# Taxation Cost-Minimisation in Barro (1979)

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In class we solved Problem Set 4 (PS4) by specifying the benevolent government's utility maximisation problem. Instead, Barro (1979) takes a cost-minimisation approach. Of course, the two are entirely equivalent. Barro (pp. 944, par. 3) chooses $\{T_{t+s}\}_{s=0}^{\infty}$ to minimise the PDV of dead-weight losses from taxation (pp. 943, eq. 4) subject to the IBC of the government (pp. 942, eq. 2). In the context of PS4 this writes as:

$$\max_{\{T_{t+s}\}_{s=0}^{\infty}} \left[ -E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \frac{aT_{t+s}^2}{2} - \lambda E_t \left( \sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} - \sum_{s=0}^{\infty} \frac{T_{t+s}}{(1+r)^s} - (1+r)B_t^G \right) \right]$$

The FOCs are:

$$\frac{aE_t\left[T_{t+s}\right]}{(1+r)^s} = \frac{\lambda}{(1+r)^s}$$

$$\frac{aE_t\left[T_{t+s+1}\right]}{(1+r)^{s+1}} = \frac{\lambda}{(1+r)^{s+1}}$$

Combing the FOCs yields:

$$E_t\left[T_{t+s+1}\right] = E_t\left[T_{t+s}\right] \qquad ,\forall s \geq 0$$

which implies:

$$E_t\left[T_{t+s}\right] = T_t \qquad ,\forall s \geq 0 \tag{1}$$

So, the benevolent government will optimally smooth taxation and hence Ricardian equivalence does not hold.

Substituting the Euler equation for taxation (eq. (1)) into the IBC of the government yields:

$$T_t = \frac{r}{1+r} \left( \sum_{s=0}^{\infty} \frac{E_t G_{t+s}}{(1+r)^s} - (1+r)B_t^G \right) \tag{2}$$

Equation (2) tells us the following. *Expected* (predictable) variations in $G_{t+s}$, $\forall s \geq 0$ will be financed by budget deficits or surpluses as required in order to smooth taxation, and there will be *no* change in the expected plan for taxation. An *unexpected*, *transitory change* in $E_t G_{t+s}$ (e.g. outbreak of war) will result in a *negligible change* in the expected plan for taxation $\left( \frac{dT_t}{dE_t G_{t+s}} = \frac{r}{1+r} \frac{1}{(1+r)^s} < 1 \right)$. Finally, an *unexpected*, *permanent change* in $E_t G_{t+s}$ will result in a *one-for-one* change in the level of expected taxation in every period $\left( \frac{dT_t}{\{dE_t G_{t+s}\}_{s=0}^{\infty}} = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} = 1 \right)$.