The Liquidity Service of Benchmark Securities

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February 2005

Abstract

We demonstrate that benchmark securities allow heterogeneously informed investors to create trading strategies that are perfectly aligned with their signals. Investors who are informed about security-specific risks but uninformed about systematic risks can take an offsetting position in benchmark securities to eliminate exposure to adverse selection in systematic risks, while investors who are informed about systematic risks but uninformed about security-specific risks can trade systematic risks exclusively using benchmark securities. We further show that introduction of benchmark securities encourages more investors to acquire both security-specific and systematic-factor information, which leads to increased liquidity and price informativeness for all individual securities. (JEL: G10, G12, G14)

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[*]Acknowledgements: This paper is based on a chapter in my dissertation at the Massachusetts Institute of Technology. I thank my advisors Paul Krugman, Jeremy Stein, and Jiang Wang, as well as Dimitri Vayanos, Andrea Eisenfeld, Emre Ozdenoren, Paolo Pasquariello, Matthew Spiegel, Avanidhar Subrahmanyam, and the seminar participants at the Massachusetts Institute of Technology, the University of Michigan, the NBER 2001 Summer Institute, the 2002 American Finance Association Meetings, and the 2002 Utah Winter Finance Conference for helpful comments. I also thank Franklin Allen, the guest editor, Patrick Bolton, the editor, and an anonymous referee for comments that improved the original paper. Any errors are mine.

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1 Introduction

The introduction of benchmark securities, i.e., securities in which the inherent risks are completely systematic, has been a major recent innovation of financial markets. In equity markets, trading in securities based on equity market indexes (such as S&P 500 index futures and exchange-traded-funds (ETFs) on major indexes) has immense liquidity and popularity. Globally, the number of ETFs has grown from fewer than 50 in 1998 to more than 300 in 2004. As of June 30, 2004, $178 billion is invested in ETFs in the U.S., while in Asia, Japan alone has $30 billion in ETFs. ETFs tracking the DAX index are so popular that their bid-ask spreads have narrowed by more than half within a year of their introduction. In China, the first ETF, based on the Shanghai 50 index, is to be launched by the end of 2004. Barclays Global Investors (BGI), the world's leading purveyor of the ETF funds, is reported to pull in close to $1 billion a week globally to be invested in ETFs.[1] In the fixed-income market, introduction of benchmark securities such as inflation-protected bonds has been proven to be successful in times of severe inflation risks (Shiller 2003). Issuances of dollar-denominated sovereign bonds from emerging markets have been shown to increase the liquidity of dollar-denominated corporate bonds from the same country (Dittmar and Yuan 2005). The loss of such securities, on the other hand, can create havoc in the market. For example, when the U.S. Treasury announced the suspension of all sales of new 30-year T-bonds due to the outlook for long-term budget surpluses in October 2001, traders lamented that its loss may make the pricing of bonds more difficult, thereby creating more volatility and dampening

[1] See "Entirely too frenetic? The lastest in fund fashion" in *The Economist*, July 29, 2004.

appetite for risk.[2] Subsequently, agency debt securities, high-grade corporate debt securities, and interest rate swaps have been used to replace treasuries as reference and hedging benchmarks among market participants (Fleming 2002; Schinasi, Kramer, and Smith 2001).

In a frictionless world, these benchmark securities would be redundant. Furthermore, the popularity of such benchmark securities seems puzzling since investors, on their own, can apparently obtain the same cash flows by taking positions in a diversified portfolio of existing securities and "construct" their own benchmark securities. To explain this puzzle, Subrahmanyam (1991) and Gorton and Pennacchi (1993) have demonstrated theoretically that benchmark securities are popular among *uninformed liquidity* investors because these securities offer lower adverse selection costs. In this paper, we offer an alternative explanation, that benchmark securities are popular among *informed* investors because these securities allow informed investors to create trading strategies that are perfectly aligned with their signals. Besides offering an alternative explanation of the popularity of benchmark securities, we find that introduction of liquid benchmark securities increases liquidity and price informativeness of all existing individual securities.

We extend the single-trader and single-security model of Kyle (1989), the K-trader and N-security models of Subramanyam (1991) and Cabelle and Krishnan (1994) to a G-K_i-trader ($i = 1, ..., N$), N-security and two-factor model to study the liquidity impact of benchmark securities. Our model has the basic setup of the Kyle-type models: In each market, risk-neutral informed traders, who are strategic, and liquidity traders are assumed to submit their orders to risk-neutral market-markets, who set prices expecting to earn zero profits. Our model has three additional assumptions: 1) in-

[2]See the article by Bream and Wiggins in *Financial Times*, November 5, 2001.

formed investors are heterogeneously informed. They possess either security-specific or systematic-factor information; 2) risk-neutral informed investors have to pay a fixed cost to acquire an information signal and expect to earn zero profits; and 3) market markers observe order flows from other markets. Support for these additional assumptions is based on the empirical observation that, in practice, information acquisition is specialized and costly; and market-markers normally make markets for multiple securities.[3]

We find that, in this economy, informed investors adopt unique trading strategies. Systematic-factor informed traders invest in all available securities to gain the systematic risk exposure, while security-i-specific informed investors invest in security i, the security they know the best, and use all other securities to eliminate the systematic risk exposure. The intuition for this result is as follows. In our setup, informed investors are heterogeneously informed. Although security-i-specific informed investors are endowed with an informative signal about security i's idiosyncratic risk, they are uninformed about the systematic risk and other security-specific risks. Hence, they may face adverse selection when exposed to risks other than security i's idiosyncratic risk, that is, they may trade with someone who knows more than they do. Similarly, systematic-factor informed investors are at information disadvantage when exposed to security-specific risks. To avoid adverse selection costs, both types of informed investors would like to trade the security best aligned with their signals. For systematic-factor informed investors, the security that is best aligned with their signal is a security with

[3]For example, market makers for emerging market bonds quote bid and offer prices for both sovereign and corporate bonds from emerging market countries. Although the specialists on the floor of the New York Stock Exchange are only responsible to make the market for a few stocks (The number of stocks covered by each specialist firms in NYSE ranges from 27 to 430 stocks in the last three months of 1998 (Table 1A in Huang and Liu (2004))), order flow information for other stocks and the aggregate market are easily accessible.

only the systematic factor risk, i.e., the benchmark security. To synthesize a benchmark security, systematic-factor informed investors trade all existing securities. For security-specific informed investors, the security that is best aligned with their signals is a security with the pure security-specific risk. To synthesize such a security, security-i-specific informed investors need to eliminate the systematic risk factor in security i. To do so, they construct a synthetic benchmark security using all existing securities and take an offsetting position on it.

Yet, the synthetic benchmark security constructed using existing securities does not allow informed investors to eliminate adverse selection costs perfectly. In addition to transaction costs, by trading all primitive securities underlying the synthetic benchmark, systematic-factor informed investors are exposed to adverse selection costs in the market of each primitive security. Similarly, security-i-informed investors are exposed to adverse selection costs in markets of all other primitive securities.[4]

Therefore, when benchmark securities are introduced, security-specific informed investors take an offsetting position on benchmark securities, rather than synthetic benchmark securities, i.e, the primitive securities that underlie the benchmark. In doing so, they are able to eliminate adverse selection costs altogether with systematic risk and concentrate on trading security-specific risks. Similarly, systematic-factor informed investors are able to eliminate

[4]In addition, in some cases, synthetic benchmark securities constructed using a portfolio of existing securities may fail to only embody the systematic-factor risk either because the number of existing individual securities, N, is small, or because they are created using only traded securities, a subset of all securities (since not all securities are publicly traded). For example, in the dollar-denominated bond market, sovereign bonds are regarded as the benchmark securities for corporate bonds issued from the same country. However, if there are no sovereign bonds, it would be difficult to construct synthetic benchmarks since, for instance, the total number of outstanding dollar-denominated corporate bonds does not even exceed 10 in many emerging market countries (Dittmar and Yuan 2005).

adverse selection costs and concentrate on trading the systematic risk only. The increased expected profit encourages more investors to acquire systematic and security-specific information. This leads to more informative prices, less information asymmetry, and higher liquidity for *all* securities in the market. We find that the liquidity service of benchmark securities is greater when benchmark securities are liquid, systematic risks are high, and firm idiosyncratic risks are large. Finally, we find that the liquidity service of benchmark securities on individual securities is increasing in the magnitude of systematic-factor loadings of individual securities. For securities with low systematic-factor loadings, the liquidity impact is small, but still positive.

