The Distribution of Investor Beliefs, Stock Ownership and Stock Returns

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Abstract

We study the relationship between investor beliefs, breadth of ownership and expected returns in a model where stocks differ in the intensity of disagreement between optimists and pessimists and in the polarization of beliefs measured by the number of optimists and pessimists relative to moderates. Differences in polarization explain the size-dependent relationship between breadth and expected returns that we find empirically: positive for large stocks and stronger negative for small stocks. We also find empirical support for the underlying mechanism: stocks with polarized beliefs earn lower expected returns, and are held more broadly if small and less broadly if large.

JEL: G10, G11, G12, G23

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1 Introduction

Why do some assets have a narrow and some a broad investor base? Do assets with a broad base have different expected returns than assets with a narrow base? Two influential theories characterize the breadth of ownership and its relationship to expected returns, and yield opposite predictions. The first theory, formulated by Merton (1987) as the investor-recognition hypothesis, emphasizes costs of entering asset markets. According to that theory, an asset for which entry costs are high attracts few investors and trades at a deep discount because of imperfect risk-sharing. Hence, a narrow investor base indicates an undervalued asset with high expected return. The second theory, formulated by Miller (1977) and Harrison and Kreps (1978), and further developed by Chen, Hong, and Stein (2002, CHS), Scheinkman and Xiong (2003) and Hong and Stein (2007), emphasizes differences of opinion across investors, combined with short-sale constraints. According to that theory, large disagreements across investors about an asset's payoff result in the asset being held only by the most optimistic investors. Since optimists push the asset price up, a narrow investor base indicates an overvalued asset with low expected return.

Some empirical findings are consistent with the entry-cost theory. For example, Hong and Kacperczyk (2009) find that social norms prevent some institutional investors from holding stocks in "sin industries" (alcohol, gaming and tobacco), and this raises the stocks' expected returns. Lou (2014) finds that increased advertising by firms brings in more investors, raises their stock prices and lowers their expected returns. The overall cross-section of stocks, however, seems better described by the difference-of-opinion theory. CHS find that stocks with a narrow investor base earn low expected returns in the cross-section. Diether, Malloy, and Scherbina (2002, DMS) find that stocks for which financial analysts disagree the most, thus exhibiting more extreme optimism or pessimism, earn low expected returns.

In this paper we re-examine the empirical relationship between breadth of ownership and expected returns, and show that it remains puzzling for the leading formulations of the two theories. We also propose a richer formulation of the difference-of-opinion theory that can explain the puzzle. Our formulation has implications not only for the relationship between breadth of ownership and expected returns but also for how each variable relates to investor beliefs. We test for these implications using analyst forecasts as a proxy for investor beliefs, and find empirical support.

We show that the empirical relationship between breadth of ownership and expected returns depends critically on stock size. For large stocks, a narrow investor base predicts low future returns, consistent with the difference-of-opinion theory and the findings of CHS. For small stocks, by contrast, a narrow base predicts high future returns, and this effect is stronger than its opposite for large stocks. This is inconsistent with the difference-of-opinion theory, and is especially puzzling because that theory should be more relevant for small stocks. Indeed, since optimists are better able to absorb a smaller supply, the overvaluation that they generate should be more severe for small stocks (Hong, Scheinkman, and Xiong (2006)).

We solve the puzzle in a model where the distribution of investor beliefs is described by two dimensions. In CHS and most other papers in the literature, differences of opinion are described by the intensity of disagreement, as measured by the difference between optimists' and pessimists' beliefs. This one-dimensional description also underlies the empirical exercise in DMS, in which returns are predicted by the standard deviation of the distribution of analyst forecasts. In our model instead, stocks vary not only in the intensity of disagreement but also in how polarized beliefs are: holding optimists' and pessimists' beliefs constant, there is variation in the number of optimists and pessimists relative to investors with moderate beliefs. Polarization has a monotone effect on expected returns but a non-monotone effect on the distribution of asset ownership. These effects, for which we find empirical support, drive the size-dependent relationship between asset ownership and expected returns.

In our model, presented in Section 2, there are multiple stocks and multiple investors with different beliefs and short-sale constraints. Beliefs for each stock are described by a symmetric three-point distribution: there are equal numbers of optimists and pessimists, with some rational investors in the middle. The intensity of disagreement maps to the *range* of the distribution. The extent of polarization maps to the distribution's *kurtosis*. Holding range constant, higher kurtosis indicates fewer optimists and pessimists relative to rationals. Stocks differ in their range and kurtosis, as well as in their size, idiosyncratic variance, and exposure to systematic risk.

Stocks' expected returns in equilibrium are negatively related to range and positively to kurtosis. The effect of range follows the same logic as in CHS and DMS. Stocks for which the range is high exhibit more disagreement: more extreme positive beliefs by optimists and negative beliefs by pessimists. Therefore, when short-sale constraints keep pessimists out of the market, optimists render the prices of these stocks higher than of otherwise identical stocks with less extreme beliefs. The effect of kurtosis follows a different logic. Stocks for which the kurtosis is low exhibit more polarization: more optimists and pessimists, and fewer rationals. Therefore, when pessimists drop out of the market, optimists push the prices of these stocks higher than of otherwise identical stocks with fewer optimists.

Following CHS, we measure breadth of ownership by the fraction of investors holding a stock. For small stocks, breadth is independent of range. Indeed, since optimists can easily absorb the small stocks' supply, these stocks are held only by optimists regardless of how extreme optimistic beliefs are. For large stocks instead, breadth is negatively related to range. Indeed, large stocks are held only by optimists when optimistic beliefs are extreme, and by both optimists and rationals when beliefs are less extreme. The effect of kurtosis also depends on size, and crucially for our empirical analysis, changes sign. For small stocks, breadth is negatively related to kurtosis because these stocks are held only by optimists, and high kurtosis indicates few optimists. For large stocks instead, breadth is positively related to kurtosis, because these stocks are held by both optimists and rationals, and high kurtosis indicates few pessimists, who are the only investors not holding the stocks.

Our theoretical results yield three main empirical hypotheses. The first hypothesis concerns the relationship between investor beliefs and expected returns. Expected returns should be negatively related to range holding kurtosis constant, and positively related to kurtosis holding range constant. The second hypothesis concerns the relationship between investor beliefs and breadth of ownership. Holding kurtosis constant, breadth should be unrelated to range for small stocks and negatively related for large stocks. Holding range constant, breadth should be negatively related to kurtosis for small stocks and positively related for large stocks. The third hypothesis follows from the first two, and concerns the relationship between breadth of ownership and expected returns. For small stocks, that relationship is driven by kurtosis since range has no effect on breadth, and is negative. For large stocks, it is driven by both range and kurtosis, and both variables imply a positive relationship.

We test the three hypotheses using CRSP data on US stock prices and returns, Thompson Reuters data on holdings by 13-F institutional investors, and I/B/E/S data on analyst forecasts. We proxy ownership patterns by those within 13-F investors, computing breadth as the number of 13-F investors holding a stock, divided by the total number of 13-F investors. We proxy investor beliefs by analyst forecasts, and compute the range and kurtosis of the distribution of forecasts across analysts. Computing kurtosis requires four or more analysts for a stock. We measure size by market capitalization and by a proxy directly implied by our model, which is equal to market capitalization adjusted by idiosyncratic variance, range and kurtosis. We describe our dataset and empirical measures in Section 3.

Sections 4 and 5 present our empirical findings on the first and second hypotheses. When regressing breadth on range, kurtosis and their interactions with size, we find that the coefficient of kurtosis is negative, and the coefficient of the interaction between kurtosis and size is positive and larger in magnitude. Thus, kurtosis is negatively related to breadth for small stocks and positively for large stocks. When regressing expected returns on range and kurtosis, we find that kurtosis is positively related to expected returns. These results provide direct evidence for the role of kurtosis and for our model's new mechanism. The effects of range are also broadly in line with our model. Partly because of the restriction to four or more analysts, we find weaker effects of range on expected returns than in DMS.

Section 6 presents our empirical findings on the third hypothesis. We perform two related tests, in which, following CHS, we use the first difference ΔB of breadth rather than the level to account for the high autocorrelation of breadth and its correlation with size. One test is to regress future returns on ΔB, its interaction with size, and risk controls. The other test is to double-sort stocks into portfolios based on size and ΔB, and compare the returns and alphas of the high- relative to the low-ΔB portfolio across size groups. Both tests paint a consistent picture: ΔB predicts negatively the returns of small stocks and positively those of large stocks, with the effect of ΔB for small stocks being larger in absolute value than the effect for large stocks.

We subject the findings in Section 6 to a series of robustness tests. The findings hold both in the full sample and when the fraction of a stock held by 13-F investors must exceed a threshold. The findings, shown on annual returns, become stronger for returns over longer horizons. The findings do not hold for quarterly returns, and weaken in the first half of the sample and when using the level of breadth rather than the first difference. The findings become uniformly stronger when measuring breadth of ownership by the Herfindahl index rather than by the fraction of investors holding a stock.

A final robustness test, in Section 7, is to perform the analysis at the level of investment styles rather than individual investors. Our model can be applied to styles by assuming that each style is held by a disjoint group of investors. The findings in Section 6 become stronger at the level of styles. This rules out alternative explanations for the relationship between asset ownership and expected returns that apply to the level of investors but not to styles, such as monitoring or rent extraction by large shareholders, and asymmetric information by corporate insiders.

Our paper is most closely related to CHS. CHS describe the distribution of investor beliefs by the intensity of disagreement, and show theoretically and empirically that breadth is positively related to expected returns. We find instead a negative relationship for small stocks and a weaker positive one for large stocks, and explain this finding in a model where beliefs are described by two dimensions, intensity of disagreement and polarization. Unlike CHS, we also test for the direct implications of our model for how the distribution of beliefs relates to breadth and expected returns.

DMS examine empirically how intensity of disagreement relates to expected returns, thus focusing on the same one-dimensional description of beliefs as in CHS. Proxying investor beliefs by analyst forecasts, they find that stocks for which disagreement is large earn low returns. We show that beliefs can be more plausibly described by two dimensions, and that the second dimension—polarization—is useful in predicting returns. Unlike DMS, we also test for the relationship between the distribution of beliefs and the breadth of ownership.

Cen, Lu, and Yang (2013, CLY) show theoretically and empirically that the relationship between breadth and expected returns turns negative when investor sentiment is volatile. Sentiment in their model is driven by irrational investors, who trade with rational arbitrageurs. When irrational investors become optimistic, breadth increases and expected return decreases. The variation in the number of optimists is central to our model as well. We emphasize its cross-sectional implications, while CLY focus on the time-series.

While our model is based on differences of opinion and short-sale constraints, it can be related to the alternative theory based on entry costs (Merton (1987)). Small stocks in our model are held only by the optimists, who correspond to the participating investors in the entry-cost theory. Breadth of ownership within small stocks is driven by variation in the optimists' number, and thus by the entry-cost mechanism.

Barberis and Shleifer (2003) show that style investing affects asset prices and returns through the flows of funds across styles. Flows in their model generate return predictability in the form of momentum, reversal and lead-lag effects. Similar mechanisms are at play with rational investors in Vayanos and Woolley (2013). Our style-level findings indicate that variables associated to styles predict stock returns over horizons longer than those of momentum and lead-lag effects. Moreover, the direction of the predictability switches sign as stock size increases.

2 Theory

We derive our empirical hypotheses from a model in which different investors value stocks differently and there are short-sale constraints. Stocks can differ in size, riskiness and the distribution of investor valuations. We describe the distribution of valuations by range and kurtosis. Our model's main results characterize how range and kurtosis relate to expected stock returns and breadth of ownership, and how these relationships change with stock size and riskiness.

2.1 Model

There are two periods 0 and 1. There are $I+1$ assets, indexed by $i = 0, 1, .., I$, which pay off in period 1. Asset 0 is riskless. We take it as the numeraire and set its price in period 0 and its payoff in period 1 equal to one. Assets $1, .., I$ are risky and we refer to them as stocks. Stock i trades at price S_i per share in period 0, pays dividend D_i per share in period 1, and is in supply of $\theta_i > 0$ shares. Dividends have the one-factor structure

$$D_i = \bar{D}_i + b_i\eta + \sigma_i\eta_i, \tag{2.1}$$

where $\bar{D}_i$ is stock i's expected dividend, η is a systematic risk factor, $b_i \geq 0$ is stock i's sensitivity to that factor, η_i represents idiosyncratic risk, and σ_i is stock i's sensitivity to that risk. We assume that the variables $(\eta, \{\eta_i\}_{i=1,..,I})$ are mutually independent and normally distributed. We set their mean to zero, which is without loss of generality because we can redefine $\{\bar{D}_i\}_{i=1,..,I}$, and their variance to one, which is without loss of generality because we can redefine $\{b_i, \sigma_i\}_{i=1,..,I}$. We set $\bar{D}_i$ to a value $\bar{D}$ common across stocks, which is without loss of generality because we can redefine the number of shares θ_i. We assume that $\bar{D}$ is large enough so that equilibrium prices are always positive, and define the return of stock i as $\frac{D_i - S_i}{S_i}$ and the stock's expected return as $\frac{\bar{D} - S_i}{S_i}$.

