

On the Fragility of DeFi Lending

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Abstract

We develop a dynamic model of DeFi lending incorporating the following key features: 1) borrowing and lending are decentralized, anonymous, overcollateralized, and backed by the market value of crypto assets where contract terms are pre-specified and rigid; and 2) information friction exists between borrowers and lenders. We identify a price-liquidity feedback: the market outcome in any given period depends on agents' expectations about lending activities in future periods, higher future price expectation leading to more lending and higher price today. Due to the rigidity in smart contracts, this feedback leads to multiple self-fulfilling equilibria where DeFi lending and asset prices co-move according to market sentiment. Price impacts from temporary firesales can be permanent. We show that flexible updates of smart contracts can restore equilibrium uniqueness. This highlights the difficulty of achieving stability and efficiency in a decentralized environment without a liquidity backstop.

Keywords: Decentralized Finance; DeFi, Smart Contracts; Dynamic Price Feedback; Financial Fragility; Adverse Selection; DeFi trilemma, Stability, Efficiency, and Decentralization Tradeoff.

JEL classification: G10, G01

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1 Introduction

Decentralized finance (DeFi) is an umbrella term for a variety of financial service protocols and applications on blockchain. They are anonymous permission-less financial arrangements implemented via smart contracts – immutable, deterministic computer programs – on a blockchain that aim to replace traditional financial intermediaries (TradFi). With the rise of DeFi, crypto assets evolve from being solely payment instruments to becoming financial instruments as they are traded in exchanges and can also be used as collaterals in financial transactions. As DeFi grows in scale and scope and becomes more connected to the real economy, its vulnerabilities might undermine both crypto and formal financial sector stability (Aramonte, Huang, and Schrimpf (2021))¹ Yet, formal economic analysis on the interaction of crypto asset prices and its use as collateral in raising funding liquidity is still very limited. In this paper, we examine the feedback mechanism of DeFi lending protocols and exchanges and implications for the instability in DeFi system.

Empirically, we do observe that DeFi lending is indeed much more volatile relative to traditional lending and exhibits a strong “pro-cyclicality” – the comovement between crypto prices and lending activities, as shown in Figure 1.² Recently, Lehar and Parlour (2022a) show that there are permanent negative price impacts when collaterals are sold in the decentralized exchanges via the liquidation mechanism in the DeFi lending protocol, highlighting the fire-sale channel as a potential source of instability in the DeFi system. Their finding of a permanent price impact is puzzling since amplification effects from fire-sales should be transitory especially considering the fast moving capital in DeFi.³

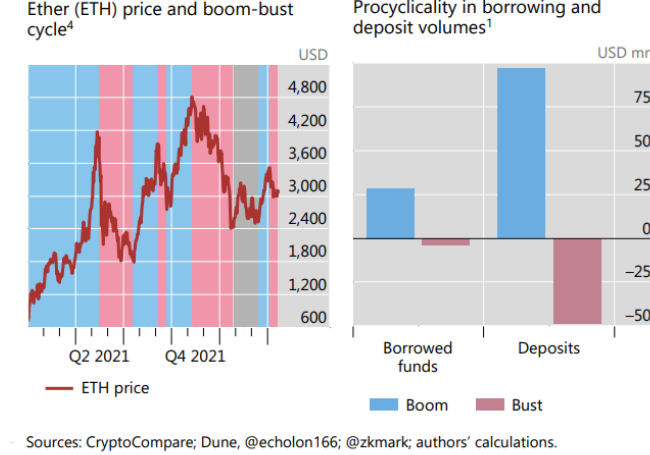
To reconcile these empirical facts and to capture dynamic feedback of crypto asset prices and lending protocols, we develop a dynamic model of DeFi lending building upon the canonical framework of Lagos and Wright (2005) adding a lending market, similar to Ozdenoren, Yuan, and Zhang (2021). Our model incorporates special DeFi institutional features such as rigid contract terms and severe information

¹According to data aggregator DeFiLlama, the total value locked (TVL) of DeFi has reached 230 billion U.S. dollars as of April 2022, up from less than one billion two years ago. The collapse of the cryptocurrency exchange FTX – a unregulated centralized blockchain trading firm, has further pushed investors away from centralized blockchain platforms towards self-custodial DeFi platforms. For example, it is reported that Uniswap, one of the largest decentralized exchanges registered a significant spike in trading volume on November 11 2022, the day FTX filed for bankruptcy. Its subsequent increase in trading volume is much higher than many centralized exchanges (<https://cointelegraph.com/news/after-ftx-defi-can-go-mainstream-if-it-overcomes-its-flaws>).

²For instance, the coefficients of variation for the total values of Aave v2 loans and deposits are respectively 73 and 65 in 2021. The corresponding statistics for the US demand deposits and C&I loans are respectively 10.4 and 2.7.

³The DeFi literature has documented active arbitrage activities cross-chains or on-chains enabled by Miner Extractable Value (MEV) bots or flash loans, indicating that DeFi capitals are fast moving (Qin, Zhou, and Gervais (2022)).

Figure 1: Crypto price boom-bust cycle and pro-cyclicality in DeFi lending



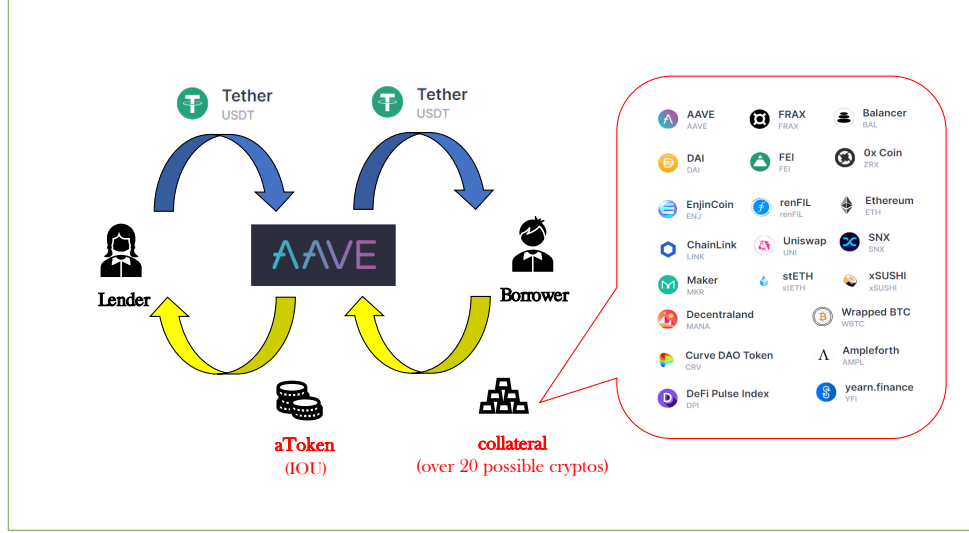
Source: Aramonte et al. (2022)

frictions. Unlike Ozdenoren, Yuan, and Zhang (2021), we show the dynamic feedback of crypto asset price and lending activities leads to multiple equilibria and volatility, driven by sentiments. These multiple equilibria can be viewed as runs on DeFi lending platforms. Unlike the run in Diamond and Dybvig (1983), the type of run we identify results from the dynamic coordination with the future agents and are driven by self-fulfilling beliefs. We also extend the basic model to incorporate fire sales and show that the model can rationalize the permanent price impact of fire sales.

Next, we turn to the construction of the model elements, each of which corresponds to a special institutional feature of DeFi lending protocols. We start with a stylized description as shown in Figure 2. Anonymous lenders deposit their crypto assets (e.g. Tether) via a lending smart contract to the lending pool of the corresponding crypto asset under a lending protocol (e.g., AAVE). In return, they receive a deposit receipt (IOU) in the form of an AAVE token (e.g., aTether) which accumulates interest continuously. Since borrowers are anonymous, credit checks and other borrower-specific evaluations are not feasible. Anonymous borrowers, however, can borrow the crypto asset (e.g, Tether) from the Tether liquidity pool by pledging *any* crypto collateral accepted by the protocol via a borrowing smart contract. Collateral assets have to be tokenized and compatible with smart contracts. The borrowing and lending interactions in a lending protocol are peer to pool, i.e. both borrowers and lenders interact with the liquidity pool, rather than peer to peer, preserving anonymity of both lenders and borrowers.

A special feature of this arrangement is that the collateral composition of a liquidity pool is not readily observable. DeFi borrowers can choose to pledge any acceptable collateral assets while lenders

Figure 2: Stylized Structure of a DeFi Lending Protocol



cannot control or easily monitor the composition of the underlying collateral pool. This means that borrowers are better informed about the collateral quality than the lenders, giving rise to information friction between borrowers and lenders. In fact, Heimbach and Huang (2023) show that borrowers with high leverage are more likely to tilt towards pledging volatile collateral when their debt positions are about to be liquidated.⁴

Another special feature of DeFi lending is the decentralized nature of contractual arrangements. There is no centralized trusted third party. In DeFi applications, smart contracts are used to replace human judgment and all terms need to be pre-programmed and can only be contingent on a small set of quantifiable real-time information. As a result, DeFi lending typically involves a linear, non-recourse

⁴Borrowers can also have an information advantage relative to the lending protocol when smart contracts rely on an inaccurate price oracle. The price feed of an oracle has to trade off latency and accuracy. For example, the reference implementation to Uniswap's oracle averages prices over a twenty-four hour window, meaning that short-lived shocks to the price are largely ignored and even a large and sustained shock (e.g. 20% for an hour) will move the oracle price by less than 1%. When the price falls because of falling fundamentals, the oracle price will lag the "true" price of the asset significantly. Since crypto is a volatile asset class, with frequent intraday spikes and drops, informed borrowers can take out large loans backed by a crypto asset with a sudden inflated price from a delayed oracle and default on loan obligations, leaving the lending protocol with a collateral whose value is far below the face value of the loan. In Appendix D, we discuss some exploit incidents during the Terra collapse in May 2022 and other price exploits due to inflated on-chain collateral prices.

debt contract, featuring over-collateralization as the only risk control. Collateral assets are valued based on price feeds provided by an oracle which can be either on-chain or off-chain. The rules for setting key parameters (e.g., interest rate formula and haircuts) are pre-programmed in the smart contracts. Since the protocol is governed by holders of governance tokens in a decentralized fashion, even a slight modification of the contract can involve a lengthy decision process among the governance token holders. Consequently, terms of smart contracts are modified only occasionally and appear inflexible and rigid.⁵

Moreover, the DeFi ecosystem is closely inter-connected, inter-operable and arbitrage capital is fast moving. Based on cryptography, crypto assets are created as payment tokens for secure settlements. With the introduction of DeFi lending protocols, they also serve as collaterals where the collateral values are determined in the decentralized exchange protocols. The movement of tokens across these on-chain protocols are fast and seamless, in contrast to slow moving capital in the traditional financial system. For example, in a study by M. Griffoli et al. (2023), it is demonstrated that the lending volume of a crypto asset within DeFi lending protocols exhibits a positive correlation with the crypto asset’s dominance status in DeFi exchange protocols.

Our dynamic model of DeFi lending incorporating these special features. Borrowing is decentralized, over-collateralized, backed by the market value of various crypto assets, governed by a linear borrowing contract, while terms of the borrowing contract (such as the rule for haircuts) are pre-specified and rigid. Borrowers are better informed about the fundamental values of the crypto collaterals than lenders. Additionally, lending activities in the DeFi lending protocol and crypto asset prices in the decentralized exchanges are mutually dependent.⁶

Specifically, in our model, borrowers would like to borrow funds (e.g., stablecoins) from lenders through a lending platform, using their crypto asset holdings as collateral. There is asymmetric information about the asset’s quality between borrowers and lenders.⁷ Since lenders cannot control the

⁵To amend or upgrade smart contracts, proxy contracts and implementation contracts are deployed to swap the old for the new smart contract. However, DeFi protocols are typically controlled using on-chain governance, where token holders vote to modify different parameters of the smart contracts, resulting in only occasional risk parameter changes. We find that, for example, AAVE protocol only had 13 risk parameter changes in its first two years of operation. There are calls for technology developments to make decentralized governance semi-automatic and data driven. However, up till now choosing these parameters has been a manual process (See Xu (2022)). DeFi lending is typically short-term since all lending and borrowing can be terminated at any minute.

⁶In fact, this information friction does not need to be about the quality of the asset. In the appendix, we show that an alternative information friction, unobservable private valuation, leads to the same price-liquidity feedback effects and multiple equilibria in outcomes.

⁷Asymmetric information arises because borrowers are privately informed that their crypto assets are low or high quality. This quality could be about borrowers’ private information about interest payoff and/or survival probability in the future.

collateral mix directly, this information friction results in the classic lemons problem (Akerlof (1970)) and can severely reduce the gains from trade by driving out the high quality borrowers. The DeFi platform imposes a haircut on the crypto asset which decreases the information sensitivity of the loan and mitigates the adverse selection problem.

In this setting, we find that rigidity in smart contracts leads to multiple self-fulfilling equilibria giving rise to the fragility of DeFi lending. There is a feedback loop between crypto prices and the amount of DeFi lending in this dynamic model. Optimistic expectation about future crypto asset prices improves DeFi lending and supports higher crypto prices today while pessimistic expectation about future crypto asset prices worsens DeFi lending and justifies lower crypto prices today. These self-fulfilling beliefs allow for the existence of “sentiment” equilibria: agents dynamically “coordinate” their beliefs based on sunspots which generate fluctuations in crypto asset prices and DeFi lending volume. These findings align with the pro-cyclical pattern shown in Figure 1. In our model, the possibility of a fire sale depresses price due to the direct price impact from slow moving capital. However, even if the impact of fire sale is temporary, we show that this negative price impact can become permanent due to agents dynamically “coordinate” their pessimistic beliefs in an environment with heightened adverse selection. Therefore, our model rationalizes not only the empirical findings of permanent price impact in Lehar and Parlour (2022a) and outline conditions giving rise to such occasions.⁸

It is widely acknowledged that crypto-assets can be inherently fragile due to their money-like nature and susceptibility to the formation of rational bubbles. In our model, however, we identify a distinct and unique source of fragility within the crypto asset ecosystem, stemming from the introduction of lending services collateralized by prices of crypto tokens. In contrast to traditional bank runs, the multiple equilibria in our model result from (mis-)coordination between present and future market participants. This means that sudden stops, such as a transition from a positive to a negative sentiment equilibrium, are possible, driven by abrupt changes in self-fulfilling beliefs. Importantly, this new form of fragility exists even in the absence of fire-sale externalities across exchange and lending protocols.

An interesting query arises: Is the fragility identified in our model exclusive to DeFi, or does it extend to other collateralized borrowing markets, like Treasury repo and the housing/mortgage sector? The Treasury repo market has considerable liquidity, with flexible haircut/leverage adjustments throughout the day to reflect the prevailing information and risk environment. This market, unlike DeFi, isn’t bound by contract rigidity, making the occurrence of multiple equilibria unlikely.

It could be micro-founded based on borrowers’ private valuation of the crypto assets’ future convenience yields.

⁸In our model, the collateralized lending market is competitive and lenders break even. This means that there are liquidations but the ratios of non-performing loan are low across equilibrium outcomes.

In the mortgage/housing market, while information friction exists and borrowing contracts are generally rigid due to regulatory constraints, an additional friction is at play -- slow-moving capital. The high transaction costs for arbitraging housing prices result in prolonged impacts from fire sales, potentially causing fragility in this market segment.

However, in DeFi, capital moves swiftly due to reduced arbitrage costs facilitated by flash loans and protocol interoperability. Consequently, when fire-sales occur in isolation within the DeFi space, their price impact tends to be transient. Our model indicates that only when paired with information friction and contract rigidity does the price impact of fire-sales acquire a lasting component. Thus, the self-fulfilling beliefs and dynamic DeFi runs elucidated by our model constitute a more relevant economic mechanism underlying DeFi fragility.

We further show that the decentralized nature of the contractual arrangements leads to the contract rigidity and underlies the source of fragility identified by our model. Under a flexible smart contract where the haircut can be updated nonlinearly in response to changes in market prices and both soft and hard information, it is possible to support a unique equilibrium with high and stable lending volume and asset price. However, these contracts are costly and difficult to implement in the decentralized environment, pinpointing the inherent fragility of the DeFi lending protocols. To improve stability, it is necessary to give up certain degree of decentralization in DeFi. For example, some platforms need to re-introduce human actors to provide real-time risk management – an arrangement that forces the decentralized protocol to rely on a trusted third party. These findings could potentially guide policymakers and regulators who are concerned about the financial stability implications of DeFi in designing safe-guarding rules and regulations (FSB 2022; IOSCO 2022).⁹ While decentralization in participation and governance is fundamental to DeFi’s exciting prospects in democratizing finance, our findings also highlight that decentralization also imposes limitations on DeFi’s size, efficiency, complexity and flexibility, questioning its ability to significantly challenge the TradFi.

The rest of the paper is organized as follows. In Section 2, we provide a brief description of features and frictions of lending protocol using Aave as an example to motivate the model assumptions. We describe the model setup in Section 3 and derive the equilibrium lending market in Section 4. In Section 5, we establish the inherent fragility of DeFi lending, discuss how flexible contract design can improve stability and efficiency, and study the impact of temporary firesale. Section 6 provides related literature, additional discussion and concludes. In the Online Appendix, we report some evidence to support the

⁹URLs of reports: https://g20.org/wp-content/uploads/2022/02/FSB-Report-on-Assessment-of-Risks-to-Financial-Stability-from-Crypto-assets_.pdf and <https://www.iosco.org/library/pubdocs/pdf/IOSCOPD699.pdf>

case that our model can be useful for understanding the relationship between DeFi lending, crypto prices and market sentiment.