Existing literature on benchmark securities, such as Subrahmanyam (1991) and Gorton and Pennacchi (1993), offers a different rationale for the existence and the popularity of such securities. According to Subrahmanyam (1991) and Gorton and Pennacchi (1993), benchmark securities are securities whose information sensitivities are minimized. Benchmark securities are liquid and popular among uninformed investors because they offer lower adverse selection costs. We identify a different rationale for the popularity of benchmark securities. Namely, in a world with heterogeneously informed agents, benchmark securities, by maximizing information sensitivity, encourage information production and liquidity. Besides identifying a different rationale for the popularity of benchmark securities, we also find that benchmark securities have a liquidity service on all individual securities. This *universal* positive liquidity impact of benchmark securities differs sharply from the predictions in Subrahmanyam (1991) and Gorton and Pennacchi (1993). Subrahmanyam (1991) found that the introduction of a benchmark security may lower the number of security-specific informed investors for securities that have lower weights in the benchmark. Gorton and Pennacchi (1993) found that the introduction of a benchmark security eliminates all trading in the individual securities when traders have homogenous preferences and endowment distributions.

In addition to benchmark securities that are composites of existing securities, Shiller (1993) proposed to establish markets for claims on large income aggregates. These claims embody systematic risks, similar to the benchmark securities in our model. Shiller (1993) argued that these securities help individuals to hedge against major income risks and complete the market. In our model, investors have hedging needs even though they are risk neutral. The reason is that they are heterogeneously informed and face adverse selection when trading securities they are less informed about. Similar to the rationale provided in Shiller (1993), in our setting benchmark securities help to complete the market and allow heterogeneously informed investors to perfectly hedge against adverse selection.

Even with the popularity of the existing benchmark securities, the supply of benchmark securities is limited in reality. For example, we do not observe markets in claims on national incomes, which are important benchmarks, nor markets in claims on major components of incomes (such as real estate service flows). Theoretically, the limited supply of benchmark securities can be explained by adverse selection concerns in security design (DeMarzo and Duffie 1999; Rahi 1996; Yuan 2001). DeMarzo and Duffie (1999) found that the security issuer's private information regarding the payoff of the security may cause illiquidity for the security. Similarly, Rahi (1996) demonstrated that the issuer may not find it profitable to float an asset that affords her an information advantage in the presence of rational uninformed outside investors. Yuan (2001) showed that issuing a benchmark security is costly because the issuer has to compensate uninformed investors to acquire information on the issued security to lower their adverse selection costs.

In the remainder of the paper, we present a model that formalizes the preceding discussion. The model setup is introduced in Section 2. In Section 3, we analyze market liquidity and price information in the absence of benchmark securities. Section 4 then considers the introduction of benchmark securities and demonstrates improved market liquidity and price informativeness. Section 5 concludes.

2 The Model

In this section we describe our basic model. We extend the single-trader and single-security model of Kyle (1989), the K-trader and N-security models of Subramanyam (1991) and Cabelle and Krishnan (1994) to study the liquidity impact of benchmark securities. We first detail the main features of the assets in the model. We then introduce agents and the information structure. Finally, we discuss the equilibrium notion used in the paper and introduce the equilibrium concept.

Assets

We consider an economy with N assets, which are indexed by $i = 1, ..., N$ and are simultaneously traded over a single period. Trading takes place at time 0, and the securities are liquidated at time 1. The vector of liquidation values, $\widetilde{v}$, is characterized by the following linear factor structure:

$$\widetilde{v} = \overline{v} + \beta\widetilde{\gamma} + \widetilde{\mu}, \tag{1}$$

where $\overline{v}$, is an $N \times 1$ vector of mean liquidation values, $\widetilde{\gamma}$ is a normally distributed random variable with a mean of zero, and a variance of σ_γ^2, $\widetilde{\mu}$ is an $N \times 1$ random vector of idiosyncratic shocks, and β is a non-zero

$N \times 1$ vector of systematic factor loading. We interpret $\widetilde{\gamma}$ and $\widetilde{\mu}$ as the systematic (market) and security-specific (idiosyncratic) components of the security value innovation, respectively. We assume that $\widetilde{\mu}$ is multivariate normally (MN) distributed with a mean vector of 0, and a diagonal and non-singular covariance matrix Σ_μ. Hence, $\Sigma_\upsilon = \Sigma_\mu + \sigma_\gamma^2 \beta\beta'$, which is non-singular.

Traders

There are two types of traders in this economy: informed traders and liquidity traders. Informed traders are risk-neutral and have access to an information-production technology. This technology enables them to acquire, at time 0, either a noisy signal ($\widetilde{s}_m$) on the systematic factor ($\widetilde{\gamma}$), or a noisy signal ($\widetilde{s}_i$) on the security-specific factor ($\widetilde{\mu}_i$), at a fixed cost, c.[5] Two signals are,

$$\widetilde{s}_m = \widetilde{\gamma} + \widetilde{\epsilon_\gamma}, \quad \widetilde{s}_i = \widetilde{\mu}_i + \widetilde{\epsilon}_i, \tag{2}$$

where $\widetilde{\epsilon_\gamma}$ is a normally distributed random variable with a mean of zero and a variance of $\sigma^2_{\epsilon_\gamma}$, and $\widetilde{\epsilon}_i$ is a normally distributed random variable with a mean of zero and a variance of $\sigma^2_{\epsilon_i}$. We assume there are G systematic-factor informed traders and K_i security-i-factor informed traders, where G and K_is are positive integers. Numbers of different factor-informed traders are determined endogenously through the zero-expected-profit condition. Systematic-factor informed trader g ($g = 1, ..., G$) and security-i-specific-factor informed trader k_i ($k_i = 1, ..., K_i$) submit $N \times 1$ vectors of market order of $x_m(\widetilde{s}_m)$ and $x_{k_i}(\widetilde{s}_i)$, respectively.

[5] To demonstrate the liquidity service of benchmark securities, we just need to assume that traders are heterogeneously informed. That is, traders cannot acquire the signal on the systematic-factor, $\widetilde{s}_m$, and, at the same time, all N security-specific signals, $\sum_{i=1}^N \widetilde{s}_i$, This assumption is reasonable. For example, in practice, not a single investment bank has an analyst for every publicly traded firm. Assuming that a trader acquires either $\widetilde{s}_m$ or $\widetilde{s}_i$ is made for expositional clarity and without loss of generality.

Liquidity traders are uninformed about liquidation values of securities. They submit a random vector of market orders $\widetilde{z}$, which is $MN(0, \Sigma_z)$. We assume the covariance matrix Σ_z is diagonal (so that the correlation in asset prices comes from the correlation in asset factor exposures rather than liquidity trading) and non-singular (so to provide camouflage for informed trading).