There are N competitive investors indexed by $n = 1,..,N$. All investors have CARA utility with risk-aversion coefficient a. Different investors value stocks differently. We model the differences in valuations as an additional component of dividends that is private to each investor: from the viewpoint of investor n, the dividend of stock i is $D_i + \epsilon_{in}$ instead of D_i. The valuation ϵ_{in} could reflect different opinions or hedging benefits. Adopting the different opinions interpretation from now on, we refer to ϵ_{in} as the opinion or belief of investor n. We also refer to investors with $\epsilon_{in} > 0$ as optimists for stock i, to investors with $\epsilon_{in} < 0$ as pessimists, and to investors with $\epsilon_{in} = 0$ as rationals. We denote by x_{in} the number of shares of stock i held by investor n. Investors are subject to short-sale constraints: x_{in} must be non-negative.

The distribution of investors' beliefs for stock i is symmetric around zero and takes the form

$$\epsilon_{in} = \begin{cases} \epsilon_i & \text{for } K_i \leq \frac{N}{2} \text{ investors,} \\ 0 & \text{for } N - 2K_i \text{ investors,} \\ -\epsilon_i & \text{for } K_i \text{ investors.} \end{cases} \tag{2.2}$$

There are K_i optimists with valuation ϵ_i, K_i pessimists with valuation $-\epsilon_i$, and $N - 2K_i$ rationals. The range of the distribution is $2\epsilon_i$, the standard deviation is $\sqrt{\frac{2K_i}{N}}\epsilon_i$, and the kurtosis is $\frac{N}{2K_i}$. An increase in ϵ_i does not affect the kurtosis but raises the range and the standard deviation. An increase in K_i does not affect the range. It raises the standard deviation because there are more optimists and more pessimists. It lowers the kurtosis because the extreme values ϵ_i and $-\epsilon_i$ become smaller when measuring distance in units of the higher standard deviation. Since ϵ_i affects the range and not the kurtosis, while the opposite is true for K_i, we can map the effects of ϵ_i to those of range and the effects of K_i to those of kurtosis. The standard deviation captures effects of both ϵ_i and K_i, which is why we do not use it in our analysis.

2.2 Equilibrium

Using CARA utility, the dividends' one-factor structure (2.1) and their normality, and the parameter normalizations, we can write the maximization problem of investor n in the mean-variance form

$$\max_{\substack{\{x_{in}\}_{i=1,..,N}\\ x_{in}\geq 0}} \sum_{i=1}^{I}\left(\bar{D}+\epsilon_{in}-S_i\right)x_{in}-\frac{a}{2}\left[\left(\sum_{i=1}^{I} b_i x_{in}\right)^2+\sum_{i=1}^{I}\sigma_i^2 x_{in}^2\right].$$

The first-order condition for stock i is

$$\begin{aligned}
&\bar{D}+\epsilon_{in}-S_i-b_i\lambda_n-a\sigma_i^2 x_{in}=0 \quad \text{if} \quad x_{in}>0,\\
&\bar{D}+\epsilon_{in}-S_i-b_i\lambda_n-a\sigma_i^2 x_{in}\leq 0 \quad \text{if} \quad x_{in}=0,
\end{aligned}$$

where $\lambda_n \equiv a\sum_{j=1}^{I} b_j x_{jn}$. Using the first-order condition, we can write the investor's demand for stock i as

$$x_{in}=\max\left\{\frac{\bar{D}+\epsilon_{in}-S_i-b_i\lambda_n}{a\sigma_i^2},0\right\}. \tag{2.3}$$

Demand is positive if the investor's expectation $\bar{D}+\epsilon_{in}$ of the stock's dividend, minus a systematic risk premium $b_i\lambda_n$, exceeds the stock's price S_i. The systematic risk premium is the product of the stock's sensitivity b_i to systematic risk times an investor-specific premium λ_n. The investor premium λ_n is the product of the investor's risk aversion a times the investor's portfolio sensitivity $\sum_{j=1}^{I} b_j x_{jn}$ to systematic risk.

Aggregating across investors and using market clearing

$$\sum_{n=1}^{N} x_{in}=\theta_i, \tag{2.4}$$

we find that the equilibrium price S_i of stock i solves

$$\sum_{n=1}^{N}\max\left\{\frac{\bar{D}+\epsilon_{in}-S_i-b_i\lambda_n}{a\theta_i\sigma_i^2},0\right\}=1. \tag{2.5}$$

Solving for equilibrium amounts to solving for stock prices $\{S_i\}_{i=1,..,I}$ and investor premia $\{\lambda_n\}_{n=1,..,N}$. Proposition 2.1 characterizes the equilibrium when the premia $\{\lambda_n\}_{n=1,..,N}$ are equal across investors. It also derives sufficient conditions on the model's primitives for the premia to be equal.

Proposition 2.1. *Suppose that in equilibrium the premia $\{\lambda_n\}_{n=1,..,N}$ are equal across investors. The price S_i of asset i is*

$$S_i=\bar{D}-\frac{ab_i}{N}\sum_{j=1}^{I} b_j\theta_j+a\theta_i\sigma_i^2\phi_i, \tag{2.6}$$

where ϕ_i is the unique solution of

$$\sum_{n=1}^{N} \max\left\{\frac{\epsilon_{in}}{a\theta_i\sigma_i^2} - \phi_i, 0\right\} = 1. \tag{2.7}$$

Asset holdings for asset i depend only on investor beliefs for that asset, and are given by

$$x_{in} = \max\left\{\frac{\epsilon_{in}}{a\sigma_i^2} - \theta_i\phi_i, 0\right\} \tag{2.8}$$

for investor n. The premia $\{\lambda_n\}_{n=1,..,N}$ are equal across investors under either of the two sufficient conditions:

(A) There is no systematic risk ($b_i = 0$ for all i).

(B) For each stock i, there are $\frac{N}{K_i}$ stocks sharing its characteristics $(\theta_i, b_i, \sigma_i, \epsilon_i, K_i)$, and each investor's distribution of private valuations across this subset of stocks matches the population distribution of private valuations across the subset.

The equilibrium price S_i of stock i is equal to the stock's expected dividend, minus a systematic risk premium $\frac{ab_i}{N}\sum_{j=1}^{I} b_j\theta_j$, plus a term $a\theta_i\sigma_i^2\phi_i$ that captures the joint effects of differences of opinion and idiosyncratic risk. The systematic risk premium is the product of the stock's sensitivity b_i to systematic risk times the common value of the investor-specific premia $\{\lambda_n\}_{n=1,..,N}$. The common value of $\{\lambda_n\}_{n=1,..,N}$ is derived by multiplying investor risk aversion a times portfolio sensitivity to systematic risk. Portfolio sensitivity is $\frac{1}{N}\sum_{j=1}^{I} b_j\theta_j$ and is the same as if each investor were holding $\frac{1}{N}$'th of the supply of each asset.

The term $a\theta_i\sigma_i^2\phi_i$, which captures how differences of opinion and idiosyncratic risk impact the price of stock i, is central to our analysis. We determine how it depends on the range and kurtosis of the distribution of beliefs in Section 2.3. Using $a\theta_i\sigma_i^2\phi_i$, we also determine how range and kurtosis impact the breadth of ownership of stock i. Investor holdings of stock i depend only on investor beliefs about that stock and not on their beliefs about other stocks. We denote the common value of holdings for all optimists for stock i by x_{iO}, for all pessimists by x_{iP}, and for all rationals by x_{iR}.

The equality of the premia $\{\lambda_n\}_{n=1,..,N}$ across investors implies that the equilibrium has a simple and intuitive form. Indeed, premia would differ across investors if investors differed in their average optimism across all stocks. Investors with higher average optimism would hold larger long positions and would be more exposed to systematic risk. As a result, they would hold a smaller position in a given stock i relative to investors with lower average optimism but same belief for stock i. An investor's holdings of stock i would thus depend on the investor's beliefs for all other stocks, complicating the equilibrium. By the same logic, a stock might be held primarily by the investors who are most pessimistic about it, if these investors are even more pessimistic about other stocks. Focusing on equilibria where $\{\lambda_n\}_{n=1,..,N}$ are equal across investors eliminates these complicating features.

Proposition 2.1 provides two sufficient conditions on the model's primitives for the premia $\{\lambda_n\}_{n=1,..,N}$ to be equal across investors. One obvious condition is that there is no systematic risk ($b_i = 0$ for all i) because the premia are then all equal to zero. Another condition is that there is systematic risk but investors are symmetric in their average optimism across stocks in the sense of Condition (B) in the proposition. Condition (B) requires that if an investor is optimistic about a stock i, then there are other stocks with the same characteristics $(\theta_i, b_i, \sigma_i, \epsilon_i, K_i)$ as i such that the investor is rational or pessimistic about them. Moreover, the distribution of the investor's beliefs within the subset of stocks with characteristics $(\theta_i, b_i, \sigma_i, \epsilon_i, K_i)$ matches the population distribution of beliefs within that subset. Condition (B) ensures, in particular, that investors are identical in their average optimism across all stocks. Differences of opinion can be interpreted as pertaining to the idiosyncratic component of stocks' dividends rather than to the systematic component.

2.3 Expected Returns and Breadth of Ownership

Range and kurtosis have monotone effects on stock prices and expected returns. Consider stocks i and i' such that the range of the distribution of beliefs is higher for stock i ($\epsilon_i > \epsilon_{i'}$) and the other characteristics are identical across the two stocks ($(\theta_i, b_i, \sigma_i, K_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, K_{i'})$). If the two stocks are held by all investors, then they trade at the same price because the average belief across investors is zero ($\frac{\sum_{n=1}^{N} \epsilon_{in}}{N} = \frac{\sum_{n=1}^{N} \epsilon_{i'n}}{N} = 0$). If, however, pessimists drop out, then stock i trades at a higher price than stock i' and earns lower expected return. This is because the optimists for stock i are more optimistic than those for stock i'.

Suppose next that stocks i and i' differ only in the kurtosis of the distribution of beliefs, with the kurtosis being higher for stock i ($K_i < K_{i'}$). If the two stocks are held by all investors, then they trade at the same price. If, however, pessimists drop out, then stock i trades at a lower price than stock i' and earns higher expected return. This is because there are fewer optimists for stock i than for stock i'.

The condition for pessimists to drop out of a stock can be derived from Proposition 2.1. Equation (2.8) implies that pessimists hold stock i if $-\frac{\epsilon_i}{a\sigma_i^2} - \theta_i\phi_i > 0$. Equation (2.7) implies that when stock i is held by all investors, $\phi_i = -\frac{1}{N}$. Pessimists drop out of stock i if the inequality that results from substituting $\phi_i = -\frac{1}{N}$ into $-\frac{\epsilon_i}{a\sigma_i^2} - \theta_i\phi_i > 0$ holds in the opposite direction, i.e.,

$$-\frac{\epsilon_i}{a\sigma_i^2} + \theta_i\frac{1}{N} < 0 \Leftrightarrow \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}. \tag{2.9}$$

Pessimists drop out of the stock if the range ϵ_i of the distribution of beliefs is sufficiently large relative to the stock's supply θ_i and idiosyncratic variance σ_i^2. Pessimists' reward from holding the stock is the premium from bearing the stock's idiosyncratic risk. When differences of opinion are sufficiently large, that reward is small relative to the pessimists' negative belief. Differences of opinion in that case overcome the gains from risk-sharing.