2 Protocol For Loanable Funds: Features and Frictions

To motivate our model setup, we now describe some key features and frictions of DeFi lending protocols based on Aave, the largest DeFi lending protocol.

Key players. The Aave eco-system consists of various types of participants. Depositors can deposit a crypto asset into the corresponding pool of the Aave protocol and collect interest over time. Borrowers can borrow these funds from the pool by pledging any acceptable crypto assets as collateral to back the borrow position. A borrower repays the loan in the same asset borrowed. There is no fixed time period to pay back the loan. Partial or full repayments can be made anytime. As long as the position is safe, the loan can continue for an undefined period. However, as time passes, the accrued interest of an unpaid loan will grow, which might result in the deposited assets becoming more likely to be liquidated by liquidators. In the eco-system, there are also AAVE token holders. Like “shareholders”, they act as residual claimants and vote when necessary to modify the protocol. The daily operations are governed by smart contracts stored on a blockchain that run when predetermined conditions are met.

Loan rate and liquidation threshold. The loan and the deposit rates are set based on the current supply and demand in the pool according to formulas specified in the smart contracts. In particular, as the utilization rate of the deposits in a pool goes up (i.e., a larger fraction of deposits are borrowed), both rates will rise in a deterministic fashion. The Loan to Value (LTV) ratio defines the maximum amount that can be borrowed with a specific collateral. For example, at $LTV = .75$, for every 1 ETH worth of collateral, borrowers will be able to borrow 0.75 ETH worth of funds. The protocol also defines a liquidation threshold, called the health factor. When the health factor is below 1, a loan is considered undercollateralized and can be liquidated by collateral liquidators. This collateral liquidation mechanism of LTV and health ratio is akin to initial margin and maintenance margin in the traditional margin borrowing. The part of difference of the maintenance margin is given to liquidators as incentive to activate the dormant liquidation (smart) contract for a levered position whenever its health factor falls below 1.¹⁰ The collateral assets are valued based on price feed provided by Chainlink’s decentralized oracles.

Risky collateral. Aave currently accepts over 20 different crypto assets as collateral including

¹⁰In practice, small loans are often not liquidated when the high gas fees make a liquidation unprofitable.

WETH, WBTC, USDC and UNI. Most non-stablecoin collateral assets have very volatile market value. As shown in table 3 in the Appendix, the prices of stablecoins such as USDC and DAI (top panel), are not so volatile and they are typically loaned out by lenders. Other crypto assets, which are used as collaterals to back the borrowings, are extremely volatile relative to collateral assets commonly used in traditional finance (bottom panel). For example, ETH, which accounts for about 50% of use non-stablecoin deposits in Aave, has a daily volatility of 5.69%. The maximum daily price drop was over 26% during the sample period. The most volatile one is CRV, the governance token for the decentralized exchange and automated market maker protocol Curve DAO. For CRV the maximum price change within a day was over 40%. For risk management purposes, Aave has imposed very high haircuts on these crypto assets. For example, the haircuts for YFI and SNX are respectively 60% and 85%.¹¹

Collateral pool. Loans are backed by a pool of collateral assets. While the borrower can pledge any one of the acceptable assets as a collateral, the lenders cannot control or easily monitor the quality of the underlying collateral pool. As a result, DeFi lending is subject to asymmetric information: borrowers can freely modify the underlying collateral mix without notifying the lenders. Naturally, borrowers and lenders have asymmetric incentives to spend effort acquiring information about the collateral pledged (e.g., monitor new information, conduct data analytics).

Pre-specified loan terms. Aave lending pools follow pre-specified rules to set loan rates and haircuts. As a smart contract is isolated from the outside world, it cannot be contingent on all available real-time information. While asset prices are periodically queried from an oracle (Chainlink), the loan terms do not depend on other soft information (e.g., regulatory changes, projections, statements of future plans, rumors, market commentary) as they cannot be readily quantified and fed into the contract.

Decentralized governance. Like many other DeFi protocols, Aave has released the governance to the user community by setting up a decentralized autonomous organization or DAO. Holders of the AAVE token can vote on matters such as adjustments of interest rate functions, addition or removal of assets, and modification of risk parameters such as margin requirements. To implement such changes to the protocol, token holders need to make proposals, discuss with the community, and obtain enough support in a vote. This process helps protect the system against censorship and collusion. However,

¹¹More recently, Aave has started to accept real world asset (RWA) as collateral, allowing businesses to finance their tokenized real estate bridge loans, trade receivables, cargo & freight forwarding invoices, branded inventory financing, and revenue based financing (<https://medium.com/centrifuge/rwa-market-the-aave-market-for-real-world-assets-goes-live-48976b984dde>). Aave also plans to accept non-fungible tokens (NFTs) as collateral (<https://twitter.com/StaniKulechov/status/1400638828264710144>). Being non-standardized, NFTs are likely to be subject to even high informational frictions. Popular DeFi lending platforms for NFTs include NFTfi, Arcade, and Nexo.

decentralized governance by a large group of token holders is both time and resource costly. Hence it is not possible to update the protocol or the smart contract terms very frequently. As a result, relative to a centralized organization, a DeFi protocol may be slower to make necessary adjustments to respond to certain unexpected external changes (e.g., changes in market sentiments) in a timely manner. This problem is well documented. For instance, a risk assessment report of Aave in April 2021 pointed out that “As market conditions change, the optimal parameters and suggestions will need to dynamically shift as well. Our results suggest that monitoring and adjustment of protocol parameters is crucial for reducing risk to lenders and slashing in the safety module.”¹² In practice, since the setup of Aave v2 in late 2020 until May 2022, the risk parameters have been updated only 13 times (see Table 2 in the Appendix for some of the key changes). All were conducted after Aave DAO elected Gauntlet, a centralized entity, to provide dynamic risk parameters recommendations.

These features of Aave are common among the DeFi lending protocols, highlighting three key frictions in the DeFi lending. First, there is lack of commitment from DeFi borrowers and hence the borrowings have to be (over-)collateralized. Second, there is potentially information asymmetry between DeFi borrowers and lenders because lenders cannot control the collateral mix in the collateral pool. Third, DeFi contracts are rigid and based on quantifiable information on blockchain.

3 The Model Setup

The economy is set in discrete time and lasts forever.¹³ There are many potential borrowers with identical preferences. There is a fixed set of crypto assets. Each borrower can hold at most one unit of the asset.¹⁴ There are also many potential lenders who lend funds to a liquidity pool each period. The lending protocol intermediates DeFi lending via smart contracts. All agents can consume/produce a numeraire good at the end of each period with a constant per unit utility/cost.

Gains from Trade and the Lending Platform A borrower needs funding that can be provided by lenders. There are gains from trade as the value per-unit of funding to a borrower is $z > 1$, while the per-unit cost of providing funding by lenders is normalized to one. In the DeFi setting, borrowers are anonymous and cannot commit to paying their debt. To overcome the commitment problem, loans

¹²Source: <https://gauntlet.network/reports/aave>

¹³In reality, interest payment on the borrowing in the lending protocols is continuously compounded and can be terminated at any time. Therefore, we can interpret that each time period in our model is relatively short.

¹⁴One might interpret this asset as a portfolio of crypto assets used as collateral to borrow from a DeFi liquidity pool.

must be backed by collateral. DeFi lending relies on a smart contract to implement a collateralized loan. The DeFi intermediary determines the terms of the smart contract. Collateral is locked into the smart contract and released to the borrower if and only if a repayment is received.¹⁵

In DeFi lending protocols such as Aave, borrowers predominantly borrow stablecoins such as USDT and USDC using risky crypto assets as collaterals (e.g. ETH, BTC, YFI, YNX). As stablecoins are regarded as medium of exchange and unit of account in DeFi, they are used to fund various transactions or to increase leverage in crypto investment. We can interpret z as the value accrued to borrowers when using stablecoins borrowed from lenders for speculative or productive purposes.¹⁶

Crypto Asset’s Properties and Information Environment We assume that all crypto assets are ex-ante identical and pay random dividend $\tilde{\delta}$ at each period and survive to the next period with random probability \tilde{s} .¹⁷

If crypto assets do not pay dividend, crypto assets are purely speculative assets. We do not take such an extreme position, mainly because we think of “dividend” more broadly. For example, crypto asset’s dividend value may come from its role in enabling payments. Crypto assets, especially utility tokens of proprietary blockchains, act as mediums of exchanges for protocols developed on respective chains. Since search and matching technologies vary across chains, these crypto assets yield different convenience yields. In an on-line appendix, we micro found random dividend $\tilde{\delta}$ by showing that crypto assets generate a form of dividend endogenously arising from convenience yields because these crypto assets can be exchanged for consumption goods in future periods in a framework based on Lagos and Wright (2005). A crypto asset with a more efficient matching technology has a larger probability to be exchanged for consumption goods, and hence yields a larger utility gain from this convenience. There are other private benefits that might accrue from holding a crypto asset such as governance rights.

Additionally, crypto assets might generate pecuniary payoffs. For example, some protocols offer staking returns to asset holders. Certain assets are in high demand and able to generate rental income. These non-pecuniary and pecuniary benefits are random for a host of reasons which we capture through the randomly evolving quality of the asset.

We assume that the beginning of a period, each asset receives an iid quality shock. Specifically, with

¹⁵Chiu, Kahn, and Koepl (2022) study how a smart contract helps mitigate commitment problems in decentralized lending.

¹⁶It is straight-forward to introduce governance tokens issued by the intermediary - the lending platform. Governance token holders then provide insurance to lenders by acting as residual claimants. Given risk neutrality, the equilibrium outcome remains the same.

¹⁷We use $\tilde{\cdot}$ to denote random variables.

probability $1 - \lambda$, the quality of an asset is high (H) and probability λ it is low (L). The distribution of $(\tilde{\delta}, \tilde{s})$ is F_Q if asset quality is $Q \in \{H, L\}$. We assume F_H first-order stochastically dominates F_L and denote expectation with respect to F_Q with \mathbb{E}_Q .

To simplify the analysis we make further assumptions on the distributions. We assume that a high-quality asset pays dividend $\delta > 0$ at the end of the period and survives to the next period with probability $s = 1$. A low-quality asset does not pay any dividends today ($\delta = 0$) and it survives to the next period with probability $s \in [0, 1]$. Here, $1 - s$ captures whether the quality shock has persistent effects on the dividend flow of the crypto asset, also reflecting the volatility of the survival probability of a crypto asset.

We assume that the crypto asset pays positive dividend in some states (that is, when it is high quality). The main role of this assumption is to eliminate non-monetary equilibrium (the equilibrium where crypto asset is worthless). In our model the asset has collateral service and can have positive price even if it does not pay any dividend. However, there can also be an equilibrium where the asset is worthless because current lenders believe future lenders will not accept the asset the asset as collateral. Positive dividend eliminates the latter equilibrium.

Next, we model asymmetric information between borrowers and lenders. The source of private information could be multitude. As we motivated earlier in the introduction section, the delay of Oracle in updating asset value might give collateral asset holders an information advantage.¹⁸

Owners might also have a better information about future convenience benefits generated by the crypto assets. Asymmetric information might be about private valuation of the asset rather than the dividend payoff.¹⁹

Specifically, we assume that at the beginning of each period, the borrower of a crypto asset privately learns the asset's quality (i.e., whether it is high or low). After observing the quality shock, the borrower decides whether and how much to borrow from the platform. The borrower then receives the private return from the loan (which is z times the loan size), and observes the realization of $(\tilde{\delta}, \tilde{s})$. Given the information, the borrower decides whether to repay the loan or default. The asset's quality and the state $(\tilde{\delta}, \tilde{s})$ are both publicly revealed at the end of each period. In the next period, some low-type assets do

¹⁸Instead of selling the overvalued (by the Oracle) asset in the DeFi exchange and incur a price impact, borrowing against it yields a larger return for the asset owners.

¹⁹Our results do not depend on the asymmetric information on the common value component of the dividends. In Appendix A.8, we explore an alternative setup where there is asymmetric information concerning borrowers' private valuation. The main results hold. In Appendix A.9, we show that our setup can also be extended to time-varying information friction.

not survive and are replaced by new ones that are ex-ante identical. In the main model, we assume that borrowers receive private information every period and low quality asset pays lower dividend and may also die. The critical source of asymmetric information in our model, as evidenced by our assumption, is about dividend. In the Appendix, we consider the more general case where private information arrives only infrequently with probability χ , which can capture the degree of information imperfection.

Asset Price At the end of each period, agents meet in an exchange market to trade the assets by transferring the numeraire good. At this point, the private information is revealed publicly. The end-of-period ex-dividend price of a crypto asset that will survive to the next period is denoted as ϕ_t . At the end of the period, a high quality asset receives $\delta + \phi_t$ and a low quality asset receives ϕ_t with survival probability s . In the exchange market, each borrower can acquire at most one unit of crypto asset to the next period.²⁰

Smart Contract As discussed in the introduction, DeFi lending is anonymous and collateralized via a smart contract. The smart contract is a debt contract that specifies, at each time t , the haircut and interest rate (h, R_t) set by the lending protocol. The haircut defines the debt limit per unit of collateral according to:

$$D_t \equiv \Phi_t(1 - h) \tag{1}$$

where Φ_t is the contractual price underlying the DeFi debt contract. The borrowing limit is set by applying a pre-specified haircut on Φ_t . In many real world settings (such as Libor contracts and DeFi lending contracts) the contractual price Φ_t is set by traders in a forward looking manner.²¹ In DeFi lending the contractual price Φ_t comes from an Oracle that scans price quotes from many (centralized or decentralized) exchanges. All these contractual prices share common characteristics – they are forward looking and reflect both future ex-dividend price and some amount of the asset’s promised dividend/convenience yield, δ_t , received at the end of the period.

In our model, these two components of Φ_t are constructed differently. The dividend/convenience yield component is exogenously given, risky and subject to adverse selection, while the ex-dividend price component is determined in equilibrium. This distinction is important because **in equilibrium**, agents anticipate future (ex-dividend) price ϕ_t correctly even though borrowers and lenders are asymmetrically

²⁰The dynamic structure of the model is based on Lagos and Wright (2005).

²¹If trading is synchronous, Φ_t should be the “current” traded price. In our model, the current price, at the beginning of the period when the DeFi contract is set, is **not** the past ex-dividend price ϕ_{t-1} nor the future ex-dividend price ϕ_t since neither of these prices capture the value of the asset at the beginning of the period.

informed about the dividend. So it is also **rational** for agents to set the forward looking ex-dividend price component of Φ_t , to be the same as ϕ_t .

We choose the promised dividend/interest/convenience yield of the asset, that is δ , to compute the dividend component of Φ_t . This is to match the current industry practice in setting the haircut and margin loan limit. For example, for repo contracts, the haircut is on the “dirty” price of an asset. This price includes the asset’s “clean” price quote (or ex-dividend price quoted on exchanges), and accrued interest (or dividend) even when the promised interest (or dividend) payment is risky and the realized value might be lower. This is especially true for fixed-income securities, such as exchange rate, bills and bonds. In DeFi, one can map prices quoted on Binance (a centralized exchange) or Uniswap/Curve (decentralized exchanges) as the sum of a risky convenience yield component of these crypto assets that is subject to asymmetric information and a clean price component. For these reason we choose to specify the contractual price as $\Phi_t = \delta + \phi_t$. Note that our main results hold as long as the specification of the contractual price has an endogenously determined equilibrium price and a dividend component (which is subject to asymmetric information). ²²

In practice, the DeFi loan interest rate in the smart contract is a function of the utilization ratio i.e., the ratio of demand and supply for funding, and the collateral specific haircut is *infrequently* updated. To capture the economic impact of these features, we assume in our main model that the smart contract specifies a *flexible* market clearing interest rate and a *fixed* haircut. We investigate the flexible haircut case in an extension.

DeFi Lending & Borrowers In each period, if the borrower borrows ℓ_t units of funding, the face value of the debt is $R_t \ell_t$. After observing the asset quality, the borrower raises funding from a DeFi protocol by executing the lending contract. Given (R_t, D_t) , a type $Q = H, L$ borrower chooses how much collateral a_t to pledge and how much loan ℓ_t to borrow from the pool:

$$\max_{a_t, \ell_t} z \ell_t - \mathbb{E}_Q \min\{\ell_t R_t, a_t(\delta + \tilde{s}\phi_t)\}$$

subject to a collateral constraint

$$\ell_t R_t \leq a_t D_t$$

where D_t is the debt limit pinned down by (1). By borrowing ℓ_t and pledging a_t , the borrower obtains $z \ell_t$ from the loan but needs to either repay $\ell_t R_t$ or lose the collateral value $a_t(\tilde{\delta} + \tilde{s}\phi_t)$. The collateral

²²Besides matching industry practice, this specification also leads to clean expositions. We could also use some other function of δ (for example expected dividend) such as $\Phi_t = E[\delta] + \phi_t$. This modification would not affect our main results since this function of dividend is subject to some degree of adverse selection.

value discounted by the haircut needs to be sufficiently high to cover the loan repayment. Note that, without loss of generality, we can assume that the collateral constraint is binding: $\ell_t R_t = a_t D_t$.²³ So the solution for the borrowing decision is given by

$$a_{it} \in \arg \max_{a_t \in [0,1]} a_t [z D_t / R_t - \mathbb{E}_Q \min\{D_t, \tilde{\delta} + \tilde{s} \phi_t\}]. \quad (2)$$

Hence, it is optimal to set $a_t \in \{0, 1\}$. When the term inside the square bracket is positive, the borrower pledges $a_t = 1$ to borrow $\ell_t = D_t / R_t$ and promises to repay D_t . Default happens whenever $D_t > \tilde{\delta} + \tilde{s} \phi_t$. When the term inside the square bracket is non-positive, the borrower does not borrow: $a_t = \ell_t = 0$. Since $\mathbb{E}_H \min\{D_t, \delta + \phi_t\} = D_t \geq \mathbb{E}_L \min\{D_t, \tilde{s} \phi_t\}$, we have $a_{Lt} \geq a_{Ht}$ and $\ell_{Lt} \geq \ell_{Ht}$. That is, the low-type borrowers have higher incentives to borrow than the high-type. When both types borrow, we have a *pooling* outcome. When only the low-type borrows, we have a *separating* outcome.

