaller values of ϵ the Matlab integration routines become inaccurate.

Equations (3.2), (3.8), (3.9) and (3.11) imply that conditional return volatility is

$$\sqrt{\frac{\mathbb{V}\mathrm{ar}_t(dR_{nt})}{dt}} = \frac{\sqrt{b_n^2(\sigma^s a_1^s)^2 D_t^s + (\sigma_n^i a_{n1}^i)^2 D_{nt}^i}}{\frac{\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa_n^i \bar{D}_n^i + rD_{nt}^i)}{r}}. \tag{B.4}$$

Conditional return variance is the square of (B.4). Unconditional return variance is the expectation of conditional variance

$$\frac{\mathbb{V}\mathrm{ar}(dR_{nt})}{dt} = r^2 \mathbb{E}\left\{\frac{b_n^2(\sigma^s a_1^s)^2 D_t^s + (\sigma^i a_{n1}^i)^2 D_{nt}^i}{[\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa^i \bar{D}^i + rD_{nt}^i)]^2}\right\}, \tag{B.5}$$

because infinitesimal dR_{nt} implies that $\mathbb{E}(dR_{nt}^2)$ and $\mathbb{E}_t(dR_{nt}^2)$ are equal to $\mathbb{V}\mathrm{ar}(dR_{nt})$ and $\mathbb{V}\mathrm{ar}_t(dR_{nt})$, respectively, plus smaller-order terms. We calculate the expectation in (B.5) by writing the double integral as a sum of four terms, as in the case of expected return.

Unconditional CAPM beta is

$$\beta_{nt}^{\mathrm{CAPM}} = \frac{\frac{\mathbb{C}\mathrm{ov}(dR_{nt}, dR_{Mt})}{dt}}{\frac{\mathbb{V}\mathrm{ar}(dR_{Mt})}{dt}}, \tag{B.6}$$

where dR_{Mt} denotes the return on the index. The numerator of (B.6) is

$$\frac{\mathbb{C}\mathrm{ov}(dR_{nt}, dR_{Mt})}{dt} =$$
$$r^2 \mathbb{E}\left\{\frac{b_n(\sum_{m=1}^N \eta'_m b_m)(\sigma^s a_1^s)^2 D_t^s + \eta'_n(\sigma^i a_{n1}^i)^2 D_{nt}^i}{[\bar{D}_n + b_n a_1^s(\kappa^s + rD_t^s) + a_{n1}^i(\kappa^i \bar{D}^i + rD_{nt}^i)][\sum_{m=1}^N \eta'_m[\bar{D}_m + b_m a_1^s(\kappa^s + rD_t^s) + a_{m1}^i(\kappa^i \bar{D}^i + rD_{mt}^i)]}\right\}. \tag{B.7}$$

Computing the expectation in (B.7) requires integrating over $(D_t^s, \{D_{mt}^i\}_{m=1,..,N})$, i.e., $N+1$ random variables. To keep the integration manageable, we replace $\{D_{mt}^i\}_{m\neq n}$ by their expectations $\bar{D}^i$, thus applying the law of large numbers. We then calculate the expectation over (D_t^s, D_{nt}^i) by writing the double integral as a sum of four terms, as in the case of expected return. The denominator of

(B.6) is

$$\frac{\mathbb{V}\mathrm{ar}(dR_{Mt})}{dt} = r^2 \mathbb{E}\left\{ \frac{(\sum_{m=1}^{N} \eta'_m b_m)^2 (\sigma^s a_1^s)^2 D_t^s + \sum_{m=1}^{N} (\eta'_m)^2 (\sigma^i a_{m1}^i)^2 D_{mt}^i}{[\sum_{m=1}^{N} \eta'_m [\bar{D}_m + b_m a_1^s (\kappa^s + r D_t^s) + a_{m1}^i (\kappa^i \bar{D}^i + r D_{mt}^i)]^2} \right\}. \tag{B.8}$$

We replace $\{D_{mt}^i\}_{m=1,..,N}$ by their expectations $\bar{D}^i$, and calculate the expectation over D_t^s by writing the integral as a sum of two terms, with integration domains $[0, \epsilon]$ and $[\epsilon, M]$. We do not distinguish between stock n and stocks $m \neq n$ because all stocks are symmetric in (B.8).

CAPM R-squared is

$$R^{2,\mathrm{CAPM}} = \frac{\left[\frac{\mathbb{C}\mathrm{ov}(dR_{nt}, dR_{Mt})}{dt}\right]^2}{\frac{\mathbb{V}\mathrm{ar}(dR_{nt})}{dt} \frac{\mathbb{V}\mathrm{ar}(dR_{Mt})}{dt}} = \left(\beta_{nt}^{\mathrm{CAPM}}\right)^2 \frac{\frac{\mathbb{V}\mathrm{ar}(dR_{Mt})}{dt}}{\frac{\mathbb{V}\mathrm{ar}(dR_{nt})}{dt}}$$

and can be computed from the previous moments. Unconditional idiosyncratic variance is

$$\frac{\mathbb{V}\mathrm{ar}^i(dR_{nt})}{dt} = \frac{\mathbb{V}\mathrm{ar}(dR_{nt})}{dt} - \left(\beta_{nt}^{\mathrm{CAPM}}\right)^2 \frac{\mathbb{V}\mathrm{ar}(dR_{Mt})}{dt} = \frac{\mathbb{V}\mathrm{ar}(dR_{nt})}{dt} \left(1 - R^{2,\mathrm{CAPM}}\right)$$

and can be computed from the previous moments.

C Volatility of Dividends Per Share

For each quarter and for each stock in the S&P500 index, we compute ordinary cash dividends per share, adjusting for stock splits. We source dividends per share from the monthly CRSP file and aggregate them to the quarterly frequency. We compute additionally the standard deviation of quarterly changes in dividends per share over the next three years. We begin our sample in the first quarter of 1996. We end it in the fourth quarter of 2017 because our main sample ends in the fourth quarter of 2020 and standard deviation of dividend changes is computed three years ahead. In each quarter, we sort the S&P500 stocks into quintile portfolios based on the level of dividends per share. We compute dividends per share and volatility of dividends per share for each portfolio by averaging the stock-level variables. Table C.1 and Figure C.1 present the quintile averages.