Equilibrium

All traders submit their orders for security i to a market-maker, without knowing the market clearing price when they do so. Unlike Subramanyam (1991), we assume the market-maker observes the net total aggregate demands of informed and liquidity traders for all securities, $\widetilde{q}$, where $\widetilde{q}$ is an $N \times 1$ vector of order flows and $\widetilde{q} = \sum_{g=1}^{G} x_m + \sum_{i=1}^{N} \sum_{k_i=1}^{K_i} x_{k_i} + \widetilde{z}$. The motivation for this assumption is twofold. First, for over-the-counter markets, one market maker normally provides price quotes for all securities in his/her specialty market. For example, the emerging-market bond trading desk in investment banks normally makes markets for almost all emerging market bonds and hence observes order flows for all these securities. Second, even on organized exchanges, specialists on the same trading floor communicate with each other and hence are informed about order flows in each other's market.

We also assume that competition among market-makers forces the expected profits of market-markers to zero. Therefore, the price vector set by the market-maker at time 0 is,

$$p = p(\widetilde{q}) = E[\widetilde{v}|\widetilde{q}]. \tag{3}$$

We use the following notion of noisy rational expectations equilibrium.

Definition 1 *An equilibrium of our model is a strategy (x_m or x_i), a pricing function $p(\widetilde{q})$, and the number of traders acquiring the systematic-factor signal (G) or the security-i-specific-factor signal (K_i) such that,*

- *Given $p(\widetilde{q})$, informed traders solve*

$$Max_{x_m} E\{(\widetilde{v} - p)^T x_m | \widetilde{s}_m\} \quad or \quad Max_{x_i} E\{(\widetilde{v} - p)^T x_i | \widetilde{s}_i\} \tag{4}$$

- *Given x_m, x_i, G, K_i, and $\widetilde{z}$, market-makers set the following price to clear the market:*

$$p = E[\widetilde{v}|\widetilde{q}].$$

- *Ex-ante expected profit of each factor informed trader is zero.*

3 Price Informativeness and Market Liquidity in the Absence of Benchmark Securities

We now provide an explicit characterization of a linear equilibrium in Proposition 1.

Proposition 1 *The price function is*

$$p(\widetilde{q}) = \overline{v} + A\widetilde{q}, \tag{5}$$

$$A = \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2},$$

where $M \equiv \Sigma_z^{1/2} F \Sigma_z^{1/2}$, 1_i is the ith column vector of the $N \times N$ identity matrix, and F is defined in the following expression:

$$F \equiv \left(\frac{G}{(G+1)^2} \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2} \beta\beta^T + \sum_{i=1}^{N} \frac{K_i}{(K_i+1)^2} \frac{\sigma_i^4}{\sigma_i^2 + \sigma_{\epsilon_i}^2} 1_i 1_i^T \right).$$

Systematic-factor-informed and security-i-informed traders submit $x_m(\widetilde{s}_m) = B_m\widetilde{s}_m$ *and* $x_i(\widetilde{s}_i) = B_i\widetilde{s}_i$*, respectively, where*

$$B_m = \frac{1}{G+1}\frac{\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_{\epsilon_\gamma}}A^{-1}\beta, \tag{6}$$

$$B_i = \frac{1}{K_i+1}\frac{\sigma_i^2}{\sigma_i^2+\sigma_{\epsilon_i}}A^{-1}1_i. \tag{7}$$

The number of equilibrium systematic-factor traders is the largest integer smaller than or equal to X *, i.e.,* $G = \lfloor X \rfloor$*, while* X *solves the zero-expected-profit condition in equation (8). The number of security-i-specific informed traders is similarly defined as* $K_i = \lfloor Y \rfloor$*, while* Y *is determined by the zero-expected-profit condition in equation (9).*

$$c = \frac{1}{(X+1)^2}\frac{\sigma_\gamma^4}{\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2}\beta^T A^{-1}\beta, \tag{8}$$

$$c = \frac{1}{(Y+1)^2}\frac{\sigma_i^4}{\sigma_i^2+\sigma_{\epsilon_i}^2}1_i^T A^{-1}1_i. \tag{9}$$

It should be noted that in equilibrium the matrix A is positive definite and symmetric. This has two implications. The first implication is that the price of each asset is increasing in and relatively more sensitive to its own order flows. The sensitivity (measured by the (i, i) term in matrix A) is decreasing in the number of informed investors. In other words, the liquidity of the asset i (which is measured by the inverse of the price sensitivity to its own order flow as in Kyle (1989)) is increasing in the number of systematic-factor informed investors, G, and the number of security-i informed investors, K_i. The second implication is that the ith price responds to the jth order flow exactly as the jth price responds to the ith order flow, which is also shown in Caballe and Krishnan (1994). The symmetry result is due to the strategic behavior of informed investors. Suppose that asset i is characterized by a high level of liquidity trading; then the ex ante expected profit of a security-i-informed trader will be high, which will result in more traders acquiring information on asset i idiosyncratic risk, μ_i, and more aggressive trading in asset i. This makes the informativeness of order flows the same for both assets: Order flow q_i is as informative in predicting payoff $\widetilde{v}_j$ as is order flow q_j in predicting $\widetilde{v}_i$.

The following corollary shows that the information content of the equilibrium price vector (which is denoted by I_p and measured by $Var(\widetilde{v})-Var(\widetilde{v}|\widetilde{p})$) is independent of the variance of liquidity trading and increasing in the number of systematic-factor or the number of security-i-specific informed investors. Similar results are shown in Kyle (1989), Subramanyam (1991), and Cabelle and Krishnan (1994). In this paper, we generalize the results to a setting with N assets of multiple factors. We also allow endogenous information acquisition by informed investors on different factors and market markers to observe order flows from all assets.

Corollary 1 *The informativeness of prices, I_p, is given by the following expression:*

$$\frac{G}{G+1}\frac{\sigma_\gamma^4}{\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2}\beta\beta^T+\sum_{i=1}^{N}\frac{K_i}{K_i+1}\frac{\sigma_i^4}{\sigma_i^2+\sigma_{\epsilon_i}^2}1_i1_i^T, \tag{10}$$

which is independent of Σ_z and increasing in the number of systematic-factor and the number of security-specific informed investors, G and K_i ($i=1,...,N$).

In this economy, systematic-factor informed traders will invest in all available securities, while security-i-specific informed traders will trade security i, the security they know best, and use all other securities to eliminate the exposure to adverse selection in trading the systematic factor risk. We illustrate this point using a numerical example with two securities. More specifically, we assume the liquidation values of the two securities follow the following expression,

$$\widetilde{v}_1 = 0.5\widetilde{\gamma} + \widetilde{\mu}_1,\ \widetilde{v}_2 = 1.5\widetilde{\gamma} + \widetilde{\mu}_2,$$

where $\sigma_\gamma = 19.2\%$, which is chosen to reflect the annualized volatility of common stock indices; and $\sigma_{\mu_1} = \sigma_{\mu_2} = 35\%$, which are selected so that a stock with a β of 1 has an annualized volatility of 40%. We assume the signal quality of all informed investors is the same: The signal-to-noise ratio is 5, which means that $\sigma_{\epsilon_\gamma} = 3.84\%$, and $\sigma_{\epsilon_{\mu_1}} = \sigma_{\epsilon_{\mu_2}} = 7\%$.

We also assume that the liquidity trading in two-security markets is uncorrelated with the following variance-covariance matrix $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$. In this setup, systematic-factor and security-specific informed investors have the same signal precision regarding the risk factor in which they are interested. Finally, we assume that the cost of acquiring a signal is 0.2.

We find that the number of investors acquiring a systematic-factor signal, a security-1-specific signal, or a security-2-specific signal, is 2, 8, and 4 respectively. Their demand schedules for each security are shown in Table 1.

[Insert Table 1 about here.]

