# Fund Flows and Asset Prices: A Baseline Model

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**Abstract**

We study flows between investment funds and their effects on asset prices in a simple two-period version of Vayanos and Woolley (2010, VW). As in VW, flows cause assets to comove in ways unrelated to fundamentals, affect assets with high idiosyncratic risk the most, and raise the expected returns of funds experiencing outflows. We sketch how adding periods can generate other results of VW such as momentum, reversal, amplification, and commercial-risk management. We also extend the VW framework to study how index redefinitions affect the price level and the extent of comovement.

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# 1 Introduction

Institutional investors, such as mutual, hedge and pension funds, hold a large fraction of financial assets.[1] Flows by these investors have significant price effects, which are often hard to explain based only on asset fundamentals. For example, "fire sales" of stocks by mutual funds experiencing large outflows drive the prices of these stocks down to a level where subsequent returns are abnormally high; redefinitions of market indices raise the prices of stocks whose weight increases and lower those of stocks whose weight decreases; and comovement between stocks increases when they become part of the same market index or are held by many mutual funds in common.[2]

Vayanos and Woolley (2010, VW) develop a model to study fund flows and their price effects. Fund flows are shown to generate momentum and reversal, two of the most prominent market anomalies. They also amplify shocks to asset fundamentals, generate lead-lag effects, and cause assets to comove in ways unrelated to fundamentals. The effects of flows are larger for assets with high idiosyncratic risk. Moreover, flows affect prices not only through contemporaneous price pressure, but also through fund managers' willingness to hedge against the commercial risk associated with future outflows.

VW is set in continuous time and infinite horizon, and hence requires technical tools such as Ito's lemma and dynamic programming. In this paper we present a simple two-period version of VW that illustrates the basic model and some of the main mechanisms. Two periods suffice to generate flow-driven comovement and the effects of idiosyncratic risk. Other results, such as momentum, reversal, amplification, and the effects of commercial risk require more than two periods, and we sketch how they can be derived. In addition to providing a simplified version of VW, we derive additional results that are not in VW. We study, in particular, how changes in index weights, assumed constant in VW, affect the price level and the extent of comovement.

Section 2 presents the model. We consider an economy lasting over two periods: Period 1 in which financial assets are traded and Period 2 in which they pay off. There are multiple risky assets, which we refer to as stocks, and one riskless asset. A competitive investor can invest in stocks through two investment funds. We assume that one of these funds tracks mechanically a market index. This is for simplicity, so that portfolio optimization concerns only the other fund, which we refer to as the active fund. To ensure that the investor has a motive to move across

[1] For example, according to the New York Stock Exchange Factbook, the fraction of US stocks held directly by individuals in 2002 was less than 40%.

[2] See, for example, Coval and Stafford (2007), Jotikasthira, Lundblad and Ramadorai (2010) and Lou (2010) for evidence on fire sales; Harris and Gurel (1986), Shleifer (1986) and Chen, Noronja and Singal (2004) for evidence on index additions and deletions; Greenwood (2005) and Hau (2010) for evidence on more general index redefinitions; Vijh (1994), Barberis, Shleifer and Wurgler (2005) and Boyer (2010) for evidence on how index membership affects comovement; and Anton and Polk (2010) and Greenwood and Thesmar (2010) for evidence on how comovement depends on mutual-fund holdings. See also Duffie's (2010) presidential address to the American Finance Association, and the survey by Gromb and Vayanos (2010), for additional discussion and references.

funds, we assume that she suffers a cost from investing in the active fund. Changes in this cost generate fund flows. The interpretation of the cost that best fits our model is as a managerial perk, although other interpretations such as managerial ability could fit more complicated versions of the model. The active fund is run by a competitive manager, who can also invest his personal wealth in stocks through the fund. The latter assumption is for parsimony: in addition to choosing the active portfolio, the manager acts as trading counterparty to the investor's flows, and this eliminates the need to introduce additional agents into the model. The investor and the active-fund manager are risk averse and maximize expected utility of wealth in Period 2.

Section 3 computes the equilibrium by solving the investor's and manager's optimization problems and imposing market clearing. The optimization problems take a simple mean-variance form. Equilibrium also takes a simple form, with stock prices and investor holdings of the active fund being linear functions of the cost $C$.

Section 4 examines how changes in $C$ affect fund flows, and how these flows affect the prices and expected returns of the stocks and the investment funds. Following an increase in $C$, the investor flows out of the active and into the index fund. Through these flows, she effectively sells stocks that the active fund overweights relative to the index fund, and buys stocks that it underweights. These transactions are equivalent to selling a slice of a "flow portfolio," which is independent of $C$. Long positions in that portfolio correspond to stocks that the active fund overweights and the investor effectively sells. Conversely, short positions correspond to stocks that the active fund underweights and the investor effectively buys.

The price effects of flows depend on the covariance with the flow portfolio. Following an increase in $C$, stocks' expected returns must change so that the risk-averse manager is induced to buy a slice of the flow portfolio from the investor. Stocks that covary positively with the flow portfolio offer higher expected returns and their price decreases, while stocks that covary negatively offer lower expected returns and their price increases. We illustrate these effects using standard portfolio frontiers in a simple two-stock example.

A stock's covariance with the flow portfolio is high in absolute value for stocks with high idiosyncratic risk, meaning that these stocks are more affected by fund flows. The intuition is that changes in $C$ induce the investor to rebalance across funds but not to change her overall exposure to the market index. Therefore, the investor's willingness to carry risk perfectly correlated with the index does not change. The investor's exposure to individual stocks or industry sectors changes, however, and the resulting price effects are larger for stocks or sectors with high idiosyncratic risk.

Since fund flows affect prices, they induce comovement between stocks. This comovement is large in absolute value when the stocks have high idiosyncratic risk because they are more affected by fund flows. Moreover, it can be positive or negative: positive for stock pairs whose covariance with the flow portfolio has the same sign, and negative otherwise. Intuitively, two stocks move in the same direction in response to fund flows if they are both overweighted or both underweighted by the active fund, but move in opposite directions if one is overweighted and the other underweighted. Comovement thus depends on the pattern of stock holdings across funds.

Changes in $C$ affect not only the expected returns of stocks, but also that of the active fund. Holding stock prices constant, an increase in $C$ reduces the net-of-cost return that the active fund offers to the investor. At the same time, the increase in $C$ also triggers outflows from the active fund, which tend to raise the fund's gross expected return because of price pressure: stocks that the active fund overweights relative to the index fund experience a price drop and an increase in expected return, while the converse is true for underweighted stocks. The first effect dominates the second: an increase in $C$ lowers the active fund's net expected return, even taking into account the price pressure caused by outflows.

Section 5 outlines two extensions of the model. One extension is to allow the index weights to vary, and study how index redefinitions affect prices. Consistent with the empirical evidence, we show that stocks whose index weight increases go up in price and become more correlated with other stocks with high index weight. A more difficult extension, which we only sketch, is to allow for more than two periods. Adding periods makes it possible to address momentum, reversal, amplification, and commercial-risk management, results which are explicitly derived in continuous time in VW.

Our analysis of how index redefinitions affect prices is related to a number of papers. Brennan (1993) shows in a two-period model that when some investors care about their wealth relative to an index, inclusion of a stock in the index raises its price. Cuoco and Kaniel (2010) show a similar result in a continuous-time model, and also study the effect of index inclusion on volatility. Basak and Pavlova (2010) show in a continuous-time model that inclusion in an index not only raises a stock's price but also makes the stock more correlated with other stocks in the index. Barberis and Shleifer (2003) show in a multi-period model that stocks within the same investment style comove more. Their definition of a style is behavioral and could encompass an index.