The volatility of dividends per share increases with the level of dividends per share. Moreover, the increase is concave rather than linear. These findings are consistent with the square-root

	1	2	3	4	5
Dividend	0.051	0.128	0.212	0.321	0.780
$\sigma(\Delta Dividend)$	0.024	0.045	0.046	0.062	0.085

Table C.1: Dividend Level and Volatility.

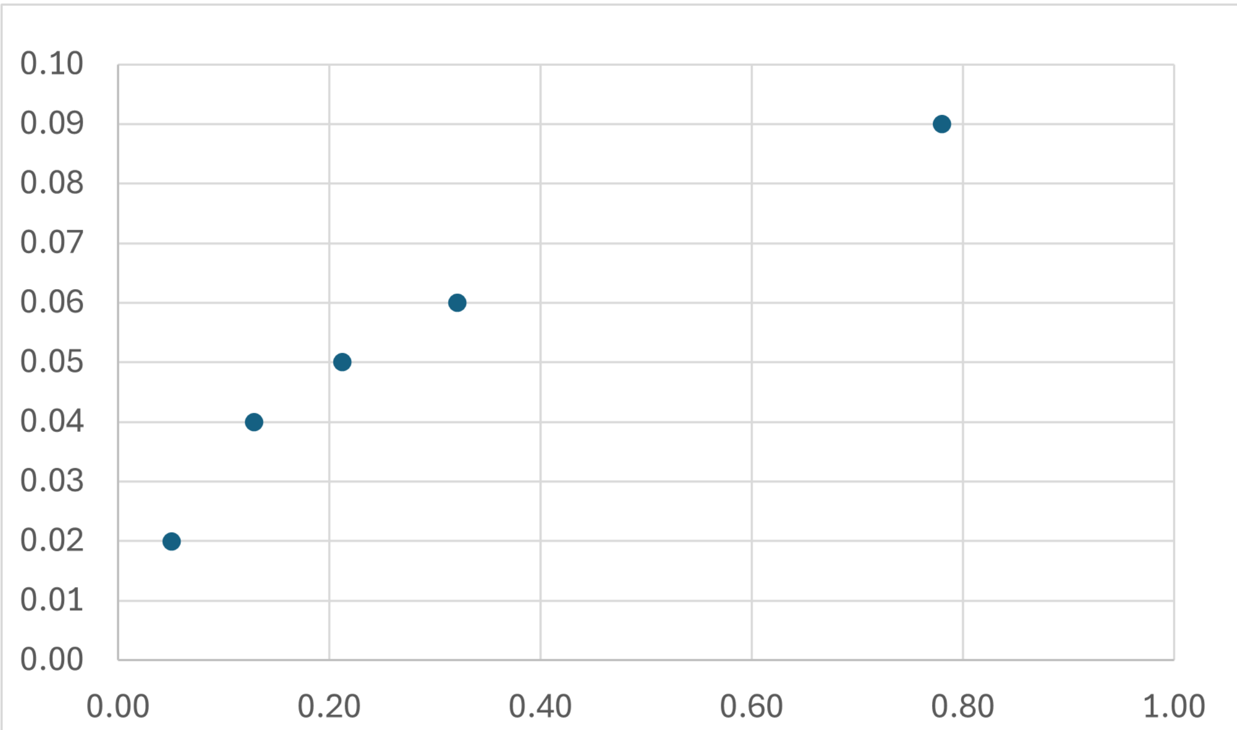

Figure C.1: **Dividend Level and Volatility.** Dividend level, computed at a quarterly frequency, is on the x-axis. Dividend volatility, computed as the standard deviation of quarterly dividend change in the next three years, is on the y-axis.

specification in our model, with the caveat that they could be partly driven by variation across firms rather than across time for a given firm.

D Three-Period Model

We assume that there are three periods 0, 1 and 2, and that stock n pays a single dividend

$$D_n = b_n \epsilon_1^s \epsilon_2^s + D_{n0}^i \epsilon_{n1}^i \epsilon_{n2}^i$$

in period 2, where $\{b_n, D^i_{n0}\}_{n=1,..,N}$ are positive constants and $(\epsilon^s_1, \epsilon^s_2, \{\epsilon^i_{n1}, \epsilon^i_{n2}\}_{n=1,..,N})$ are mutually independent random variables. We consider two specifications. In the Geometric Random Walk (GRW) specification, $(\epsilon^s_1, \epsilon^s_2)$ are normal with mean μ^s and variance $(\sigma^s)^2$, and $(\epsilon^i_{n1}, \epsilon^i_{n2})$ are normal with mean μ^i_n and variance $(\sigma^i)^2$. In the Square Root (SR) specification $(\epsilon^s_1, \epsilon^s_2)$ are normal with mean μ^s and variances $\left((\sigma^s)^2, \frac{(\sigma^s)^2}{|\epsilon^s_1|}\right)$, and $(\epsilon^i_{n1}, \epsilon^i_{n2})$ are normal with mean μ^i_n and variances $\left(\frac{(\sigma^i)^2}{D^i_{n0}}, \frac{(\sigma^i)^2}{D^i_{n0}|\epsilon^i_{n1}|}\right)$. For simplicity, we set μ^s and μ^i_n to one and the riskless rate r to zero. We normalize the expected dividend to one by assuming

$$b_n + D_{n0} = 1$$

Experts and non-experts maximize CARA utility over wealth in period 2 and start with wealth W in period 0.