DeFi Lending & Lenders The intermediary has no initial funding. It obtains funding q_t from the lenders to finance loans to borrowers. When the loan matures, the intermediary passes the cash flows – either the repayment of the borrowers or the resale value of the collateral (in case of a default) – to the lenders, after collecting an intermediation fee (discussed below).²⁴ Note that the borrower’s borrowing decision, $a_{i,t}$ where $i \in \{L, H\}$, is quality dependent, meaning that lenders face adverse selection in DeFi lending. Since lenders are not able to distinguish between low and high quality borrowers at the time of lending, the choice of funding size q_t does not depend on the underlying asset quality. Of course, in equilibrium, lenders take into account the expected quality of the collateral mix backing the loan.

We assume that the lending market is competitive. That is, given $\{a_{i,t}\}_{i \in \{L, H\}}$, D_t , and ϕ_t , funding supply q_t satisfies the following zero profit condition:

$$q_t = \frac{1}{1+f} \left\{ \frac{1}{a_{L,t} \lambda + a_{H,t} (1-\lambda)} [a_{L,t} \lambda \mathbb{E}_L \min\{D_t, \tilde{s} \phi_t\} + a_{H,t} (1-\lambda) \min\{D_t, \delta + \phi_t\}] \right\} \quad (3)$$

where $f < z - 1$ is a fixed fee charged by the intermediary per unit of loan.²⁵

²³To see this, suppose (ℓ^*, a^*) is optimal and $\ell^* R < a^* D$. Since the objective function is (weakly) decreasing in a , lowering a (weakly) increases the objective. The increase is strict if $a s \phi < \ell R$ for some realization of s .

²⁴In reality, some lending protocols have a backstop provided by “equity holders”. For example, Aave’s Safety Module incentivizes its governance token holders to lock their AAVE tokens as a mitigation tool in case of a shortfall event. The feature can be incorporated into our model easily by introducing some risk neutral agents who absorb the default risk and promise a constant payoff q to the lenders. Risk neutrality and zero profit condition imply that we will get exactly the same result.

²⁵When the loan matures the intermediary takes qf either from the repayment or from the resale value of the collateral. The remaining amount goes to the lender. The assumption of $f < z - 1$ ensures that the net gain from loans is positive.

When $a_{L,t} = a_{H,t} = 1$ (both types are borrowing) or when $a_{L,t} = 1$, $a_{H,t} = 0$ and the realized type is L , the funding supply is fully utilized and the funding market clears. In the separating case, if the realized type is H then there is no demand for funding. In this case, we assume the intermediary returns the funding supply to the lenders without charging a fee.

The intermediary's payoff is given by

$$f[\lambda a_{L,t} + (1 - \lambda) a_{H,t}] q_t. \quad (4)$$

In section 5.5, we consider the case where the intermediary flexibly chooses the haircut. In that case, the intermediary chooses h_t to maximize (4) taking $(a_{i,t})_{i \in \{L,H\}}$ and ϕ_t as given.

Determination of the Crypto Asset Price At the end of each period, borrowers bid for the crypto asset to use as collateral for future period. Therefore, the price of a crypto asset at the end of period t , ϕ_t , is given by its continuation value to the borrower:

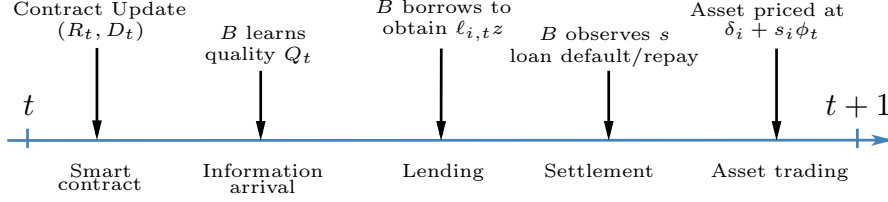
$$\begin{aligned} \phi_t = \beta \left\{ \underbrace{\lambda (a_{L,t+1} \mathbb{E}_L (z D_{t+1} / R_{t+1} - \min\{D_{t+1}, \tilde{s} \phi_{t+1}\}))}_{\text{Collateral Value}} \right. \\ \left. + \underbrace{(1 - \lambda) a_{H,t+1} (z D_{t+1} / R_{t+1} - \min\{D_{t+1}, \delta + \phi_{t+1}\})}_{\text{Fundamental Value}} \right\} \end{aligned} \quad (5)$$

where β is the discount factor. We assume $0 < \beta < 1/z$.²⁶ The continuation value of the asset, is simply the sum of two terms: the fundamental value of the asset which is the discounted value of future dividend and asset resale price, and the collateral value. Importantly, the collateral value of the asset depends on endogenous variables, $(a_{i,t+1})_{i \in \{L,H\}}$, D_{t+1} , R_{t+1} and ϕ_{t+1} , which in turn depend on the extent of asymmetric information in future DeFi lending markets.

Timing The time-line is summarized in Figure (3). In the beginning of each period, the smart contract specifies the debt limit D_t (or equivalently the haircut h) and the loan interest rate. Next, borrower receives private information about the quality of the asset and decides whether to borrow from the lending platform by pledging collateral to the smart contract and lenders supply funding subject to zero profit condition. After this stage, the borrower's type is revealed, and the borrower either repays the loan or defaults and loses the collateral. If the asset survives then its price is determined, consumption takes place and the borrower works to acquire assets for the next period.

²⁶This assumption is to make sure that the equilibrium asset prices are bounded.

Figure 3: Timeline



Note that in this timeline, the lending platform is exposed to information friction and the asset market is frictionless, and we assume that they do not open simultaneously, which reflects the natural timing of information revelation process. In reality, a privately informed borrower can choose to offload the underlying asset in a lending platform by borrowing a stablecoin loan against it or conduct an outright sale in an exchange (that is, an asset market). However, theoretically, adverse selection problem is more severe in an asset exchange since the borrower is selling an equity contract and less so in a lending platform since the borrower is selling a debt contract.²⁷ Empirically, there are other technical frictions in selling crypto assets on decentralized and centralized exchanges on blockchains. Transferring crypto assets to an off-chain centralized exchange is often subject to a long time lag before the assets can be traded, while transactions on an on-chain decentralized exchange are often subject to market illiquidity and price slippage. Therefore, for expositional clarity and without loss of generality, we assume that the asset market with frictions does not open simultaneously with the lending platform.

Equilibrium Definition Given haircut h and fee f , an equilibrium consists of asset prices $\{\phi_t\}_{t=0}^{\infty}$, debt thresholds $\{D_t\}_{t=0}^{\infty}$, loan rates $\{R_t\}_{t=0}^{\infty}$, funding size $\{q_t\}_{t=0}^{\infty}$ and collateral quantities $\{a_{Lt}, a_{Ht}\}_{t=0}^{\infty}$ such that

1. borrowers' loan decisions are optimal (condition 2),
2. lenders earn zero profit (condition 3),
3. funding supply equals funding demand, i.e. $q_t = D_t/R_t$, and
4. the asset pricing equation is satisfied (condition 5).

²⁷Ozdenoren, Yuan, and Zhang (2021) have shown the optimal security for privately informed borrowers to sell in a similar setting consists of a debt contract (which both high and low quality borrowers sell) and a residual equity contract (which only the low quality borrowers sell).

4 Equilibrium in Lending Market

We begin the analysis by describing the equilibrium in the DeFi lending market for a given asset price ϕ .²⁸ To study the borrowers' decision, we first define the degree of *information insensitivity* as the ratio of the expected value of the debt contract for types L and H , i.e., $\zeta(\phi; h) = \mathbb{E}_L \min\{D, \tilde{s}\phi\} / D \in (0, 1]$ where $D = ((1 - \lambda)\delta + \phi)(1 - h)$. As this ratio increases, the expected values of the debt under the low versus high become closer, and the adverse selection problem becomes less severe.

There are two cases depending on whether the high-type borrowers are active. In the pooling case, condition (3) implies that the equilibrium funding supplied by lenders is

$$q^P = \frac{1}{1+f} [\lambda \mathbb{E}_L \min\{D, \tilde{s}\phi\} + (1-\lambda)D].$$

Interest rate is pinned down by $q^P = D/R^P$, that is,

$$R^P = \frac{D(1+f)}{\lambda \mathbb{E}_L [\min\{D, \tilde{s}\phi\}] + (1-\lambda)D}.$$

In the separating case, the funding from lenders is given by

$$q^S = \frac{1}{1+f} \mathbb{E}_L \min\{D, \tilde{s}\phi\}.$$

and the interest rate pinned down by $q^S = D/R^S$, that is,

$$R^S = \frac{D(1+f)}{\mathbb{E}_L [\min\{D, \tilde{s}\phi\}]}.$$

Define $\bar{\zeta} \equiv 1 - \frac{z-1-f}{z\lambda}$. The next proposition characterizes the equilibrium in the DeFi lending market for a given asset price ϕ .

Proposition 1. *Given asset price ϕ , if the degree of information insensitivity $\zeta(\phi; h) > \bar{\zeta}$, then borrowers' equilibrium funding obtained from DeFi lending is $q = q^P$, interest rate is $R = R^P$ and collateral choices for H type borrower and L type borrower are $a_L = a_H = 1$. If the degree of information insensitivity $\zeta(\phi; h) < \bar{\zeta}$, then borrowers' equilibrium funding from DeFi lending is $q = q^S$, interest rate is $R = R^S$, and collateral choices for H type borrower and L type borrower are $a_L = 1$ and $a_H = 0$. The former condition, for a pooling equilibrium, is easier to satisfy when asset price ϕ , haircut h or productivity from borrowers' private investment z is higher.*

Proposition 1 implies that, given asset price ϕ , there is a unique equilibrium in DeFi lending. It is a pooling (separating) outcome when the debt contract is sufficiently informationally insensitive (sensitive).

²⁸In this section we drop the time subscript t from all the variables to ease the notation.

In particular, when the degree of information insensitivity $\zeta(\phi; h)$ is above the threshold $\bar{\zeta}$, the adverse selection problem is not too severe and both types borrow. In this case, the loan size is the pooling quantity $q = q^P$. When the degree of information insensitivity is below the threshold, the adverse selection problem is severe and only the low type borrows. In this case, the loan size is the separating amount $q = q^S$. Furthermore, the loan rate in a pooling equilibrium is lower than that in a separating equilibrium.

Note that $\zeta(\phi; h) = \mathbb{E}_L \min\{1, \frac{\tilde{s}\phi}{(\delta+\phi)(1-h)}\}$. As a result, the debt contract becomes informationally less sensitive for a high ϕ and for a high h . The above proposition also indicates that in addition to the parameter λ that characterizes type heterogeneity, the net gains from trade, $z/(1+f)$, is also an important determinant of adverse selection: a lower $z/(1+f)$ leads to a higher $\bar{\zeta}$. In particular, even if there is very little asymmetric information about the quality of the debt contract (i.e., when $\zeta(\phi; h)$ is slightly below 1), as $z/(1+f)$ approaches 1 (so that $\bar{\zeta}$ is close 1), the DeFi lending will be in a separating equilibrium. In other words, when net gains from trade is low, even a slight amount of asymmetric information results in adverse selection problem.

5 Multiple Equilibria in Dynamic DeFi Lending

The analysis in the previous section takes the asset price as given. In this section, we characterize the stationary equilibrium where asset prices are endogenously determined. We demonstrate that DeFi lending is fragile in the sense that it exhibits dynamic multiplicity in prices. Specifically, we show that there might be multiple equilibria in the DeFi lending market justified by different crypto asset prices. The multiple asset prices are in turn justified by the different equilibria in DeFi lending. Since we are focusing on stationary equilibria, we drop the time subscripts.

5.1 Characterization of Stationary Equilibria

5.1.1 Pooling equilibrium

In a stationary pooling equilibrium, all borrowers borrow ($a_L = a_H = 1$). This equilibrium exists when there is an asset price ϕ^P satisfying the equation

$$\phi^P = \beta [(z-1-f)q^P] + \beta(1-\lambda)\delta + \beta(\lambda\mathbb{E}_L\tilde{s} + (1-\lambda))\phi^P. \quad (6)$$

The loan size is given by

$$q^P = \frac{1}{1+f} (\lambda\mathbb{E}_L [\min\{D^P, \tilde{s}\phi^P\}] + (1-\lambda)D^P),$$

where $D^P = (\delta + \phi^P)(1 - h)$. In addition, it has to satisfy the high-type borrowers' incentive constraint to pool:

$$\zeta(\phi^P; h) = \mathbb{E}_L \min\{1, \frac{\tilde{s}\phi^P}{(\delta + \phi^P)(1 - h)}\} \geq \bar{\zeta}. \quad (7)$$

5.1.2 Separating Equilibrium

In a separating equilibrium, only the low-type borrowers borrow (i.e., $a_H = 0, a_L = 1$). This equilibrium exists when there is an asset price ϕ^S satisfying the equation

$$\phi^S = \beta(\lambda(z - 1 - f)q^S + (1 - \lambda)\delta + (\lambda\mathbb{E}_L\tilde{s} + (1 - \lambda))\phi^S). \quad (8)$$

The loan size is given by

$$\frac{D^S}{R} = q^S = \frac{1}{1 + f} \mathbb{E}_L [\min\{D^S, \tilde{s}\phi^S\}],$$

where $D^S = (\delta + \phi^S)(1 - h)$. In addition, high-type's incentive constraint to pool is violated:

$$\zeta(\phi^S; h) < \bar{\zeta}. \quad (9)$$

5.2 Existence and Uniqueness

We first focus on the asset pricing equations (6) and (8).

Lemma 1. *Equation (6) has a unique solution ϕ^P and equation (8) has a unique solution ϕ^S . Also, $\phi^P \geq \phi^S$.*

Lemma 1 implies that there exists at most one pooling and one separating stationary equilibrium. If they co-exist, the price in the pooling equilibrium is higher than that in the separating equilibrium. It is also easy to show that both prices are higher than the fundamental price of the asset in autarky, $\underline{\phi} = \frac{\beta(1-\lambda)\delta}{1-\beta(\lambda\mathbb{E}(s_L)+(1-\lambda))}$. This means that the introduction of DeFi lending raises the equilibrium asset price above its fundamental level. Lemma 1 implies that $\zeta(\phi^P; h) \geq \zeta(\phi^S; h)$. Hence, we have the following proposition.

Proposition 2. *There always exists at least one stationary equilibrium:*

- it is a unique pooling equilibrium when $\bar{\zeta} < \zeta(\phi^S; h)$,
- it is a unique separating equilibrium when $\bar{\zeta} > \zeta(\phi^P; h)$,
- a pooling equilibrium and a separating equilibrium coexist when $\bar{\zeta} \in [\zeta(\phi^S; h), \zeta(\phi^P; h)]$.

In the next section, we examine the conditions under which the multiplicity arises.

5.3 Haircut and Multiplicity

In Proposition 2, multiplicity arises due to a dynamic price feedback effect. When the collateral asset price is high, the degree of information insensitivity of the debt contract, $\zeta(\phi^P; h)$, is above the threshold $\bar{\zeta}$. Hence, the adverse selection problem is mild and the high-type borrowers are willing to pool with the low type. In turn, if agents anticipate a pooling equilibrium in future periods, the expected liquidity value of the asset in the next period is large, hence the asset price today is high. Conversely, when the asset price is low, the degree of information insensitivity of the debt contract, $\zeta(\phi^S; h)$, is below the threshold $\bar{\zeta}$. Therefore, the adverse selection problem is severe and the high type retains the asset and chooses not to borrow. In turn, if agents anticipate a separating equilibrium in future periods, the liquidity value of the asset is limited and thus the asset price today is low. As a result, the asset prices are self-fulfilling in this economy.

The haircut is a key parameter controlling the degree of information sensitivity. Setting a lower haircut makes the debt contract informationally more sensitive, magnifying the adverse selection problem. Defining two thresholds

$$\begin{aligned}\kappa_P &\equiv \frac{\zeta}{\beta z[(1-\lambda) + \zeta\lambda]} \\ \kappa_S &\equiv \frac{\zeta}{\beta[(1-\lambda) + \zeta\lambda z]} < \kappa_P,\end{aligned}$$

we have the following result.

Proposition 3. *Suppose the expected survival probability of the crypto asset satisfies $\mathbb{E}_L \tilde{s} \in (\kappa_P, \kappa_S)$. There exists a threshold for haircut such that when the haircut h is below this threshold, there are multiple equilibria.*

5.3.1 Example: Two-point distribution

We now use an example to illustrate the effects of h on the equilibrium outcome. The full analysis is given in the Appendix. Suppose \tilde{s} is drawn from a two-point distribution such that $s = 1$ with probability π , and $s = 0$ with probability $1 - \pi$. Consider the separating equilibrium. When $s = 0$, a low-type borrower always defaults. When $s = 1$, the low-type defaults if $D^S = (\delta + \phi^S)(1 - h) > \phi^S$ and repays if $D^S \leq \phi^S$. We can rewrite this condition to show that there exists a threshold level \underline{h}^S such that when $s = 1$, the low-type defaults if $h < \underline{h}^S$ and repays if $h \geq \underline{h}^S$. In the former case, the low type always defaults so the face value of the loan and consequently the loan size do not depend on the haircut. In the latter case, the low type repays the loan in the good state (i.e., $s = 1$), hence the loan size depends

on the face value of the debt. Since the face value of debt declines as the haircut increases, the loan size decreases in h .