## 2 Model

There are two periods, $t = 1, 2$. The financial market consists of one riskless and $N$ risky assets that pay off in Period 2. We refer to the risky assets as stocks, but they could also be interpreted as industry-level portfolios, asset classes, etc. The riskless asset pays off one unit with certainty, and stock $n = 1, .., N$ pays off $D_n$ units. The random vector $D \equiv (D_1, .., D_N)'$ is normal with mean

$\bar{D} \equiv (\bar{D}_1, .., \bar{D}_N)'$ and covariance matrix $\Sigma$. (The vector $v'$ is the transpose of $v$.) Using the riskless asset as the numeraire, we denote by $S_n$ the price of stock $n$ in Period 1, and set $S \equiv (S_1, .., S_N)'$. We denote by $\pi_n$ the supply of stock $n$ in terms of number of shares.

A competitive investor can invest in the riskless asset and in the stocks. The investor can access the stocks only through two investment funds. We assume that the first fund is passively managed and tracks mechanically a market index. This is for simplicity, so that portfolio optimization concerns only the other fund, which we refer to as the active fund. We assume that the market index includes a fixed number $\eta_n$ of shares of stock $n$. Thus, if the vectors $\pi \equiv (\pi_1, .., \pi_N)$ and $\eta \equiv (\eta_1, .., \eta_N)$ are collinear, the market index is capitalization-weighted and coincides with the market portfolio.

To ensure that the active fund can add value over the index fund, we assume that the market index differs from the true market portfolio characterizing equilibrium asset returns. This can be because the market index does not include some stocks. Alternatively, the market index can coincide with the market portfolio, but unmodelled buy-and-hold investors, such as firms' managers or founding families, can hold a portfolio different from the market portfolio. That is, buy-and-hold investors hold $\hat{\pi}_n$ shares of stock $n$, and the vectors $\pi$ and $\hat{\pi} \equiv (\hat{\pi}_1, .., \hat{\pi}_N)$ are not collinear. To nest the two cases, we define a vector $\theta \equiv (\theta_1, .., \theta_N)$ to coincide with $\pi$ in the first case and $\pi - \hat{\pi}$ in the second. The vector $\theta$ represents the residual supply left over from buy-and-hold investors, and is the true market portfolio characterizing equilibrium asset returns. We assume that $\theta$ is not collinear with the market index $\eta$.

The investor determines how to allocate her wealth between the riskless asset, the index fund, and the active fund. She maximizes expected utility of wealth in Period 2. Utility is exponential, i.e.,

$$-E_1 \exp(-aW_2), \tag{2.1}$$

where $a$ is the coefficient of absolute risk aversion and $W_2$ is wealth. The investor's control variables are the number of shares $x$ and $y$ of the index and active fund, respectively.

The active fund is run by a competitive manager, who can also invest his personal wealth in the fund. The manager determines the active portfolio and the allocation of his wealth between the riskless asset and the fund. He maximizes expected utility of wealth in Period 2. Utility is exponential, i.e.,

$$-E_1 \exp(-\bar{a}\bar{W}_2), \tag{2.2}$$

where $\bar{a}$ is the coefficient of absolute risk aversion and $\bar{W}_2$ is wealth. The manager's control variables are the number of shares $\bar{y}$ of the active fund, and the active portfolio $z \equiv (z_1, .., z_N)$, where $z_n$ denotes the number of shares of stock $n$ included in one share of the active fund.

The assumption that the manager can invest his personal wealth in the active fund is for parsimony: it generates a simple objective that the manager maximizes when choosing the fund's portfolio, and ensures that the manager acts as trading counterparty to the investor's flows. Under the alternative assumption that the manager must invest his wealth in the riskless asset, we would need to introduce two new elements into the model: a performance fee to provide the manager with incentives for portfolio choice, and an additional set of agents who could access stocks directly and act as counterparty to the investor's flows. This would complicate the model without changing the main intuitions. The manager in our model can be viewed as the aggregate of all agents absorbing the investor's flows.

Under the assumptions introduced so far, and in the absence of other frictions, the investor holds stocks only through the active fund since its portfolio dominates the index portfolio. We introduce a tradeoff between the two funds by assuming that the investor's return from the active fund is equal to the gross return, made of the returns of the stocks held by the fund, net of a cost. Changes in this cost generate fund flows. An empirical counterpart for the cost is the return gap, defined as the difference between a mutual fund's return over a given quarter and the return of a hypothetical portfolio invested in the stocks that the fund holds at the beginning of the quarter. Empirical studies of the return gap include Grinblatt and Titman (1989), Wermers (2000), and Kacperczyk, Sialm and Zhang (2008). They attribute the return gap mostly to operational costs, agency costs, and managerial stock-picking ability.

All three interpretations of the return gap—with agency costs and ability in reduced form—fit the more complicated version of our model where the manager must invest his wealth in the riskless asset. Because, however, we are assuming (for parsimony) that the manager can also invest in the active fund, we need to specify how his own investment in the fund is affected by the cost. The most convenient assumption is that the manager does not suffer the cost on his investment: this ensures, in particular, that changes in the cost generate flows between the investor and the manager. This assumption rules out the operational-cost and ability interpretations of the cost, which imply that the cost hurts the manager. We adopt instead the agency-cost interpretation, assuming that the cost is a perk that the manager can extract from the investor. Examples of perks in a delegated portfolio management context are late trading and soft-dollar commissions. The main intuitions coming out of our model, however, are broader than the managerial-perk interpretation.

We assume that the index fund entails no cost, so its gross and net returns coincide. This is for simplicity, but also fits the interpretations of the return gap. Indeed, managing an index fund involves no stock-picking ability, and operational and agency costs are smaller than for active funds.

We take the active fund's cost to be proportional to the number of shares $y$ that the investor holds in the fund, and denote the coefficient of proportionality by $C \geq 0$. We assume that the investor observes $C$ perfectly; unobservability of $C$ matters for our analysis only when there are more than two periods, as we explain in Section 5.2.

To remain consistent with the managerial-perk interpretation of the cost, we should allow the manager to derive a benefit from the investor's participation in the active fund. The benefit, however, matters for our analysis only when there are more than two periods, so we ignore it until Section 5.2.

The cost $Cy$ is assumed proportional to $y$ for analytical convenience. At the same time, it is sensitive to how shares of the active fund are defined (e.g., it changes with a stock split). We define one share of the fund by the requirement that its market value equals the equilibrium market value of the entire fund. Under this definition, the number of fund shares held by the investor and the manager in equilibrium sum to one, i.e.,

$$y + \bar{y} = 1. \tag{2.3}$$

We define one share of the index fund to coincide with the market index $\eta$. We define the constant

$$\Delta \equiv \theta\Sigma\theta'\eta\Sigma\eta' - (\eta\Sigma\theta')^2,$$

which is positive and becomes zero when the vectors $\eta$ and $\theta$ are collinear. Figure 1 summarizes our model's basic structure.