We limit our analysis to three periods because the analysis of the GRW specification becomes intractable when adding more periods. We refer to the second specification as the SR specification because the standard deviation of systematic dividends as of period 1 is

$$\sqrt{b_n^2(\epsilon^s_1)^2\frac{(\sigma^s)^2}{|\epsilon^s_1|}} = b_n\sigma^s\sqrt{|\epsilon^s_1|}$$

and of idiosyncratic dividends is

$$\sqrt{(D^i_{n0})^2(\epsilon^i_{n1})^2\frac{(\sigma^i)^2}{|\epsilon^i_{n1}|}} = D^i_{n0}\sigma^s\sqrt{|\epsilon^i_{n1}|}.$$

They are proportional, respectively, to the square root of the expected systematic dividend $b_n\epsilon^s_1$ as of period 1 provided that $\epsilon^s_1 > 0$, and to the square root of the expected idiosyncratic dividend $D^i_{n0}\epsilon^i_{n1}$ provided that $\epsilon^i_{n1} > 0$.

D.1 GRW Specification

Proposition D.1 characterizes the equilibrium under the GRW specification. Before proving the proposition, we prove a useful lemma.

Lemma D.1. *Let x be an $n \times 1$ normal vector with mean zero and covariance matrix Σ, A a scalar, B an $n \times 1$ vector, C an $n \times n$ symmetric matrix, I the $n \times n$ identity matrix, and $|M|$ the determinant of a matrix M. Then,*

$$\mathbb{E}_x \exp\left\{-\alpha\left[A + B'x + \frac{1}{2}x'Cx\right]\right\} = \exp\left\{-\alpha\left[A - \frac{1}{2}\alpha B'\Sigma(I + \alpha C\Sigma)^{-1}B\right]\right\}\frac{1}{\sqrt{|I + \alpha C\Sigma|}}. \quad (D.1)$$

Proof of Lemma D.1. When $C = 0$, (D.1) gives the moment-generating function of the normal distribution. We can always assume $C = 0$ by also assuming that x is a normal vector with mean 0 and covariance matrix $\Sigma(I + \alpha C\Sigma)^{-1}$. □

Proposition D.1. *Under the GRW specification, the price of stock n in period 1 is*

$$S_{n1} = b_n\epsilon_1^s + D_{n0}^i\epsilon_{n1}^i - \rho\left[b_n(\epsilon_1^s)^2G^s + (D_{n0}^i\epsilon_{n1}^i)^2G_n^i\right], \quad (D.2)$$

and in period 0 is

$$S_{n0} = 1 - \rho\left[b_nG^s\left(\frac{2 + \rho G^s}{(1 + \rho G^s)^2} + \frac{(\sigma^s)^2}{1 - (\rho G^s)^2}\right) + (D_{n0}^i)^2G_n^i\left(\frac{2 + \rho D_{n0}^iG_n^i}{(1 + \rho D_{n0}^iG_n^i)^2} + \frac{(\sigma_n^i)^2}{1 - (\rho D_{n0}^iG_n^i)^2}\right)\right], \quad (D.3)$$

where

$$G^s = \left(\sum_{m=1}^{N}\frac{\eta_m - \mu_2\lambda\eta_m' - u_m}{\mu_1}b_m\right)(\sigma^s)^2,$$
$$G_n^i = \frac{\eta_n - \mu_2\lambda\eta_n' - u_n}{\mu_1}(\sigma_n^i)^2,$$

λ solves

$$\lambda\left[\left(\frac{H^s}{\sigma^s}\right)^2\left(\frac{2 + \lambda\rho H^s}{(1 + \lambda\rho H^s)^2} + \frac{(\sigma^s)^2}{1 - (\lambda\rho H^s)^2}\right) + \sum_{m=1}^{N}\left(\frac{D_{m0}^iH_m^i}{\sigma_m^i}\right)^2\left(\frac{2 + \rho D_{m0}^iH_m^i}{(1 + \rho D_{m0}^iH_m^i)^2} + \frac{(\sigma_m^i)^2}{1 - (\rho D_{m0}^iH_m^i)^2}\right)\right]$$

$$= \frac{H^s G^s}{(\sigma^s)^2}\left(\frac{2+\rho G^s}{(1+\rho G^s)^2}+\frac{(\sigma^s)^2}{1-(\rho G^s)^2}\right)+\sum_{m=1}^{N}\frac{(D^i_{m0})^2 H^i_m G^i_m}{(\sigma^i_m)^2}\left(\frac{2+\rho D^i_{m0}G^i_m}{(1+\rho D^i_{m0}G^i_m)^2}+\frac{(\sigma^i_n)^2}{1-(\rho D^i_{n0}G^i_m)^2}\right). \tag{D.4}$$

and

$$H^s = \left(\sum_{m=1}^{N}\eta'_m b_m\right)(\sigma^s)^2,$$
$$H^i_n = \eta'_n(\sigma^i_n)^2.$$

Proof of Proposition D.1. The expected utility of experts as of period 1 is

$$\begin{aligned}
&-\mathbb{E}_1 \exp\left[-\rho\left(W_{11}+\sum_{n=1}^{N} z_{1n1}\left(D_n - S_{n1}\right)\right)\right] \\
&= -\mathbb{E}_1 \exp\left[-\rho\left(W_{11}+\sum_{n=1}^{N} z_{1n1}\left(b_n\epsilon^s_1\epsilon^s_2 + D^i_{n0}\epsilon^i_{n1}\epsilon^i_{n2} - S_{n1}\right)\right)\right] \\
&= -\exp\left[-\rho\left(W_{11}+\left(\sum_{n=1}^{N} z_{1n1}b_n\right)\epsilon^s_1 + \sum_{n=1}^{N} z_{1n1}D^i_{n0}\epsilon^i_{n1} - \sum_{n=1}^{N} z_{1n1}S_{n1}\right.\right. \\
&\quad \left.\left. -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n1}b_n\right)^2(\epsilon^s_1\sigma^s)^2 + \sum_{n=1}^{N}(z_{1n1}D^i_{n0}\epsilon^i_{n1}\sigma^i_n)^2\right)\right)\right],
\end{aligned} \tag{D.5}$$

where the last step follows from normality. Taking the first-order condition in (D.5) and substituting $\{z_{1n1}\}_{n=1,..,N}$ from the market-clearing equation

$$\mu_1 z_{1n1} + \mu_2\lambda\eta'_n + u_n = \eta_n,$$

we find (D.2).

