A Model with an Expanding Variety of Products: Solution Recipe

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This is a one page recipe for solving the endogenous technological progress model presented in slides 43-53 of the lecture notes (http://econ.lse.ac.uk/staff/dquah/c/ec413/Ec413-2009T1-2-Growth+Technology-DQ.pdf).

The starting point would be to list the assumptions for the two types of firms (manufacturing firms, aka final good producers, and research firms, aka intermediate good producers). Recall the research firms have two production functions, one to produce ideas and another to produce the intermediate good associated with an idea. Then write down the steps to solve the model. We proceed by solving the model *backward*:

Step 1: consider the profit maximisation problem of manufacturing firms and derive their demand for intermediate $X(\omega)$ as a function of p_X.

Step 2: write down the profit maximisation problem for the research firm that has *already invented idea* ω. This research firm has monopoly power to choose price p_X (or quantity $X(\omega)$) subject to the demand curve derived in Step 1. Solve for p_X and $X(\omega)$. State that p_X exceeds the marginal cost of production $r\eta$.

Step 3: write down an expression for the value of the research firm that has *already invented idea* ω, which is the present discounted value of profits. Call this $p_A(\omega)$.

Step 4: we need to find out the quantity of resources (that is, skilled labour H_A) devoted to developing new ideas. A research firm receives a benefit p_A from developing a new idea (derived in Step 3) and its production function for developing new ideas is $\dot{A} = \xi A H_A$. So the only input here is skilled labour, which the research firm pays a per person wage w_{H_A}. The research firms take this wage as given. A research firm chooses H_A to maximise its profits from new ideas. The FOC sets $w_{H_A} = \xi A p_A$.

It is important to notice that, in this model, technological progress takes the form of increases in A, that is, the total number of ideas. Why? An increase in A results in an equivalent increase in the number of intermediate goods. Since final output Y is increasing in the number of intermediate goods (there is diminishing returns to each intermediate), then Y increases. In other words, there is economic growth from increases in A, as well as increases in the other inputs N and H.

It would be worthwhile discussing ex-ante incentives and ex-post efficiency in this model. Compare the government's ex-post choice of p_X and X with that of the monopolist.

[1] Any errors are my own.