# 3 Equilibrium

## 3.1 Manager's Optimization

The manager chooses the active fund's portfolio $z$ and the number $\bar{y}$ of fund shares that he owns to maximize the expected utility (2.2). He is subject to the budget constraint

$$\bar{W}_2 = \bar{W}_1 + \bar{y}z(D - S),$$

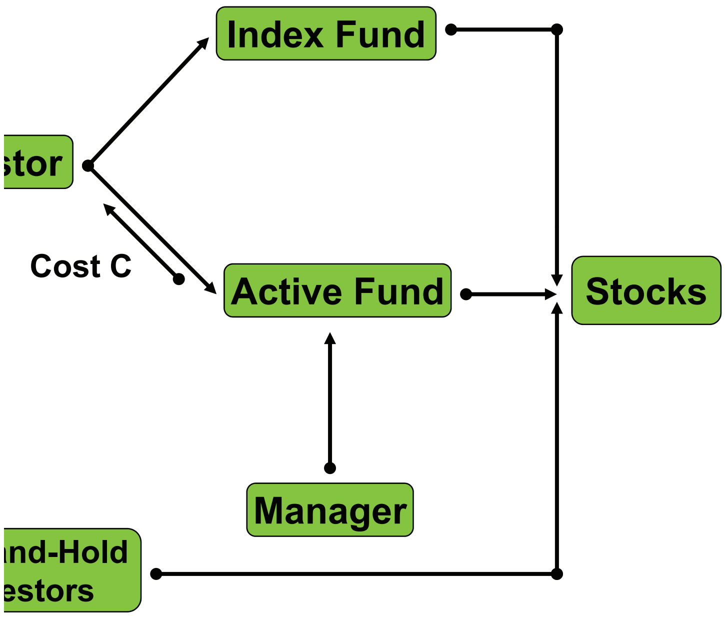


Figure 1: Agents and assets.

where the second term is his capital gain from investing in the active fund. Given exponential utility and normality, the objective (2.2) is equivalent to the mean-variance objective

$$\bar{y}z(\bar{D} - S) - \frac{\bar{a}}{2}\bar{y}^2 z\Sigma z'.$$

This objective depends on $(\bar{y}, z)$ only through the product $\bar{y}z$, and the first-order condition is

$$\bar{D} - S = \bar{a}\bar{y}\Sigma z'. \tag{3.1}$$

Eq. (3.1) links stocks' expected returns $\bar{D} - S$ to the risk faced by the manager. The expected return that the manager requires from a stock depends on the stock's covariance with the manager's portfolio $\bar{y}z$.

## 3.2 Investor's Optimization

The investor chooses the number of shares $x$ and $y$ of the index and active fund, respectively, to maximize the expected utility (2.1). She is subject to the budget constraint

$$W_2 = W_1 + x\eta(D - S) + yz(D - S) - Cy,$$

where the second and third terms are her capital gains from investing in the index and active fund, respectively. Given exponential utility and normality, the objective (2.1) is equivalent to the mean-variance objective

$$(x\eta + yz)(\bar{D} - S) - \frac{a}{2}(x\eta + yz)\Sigma(x\eta + yz)' - Cy.$$

The first-order conditions with respect to $x$ and $y$ are

$$\eta(\bar{D} - S) = a\eta\Sigma(x\eta + yz)', \tag{3.2}$$

$$z(\bar{D} - S) - C = az\Sigma(x\eta + yz)', \tag{3.3}$$

respectively. Eqs. (3.2) and (3.3) are analogous to the manager's first-order condition (3.1) in that they equate expected returns to risk. The difference with (3.1) is that the investor is constrained to two portfolios rather than $N$ individual stocks. Eq. (3.1) is a vector equation with $N$ components, while (3.2) and (3.3) are scalar equations derived by pre-multiplying expected returns with the vectors $\eta$ and $z$ of index- and active-fund weights. Note that the investor's expected return from the active fund in (3.3) is net of the cost $C$.

## 3.3 Market Clearing

In equilibrium, the active fund holds $\theta_n - x\eta_n$ shares of stock $n$: the number of shares $\theta_n$ in the true market portfolio, minus the number of shares $x\eta_n$ that the investor holds through the index fund. Since one share of the active fund includes $z_n$ shares of stock $n$, and is normalized to coincide with the total holdings of the active fund, market clearing implies that

$$z = \theta - x\eta. \tag{3.4}$$

Combining (2.3), (3.1), (3.2), (3.3) and (3.4), we can compute equilibrium prices and investment levels.

**Proposition 3.1** *In equilibrium,*

$$x = \frac{C\eta\Sigma\theta'}{a\Delta + C\eta\Sigma\eta'}, \tag{3.5}$$

$$y = \frac{\bar{a}}{a+\bar{a}} - \frac{C\eta\Sigma\eta'}{(a+\bar{a})\Delta}, \tag{3.6}$$

$$\bar{y} = \frac{a}{a+\bar{a}} + \frac{C\eta\Sigma\eta'}{(a+\bar{a})\Delta}, \tag{3.7}$$

$$S = \bar{D} - \frac{a\bar{a}}{a+\bar{a}}\Sigma\theta' - \frac{\bar{a}\eta\Sigma\eta' C}{(a+\bar{a})\Delta}\Sigma p_f', \tag{3.8}$$

*where*

$$p_f \equiv \theta - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta. \tag{3.9}$$

Eqs. (3.5)-(3.8) simplify in the benchmark case where the investor's cost $C$ from investing in the active fund is zero. The investor holds $x = 0$ shares of the index fund, thus investing exclusively in the active fund. This is because the active fund offers a superior portfolio than the index fund at no cost. The relative share of the investor and the manager in the active fund is $y/\bar{y} = \bar{a}/a$, which corresponds to the optimal risk-sharing rule. Stocks' expected returns are

$$\bar{D} - S = \frac{a\bar{a}}{a+\bar{a}}\Sigma\theta',$$

and hence are determined by the covariance with the true market portfolio. The intuition is that since the index fund receives zero investment, the true market portfolio coincides with the active portfolio $z$, which is also the portfolio held by the manager. Since the manager determines the cross section of expected returns through the first-order condition (3.1), the true market portfolio is the only pricing factor.

We next examine how changes in the cost $C$ affect fund flows (Section 4.1), and how these flows affect stock prices and expected returns (Section 4.2). We show that fund flows have larger effects on stocks with high idiosyncratic risk (Section 4.3) and induce comovement between stocks (Section 4.4). We finally examine how changes in $C$ and the resulting fund flows affect the expected returns of the index and active funds (Section 4.5).

# 4 Fund Flows and Price Effects

## 4.1 Fund Flows

Following an increase in the cost $C$ of investing in the active fund, the investor flows out of the active and into the index fund. Indeed, (3.5) implies that the number of shares $x$ held by the investor in the index fund increases in $C$, and (3.6) implies that the number of shares $y$ held in the active fund decreases in $C$.[3] Using (3.5) and (3.6) we can also determine the change in the investor's indirect stock holdings, i.e., the shares of the stocks that the investor owns through the funds. Indirect stock holdings are $x\eta + yz$ and they change by

$$\frac{\partial}{\partial C}(x\eta + yz) = \frac{\partial}{\partial C}\left[x\eta + y(\theta - x\eta)\right] = -\frac{1}{(a+\bar{a})\Delta}\left(\eta\Sigma\eta'\theta - \eta\Sigma\theta'\eta\right) = -\frac{\eta\Sigma\eta'}{(a+\bar{a})\Delta}p_f,$$

where the second step follows from (3.5) and (3.6), and the third from (3.9). The net change in indirect stock holdings is proportional to the portfolio $p_f$ defined in (3.9). We refer to $p_f$ as the flow portfolio because it characterizes fund flows. The flow portfolio consists of the true market portfolio $\theta$, plus a position in the market index $\eta$ that renders the covariance with the index equal to zero.[4] The intuition why the flow portfolio characterizes fund flows is as follows. Following an increase in $C$, the investor reduces her investment in the active fund, thus selling a slice of the true market portfolio. She also increases her investment in the index fund, thus buying a slice of the index. Because investing in the index fund is costless, the investor maintains a constant overall exposure to the index. Therefore, the net change in her portfolio is uncorrelated with the index, which means that she is selling a slice of the flow portfolio.