The expected utility of experts as of period 0 likewise is

$$\begin{aligned}
&-\mathbb{E}\exp\left[-\rho\left(W+\sum_{n=1}^{N} z_{1n0}\left(D_n - S_{n0}\right)\right)\right] \\
&= -\mathbb{E}\exp\left[-\rho\left(W+\sum_{n=1}^{N} z_{1n0}\left(b_n\epsilon^s_1\epsilon^s_2 + D^i_{n0}\epsilon^i_{n1}\epsilon^i_{n2} - S_{n0}\right)\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&= -\mathbb{E}\left\{\mathbb{E}_1 \exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}\left(b_n\epsilon_1^s\epsilon_2^s + D_{n0}^i\epsilon_{n1}^i\epsilon_{n2}^i - S_{n0}\right)\right)\right]\right\} \\
&= -\mathbb{E}\exp\left[-\rho\left(W + \left(\sum_{n=1}^{N} z_{1n0}b_n\right)\epsilon_1^s + \sum_{n=1}^{N} z_{1n0}D_{n0}^i\epsilon_{n1}^i - \sum_{n=1}^{N} z_{1n0}S_{n0}\right.\right. \\
&\quad \left.\left. -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2(\epsilon_1^s\sigma^s)^2 + \sum_{n=1}^{N}(z_{1n0}D_{n0}^i\epsilon_{n1}^i\sigma_n^i)^2\right)\right)\right] \\
&= -\frac{1}{\sqrt{\left[1-\left(\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2\right)^2\right]\prod_{n=1}^{N}\left[1-\left(\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2\right)^2\right]}} \\
&\quad \times \exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}b_n + \sum_{n=1}^{N} z_{1n0}D_{n0}^i - \sum_{n=1}^{N} z_{1n0}S_{n0}\right.\right. \\
&\quad -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2(\sigma^s)^2 + \sum_{n=1}^{N}(z_{1n0}D_{n0}^i\sigma_n^i)^2\right) \\
&\quad \left.\left. -\frac{\rho}{2}\left(\frac{\left(\sum_{n=1}^{N} z_{1n0}b_n - \rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2(\sigma^s)^2\right)^2(\sigma^s)^2}{1-\left(\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2\right)^2} + \sum_{n=1}^{N}\frac{\left(z_{1n0}D_{n0}^i - \rho(z_{1n0}D_{n0}^i\sigma_n^i)^2\right)^2(\sigma_n^i)^2}{1-\left(\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2\right)^2}\right)\right)\right] \\
&= -\frac{1}{\sqrt{\left[1-\left(\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2\right)^2\right]\prod_{n=1}^{N}\left[1-\left(\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2\right)^2\right]}} \\
&\quad \times \exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}b_n + \sum_{n=1}^{N} z_{1n0}D_{n0}^i - \sum_{n=1}^{N} z_{1n0}S_{n0}\right.\right. \\
&\quad \left.\left. -\rho\left(\frac{\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2(\sigma^s)^2}{1+\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2} + \sum_{n=1}^{N}\frac{(z_{1n0}D_{n0}^i\sigma_n^i)^2}{1+\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2}\right)\right)\right],
\end{aligned}
\tag{D.6}
$$

where the third step follows from (D.5) by replacing (W, z_{1n1}, S_{n1}) by $(W_{11}, z_{1n0}, S_{n0})$, and the fourth step follows from Lemma D.1. Taking the first-order condition in (D.6), we find

$$
b_n + D_{n0}^i - S_{n0} - \rho\left(\frac{2b_n\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2}{1+\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2} + \sum_{n=1}^{N}\frac{2z_{1n0}(D_{n0}^i\sigma_n^i)^2}{1+\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2}\right.
$$

$$
\begin{aligned}
&-\frac{\rho b_n\left(\sum_{n=1}^N z_{1n0}b_n\right)^2(\sigma^s)^4}{\left[1+\rho\left(\sum_{n=1}^N z_{1n0}b_n\right)(\sigma^s)^2\right]^2}-\sum_{n=1}^N\frac{\rho z_{1n0}^2(D_{n0}^i)^3(\sigma_n^i)^4}{\left[1+\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2\right]^2}\\
&\left.+\frac{b_n\left(\sum_{n=1}^N z_{1n0}b_n\right)(\sigma^s)^4}{1-\left(\rho\left(\sum_{n=1}^N z_{1n0}b_n\right)(\sigma^s)^2\right)^2}+\sum_{n=1}^N\frac{z_{1n0}(D_{n0}^i(\sigma_n^i)^2)^2}{1-\left(\rho z_{1n0}D_{n0}^i(\sigma_n^i)^2\right)^2}\right)=0. \qquad \text{(D.7)}
\end{aligned}
$$

Substituting $\{z_{1n0}\}_{n=1,..,N}$ from the market-clearing equation

$$\mu_1 z_{1n0}+\mu_2\lambda\eta_n'+u_n=\eta_n$$

into (D.7) and using the normalization $b_n+D_{n0}^i=1$ and the definitions of $(G^s,\{G_n^i\}_{n=1,..,N})$, we find (D.3).

The first-order condition of non-experts can be derived from (D.7) by replacing z_{1n0} by $\lambda\eta_n'$, multiplying by η_n' and summing over n. Using the definitions of $(H^s,\{H_n^i\}_{n=1,..,N})$, we find

$$
\begin{aligned}
&\sum_{m=1}^N\eta_m'b_m+\sum_{m=1}^N\eta_m'D_{m0}^i-\sum_{m=1}^N\eta_m'S_{m0}-\lambda\rho\left[\left(\frac{H^s}{\sigma^s}\right)^2\left(\frac{2+\lambda\rho H^s}{(1+\lambda\rho H^s)^2}+\frac{(\sigma^s)^2}{1-(\lambda\rho H^s)^2}\right)\right.\\
&\left.+\sum_{m=1}^N\left(\frac{D_{m0}^iH_m^i}{\sigma_m^i}\right)^2\left(\frac{2+\rho D_{m0}^iH_m^i}{(1+\rho D_{m0}^iH_m^i)^2}+\frac{(\sigma_m^i)^2}{1-(\rho D_{m0}^iH_m^i)^2}\right)\right]=0. \qquad \text{(D.8)}
\end{aligned}
$$