Although the signal precisions are the same across the signals, we find that more investors choose to become informed about the security 1 idiosyncratic risk factor. Investors prefer to acquire signals on security-specific risks over the systematic risk, because signals on volatile risk components are more valuable. In this case, the standard deviation of each security-specific risk is 35%, while that of systematic risk is 19.2%. There are two reasons behind the popularity of the security-1-specific signal. The first reason is that security-specific informed investors are uninformed about the systematic risk. The smaller the systematic factor loading an asset has, the more valuable the security-specific signal will be. The second reason comes from the trading strategies of security-specific investors. The numerical example shows that security-specific informed investors trade differently from systematic-factor informed investors. Systematic-factor informed investors trade the systematic-factor component in both securities, more aggressively in security 2 where the systematic-factor loading is large. Effectively, systematic-factor informed investors are trading a synthetic benchmark security which embodies mostly the systematic risk. This synthetic benchmark security is a portfolio of securities 1 and 2 with a higher weight on security 2 whose systematic-factor loading is large. By contrast, security-specific informed investors face adverse selection if they take a position on the systematic risk since they are less informed about the systematic risk component than systematic-factor informed investors. They prefer to trade the security they know the best and eliminate the exposure to the systematic risk. To do so, they take an offsetting position in the synthetic benchmark security. For example, if security-1-informed investors receive a positive signal, they will long security 1, short security 2 to eliminate the systematic risk. The example also shows that the offsetting demand of security-2-specific informed investors is larger due to its high systematic-factor loading. This offsetting demand also exposes security-specific informed investors to the security-specific risk and the liquidity risk in the offsetting instrument, and reduces their expected profits. Obviously, these risks from offsetting are greater if the offsetting need, β, is greater. This is the second reason why relatively fewer investors choose to acquire the security-2-specific signal.

It should be noted that our finding of this particular trading strategy of informed investors (which is to trade the security they have the information on and eliminate the exposure to risks they are less informed about) is unique. This finding comes from two assumptions: 1) informed investors are heterogeneously informed; 2) market makers observe all order flows. Since investors are heterogeneously informed, they face adverse selection when trading the risks they are less informed about and have incentives to eliminate the exposure to these risks. Subrahmanyam (1991) also assumed heterogeneously informed investors. However, in Subrahmanyam (1991), each market maker is assumed to only observe one order flow. This assumption, for example, eliminates the security-specific informed investor's incentive to take off-setting positions since market makers do not observe other order flows and cannot infer about the security-specific informed investors' offsetting demand when setting prices.

It also should be noted that trading in all other individual securities by security-specific informed investors is not strategic. Security-i-specific informed investors are uninformed about the systematic risk and other security-specific risks and may face adverse selection when exposed to these risks. In our setup, security-i-specific informed investors are strategic in affecting the market maker's inference about security i's idiosyncratic factor but not the systematic factor. They are price-takers in securities other than i and take positions in these securities to reduce the systematic factor risk exposure and to lower the adverse selection cost.[6]

In our model, to set a price the market marker will condition on not only the order flow from the corresponding market, but also the order flow from all other markets. The following pricing rule shows that he/she puts a higher

[6]In a typical Kyle (1989) setup, if risk-neutral investors receive informed signals on all sources of risks (i.e., systematic and security-specific risks), their trading activities across all securities would be driven by strategic considerations. An example is the setup in Pasquariello (2004), where he also assumes that the market-makers observe all order flows. However, he further assumes that informed investors are informed about all sources of risks, while in our setup informed investors are heterogeneously informed of different sources of risks.

weight on the order flow from the corresponding market:

$$P_1 = 0.0575\widetilde{q}_1 + 0.0097\widetilde{q}_2, \quad P_2 = 0.0097\widetilde{q}_1 + 0.0951\widetilde{q}_2.$$

The informativeness of the prices in this two-security economy is:

$$\begin{pmatrix} 0.1317 & 0.0504 \\ 0.0504 & 0.2054 \end{pmatrix} - \begin{pmatrix} 0.1106 & 0.0177 \\ 0.0177 & 0.1474 \end{pmatrix} = \begin{pmatrix} 0.2111 & 0.0327 \\ 0.0327 & 0.0580 \end{pmatrix},$$

where the biggest variance reduction occurs in security 2, the most volatile security.

4 The Effect of Benchmark Securities

We now consider the case when a benchmark security, which embodies the pure systematic factor risk, γ , is introduced. Note that this benchmark security is a claim issued against the systematic risk of the economy. On-the-run treasury securities in the U.S. can be regarded as examples of this type. The government can issue this security more easily than private firms, since it can use audit and tax information to obtain economy-wide information and thus has greater credibility. This benchmark security also can be regarded as a weighted average of the payoffs of individual securities:

$$\widetilde{S}_m = \sum_{i=1}^{N} w_i \overline{v}_i \quad + \quad \sum_{i=1}^{N} w_i \beta_i \widetilde{\gamma} + \sum_{i=1}^{N} w_i \widetilde{\mu}_i,$$

where w_i is the weight of security i such that $0 < w_i < 1$ and $\sum_{i=1}^{N} w_i = 1$. Note that in this case the benchmark security is a claim issued against only publicly traded securities in this economy and may have a β greater or less than 1.[7] Some real-world examples include: S&P 500 index futures and IPC, the Mexican stock index futures traded on the Chicago Mercantile Exchange.[8]

[7]Our results will remain qualitatively the same if a benchmark security, $\beta_m \widetilde{\gamma}$, where $\beta_m \neq 1$, is introduced.

[8]IPC was introduced by Chicago Mercantile Exchange on May 30, 1995.

In this economy, market makers have one more signal, $\widetilde{w}$, the order flow of the benchmark security, to set prices. Systematic-factor informed investors can trade on their signal, and security-specific informed investors can eliminate systematic risk better using the benchmark security. The following proposition characterizes the impact of the benchmark security on the market liquidity and price informativeness of existing securities.[9]

Proposition 2 *Upon introduction of the benchmark security, γ, the number of investors acquiring systematic-factor information, G, increases, and the amount of increase is larger when either the systematic factor risk, σ_γ^2, is higher, or the precision of the informed signal, measured by $\sigma_\gamma^2/(\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2)$, is higher, or the liquidity of the benchmark, measured by $\sigma_{z_m}^2$, is higher. The number of investors acquiring security-i-specific, K_i, also increases for all securities ($i = 1, ..., N$), with bigger increases occurring in securities with either large systematic factor loadings (that is, β), or more precise signals (that is, larger $\sigma_i^2/(\sigma_i^2 + \sigma_{\epsilon_i}^2)$), or large idiosyncratic risks (that is, σ_i^2).*

It should be noted that this finding is true regardless of the magnitude of βs on systematic risk. For example, when we introduce a benchmark security to the previous numerical example with the liquidity trading in benchmark securities having the same normal distribution as in other markets ($N(0, 4)$), we find that numbers of investors acquiring a systematic-factor signal, a security-1-specific signal, or a security-2-specific signal have *all* increased. Since introduction of a benchmark security allows security-specific informed investors to eliminate systematic risk perfectly and to be less exposed to adverse selection in the offsetting instrument, we expect that the amount

[9] The explicit characterization of a linear equilibrium in this case is similar to Proposition 1. The difference is that there are $N + 1$ (instead of N) securities with the $N + 1$th security having a β of 1 and a variance of 0 on the idiosyncratic risk.

of increase in the number of security-specific informed investors is larger in securities with higher βs. We find this result in the numerical example, where the number of security-2-informed investors increases from 4 to 7 while the number of security-1-informed investors increases marginally from 8 to 9. The largest increase occurs in the number of the systematic-factor informed investors, from 2 to 7.

