Problem Set 2

The derivative of the PDV of income wrt. r

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In part 2 of Problem Set 2, we defined the present discounted value of income as:

$$y - I + \frac{I + I^{\alpha}}{1 + r} \tag{1}$$

where $I = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$ from part 1.

The derivative of (1) with respect to r is:

$$-\frac{dI}{dr} + \frac{\frac{dI}{dr}\left(1 + \alpha I^{\alpha-1}\right)(1 + r) - (I + I^{\alpha})}{(1 + r)^2}$$

Using the fact that from part 1 the optimal choice of I is given by the condition $r = \alpha I^{\alpha-1}$, then the above expression becomes:

$$\frac{-(I + I^{\alpha})}{(1 + r)^2}$$

which is negative. So $\frac{d}{dr}\left(y - I + \frac{I+I^{\alpha}}{1+r}\right) < 0$.