Substituting $\{S_{n0}\}_{n=1,..,N}$ from (D.3) into (D.8) and using the normalization $b_n+D_{n0}^i=1$, we find (D.4). $\square$

We calibrate the model as in Section 5. The values of (r,N,μ_1,μ_2,ρ), the size distribution of firms as described by number of shares as function of η, the cases for index composition, and the cases for noise-trader demand are as in Section 5. As in Section 5, we also set $\sigma_n^i=\sigma^s$ for all n and $b_n=\bar{b}-(m-3)\Delta b$ for all stocks n in size group m. Since $b_n+\bar{D}_n^i=1$ for all n, the parameters left to calibrate are $(\bar{b},\Delta b,\sigma^s,\eta)$. We calibrate them based on stocks' expected returns, return variances, CAPM betas and CAPM R-squareds. We compute these moments for the returns between period 0 and 2, expressing expected returns and return variances in per period terms. As in Section 5, we target expected returns (in excess of the riskless rate) in the baseline to average to 4% across all stocks and CAPM R-squareds to average to their empirical counterpart. We target return volatilities to average 30%. We consider only the varying-b_n case and target the spread in CAPM betas across size groups to march its empirical counterpart. CAPM beta decreases from 1.22 for size group 1 to 0.92 for size group 5. The values of $(\bar{b}, \Delta b, \sigma^s, \eta)$ are $(0.4, 0.04, 0.37, 0.00012)$.

Table D.1 shows how flows into passive funds affect stock prices in the baseline and when the funds track a large-stock index in the absence of noise traders. The J-shape shown in Section 5 appears when passive flows are due to entry by new investors into the stock market. The increasing part of the J-shape is small, however, because with only three periods the effect of different discount rates for idiosyncratic and systematic dividends is small. We show below that the same result holds under the SR specification with three periods.

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	All Firms in Index	**Size Groups 3-5 in Index**	**All Firms in Index**	**Size Groups 3-5 in Index**
1 (Smallest)	3.00	2.91	0	-0.37
2	2.73	2.65	0	-0.34
3	2.47	2.40	0	-0.28
4	2.27	2.23	0	-0.15
5 (Largest)	2.32	2.42	0	0.43

Table D.1: Percentage Price Change Caused by Passive Flows – GRW specification.

Table D.2 repeats the analysis with noise traders. As in Section 5, a pure switch from active to passive tracking a large-stock index has its largest positive effects on high-demand stocks in size group 5. With only three periods, however, the effects on large high-demand stocks are not sufficiently large to cause the aggregate market to rise. We show below that the same result holds under the SR specification with three periods.

D.2 SR Specification

Proposition D.2 characterizes the equilibrium under the SR specification. For tractability, we ignore negative values for $(\epsilon_1^s, \{\epsilon_{n1}^i\}_{n=1,..,N})$, assuming that $(\sigma^s, \{\sigma_n^i\}_{n=1,..,N})$ are sufficiently small so that negative values have low probability.

Size Group	Noise-Trader Demand	Entry into the Stock Market		Switch from Active to Passive	
		All Firms in Index	Size Groups 3-5 in Index	All Firms in Index	Size Groups 3-5 in Index
1 (Smallest)	Low	2.60	2.52	0.00	-0.31
	High	2.60	2.52	0.00	-0.31
2	Low	2.37	2.30	0.00	-0.29
	High	2.37	2.30	0.00	-0.29
3	Low	2.15	2.09	-0.01	-0.24
	High	2.14	2.09	0.01	-0.23
4	Low	1.97	1.94	-0.04	-0.17
	High	1.96	1.93	0.04	-0.09
5 (Largest)	Low	2.00	2.09	-0.22	0.14
	High	2.00	2.09	0.22	0.59

Table D.2: Percentage Price Change Caused by Passive Flows with Noise Traders – GRW specification.

Proposition D.2. *Under the SR specification, the price of stock n in period 1 is*

$$S_{n1} = b_n\epsilon_1^s + D_{n0}^i\epsilon_{n1}^i - \rho\left[b_n\epsilon_1^s G^s + (D_{n0}^i)^2\epsilon_{n1}^i G_n^i\right], \tag{D.9}$$

and in period 0 is

$$S_{n0} = 1 - \rho\left[b_n G^s\left(2 - \frac{3}{2}\rho G^s + \frac{1}{2}(\rho G^s)^2\right) + D_{n0}^i G_n^i\left(2 - \frac{3}{2}\rho G_n^i + \frac{1}{2}(\rho G_n^i)^2\right)\right], \tag{D.10}$$

where λ solves

$$\begin{aligned}
&\lambda\left[\left(\frac{H^s}{\sigma^s}\right)^2\left(2 - \frac{3}{2}\rho H^s + \frac{1}{2}(\rho H^s)^2\right) + \sum_{m=1}^N D_{m0}^i\left(\frac{H_m^i}{\sigma_m^i}\right)^2\left(2 - \frac{3}{2}\rho H_m^i + \frac{1}{2}(\rho H_m^i)^2\right)\right] \\
&= \frac{H^s G^s}{(\sigma^s)^2}\left(2 - \frac{3}{2}\rho G^s + \frac{1}{2}(\rho G^s)^2\right) + \sum_{m=1}^N D_{m0}^i\frac{H_m^i G_m^i}{(\sigma_m^i)^2}\left(2 - \frac{3}{2}\rho G_m^i + \frac{1}{2}(\rho G_m^i)^2\right).
\end{aligned} \tag{D.11}$$