In our numerical example, we vary the volatility of the systematic risk, σ_γ, and find that the increase in the number of informed investors after the benchmark introduction is positively related to σ_γ (Figure 1). This relationship is most significant in the case of systematic-factor informed investors, less so in the case of security-2-specific informed investors, and the increase in the number of security-1-specific informed investors remains at the same level regardless of the magnitude of systematic risk. A volatile systematic factor indicates that the offsetting need of security-specific informed investors is high (especially for securities with large βs), and the systematic-factor informed investor's signal is more valuable. Introduction of a benchmark security allows security-specific informed investors to eliminate systematic factor risks perfectly and systematic-factor informed investors to trade the security that is aligned with their signal.

[Insert Figure 1 about here.]

In Figure 2, we also find that the amount of increase in the number of informed investors after the benchmark introduction depends on the liquidity of the benchmark security, which can be measured by σ_{z_m}. A liquid benchmark allows systematic-factor informed investors to profit more from trading and security-specific informed investors to suffer smaller losses from offsetting.

[Insert Figure 2 about here.]

In Figure 3, we find that the amount of increase in the number of security-specific informed investors after the benchmark introduction is positively related to the magnitude of the respective idiosyncratic risk. This is due to the fact that security-specific informed investors can profit more from a volatile idiosyncratic risk factor when they can offset systematic risks perfectly using the benchmark security. We find this relationship is more significant for security-2-specific informed investors, because the offsetting benefit is larger to security-2-specific informed investors.

[Insert Figure 3 about here.]

Furthermore, the following corollary shows that the liquidity and price informativeness impact of a benchmark security is also universal among existing securities.

Corollary 2 *Upon introduction of the benchmark security, the market liquidity and the price informativeness increase for all existing securities* $i = 1, ..., N$. *The amount of increase is larger when the systematic risk* (σ_γ^2), *the systematic factor loading* (β), *the liquidity of the benchmark security* ($\sigma_{z_m}^2$), *and the idiosyncratic risk* (σ_i^2) *is higher.*

In Table 2, we find that systematic-factor informed investors have reduced their trading in securities 1 and 2 substantially and concentrate on the security they have the best signal on: the benchmark security. The benchmark security is also the preferred offsetting instrument for security-specific informed investors. In fact, security-specific informed investors have reduced the use of non-benchmark securities for offsetting purposes to close to zero.

[Insert Table 2 about here.]

To set prices, the market maker has one more order flow to condition on. The following price-setting rule shows again that he/she puts the largest weight on the order flow from the corresponding market:

$$
\begin{aligned}
P_1 &= 0.0533\widetilde{q}_1 + 0.0052\widetilde{q}_2 + 0.0051\widetilde{w}, \\
P_2 &= 0.0052\widetilde{q}_1 + 0.0719\widetilde{q}_2 + 0.0144\widetilde{w}, \\
P_\gamma &= 0.0051\widetilde{q}_1 + 0.0144\widetilde{q}_2 + 0.0271\widetilde{w}.
\end{aligned}
$$

The coefficients on $\widetilde{q}_1$ and $\widetilde{q}_2$ are smaller, which indicates the liquidity in both markets is now higher.

Upon introduction of the benchmark security, the informativeness of the security 1 and 2 prices will be:

$$
\begin{pmatrix} 0.1317 & 0.0504 \\ 0.0504 & 0.2054 \end{pmatrix} - \begin{pmatrix} 0.1138 & 0.0233 \\ 0.0233 & 0.1729 \end{pmatrix} = \begin{pmatrix} 0.0179 & 0.0271 \\ 0.0271 & 0.0325 \end{pmatrix}.
$$

It is evident the price informativeness is improved for both securities, with the biggest improvement happening in security 2, the security with a larger systematic factor loading, β.

By contrast, Subramanyam (1991) found the number of security-specific informed investors tends to increase for securities with higher weights on the systematic factor, but not for securities with lower weights. In other words, introduction of benchmark securities may crowd out liquidity or price informativeness for some individual securities while benefiting those of other securities. Gorton and Pennacchi (1993) found that the introduction of a benchmark security eliminates all trading in the individual securities when traders have homogenous preferences and endowment distributions. The difference in findings comes from different mechanisms of liquidity improvement. Gorton and Pennacchi (1993) as well as Subramanyam (1994) endogenized liquidity trading and found that liquidity traders prefer benchmark securities and securities with higher weights in the systematic factor since adverse selection costs in these securities are lower. The offsetting demand of informed investors is not modeled since market makers do not condition on order flow information from other markets. In the model presented in this paper, order flow information is accessible to market makers, and hence security-specific informed investors have incentives to take offsetting positions. Since all security-specific informed investors have incentives to eliminate systematic risks regardless of the magnitudes, introduction of a better offsetting instrument, such as a benchmark security, improves liquidity for every traded security.

5 Conclusion

We have presented a theory of trading in benchmark securities and have demonstrated that benchmark securities promote both systematic and security-specific information production, which leads to more liquidity for all traded securities. In our theory the offsetting demand of heterogeneously informed investors is explicitly modeled. Since benchmark securities are preferred trading vehicles for systematic-factor informed investors and perfect offsetting instruments for security-specific informed investors, their introduction increases expected revenue from informed trading and provides incentives for more investors to acquire information. Our analysis also identifies situations when benchmark securities provide greater liquidity services. For the whole economy, that is when the systematic risk is large; for individual securities, that is when the factor loading on systematic risk is high and the idiosyncratic risk is large.

Some empirical implications of the analysis are:

- The introduction of benchmark securities increases the number of both systematic-factor and security-specific informed investors. The amount of the increase in the number of systematic-factor informed investors is positively related to the magnitude of the systematic risk. The amount of the increase in the number of security-specific informed investors is positively related to the magnitude of factor loading on the systematic risk. This implication can be tested, for example, by analyzing the extent of analyst coverage (e.g., number of analysts) or number of market-markers for individual securities, before and after a benchmark security is introduced.

- The introduction of benchmark securities increases the liquidity for all individual securities. The amount of increase is larger when the systematic risk, the systematic-factor loading, or the idiosyncratic risk is higher. This implication can be tested by analyzing the cross-sectional variation in bid-ask or trading volume of individual securities before and after the introduction of a benchmark security.

- The introduction of benchmark securities increases the price informativeness for all traded securities. The amount of increase is larger when the systematic-factor risk, the systematic-factor loading, or the idiosyncratic risk is higher. This implication can be tested by analyzing price discovery in the benchmark and individual securities. Since benchmark securities are major offsetting instruments, price discovery in benchmarks is expected to be faster than in individual securities.

- The liquidity of benchmark securities is positively related to the liquidity and price informativeness of individual securities. We can test this implication by analyzing the correlation between bid-ask spreads or trading volumes of benchmark securities with those of individual securities.