We define $\zeta^S(h) \equiv \zeta(\phi^S(h); h)$. That is, we obtain $\zeta^S(h)$ by substituting the price ϕ^S as a function of haircut given fixed values for all other exogenous variables. We define $\zeta^P(h)$ similarly. Using (9), a separating equilibrium exists if $\zeta^S(h) \leq \bar{\zeta}$. The threshold $\zeta^S(h)$ is strictly increasing in h for $h < \underline{h}^S$. The reason is that the high type never defaults, so the expected value of the contract under the high type declines as h increases. The low type, on the other, always defaults and the expected value of the contract under the low type is independent of h . Hence, the information sensitivity of the contract decreases as h increases and it becomes harder to support a separating equilibrium. For $h \geq \underline{h}^S$, $\zeta^S(h) = \pi$ and a separating equilibrium exists whenever $\pi < \bar{\zeta}$. That is, once the haircut is large enough, increasing it further does not affect the information sensitivity of the contract. The reason is that, in this case, the high type always pays the face value and the low type pays the face value only in the good state. As the haircut increases, the face value decreases but the value of the contract declines at the same rate for both types so its information sensitivity remains constant.

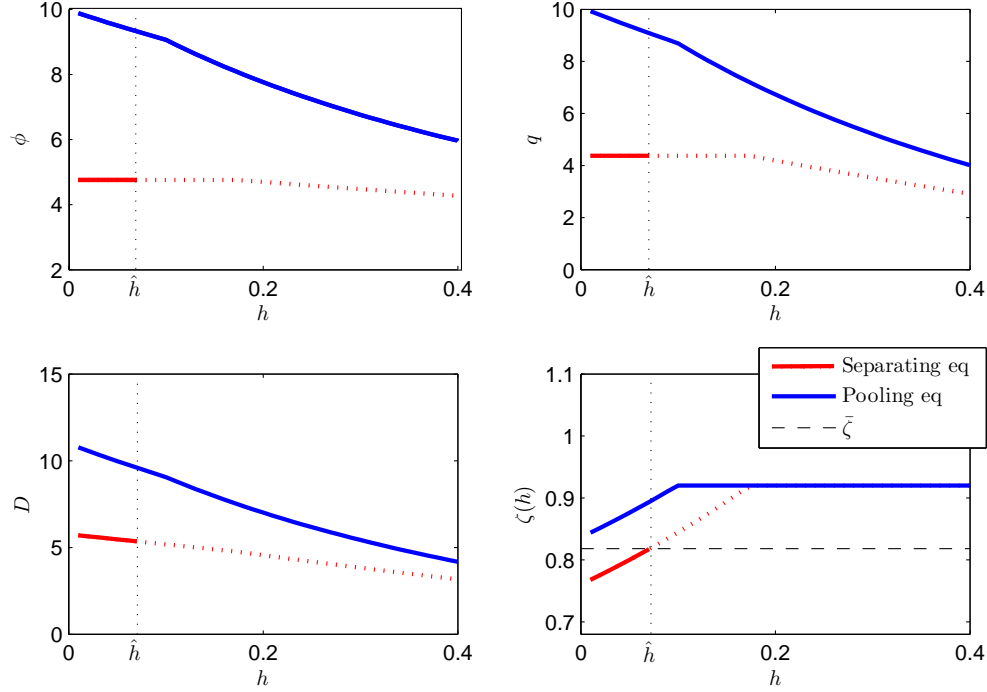
We analyze the pooling equilibrium similarly, and find a threshold $\underline{h}^P < \underline{h}^S$ such that when $s = 1$, the low-type defaults if $h < \underline{h}^P$ and repays if $h \geq \underline{h}^P$. A pooling equilibrium exists if $\zeta^P(h) \geq \bar{\zeta}$. The threshold $\zeta^P(h)$ is strictly increasing in h and $\zeta^P(h) > \zeta^S(h)$ for $h < \underline{h}^P$. For $h \geq \underline{h}^P$, $\zeta^P(h) = \pi$ and a pooling equilibrium exists whenever $\pi > \bar{\zeta}$.

Putting these facts together we see that whenever $h < \underline{h}^S$, we have $\zeta^S(h) < \zeta^P(h)$. Hence when $\bar{\zeta}$ is in this range the two equilibria coexist. When the haircut exceeds \underline{h}^S , there can only be a unique equilibrium depending on whether $\bar{\zeta}$ is above or below π .

Figure 4 plots the effects of h on the asset price, the loan size, the debt limit and the degree of information insensitivity of the contract. The red and blue curves indicate respectively the separating and pooling equilibria, assuming their existence. The parameter values used are $z = 1.1$, $\lambda = 0.5$, $\beta = 0.9$, $\delta = 1$, $\pi = 0.92$, $f = 0$, which satisfy the condition $\mathbb{E}_L \tilde{s} \in (\kappa_P, \kappa_S)$ in Proposition 3. The bottom right plot compares the degrees of information insensitivity to the threshold $\bar{\zeta}$ which is captured by the horizontal dash line. When h is close to zero, the dash line appears above the red curve and below the blue curve, confirming the multiplicity result in Proposition 3. The other three plots also confirm the earlier result that the asset price, loan size and debt limit are higher in a pooling equilibrium. In this example, multiplicity can be ruled out and pooling can be supported by setting $h > \hat{h} = 7.1\%$ where $\bar{\zeta} = \zeta^S(\hat{h})$.²⁹

²⁹When $h > \hat{h}$, separating equilibrium cannot be sustained and hence in Figure 4 red lines depicting separating equilibria become red dotted lines in this region.

Figure 4: Effects of Haircut h



5.4 Sentiment Equilibrium

In the middle region where multiple self-fulfilling equilibria coexist, it is possible to construct *sentiment equilibria* where agents' expectations depend on non-fundamental sunspot states. In a static game with multiple equilibria, as in Diamond and Dybvig (1983), an equilibrium is chosen by a sunspot. In a dynamic setting, the economy can switch between equilibria (based on a sunspot) and we refer to different phases as different sentiments. Clearly, in a dynamic setting construction is more delicate because one has to take into account that equilibrium can switch again in the future. We are not the first to use this dynamic notion of sentiment equilibrium. Similar notions have been used by among others including Hassan and Mertens (2011), Benhabib, Wang, and Wen (2015), Asriyan, Fuchs, and Green (2017), etc. By constructing sentiment equilibria, we offer potential empirical testable hypotheses relating measurable sentiment index with equilibrium price and quantities.

Suppose that there are K sentiment states indexed from 1 to K . We let $\sigma_{kk'}$ be the Markov transition probability from sentiment state k to k' . In the presence of sentiments we modify the model as follows.

Let ϕ^k be the price of the asset, R^k be the loan rate, and $D^k = (\delta + \phi^k)(1 - h)$ be the debt limit in sentiment state k . Quantities of collateral a_L^k, a_H^k chosen by each type must be optimal given the price and rate at each sentiment state k . The loan size chosen by the lender in sentiment state k is given by:

$$q^k = \lambda E_L [\min\{D^k, s\phi^k\}] + (1 - \lambda)D^k$$

The price of crypto asset in sentiment state k is given by:

$$\begin{aligned} \phi^k = \beta \sum_{k=1}^K \sigma_{kk'} & \left\{ \lambda \int_{\underline{s}}^{\bar{s}} s_L \phi^{k'} dF(s_L) + (1 - \lambda) (\delta + \phi^{k'}) \right. \\ & \left. + \lambda a_L^{k'} \int_{\underline{s}}^{\bar{s}} (zD^{k'}/R^{k'} - \min\{D^{k'}, s_L \phi^{k'}\}) dF(s_L) + (1 - \lambda) a_H^{k'} (zD^{k'}/R^{k'} - D^{k'}) \right\}. \end{aligned}$$

We want to construct a *non-trivial sentiment equilibrium* such that the economy supports a pooling outcome in states $k = 1, \dots, \bar{k}$ and a separating outcome in states $k = \bar{k} + 1, \dots, K$. By continuity, one can obtain the following result.

Proposition 4. *Suppose $\mathbb{E}(s) \in (\kappa_P, \kappa_S)$ and haircut is not too big. Then for σ_{kk} large enough, there exists a non-trivial sentiment equilibrium.*

To demonstrate non-trivial sentiment equilibrium and examine equilibrium properties, we provide the following two numerical examples. In both examples we assume \tilde{s} is drawn from a two-point distribution such that $s = 1$ with probability π , and $s = 0$ with probability $1 - \pi$.

Example 1. Suppose $K = 3$ and $\bar{k} = 1$. The economy stays in the same state with probability σ and moves to the next state with probability $1 - \sigma$ where the next state from 1 is 2, from 2 is 3 and from 3 is 1. We can interpret the three states as follows:

- $k = 1$: Boom state

$$- a_L^1 = a_H^1 = 1, q^1 = \lambda \pi \min\{(\delta + \phi^1)(1 - h), \phi^1\} + (1 - \lambda)(\delta + \phi^1)(1 - h)$$

- $k = 2$: Crash state

$$- a_L^2 = 1, a_H^2 = 0, q^2 = \pi \min\{(\delta + \phi^2)(1 - h), \phi^2\}$$

- $k = 3$: Recovery state

$$- a_L^3 = 1, a_H^3 = 0, q^3 = \pi \min\{(\delta + \phi^3)(1 - h), \phi^3\}$$

The asset prices are then given by

$$\begin{aligned}\phi^k = & \beta\sigma_{k1} [(z-1)q^1 + (1-\lambda)\delta + (\lambda\pi + (1-\lambda))\phi^1] \\ & + \beta\sigma_{k2} [\lambda(z-1)q^2 + (1-\lambda)\delta + (\lambda\pi + (1-\lambda))\phi^2] \\ & + \beta\sigma_{k3} [\lambda(z-1)q^3 + (1-\lambda)\delta + (\lambda\pi + (1-\lambda))\phi^3]\end{aligned}$$

Figure 5 below plots the effects of sentiment states on asset prices and total lending. When $\sigma = 0.95$, the sentiment state is sufficiently persistent so that the above sentiment equilibrium exists. As shown, the sentiment dynamics drive the endogenous asset price cycle: The asset price declines when the economy enters the crash state, jumps up when the economy moves from the crash state to the recovery state, and jumps up further when the economy returns to the boom state. Note that the total lending, $(\lambda a_L^k + (1-\lambda)a_H^k)q^k$ is “pro-cyclical” in the sense that it is positively correlated with the asset price.

Next, we show a similar pro-cyclical pattern of lending and asset prices in an example where there are more (than three) states and a state moves to an up or a down state with an equal probability. In this example, equilibrium lending and asset prices are more volatile.

Example 2. Let $K = 10$. If the economy is in state k in a given period, in the next period sentiment stays the same with probability σ . From states $k \in \{2, \dots, K-1\}$ economy moves to state $k-1$ with probability $(1-\sigma)/2$ and to state $k+1$ with probability $(1-\sigma)/2$. From state 1 economy moves to state 2 with probability $1-\sigma$. From state K economy moves to state $K-1$ with probability $1-\sigma$. Figure 6 plots a simulation for 5000 periods when $\sigma = 0.95$ and $\bar{k} = 6$.

5.5 Uniqueness under Flexible Design of Debt limit

We have shown that DeFi lending subject to a rigid haircut can lead to multiplicity when the debt contract is too informationally sensitive. In this extension, we show that a flexible contract design supports a unique equilibrium in the case and generates higher social surplus from lending compared to the case with a rigid haircut.

Under flexible design, the smart contract is no longer subject to constraint (1). Instead, in each period, the intermediary, in this case, the DeFi protocol, can choose any feasible debt contract, $y(D_t, \tilde{\delta} + \tilde{s}\phi_t) = \min(D_t, \tilde{\delta} + \tilde{s}\phi_t)$ for $0 \leq D_t \leq \delta + \phi_t$. Let \hat{z} denote the marginal value of obtaining funding from lenders deducting the intermediation fee f to the intermediary,

$$\hat{z} = \frac{z}{1+f}.$$

Figure 5: Sentiment Equilibrium Example 1

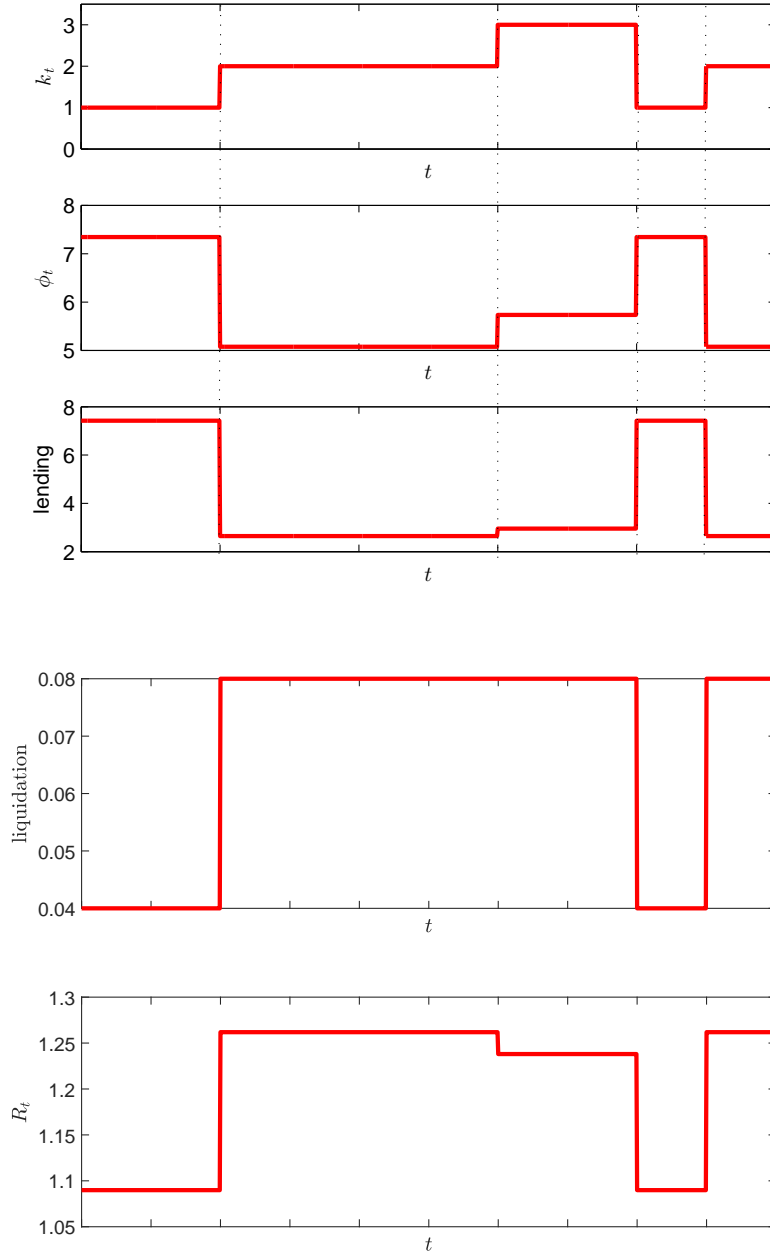
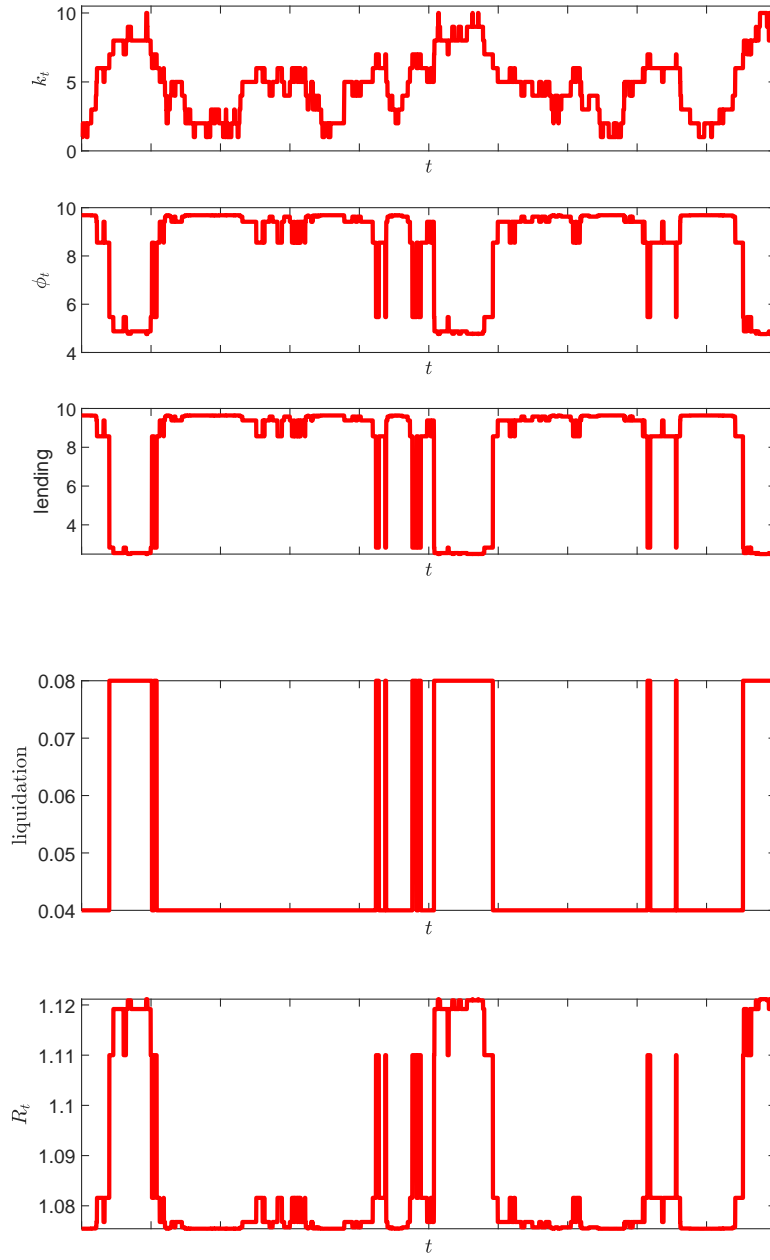