Proof of Proposition D.2. The expected utility of experts as of period 1 is

$$-\mathbb{E}_1 \exp\left[-\rho\left(W_{11} + \sum_{n=1}^N z_{1n1}\left(D_n - S_{n1}\right)\right)\right]$$

$$
\begin{aligned}
&= -\mathbb{E}_1 \exp\left[-\rho\left(W_{11} + \sum_{n=1}^{N} z_{1n1}\left(b_n\epsilon_1^s\epsilon_2^s + D_{n0}^i\epsilon_{n1}^i\epsilon_{n2}^i - S_{n1}\right)\right)\right] \\
&= -\exp\Bigg[-\rho\Bigg(W_{11} + \left(\sum_{n=1}^{N} z_{1n1}b_n\right)\epsilon_1^s + \sum_{n=1}^{N} z_{1n1}D_{n0}^i\epsilon_{n1}^i - \sum_{n=1}^{N} z_{1n1}S_{n1} \\
&\quad -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n1}b_n\right)^2 \epsilon_1^s(\sigma^s)^2 + \sum_{n=1}^{N} z_{1n1}^2 D_{n0}^i\epsilon_{n1}^i(\sigma_n^i)^2\right)\Bigg)\Bigg],
\end{aligned} \tag{D.12}
$$

where the last step follows from normality and from $(\epsilon_1^s, \{\epsilon_{n1}^i\}_{n=1,..,N})$ being positive. Taking the first-order condition in (D.12) and substituting $\{z_{1n1}\}_{n=1,..,N}$ from the market-clearing equation

$$
\mu_1 z_{1n1} + \mu_2\lambda\eta_n' + u_n = \eta_n,
$$

we find (D.9).

The expected utility of experts as of period 0 likewise is

$$
\begin{aligned}
&- \mathbb{E}\exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}\left(D_n - S_{n0}\right)\right)\right] \\
&= -\mathbb{E}\exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}\left(b_n\epsilon_1^s\epsilon_2^s + D_{n0}^i\epsilon_{n1}^i\epsilon_{n2}^i - S_{n0}\right)\right)\right] \\
&= -\mathbb{E}\left\{\mathbb{E}_1\exp\left[-\rho\left(W + \sum_{n=1}^{N} z_{1n0}\left(b_n\epsilon_1^s\epsilon_2^s + D_{n0}^i\epsilon_{n1}^i\epsilon_{n2}^i - S_{n0}\right)\right)\right]\right\} \\
&= -\mathbb{E}\exp\Bigg[-\rho\Bigg(W + \left(\sum_{n=1}^{N} z_{1n0}b_n\right)\epsilon_1^s + \sum_{n=1}^{N} z_{1n0}D_{n0}^i\epsilon_{n1}^i - \sum_{n=1}^{N} z_{1n0}S_{n0} \\
&\quad -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2 \epsilon_1^s(\sigma^s)^2 + \sum_{n=1}^{N} z_{1n0}^2 D_{n0}^i\epsilon_{n1}^i(\sigma_n^i)^2\right)\Bigg)\Bigg] \\
&= -\exp\Bigg[-\rho\Bigg(W + \sum_{n=1}^{N} z_{1n0}b_n + \sum_{n=1}^{N} z_{1n0}D_{n0}^i - \sum_{n=1}^{N} z_{1n0}S_{n0} \\
&\quad -\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2 (\sigma^s)^2 + \sum_{n=1}^{N} z_{1n0}^2 D_{n0}^i(\sigma_n^i)^2\right)
\end{aligned}
$$

$$
-\frac{\rho}{2}\left(\left(\sum_{n=1}^{N} z_{1n0}b_n - \frac{\rho}{2}\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2 (\sigma^s)^2\right)^2 (\sigma^s)^2 + \sum_{n=1}^{N}\left(z_{1n0} - \frac{\rho}{2}(z_{1n0}\sigma_n^i)^2\right)^2 D_{n0}^i(\sigma_n^i)^2\right)\Bigg)\Bigg],
\tag{D.13}
$$

where the third step follows from (D.12) by replacing (W, z_{1n1}, S_{n1}) by $(W_{11}, z_{1n0}, S_{n0})$, and the fourth step follows from normality and from $(\epsilon_1^s, \{\epsilon_{n1}^i\}_{n=1,..,N})$ being positive. Taking the first-order condition in (D.13), we find

$$
\begin{aligned}
& b_n + D_{n0}^i - S_{n0} - \rho\left(b_n\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2 + \sum_{n=1}^{N} z_{1n0}D_{n0}^i(\sigma_n^i)^2\right. \\
& + b_n\left(\sum_{n=1}^{N} z_{1n0}b_n - \frac{\rho}{2}\left(\sum_{n=1}^{N} z_{1n0}b_n\right)^2(\sigma^s)^2\right)(\sigma^s)^2\left(1-\rho\left(\sum_{n=1}^{N} z_{1n0}b_n\right)(\sigma^s)^2\right) \\
& \left.+ D_{n0}^i\left(z_{1n0} - \frac{\rho}{2}(z_{1n0}\sigma_n^i)^2\right)(\sigma_n^i)^2\left(1-\rho z_{1n0}(\sigma_n^i)^2\right)\right) = 0.
\end{aligned}
\tag{D.14}
$$

Substituting $\{z_{1n0}\}_{n=1,..,N}$ from the market-clearing equation

$$
\mu_1 z_{1n0} + \mu_2\lambda\eta_n' + u_n = \eta_n
$$

into (D.14) and using the normalization $b_n + D_{n0}^i = 1$ and the definitions of $(G^s, \{G_n^i\}_{n=1,..,N})$, we find (D.10).

The first-order condition of non-experts can be derived from (D.14) by replacing z_{1n0} by $\lambda\eta_n'$, multiplying by η_n' and summing over n. Using the definitions of $(H^s, \{H_n^i\}_{n=1,..,N})$, we find

$$
\begin{aligned}
& \sum_{m=1}^{N}\eta_m' b_m + \sum_{m=1}^{N}\eta_m' D_{m0}^i - \sum_{m=1}^{N}\eta_m' S_{m0} - \lambda\rho\left[\left(\frac{H^s}{\sigma^s}\right)^2\left(2 - \frac{3}{2}\rho H^s + \frac{1}{2}(\rho H^s)^2\right)\right. \\
& \left.+ \sum_{m=1}^{N} D_{m0}^i\left(\frac{H_m^i}{\sigma_m^i}\right)^2\left(2 - \frac{3}{2}\rho H_m^i + \frac{1}{2}(\rho H_m^i)^2\right)\right] = 0.
\end{aligned}
\tag{D.15}
$$

Substituting $\{S_{n0}\}_{n=1,..,N}$ from (D.10) into (D.15) and using the normalization $b_n + D_{n0}^i = 1$, we find (D.11). □

We calibrate the model as in Section D.1. CAPM beta decreases from 1.23 for size group 1 to

0.92 for size group 5. The values of $(\bar{b}, \Delta b, \sigma, \eta)$ are $(0.4, 0.041, 0.32, 0.00016)$.