Proposition 2.2. *The range and kurtosis of the distribution of investor beliefs have monotone effects on stock prices and expected returns.*

- *A stock i with higher range than another stock i' and same other characteristics ($\epsilon_i > \epsilon_{i'}$ and $(\theta_i, b_i, \sigma_i, K_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, K_{i'})$) trades at a higher price and earns lower expected return when $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$. When instead $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2}$, price and expected return are equal across the two stocks.*
- *A stock i with higher kurtosis than another stock i' and same other characteristics ($K_i < K_{i'}$ and $(\theta_i, b_i, \sigma_i, \epsilon_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, \epsilon_{i'})$) trades at a lower price and earns higher expected return when $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$. When instead $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2}$, price and expected return are equal across the two stocks.*

We next examine how range and kurtosis affect breadth of ownership, measured by the fraction of investors holding a stock. The breadth for stock i is

$$B_i \equiv \frac{K_i 1_{\{x_{iO}>0\}} + (N-2K_i)1_{\{x_{iR}>0\}} + K_i 1_{\{x_{iP}>0\}}}{N},$$

where the indicator function $\{1_{\{x_{iZ}>0\}}\}_{Z=O,P,R}$ is equal to one if investors $Z = O, R, P$ hold the stock ($x_{iZ} > 0$) and zero if they do not ($x_{iZ} = 0$). Breadth of ownership can also be measured using the Herfindahl index, defined by squaring the fraction of the stock held by each investor and summing across investors. The Herfindahl index for stock i is

$$H_i \equiv K_i\left(\frac{x_{iO}}{x_{iO}+x_{iR}+x_{iP}}\right)^2 + (N-2K_i)\left(\frac{x_{iR}}{x_{iO}+x_{iR}+x_{iP}}\right)^2 + K_i\left(\frac{x_{iP}}{x_{iO}+x_{iR}+x_{iP}}\right)^2.$$

The Herfindahl index is high when few investors hold most of the stock, and thus is inversely related to breadth. While we focus on B_i in our theoretical and empirical analysis, we show in Section 6.3 that our empirical findings carry through to H_i.

Range has a monotone effect of breadth, but the effect of kurtosis is non-monotone and depends on stock size and riskiness. Consider stocks i and i' that differ only in the range of the distribution of beliefs, with the range being higher for stock i ($\epsilon_i > \epsilon_{i'}$). Since the pessimists for stock i are more pessimistic than those for stock i', they drop out first, i.e., stock i is not held by its pessimists if the same is true for stock i'. After the pessimists drop out, the rationals are also first to drop out for stock i. This is because the optimists for stock i are more optimistic than those for stock i', and thus push the price of stock i higher than of stock i'. Since the pessimists and the rationals for stock i drop out first relative to their counterparts for stock i', breadth for stock i is smaller or equal than for stock i'.

Suppose next that stocks i and i' differ only in the kurtosis of the distribution of beliefs, with the kurtosis being higher for stock i ($K_i < K_{i'}$). Since there are fewer optimists for stock i than for stock i', stock i has smaller breadth than stock i' when the two stocks are held only by their optimists. When, however, stock i is also held by its rationals, it has larger breadth than stock i'. Indeed, in that case, stock i' is held only by its optimists, or by its optimists and its rationals. Moreover, since there are fewer pessimists for stock i than for stock i', the combined number of optimists and rationals for stock i exceeds its counterpart for stock i'. Breadth is equal across the two stocks only when they are also held by their pessimists.

The condition for a stock to be held only by its optimists can be derived from Proposition 2.1. Equation (2.8) implies that rationals do not hold stock i if $\phi_i > 0$. Equation (2.7) implies that when stock i is held only by its optimists, $\phi_i = \frac{\epsilon_i}{a\theta_i\sigma_i^2} - \frac{1}{K_i}$. Therefore, stock i is held only by its optimists if

$$\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}. \tag{2.10}$$

For the rationals to drop out, the range ϵ_i of the distribution of beliefs must be sufficiently large relative to the stock's supply θ_i and idiosyncratic variance σ_i^2. The required threshold that the ratio $\frac{\epsilon_i}{a\theta_i\sigma_i^2}$ must exceed is $\frac{1}{K_i}$, which is larger than the threshold $\frac{1}{N}$ required by (2.9) for the pessimists to drop out. Intuitively, the rationals' reward from holding a stock is the premium from bearing the stock's idiosyncratic risk. When differences of opinion are sufficiently large, optimists push up the stock's price to a level where that reward is small relative to the stock's overpricing. Equation (2.10) is easier to satisfy for stocks that are in small supply, have low idiosyncratic variance, or have high range of the distribution of beliefs. For such stocks, kurtosis is negatively related to breadth. Conversely, kurtosis is positively related to breadth for stocks that are in large supply, have high variance or have low range.

Proposition 2.3. *The range of the distribution of investor beliefs has a monotone effect on the* *breadth of ownership, but the effect of kurtosis is non-monotone.*

- *A stock i with higher range than another stock i' and same other characteristics ($\epsilon_i > \epsilon_{i'}$ and $(\theta_i, b_i, \sigma_i, K_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, K_{i'})$) has smaller breadth when $\frac{\epsilon_i}{\epsilon_{i'} K_i} > \frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{1}{K_i}$ or $\frac{\epsilon_i}{\epsilon_{i'} N} > \frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{1}{N}$. Otherwise, breadth is equal across the two stocks.*
- *A stock i with higher kurtosis than another stock i' and same other characteristics ($K_i < K_{i'}$ and $(\theta_i, b_i, \sigma_i, \epsilon_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, \epsilon_{i'})$) has smaller breadth when $\frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{1}{K_i}$ and larger breadth when $\frac{1}{K_i} > \frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{1}{N}$. When instead $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i \sigma_i^2}$, breadth is equal across the two stocks.*

A final proposition concerns the effect of stock size on breadth. Consider stocks i and i' such that the number of shares is larger for stock i ($\theta_i > \theta_{i'}$) and the other characteristics are identical across the two stocks ($(b_i, \sigma_i, \epsilon_i, K_i) = (b_{i'}, \sigma_{i'}, \epsilon_{i'}, K_{i'})$). We interpret stock i as the larger stock: its market capitalization $\theta_i S_i$ exceeds that of stock i' if the common expected dividend $\bar{D}$ of the two stocks is large enough. Since pessimists and rationals drop out of a stock if the range of the distribution of beliefs is large relative to the stock's supply and idiosyncratic variance, the pessimists and rationals for stock i' drop out first relative to their counterparts for stock i. Therefore, breadth for stock i is larger or equal than for stock i'.

Proposition 2.4. *Stock size has a monotone effect on the breadth of ownership. A stock i that is larger number of shares another stock i' and same other characteristics ($\theta_i > \theta_{i'}$ and $(b_i, \sigma_i, \epsilon_i, K_i) = (b_{i'}, \sigma_{i'}, \epsilon_{i'}, K_{i'})$) has larger breadth when $\frac{1}{K_i} > \frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{\theta_{i'}}{\theta_i K_i}$ or $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i \sigma_i^2} > \frac{\theta_{i'}}{\theta_i N}$. Otherwise, breadth is equal across the two stocks.*

Using Propositions 2.2-2.4, we derive our empirical hypotheses. Our first hypothesis follows from Proposition 2.2 and concerns the relationship that range and kurtosis have with expected returns.

Hypothesis 1 (Investor beliefs and expected returns). *Expected returns are predictable from the range and the kurtosis of the distribution of investor beliefs as follows:*

- *Expected returns are negatively related to range.*
- *Expected returns are positively related to kurtosis.*

Our second hypothesis follows from Proposition 2.3 and concerns the relationship that range and kurtosis have with breadth. The monotonicity of the effect of kurtosis depends on the comparison between $\frac{\epsilon_i}{a\theta_i\sigma_i^2}$ and $\frac{1}{K_i}$, or equivalently between one and $\frac{a\theta_i\sigma_i^2}{\epsilon_i K_i}$. The variable $\frac{a\theta_i\sigma_i^2}{\epsilon_i K_i}$ is a composite involving stock i's supply θ_i, idiosyncratic variance σ_i^2, range $2\epsilon_i$, and kurtosis $\frac{N}{2K_i}$. We refer to it as adjusted size, in the sense that it can be viewed as equal to stock i's supply adjusted for idiosyncratic variance, range and kurtosis. Proposition 2.3 implies that breadth is negatively related to kurtosis for stocks whose adjusted size is small, and is positively related to kurtosis for stocks whose adjusted size is large.

Proposition 2.3 implies additionally that breadth is unrelated to range for stocks whose adjusted size is so small that are held only by optimists or is so large that are held by all investors. Breadth is instead negatively related to range for intermediate values of adjusted size. Assuming that stocks are not held by their pessimists, i.e., (2.9) holds for all stocks, the cross-effect between range and adjusted size is negative: range has no effect on breadth for stocks whose adjusted size is small, and has a negative effect when adjusted size is larger.

Hypothesis 2 (Investor beliefs and breadth of ownership). *Breadth of ownership is related to the range and the kurtosis of the distribution of investor beliefs as follows:*

- *Breadth is unrelated to range for stocks whose adjusted size is small. The relationship turns negative for stocks whose adjusted size is large.*
- *Breadth is negatively related to kurtosis for stocks whose adjusted size is small. The relationship turns positive for stocks whose adjusted size is large.*

Our third hypothesis, which we term Hypothesis 2a because it relates closely to Hypothesis 2, concerns the relationship between stock size and breadth. The monotonicity of this relationship follows from Proposition 2.4. The cross-effects are related to those in Hypothesis 2.

Hypothesis 2a (Stock size and breadth of ownership). *Breadth of ownership is increasing in stock size. The effect of size on breadth is stronger for stocks with:*

- *A low range of the distribution of investor beliefs.*
- *A high kurtosis of the distribution of investor beliefs.*

Our final hypothesis concerns the relationship between breadth and expected returns. That hypothesis follows by combining Propositions 2.2 and 2.3, or equivalently, Hypotheses 1 and 2. Suppose that there is independent cross-sectional variation in range and kurtosis. For stocks whose adjusted size is small, breadth is negatively related to kurtosis and is unrelated to range (Hypothesis

2). Since expected returns are positively related to kurtosis (Hypothesis 1), they are negatively related to breadth. For stocks whose adjusted size is large, breadth is negatively related to range and positively to kurtosis (Hypothesis 2). Since expected returns are negatively related to range and positively to kurtosis (Hypothesis 1), they are positively related to breadth.

Hypothesis 3 (Breadth of ownership and expected returns). *Expected returns are predictable from the breadth of ownership as follows:*

- *Expected returns are negatively related to breadth for stocks whose adjusted size is small.*
- *Expected returns are positively related to breadth for stocks whose adjusted size is large.*

In our empirical tests of Hypotheses 2 and 3, we use two measures of adjusted size. The first measure is simply size, defined as market capitalization. The second measure is our model-implied version, derived by multiplying market capitalization with idiosyncratic variance and kurtosis, and dividing by range.

3 Data Sources and Variables

Our sample consists of common stocks (codes 10 and 11 of CRSP) trading on NYSE, NASDAQ and AMEX between the first quarter of 1980 and the fourth quarter of 2018. The frequency of the sample is quarterly. The range and frequency of the sample are driven by the availability of the ownership data. Ownership data pertaining to investment styles are available only between the first quarter of 1997 and the fourth quarter of 2015, so our analysis of style-level ownership is limited accordingly.

3.1 Stock Returns

We source data on stock prices, stock returns including dividends, trading volume, and number of outstanding shares from CRSP. We calculate a stock's return over any given horizon by compounding the stock's monthly returns during that horizon. We measure a stock's size in any given quarter by market capitalization, which we calculate by multiplying the stock's share price at the end of the quarter times the number of outstanding shares on the same day. We define small stocks as those with size below the 30th percentile of our sample, mid-cap stocks as those with size between the 30th and the 70th percentile, and large stocks as those with size above the 70th percentile.

We construct a number of stock-level variables that we use as controls. These include idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. We calculate market beta and idiosyncratic volatility in any given quarter using a within-quarter time-series regression of daily excess stock returns over the riskless rate on the daily market excess return. Idiosyncratic volatility is the standard deviation of the regression residuals, and idiosyncratic variance is its square. If more than ten observations are missing within the quarter, then we treat the market beta and idiosyncratic volatility observations as missing. We source the riskless rate and the market return from Kenneth French's website.

We source the ratio of book value of equity to market value from the Financial Ratios Suite of WRDS. WRDS calculates the book-to-market ratio on a quarterly basis and lags all observations by two months to ensure no look-ahead biases. We construct our momentum variable in any given quarter by compounding monthly returns during the nine-month period ending at the end of the quarter.

We construct turnover in each month by dividing the number of shares traded in that month by the total outstanding shares in the same month. Because reported share volume is estimated differently by NYSE/AMEX and NASDAQ (Atkins and Dyl (1997)), with the latter roughly double-counting, we divide NASDAQ share volume by two (Nagel (2005)).

3.2 Institutional Ownership

We source data on institutional ownership from Thomson Reuters (TR). That data are derived from institutional investors' 13-F filings. Institutional investors with more than $100 million in assets are required to report their stock-level holdings to the SEC on a quarterly basis, within 45 days from the end of the quarter.

We use two different databases of TR. From the first database, TR Stock Ownership, available in WRDS, we source the number of 13-F institutional investors who hold any given stock, the total number of 13-F investors, the fraction of the stock held by all 13-F investors, and the stock's Herfindahl index H. The fraction of the stock held by all 13-F investors, which we term institutional ownership (IO), is calculated by dividing the number of shares held by all 13-F investors by the total number of outstanding shares of the stock. The Herfindahl index H is calculated by dividing the number of shares held by a given 13-F investor by the number of shares held by all 13-F investors, squaring that fraction, and summing across investors.