Figure 6: Sentiment Equilibrium Example 2



Recall from (4) that intermediary maximizes the expected loan size times the intermediation fee:

$$f[\lambda + (1 - \lambda) a_{H,t}] q_t \left(y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \right)$$

The loan size is:

$$q_t \left(y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \right) = \frac{1}{1 + f} \frac{[\lambda \mathbb{E}_L + a_{H,t} (1 - \lambda) \mathbb{E}_H] y(D_t, \tilde{\delta} + \tilde{s}\phi_t)}{\lambda + a_{H,t} (1 - \lambda)} \quad (10)$$

where

$$a_{H,t} = \begin{cases} 1 & \text{if } \hat{z}[\lambda \mathbb{E}_L + (1 - \lambda) \mathbb{E}_H] y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \geq \mathbb{E}_H y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \\ 0 & \text{otherwise} \end{cases}. \quad (11)$$

Equivalently the intermediary maximizes

$$[\lambda \mathbb{E}_L + a_{H,t} (1 - \lambda) \mathbb{E}_H] y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \quad (12)$$

subject to (11). In words, the intermediary takes the price ϕ_t as given and sets the debt threshold D to maximize the expected loan size taking into account the impact of the contract on the funding that the lenders are willing to supply. The value of the asset to the borrower is:

$$\begin{aligned} V_t = \max_{0 \leq D \leq \tilde{\delta} + \phi_t} & \lambda \left[\hat{z} q_t \left(y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \right) - \mathbb{E}_L y(D_t, \tilde{\delta} + \tilde{s}\phi_t) + \mathbb{E}_L \left(\tilde{\delta} + \tilde{s}\phi_t \right) \right] \\ & + (1 - \lambda) \left[a_{H,t} \left\{ \hat{z} q_t \left(y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \right) - \mathbb{E}_H y(D_t, \tilde{\delta} + \tilde{s}\phi_t) \right\} + \mathbb{E}_H \left(\tilde{\delta} + \tilde{s}\phi_t \right) \right] \end{aligned} \quad (13)$$

Given the optimal design, the asset price at the end of the previous period equals

$$\phi_{t-1} = \beta V_t. \quad (14)$$

An equilibrium under flexible design of smart contracts is debt face value D_t , the borrower's value for the asset at the beginning of period t V_t , and the resale price of the asset at the end of period t ϕ_t such that (i) D_t maximizes (12) taking ϕ_t as given and, (ii) V_t , and ϕ_t satisfy (13) and (14).

We also make the same simplifying assumptions on the distribution of $(\tilde{\delta}, \tilde{s})$ that we make in the rigid haircut case. That is, we assume that a high-quality asset pays dividend $\delta > 0$ at the end of the period and survives to the next period with certainty which implies:

$$\mathbb{E}_H y(D_t, \tilde{\delta} + \tilde{s}\phi_t) = y(D_t, \delta + \phi_t);$$

and the low type asset does not pay any dividends and it survives to the next period with probability $s \in [0, 1]$ which is drawn from a distribution F which implies:

$$\mathbb{E}_L y(D_t, \tilde{\delta} + \tilde{s}\phi_t) = \int_s^{\bar{s}} y(D_t, s_L \phi_t) dF(s_L).$$

The following proposition describes the optimal debt threshold and the implied haircut as a function of the asset price ϕ_t .

Proposition 5. *If $\mathbb{E}_L s < 1 + \frac{1}{\lambda \hat{z}} - \frac{1}{\lambda}$ then let s^* be the unique solution to:*

$$\hat{z}[\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^*] = s^*.$$

In this case, the equilibrium contract is a pooling one ($a_{H,t} = 1$) with face value $D_t = s^ \phi_t$ when*

$$\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^* - \lambda \mathbb{E}_L s \geq 0.$$

Otherwise, the equilibrium contract is a separating one ($a_{H,t} = 0$) with face value $D_t = \delta + \phi_t$. The implied haircut is:

$$h_t = \begin{cases} 0 & \text{if } \lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^* - \lambda \mathbb{E}_L s < 0, \\ 1 - \frac{s^* \phi_t}{(1 - \lambda)\delta + \phi_t} & \text{if } \lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^* - \lambda \mathbb{E}_L s \geq 0. \end{cases}$$

If $\mathbb{E}_L s > 1 + \frac{1}{\lambda \hat{z}} - \frac{1}{\lambda}$, the equilibrium contract is a pooling one with face value $D = d^ + \phi$ where*

$$d^* = \min \left\{ \delta, \frac{\hat{z}[\lambda \mathbb{E}_L s + (1 - \lambda)] - 1}{1 - \hat{z}(1 - \lambda)} \phi \right\}.$$

The implied haircut is

$$h_t = \max \left\{ 0, 1 - \frac{\hat{z} \lambda \mathbb{E}_L s}{1 - \hat{z}(1 - \lambda)} \frac{\phi_t}{\delta + \phi_t} \right\}.$$

Moreover, given any end-of-period price ϕ_t , the asset price in the previous period and the lending volume are higher than those under the rigid DeFi contract.

Note that the optimal haircut rule is not a fixed number or a simple linear rule but non-linear in price ϕ_t . The proposition shows that the flexible contract generates more social surplus. For example, when ϕ_t is high (which makes the debt contract informationally less sensitive), the intermediary can increase D_t to induce a higher lending volume which raises the surplus from lending. In contrast, when ϕ_t is low (which makes the contract informationally more sensitive), the intermediary may choose to lower D_t to maintain a pooling outcome. Depending on the parameter values, the intermediary may also choose to raise D_t to induce a separating equilibrium. This flexibility in adjusting D_t implies that, given any end-of-period price ϕ_t , the price of asset in the previous period and the loan size are weakly greater than those under the rigid DeFi contract.

The following proposition shows that the flexibility in setting the haircut optimally in response to changes in the asset price leads to a unique stationary equilibrium with a fixed realized equilibrium haircut.

Proposition 6. *Under flexible optimal debt limit there exists a unique stationary equilibrium that Pareto dominates the one under DeFi.*

The above result suggests that the rigid haircut rule (1) imposed by the DeFi smart contract generates financial instability in the form of multiple equilibria, and potential sentiment driven equilibria (e.g. Asriyan, Fuchs, and Green (2017)), and lowers welfare. Can a DeFi smart contract be pre-programmed to replicate the flexible contract design? This can be challenging in practice. First, flexible contract cannot be implemented using simple linear hair-cut rules that are typically en-coded in DeFi contracts. Second, the optimal debt threshold depends on information that may not be readily available on-chain (e.g., z, λ). Alternatively, the lending protocol can replace the algorithm by a human risk manager who can adjust risk parameters in real time according to the latest information. Relying fully on a trusted third party, however, can be controversial for a DeFi protocol. Our results highlight the difficulty in achieving stability and efficiency in a decentralized environment subject to informational frictions.

5.6 Liquidation and Fire Sale

In this section, we incorporate the price impact of liquidation sale of defaulted contracts from DeFi lending protocol that is empirically investigated in Lehar and Parlour (2022a). To incorporate the possibility of fire-sales we modify the model as follows. There are two states: normal and fire-sale with probabilities σ and $1 - \sigma$, respectively, known to all agents at the beginning of the period. In the fire-sale state, DeFi capital is not fast moving enough within the period to correct the temporary price impact due to the collateral liquidation from the defaulted DeFi debt contracts and fire-sale happens. These probabilities are iid across periods. We denote by ϕ and ϕ^f the price of the collateral asset in the normal state and the fire-sale state, respectively. Since the collateral price is lower in the fire-sale state and the states are known to all participants at the beginning of the period, the amount of collateralizable lending is different between the normal and the fire-sale state. We denote R^f the fire-sale loan rate, $D^f = (\delta + \phi^f)(1 - h)$ the debt limit, a_L^f and a_H^f the amount of collateral pledged by types L and H in the fire-sale state. We denote by \mathcal{M} is the amount of collateral being liquidated, and Λ the price impact of the liquidation.

As in the main model, the loan size chosen by the lender in the normal state based on the haircut rule $D = (\delta + \phi)(1 - h)$ is given by the lender's break-even zero profit condition as follows:

$$q_t = \frac{1}{1 + f} \left\{ \frac{1}{a_{L,t}\lambda + a_{H,t}(1 - \lambda)} [a_{L,t}\lambda \mathbb{E}_L \min \{D_t, s\phi_t\} + a_{H,t}(1 - \lambda) \min \{D_t, \delta + \phi_t\}] \right\},$$

and the price of crypto asset in the normal state is given by:

$$\begin{aligned}
\phi_t = & \beta\sigma\lambda (a_{L,t+1}\mathbb{E}_L (zD_{t+1}/R_{t+1} - \min\{D_{t+1}, s\phi_{t+1}\})) \\
& + \beta\sigma (1 - \lambda) a_{H,t+1} (zD_{t+1}/R_{t+1} - D_{t+1}) \\
& + \beta\sigma \{ \lambda (\mathbb{E}_L s) \phi_{t+1} + (1 - \lambda) (\delta + \phi_{t+1}) \} \\
& + \beta (1 - \sigma) \lambda \left(a_{L,t+1}^f \mathbb{E}_L \left(zD_{t+1}^f/R_{t+1}^f - \min\{D_{t+1}^f, s\phi_{t+1}^f\} \right) \right) \\
& + \beta (1 - \sigma) (1 - \lambda) \left(a_{L,t+1}^f \mathbb{E}_L \left(zD_{t+1}^f/R_{t+1}^f - \min\{D_{t+1}^f, \delta + \phi_{t+1}^f\} \right) \right) \\
& + \beta (1 - \sigma) \left\{ \lambda (\mathbb{E}_L s) \phi_{t+1}^f + (1 - \lambda) (\delta + \phi_{t+1}^f) \right\}.
\end{aligned}$$

Similarly, the loan size chosen by the lender in the distressed state is given by:

$$q^f = \lambda \frac{1}{1+f} \left\{ \frac{1}{a_{L,t}\lambda + a_{H,t}(1-\lambda)} \left[a_{L,t}\lambda \mathbb{E}_L \min \left\{ D_t^f, s\phi_t^f \right\} + a_{H,t}(1-\lambda) \min \left\{ D_t^f, \delta + \phi_t^f \right\} \right] \right\},$$

but the price of crypto asset in the distressed state with fire sale has a price impact discount which is specified as follows:

$$\begin{aligned}
\phi_t^f = & \beta\sigma\lambda (a_{L,t+1}\mathbb{E}_L (zD_{t+1}/R_{t+1} - \min\{D_{t+1}, s\phi_{t+1}\})) \\
& + \beta\sigma (1 - \lambda) a_{H,t+1} (zD_{t+1}/R_{t+1} - D_{t+1}) \\
& + \beta\sigma \{ \lambda (\mathbb{E}_L s) \phi_{t+1} + (1 - \lambda) (\delta + \phi_{t+1}) \} \\
& + \beta (1 - \sigma) \lambda \left(a_{L,t+1}^f \mathbb{E}_L \left(zD_{t+1}^f/R_{t+1}^f - \min\{D_{t+1}^f, s\phi_{t+1}^f\} \right) \right) \\
& + \beta (1 - \sigma) (1 - \lambda) \left(a_{L,t+1}^f \mathbb{E}_L \left(zD_{t+1}^f/R_{t+1}^f - \min\{D_{t+1}^f, \delta + \phi_{t+1}^f\} \right) \right) \\
& + \beta (1 - \sigma) \left\{ \lambda (\mathbb{E}_L s) \phi_{t+1}^f + (1 - \lambda) (\delta + \phi_{t+1}^f) \right\} \\
& - \underbrace{\Lambda \mathcal{M}_{t+1}}_{\text{Fire-sale Discount}}
\end{aligned}$$

The key difference between the normal and fire-sale prices is the last term in the fire-sale price which captures the fire-sale price impact of collateral liquidations. This price impact is temporary and within the period. We assume that $\Lambda \leq \beta(1-\lambda)\delta$. This assumption is to make sure that the price impact does not dominate asset fundamentals completely. That is, the fire sale does not lead to a negative asset price but results in a significant discount.

Next, we first solve for equilibrium outcomes in the special case where fire sale is certain every period ($\sigma = 0$) to demonstrate the direct impact of a fire sale. In this case, the stationary pooling equilibrium asset price ϕ^{fP} satisfies the equation:

$$\phi^{fP} = \beta [(z - 1 - f)q^P] + \beta(1 - \lambda)\delta + \beta(\lambda\mathbb{E}_L\tilde{s} + (1 - \lambda))\phi^{fP} - \Lambda\mathcal{M}, \quad (15)$$

where

$$\mathcal{M}^P = \lambda \Pr(\tilde{s}\phi^{fP} < D^{fP}) = \lambda \Pr\left(\tilde{s} < \frac{D^{fP}}{\phi^{fP}}\right) = \lambda F_L\left(\frac{D^{fP}}{\phi^{fP}}\right).$$

and the stationary separating equilibrium asset price ϕ^{fS} satisfies the equation:

$$\phi^{fS} = \beta (\lambda(z - 1 - f)q^S + (1 - \lambda)\delta + (\lambda\mathbb{E}_L\tilde{s} + (1 - \lambda))\phi^{fS}) - \Lambda\mathcal{M}, \quad (16)$$

$$\mathcal{M}^S = \lambda \Pr(\tilde{s}\phi^{fS} < D^{fS}) = \lambda \Pr\left(\tilde{s} < \frac{D^{fS}}{\phi^{fS}}\right) = \lambda F_L\left(\frac{D^{fS}}{\phi^{fS}}\right).$$

It is obvious that the size of fire-sale discount is related to the magnitude of the price impact Λ and the amount of default.

Lemma 2. *Equations (15) and (16) each have at least one solution. The largest solution of (15) is larger than all solutions of (16) and the smallest solution (16) is less than all solutions of (15). Consequently, when these equations both have a unique solution, $\phi^{fP} \geq \phi^{fS}$.*

Let $\underline{\phi}^{fS}$ be the smallest separating equilibrium price and $\bar{\phi}^{fP}$ be the largest pooling equilibrium price. Following the same steps as in the proofs of the main model, we establish the following existence result.

Proposition 7. *In the case of fire sales every period ($\sigma = 0$), there always exists at least one stationary equilibrium. Moreover,*

- only pooling equilibria exist when $\bar{\zeta} < \zeta(\underline{\phi}^{fS}; h)$,
- only separating equilibria exist when $\bar{\zeta} > \zeta(\bar{\phi}^{fP}; h)$,
- there is at least one pooling equilibrium and one separating equilibrium when $\bar{\zeta} \in [\zeta(\underline{\phi}^{fS}; h), \zeta(\bar{\phi}^{fP}; h)]$.

However, the conditions under which the multiplicity arises would be different from the case without any fire sale presented in the main model. This is because lower collateral asset prices make DeFi debts more information sensitive, resulting in more adverse selection. The two thresholds for multiplicity are now:

$$\tilde{\kappa}_S = \frac{\bar{\zeta}(\delta - \Lambda\lambda)}{\beta\delta((1 - \lambda) + \bar{\zeta}\lambda z) - \Lambda\lambda}$$

and

$$\check{\kappa}_P = \frac{\bar{\zeta}(\delta - \Lambda\lambda)}{\beta z \delta ((1 - \lambda) + \bar{\zeta}\lambda) - \Lambda\lambda} < \check{\kappa}_S.$$

Now, we can prove the counterpart of Proposition 3 in the setting with fire sales.

Proposition 8. *Suppose fire sales occur every period ($\sigma = 0$), and the expected survival probability of the crypto asset satisfies $\mathbb{E}_L \tilde{s} \in (\check{\kappa}_P, \check{\kappa}_S)$. There exists a threshold for haircut such that when the haircut h is below this threshold, there are multiple equilibria.*

It is easy to see that $\kappa_S < \check{\kappa}_S$ and $\kappa_P < \check{\kappa}_P$ which implies together with Proposition 8 that the region for multiplicity shifts up in the presence of fire sale. That is, multiple equilibria might occur for crypto assets with better fundamentals (that is, a higher survival probability), indicating that adverse selection is more severe with fire sale. Besides a direct impact on crypto asset price (temporary or not), fire sale in our model also triggers the feedback loop identified earlier: a lower asset price makes DeFi debt more information sensitive, results in more adverse selection in the DeFi lending market, leading to the withdrawal of the funding from the lenders, which in turn justifies even a lower asset price, and so on. This feedback loop might result in equilibrium multiplicity and hence amplifies the downward price impact from fire sale leading to more volatile equilibrium outcomes.

Finally, we study the case when fire sales might be temporary ($1 > \sigma > 0$). In this case, the impact of liquidation is only within the period and with probability σ , the state next period will return to normal. Following proposition 8, it is straightforward to show that the result of multiple equilibria is robust. Furthermore, we show that the anticipation of multiplicity in the fire-sale state might lead to multiplicity in the normal state, which is another channel that information friction amplifies the fire sale which making the price impact of the sale permanent even in the case when the fire sales are temporary. Hence the following theoretical result of price multiplicity offers a potential explanation of the empirical finding of permanent price impact of fire sale in Lehar and Parlour (2022a).