Tables D.3 and D.4 are the counterparts of Tables D.1 and D.2 under the SR specification. As in Table D.1, the J-shape appears when passive flows are due to entry by new investors into the stock market. The increasing part of the J-shape is larger than in Table D.1, but remains small relative to the decreasing part. As in Table D.2, a pure switch from active to passive tracking a large-stock index has its largest positive effects on high-demand stocks in size group 5, but the aggregate market drops. Overall, the effects in Tables D.3 and D.4 are similar in magnitude to their counterparts in Tables D.1 and D.2.

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	All Firms in Index	**Size Groups 3-5 in Index**	**All Firms in Index**	**Size Groups 3-5 in Index**
1 (Smallest)	3.08	2.96	0	-0.51
2	2.80	2.69	0	-0.47
3	2.54	2.44	0	-0.38
4	2.34	2.29	0	-0.20
5 (Largest)	2.49	2.63	0	0.58

Table D.3: Percentage Price Change Caused by Passive Flows – SR specification.

Size Group	**Noise-Trader Demand**	**Entry into the Stock Market**		**Switch from Active to Passive**	
		All Firms in Index	**Size Groups 3-5 in Index**	**All Firms in Index**	**Size Groups 3-5 in Index**
1 (Smallest)	Low	2.64	2.53	0.00	-0.43
	High	2.64	2.53	0.00	-0.43
2	Low	2.40	2.30	0.00	-0.40
	High	2.40	2.30	0.00	-0.40
3	Low	2.17	2.10	-0.01	-0.33
	High	2.17	2.09	0.01	-0.31
4	Low	2.01	1.97	-0.06	-0.22
	High	2.00	1.96	0.06	-0.11
5 (Largest)	Low	2.13	2.25	-0.31	0.18
	High	2.13	2.24	0.31	0.80

Table D.4: Percentage Price Change Caused by Passive Flows with Noise Traders – SR specification.

E Additional Results

E.1 Alternative Calibration

We report calibration results for $s^s = 2.2$. Table E.1 is the counterpart of Table 5.1. CAPM beta and CAPM R-squared have the same properties as in Table 5.1. Prices are lower and return volatilities are higher than in Table 5.1.

Panel A: Constant-b_n case

Size Group	Price	Expected Return (%)	Return Volatility (%)	CAPM Beta	CAPM R^2 (%)
1 (Smallest)	4.79	3.79	21.77	0.89	24.85
2	4.76	3.80	21.77	0.89	25.08
3	4.64	3.85	21.76	0.91	26.14
4	4.22	4.05	21.74	0.98	30.45
5 (Largest)	3.43	4.53	21.72	1.16	42.85

Panel B: Varying-b_n case

Size Group	Price	Expected Return (%)	Return Volatility (%)	CAPM Beta	CAPM R^2 (%)
1 (Smallest)	3.82	5.24	21.79	1.31	22.56
2	4.58	4.53	18.21	1.10	22.61
3	5.21	4.05	15.63	0.96	23.59
4	5.53	3.79	13.16	0.90	28.88
5 (Largest)	5.61	3.68	10.47	0.88	44.21

Table E.1: Price and Return Moments.

Table E.2 is the counterpart of Table 5.2. When passive flows are due to entry by new investors into the stock market, the percentage price increase that they generate is larger for larger stocks in the constant-b_n case, and is a J-shaped function of stock size in the varying-b_n case. When passive flows are due to a switch by investors from active to passive, they do not affect stock prices. These results are as in Table 5.2.

Table E.3 is the counterpart of Table 5.3. The effect of passive flows on small stocks is less

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	7.44	6.77	0	0
2	7.66	5.78	0	0
3	8.66	5.89	0	0
4	11.85	7.09	0	0
5 (Largest)	16.75	7.26	0	0

Table E.2: Percentage Price Change Caused by Passive Flows in the Baseline.

dependent on index composition than the effect on large stocks. This result is as in Table 5.3, but the dependence on index composition is stronger. A pure switch from active to the passive large-stock index raises the aggregate market, with the effect being stronger than in Table 5.3.

Panel A: Index Includes Only Top Three Size Groups

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	7.29	6.56	-0.51	-0.77
2	7.34	5.33	-1.24	-1.72
3	8.98	6.38	1.36	1.93
4	13.39	8.60	5.67	5.23
5 (Largest)	20.40	9.38	11.95	6.70

Panel B: Index Includes Only Bottom Three Size Groups

Size Group	**Entry into the Stock Market**		**Switch from Active to Passive**	
	Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	7.46	6.82	0.11	0.23
2	7.85	6.07	0.82	1.24
3	9.65	7.11	4.30	5.26
4	8.44	4.26	-9.27	-6.66
5 (Largest)	10.48	4.07	-12.57	-5.75

Table E.3: Percentage Price Change Caused by Passive Flows into a Partial Index

Table E.4 is the counterpart of Table 5.4. The effect of passive flows on small stocks is less dependent on noise-trader demand than the effect on large stocks. This result is as in Table 5.3, but the dependence on demand is stronger for large stocks than in Table 5.4. A pure switch from active to passive raises the aggregate market, with the effect being stronger than in Table 5.4.

