Let t denote the first period, s denote the distance in time from period t, and define time $\nu \equiv t + s$.

Then:

$$\int_{t}^{\infty} \left[M(\nu) - X\left(\nu\right) \right] e^{-\int_{t}^{\nu} r(z).dz}.d\nu = \int_{0}^{\infty} \left[M(t+s) - X\left(t+s\right) \right] e^{-\int_{0}^{s} r(t+z).dz}.ds \tag{1}$$

If r is constant then the above is equivalent to:

$$\int_{t}^{\infty} \left[M(\nu) - X\left(\nu\right) \right] e^{-r(\nu - t)}.d\nu = \int_{0}^{\infty} \left[M(t+s) - X\left(t+s\right) \right] e^{-rs}.ds \tag{2}$$

As a quick check, make sure that when the discount factor is evaluated in the first period it is equal to 1 (i.e. no discount).