# Deriving the Wage Curve (Approach 2)

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This note derives equation (20) from (18) in slide 21 of the lecture note entitled "The Labour Market I".

We have the following Flow Value Equations:

$$
\begin{aligned}
rU &= z + \theta q(\theta)(W - U) \quad (1)\\
rW &= w + s(U - W) \quad (2)\\
rV &= -c + q(\theta)(J - V) \quad (3)\\
rJ &= p - w - sJ \quad (4)
\end{aligned}
$$

After rearranging (2):

$$W = \frac{w + sU}{r + s} \quad (5)$$

After rearranging (4):

$$J = \frac{p - w}{r + s} \quad (6)$$

From equation (18) of the lecture slides:

$$
\begin{aligned}
W - U &= \beta(W - U + J - V)\\
(1 - \beta)W &= (1 - \beta)U + \beta(J - V)
\end{aligned}
$$

Substituting the free entry condition ($V = 0$) and equations (5) and (6) we obtain:

$$(1 - \beta)\left(\frac{w + sU}{r + s}\right) = (1 - \beta)U + \beta\left(\frac{p - w}{r + s}\right)$$

or

$$(1 - \beta)(w + sU) = (1 - \beta)(r + s)U + \beta(p - w)$$

[1] Any errors are my own.

Subtract $(1-\beta) sU$ from both sides:

$$(1-\beta) w = (1-\beta) rU + \beta (p - w)$$

$$\begin{aligned} w &= (1-\beta) rU + \beta p \quad (7) \\ w &= rU + \beta (p - rU) \end{aligned}$$

Using the free entry condition $(V=0)$, equation (3) gives $J = \frac{c}{q(\theta)}$. Plugging this into the FOC of wage determination $W - U = \beta (W - U + J - V)$ and rearranging yields:

$$W - U = \frac{\beta}{(1-\beta)} \frac{c}{q(\theta)}$$

Plugging the above expression for $W - U$ into equation (1) gives:

$$rU = z + \theta q(\theta) \left( \frac{\beta}{(1-\beta)} \frac{c}{q(\theta)} \right)$$

$$rU = z + \frac{\beta}{(1-\beta)} c\theta$$

Substituting this into equation (7) yields:

$$w = (1-\beta) z + \beta c\theta + \beta p$$

or

$$w = z + \beta (p - z + c\theta)$$

which is equation (20) of the lecture slides.