Proposition 9. *When the probability of fire sale at each period is ($0 < \sigma < 1$), suppose the expected survival probability of the crypto asset satisfies $\mathbb{E}_L \tilde{s} \in (\check{\kappa}_P, \check{\kappa}_S)$. There exists a threshold for haircut such that when the haircut h is below this threshold, there are multiple equilibrium outcomes in both normal and distressed states.*

6 Related Literature and Discussion

We have developed a dynamic equilibrium model for studying decentralized lending protocols such as Aave and Compound. While there is a young and growing literature on decentralized finance, there is relatively limited work on DeFi lending platforms. Besides those mentioned earlier, most studies focus on the special institutional arrangements in DeFi. Mueller (2022) find that the liquidation mechanism in the DeFi lending protocol increases cost and risk for leveraged investment. Heimbach and Huang (2023) examine leverage decisions (eg collateral use, leverage ratio) of DeFi borrowers and their determinants. Rivera, Saleh, and Vandeweyer (2023) study the programmable interest rate function in the DeFi lending protocols that implements optimal competitive equilibrium. Most existing DeFi papers study decentralized exchanges to understand how automated market makers (e.g., Uniswap) function differently from a traditional exchange (e.g., see Aoyagi and Itoy (2021), Capponi and Jia (2021), Lehar and Parlour (2021), Park (2021)). There are also papers investigating the structure of decentralized stablecoins such as Dai issued by the MakerDAO (e.g., d’Avernas, Bourany, and Vandeweyer (2021), Li and Mayer (2021), Kozhan and Viswanath-Natraj (2021)). Lehar and Parlour (2022b) study empirically the puzzling permanent impact of collateral liquidations on asset prices. For a general overview of DeFi architecture and applications, see Harvey et al. (2021) and Schar (2021). Chiu, Kahn, and Koepl (2022) study the value propositions and limitations of DeFi. Vulnerabilities that make DeFi lending protocols fragile (e.g., price oracle exploits by borrowers) are studied in the recent computer science literature. These computer science papers focus mainly on the efficiency of design features of these protocols (e.g., see Gudgeon et al. (2020), Perez et al. (2021), Qin et al. (2020), Qin et al. (2021)).

A key distinct friction in our model from the existing DeFi literature is asymmetric information about the asset’s future payoff between the borrowers and the lenders. One might question the nature of this information asymmetry: is it possible for the owner of the crypto assets to know more about the future payoff than the other participants in the DeFi system? As discussed previously, borrowers can have information advantage relative to the lending protocol when the smart contract relies on a stale price feed from the Oracle. Since crypto asset prices are volatile and most lenders have little knowledge of the collateral mix used by the borrowers, the owners of the cryptoassets, who are also borrowers, are more likely to have better information about the crypto asset value. Alternatively, if the dividend payoff from cryptoassets takes the form of convenience yields (e.g, resulting from improved searching and matching technology among buyers and sellers), asset owners may be privately informed about the extent of the convenience yield since they know whether they need to transact in the future or not. Moreover, by incorporating information friction, our model explains how firesale might have permanent price impact

even when DeFi capital is fast moving. The price-funding liquidity feedback channel, compounds by firesale, as shown in the model, could potentially destabilize the DeFi ecosystem.

Additionally, our model is related to existing theoretical works on collateralized borrowing in a general equilibrium setting such as Geanakoplos (1997), Geanakoplos and Zame (2002), Geanakoplos (2003), Fostel and Geanakoplos (2012) and Ozdenoren, Yuan, and Zhang (2021). In Ozdenoren, Yuan, and Zhang (2021), the focus is on how flexible and optimal security design eliminates equilibrium multiplicity. In this paper, we take contract rigidity seriously, match it with institutional details of the DeFi system, and study its implications for sentiment-driven sunspot equilibrium. Applying the concept of sentiment helps us to understand the nature of pro-cyclicality in the crypto ecosystem.

To summarize, in this paper, we investigate the operation of DeFi lending protocols and uncover a distinctive source of fragility that emerges with their adoption. Crypto-assets, by nature, are inherently volatile as payment instruments. With the advent of DeFi lending services, these tokenized crypto-assets can also serve as collateral. As DeFi lending expands and more crypto assets are utilized as collateral, there is an increase in system-wide leverage, liquidity, value creation, and interconnectedness. The unique institutional characteristics of DeFi, compounded by underlying information frictions, can potentially trigger self-fulfilling sentiment-driven cycles and runs on DeFi, ultimately introducing fragility into the crypto ecosystem.

Moreover, the inherent DeFi fragility we have demonstrated through our model underscores the need for policies that safeguard the stability of the DeFi ecosystem. The current risk management practice established by individual protocols may not be adequate. The standard approach is to require haircuts/margins not just at loan initiation but also throughout the loan’s duration. Margins are maintained by liquidators who are incentivized to liquidate loans nearing insolvency.³⁰ Additionally, some protocols set up reserves funded by transaction fees and ask equity holders to absorb the initial loss if there is any. However, there are countervailing incentives because increasing haircuts or reserve fees

³⁰Note that price volatility and information problem associated with collateral does not necessarily lead to high liquidations and bad loans in equilibrium as lending terms are endogenously determined. In practice, large haircuts (as high as 70-80% for some tokens) are imposed to protect lenders, implying that the realized liquidation rate is not too high. For example, between November 2021 and December 2022, on average only 0.09% of the outstanding loans are liquidated in the Aave USDT pool. The liquidation rate is higher (at 0.21%) when the market sentiment is bad (in the “extreme fear” category of the FG Index) and the liquidation rate is lower (at 0.10%) when the market sentiment is good (in the “extreme greed” category of the FG Index). This is a feature captured by our model as the lending market responds to different sentiment levels. In addition, to further protect the lenders, AAVE holders are incentivized to lock their governance tokens into a Safety Module to share the loss in case of a shortfall event. As discussed, this feature can be incorporated easily into our model.

can discourage participation and reduce protocol profitability. These conflicts highlight the challenge faced by protocols in internalizing the costs associated with systemic failures when establishing their own backstops.

More important, achieving stability in DeFi requires flexible and state-contingent smart contracts, which often take complex forms. It is imperative that the oracle for DeFi applications is reliable and provides real-time hard and soft information from both on- and off-chain worlds. To achieve these, DeFi lending may need to compromise on complete decentralization and reintroduce human intervention for real-time risk management – an arrangement that forces the protocol to rely on a trusted third party. This discussion highlights a trilemma faced by DeFi protocols: the challenge of simultaneously achieving simplicity in smart contracts, stability in asset prices, and a high degree of decentralization. We defer to future studies to explore financial regulations that optimize these economic trade-offs.

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A Appendix

A.1 Proof of Proposition 1

Condition (2) implies that, in a pooling equilibrium, the high-type borrower is willing to borrow if and only if

$$zq^P \geq \mathbb{E} \min\{D, \delta + \phi\},$$

which is equivalent to

$$\mathbb{E}y_L(s_L, \phi)/\mathbb{E}y_H(\phi) \geq \bar{\zeta}.$$

If $\mathbb{E}y_L(s_L, \phi)/\mathbb{E}y_H(\phi) > \bar{\zeta}$ then it is optimal for the intermediary to set $R = R^P$. To see this, note that at this rate lenders provide loan q^P and, by assumption, the high type borrower indeed chooses to borrow. This is clearly optimal because setting a higher rate lowers total lending and at a lower rate lenders do not break even. If $\mathbb{E}y_L(s_L, \phi)/\mathbb{E}y_H(\phi) < \bar{\zeta}$ then the intermediary's problem is solved by setting $R = R^S$. In this case, if the intermediary lowers the rate sufficiently below R^P then the high type would borrow. However, at that rate lenders would make negative profit.

Since $\mathbb{E}y_L(s_L, \phi)/\mathbb{E}y_H(\phi) = \mathbb{E} \min\{1, \frac{s_L \phi}{(\delta + \phi)(1-h)}\}$, a higher ϕ or h make the condition for the pooling outcome easier to satisfy.

A.2 Proof of Lemma 1

First, we define functions

$$\begin{aligned} \hat{q}^S(\phi) &= \frac{1}{1+f} \mathbb{E}_L [\min\{(1-h)(\phi + \delta), s\phi\}], \\ \hat{q}^P(\phi) &= \frac{1}{1+f} [\lambda \mathbb{E}_L [\min\{(1-h)(\phi + \delta), s\phi\}] + (1-\lambda)(1-h)(\phi + \delta)]. \end{aligned}$$

$$\mathbb{E}_L [\min\{(1-h)(\phi + \delta), s\phi\}] = \left(\int_0^{\hat{s}(\phi)} s dF_L(s) \right) \phi + (1 - F_L(\hat{s}(\phi))) (1-h)(\phi + \delta)$$

where $\hat{s}(\phi) = \frac{(1-h)(\phi + \delta)}{\phi}$ if $(1-h)\delta < h\phi$ and $\hat{s}(\phi) = 1$ otherwise. Note $\hat{s}'(\phi) = -\frac{(1-h)\delta}{\phi^2}$ if $(1-h)\delta < h\phi$ and $\hat{s}'(\phi) = 0$ otherwise.

$$\begin{aligned} (1+f) \hat{q}^{P'}(\phi) &= \lambda \left[\left(\int_0^{\hat{s}(\phi)} s dF_L(s) \right) + (1 - F_L(\hat{s}(\phi))) (1-h) \right] \\ &\quad + \lambda f_L(\hat{s}(\phi)) \hat{s}'(\phi) (\hat{s}(\phi) \phi - (1-h)(\phi + \delta)) + (1-\lambda)(1-h) \end{aligned}$$

If $(1-h)\delta < h\phi$ we have $\hat{s}(\phi) \phi - (1-h)(\phi + \delta) = 0$. If $(1-h)\delta \geq h\phi$ we have $\hat{s}'(\phi) = 0$. Hence

$$\begin{aligned} (1+f) \hat{q}^{P'}(\phi) &= \lambda \left[\left(\int_0^{\hat{s}(\phi)} s dF_L(s) \right) + (1 - F_L(\hat{s}(\phi))) (1-h) \right] \\ &\quad + (1-\lambda)(1-h) < 1. \end{aligned}$$

Similarly

$$(1+f) \hat{q}^{S'}(\phi) = \left[\left(\int_0^{\hat{s}(\phi)} s dF_L(s) \right) + (1 - F_L(\hat{s}(\phi))) (1-h) \right] < 1.$$

Note that their difference is

$$\begin{aligned}
& \hat{q}^P(\phi) - \hat{q}^S(\phi) \\
&= \frac{1-\lambda}{1+f} [(1-\lambda)(1-h)(\phi + \delta) - \mathbb{E} \min\{(1-\lambda)(1-h)(\phi + \delta), s_L \phi\}] \\
&\geq 0,
\end{aligned}$$

Similarly, we define functions

$$\begin{aligned}
\hat{\phi}^P(\phi) &= \beta [(z-1-f)\hat{q}^P(\phi)] + \beta(1-\lambda)\delta + \beta(\lambda\mathbb{E}(s_L) + (1-\lambda))\phi, \\
\hat{\phi}^S(\phi) &= \beta\lambda(z-1-f)\hat{q}^S(\phi) + \beta(1-\lambda)\delta + \beta(\lambda\mathbb{E}(s_L) + (1-\lambda))\phi.
\end{aligned}$$

Note:

$$\begin{aligned}
\hat{\phi}^{P'}(\phi) &= \beta [(z-1-f)\hat{q}^{P'}(\phi)] + \beta(\lambda\mathbb{E}(s_L) + (1-\lambda)) < 1, \\
\hat{\phi}^{S'}(\phi) &= \beta\lambda(z-1-f)\hat{q}^{S'}(\phi) + \beta(\lambda\mathbb{E}(s_L) + (1-\lambda)) < 1,
\end{aligned}$$

$$\begin{aligned}
\hat{\phi}^P(0) &= \beta(1-\lambda)\delta + \beta \frac{(z-1-f)(1-\lambda)(1-h)\delta}{1+f} > \beta(1-\lambda)\delta = \hat{\phi}^S(0), \\
\hat{\phi}^{P'}(\phi) &> 0 \text{ and } \hat{\phi}^{S'}(\phi) > 0.
\end{aligned}$$

Furthermore, the difference between the two functions is

$$\begin{aligned}
& \hat{\phi}^P(\phi) - \hat{\phi}^S(\phi) \\
&= \beta(1-\lambda)(z-1-f)\hat{q}^P(\phi) + \beta\lambda(z-1-f)(\hat{q}^P(\phi) - \hat{q}^S(\phi)) > 0.
\end{aligned}$$

The above properties imply that both functions have a unique fixed point and that $\phi^P > \phi^S$.

A.3 Proof of Proposition 3

For simplicity we set $f = 0$ but the result also holds for $f > 0$. In a separating equilibrium debt limit, loan size and asset price when $h = 0$ are given by:

$$D^S = (\delta + \phi^S)$$

$$q^S = \mathbb{E}_L(s)\phi^S$$

$$\phi^S = \frac{\beta(1-\lambda)\delta}{1 - \beta[\lambda z \mathbb{E}_L(s) + (1-\lambda)]}$$

Plugging the asset price into the condition for the existence of a separating equilibrium we obtain:

$$\zeta(\phi^S; 0) = \frac{\mathbb{E}_L(s)\phi^S}{(\delta + \phi^S)} < \bar{\zeta}$$

Rearranging we find that a separating equilibrium exists at $h = 0$ when

$$\mathbb{E}_L(s) < \frac{\bar{\zeta}}{\beta[(1-\lambda) + \bar{\zeta}\lambda z]} \equiv \kappa_S.$$

Furthermore, if the above condition holds, a separating also exists in a neighborhood of $h = 0$.

Similarly, in a pooling equilibrium debt limit, loan size and asset price when $h = 0$ are given by:

$$D^P = (\delta + \phi^P)$$

$$q^P = \lambda \mathbb{E}_L(s)\phi^P + (1-\lambda)(\delta + \phi^P)$$

$$\phi^P = \frac{\beta z(1-\lambda)\delta}{1 - \beta z[\lambda \mathbb{E}(s) + (1-\lambda)]}$$

Plugging the asset price into the condition for the existence of a pooling equilibrium we obtain:

$$\zeta(\phi^P; 0) = \frac{\mathbb{E}_L(s)\phi^P}{(\delta + \phi^P)} > \bar{\zeta}$$

Rearranging we find that a pooling equilibrium exists at $h = 0$ when

$$\mathbb{E}_L(s) > \frac{\bar{\zeta}}{\beta z[(1-\lambda) + \bar{\zeta}\lambda]} \equiv \kappa_P < \kappa_S$$

Furthermore, if the above condition holds, a pooling also exists in a neighborhood of $h = 0$.

Therefore, when $\mathbb{E}(s) \in (\kappa_P, \kappa_S)$, there are multiple equilibria in a neighborhood of $h = 0$ which implies that there is a threshold for haircut below which multiple equilibria exist.

A.4 Two-point Distribution Example

A.4.1 Separating Equilibrium

Suppose $s_L = 1$ w.p. π , and $s_L = 0$ w.p. $1 - \pi$.

In a separating equilibrium:

Debt limit:

$$D^S = (\delta + \phi^S)(1 - h)$$

Loan size:

$$\ell_L = q^S = \mathbb{E}[\min\{D^S, s\phi^S\}] = \pi \min\{D^S, \phi^S\}$$

There are two cases.

Case (i) $D^S > \phi^S$

This is true when

$$\delta \frac{1-h}{h} > \phi^S.$$

We then have

$$\begin{aligned} q^S &= \pi \phi^S, \\ \phi^S &= \frac{\beta(1-\lambda)\delta}{1 - \beta[\lambda z\pi + (1-\lambda)]}. \end{aligned}$$

The existence of separating equilibrium requires

$$\zeta^S(h) = \frac{\pi \phi^S}{(\delta + \phi^S)(1-h)} < \zeta.$$

We define a threshold

$$\underline{h}^S \equiv \frac{\delta}{\phi^S + \delta} = \frac{1 - \beta[\lambda z\pi + (1-\lambda)]}{1 - \beta\lambda z\pi}.$$

When the haircut is lower than the threshold \underline{h} , the low type borrowers default even when $s_L = 1$. In this case, the loan size is equal to the expected value of the asset, $\pi \phi^S$, which does not depend on the haircut. Hence, the asset price is also independent of h . An increase in h , however, makes it harder to support a separating equilibrium as the contract becomes less information sensitive.

Case (ii) $D^S < \phi^S$

This is true when

$$\delta \frac{1-h}{h} < \phi^S.$$

We then have

$$\begin{aligned} q^S &= \pi(\delta + \phi^S)(1-h) \\ \phi^S &= \frac{\beta(\lambda(z-1)\pi(1-h) + (1-\lambda))\delta}{1 - \beta[\lambda(z-1)\pi(1-h) + (1-\lambda) + \lambda\pi]}. \end{aligned}$$

The existence of separating equilibrium requires

$$\zeta^S(h) = \pi < \zeta.$$

When the haircut is higher than the threshold \underline{h} , the low type pays back the loan to retain the collateral when $s_L = 1$. In this case, the loan size is equal to the πD . Hence, the asset price is decreasing in h . A separating equilibrium exists whenever $\pi < \zeta$ as h does not affect the information sensitivity of the contract.

A.4.2 Pooling Equilibrium

In a pooling equilibrium:

Debt limit:

$$D^P = (\delta + \phi^P)(1 - h)$$

Loan size:

$$q^P = \lambda \mathbb{E}[\min\{D^P, s\phi^P\}] + (1 - \lambda)D^P = \lambda \pi \min\{D^P, \phi^P\} + (1 - \lambda)D^P$$

There are two cases.