In selling a slice of the flow portfolio, the investor is effectively selling some stocks and buying others. The stocks being sold correspond to long positions in the flow portfolio. Therefore, they correspond to large components of the vector $\theta$ relative to $\eta$, and are overweighted by the active fund relative to the index fund. Conversely, the stocks being bought correspond to short positions in the flow portfolio, and are underweighted by the active fund.

[3] The result that $x$ increases in $C$ requires the additional assumption that $\eta\Sigma\theta' > 0$, i.e., the market index $\eta$ and the true market portfolio $\theta$ covary positively. This assumption is satisfied, for example, if the elements of the vector $\theta$ and of the matrix $\Sigma$ are positive.

[4] The covariance between the return $\eta(D - S)$ of the index and the return $p_f(D - S)$ of the flow portfolio is

$$Cov(\eta D, p_f D) = \eta\Sigma p_f' = \eta\Sigma\left(\theta - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta\right)' = \eta\Sigma\theta' - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta\Sigma\eta' = 0.$$

## 4.2 Stock Prices and Returns

The flows generated by the investor affect stock prices and expected returns. Eq. (3.8) implies that following an increase in $C$, prices change by

$$\frac{\partial S}{\partial C} = -\frac{\bar{a}\eta\Sigma\eta'}{(a+\bar{a})\Delta}\Sigma p_f' = -\frac{\bar{a}\eta\Sigma\eta'}{(a+\bar{a})\Delta}Cov(D, p_f D). \tag{4.1}$$

Prices of stocks that covary positively with the flow portfolio decrease and prices of stocks that covary negatively increase. These effects are through fund flows. Indeed, when $C$ increases, the investor sells a slice of the flow portfolio, which is acquired by the manager. Since the manager is risk averse, he requires higher expected returns from stocks that covary positively with the flow portfolio, and the price of these stocks decreases. Conversely, the expected returns of stocks that covary negatively with the flow portfolio decrease, and their price increases.

The effects of $C$ on fund flows, prices and expected returns can be illustrated through simple portfolio-frontier diagrams. Suppose that there are two stocks, 1 and 2, whose payoffs have the same mean and variance ($\bar{D}_1 = \bar{D}_2$ and $\Sigma_{11} = \Sigma_{22}$). Suppose that the market index consists of one share of each stock ($\eta_1 = \eta_2 = 1$), but the true market portfolio is $(\theta_1, \theta_2) = (1, 0.6)$. Figures 2 and 3 plot portfolio frontiers involving the two stocks. Returns are computed per share ($D_n - S_n$) rather than per dollar ($(D_n - S_n)/S_n$), and so portfolio weights are computed in terms of shares rather than dollars. Since the two stocks have the same payoff variance, they are on the same vertical line. Moreover, stock 1 is higher in that line: it is cheaper than stock 2 since its weight in the true market portfolio is higher.

The blue (dark) solid line in Figure 2 is the portfolio frontier of stocks 1 and 2 when $C = 0$. The portfolio tangent to that line is the active portfolio. This is because the active portfolio is also the personal stock portfolio of the manager, who maximizes a mean-variance objective and can invest in the riskless asset. When $C = 0$, the investor can achieve the same point on the frontier as the manager by investing exclusively in the active fund. Note that since the index fund receives zero investment, the active portfolio coincides with the true market portfolio. The latter portfolio has weights $(5/8, 3/8)$ since the fraction of shares of stock 1 is $\theta_1/(\theta_1 + \theta_2) = 1/(1 + 0.6) = 5/8$. The index portfolio has weights $(1/2, 1/2)$ since the fraction of shares of stock $n = 1, 2$ is $\eta_n/(\eta_1 + \eta_2) = 1/(1 + 1) = 1/2$.

An increase in $C$, holding stock prices constant, lowers the return of the active fund to the investor: the point representing the active fund from the investor's viewpoint shifts downwards by an amount equal to $C$. Therefore, the investor can no longer access the same point on the frontier

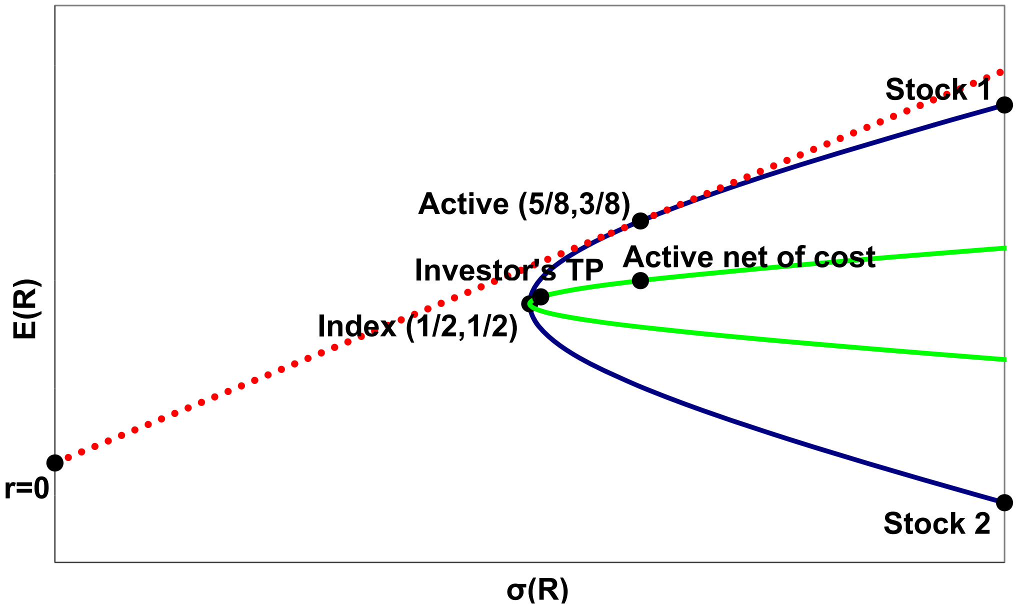


Figure 2: Effect of $C$ on fund flows, prices and expected returns. The blue (dark) solid line is the portfolio frontier of stocks 1 and 2 when $C = 0$. The portfolio tangent to that line is the active portfolio. This is the manager's personal stock portfolio, as well as that of the investor when $C = 0$. When $C$ increases, holding prices constant, the investor holds the portfolio tangent to his new frontier, which is the green (light) solid line. The investor's new portfolio gives less than full weight to the active fund, and positive weight to the index fund. The investor effectively sells stock 1, which the active fund overweights relative to the index fund, and buys stock 2, which the active fund underweights.

as the manager, but instead is limited to the more restricted frontier formed by the index fund and the active fund net of the cost. The latter frontier is the green (light) solid line, and the investor holds the portfolio tangent to that line. The investor's new portfolio gives less than full weight to the active fund, and positive weight to the index fund. (In fact, it almost coincides with the index fund in Figure 2.) Figure 2 thus confirms that an increase in $C$ causes the investor to flow out of the active and into the index fund. The investor effectively sells stock 1, which the active fund overweights relative to the index fund, and buys stock 2, which the active fund underweights. Since the manager's portfolio does not change, markets do not clear, and prices have to adjust.