Appendix A: Proof of Proposition 1

Let us check that the pricing rule and the demand strategies given the statement of the theorem constitute an equilibrium. First, note that the equilibrium takes the following linear functional form:

$$p(\widetilde{q}) = A_0 + A_1\widetilde{q},\ x_m(\widetilde{s}_m) = B_m\widetilde{s}_m,\ \text{and } x_i(\widetilde{s}_i) = B_i\widetilde{s}_i,\ \text{for } i = 1, ..., N.$$

The quantities x_i demanded by each factor-i-informed agent j ($j = 1, ..., K_i$) must maximize

$$E\left[\left(\widetilde{v} - A_0 - A_1\left(\sum_{j\neq i}^{N} K_jB_j\widetilde{s}_j + (K_i-1)B_i\widetilde{s}_i + x_i + GB_m\widetilde{s}_m + \widetilde{z}\right)\right)^T x_i|\widetilde{s}_i\right].$$

The first order condition is

$$E\left[\widetilde{v}|\widetilde{s}_i\right] - A_0 - A_1(K_i-1)B_i\widetilde{s}_i = 2A_1x_i, \tag{A1}$$

where $E\left[\widetilde{v}|\widetilde{s}_i\right] = \overline{v} + d_i1_i\widetilde{s}_i$ and $d_i = \sigma_i^2/(\sigma_i^2 + \sigma_{\epsilon_i}^2)$. Equating coefficients, we obtain

$$\overline{v} = A_0 \tag{A2}$$

$$(K_i+1)A_1B_i = d_i1_i. \tag{A3}$$

The quantities x_m demanded by each systematic-factor-informed agent m ($m = 1, ..., G$) must maximize

$$E\left[\left(\widetilde{v} - A_0 - A_1\left(\sum_{j=1}^{N} K_jB_j\widetilde{s}_j + (G-1)B_m\widetilde{s}_m + x_m + \widetilde{z}\right)\right)^T x_m|\widetilde{s}_m\right].$$

The first order condition is

$$E\left[\widetilde{v}|\widetilde{s}_m\right] - A_0 - A_1(G-1)B_m\widetilde{s}_m = 2A_1x_m, \tag{A4}$$

where $E[\widetilde{v}|\widetilde{s}_m] = \overline{v} + d_m \widetilde{s}_m \beta$ and $d_m = \sigma_\gamma^2/(\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2)$. Equating coefficients, we obtain

$$\begin{aligned} \overline{v} &= A_0 & \text{(A5)} \\ (G+1)A_1 B_m &= d_m \beta. & \text{(A6)} \end{aligned}$$

The vector of order flows $\widetilde{m} = A_0 + \sum_{j=1}^{N} B_j \widetilde{s}_j + G B_m \widetilde{s}_m + \widetilde{z}$ is a multivariate normally (MN) distributed as $\widetilde{q} \sim MN(0, \sum_{j=1}^{N} K_j^2 B_j B_j^T (\sigma_j^2 + \sigma_{\epsilon_j}^2) + G^2 B_m B_m^T (\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2) + \Sigma_z)$, and $Cov(\widetilde{v}, \widetilde{q}) = G\beta B_m^T \sigma_\gamma^2 + \sum_{j=1}^{N} K_j 1_j B_j^T \sigma_j^2$. Therefore,

$$\begin{aligned} E[\widetilde{v}|\widetilde{q}] &= \overline{v} + \left(G\beta B_m^T \sigma_\gamma^2 + \sum_{j=1}^{N} K_j 1_j B_j^T \sigma_j^2 \right) & \text{(A7)} \\ & \left(G^2 B_m B_m^T (\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2) + \sum_{j=1}^{N} K_j^2 B_j B_J^T (\sigma_j^2 + \sigma_{\epsilon_j}^2) + \Sigma_z \right)^{-1} \widetilde{q}. \end{aligned}$$

Since $p(\widetilde{q}) = E[\widetilde{v}|\widetilde{q}]$ and we conjecture that $p(\widetilde{w}) = A_0 + A_1 \widetilde{q}$, we can equate the coefficients to obtain

$$\begin{aligned} A_0 &= \overline{v} & \text{(A8)} \\ A_1 &= \left(G\beta B_m^T \sigma_\gamma^2 + \sum_{j=1}^{N} K_j 1_j B_j^T \sigma_j^2 \right) & \text{(A9)} \\ & \left(G^2 B_m B_m^T (\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2) + \sum_{j=1}^{N} K_j^2 B_j B_J^T (\sigma_j^2 + \sigma_{\epsilon_j}^2) + \Sigma_z \right)^{-1}. \end{aligned}$$

Solving B_m and B_m^T in (A6), we obtain

$$\begin{aligned} B_m &= \frac{d_m}{G+1} A_1^{-1} \beta & \text{(A10)} \\ B_m^T &= \frac{d_m}{G+1} \beta^T A_1^{-1}. & \text{(A11)} \end{aligned}$$

Solving B_i and B_i^T in (A3), we obtain

$$
\begin{aligned}
B_i &= \frac{d_i}{K_i+1} A_1^{-1} 1_i && \text{(A12)} \\
B_i^T &= \frac{d_i}{K_i+1} 1_i^T A_1^{-1}. && \text{(A13)}
\end{aligned}
$$

Plugging (A10)-(A13) into (A9), we obtain

$$
\begin{aligned}
A_1 = & \left(\frac{G d_m \sigma_\gamma^2}{G+1} \beta\beta^T A_1^{-1} + \sum_{i=1}^{N} \frac{K_i d_i \sigma_i^2}{K_i+1} 1_i 1_i^T A_1^{-1} \right) && \text{(A14)} \\
& \left(\frac{G^2 d_m^2}{(G+1)^2} A_1^{-1} \beta\beta^T A_1^{-1} (\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2) + \sum_{i=1}^{N} \frac{K_i^2 d_i^2}{(K_i+1)^2} A_1^{-1} 1_i 1_i^T A_1^{-1} (\sigma_i^2 + \sigma_{\epsilon_i}^2) + \Sigma_z \right)^{-1},
\end{aligned}
$$

which, after some algebra and using the symmetry of A_1, simplifies to

$$
A_1 \Sigma_z = F A_1^{-1}, \tag{A15}
$$

where F is defined in the following expression:

$$
F \equiv \left(\frac{G}{(G+1)^2} \frac{\sigma_\gamma^4}{\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2} \beta\beta^T + \sum_{i=1}^{N} \frac{K_i}{(K_i+1)^2} \frac{\sigma_i^4}{\sigma_i^2 + \sigma_{\epsilon_i}^2} 1_i 1_i^T \right). \tag{A16}
$$

F is symmetric positive definite by Rayleigh's principle, and A_1 must be a symmetric positive definite solution to (A15). To solve for A_1, write (A15) as

$$
\Sigma_z^{1/2} A_1 \Sigma_z^{1/2} \Sigma_z^{1/2} A_1 \Sigma_z^{1/2} = \Sigma_z^{1/2} F \Sigma_z^{1/2}, \tag{A17}
$$

where $\Sigma_z^{1/2}$ is the unique symmetric positive definite square root of Σ_z. Since the LHS of (A17) is symmetric positive definite, $\Sigma_z^{1/2} A_1 \Sigma_z^{1/2}$ is its unique symmetric positive definite square root. Therefore,

$$
A_1 = \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2} = A, \tag{A18}
$$

where $M \equiv \Sigma_z^{1/2} F \Sigma_z^{1/2}$. Finally, we obtain

$$B_i = \frac{d_i}{K_i + 1} \Sigma_z^{-1/2} M^{-1/2} \Sigma_z^{-1/2} 1_i \tag{A19}$$

$$B_m = \frac{d_m}{G + 1} \Sigma_z^{-1/2} M^{-1/2} \Sigma_z^{-1/2} \beta. \tag{A20}$$

As the next step, we verify that both A_1 and B_1 are invertible. The equilibrium number of systematic-factor informed investors is the largest integer smaller than or equal to X, i.e., $G = \lfloor X \rfloor$, while X solves the zero-expected-profit condition in equation (A21). The number of security-i-specific informed traders is similarly defined as $K_i = \lfloor Y \rfloor$, while Y is determined by the zero-expected-profit condition in equation (A22).

$$\begin{aligned}
c &= E\left[\left(\widetilde{v} - A_0 - A_1\left(\sum_{j=1}^{N} K_j B_j \widetilde{s}_j + X B_m \widetilde{s}_m + \widetilde{z}\right)\right)^T x_m | \widetilde{s}_m\right] \\
&= \frac{d_m^2(\sigma_\gamma^2 + \sigma_{\epsilon_\gamma}^2)}{(X+1)^2} \beta^T A^{-1} \beta && \text{(A21)} \\
c &= E\left[\left(\widetilde{v} - A_0 - A_1\left(\sum_{j \neg i}^{N} K_j B_j \widetilde{s}_j + Y B_i \widetilde{s}_i + G B_m \widetilde{s}_m + \widetilde{z}\right)\right)^T x_i | \widetilde{s}_i\right] \\
&= \frac{d_i^2(\sigma_i^2 + \sigma_{\epsilon_i}^2)}{(Y+1)^2} 1_i^T A^{-1} 1_i. && \text{(A22)}
\end{aligned}$$