Case (i) $D^P > \phi^P$

This is true when

$$\delta \frac{1 - h}{h} > \phi^P.$$

We then have

$$\begin{aligned} q^P &= \lambda \pi \phi^P + (1 - \lambda)D^P \\ \phi^P &= \frac{\beta(1 - \lambda)\delta[(z - 1)(1 - h) + 1]}{1 - \beta[\lambda(z - 1)\pi + (z - 1)(1 - \lambda)(1 - h) + \lambda\pi + 1 - \lambda]} \end{aligned}$$

The existence of separating equilibrium requires

$$\zeta^P(h) = \frac{\pi \phi^P}{(\delta + \phi^P)(1 - h)} > \zeta.$$

We can again define a threshold

$$\underline{h}^P \equiv \frac{1 - \beta[\lambda(z - 1)\pi + (z - 1)(1 - \lambda) + \lambda\pi + 1 - \lambda]}{1 - z\beta\lambda\pi - \beta(z - 1)(1 - \lambda)} < \underline{h}^S$$

such that this case holds when $h < \underline{h}^P$.

Case (ii) $D^P < \phi^P$

This is true when

$$\delta \frac{1-h}{h} < \phi^P.$$

We then have

$$\begin{aligned} q^P &= \lambda \pi D^P + (1-\lambda) D^P \\ \phi^P &= \beta \delta \frac{(z-1)(\lambda \pi + 1 - \lambda)(1-h) + (1-\lambda)}{1 - \beta[(z-1)(\lambda \pi + 1 - \lambda)(1-h) + \lambda \pi + 1 - \lambda]} \end{aligned}$$

The existence of pooling equilibrium requires

$$\zeta^P(h) = \pi > \zeta.$$

A.5 Proof of Uniqueness Under a Flexible Smart Contract

Denote the debt contract $y(D, \tilde{\delta} + \tilde{s}\phi) = \min(D, \tilde{\delta} + \tilde{s}\phi)$. We prove the result for the main model where

$$\mathbb{E}_H y(D, \tilde{\delta} + \tilde{s}\phi) = y(D, \delta + \phi);$$

and

$$\mathbb{E}_L y(D, \tilde{\delta} + \tilde{s}\phi) = \int_{\underline{s}}^{\bar{s}} y(D, s\phi) dF(s).$$

The arguments, however, generalize to the more general case with some modifications.

Denote $D^* \leq \delta + \phi$ the maximum face value so that the incentive constraint of the high type borrower is satisfied

$$\hat{z} \left[\lambda \mathbb{E}_L y(D, \tilde{\delta} + \tilde{s}\phi) + (1-\lambda) \mathbb{E}_H y(D, \tilde{\delta} + \tilde{s}\phi) \right] \geq \mathbb{E}_H y(D, \tilde{\delta} + \tilde{s}\phi)$$

in which case there is a pooling equilibrium.

When the intermediary designs the smart deposit contract flexibly, it aims to maximize the expected trading volume. Specifically, the intermediary chooses D , or equivalently haircut, to maximize expected trade volume $[\lambda \mathbb{E}_L + a_{H,t} (1-\lambda) \mathbb{E}_H] \min(D, \tilde{\delta} + \tilde{s}\phi)$ taking ϕ as given. Note that the intermediary's payoff is increasing in D as long as the equilibrium does not switch from pooling to separating. Hence, if the intermediary chooses a contract that leads to a pooling outcome, then $D = D^*$, and if the intermediary chooses a contract that leads to a separating outcome, then $D = \delta + \phi$.

Next we look at the two cases:

Pooling case:

If $D < \phi$, we can denote $\hat{s} = D/\phi$. In this case, all terms in the incentive constraint for the high type are proportional to the asset price ϕ , which drops out of the constraint. So, the high type's incentive constraint is satisfied iff

$$\hat{z}[\lambda \mathbb{E}_L \min(\hat{s}, s) + (1 - \lambda)\hat{s}] \geq \hat{s}$$

Let $\mathcal{F}(\hat{s}) \equiv \hat{z}[\lambda \mathbb{E}_L \min(\hat{s}, s) + (1 - \lambda)\hat{s}] - \hat{s}$ and note the high type's incentive constraint is satisfied iff $\mathcal{F}(\hat{s}) \geq 0$. $\mathcal{F}(\hat{s})$ has the following properties:

$$\begin{aligned}\mathcal{F}(0) &\geq 0 \\ \mathcal{F}'(0) &= \hat{z} - 1 > 0 \\ \mathcal{F}''(\hat{s}) &= -\hat{z}\lambda f(\hat{s}) < 0\end{aligned}$$

So $\mathcal{F}(\hat{s})$ is concave and strictly positive when \hat{s} is close to 0. Suppose the information friction is severe enough so that $\mathcal{F}(1) = \hat{z}(\lambda \mathbb{E}_L s + (1 - \lambda)) - 1 < 0$, or equivalently $\mathbb{E}_L s < \frac{1 - (1 - \lambda)\hat{z}}{\lambda \hat{z}} = 1 + \frac{1}{\lambda \hat{z}} - \frac{1}{\lambda} < 1$. In this case, there exists a unique threshold $0 < s^* < 1$ such that $\mathcal{F}(s^*) = 0$. Since the asset price ϕ drops out, threshold s^* does not depend on ϕ .

Taking next period asset price ϕ as given, the asset price in the current period under pooling equilibrium is

$$\phi^P(\phi) = \beta [(\hat{z} - 1)(\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^*)\phi + \lambda \phi \mathbb{E}_L s + (1 - \lambda)(\delta + \phi)] \quad (\text{A.1})$$

which has the following property

$$\begin{aligned}\frac{\partial \phi^P(\phi)}{\partial \phi} &= \beta [(\hat{z} - 1)(\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda)s^*) + \lambda \mathbb{E}_L s + (1 - \lambda)] < 1 \\ \phi^P(0) &= \beta(1 - \lambda)\delta.\end{aligned}$$

So, $\phi^P(\phi)$ is a straight line with slope $\frac{\partial \phi^P(\phi)}{\partial \phi}$ and intercept $\phi^P(0) = \beta(1 - \lambda)\delta$. Hence there is a unique steady state price satisfying $\phi^P(\phi) = \phi$.

Suppose information friction is not so severe so that $\mathcal{F}(1) > 0$, or equivalently, $1 > \mathbb{E}_L s > 1 + \frac{1}{\lambda \hat{z}} - \frac{1}{\lambda}$. In this case, the face value of the debt is $D^* \geq \phi$. Let $d^*(\phi) = D^* - \phi$. There are two possibilities: either high type's incentive constraint is binding and there is $d^*(\phi) \leq \delta$ that satisfies:

$$\hat{z}[\lambda \phi \mathbb{E}_L s + (1 - \lambda)(d^*(\phi) + \phi)] = d^*(\phi) + \phi$$

or the high-type's incentive constraint is slack for all D . In the former case

$$d^*(\phi) = \frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)] - 1}{1 - \widehat{z}(1-\lambda)}\phi.$$

In the latter case $d^*(\phi) = \delta$. If $\frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)] - 1}{1 - \widehat{z}(1-\lambda)}\phi < \delta$,

$$\phi^P(\phi) = \beta \left[\frac{\lambda\widehat{z}}{1 - \widehat{z}(1-\lambda)} \lambda\mathbb{E}_L s \phi + (1-\lambda)(\delta + \phi) \right]. \quad (\text{A.2})$$

Note,

$$\begin{aligned} \phi^P(0) &= \beta(1-\lambda)\delta, \\ \frac{\partial \phi^P(\phi)}{\partial \phi} &= \beta \left(\frac{\lambda\widehat{z}}{1 - \widehat{z}(1-\lambda)} \lambda\mathbb{E}_L s + 1 - \lambda \right). \end{aligned}$$

Hence $\phi^P(\phi)$ is a straight line with slope $\frac{\partial \phi^P(\phi)}{\partial \phi}$ and intercept $\phi^P(0)$.

If $\frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)]}{1 - \widehat{z}(1-\lambda)}\phi > \delta$,

$$\begin{aligned} \phi^P(\phi) &= \beta\widehat{z}[\lambda\mathbb{E}_L s \phi + (1-\lambda)(\delta + \phi)] \\ &= \beta\widehat{z}[(1-\lambda)\delta + (\lambda\mathbb{E}_L s + 1 - \lambda)\phi]. \end{aligned}$$

Note,

$$\begin{aligned} \phi^P(0) &= \beta\widehat{z}(1-\lambda)\delta, \\ \frac{\partial \phi^P(\phi)}{\partial \phi} &= \beta\widehat{z}(\lambda\mathbb{E}_L s + 1 - \lambda) < 1 \end{aligned}$$

By comparing the slopes of $\phi^P(\phi)$ when $\frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)]}{1 - \widehat{z}(1-\lambda)}\phi$ is below and above δ , we can see that $\phi^P(\phi)$ is concave with slope less than 1 when $\frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)]}{1 - \widehat{z}(1-\lambda)}\phi > \delta$.

Note that when $D^* \geq \phi$ in a pooling equilibrium or $\mathbb{E}_L s > 1 + \frac{1}{\lambda\widehat{z}} - \frac{1}{\lambda}$, the value of a pooling contract is always greater than that of a separating contract. This is because the intermediary designs the contract optimally to maximize the expected trade volume. The expected value of a loan to a low type is the same in a separating equilibrium and a pooling equilibrium when $D^* \geq \phi$. So the intermediary strictly prefers designing a pooling contract as the revenue from the pooling contract strictly dominates that of a separating contract.

Hence when $\mathbb{E}_L s > 1 + \frac{1}{\lambda\widehat{z}} - \frac{1}{\lambda}$, we can focus on the pooling equilibrium. From the analysis above, $\phi^P(\phi)$ is concave with slope less than 1 when $\frac{\widehat{z}[\lambda\mathbb{E}_L s + (1-\lambda)]}{1 - \widehat{z}(1-\lambda)}\phi > \delta$. Hence, in this part of the parameter space there exists a unique equilibrium where the loan is traded in a pooling equilibrium.

Separating case:

As argued above, when analyzing the optimal contract in a separating equilibrium, we can focus on the parameter space where

$$E_L s < 1 + \frac{1}{\lambda \hat{z}} - \frac{1}{\lambda}. \quad (\text{A.3})$$

If the optimal contract supports a separating equilibrium, the intermediary would set $D = \delta + \phi$ to maximize the loan size to the low type. In the special parametrization of the model, any face value between ϕ and $\delta + \phi$ generates the same revenue from borrowing because a low quality asset does not pay any dividend. More generally, low quality assets could pay positive dividend. So the maximum face value $D = \delta + \phi$ is a more robust form of debt design in the separating case.

Given the face value $D = \delta + \phi$, the incentive constraint for the high type not to borrow is

$$\delta + \phi \geq \hat{z} E_L s \phi \quad (\text{A.4})$$

Note that condition (A.3) implies that

$$\hat{z} E_L s < 1 + (\hat{z} - 1) \left(1 - \frac{1}{\lambda}\right) < 1.$$

The condition for the existence of a separating equilibrium, (A.4), always holds.

In a separating equilibrium, the asset price is

$$\phi^S(\phi) = \beta [(\hat{z} - 1) \lambda E_L s \phi + \lambda E_L s \phi + (1 - \lambda)(\delta + \phi)] \quad (\text{A.5})$$

which has the following property

$$\begin{aligned} \phi^S(0) &= \beta(1 - \lambda)\delta \\ \frac{\partial \phi^S(\phi)}{\partial \phi} &= \beta(\hat{z} \lambda E_L s + 1 - \lambda) \end{aligned}$$

So in this case, $\phi^S(\phi)$ is a straight line with slope $\frac{\partial \phi^S(\phi)}{\partial \phi}$ and intercept $\phi^S(0) = \beta(1 - \lambda)\delta$.

The intermediary chooses the pooling contract if and only if

$$[\lambda E_L + (1 - \lambda) E_H] y(D, \tilde{\delta} + \tilde{s} \phi^P) \geq \lambda E_L y(D, \tilde{\delta} + \tilde{s} \phi^S)$$

or

$$[\lambda E_L \min(s^*, s) + (1 - \lambda) s^*] \phi^P \geq \phi^S \lambda E_L s$$

where s^* is the unique solution to

$$\hat{z} [\lambda E_L \min(s^*, s) + (1 - \lambda) s^*] = s^*.$$

Plugging in for ϕ^P and ϕ^S we can rewrite the inequality as

$$\frac{[\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda) s^*]}{1 - \beta [(\widehat{z} - 1)(\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda) s^*) + \lambda \mathbb{E}_L s + (1 - \lambda)]} \geq \frac{\lambda \mathbb{E}_L s}{1 - \beta [(\widehat{z} - 1)\lambda \mathbb{E}_L s + \lambda \mathbb{E}_L s + (1 - \lambda)]}$$

which holds iff

$$\lambda \mathbb{E}_L \min(s^*, s) + (1 - \lambda) s^* - \lambda \mathbb{E}_L s \geq 0. \quad (\text{A.6})$$

In either case, the equilibrium is unique.

To summarize the equilibrium characterization, when $\mathbb{E}_L s < 1 + \frac{1}{\lambda \widehat{z}} - \frac{1}{\lambda}$, the equilibrium contract is a pooling one with face value $D = s^* \phi < \phi$ when condition (A.6) holds. Otherwise, the equilibrium contract is a separating one with face value $D = \delta + \phi$.

When $\mathbb{E}_L s > 1 + \frac{1}{\lambda \widehat{z}} - \frac{1}{\lambda}$, the equilibrium contract is a pooling one with face value $D = d^* + \phi$ where

$$d^* = \min \left\{ \delta, \frac{\widehat{z} [\lambda \mathbb{E}_L s + (1 - \lambda)] - 1}{1 - \widehat{z}(1 - \lambda)} \phi \right\}.$$

A.6 Proof of Lemma 2

We define $\hat{q}^S(\phi)$ and $\hat{q}^P(\phi)$ exactly as in the proof of Proposition 1 and following similar steps obtain $0 < \hat{q}^{S'}(\phi) < 1$, $0 < \hat{q}^{P'}(\phi) < 1$, $\hat{q}^P(\phi) \geq \hat{q}^S(\phi)$.

Next, we define functions

$$\hat{\phi}^P(\phi) = \beta [(z - 1 - f)\hat{q}^P(\phi)] + \beta(1 - \lambda)\delta + \beta(\lambda \mathbb{E}(s_L) + (1 - \lambda))\phi - \Lambda \lambda f_L \left(\frac{(1 - h)(\phi + \delta)}{\phi} \right),$$

$$\hat{\phi}^S(\phi) = \beta \lambda (z - 1 - f)\hat{q}^S(\phi) + \beta(1 - \lambda)\delta + \beta(\lambda \mathbb{E}(s_L) + (1 - \lambda))\phi - \Lambda f_L \left(\frac{(1 - h)(\phi + \delta)}{\phi} \right),$$

which have the following properties:

$$\hat{\phi}^P(0) = \beta(1 - \lambda)\delta + \beta \frac{(z - 1 - f)(1 - \lambda)(1 - h)\delta}{1 + f} - \Lambda \lambda > \beta(1 - \lambda)\delta - \Lambda = \hat{\phi}^S(0) > 0$$

where the last inequality follows because $\Lambda < \beta(1 - \lambda)\delta$ by assumption. Note:

$$\hat{\phi}^{P'}(\phi) = \beta [(z - 1 - f)\hat{q}^{P'}(\phi)] + \beta(\lambda \mathbb{E}(s_L) + (1 - \lambda)) + \Lambda \lambda f_L \left(\frac{(1 - h)(\phi + \delta)}{\phi} \right) \left(\frac{(1 - h)\delta}{\phi^2} \right),$$

$$\hat{\phi}^{S'}(\phi) = \beta \lambda (z - 1 - f)\hat{q}^{S'}(\phi) + \beta(\lambda \mathbb{E}(s_L) + (1 - \lambda)) + \Lambda f_L \left(\frac{(1 - h)(\phi + \delta)}{\phi} \right) \left(\frac{(1 - h)\delta}{\phi^2} \right),$$

and

$$\hat{\phi}^{P'}(\phi) > 0 \text{ and } \hat{\phi}^{S'}(\phi) > 0.$$

Note that both $\hat{\phi}^{P'}(\phi)$ and $\hat{\phi}^{S'}(\phi)$ are strictly less than 1 for large ϕ . To see this note that the first two terms are strictly less than 1 and the last term becomes small as ϕ increases. Finally, the difference between the two functions is

$$\begin{aligned} & \hat{\phi}^P(\phi) - \hat{\phi}^S(\phi) \\ &= \beta(1-\lambda)(z-1-f)\hat{q}^P(\phi) + \beta\lambda(z-1-f)(\hat{q}^P(\phi) - \hat{q}^S(\phi)) \\ &+ (1-\lambda)\Lambda F_L\left(\frac{(1-h)(\phi+\delta)}{\phi}\right) > 0. \end{aligned}$$

The above properties imply that both functions have at least one fixed point and the largest fixed point is larger for $\hat{\phi}^P(\phi)$ than $\hat{\phi}^S(\phi)$. So when fixed points are unique pooling price exceeds separating price and otherwise the largest pooling equilibrium price exceeds all separating prices.