The second database, Thomson Eikon, groups 13-F institutional investors into investment styles based on their portfolio characteristics and/or their business type. From that database, we source the number of investment styles that hold any given stock and the fraction of the stock held by each style. In our sample, stocks are held by 32 different styles. The 32 styles include seventeen general styles (e.g. aggressive growth, core growth, core value, deep value, index, etc) and fifteen hedge fund styles. Appendix B provides more details on the styles and the TR classification procedure.

We construct Breadth B and Herfindahl Index H at the investor and at the style level. The investor-level variables are calculated as follows. Breadth for stock i and quarter t is the number of 13-F investors who hold the stock in that quarter, divided by the total number of 13-F investors in the same quarter. Herfindahl Index for stock i in quarter t is calculated by TR as described above. The style-level variables are calculated as follows. Breadth for stock i and quarter t is the number of different styles that hold the stock in that quarter. (We do not divide by the total number of styles as it is constant over time in our sample.) Herfindahl Index for stock i and quarter t is calculated by dividing the number of shares of the stock held by any given style by the number of shares held by all styles, squaring that fraction, and summing across styles.

3.3 Analyst Forecasts

We source data on analyst forecasts from the Detail History file of the I/B/E/S database, which is provided by TR. The data cover the period between the second quarter of 1982 and the fourth quarter of 2018. We use analyst forecasts for earnings per share (EPS) one fiscal year ahead (FY1). We examine the EPS FY1 forecasts that appear in each month for each stock. When an analyst reports more than one forecast for the same stock in the same month, we use only the most recent forecast.

For any given stock and month, we standardize analyst forecasts by dividing them by the absolute value of the mean forecast (Diether, Malloy, and Scherbina (2002)). This allows us to express the dispersion in forecasts in relative terms: a given dispersion in dollar terms is more significant economically when EPS is low. We calculate the range and kurtosis of the stock's standardized forecasts. Range is the difference between the maximum and the minimum standardized forecast. Kurtosis is the fourth central moment divided by the square of the variance. We apply a finite sample correction to kurtosis to avoid a mechanical relationship between it and the number of analysts.[1] The finite sample correction requires a sample size of at least four, so we include in our analysis only stock/month observations with at least four analysts. Not applying the finite sample correction strengthens our results.

Dividing forecasts by the absolute value of the mean forecast can generate inflated standardized forecasts, and thus an inflated value of the range, when the mean forecast is close to zero. We

[1] We calculate kurtosis as

$$\frac{n-1}{(n-2)(n-3)}\left[(n+1)\frac{m_4}{m_2^2}-3(n-1)\right]+3,$$

where n is the sample size (number of analysts), m_4 is the fourth central moment and m_2 is the variance. Kurtosis without the finite sample correction is $\frac{m_4}{m_2^2}$. (See `https://www.mathworks.com/help/stats/kurtosis.html`.)

mitigate the effects of inflated values by mapping observations into deciles. Following Nagel (2005), we transform range and kurtosis into deciles across the population of stocks in any given month, and normalize the units so that the smallest decile corresponds to zero and the largest to one. We perform the same transformation for most other variables in our regressions. The decile transformation removes any time trends from our regression results. We calculate the quarterly values of the transformed range and kurtosis by averaging over the non-missing months of the quarter. We treat stock-months with zero mean forecast as missing.

We update the values of range and kurtosis each quarter. We do the same for the ownership variables B, H and IO, and for their first differences, as well as for momentum. For size, share price, idiosyncratic volatility, market beta, book-to-market ratio and turnover, we use the values corresponding to the second quarter of each year. Our results remain similar when updating the values of these variables each quarter as well. We compute first differences corresponding to quarter t by subtracting values corresponding to quarter $t-1$ from values corresponding to quarter t.

3.4 Sample Size

The full sample includes stock/quarters for which the following criteria are met. There should be data on the return, size and IO of the stock during the quarter. There should additionally be data on the breadth B of the stock during the quarter and the previous quarter, so that we can compute the change ΔB. Finally, the IO of the stock during any of the five quarters surrounding quarter t (quarters $t-2$, $t-1$, t, $t+1$ and $t+2$) should not exceed 100%. These criteria leave us with a total of 19096 stocks, 617309 stock/quarters, and an average of 4088 stocks per quarter. When using additional controls, the number of stock/quarters drops to a minimum of 511246 and the average number of stocks per quarter drops to a minimum of 3386.

We additionally report results for a more restricted sample in which we include a stock/quarter if the IO of the stock during the quarter, the two previous quarters and the two subsequent quarters is larger or equal than 30%. We impose that criterion because B being the fraction of 13-F investors holding a stock, can become an imprecise measure of how widely the stock is held across all investors (including non-13-F ones) for low values of IO. Imposing the $IO \geq 30\%$ criterion reduces our sample to 264356 stock/quarters, and an average of 1751 stocks per quarter. When using additional controls, the number of stock/quarters drops to a minimum of 243003 and the average number of stocks per quarter drops to a minimum of 1609. The excluded stock/quarters with the $IO \geq 30\%$ criterion constitute 91% of the total stock/quarters corresponding to small stocks, 59% of those corresponding to mid-cap stocks, and 24% of those corresponding to large stocks. The results for the $IO \geq 30\%$ sample are generally similar to those for the main sample.

In Sections 4 and 5, where we use data on analyst forecasts, the sample is reduced to 182941 stock/quarters (150467 for $IO \geq 30\%$). This is because we exclude stock/month observations with fewer than four analysts. The excluded stock/quarters constitute 98% of the total stock/quarters corresponding to small stocks, 80% of those corresponding to mid-cap stocks, and 33% of those corresponding to large stocks. When using additional controls, the number of stock/quarters drops to a minimum of 175009 stock/quarters (150241 for $IO \geq 30\%$).

We use data on analyst forecasts in Section 6 as well. Unlike in Sections 4 and 5, where range and kurtosis are the main explanatory variables, in Section 6 they are needed only to construct adjusted size, which enters only in some of the regressions. For that reason, we do not exclude in Section 6 stock/month observations with fewer than four analysts. We fill in the missing observations for range and kurtosis by linearly predicting them from size and idiosyncratic variance.

4 Analyst Forecasts and Breadth of Ownership

In this section we test Hypotheses 2 and 2a. Hypothesis 2 concerns the relationship between the breadth of ownership B and the distribution of investor beliefs. Hypothesis 2a concerns the relationship between B and stock size. Because we do not have direct data on investor beliefs, we proxy them by the forecasts made by financial analysts.

Table I presents descriptive statistics of breadth B, its first difference ΔB, idiosyncratic volatility, the range of the distribution of analyst forecasts, the kurtosis of that distribution, and the number of analysts. We report statistics over the full sample and over the subsamples of stock/quarters involving small, mid-cap and large stocks. The mean of B across the full sample is 4.62%, meaning that the average stock is held by only 4.62% of 13-F investors. Consistent with Hypothesis 2a, there is a strong positive relationship between B and size. The mean of B for small stocks is 0.52%, for mid-cap stocks is 2.13% and for large stocks is 11.68%. In the subsample of stock/quarters where institutional ownership IO exceeds 30%, the mean of B rises to 8.59%, reflecting the higher fraction of large stocks in that subsample. Table I shows additionally a strong negative relationship between idiosyncratic volatility and size and between range and size.

Table II presents results from contemporaneous pooled OLS regressions of B on the range of the distribution of analyst forecasts, the kurtosis of that distribution, stock size or adjusted size, IO, and the interactions of range and kurtosis with size or adjusted size. The regressions involving size are in Panel A and those involving adjusted size are in Panel B. The coefficients of the interactions of range and kurtosis with size are informative about Hypothesis 2a because they reveal how the positive relationship between B and size, shown in Table I, depends on range and kurtosis. The coefficients of range and kurtosis and of their interactions with size or adjusted size are informative

Table I: **Descriptive statistics of breadth, idiosyncratic volatility, range and kurtosis**

	Panel A: Full Sample				Panel B: $IO \geq 30\%$			
	Full	Small	Mid-cap	Large	Full	Small	Mid-cap	Large
				Breadth (B)				
mean	4.62%	0.52%	2.13%	11.68%	8.59%	1.10%	3.07%	13.36%
stdev	7.57%	0.42%	1.38%	10.45%	9.60%	0.57%	1.30%	10.77%
				ΔB				
mean	0.02%	0.01%	0.03%	0.02%	0.02%	0.01%	0.03%	0.02%
stdev	0.64%	0.16%	0.38%	1.05%	0.75%	0.20%	0.40%	0.95%
				Idiosyncratic volatility (idvol)				
mean	3.07%	4.38%	2.98%	1.96%	2.29%	3.36%	2.69%	1.88%
stdev	2.34%	3.15%	1.78%	1.21%	1.42%	2.20%	1.50%	1.07%
				Range				
mean	48.40%	80.30%	70.54%	38.79%	44.37%	81.57%	66.82%	35.81%
stdev	129.16%	147.47%	160.99%	112.16%	123.12%	147.31%	157.05%	106.81%
				Kurtosis				
mean	4.00	3.54	3.73	4.12	4.05	3.56	3.76	4.16
stdev	2.70	2.51	2.59	2.74	2.72	2.47	2.58	2.77
				Number of analysts				
mean	7.73	4.94	5.91	8.53	8.07	5.02	6.06	8.83
stdev	4.22	1.34	2.40	4.57	4.35	1.37	2.45	4.65
				Number of observations				
$B/\Delta B$	617309	177981	248405	190923	264356	16880	102396	145080
idvol	614808	177363	246980	190465	263931	16835	102141	144955
ran./kurt.	182941	3431	50905	128605	150467	1738	38956	109773

Note: Mean and standard deviation of breadth B, the first difference ΔB of B, idiosyncratic volatility, the range of the distribution of analyst forecasts, the kurtosis of that distribution, and the number of analysts. Panel A reports statistics for the full sample, broken down by stock size. Panel B reports statistics for the subsample where institutional ownership IO exceeds 30%.

about Hypothesis 2 because they reveal how B is related to range and kurtosis and how the effects depend on size or adjusted size.

As described in Section 3.4, we transform range, kurtosis, B, size and adjusted size into deciles across the population of stocks, and normalize the units so that the smallest decile corresponds to zero and the largest to one. (We calculate adjusted size as the product of size, idiosyncratic variance and kurtosis, divided by range, before transforming variables into deciles.) Thus, the coefficient of range or kurtosis measures the relationship between the respective variable and B for stocks in the bottom size or adjusted size decile. Moreover the sum of that coefficient and

Table II: **Breadth on range and kurtosis of analyst forecasts**

	Panel A: Size		Panel B: Adjusted size	
	(1): Full sample	(2): $IO \geq 30\%$	(1): Full sample	(2): $IO \geq 30\%$
range	0.055***	0.028***	-0.007	-0.000
	(5.48)	(3.32)	(-1.49)	(-0.01)
range*size	-0.097***	-0.057***		
	(-7.40)	(-5.81)		
range*adjsize			0.006	-0.000
			(0.89)	(-0.01)
kurtosis	-0.016***	-0.007*	-0.012***	-0.010***
	(-4.03)	(-1.80)	(-4.90)	(-5.08)
kurtosis*size	0.027***	0.017***		
	(5.22)	(3.64)		
kurtosis*adjsize			0.007**	0.007***
			(2.19)	(2.96)
size	0.768***	0.805***	0.695***	0.750***
	(59.75)	(94.21)	(58.46)	(109.05)
adjsize			0.033***	0.034***
			(6.36)	(8.82)

Note: Contemporaneous pooled OLS regressions of breadth B on the range of the distribution of analyst forecasts, the kurtosis of that distribution, stock size or adjusted size, IO, the interactions of range and kurtosis with size or adjusted size, and quarterly dummies. Adjusted size is the product of size, idiosyncratic variance and kurtosis, divided by range. Stock size and idiosyncratic variance are measured at the end of the last June. Range, kurtosis and IO are measured at the end of quarter t. Range, kurtosis, B, size, adjusted size and IO are transformed into deciles across the population of stocks at the time when each variable is measured, and the units are normalized so that the smallest decile corresponds to zero and the largest to one. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter. In this and subsequent tables, three asterisks (***) denote statistical significance at the 1% level, two asterisks (**) at the 5% level, and one asterisk (*) at 10% level.

of the coefficient of the interaction with size or adjusted size measures the relationship between the respective variable and B for stocks in the top decile. The decile transformation removes any time trends from our regression results. For the same reason, we include quarterly dummies in our regressions. The t-statistics are calculated using robust standard errors double-clustered by stock to address autocorrelation at the stock level (Petersen (2009)) and by quarter to address contemporaneous correlation across stocks.