The blue (dark) dashed line in Figure 3 is the portfolio frontier of stocks 1 and 2 after the price adjustment. Because the investor effectively sells stock 1 and buys stock 2, stock 1 becomes cheaper and stock 2 more expensive. Therefore, the points representing stocks 1 and 2 shift upwards and downwards, respectively. The portfolio tangent to the new frontier of stocks 1 and 2 gives increased weight to stock 1 and decreased weight to stock 2. Since the portfolio tangent to the frontier of stocks 1 and 2 is the manager's personal stock portfolio (as well as the active portfolio), the manager buys stock 1 and sells stock 2. Note that the price adjustment also causes the investor's frontier to

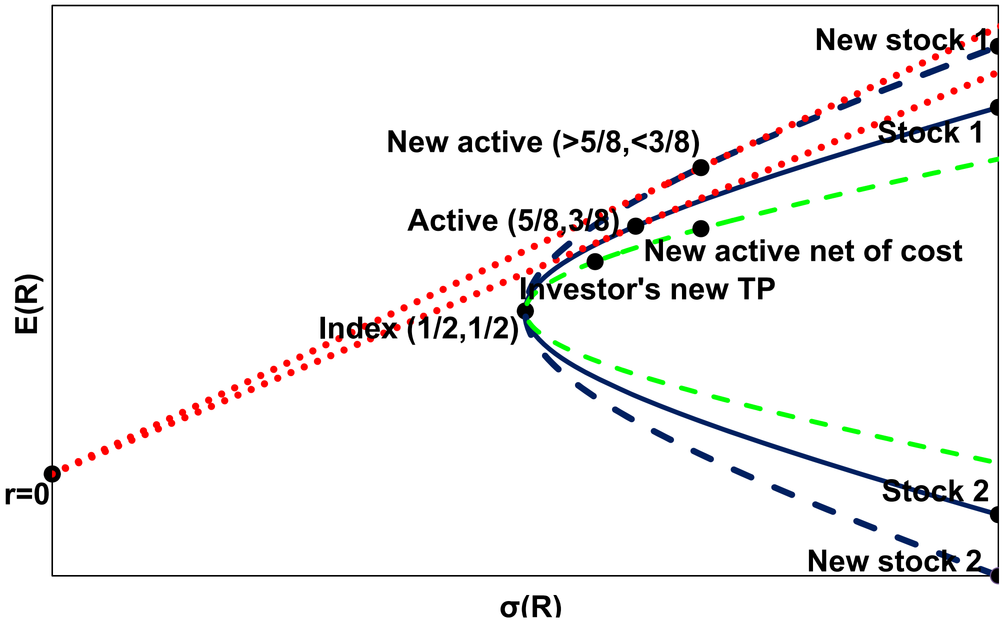


Figure 3: Effect of $C$ on fund flows, prices and expected returns. When $C$ increases, the investor effectively sells stock 1 and buys stock 2. Therefore, stock 1 becomes cheaper and stock 2 more expensive. The frontier of stocks 1 and 2 moves from the blue (dark) solid line to the blue dashed line. The portfolio tangent to the new frontier gives increased weight to stock 1 and decreased weight to stock 2, meaning that the manager buys stock 1 and sells stock 2. The green (light) dashed line is the new frontier of the investor. The manager's trades offset those of the investor.

change, from the green (light) solid line in Figure 2 to the green dashed line in Figure 3. The price adjustment and resulting changes in frontiers are such that the trades of the manager offset those of the investor.

In the example illustrated in Figures 2 and 3 an increase in $C$ lowers the price of stock 1, which is overweighted by the active fund relative to the index fund, and raises the price of stock 2, which is underweighted. Recall from (4.1) that the price effect is determined by the stock's covariance with the flow portfolio rather than by the stock's relative weight across the two funds. Stocks that the active fund overweights, however, are likely to covary positively with the flow portfolio, which involves long positions in such stocks. Conversely, stocks that the active fund underweights are likely to covary negatively because they correspond to short positions. This is the case for the example illustrated in Figures 2 and 3. Indeed, the flow portfolio involves a long position in stock 1 and a short position of the same size in stock 2. Stock 1 covaries positively with the flow portfolio and stock 2 covaries negatively.

### 4.3 Idiosyncratic Risk

While a stock's relative weight across the two funds influences the sign of a stock's covariance with the flow portfolio, idiosyncratic risk influences the magnitude: stocks with high idiosyncratic risk have higher covariance with the flow portfolio in absolute value, and hence are more affected by fund flows. To derive this effect, we regress the vector $D-S$ of stock returns on the return $\eta(D-S)$ of the market index:

$$D - S = \alpha + \beta\eta(D - S) + \epsilon.$$

Taking covariances of both sides with the return $p_f(D - S)$ of the flow portfolio, and noting that the market index is uncorrelated with the flow portfolio, we find

$$\Sigma p_f' = Cov(D, p_f D) = \mathrm{Cov}(\epsilon, p_f D),$$

i.e., the covariance between a stock and the flow portfolio is equal to that between the idiosyncratic component $\epsilon$ of the stock's payoff and the flow portfolio.[5] Therefore, stocks with high idiosyncratic risk have higher covariance with the flow portfolio in absolute value, and hence are more affected by changes in $C$. To explain the intuition, we recall that changes in $C$ induce the investor to rebalance across funds but not to change her overall exposure to the market index. Therefore, the investor's willingness to carry risk perfectly correlated with the index does not change, and neither does the price of the market index or of a stock that correlates perfectly with the index. On the other hand, changes in $C$ induce the investor to change her exposure to individual stocks or industry sectors. The price effects caused by these flows are larger for stocks or sectors with large idiosyncratic risk.

### 4.4 Comovement

Since fund flows affect prices, they induce comovement between stocks. To study comovement between prices in Period 1, we introduce a Period 0 as of which these prices are uncertain. We allow the uncertainty to concern asset payoffs and fund flows since they both affect prices. Prices in Period 1 depend on payoffs through the expected payoff vector $\bar{D} \equiv (\bar{D}_1, .., \bar{D}_N)'$. Moreover, fund flows are triggered by changes in the cost $C$. We assume that $\bar{D}$ and $C$ are random as of Period 0, independent of each other, and with covariance matrix $\Sigma$ and variance $s^2$, respectively. Eq. (3.8)

[5] Note that we consider idiosyncratic risk relative to the market index $\eta$ and not relative to the market portfolio $\pi$. This is typically how idiosyncratic risk is computed in empirical studies.

implies that the covariance matrix, as of Period 0, of prices in Period 1 is

$$Cov_0(S, S') = \Sigma + \left[\frac{\bar{a}\eta\Sigma\eta' s}{(a+\bar{a})\Delta}\right]^2 \Sigma p_f' p_f \Sigma. \tag{4.2}$$

The matrix $\Sigma$ represents the covariance driven by payoffs, and we refer to it as fundamental covariance. The matrix $\left[\frac{\bar{a}\eta\Sigma\eta' s}{(a+\bar{a})\Delta}\right]^2 \Sigma p_f' p_f \Sigma$ represents the additional covariance introduced by fund flows, and we refer to it as non-fundamental covariance. The non-fundamental covariance between a pair of stocks is proportional to the product of the covariances between each stock in the pair and the flow portfolio. It is thus large in absolute value when the stocks have high idiosyncratic risk, because they are more affected by fund flows. Moreover, it can be positive or negative: positive for stock pairs whose covariance with the flow portfolio has the same sign, and negative otherwise. Intuitively, two stocks move in the same direction in response to fund flows if they are both overweighted or both underweighted by the active fund, but move in opposite directions if one is overweighted and the other underweighted.