Appendix B: Proof of Corollary 1

First, denote the variance matrix of $\widetilde{v}$ as Σ_v, the covariance matrix $Cov(\widetilde{v}, \widetilde{q})$ as Σ_{vq}, and the covariance matrix $Cov(\widetilde{q}, \widetilde{v})$ as Σ_{qv}. Then we can express the conditional variance as

$$Var(\widetilde{v}|\widetilde{q}) = \Sigma_v - \Sigma_{vq}\Sigma_q^{-1}\Sigma_{qv} = \Sigma_v - \Sigma_{vq}\Sigma_q^{-1}A^{-1}A\Sigma_{qv}.$$

Using the fact that $A_1 = \Sigma_{vq}\Sigma_q^{-1}$ from Equation A7, we can rewrite the above as

$$Var(\widetilde{v}|\widetilde{q}) = \Sigma_v - \Sigma_{vq}A\Sigma_{vq}^{-1}A^{-1}A\Sigma_{qv} = \Sigma_v - (A\Sigma_{qv})^T.$$

From Equation A3, we obtain

$$A_1 B_i 1^T = \frac{K_i}{K_i + 1} d_i 1_i 1_i^T.$$

From Equation A6, we obtain

$$A_1 B_m \beta^T = \frac{G}{G+1} d_m \beta\beta^T.$$

We use $Cov(\widetilde{q}, \widetilde{v})$ given in the previous proof to obtain

$$\begin{aligned} A_1\Sigma_{qv} &= G\sigma_m^2 A_1 B_m \beta^T + \sum_{i=1}^{N} K_i \sigma_i^2 A_1 B_i 1_i^T \\ &= \frac{G}{G+1} d_m \sigma_\gamma^2 \beta\beta^T + \sum_{i=1}^{N} \frac{K_i}{K_i+1} d_i \sigma_i^2 1_i 1_i^T. \end{aligned} \tag{B1}$$

The results then follow.

Appendix C: Proofs of Proposition 2 and Corollary 2

Let us first write matrix A as a partitioned matrix,

$$A = \begin{bmatrix} A_{11} & A_{m1} \\ A_{1m} & A_{mm} \end{bmatrix}, \tag{C1}$$

where the benchmark security, γ, is denoted by m. The inverse of this partitioned matrix is

$$\begin{bmatrix} A_{11} & A_{m1} \\ A_{1m} & A_{mm} \end{bmatrix}^{-1} = \begin{bmatrix} A_{11}^{-1}(I + A_{m1}F_2A_{1m}A_{11}^{-1}) & -A_{11}^{-1}A_{m1}F_2 \\ -F_2A_{1m}A_{11}^{-1} & F_2 \end{bmatrix}, \tag{C2}$$

where $F_2 \equiv \left(A_{mm} - A_{1m}A_{11}^{-1}A_{m1}\right)^{-1}$ and $F_2 > 0$ by the property of A (Equation A18).

We prove the result by contradiction. The steps are as follows: First, we assume toward contradiction that the equilibrium for N assets stays the same (i.e., G and K are the same as in Proposition 1 and A_{11} is the same as A in Proposition 1) after the benchmark security, γ, is introduced. Second, we show that in this case, both the expected revenue of a systematic-factor informed investor and the expected revenue of a security-specific informed investor exceed c, the fixed cost of acquiring the respective signal, which violates the zero-expected-profit condition of informed investors in the equilibrium concept (Definition 1). Hence, we conclude that the equilibrium solution (G, K, A_{11}) when the benchmark security is absent no longer constitutes an equilibrium after the benchmark security is introduced. Since expected revenues of informed investors are decreasing in G and K (by Proposition 1, Corollary 1, and Equations A22 and A21), for the market to reach an equilibrium, G and K have to be higher, which means that more investors will acquire systematic or security-specific signals as expected revenues are greater than the fixed cost.

Let us first suppose that the equilibrium for N assets stays the same (which is, G, K, A_{11} stay the same) after the benchmark security, γ, is introduced. By symmetry of A and Equation A22, we obtain the expected revenue for security-i-specific informed investors.

$$
\begin{aligned}
& \frac{d_i^2(\sigma_i^2+\sigma_{\epsilon_i}^2)}{(K_i+1)^2}[1_i^T,0]A^{-1}\left[\begin{array}{c} 1_i \\ 0 \end{array}\right] \\
= \; & \frac{d_i^2(\sigma_i^2+\sigma_{\epsilon_i}^2)}{(K_i+1)^2}[1_i^T,0]\left[\begin{array}{cc} A_{11}^{-1}(I+A_{m1}F_2A_{1m}A_{11}^{-1}) & -A_{11}^{-1}A_{m1}F_2 \\ -F_2A_{1m}A_{11}^{-1} & F_2 \end{array}\right]\left[\begin{array}{c} 1_i \\ 0 \end{array}\right] \\
= \; & \frac{d_i^2(\sigma_i^2+\sigma_{\epsilon_i}^2)}{(K_i+1)^2}1_i^TA_{11}^{-1}1_i+\frac{d_i^2(\sigma_i^2+\sigma_{\epsilon_i}^2)}{(K_i+1)^2}(1_i^TA_{11}^{-1}A_{m1}A_{1m}A_{11}^{-1}1_i)F_2 \\
= \; & c+\underbrace{\frac{d_i^2(\sigma_i^2+\sigma_{\epsilon_i}^2)}{(K_i+1)^2}(1_i^TA_{11}^{-1}A_{m1}A_{1m}A_{11}^{-1}1_i)F_2}_{\text{Extra Profit}}>c
\end{aligned}
$$

Obviously, the extra profit is larger for securities with either larger idiosyncratic risks, σ_i^2, or sharper informed signals (that is, larger d_is), or large systematic factor loadings (that is, β).

By symmetry of A and Equation A21, we obtain the expected revenue for systematic-factor informed investors.

$$
\begin{aligned}
& \frac{d_m^2(\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2)}{(G+1)^2}[\beta^T,1]A^{-1}\left[\begin{array}{c} \beta \\ 1 \end{array}\right] \\
= \; & \frac{d_m^2(\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2)}{(G+1)^2}[\beta^T,1]\left[\begin{array}{cc} A_{11}^{-1}(I+A_{m1}F_2A_{1m}A_{11}^{-1}) & -A_{11}^{-1}A_{m1}F_2 \\ -F_2A_{1m}A_{11}^{-1} & F_2 \end{array}\right]\left[\begin{array}{c} \beta \\ 1 \end{array}\right] \\
= \; & \frac{d_m^2(\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2)}{(G+1)^2}\beta^TA_{11}^{-1}\beta+\frac{d_m^2(\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2)}{(G+1)^2}(A_{1m}A_{11}^{-1}\beta-1)(\beta^TA_{11}^{-1}A_{m1}-1)F_2 \\
= \; & c+\underbrace{\frac{d_m^2(\sigma_\gamma^2+\sigma_{\epsilon_\gamma}^2)}{(G+1)^2}(A_{1m}A_{11}^{-1}\beta-1)^2F_2}_{\text{Extra Profit}}>c
\end{aligned}
$$

Obviously, the extra profit is larger with either a larger systematic risk (that is, σ_γ^2), or a sharper informed signal, (that is, a larger d_m), or a more liquid benchmark security (that is, a larger $\sigma_{z_m}^2$).