The findings in Table II are generally consistent with Hypothesis 2 in the case of size. Consider first the effect of range. According to Hypothesis 2, B should be unrelated to range for small stocks and negatively related for large stocks. The coefficient of range, which measures the effect of range on B for stocks in the bottom size decile is positive, at odds with Hypothesis 2. Consistent with Hypothesis 2, however, the coefficient of the interaction between range and size is negative, meaning that the positive effect of range on B weakens as size increases. Moreover, the sum of the interaction coefficient and of the coefficient of range is negative, meaning that range and B are negatively related for stocks in the top size decile. The effect of range on B is negative and significant at the 5% level for size deciles 7-10.

Consider next the effect of kurtosis. According to Hypothesis 2, B should be negatively related to kurtosis for small stocks and positively related for large stocks. The coefficient of kurtosis, which measures the effect of kurtosis on B for stocks in the bottom size decile, is negative. Thus, consistent with Hypothesis 2, kurtosis and B are negatively related for small stocks. Also consistent with Hypothesis 2, the coefficient of the interaction between kurtosis and size is positive, meaning that the negative effect of kurtosis on B weakens as size increases. The sum of the interaction coefficient and of the coefficient of kurtosis is positive, also consistent with Hypothesis 2. The effect of kurtosis on B is negative and significant at the 5% level for size deciles 1-5 (slightly below the 10% level for decile 1 for $IO \geq 30\%$), and is positive and significant for deciles 8-10 (7-10 for $IO \geq 30\%$).

The signs of the interaction terms in the Breadth regressions are consistent not only with Hypothesis 2 but also with Hypothesis 2a. The negative (positive) coefficient on the interaction between range (kurtosis) and size means that the positive relationship between B and size strengthens for low range (high kurtosis) stocks.

The findings weaken when replacing size by adjusted size. The coefficient of range is insignificant, consistent with Hypothesis 2, but the coefficient of the interaction term is also insignificant, at odds with Hypothesis 2. The coefficient of kurtosis is negative and significant, consistent with Hypothesis 2, and the coefficient of the interaction between kurtosis and adjusted size is positive, also consistent with Hypothesis 2. The sum of the interaction coefficient and of the coefficient of kurtosis remains negative, however, at odds with Hypothesis 2. The weaker effects for adjusted size may be due in part to idiosyncratic volatility being strongly negatively related to size. This lowers the variability of adjusted size, rendering it a possibly noisy measure.

5 Analyst Forecasts and Expected Returns

In this section we test Hypothesis 1, which concerns the relationship between expected returns and the distribution of investor beliefs. As in Section 4, we proxy investor beliefs by the forecasts made by financial analysts.

Table III presents results from pooled OLS regressions of stock returns on the range and the

Table III: **Returns on range and kurtosis of analyst forecasts**

	All Stocks	Below Size Median	Above Size Median	All Stocks	Below Size Median	Above Size Median
	Panel A: No extra controls			Panel B: Extra controls		
1Q						
range	-0.007	-0.010	-0.003	-0.007	-0.010*	-0.003
	(-0.78)	(-1.20)	(-0.29)	(-1.42)	(-1.92)	(-0.51)
kurtosis	0.003*	0.006**	0.000	0.004**	0.007**	0.001
	(1.73)	(2.28)	(0.18)	(2.20)	(2.37)	(0.42)
4Q						
range	-0.008	-0.003	-0.010	-0.020	-0.021	-0.016
	(-0.39)	(-0.14)	(-0.49)	(-1.51)	(-1.14)	(-1.23)
kurtosis	0.001	0.002	0.000	0.005	0.007	0.003
	(0.21)	(0.31)	(0.02)	(1.02)	(0.77)	(0.61)

Note: Pooled OLS regressions of stock returns in quarter $t+1$ and in the year formed by quarters $t+1$ to $t+4$ on the range and the kurtosis of the distribution of analyst forecasts in quarter t. The regressions in Panel A include as additional independent variables stock size and quarterly dummies. The regressions in Panel B additionally include share price, IO, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. Size, share price, idiosyncratic volatility, market beta, book-to-market ratio and turnover are measured at the end of the last June. Range, kurtosis, IO and momentum are measured at the end of quarter t. Both sets of variables are transformed into deciles across the population of stocks at the time when each variable is measured, and the units are normalized so that the smallest decile corresponds to zero and the largest to one. The size subsamples are constructed using the median of the NYSE size distribution as cut-off. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

kurtosis of the distribution of analyst forecasts in quarter t. In the top part of the table we regress returns in quarter $t+1$ and in the bottom part returns in the year formed by quarters $t+1$ to $t+4$. In Panel A we include as additional independent variables stock size and quarterly dummies to control for time fixed effects. In Panel B we additionally include share price, IO, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. We transform range, kurtosis, size, IO, share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover into deciles across the population of stocks, and normalize the units so that the smallest decile corresponds to zero and the largest to one. We report results for the full sample of stocks and for two size subsamples constructed using the median of the NYSE size distribution as cut-off. We compute t-statistics using robust standard errors double-clustered by stock and by quarter. Fama and MacBeth (1973) regressions for this and subsequent tables yield similar coefficients and standard errors.

The findings in Table III provide some support for Hypothesis 1. The coefficient of range is negative, consistent with Hypothesis 1, but not statistically significant. It becomes more negative in the subsample of small stocks, and significant at the 10% level for returns one quarter ahead when including the extra controls. The coefficient of kurtosis is positive, consistent with Hypothesis 1, and significant at the 5% level for returns one quarter ahead when including the extra controls (10% without the extra controls). It becomes twice as large and more statistically significant in the subsample of small stocks. The difference in expected returns between stocks in the highest and in the lowest kurtosis decile is 0.4% per quarter, rising to 0.7% for small stocks.

The effect of range on expected returns in Table III is weaker than in DMS. DMS forecast monthly returns and find that the expected returns of the stocks in the lowest range quintile exceed those in the highest quintile by 0.79% per month. One reason for the discrepancy is that we forecast quarterly or annual returns. An additional reason is that we restrict our sample to stock/quarters with four or more analysts, so that we can calculate the kurtosis, while DMS allow for two or more analysts. Our stricter criterion excludes primarily stock/quarters involving small stocks, for which DMS find stronger effects of range. Because of our stricter criterion, the estimates for the effects of range and kurtosis on expected returns in Table III may be overly conservative.

DMS's empirical finding that the effect of range on expected returns is larger for small stocks and our analogous finding in Table III on the effect of kurtosis are consistent with our model. Suppose that small stocks are held only by their optimists ($\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}$) and that large stocks are held only by their optimists and their rationals ($\frac{1}{K_i} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$). Suppose also for simplicity that there are few optimists relative to rationals (K_i is small relative to N), in which case the prices of large stocks are determined almost exclusively by the rationals. An increase in range renders the optimists more optimistic and has no effect on the rationals' beliefs. Since the prices of large stocks are determined by the rationals, they do not change. The prices of small stocks instead rise because they are determined by the optimists, implying a larger effect of range for small stocks. A decrease in kurtosis raises the number of optimists and lowers that of rationals. The prices of large stocks barely rise because they are determined by the rationals and the percentage change in the number of rationals is small. The prices of small stocks rise more because the percentage increase in the number of optimists and decrease in the risk premium that optimists require to hold small stocks are large.

6 Breadth of Ownership and Expected Returns

In this section we test Hypothesis 3, which concerns the relationship between breadth B and expected returns. Hypothesis 3 follows from Hypotheses 1 and 2 tested in Sections 5 and 4, respectively. The tests in this section can be viewed as complementary to those in Sections 4 and 5: they provide only indirect evidence on the role of investor beliefs and disagreement, but they do not require noisy proxies of beliefs such as analyst forecasts (except for the calculation of adjusted size).

6.1 Main Results

Tables IV and V present our main tests of Hypothesis 3. In these tables we use the first difference ΔB of B, rather than the level. This is because B is highly autocorrelated (first-order autocorrelation is 0.997) and highly correlated with stock size (correlation of breadth and size deciles is 0.904). Hence, using levels may confound the effects of B on returns with the effects of size. CHS use the first difference of B in their regression of returns on B for similar reasons. In Table VII we report results using the level of B rather than the first difference.

Table IV presents results from pooled OLS regressions of stock returns in the year formed by quarters $t+1$ to $t+4$ on ΔB in quarter t and the interaction between ΔB with stock size or adjusted size in the same quarter. The results for size are in Panel A, where we include as additional independent variables size, IO, and quarterly dummies to control for time fixed effects. The results for adjusted size are in Panel B, where we include as additional independent variables adjusted size, IO and quarterly dummies. In the specifications termed "Extra controls" we include as additional independent variables share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. In the specifications termed "Contemp. ΔIO" we additionally include the first difference of IO in quarters $t+1$ to $t+4$ (four variables). The variable ΔIO is contemporaneous with returns. Controlling for it, as is done also in CHS, removes a relationship between ΔB and expected returns that arises if ΔB predicts changes in IO, and these changes affect returns. As in previous tables, we transform variables into deciles and compute t-statistics using robust standard errors double-clustered by stock and by quarter.

The findings in Table IV are consistent with Hypothesis 3. Consider first the effects for small stocks. The coefficient of ΔB, which measures the effect of ΔB on expected returns for stocks in the bottom size or adjusted size decile, is negative in both Panels A and B. Thus, ΔB predicts the returns of small stocks negatively, consistent with Hypothesis 3.

Consider next the effects for large stocks. The coefficient of the interaction between ΔB with size is positive, meaning that the negative effect of ΔB on expected returns weakens as size increases. Moreover, the sum of the interaction coefficient and of the coefficient of ΔB is positive in both Panels A and B, meaning that ΔB and expected returns are positively related for stocks in the top size or adjusted-size decile. Thus, ΔB predicts the returns of large stocks positively, consistent with Hypothesis 3.

The negative effect of ΔB on expected returns is significant at the 5% level for size deciles 1-3 in the full sample with extra controls (second column in Panel A1), which we take as our baseline

Table IV: **Returns on first difference of breadth**

	Panel A: Size							
	Panel A1: Full Sample				Panel A2: $IO \geq 30\%$			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
ΔB	-0.049**	-0.052**	-0.137***	-0.119***	-0.071*	-0.058**	-0.179***	-0.147***
	(-2.03)	(-2.46)	(-5.07)	(-5.02)	(-1.78)	(-2.18)	(-4.44)	(-5.45)
ΔB*size	0.068***	0.071***	0.171***	0.172***	0.080**	0.071**	0.210***	0.191***
	(3.09)	(3.05)	(6.48)	(6.24)	(2.11)	(2.29)	(5.49)	(6.08)
size	-0.177***	-0.057***	-0.310***	-0.179***	-0.110***	-0.048**	-0.219***	-0.140***
	(-6.08)	(-3.17)	(-9.16)	(-9.23)	(-3.03)	(-2.11)	(-5.75)	(-5.91)
IO	0.068***	0.066***	0.124***	0.138***	-0.048***	-0.047***	0.074***	0.086***
	(5.37)	(4.37)	(9.78)	(9.34)	(-3.37)	(-2.83)	(5.16)	(5.65)
Extra controls	No	Yes	No	Yes	No	Yes	No	Yes
Contemp. ΔIO	No	No	Yes	Yes	No	No	Yes	Yes
	Panel B: Adjusted Size							
	Panel B1: Full Sample				Panel B2: $IO \geq 30\%$			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
ΔB	-0.032	-0.032**	-0.107***	-0.087***	-0.033	-0.020	-0.110***	-0.083***
	(-1.56)	(-1.97)	(-4.60)	(-4.68)	(-1.02)	(-0.99)	(-3.37)	(-3.97)
ΔB*adjsize	0.043**	0.041**	0.127***	0.126***	0.033	0.024	0.126***	0.112***
	(2.38)	(2.30)	(5.97)	(5.93)	(1.11)	(0.98)	(4.26)	(4.63)
adjsize	-0.129***	-0.009	-0.232***	-0.093***	-0.063**	-0.004	-0.145***	-0.061***
	(-5.95)	(-0.57)	(-10.33)	(-6.64)	(-2.26)	(-0.19)	(-5.06)	(-3.12)
IO	0.037*	0.058***	0.075***	0.119***	-0.061***	-0.050***	0.054***	0.080***
	(1.71)	(3.04)	(3.80)	(6.76)	(-3.92)	(-2.79)	(3.81)	(4.92)
Extra controls	No	Yes	No	Yes	No	Yes	No	Yes
Contemp. ΔIO	No	No	Yes	Yes	No	No	Yes	Yes

Note: Pooled OLS regressions of stock returns in the year formed by quarters $t+1$ to $t+4$ on the first difference ΔB of breadth in quarter t and the interaction of ΔB with stock size or adjusted size in the same quarter. Panel A shows the results for size. These regressions include as additional independent variables size, IO and quarterly dummies. Panel B shows the results for adjusted size. These regressions include as additional independent variables adjusted size, IO and quarterly dummies. The regressions under "Extra controls" additionally include share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. The regressions under "Contemp. ΔIO" additionally include the first difference of IO in quarters $t+1$ to $t+4$. Adjusted size is the product of size, idiosyncratic variance and kurtosis, divided by range. For stock/month observations with fewer than four analysts, we fill in the missing observations for range and kurtosis by linearly predicting them from size and idiosyncratic variance over the full sample. Size, adjusted size, share price, idiosyncratic volatility, market beta, book-to-market ratio and turnover are measured at the end of the last June. ΔB, IO and momentum are measured at the end of quarter t. Both sets of variables are transformed into deciles across the population of stocks at the time when each variable is measured, and the units are normalized so that the smallest decile corresponds to zero and the largest to one. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

specification. The positive effect of ΔB is significant at the 5% level for size decile 10 in the baseline specification.

