Deriving the Wage Curve

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This note derives equation (20) from (18) in slide 21 of the lecture note entitled "The Labour Market I".

We have the following Flow Value Equations:

$$\begin{aligned} rU &= z + \theta q(\theta)(W-U) && (1)\\ rW &= w + s(U-W) && (2)\\ rV &= -c + q(\theta)(J-V) && (3)\\ rJ &= p - w - sJ && (4) \end{aligned}$$

After rearranging,(2) − (1) gives:

$$W - U = \frac{w-z}{r+s+\theta q(\theta)} \tag{5}$$

Equation (3) and the free entry condition ($V=0$) yield $J = \frac{c}{q(\theta)}$. Equation (4) yields $J = \frac{p-w}{r+s}$. Combining these gives the Job Creation Condition:

$$\frac{p-w}{r+s} = \frac{c}{q(\theta)}$$

or

$$r+s = \frac{(p-w)\,q(\theta)}{c} \tag{6}$$

From equation (18) of the lecture slides:

$$\begin{aligned} W-U &= \beta(W-U+J-V)\\ (1-\beta)(W-U) &= \beta(J-V) \end{aligned}$$

Substituting the free entry condition ($V=0$) and equations (5) and $J = \frac{p-w}{r+s}$ we obtain:

$$(1-\beta)\left(\frac{w-z}{r+s+\theta q(\theta)}\right) = \beta\left(\frac{p-w}{r+s}\right)$$

[1] Any errors are my own.

or

$$(1-\beta)(w-z)(r+s) = \beta(p-w)(r+s+\theta q(\theta))$$

Add $\beta w(r+s)$ to both sides and rearrange:

$$(w-(1-\beta)z-\beta p)(r+s) = \beta(p-w)\theta q(\theta)$$

$$w-(1-\beta)z-\beta p = \frac{\beta(p-w)\theta q(\theta)}{r+s}$$

Substitute equation (6) for $(r+s)$:

$$w-(1-\beta)z-\beta p = \beta c\theta$$

$$w = (1-\beta)z+\beta(p+c\theta)$$

$$w = z+\beta(p-z+c\theta)$$

which is equation (20) of the lecture slides.