The non-fundamental covariance depends on the pattern of stock holdings across funds, which can be viewed as an institutional characteristic of the market. We further emphasize the role of institutional characteristics in driving comovement in Section 5.1, where we show that a redefinition of the market index changes the non-fundamental covariance.

### 4.5 Fund Returns

We next examine how changes in the cost $C$ affect the expected returns of the index and active funds. Changes in $C$ have a direct and an indirect effect on the active fund's expected return. The direct effect is that holding stock prices constant, an increase in $C$ reduces the net-of-cost return that the active fund offers to the investor. The indirect effect is that the active fund's gross return changes because the investor's outflows from the fund affect stock prices and expected returns. In particular, stocks that the active fund overweights relative to the index fund experience a price drop and an increase in expected return, while the converse is true for underweighted stocks. The indirect effect raises the active fund's gross expected return, thus attenuating the direct effect. This means that the investor's outflows from the active fund make the fund's portfolio more attractive. But while the direct effect is attenuated, it dominates the indirect effect: an increase in $C$ lowers the active fund's net expected return, even taking into account the price pressure caused by outflows.

To compute the direct and indirect effects, we need a measure of the active fund's expected return. We use the active fund's alpha, which is the fund's expected return risk-adjusted by the expected return of the market index. Alpha is the constant in the regression of the active fund's net return $(\theta - x\eta)(D - S) - C$ on the market index return $\eta(D - S)$:[6]

$$(\theta - x\eta)(D - S) - C = \alpha + \beta\eta(D - S) + \epsilon.$$

The regression yields

$$\beta = \frac{Cov\left[(\theta - x\eta)D, \eta D\right]}{Var(\eta D)} = \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'} - x, \tag{4.3}$$

$$\alpha = (\theta - x\eta)(\bar{D} - S) - C - \beta\eta(\bar{D} - S) = p_f(\bar{D} - S) - C. \tag{4.4}$$

Alpha is equal to the expected return of the flow portfolio net of the cost $C$. The intuition is that risk-adjusting the active fund's return amounts to adding to the active portfolio a position in the market index such that the covariance with the index is zero—and this exactly how the flow portfolio is constructed. Substituting (3.8) and (3.9) into (4.4), we find

$$\alpha = \frac{a\bar{a}\Delta}{(a + \bar{a})\eta\Sigma\eta'} + \left(\frac{\bar{a}}{a + \bar{a}} - 1\right) C. \tag{4.5}$$

The term $\frac{a\bar{a}\Delta}{(a+\bar{a})\eta\Sigma\eta'}$ is the active fund's alpha when $C = 0$. When $C = 0$, the active portfolio dominates the market index, and hence has positive alpha. The term $\left(\frac{\bar{a}}{a+\bar{a}} - 1\right) C$ describes how alpha varies with $C$, and is the sum of the direct and indirect effects. The direct effect is that an increase in $C$ causes a one-for-one decrease in the active fund's alpha: this corresponds to the second term in the parenthesis. The indirect effect corresponds to the first term, and is positive but dominated by the direct effect.

The direct and indirect effects cancel each other only in the special case where the manager is infinitely risk averse ($\bar{a} = \infty$). Indeed, in that case the manager's personal stock holdings are zero, and hence the investor holds the entire true market portfolio in equilibrium. Therefore, following an increase in $C$, the investor does not flow out of the active fund, but prices adjust so that the active fund's net expected return remains constant. In terms of Figure 3, the investor's portfolio frontier remains identical to the case where $C = 0$, i.e., the green dashed line coincides with the blue solid line. In the general case where $\bar{a}$ is finite, the investor flows out of the active fund, and the manager takes the other side of this trade attenuating the price effects. As a consequence, the indirect effect is smaller than when $\bar{a} = \infty$, and the active fund's net expected return decreases.

[6]Note that we consider alpha relative to the market index $\eta$ and not relative to the market portfolio $\pi$. This is typically how alpha is computed in empirical studies.

The direct and indirect effects are zero for the index fund. Indeed, since the index fund entails no cost for the investor, the direct effect is zero. Moreover, since changes in $C$ do not change the price of the market index, the indirect effect is zero. Therefore, changes in $C$ do not affect the index fund's expected return.

# 5 Extensions

## 5.1 Index Redefinitions

To study the effects of index redefinitions, we perform the following simple exercise. Suppose that two stocks $(n, n')$ are identical in all aspects except their weights in the market index. That is, they have the same payoff variance ($\Sigma_{nn} = \Sigma_{n'n'}$), the same payoff covariance with all other stocks ($\Sigma_{n\ell} = \Sigma_{n'\ell}$ for all $\ell \neq n, n'$), the same weights in the true market portfolio ($\theta_n = \theta_{n'}$), but different weights in the market index ($\eta_n \neq \eta_{n'}$). We examine the price effects of switching the index weights of the two stocks. Without loss of generality we assume that stock $n$ receives originally higher weight ($\eta_n > \eta_{n'}$), so after the switch stock $n'$ receives higher weight.

**Proposition 5.1 (Index Effects)** *Suppose that stocks $(n, n')$ are identical in all aspects except that stock $n$ receives higher index weight. Switching the index weights of the two stocks raises the price of stock $n'$ and reduces that of stock $n$ by*

$$\frac{\bar{a}\eta\Sigma\theta' C}{(a+\bar{a})\Delta}\Sigma_{nn}(1-\rho_{nn'})(\eta_n - \eta_{n'}), \tag{5.1}$$

*where $\rho_{nn'}$ is the correlation coefficient between stocks $(n, n')$*

Consistent with the empirical evidence, the stock whose index weight increases (stock $n'$) experiences a price increase, while the stock whose index weight decreases (stock $n$) experiences a price decrease.[7] The effect appears only when $C > 0$; when $C = 0$, the investor does not invest in the index fund, and hence index weights are irrelevant. Moreover, the effect strengthens when $C$ increases because the index fund then receives more investment.

To study how index composition affects comovement, we extend the previous exercise by considering two additional stocks $(m, m')$, which are also identical in all aspects except that stock $m$ receives higher index weight than stock $m'$ ($\eta_m > \eta_{m'}$). We examine how the weight switch between

[7] This result requires the additional assumption that $\eta\Sigma\theta' > 0$, i.e., the index portfolio $\eta$ and the true market portfolio $\theta$ covary positively. This assumption is satisfied, for example, if the elements of the vectors $(\eta, \theta)$ and of the matrix $\Sigma$ are positive.

stocks $(n, n')$ affects how these covary with stocks $(m, m')$. As in Section 4.4, covariance concerns prices in Period 1 and is evaluated as of Period 0.