Turning to economic significance, the coefficient -0.052 on ΔB in the baseline specification means that within the smallest size decile, a stock in the top ΔB decile ($\Delta B = 1$) earns 5.2% lower annual expected return than a stock in the bottom ΔB decile ($\Delta B = 0$). The sum of that coefficient and that of the interaction term is 0.019, meaning that within the largest size decile, a stock in the top ΔB decile earns 1.9% higher annual expected return than a stock in the bottom ΔB decile. The negative predictive effect of ΔB for small stocks is larger than the positive effect for large stocks.

The effects remain comparable in magnitude in the subsample $IO \geq 30\%$. They become somewhat weaker when replacing size by adjusted size. They become two to four times as large when controlling for ΔIO. Thus, the relationship between ΔB and expected returns that arises because ΔB predicts changes in IO, and these changes affect returns, is quantitatively important. Moreover, that relationship works in the opposite direction than the one implied by our model.

Table V complements Table IV by presenting results from portfolio sorts. We construct nine portfolios based on a 3×3 double sort, first on size and then on ΔB in quarter t. As per Section 3.1, we define small stocks as those with size below the 30th percentile of our sample, mid-cap stocks as those with size between the 30th and the 70th percentile, and large stocks as those with size above the 70th percentile. Within each size group, we define low ΔB stocks as those with ΔB below the 20th percentile, mid ΔB stocks as those between the 20th and the 80th percentile, and high ΔB stocks as those above the 80th percentile.

Panel A reports the returns of the nine portfolios. The portfolios are formed at the end of quarter t. Annual returns are measured in the subsequent year, in the year beginning one month after the end of quarter t, and in the year beginning two months after the end of quarter t. The average of the entire series of annual returns is calculated. Portfolio returns are the equally weighted averages of the returns of the stocks in the portfolio. Results for value-weighted averages are similar. Panel B reports average annual returns and alphas for long-short strategies that go long in the high ΔB portfolio and short in the low ΔB portfolio. Alphas are computed using the CAPM, the Carhart (1997) four-factor model, and the Fama and French (2015) five-factor model augmented by the Carhart (1997) momentum factor. The t-statistics are computed using Newey-West standard errors with two lags, to address the serial correlation induced by the overlap of the annual returns.

Panel A shows that for small stocks there is a clear decreasing pattern in average return when moving from the low ΔB to the high ΔB portfolio. The average annual return drops from 21.43% for low ΔB to 17.91% for high ΔB. Going long in the high ΔB portfolio and short in the low ΔB

Table V: **Portfolio double-sorts on size and first difference of breadth**

Panel A: Average returns of nine size - ΔB portfolios						
	Panel A1: Full Sample			Panel A2: $IO \geq 30\%$		
	Small	Mid-cap	Large	Small	Mid-cap	Large
Low ΔB	21.43%	14.03%	12.79%	28.54%	15.78%	13.77%
	(7.08)	(5.99)	(7.65)	(8.50)	(6.75)	(8.25)
Mid ΔB	19.01%	13.59%	13.13%	18.73%	15.56%	13.92%
	(8.33)	(7.70)	(9.37)	(8.56)	(8.63)	(9.86)
High ΔB	17.91%	13.82%	13.41%	21.23%	15.03%	13.72%
	(7.11)	(7.28)	(8.78)	(6.84)	(8.02)	(9.24)

Panel B: High ΔB minus Low ΔB								
	Panel B1: Full Sample				Panel B2: $IO > 30\%$			
	Small	Mid-cap	Large	Large minus Small	Small	Mid-cap	Large	Large minus Small
Average returns	-3.52%***	-0.20%	0.62%	4.14%***	-7.31%***	-0.76%	-0.05%	7.27%***
	(-3.34)	(-0.16)	(0.58)	(3.88)	(-2.85)	(-0.53)	(-0.04)	(2.93)
CAPM alpha	-2.45%***	0.34%	0.74%	3.19%***	-5.95%**	-0.09%	0.17%	6.12%**
	(-3.20)	(0.33)	(0.66)	(3.38)	(-2.52)	(-0.08)	(0.17)	(2.37)
Carhart-4 alpha	-4.47%***	-2.56%*	-1.58%	2.89%***	-10.11%***	-2.72%*	-2.08%**	8.03%***
	(-4.60)	(-1.80)	(-1.53)	(2.59)	(-3.61%)	(-1.85)	(-2.15)	(3.00)
FF-5 & UMD	-4.40%***	-2.47%	-0.09%	4.31%***	-9.14%***	-1.27%	-0.48%	8.66%***
alpha	(-3.73)	(-1.58)	(-0.08)	(3.23)	(-2.76)	(-0.77)	(-0.46)	(2.62)

Note: Average returns of nine portfolios formed by a 3×3 double sort, first on size and then on the first difference ΔB of breadth in quarter t. We define small stocks as those with size below the 30th percentile of our sample, mid-cap stocks as those with size between the 30th and the 70th percentile, and large stocks as those with size above the 70th percentile. Within each size group, we define low ΔB stocks as those with ΔB below the 20th percentile, mid ΔB stocks as those between the 20th and the 80th percentile, and high ΔH stocks as those above the 80th percentile. Panel A reports the returns of the nine portfolios. The portfolios are formed at the end of quarter t. Annual returns are measured in the subsequent year, in the year beginning one month after the end of quarter t, and in the year beginning two months after the end of quarter t. The average of the entire series of annual returns is calculated. Portfolio returns are the equally weighted averages of the returns of the stocks in the portfolio. Panel B reports average annual returns and alphas for long-short strategies that go long in the high ΔB portfolio and short in the low ΔB portfolio. Alphas are computed using the CAPM, the Carhart (1997) four-factor model, and the Fama and French (2015) five-factor model augmented by the Carhart (1997) momentum factor. The t-statistics, in parentheses, are computed using Newey-West standard errors with two lags.

portfolio yields an average annual return of -3.52%, with t-statistic -3.34. The strategy's CAPM, four-factor and five-factor alphas are similar to its average return.

For mid-cap stocks, there is no clear pattern in average return across the ΔB portfolios. For large stocks, a pattern reappears in the full sample and is the opposite to that for small stocks, but is not statistically significant. The change in the long-short strategies' returns when moving from small to large stocks is statistically significant, however. Going long in the large-stock long-short ΔB portfolio and short in the small-stock long-short ΔB portfolio yields an average annual return of 4.14%, with t-statistic 3.88. These findings are consistent with Hypothesis 3.

When performing the double sort using adjusted size instead of size, the results (not reported) lose their significance for value-weighted returns and remain significant in some cases for equally weighted returns. Significance is restored when value weights are computed using adjusted size rather than size.

Tables IV and V paint a consistent picture. ΔB predicts negatively the returns of small stocks and positively those of large stocks. The effect of ΔB for small stocks is statistically significant. It is also larger in absolute value than the effect for large stocks, which is not statistically significant in some specifications. The change in the effect of ΔB from small to large stocks is statistically significant.

6.2 Other Horizons

Table VI presents results from the same regressions as in the second and fourth columns of Panel A1 of Table IV (stock returns in the year formed by quarters $t+1$ to $t+4$ on ΔB in quarter t, the interaction between ΔB and size, size, IO, extra controls, on the full sample, with or without ΔIO), except that stock returns are evaluated over horizons different than one year. We evaluate returns from quarter $t+1$ to $t+k$ and consider horizons of one quarter ($k=1$), two years ($k=8$), three years ($k=12$), four years ($k=16$) and five years ($k=20$). We do not express stock returns in annualized terms, i.e., leave them as cumulative returns. As in Table IV, we transform the independent variables into deciles, and normalize the units so that the smallest decile corresponds to zero and the largest to one. We compute t-statistics using robust standard errors double-clustered by stock and by quarter. Clustering by stock addresses the autocorrelation that arises because the periods over which returns are evaluated overlap. Using Newey-West standard errors with $k-1$ lags, corresponding to the overlap, yields similar standard errors.

The findings in Table VI are consistent with Hypothesis 3 except for the quarterly horizon. Over a quarterly horizon, ΔB predicts returns of small stocks positively, while Hypothesis 3 implies a negative effect. Moreover, the effect weakens with stock size, becoming less positive, while Hypothesis 3 implies that it should become less negative. Controlling for ΔIO brings the coefficients of ΔB and of the interaction term close to zero but does not change their signs.

Over horizons longer than one year, the relationship between ΔB and future returns is not only consistent with Hypothesis 3 but becomes stronger as horizon increases. As in Table IV, B predicts returns of small stocks negatively. Moreover, the effect weakens when moving from small to large

Table VI: **Long-horizon return regressions**

	1Q		8Q		12Q		16Q		20Q	
ΔB	0.019**	0.013	-0.107***	-0.209***	-0.164***	-0.305***	-0.239***	-0.424***	-0.290***	-0.490***
	(2.31)	(1.53)	(-3.25)	(-5.84)	(-3.45)	(-5.92)	(-3.88)	(-6.14)	(-2.88)	(-4.41)
ΔB*size	-0.011	-0.001	0.161***	0.318***	0.225***	0.434***	0.315***	0.577***	0.378***	0.667***
	(-1.22)	(-0.08)	(4.36)	(7.54)	(4.26)	(7.26)	(4.71)	(7.32)	(3.34)	(5.23)
size	-0.004	-0.020**	-0.102***	-0.340***	-0.182***	-0.534***	-0.267***	-0.745***	-0.342***	-0.960***
	(-0.42)	(-2.38)	(-3.25)	(-9.35)	(-3.65)	(-8.88)	(-3.88)	(-8.88)	(-3.67)	(-8.36)
IO	0.021***	0.036***	0.113***	0.262***	0.179***	0.401***	0.276***	0.572***	0.445***	0.834***
	(3.20)	(5.54)	(4.89)	(10.79)	(5.29)	(11.64)	(6.27)	(12.70)	(6.61)	(11.48)
Extra controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Contemp. ΔIO	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Note: Pooled OLS regressions of stock returns from end of quarter t to end of quarter $t+k$ with $k = 1, 8, 12, 16, 20$, on the first difference ΔB of Breadth in quarter t and the interaction between ΔB with stock size in the same quarter. The regressions include additionally size, IO, quarterly dummies and the extra controls in Table IV. The regressions are run with or without ΔIO, as indicated. Stock returns are not expressed in annualized terms, i.e., are left as cumulative returns. Variables are transformed into deciles and normalized, as described in Table IV. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

Table VII: **Robustness tests: Sub-periods and level of breadth**

	(1) First part of sample (2Q-1980 to 4Q-1997)		(2) Second part of sample (1Q-1998 to 4Q-2017)		(3) Levels of B	
ΔB	0.023	-0.019	-0.101***	-0.189***		
	(1.42)	(-1.07)	(-2.95)	(-4.90)		
ΔB*size	-0.007	0.059***	0.122***	0.253***		
	(-0.34)	(2.75)	(3.13)	(5.52)		
B					0.115**	-0.084**
					(2.49)	(-2.00)
B*size					0.151***	0.357***
					(4.42)	(9.87)
Extra controls	Yes	Yes	Yes	Yes	Yes	Yes
Contemp. ΔIO	No	Yes	No	Yes	No	Yes

Note: In column groups (1) and (2) the sample period is split into two sub-periods, and in column group (3) the level of B is used rather than the first difference ΔB. All columns present results from pooled OLS regressions of stock returns from end of quarter t to end of quarter $t+4$ on B or ΔB in quarter t and the interaction between B or ΔB with stock size in the same quarter. The regressions include additionally size, IO, quarterly dummies and the extra controls in Table IV. The regressions are run with or without ΔIO, as indicated. Variables are transformed into deciles and normalized, as described in Table IV. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

stocks, and becomes positive for large stocks. The coefficients of ΔB and of the interaction term scale up approximately linearly with horizon. They are statistically significant at the 1% level for all horizons. Their sum (effect of ΔB on returns of size decile 10) is statistically significant at the 1% level as well.

