Ricardian Equivalence Clarification

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In part (e) of Problem Set 5 we are asked whether Ricardian equivalence holds in the OLG model with bequests. A fairly intuitive answer, albeit wrong, is to argue that because the representative individual places less weight on the utility of his children than his own utility, he would rather taxes be incurred by his children (or better still his grandchildren) than himself. If this were true, Ricardian equivalence would not hold. However, the argument is wrong and in this note we will see why.

The argument stated above is equivalent to saying that an infinitely-lived individual would prefer to incur taxes later in life because he discounts the future. Under the assumption of perfect capital markets, this, again, is false. Consider the Euler equation of an infinitely-lived representative consumer who maximises $\sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{1-\sigma}}{1-\sigma}$ subject to his IBC: $\sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = PDV(income)$. The now familiar Euler equation is:

$$c_t^{-\sigma} = \beta (1+r) c_{t+1}^{-\sigma}$$

or

$$c_{t+1} = [\beta (1+r)]^{\frac{1}{\sigma}} c_t$$

or

$$c_{t+s} = [\beta (1+r)]^{\frac{s}{\sigma}} c_t \quad ; \forall s \geq 1 \tag{1}$$

Substituting equation (1) into the IBC yields the following consumption function for c_t:

$$c_t = \left[\frac{1+r}{1+r-[\beta (1+r)]^{\frac{1}{\sigma}}} \right]^{-1} .PDV(income)$$

Now if we introduce a government with an IBC given by $\sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} = \sum_{s=0}^{\infty} \frac{T_{t+s}}{(1+r)^s}$, the representative consumer's consumption function for c_t becomes:

$$c_t = \left[\frac{1+r}{1+r-[\beta (1+r)]^{\frac{1}{\sigma}}} \right]^{-1} . \left[PDV(income) - \sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} \right] \tag{2}$$

Equation (2) tells us that Ricardian equivalence holds, because given a plan for $\{G_{t+s}\}_{s=0}^{\infty}$ the *timing* of taxation is irrelevant for the choice of consumption. What is the intuition for this? The timing of lump-sum taxation (and income) is irrelevant because the consumer is assumed to have perfect access to capital markets and therefore can borrow and lend to achieve his optimum consumption path. Only permanent disposable income matters for consumption $\left[PDV(income) - \sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} \right]$. Notice that the value of β is irrelevant for this result, and we do not need to impose the restriction that $\beta (1+r) = 1$. However, the value of β will, of course, affect the optimal time path of consumption. Consider, for example, $\beta (1+r) < 1$ (an impatient representative consumer). Then, the optimal time path for consumption will be downward sloping (see equation 1)). Nonetheless, as we have shown above, the timing of taxation remains irrelevant. To see this, imagine taxes in period 1 rise by an amount ΔT and - to balance the government's budget - taxes fall in period 2 by an amount $(1+r)\Delta T$. The representative consumer will maintain his optimal downward sloping timepath for consumption by borrowing an amount ΔT in period 1 and repaying $(1+r)\Delta T$ in period 2. The choice of consumption in every period is unchanged because the change in taxation policy has no effect on the permanent disposable income of the representative consumer.

The same intuition applies for the OLG model with bequests we considered in Problem Set 5. Due to the bequest motive, the representative consumer has an infinite horizon and hence the same time horizon as the infinitely-lived government. Since by assumption capital markets are perfect, the timing of lump-sum taxation is irrelevant for the consumption decision. Notice that the value of θ (and β) is irrelevant for this result. However, the value of θ will effect the optimal allocation of consumption across generations (and β will affect the optimal allocation of consumption across time within a generation).

We showed in part (d) of PS5 that in S.S. $\theta(1+r) = 1$. This is the condition needed to get perfect consumption smoothing across generations. Similarly, if we assume $\beta(1+r) = 1$ then we would have perfect consumption smoothing between the young and old of the same generation.

In class I have mentioned that in a general equilibrium (i.e. endogenous r) model, perfect capital markets yields the condition $\beta(1+r) = 1$ in steady state (see, for example, the Ramsey model on pp. 100 of Barro and Sala-i-Martin). This is an equilibrium, steady-state condition which is not necessary for the Ricardian equivalence result (as shown above).