**Proposition 5.2 (Index-Induced Comovement)** *Suppose that stocks $(n, n')$ are identical in all aspects except that stock $n$ receives higher index weight, and the same is true for stocks $(m, m')$. Switching the index weights of stocks $(n, n')$ raises the covariance between $(n', m)$ relative to that between $(n', m')$, and lowers the covariance between $(n, m)$ relative to that between $(n, m')$.*

Consistent with the empirical evidence, the stock whose index weight increases (stock $n'$) experiences an increase in covariance with stocks whose index weight is high (stock $m$) relative to stocks whose index weight is low (stock $m'$). Conversely, the stock whose index weight decreases (stock $n$) experiences an increase in covariance with stocks whose index weight is low (stock $m'$) relative to stocks whose index weight is high (stock $m$). The intuition is analogous to that in Section 4.4: stocks with high index weight tend to go up following inflows to the index fund, while stocks with low index weight tend to go down.

## 5.2 Multiple Periods

Our analysis so far assumes two periods: Period 1, in which stocks are traded, and Period 2, in which they pay off. Sections 4.4 and 5.1 introduce an additional Period 0, as of which the covariance of prices in Period 1 is computed. Adding more periods, and allowing $C$ to vary across periods, makes it possible to study a number of new issues. We analyze these issues informally here, and refer to Vayanos and Woolley (2010, VW) for a formal analysis in continuous time.

### 5.2.1 Momentum and Reversal

Momentum and reversal are two of the most prominent market anomalies, and have been documented extensively for a wide variety of assets. Momentum is the tendency of assets with good (bad) recent performance to continue overperforming (underperforming) in the near future. Reversal concerns predictability based on a longer performance history: assets that performed well (poorly) over a long period tend to subsequently underperform (overperform).

Our two-period model generates return predictability, which is driven by fund flows. Following an increase in $C$, the investor flows out of the active fund in Period 1. In response to these outflows, stocks that covary positively with the flow portfolio drop in price and their expected returns increase, while the opposite happens to stocks that covary negatively. In both cases, a stock's price in Period 1 predicts negatively the stock's return between Periods 1 and 2. This yields reversal, as can be seen by introducing a Period 0: a stock's return between Periods 0 and 1 predicts negatively the return between Periods 1 and 2.

The combination of momentum and reversal can be derived by introducing an additional period prior to Period 0, and assuming that fund flows exhibit inertia. Suppose, for example, that the investor cannot respond instantaneously to an increase in $C$ in Period 0, but can flow out of the active fund only in Period 1. The investor's outflows in Period 1 cause stocks that covary positively with the flow portfolio to be cheap in that period and earn high expected returns between Periods 1 and 2. Since the outflows are anticipated in Period 0, the prices of these stocks drop immediately following the increase in $C$ in Period 0. VW show, however, that the expected returns of these stocks between Periods 0 and 1 decrease, and so the underperformance in Period 0 is expected to continue. This yields the combination of momentum and reversal, as can be seen by introducing a Period -1: a stock's return between Periods -1 and 0 predicts positively the return between Periods 0 and 1, but the return between Periods -1 and 1 predicts negatively the return between Periods 1 and 2.

The result that momentum can arise is surprising: why is the manager willing to hold in Period 0 a stock that is expected to underperform in Period 1? The intuition is that the manager prefers to guarantee a "bird in the hand." Indeed, the anticipation of outflows in Period 1 causes the stock to become underpriced in Period 0 and offer an attractive return (bird in the hand) between Periods 0 and 2. The manager could earn an even more attractive return (two birds in the bush), on average, by buying the stock in Period 1, immediately after the outflows occur. This, however, exposes him to the risk that the outflows might not occur (because of an offsetting shock to $C$ in Period 1), in which case the stock would cease to be underpriced. Thus, the manager might prefer to guarantee an attractive long-horizon return, and pass up on the opportunity to exploit an uncertain short-run price drop.

A simple example illustrates the point. A stock is expected to pay off at 100 in Period 2. The stock price is 92 in Period 0, and 80 or 100 in Period 1 with equal probabilities. Buying the stock in Period 0 earns the manager a two-period expected capital gain of 8. Buying in Period 1 earns an expected capital gain of 20 if the price is 80 and 0 if the price is 100. A risk-averse manager might prefer earning 8 rather than 20 or 0 with equal probabilities, even though the expected return between Periods 0 and 1 is negative.

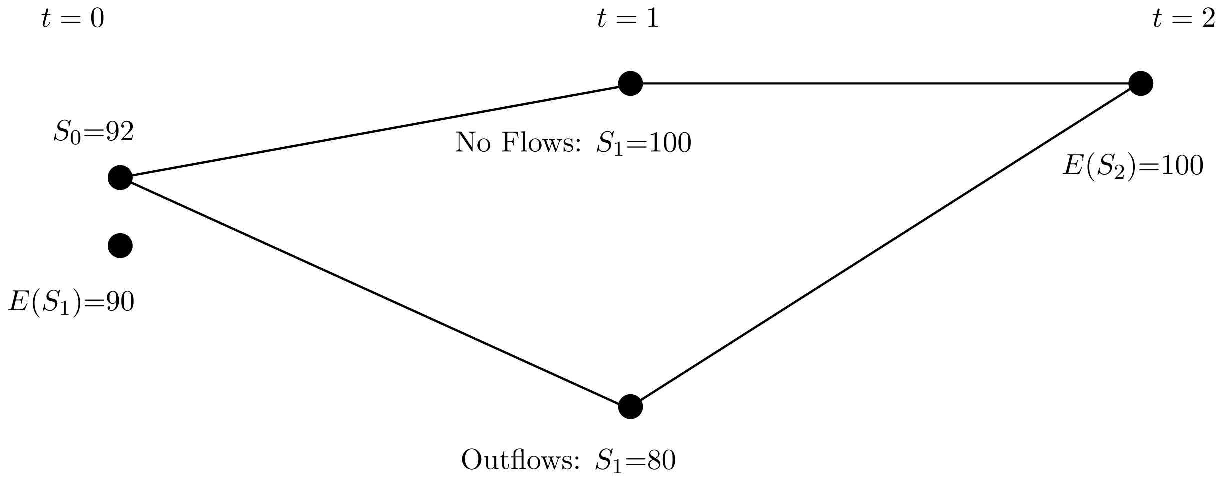


Figure 4: Momentum and reversal in a three-period example.

#### 5.2.2 Commercial Risk

The compensation of fund managers in practice increases in the size of the fund that they manage. This gives rise to commercial risk: managers are concerned with experiencing outflows. How do managers' concerns with commercial risk affect equilibrium asset prices?

To study the effects of commercial risk, we must introduce a benefit that the manager derives from managing a larger fund. Our model can accommodate a benefit that is a general function of the number of shares $y$ that the investor holds in the active fund. For example, the benefit can be assumed proportional to $y$, with coefficient of proportionality $B$. If the cost is a perk that the manager can extract efficiently, then $B = C$. VW allow $B$ to be a general affine function of $C$.

Introducing a benefit has no effect on stock prices in the two-period model. This is because stocks pay off in Period 2, so the active fund is terminated regardless of the manager's portfolio choice in Period 1. With more periods, however, the benefit matters. Indeed, the manager's concern with outflows in Period 1 influences his portfolio choice in Period 0. VW show that an increase in $B$ lowers the prices of stocks that covary positively with the flow portfolio and raises those of stocks covarying negatively. Since the former are generally stocks that the active fund overweights and the latter stocks that it underweights, this result implies that a manager concerned with losing his fee is less willing to deviate from the market index. The intuition is that a manager concerned with losing his fee seeks to hedge against increases in $C$ since these trigger outflows. Hedging can be accomplished by holding a portfolio closer to the index since changes in $C$ do not affect the index price.