These results violate the zero-expected-profit condition of informed investors in the equilibrium concept (Definition 1). Since expected revenues of informed investors are decreasing in G and K (by Proposition 1, Corollary 1, and Equations A22 and A21), for the market to reach an equilibrium after benchmark introduction, G and K have to be higher, which means that more investors will acquire systematic or security-specific signals as the expected revenue is greater than the fixed cost.

In summary, the logic of the proof is as follows: Equilibrium is defined by a function $F(G, K, A(G, K), \phi) = 0$, where $\phi \in (0, 1)$ indicates whether there is a benchmark security or not. Fixing the equilibrium for $\phi = 0$, we note that $F(G, K, A(G, K), 1) > 0$. Then, since all components of F diminish in both G and K, we infer that both G and K must increase.

To prove Corollary 2, we apply Proposition 2 to Proposition 1 and Corollary 1. The results then follow.

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Table 1: Demand Schedules of Informed Investors in Absence of Benchmark Securities

	Systematic-factor Informed	Security-1-specific Informed	Security-2-specific Informed
Demand for Security 1	$1.9700\widetilde{s}_m$	$1.8902\widetilde{s}_1$	$-0.3461\widetilde{s}_2$
Demand for Security 2	$4.8537\widetilde{s}_m$	$-0.1923\widetilde{s}_1$	$2.0568\widetilde{s}_2$

Table 2: Demand Schedules of Informed Investors after Benchmark Introduction

	Systematic-factor Informed	Security-1-specific Informed	Security-2-specific Informed
Demand for Security 1	$0.6313\widetilde{s}_m$	$1.8409\widetilde{s}_1$	$-0.0901\widetilde{s}_2$
Demand for Security 2	$1.7879\widetilde{s}_m$	$-0.0721\widetilde{s}_1$	$1.8753\widetilde{s}_2$
Demand for Benchmark	$3.3630\widetilde{s}_m$	$-0.3073\widetilde{s}_1$	$-0.9800\widetilde{s}_2$

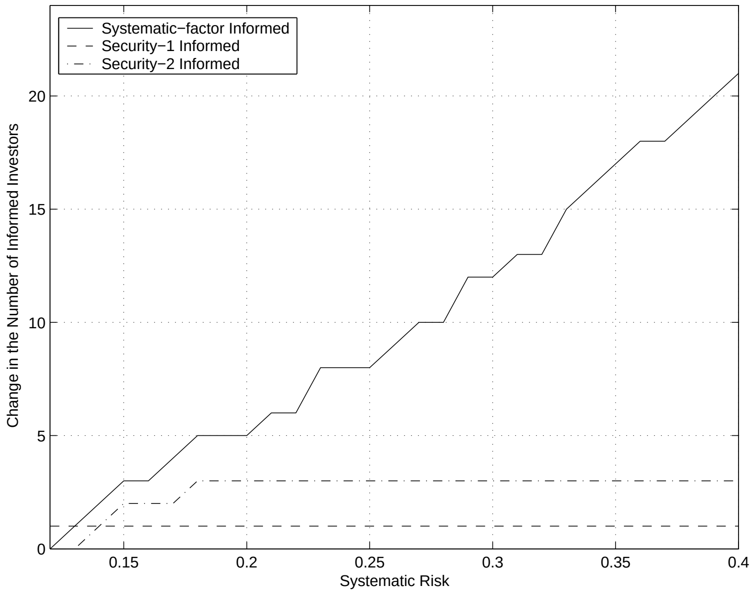

Figure 1: **Systematic risk and the change in the number of informed investors after benchmark introduction.**
For varying levels of systematic risk, σ_γ, the solid line in the figure represents the amount of change in the number of systematic-factor informed investors after benchmark introduction. The dash-dash line represents the amount of change in the number of security-1-specific informed investors. The dashed-dotted line represents the amount of change in the number of security-2-specific informed investors.

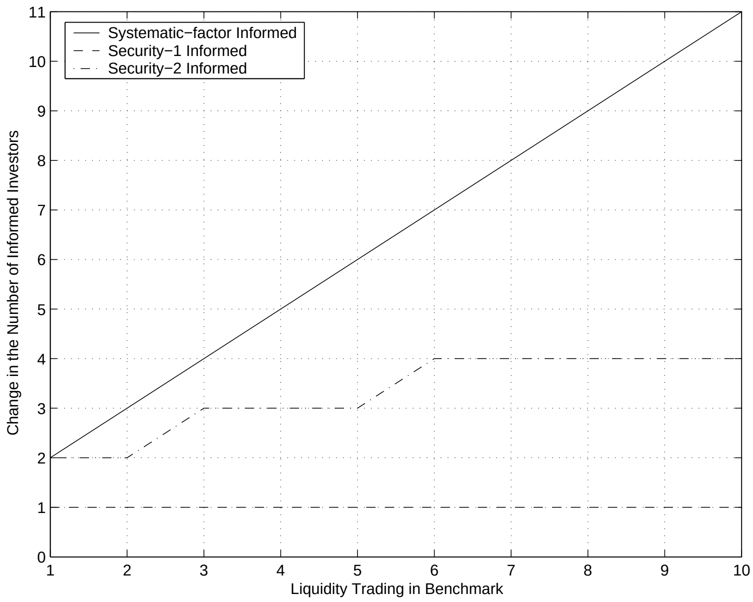

Figure 2: **Benchmark liquidity and the change in the number of informed investors.**
For varying levels of benchmark liquidity, σ_{z_m}, the solid line in the figure represents the amount of change in the number of systematic-factor informed investors after benchmark introduction. The dash-dash line represents the amount of change in the number of security-1-specific informed investors. The dashed-dotted line represents the amount of change in the number of security-2-specific informed investors.

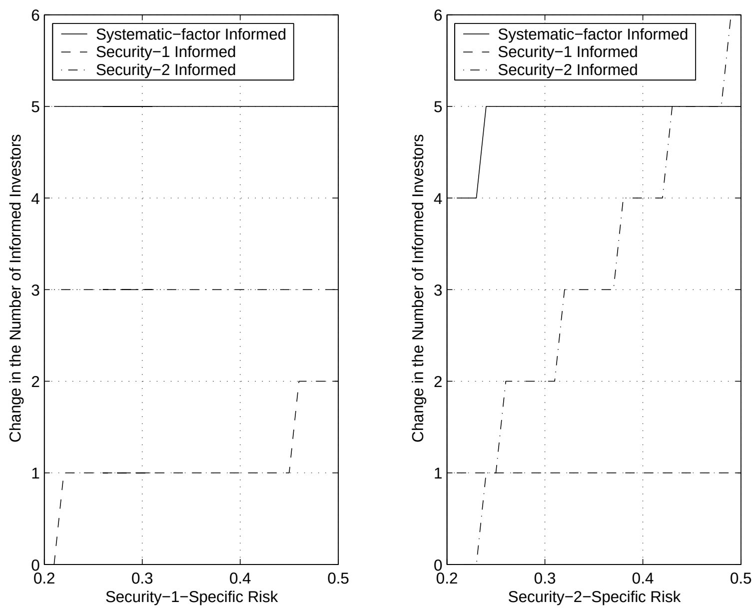

Figure 3: **Security-specific risk and the change in the number of informed investors.**
In the left graph, the solid line represents the amount of change in the number of systematic-factor informed investors after benchmark introduction; the dash-dash line represents the amount of change in the number of security-1-specific informed investors; and the dashed-dotted line represents the amount of change in the number of security-2-specific informed investors, for varying levels of security-1-specific risk, σ_1. The right graph shows the same relationships for varying levels of security-2-specific risk, σ_2.