Size Group	Noise-Trader Demand	Entry into the Stock Market		Switch from Active to Passive	
		Constant-b_n	**Varying-b_n**	**Constant-b_n**	**Varying-b_n**
1 (Smallest)	Low	7.79	7.11	-0.04	-0.08
	High	7.79	7.11	0.01	0.04
2	Low	7.99	6.03	-0.15	-0.23
	High	7.97	6.02	0.13	0.21
3	Low	8.85	5.96	-0.62	-0.73
	High	8.82	6.00	0.64	0.81
4	Low	11.56	6.81	-1.81	-1.42
	High	11.85	7.44	2.38	2.12
5 (Largest)	Low	15.75	6.85	-2.87	-1.41
	High	17.56	8.35	5.06	2.80

Table E.4: Percentage Price Change Caused by Passive Flows with Noise Traders.

Table E.5 is the counterpart of Table 5.5. A pure switch from active to passive has large positive effects on high-demand stocks in size group 5, and significantly smaller effects on all other stocks. Moreover, the aggregate market rises. These effects are stronger than in Table 5.5.

E.2 Return Volatility

Table E.6 shows the effect of passive flows on idiosyncratic volatility when flows are due to entry by new investors in the stock market. Idiosyncratic volatility can rise or fall, but rises more (or falls less) for large stocks.

E.3 Additional Empirical Tests

Table E.7 reports results from regressing changes in the three measures of S&P500 index concentration on passive flows. Results are shown with and without controls. Controls are as in Table 6.2: the S&P500 return, the one-quarter lagged S&P500 return and VIX. Also as in Table 6.2, the t-statistics, in parentheses, are based on Newey-West heteroskedasticity- and autocorrelation-

Size Group	Noise-Trader Demand	Entry into the Stock Market		Switch from Active to Passive	
		Constant-b_n	Varying-b_n	Constant-b_n	Varying-b_n
1 (Smallest)	Low	7.66	6.91	-0.62	-1.02
	High	7.65	6.91	-0.56	-0.91
2	Low	7.70	5.64	-1.33	-1.91
	High	7.68	5.62	-1.07	-1.53
3	Low	9.09	6.33	0.26	0.43
	High	9.08	6.43	1.72	2.43
4	Low	12.73	7.94	1.83	1.56
	High	13.35	9.09	8.83	9.25
5 (Largest)	Low	18.32	8.33	3.55	1.80
	High	22.16	11.47	29.81	22.15

Table E.5: Percentage Price Change Caused by Passive Flows into a Partial Index with Noise Traders.

Size Group	Noise-Trader Demand	All-Stock Index		Small-Stock Index	
		Constant-b_n	Varying-b_n	Constant-b_n	Varying-b_n
1 (Smallest)	Low	-0.52	-0.54	-0.50	-0.52
	High	-0.52	-0.54	-0.50	-0.52
2	Low	-0.52	-0.41	-0.50	-0.40
	High	-0.52	-0.41	-0.50	-0.40
3	Low	-0.51	-0.31	-0.49	-0.29
	High	-0.51	-0.31	-0.49	-0.29
4	Low	-0.48	-0.19	-0.44	-0.15
	High	-0.47	-0.19	-0.44	-0.14
5 (Largest)	Low	-0.38	-0.01	-0.30	0.09
	High	-0.34	0.03	-0.26	0.16

Table E.6: Change in Idiosyncratic Volatility Caused by Passive Flows by New Investors Entering the Stock Market.

consistent standard errors with three lags. Our findings are robust to increasing the number of lags.

Consistent with our model, the relationship between passive flows and changes in all three measures of concentration is positive and significant. An one-standard-deviation increase in $PassiveFlow$ is associated with an increase in the concentration measures ranging from 0.224 to 0.244 standard deviations, depending on the measure of concentration and on whether controls are added or not.

Table E.8 reports results from panel regressions of daily individual stock returns on a $Month\,Start$ indicator variable interacted with an indicator variable describing whether a stock is in a large-stock portfolio. The $Month\,Start$ indicator is equal to one if a trading day is within the first seven days of a month and to zero otherwise. The large-stock indicator is equal to one of a stock is in the top 10, or in the top 50, or in the top 100, or in the top 150, or in the top 200. We include Firm $\times$ Month and Month $\times$ $Month\,Start$ fixed effects. These absorb variation in stock returns across months for each stock and variation in the constituents of the interaction term. Consistent with our model, the coefficient of the interaction term is positive and significant for large-stock portfolios: large firms experience higher returns than other firms at the beginning of each month.

	$\Delta\,\widehat{\log(w_{Top\,10})}$	$\Delta\,\widehat{\log(Dispersion)}$	$\Delta\,\widehat{\log(HHI)}$	$\Delta\,\widehat{\log(w_{Top\,10})}$	$\Delta\,\widehat{\log(Dispersion)}$	$\Delta\,\widehat{\log(HHI)}$
$\widehat{PassiveFlow}$	0.244	0.239	0.230	0.235	0.233	0.224
	(2.30)	(2.02)	(1.92)	(2.74)	(2.46)	(2.35)
Constant	3.02e-09	-1.81e-09	2.74e-10	-4.75e-05	-0.0154	-0.0193
	(0.00)	(-0.00)	(0.00)	(-0.00)	(-0.12)	(-0.15)
Observations	99	99	99	99	99	99
Controls	N	N	N	Y	Y	Y
Adjusted R^2	0.060	0.057	0.053	0.121	0.126	0.125

Table E.7: Passive Flows and Index Concentration.

Variables	$R_{Top\,10}$	$R_{Top\,50}$	$R_{Top\,100}$	$R_{Top\,150}$	$R_{Top\,200}$
$Top\,10 \times Month\,Start$	0.0282				
	(2.28)				
$Top\,50 \times Month\,Start$		0.0220			
		(2.82)			
$Top\,100 \times Month\,Start$			0.0140		
			(2.10)		
$Top\,150 \times Month\,Start$				0.00647	
				(1.07)	
$Top\,200 \times Month\,Start$					0.000915
					(0.16)
Observations	2,974,202	2,974,202	2,974,202	2,974,202	2,974,202
Firm $\times$ Month FE	Y	Y	Y	Y	Y
Month $\times$ $Month\,Start$ FE	Y	Y	Y	Y	Y
Adjusted R^2	0.00656	0.00656	0.00656	0.00656	0.00655

Table E.8: Excess Returns on Large Stocks at Beginning of Month