6.3 Robustness

Tables VII and VIII present results from a series of robustness tests. In column groups (1) and (2) of Table VII the sample period is split into two sub-periods, and in column group (3) the level of B is used rather than the first difference ΔB. The regressions are the same as in the second and fourth columns of Panel A1 of Table IV (stock returns in the year formed by quarters $t+1$ to $t+4$ on ΔB or B in quarter t, the interaction between ΔB or B and size, size, IO, extra controls, with or without ΔIO, on the full sample).

Sub-period results are consistent with Hypothesis 3 when controlling for ΔIO: the coefficient of ΔB is negative, the coefficient of the interaction term is positive, and the sum of the two coefficients is positive. When not controlling for ΔIO, the results are consistent with Hypothesis 3 only in the second half of the sample. In the first half, both coefficients have the wrong signs. The results using the level of B rather than the first difference are consistent with Hypothesis 3 when controlling for ΔIO. When not controlling for ΔIO, the coefficient of the interaction term is positive, consistent with Hypothesis 3, but the coefficient of B is positive, at odds with Hypothesis 3.

Table VIII presents results using the Herfindahl index H instead of B as the measure of breadth of ownership. A fuller analysis of H and its relationship with investor beliefs and expected returns is in a previous version of this paper (Hardouvelis, Karalas, and Vayanos (2020)). Column group (1) presents results from the main regressions with annual returns. In column group (2), returns are quarterly. In column groups (3) and (4), the sample period is split into two sub-periods. In column group (5), the level of H is used rather than the first difference ΔH. The regressions in each column group are analogous to those in the second and fourth columns of Panel A1 of Table IV.

Results using H provide stronger support for Hypothesis 3 than results using B: they are consistent with Hypothesis 3 in the cases where the B results are consistent, and they become consistent with Hypothesis 3 in additional cases. Since B and H are negatively related, Hypothesis 3 implies that the coefficient of ΔH or H should be positive, the coefficient of the interaction with size should be negative, and the sum of the two coefficients should be negative. These predictions are borne out in the main regressions with annual returns. They are borne out in the second subperiod, except that the sum of the coefficients is negative in one case. They are borne out in the regressions with the level of H rather than the first difference, even when not controlling for ΔIO. They are borne out in the regressions with quarterly returns. In the latter two cases, statistical significance for some of the coefficients is lacking, but their signs are consistent with Hypothesis 3 unlike when using B.

7 Ownership at the Style Level

In this section we extend our analysis of breadth of ownership and expected returns to the level of investment styles. Investors can adopt different styles, such as value and growth, because of different preferences or beliefs. Assuming that different styles are adopted by disjoint groups of investors, we can map each style to an investor group and interpret the investors in our model as styles. With that interpretation, we can test the model using measures of ownership computed at the style rather than the investor level. The style-level analysis can be viewed as an additional robustness test. It can also help rule out alternative explanations of our findings on the relationship between breadth of ownership and expected returns that apply to the level of individual investors but not to aggregate styles. Examples are explanations based on monitoring or rent extraction by large shareholders (e.g., Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi

Table VIII: **Robustness tests: Herfindahl index**

	(1): ΔH		(2): ΔH - 1Q		(3): ΔH - First part of sample (2Q-1980 to 4Q-1997)		(4): ΔH - Second part of sample (1Q-1998 to 4Q-2017)		(5): Levels of H	
ΔH	0.017**	0.030***	0.003	0.000	-0.003	-0.001	0.023*	0.043***		
	(2.01)	(3.36)	(1.05)	(0.10)	(-0.32)	(-0.08)	(1.80)	(3.37)		
ΔH*size	-0.035**	-0.044**	-0.013*	-0.007	-0.021	-0.020	-0.026	-0.041		
	(-2.02)	(-2.51)	(-1.86)	(-1.11)	(-1.33)	(-1.22)	(-0.90)	(-1.40)		
H									0.034	0.189***
									(1.38)	(7.32)
H*size									-0.168***	-0.347***
									(-5.06)	(-9.47)
Extra controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Contemp. ΔIO	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Note: Column group (1) presents results from the main regressions with annual returns, with the first difference ΔH of the Herfindahl index rather than the first difference ΔB of breadth. In column group (2), returns are quarterly. In column groups (3) and (4), the sample period is split into two sub-periods. In column group (5), the level of H is used rather than the first difference ΔH. All columns present results from pooled OLS regressions of stock returns from end of quarter t to end of quarter $t+1$ or $t+4$ on H or ΔH in quarter t and the interaction between H or ΔH with stock size in the same quarter. The regressions include additionally size, IO, quarterly dummies and the extra controls in Table IV. The regressions are run with or without ΔIO, as indicated. Variables are transformed into deciles and normalized, as described in Table IV. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

Table IX: **Descriptive statistics of style-level breadth**

	Panel A: Full Sample				Panel B: $IO \geq 30\%$			
	Full	Small	Mid-cap	Large	Full	Small	Mid-cap	Large
	Breadth of Styles (B-styles)							
mean	9.24	5.80	9.33	11.92	11.18	8.43	10.51	12.19
stdev	3.67	2.84	2.95	2.64	2.62	2.36	2.26	2.44
	ΔB-style							
mean	0.08	0.09	0.10	0.06	0.06	0.08	0.07	0.05
stdev	0.97	0.91	1.00	0.97	0.95	0.83	0.98	0.95
	Number of Observations							
Obs.	264353	70213	107174	86966	153053	11749	65332	75972

Note: Mean and standard deviation of style-level breadth B_{style} and of its first difference ΔB_{style}. Panel A reports statistics for the full sample, broken down by stock size. Panel B reports statistics for the subsample where institutional ownership IO exceeds 30%. The statistics are based on pooled cross-sectional and time-series samples.

(1997), Bolton and Von Thadden (1998)) and on asymmetric information by corporate insiders (e.g., Kyle (1985)).

Breadth is correlated at the investor and at the style level, with a correlation of 0.48. This correlation is driven partly by size, but remains important even within size groups. The correlation within the groups of small, mid-cap and large stocks is 0.65, 0.57 and 0.25, respectively. Given the positive correlation between breadth at the investor and at the style level, we expect our findings to extend to styles.

Table IX presents descriptive statistics of B and ΔB at the style level. We denote B at the style level by B_{style}. Consistent with Hypothesis 2a and the findings in Table I, there is a positive relationship between B_{style} and size. The mean of B_{style} for small stocks is 5.80, meaning that the average small stock is held by 5.80 out of the 29 styles in our data. The mean of B_{style} rises to 9.33 for mid-cap stocks and to 11.92 for large stocks.

Table X presents results using style-level measures of ownership. Column group (1) presents results from the main regressions with annual returns. In column groups (2) and (3), returns are quarterly and bi-annual, respectively. In column group (4), the level of B_{style} is used rather than the first difference ΔB_{style}. In column group (5), the first difference ΔH_{style} of the style-level Herfindahl index is used instead of ΔB_{style}. The regressions in each column group are analogous to those in the second and fourth columns of Panel A1 of Table IV. We do not consider sub-periods because ownership data pertaining to investment styles are available only from 1997 to 2015.

Results using style-level measures of ownership provide strong support for Hypothesis 3. In all regressions, the coefficients of B_{style}, ΔB_{style} or ΔH_{style}, and of the interaction term with size,

Table X: **Returns on style-level measures of ownership**

	(1): ΔB-style - 4Q		(2): ΔB-style - 1Q		(3): ΔB-style - 8Q		(4): Level of B-style		(5): ΔH-style	
ΔB-style	-0.057***	-0.087***	-0.009	-0.016**	-0.118***	-0.170***				
	(-3.62)	(-5.08)	(-1.19)	(-2.25)	(-5.33)	(-7.10)				
ΔB-style*size	0.076***	0.110***	0.019	0.025**	0.156***	0.218***				
	(2.92)	(3.88)	(1.65)	(2.19)	(5.57)	(7.03)				
B-style							-0.040	-0.226***		
							(-0.86)	(-5.21)		
B-style*size							0.141***	0.349***		
							(2.61)	(6.25)		
ΔH-style									0.033***	0.048***
									(2.89)	(4.10)
ΔH-style*size									-0.034*	-0.043**
									(-1.79)	(-2.27)
Extra controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Contemp. ΔIO	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes

Note: Column group (1) presents results from the main regressions with annual returns, with the first difference ΔB_{style} of style-level breadth rather than ΔB. In column groups (2) and (3), returns are quarterly and bi-annual, respectively. In column group (4), the level of B_{style} is used rather than the first difference ΔB_{style}. In column group (5), the first difference ΔH_{style} of the style-level Herfindahl index is used instead of ΔB_{style}. All columns present results from pooled OLS regressions of stock returns from end of quarter t to end of quarter $t+1$ or $t+4$ on B_{style}, ΔB_{style} or ΔH_{style} in quarter t and the interaction between B_{style}, ΔB_{style} or ΔH_{style} with stock size in the same quarter. The regressions include additionally size, IO, quarterly dummies and the extra controls in Table IV. The regressions are run with or without ΔIO, as indicated. Variables are transformed into deciles and normalized, as described in Table IV. The t-statistics, in parentheses, are computed using robust standard errors double-clustered by stock and by quarter.

have signs consistent with Hypothesis 3. These coefficients are statistically significant except for two cases, and their sum has sign consistent with Hypothesis 3 except for one case.

8 Conclusion

We study theoretically and empirically the relationship between investor beliefs, breadth of ownership and expected returns. Investor beliefs in our model are described by two dimensions: the intensity of disagreement between optimists and pessimists, and the polarization of beliefs measured by the number of optimists and pessimists relative to moderates. These dimensions map, respectively, to the range and kurtosis of the distribution of beliefs across investors. Our two-dimensional description of beliefs generates a size-dependent relationship between breadth and expected returns: positive for large stocks and negative for small stocks. This relationship, which we also find in the data, is puzzling for the one-dimensional description of beliefs that is common in the literature and based on the intensity of disagreement.

Besides testing for the relationship between breadth and expected returns that our model predicts, we test for the relationship that beliefs should have with each of these variables. The size-dependent breadth-return relationship in our model arises because stocks with more polarized beliefs (1) earn lower expected returns and (2) have a broader investor base (relative to similar-size stocks with less polarized beliefs) if they are small and a narrower base if they are large. Proxying investor beliefs by analyst forecasts, we find empirical support for both predictions.

Our results suggest that a two-dimensional description of beliefs seems necessary to explain the empirical relationship between breadth and expected returns. Moreover, such a description better accounts for the relationship between beliefs and each of these variables. Incorporating the two-dimensional description suggested by our model and CLY into richer dynamic settings, and fleshing out the joint dynamics of disagreement, polarization, holdings and prices, seems an interesting direction for future research.

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Appendix

A Proofs

Proof of Proposition 2.1. Suppose $\lambda_1 = .. = \lambda_N \equiv \lambda$. Setting

$$S_i \equiv \bar{D} - b_i\lambda + a\theta_i\sigma_i^2\phi_i, \tag{A.1}$$

we can write (2.5) as (2.7). Equation (2.7) has a unique solution because when the left-hand side is positive, it is decreasing in ϕ_i. Summing $\lambda_n = a\sum_{j=1}^{I} b_j x_{jn}$ over n and using (2.4) and $\lambda_1 = .. = \lambda_N = \lambda$, we find

$$\lambda = \frac{a}{N}\sum_{j=1}^{I} b_j\theta_j. \tag{A.2}$$

Substituting (A.2) into (A.1), we find (2.6). Substituting (A.1) into (2.3) and using $\lambda = \lambda_n$, we find (2.8).