When $B$ is an increasing (affine) function of $C$, an increase in $C$ in Period 0 lowers the prices of stocks that covary positively with the flow portfolio and raises those of stocks covarying negatively because of two mutually-reinforcing effects: the price pressure caused by outflows, and the manager's increased concern with commercial risk. Thus, the manager's desire to hedge against future outflows amplifies the effect that current outflows have on prices. In other words, the manager's concern with commercial risk has the perverse effect to make returns riskier.

### 5.2.3 Asymmetric Information

Our analysis so far assumes that the investor observes $C$. A more realistic assumption, however, is that the investor does not observe $C$ but infers it from fund returns and prices. Inference generates a causal effect of fund returns on fund flows: for example, following poor returns by the active fund, the investor infers that $C$ has increased and flows out of the fund. Note that when the investor observes $C$, causality runs only from flows to returns: changes in $C$ trigger flows, and the price pressure that these generate affects returns.

Non-observability of $C$ generates a causal effect of returns on flows only when there are more than two periods. Indeed, in our two-period model, returns are realized in Period 2, but no flows are possible in that period because stocks pay off and funds are terminated. With an additional Period 0, however, returns realized in Period 1 affect flows in that period.

When causality between flows and returns runs in both directions, flows amplify the effects of shocks to asset fundamentals. Suppose, for example, that a stock experiences a negative payoff shock. If the stock is overweighted by the active fund, then the shock lowers the return of the active fund relative to the index fund. As a consequence, the investor infers that $C$ has increased, and flows out of the active and into the index fund. Since the active fund overweights the stock, the investor's flows cause the stock to be sold and push its price down. Conversely, if the stock is underweighted, then the investor infers that $C$ has decreased, and flows out of the index and into the active fund. Since the active fund underweights the stock, the investor's flows cause again the stock to be sold and push its price down. Thus, in both cases, fund flows amplify the effect that the payoff shock has on returns.

Amplification is related to comovement. Recall that when the investor observes $C$, and so causality runs only from flows to returns, non-fundamental comovement between a pair of stocks arises because flows affect the prices of both stocks in the pair. When, however, the investor does not observe $C$, and so causality runs also from returns to flows, a new channel of comovement appears: a shock to one stock's payoffs affects fund returns, and so triggers flows which affect the price of the other stock. When the two stocks coincide, this channel manifests itself as the amplification effect of the previous paragraph.

Finally, causality from returns to flows gives rise to a new channel of momentum and reversal. When the investor observes $C$, momentum and reversal are triggered only by (observable) stocks to

$C$. When, however, the investor does not observe $C$, momentum and reversal are triggered by fund returns, which are affected not only by (unobservable) shocks to $C$ but also by shocks to stocks' payoffs.

# Appendix

## A Proofs

**Proof of Proposition 3.1:** Substituting (3.4) into (3.1), and using (2.3), we find

$$\bar{D} - S = \bar{a}(1-y)\Sigma(\theta - x\eta)'. \tag{A.1}$$

Substituting (3.4) into (3.2) and (3.3), we find the equivalent system

$$\eta(\bar{D} - S) = a\eta\Sigma\left[x\eta + y(\theta - x\eta)\right]', \tag{A.2}$$

$$\theta(\bar{D} - S) - C = a\theta\Sigma\left[x\eta + y(\theta - x\eta)\right]'. \tag{A.3}$$

Substituting $S$ from (A.1) into (A.2) and (A.3), we find

$$\bar{a}(1-y)\eta\Sigma(\theta - x\eta)' = a\eta\Sigma\left[x\eta + y(\theta - x\eta)\right]', \tag{A.4}$$

$$\bar{a}(1-y)\theta\Sigma(\theta - x\eta)' - C = a\theta\Sigma\left[x\eta + y(\theta - x\eta)\right]'. \tag{A.5}$$

Eqs. (A.4) and (A.5) constitute a linear system of equations in $(y, x(1-y))$. Solving for $y$ we find (3.6), and solving for $x$ we find (3.5). Eq. (3.7) follows from (2.3) and (3.6). Eq. (3.8) follows from (3.5), (3.6) and (A.1). ∎

**Proof of Proposition 5.1:** Since stocks $(n, n')$ are identical in all aspects except their index weights, the weight switch does not affect the scalars $\eta\Sigma\eta'$ and $\eta\Sigma\theta'$. It also does not affect the vectors $p_f$ and $\Sigma p_f'$ except for their components $n$ and $n'$, which are switched. Using the subscripts $(n, n')$ to denote components before the switch, we find

$$(p_f)_n - (p_f)_{n'} = -\frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}(\eta_n - \eta_{n'}), \tag{A.6}$$

$$\begin{aligned}(\Sigma p_f')_n - (\Sigma p_f')_{n'} &= \Sigma_{nn}(1-\rho_{nn'})\left[(p_f)_n - (p_f)_{n'}\right] \\ &= -\frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\Sigma_{nn}(1-\rho_{nn'})(\eta_n - \eta_{n'}),\end{aligned} \tag{A.7}$$

where (A.6) follows from (3.9), and (A.7) from (A.6). Eq. (3.8) implies that the price of stock $n'$

increases by

$$-\frac{\bar{a}\eta\Sigma\eta' C}{(a+\bar{a})\Delta}\left[(\Sigma p'_f)_n - (\Sigma p'_f)_{n'}\right], \tag{A.8}$$

and the price of stock $n$ decreases by the same amount. Eqs. (A.7) and (A.8) imply (5.1). ∎

**Proof of Proposition 5.2:** Since $\Sigma_{m\ell} = \Sigma_{m'\ell}$ for $\ell = n, n'$, (4.2) implies that before the switch, the difference in covariances between stocks $(n, m)$ and $(n, m')$ is

$$Cov_0(S, S')_{nm} - Cov_0(S, S')_{nm'} = \left[\frac{\bar{a}\eta\Sigma\eta' s}{(a+\bar{a})\Delta}\right]^2 (\Sigma p'_f)_n \left[(\Sigma p'_f)_m - (\Sigma p'_f)_{m'}\right], \tag{A.9}$$

and that between stocks $(n', m)$ and $(n', m')$ is

$$Cov_0(S, S')_{n'm} - Cov_0(S, S')_{n'm'} = \left[\frac{\bar{a}\eta\Sigma\eta' s}{(a+\bar{a})\Delta}\right]^2 (\Sigma p'_f)_{n'} \left[(\Sigma p'_f)_m - (\Sigma p'_f)_{m'}\right]. \tag{A.10}$$

After the switch, the difference in covariances between stocks $(n, m)$ and $(n, m')$ is given by (A.10), and that between stocks $(n', m)$ and $(n', m')$ is given by (A.9). This is because stocks $(n, n')$ are identical in all aspects except their index weights. Therefore, the proposition holds if

$$\left[(\Sigma p'_f)_n - (\Sigma p'_f)_{n'}\right]\left[(\Sigma p'_f)_m - (\Sigma p'_f)_{m'}\right] > 0, \tag{A.11}$$

where the subscripts $(n, n')$ denote components before the switch. Using (A.7) for stocks $(m, m')$ and $(n, n')$, we find that (A.11) is equivalent to

$$(\eta_n - \eta_{n'})(\eta_m - \eta_{m'}) > 0,$$

which holds. ∎

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