Connectivity augmentation algorithms

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The main subject of the thesis is connectivity augmentation: we would like to make a given graph k-connected by adding a minimum number of new edges. There are four basic problems in this field, since one might consider both edge- and node-connectivity augmentation in both graphs and digraphs. The thesis wishes to contribute to three out of these four problems: directed- and undirected node-connectivity and undirected edge-connectivity augmentation. Although directed edge-connectivity augmentation is not being considered, the last chapter is devoted to a constructive characterization result related to directed edge-connectivity. Let us summarize the main results of the thesis.

- *We present a min-max formula and a combinatorial polynomial time algorithm for augmenting undirected node-connectivity by one.* The complexity status of undirected node-connectivity augmentation of arbitrary graphs is still open; already the special case of augmenting by one has attracted considerable attention. The formula proved in Chapter 3 was conjectured by Frank and Jordn in 1994.

- *We present the first combinatorial polynomial time algorithm for directed node-connectivity augmentation.* For this problem, Frank and Jordán gave a min-max formula in 1995; however, it remained an open problem to develop a combinatorial algorithm. We present two, completely different combinatorial algorithms. Chapter 2 contains one for the special case of augmenting connectivity by one (a joint work with András Frank), and Chapter 4 presents another for augmenting the connectivity of arbitrary digraphs (a joint work with András Benczúr Jr.). The latter result also gives a new, algorithmic proof of the general theorem of Frank and Jordán on covering positively crossing supermodular functions on set pairs.

- *We establish a constructive characterization of (k, ℓ)-edge-connected digraphs.* This result of Chapter 6, a joint work with Erika Renta Kovcs, settles a conjecture of Frank from 2003. The theorem gives a common generalization of a number of previously known characterizations, and naturally fits into the framework defined by splitting off and orientation theorems.

- *We present partial results concerning partition constrained undirected local edge-connectivity augmentation.* In Chapter 5, we discuss some classical results concerning undirected edge-connectivity augmentation in a unified framework, based on the technique of edge-flippings. For the partition constrained problem we formulate a conjecture and give a partial proof.