When Condition (A) is satisfied, $\lambda_n = a\sum_{j=1}^{I} b_j x_{jn}$ implies $\lambda_1 = .. = \lambda_N = 0$. When Condition (B) is satisfied, (2.3) and $\lambda_n = a\sum_{j=1}^{I} b_j x_{jn}$ imply

$$\begin{aligned}
\lambda_n &= \sum_{j=1}^{I} b_j \max\left\{\frac{\bar{D} + \epsilon_{jn} - S_j - b_j\lambda_n}{\sigma_j^2}, 0\right\} \\
&= \sum_{m\in\mathcal{I}}\sum_{j\in\mathcal{I}_m} b_j \max\left\{\frac{\bar{D} + \epsilon_{jn} - S_j - b_j\lambda_n}{\sigma_j^2}, 0\right\},
\end{aligned} \tag{A.3}$$

where $\mathcal{I}$ is a set formed by one stock in each subset of stocks sharing the same characteristics, and $\mathcal{I}_m$ is the subset corresponding to stock $m \in \mathcal{I}$. Suppose that S_i is given by (2.6) for all $i = 1, .., N$, in which case Condition (B) implies that S_j is equal across $j \in \mathcal{I}_m$. Suppose also, proceeding by contradiction, that $\lambda_n > \lambda_{n'}$ for some $n, n' \in \{1, .., N\}$. Condition (B), equality of S_j across $j \in \mathcal{I}_m$, and $b_j \geq 0$ for all $j \in \mathcal{I}_m$, imply

$$\sum_{j\in\mathcal{I}_m} b_j \max\left\{\frac{\bar{D} + \epsilon_{jn} - S_j - b_j\lambda_n}{a\sigma_j^2}, 0\right\} \leq \sum_{j\in\mathcal{I}_m} b_j \max\left\{\frac{\bar{D} + \epsilon_{jn} - S_j - b_j\lambda_{n'}}{a\sigma_j^2}, 0\right\}. \tag{A.4}$$

Summing (A.4) over $m \in \mathcal{I}$ and using (A.3), we find $\lambda_n \leq \lambda_{n'}$, which contradicts $\lambda_n > \lambda_{n'}$. Therefore, λ_n is equal across n. The first part of the proposition then implies that S_i is given by (2.6) for all $i = 1, .., N$ □

Proof of Proposition 2.2. Suppose $\epsilon_i > \epsilon_{i'}$ and $(\theta_i, b_i, \sigma_i, K_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, K_{i'})$. When $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2}$, the unique solution of (2.7) for stock i is $\phi_i = -\frac{1}{N}$. Since $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{\epsilon_{i'}}{a\theta_i\sigma_i^2}$, the unique solution of (2.7) for stock i' is also $\phi_{i'} = -\frac{1}{N}$. Therefore, (2.6) implies $S_i = S_{i'}$. When instead $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$, the unique solution of (2.7) for stock i is such that $-\frac{\epsilon_i}{a\theta_i\sigma_i^2} - \phi_i < 0$. Since $-\frac{\epsilon_i}{a\theta_i\sigma_i^2} - \phi_i < 0$ and $\epsilon_i > \epsilon_{i'}$,

$$1 = \sum_{n=1}^{N} \max\left\{\frac{\epsilon_{in}}{a\theta_i\sigma_i^2} - \phi_i, 0\right\} > \sum_{n=1}^{N} \max\left\{\frac{\epsilon_{in'}}{a\theta_i\sigma_i^2} - \phi_i, 0\right\}.$$

Therefore, the unique solution of (2.7) for stock i' satisfies $\phi_{i'} < \phi_i$, which implies from (2.6) $S_i > S_{i'}$.

Suppose next $K_i < K_{i'}$ and $(\theta_i, b_i, \sigma_i, \epsilon_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, \epsilon_{i'})$. When $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2}$, the unique solution of (2.7) for stock i is $\phi_i = -\frac{1}{N}$. Since $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} = \frac{\epsilon_{i'}}{a\theta_i\sigma_i^2}$, the unique solution of (2.7) for stock i' is also $\phi_{i'} = -\frac{1}{N}$, and (2.6) implies $S_i = S_{i'}$. When instead $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$, the unique solution of (2.7) for stock i is such that $-\frac{\epsilon_i}{a\theta_i\sigma_i^2} - \phi_i < 0$. Since $-\frac{\epsilon_i}{a\theta_i\sigma_i^2} - \phi_i < 0$ and $K_i < K_{i'}$,

$$1 = \sum_{n=1}^{N} \max\left\{\frac{\epsilon_{in}}{a\theta_i\sigma_i^2} - \phi_i, 0\right\} < \sum_{n=1}^{N} \max\left\{\frac{\epsilon_{in'}}{a\theta_i\sigma_i^2} - \phi_i, 0\right\}.$$

Therefore, the unique solution of (2.7) for stock i' satisfies $\phi_{i'} > \phi_i$, which implies from (2.6) $S_i < S_{i'}$. The expected return comparisons follow from the price comparisons because expected return $\frac{\bar{D}-S_i}{S_i}$ is decreasing in the price. □

Proof of Proposition 2.3. Suppose $\epsilon_i > \epsilon_{i'}$ and $(\theta_i, b_i, \sigma_i, K_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, K_{i'})$. The argument before the proposition implies that five cases are possible: (1) stocks i and i' are held only by their optimists, (2) stock i is held only by its optimists and stock i' is held by its optimists and additional investors, (3) stocks i and i' are held only by their optimists and their rationals, (4) stock i is held only by its optimists and its rationals and stock i' is held by all investors, and (5) stocks i and i' are held by all investors. In Cases (2) and (4), $B_i < B_{i'}$. In Cases (1), (3) and (5), $B_i = B_{i'}$. The condition for Case (2) is $\frac{\epsilon_i}{\epsilon_{i'}K_i} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}$. The condition for Case (4) is $\frac{\epsilon_i}{\epsilon_{i'}N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$.

Suppose next $K_i < K_{i'}$ and $(\theta_i, b_i, \sigma_i, \epsilon_i) = (\theta_{i'}, b_{i'}, \sigma_{i'}, \epsilon_{i'})$. If $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}$, then stock i is held only by its optimists. Since $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i} > \frac{1}{K_{i'}}$, the same is true for stock i'. Therefore, $B_i = \frac{K_i}{N}$ and $B_{i'} = \frac{K_{i'}}{N} > B_i$. If $\frac{1}{K_i} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$, then stock i is held only by its optimists and its rationals. Since $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$, stock i' is not held by its pessimists. Therefore, $B_i = \frac{N-K_i}{N}$ and $B_{i'} \le \frac{N-K_{i'}}{N} < B_i$. If $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2}$, then stocks i and i' are held by all investors, and $B_i = B_{i'} = 1$ □

Proof of Proposition 2.4. If stock i is held only by its optimists, then $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}$. Since $\frac{\epsilon_i}{a\theta_{i'}\sigma_i^2} >$

$\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{K_i}$, stock i' is also held only by its optimists. If stock i is not held by its pessimists, then $\frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$. Since $\frac{\epsilon_i}{a\theta_{i'}\sigma_i^2} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{1}{N}$, stock i' is also not held by its pessimists. Therefore, five cases are possible: (1) stocks i and i' are held only by their optimists, (2) stock i' is held only by its optimists and stock i is held by its optimists and additional investors, (3) stocks i and i' are held only by their optimists and their rationals, (4) stock i' is held only by its optimists and its rationals and stock i is held by all investors, and (5) stocks i and i' are held by all investors. In Cases (2) and (4), $B_i > B_{i'}$. In Cases (1), (3) and (5), $B_i = B_{i'}$. The condition for Case (2) is $\frac{1}{K_i} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{\theta_{i'}}{\theta_i K_i}$. The condition for Case (4) is $\frac{1}{N} > \frac{\epsilon_i}{a\theta_i\sigma_i^2} > \frac{\theta_{i'}}{\theta_i N}$. □

B Investment Styles of 13-F Investors by Thomson Reuters

Table B.I presents the 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors.

Table B.I: **The 32 investment styles in which Thomson Reuters classifies 13-F investors.**

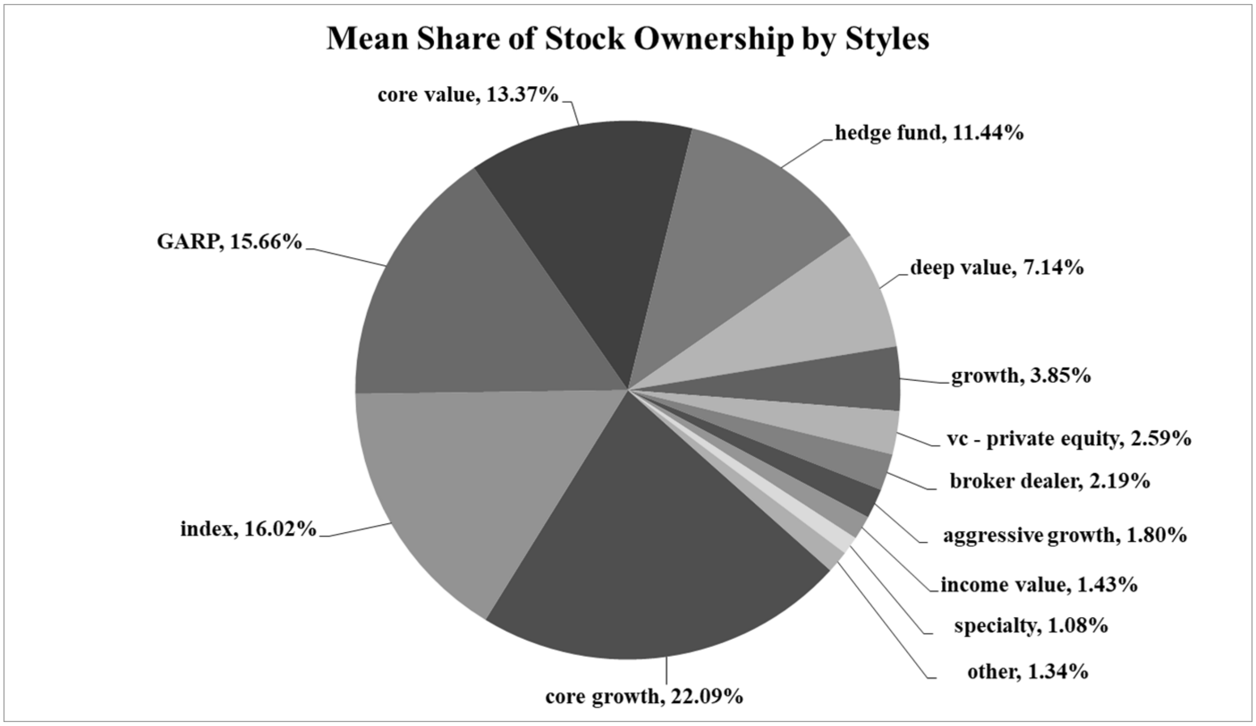
General Styles	Hedge Fund Styles
Aggressive Growth, Broker Dealer, Core Growth, Core Value, Deep Value, Emerging Markets, GARP(Growth at Reasonable Price), Growth, Hedge Fund, Income Value, Index, Mixed Style, Momentum, Sector Specific, Specialty, VC(Venture Capital)/Private Equity, Yield	Capital Structure Arbitrage, Convertible Arbitrage, CTA(Commodity Trading Advisors) Managed Futures, Distressed Securities, Emerging Markets (Hedge), Equity Hedge, Event Driven (Merger/ Risk Arbitrage), Fixed Income Arbitrage, Funds of Funds, Global Macro, Long Bias, Long-Short, Market Neutral, Multi-Strategy (Hedge), Quantitative/Statistical Arbitrage

Note: The 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors. The left column reports the seventeen general styles and the right column reports the fifteen hedge fund styles. The styles are reported alphabetically in each column. The information is available on http://banker.thomsonib.com/ta/help/webhelp/Ownership_Glossary.htm

TR classifies 13-F investors into styles based on the characteristics of the stocks that they hold, their historical investment behavior, their current transactions and their general business type. TR first classifies each stock into a certain group or style based on its price-earnings ratio, dividend yield, and the three- to five-year projected earnings-per-share growth relative to the corresponding S&P500 or sector averages. For each 13-F investor, TR then calculates the weights of the different groups or styles of stocks. The group with the biggest weight generally characterizes the investor's style.

Some classifications are more mechanical. 13-F investors whose portfolios follow the composition of certain indices (e.g. S&P 500, Russell 1000/2000/3000, etc) are classified into the Index

Figure B.1: **Mean share of stock ownership by style**



Note: Mean percentage shares in our sample of the 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors. The average shares above 1% are reported separately (twelve styles) and the average shares below 1% are reported together as "other" (twenty styles).

style. Styles such as "Broker Dealer," "Hedge Funds" and "VC/Private Equity" are assigned mainly based on the business type of the corresponding investors. Finally, some 13-F investors are classified into hedge-fund styles depending on their exact investment strategy (e.g. "Convertible Arbitrage," "Quantitative-Statistical Arbitrage," "Emerging Markets," "Fund of funds"). The relative importance of hedge-fund styles is small.

The pie chart in Figure B.1 shows the size of each of the 32 styles in our sample, defined as the asset value attributed to the style over the total asset value of all styles. There are twelve styles with size above 1%. The combined size of the remaining twenty styles is 1.34%.