A.7 Proof of Proposition 8

For simplicity we set $f = 0$ but the result also holds for $f > 0$. In a separating equilibrium with fire sale every period, debt limit, loan size and asset price when $h = 0$ are given by:

$$D^{fS} = (\delta + \phi^{fS})$$

$$q^{fS} = \mathbb{E}_L(s)\phi^{fS}$$

$$\phi^{fS} = \frac{\beta(1-\lambda)\delta - \Lambda\lambda}{1 - \beta[\lambda z \mathbb{E}_L(s) + (1-\lambda)]}$$

Plugging the asset price into the condition for the existence of a separating equilibrium we obtain:

$$\zeta(\phi^{fS}; 0) = \frac{\mathbb{E}_L(s)\phi^{fS}}{(\delta + \phi^{fS})} < \bar{\zeta}$$

Rearranging we find that a separating equilibrium exists at $h = 0$ when

$$\mathbb{E}_L(s) < \frac{\bar{\zeta}(\delta - \Lambda\lambda)}{\beta\delta((1-\lambda) + \bar{\zeta}\lambda z) - \Lambda\lambda} \equiv \tilde{\kappa}_S$$

Similarly, in a pooling equilibrium with fire sale every period, debt limit, loan size and asset price when $h = 0$ are given by:

$$D^{fP} = (\delta + \phi^{fP})$$

$$q^{fP} = \lambda \mathbb{E}_L(s) \phi^{fP} + (1 - \lambda)(\delta + \phi^{fP})$$

$$\phi^{fP} = \frac{\beta z(1 - \lambda)\delta - \Lambda\lambda}{1 - \beta z[\lambda \mathbb{E}_L(s) + (1 - \lambda)]}$$

Rearranging we find that a separating equilibrium exists at $h = 0$ when

$$\mathbb{E}_L(s) > \frac{\bar{\zeta}(\delta - \Lambda\lambda)}{\beta z\delta((1 - \lambda) + \bar{\zeta}\lambda) - \Lambda\lambda} \equiv \tilde{\kappa}_P.$$

Clearly $\tilde{\kappa}_P < \tilde{\kappa}_S$. Therefore, when $\mathbb{E}(s) \in (\tilde{\kappa}_P, \tilde{\kappa}_S)$, there are multiple equilibria in a neighborhood of $h = 0$.

A.8 An Alternative Setup with Unobservable Private Valuation

We briefly consider an alternative setup where the private information is related to borrowers' private valuation of the asset, instead of the asset's common value. We show that the main results hold.

Suppose with probability $1 - \varepsilon$, the state is good ($s = 1$) and the asset pays dividend δ . With probability ε , the state is bad ($s = 0$), it does not pay any dividends. In addition, the borrower has unobservable private valuation. A type $i = H, L$ borrower, if holding an asset, receives a private value $v_i(s)$ before the asset market opens and after the loan is settled. The type i is determined before the loan is made and the information is private. With probability λ , the borrower is of type $i = L$, and the private valuation is $v_L(1) = v$ in the good state and $v_L(0) = 0$ in the bad state. With probability $1 - \lambda$, the borrower's type is $i = H$ and the private valuation is $v_H(1) = v_H(0) = v$. After observing the private information, the borrower borrows from the platform. After observing the realization of δ , the borrower decides whether to repay or to default. After the loan is settled, the borrower, if holding the asset, receives the private valuation. At the end of the period, the asset is traded at $\delta + \phi$ in the good state and at ϕ in the bad state.

The debt limit is given by $D = (\delta + \phi)(1 - h)$. We assume that $v > \delta$. As a result, all borrowers repay in the good state. A low type borrower defaults in the bad state when $D > \phi$. Our analysis will focus on the case of $D \geq \phi$ as it is suboptimal to set $D < \phi$.

In the separating equilibrium, the loan size is

$$q^S = D^S - \varepsilon(D^S - \phi^S)$$

and the asset price is

$$\phi^S = \beta \frac{\lambda(z-1)(1-h)(1-\varepsilon)\delta + (1-\varepsilon)\delta + (1-\varepsilon\lambda)v}{1-\beta-\beta\lambda(z-1)(1-h(1-\varepsilon))}.$$

The separating equilibrium exists when

$$\frac{(1-\varepsilon)D^S + \varepsilon\phi^S}{D^S} < \zeta.$$

In the pooling equilibrium, the loan size is

$$q^P = D^P + \lambda\varepsilon(\phi^P - D^P)$$

and the asset price is

$$\phi^P = \beta \frac{(z-1)\delta(1-h)(1-\varepsilon\lambda) + \beta(1-\varepsilon)\delta + \beta(1-\varepsilon\lambda)v}{1-\beta-\beta(z-1)(1-h(1-\varepsilon\lambda))}.$$

The pooling equilibrium exists when

$$\frac{(1-\varepsilon)D^P + \varepsilon\phi^P}{D^P} > \zeta.$$

Hence we can reproduce the main multiplicity result.

Proposition 10. *For h not too large, $\phi^P > \phi^S$ and multiplicity exists when*

$$1 - \frac{\varepsilon\delta}{\delta + \phi^P} > \zeta > 1 - \frac{\varepsilon\delta}{\delta + \phi^S}.$$

A.9 Private Information Parameter $\chi < 1$

We have considered the case where there is private information in each period. We now introduce a parameter, χ , to control the degree of information imperfection. With probability $1 - \chi$, there is no private information in the sense that there are no low-quality assets (denoted by state 0). All the equilibrium conditions remain the same except that the asset prices satisfy

$$\begin{aligned} \phi_t = \beta\chi & \left\{ \lambda \left[\int_{\underline{s}}^{\bar{s}} (z\ell_{L,t+1} - \min\{\ell_{L,t+1}R_{t+1}, a_{L,t+1}s_L\phi_{t+1}\}) + s_L\phi_{t+1} \right] dF(s_L) \right. \\ & + \chi(1-\lambda) [z\ell_{H,t+1} - \min\{\ell_{H,t+1}R_{t+1}, a_{H,t+1}(\delta + \phi_{t+1})\} + \delta + \phi_{t+1}] \\ & \left. + \beta(1-\chi) [z\ell_{t+1}^0 - \min\{\ell_{t+1}^0 R_{t+1}^0, a_{t+1}^0(\delta + \phi_{t+1})\} + \delta + \phi_{t+1}] \right\} \end{aligned}$$

where $a^0 = 1$, $\ell_t^0 = q_t^0 = \frac{1}{1+f}(\delta + \phi_t)(1-h)$ and $R_t^0 = (\delta + \phi_t)(1-h)/q_t^0$. By continuity, all results hold when χ is sufficiently close to 1.

B More Details about Aave Lending Protocol

According to DeFiLlama, there are 1485 DeFi protocols running on different blockchains (e.g., Ethereum, Terra, BSC, Avalanche, Fantom, Solana) as of April 2022. The TVL of these protocols are 237 billion USD with lending protocols accounting for about 20%. (Figure 7).³¹ Table 1 reports some basic statistics about the three main lending protocols: Compound operating on Ethereum, Venus on the BSC and Aave on multiple chains. Operating on multiple blockchains, Aave is the largest among the three in terms of TVL, deposits and borrows, and market capitalization of its governance tokens. Below, we give a brief overview of some key features of the Aave lending protocol. More details can be found in the appendix.

Table 1: Major decentralized lending Platforms (April 17, 2022)

	Aave	Compound	Venus
Total value locked (USD)	13.35 B	6.35 B	1.51 B
Blockchain	Multi	Ethereum	BSC
Total deposits (USD)	15.37 B	9.51 B	1.51 B
Total borrows (USD)	5.93 B	3.21 B	0.82 B
Governance Token	AAVE	COMP	XVS
Market Cap (USD)	2.38 B	0.99 B	0.13 B

Data Source: DefiLlama; Aavewatch; Compound.finance; Venus.io; Glassnode.

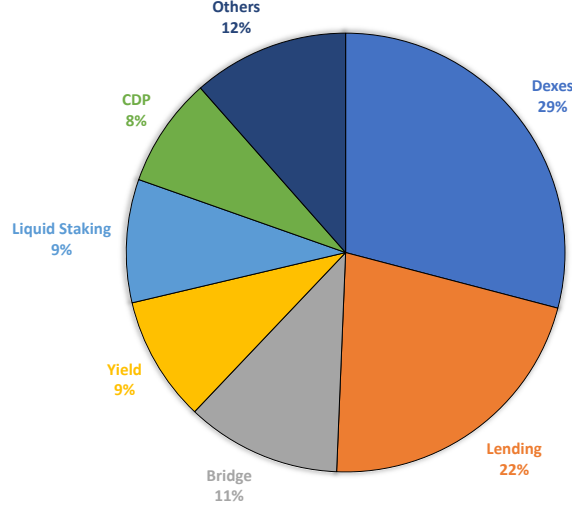
B.1 Tokens

Aave issues two types of tokens: (i) aTokens, issued to lenders so they can collect interest on deposits, and (ii) AAVE tokens, which are the native token of Aave.³² **aTokens** are interest-bearing tokens that are minted upon deposit and burned at withdraw. The aTokens’ value is pegged to the value of the corresponding deposited asset at a 1:1 ratio, and can be safely stored, transferred or traded. Withdrawals of the deposited assets burns the aTokens. **AAVE tokens** are used to vote and influence the governance of the protocol. AAVE holders can also lock (known as “staking”) the tokens to provide insurance to the protocol/depositors and earn staking rewards and fees from the protocol (more details below).

³¹Collateralized debt position (CDP), e.g., MakerDAO, accounts for 8% of the TVL. Both lending and CDP protocols support collateralized lending. The key difference is that a lending protocol lends out assets deposited by lenders while a CDP lends out assets (e.g., stablecoins) minted by the protocol.

³²One may interpret aTokens as bank deposits and AAVE tokens as bank equity shares.

Figure 7: Composition of TVL of all DeFi Protocols on all Chains (April 2022)



Data Source: DefiLlama.

B.2 Deposits and loans

By depositing a certain amount of an asset into the protocol, a **depositor** mints and receives the same amount of corresponding aTokens. All interest collected by these aTokens are distributed directly to the depositor.

Borrowers can borrow these funds with collateral backing the borrow position. A borrower repays the loan in the same asset borrowed. There is no fixed time period to pay back the loan. Partial or full repayments can be made anytime. As long as the position is safe, the loan can continue for an undefined period. However, as time passes, the accrued interest of an unpaid loan will grow, which might result in the deposited assets becoming more likely to be liquidated.

Every borrowing position can be opened with a stable or variable rate. The **loan rate** follows the model:

$$Rate = \begin{cases} R_0 + \frac{U}{U_{optimal}} R_{slope1} & , \text{ if } U \leq U_{optimal} \\ R_0 + R_{slope1} + \frac{U - U_{optimal}}{1 - U_{optimal}} R_{slope2} & , \text{ if } U > U_{optimal} \end{cases}$$

where $U = Total\ Borrow / Total\ Liquidity$ is the share of the liquidity borrowed.³³

The **variable rate** is the rate based on the current supply and demand in Aave. **Stable rates** act as a fixed rate.³⁴ The current model parameters for stable and variable interest rates are given in Figure

³³Total "liquidity" refers to the total deposits of a loanable asset.

³⁴The stable rate for new loans varies over time. However, once the stable loan is taken, borrowers will not experience

8. Figure 9 shows Dai's rate schedule as an example.

Figure 8: Current Rate Parameters

		Variable Rate			Stable Rate Rebalance if U > 95% + Average APY < 25%		
	Uoptimal	Base	Slope 1	Slope 2	Average Market Rate	Slope 1	Slope 2
BUSD	80%	0%	4%	100%			
DAI	80%	0%	4%	75%	4%	2%	75%
sUSD	80%	0%	4%	100%			
TUSD	80%	0%	4%	75%	4%	2%	75%
USDC	90%	0%	4%	60%	4%	2%	60%
USDT	90%	0%	4%	60%	4%	2%	60%
AAVE							
BAT	45%	0%	7%	300%	3%	10%	300%
ENJ	45%	0%	7%	300%			
ETH	65%	0%	8%	100%	3%	10%	100%
KNC	65%	0%	8%	300%	3%	10%	300%
LINK	45%	0%	7%	300%	3%	10%	300%
MANA	45%	0%	8%	300%	3%	10%	300%
MKR	45%	0%	7%	300%	3%	10%	300%
REN	45%	0%	7%	300%			
SNX	80%	3%	12%	100%			
UNI	45%	0%	7%	300%			
WBTC	65%	0%	8%	100%	3%	10%	100%
YFI	45%	0%	7%	300%			
ZRX	45%	0%	7%	300%	3%	10%	300%

Table Source: Aave.com

The **deposit rate** is given by

$$\text{Deposit Rate}_t = U_t(SB_t \times S_t + VB_t \times V_t)(1 - R_t)$$

where SB_t is the share of stable borrows, S_t is average stable rate, VB_t is the share of variable borrows, V_t is average variable rate, R_t is the reserve factor (a fraction of interests allocated to mitigate shortfall events discussed below). The **Loan to Value (LTV)** ratio defines the maximum amount that can be borrowed with a specific collateral. It's expressed in percentage: at $LTV = 75\%$, for every 1 ETH worth interest rate volatility. There is one caveat though: if the protocol is in dire need of liquidity, then some stable rate loans might undergo a procedure called rebalancing. In particular, it will happen if the average borrow rate is lower than 25% APY and the utilization rate is over 95%.

Figure 9: Stable vs Variable Rates for Dai

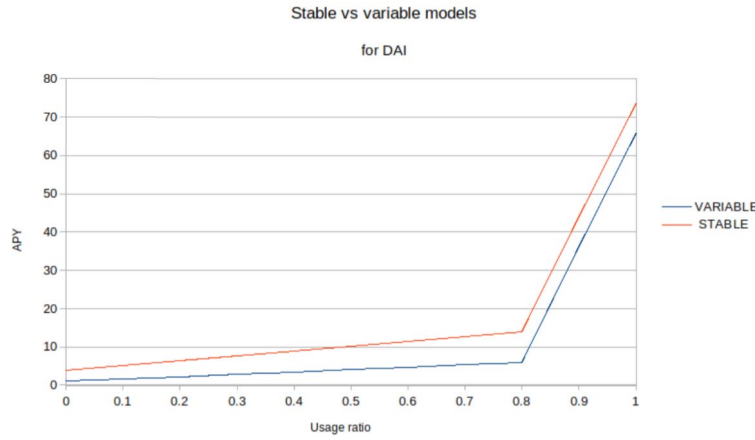


Figure Source: Aave.com

of collateral, borrowers will be able to borrow 0.75 ETH worth of the corresponding currency of the loan. The current risk parameters are given in Figure 10.

B.3 Collateral and Liquidation

The **liquidation threshold** (LQ) is the percentage at which a loan is defined as undercollateralized. For example, a LQ of 80% means that if the value rises above 80% of the collateral, the loan is undercollateralized and could be liquidated. The LQ of a borrower's position is the weighted average of those of the collateral assets:

$$LQ = \frac{\sum_i \text{Collateral } i \text{ in ETH} * LQ_i}{\text{Total Borrows in ETH}}$$

The difference between the LTV and the LQ is a safety cushion for borrowers. The values of assets are based on **price feed** provided by Chainlink's decentralized oracles. The LQ is also called the **health factor** (Hf). When $Hf < 1$, a loan is considered undercollateralized and can be liquidated. When the health factor of a position is below 1, **liquidators** repay part or all of the outstanding borrowed amount on behalf of the borrower, while receiving an equivalent amount of collateral in return plus a liquidation "bonus" (see Figure 10).³⁵ When the liquidation is completed successfully, the health factor of the

³⁵Example: Bob deposits 5 ETH and 4 ETH worth of YFI, and borrows 5 ETH worth of DAI. If Bob's Health Factor drops below 1 his loan will be eligible for liquidation. A liquidator can repay up to 50% of a single borrowed amount = 2.5 ETH worth of DAI. In return, the liquidator can claim a single collateral, as the liquidation bonus is higher for YFI (15%) than ETH (5%) the liquidator chooses to claim YFI. The liquidator claims $2.5 + 0.375$ ETH worth of YFI for repaying

Figure 10: Current Risk Parameters

	LTV	Liquidation Threshold	Liquidation Bonus	Overall Risks	Reserve Factor
BUSD				B	10%
DAI	75%	80%	5%	B	10%
sUSD				C+	20%
TUSD	75%	80%	5%	B	10%
USDC	80%	85%	5%	B+	10%
USDT				B+	10%
AAVE	50%	65%	10%	C+	
BAT	70%	75%	10%	B+	20%
ENJ	55%	60%	10%	B+	20%
ETH	80%	82.5%	5%	A+	10%
KNC	60%	65%	10%	B+	20%
LINK	70%	75%	10%	B+	20%
MANA	60%	65%	10%	B-	35%
MKR	60%	65%	10%	B-	20%
REN	55%	60%	10%	B	20%
SNX	15%	40%	10%	C+	35%
UNI	60%	65%	10%	B	20%
WBTC	70%	75%	10%	B-	20%
YFI	40%	55%	15%	B-	20%
ZRX	60%	65%	10%	B+	20%

Table Source: Aave.com

position is increased, bringing the health factor above 1.

B.4 Infrequent Updates on the Risk Parameters in Smart Contracts

Table 2: Historical AAVE V1 Risk Parameter Changes

Date	Asset	LTV	Liquidation threshold	Liquidation Bonus	Comment
10/21/20	MKR	50%	65%	10%	Decreased volatility
10/21/20	TUSD	75%	80%	5%	Following reivew of smart contract
7/22/20	LEND	50%	65%	10%	LEND cannot be borrowed due to migration incoming
7/16/20	LEND	50%	65%	10%	Improved risk parameter
7/16/20	SNX	15%	40%	10%	New Collateral
7/16/20	ENJ	55%	65%	10%	New Asset
7/16/20	REN	50%	65%	10%	New Asset
6/19/20	TUSD	1%	80%	5%	Unaudited update

2.5 ETH worth of DAI.

B.5 Shortfall Event

The primary mechanism for securing the Aave Protocol is the incentivization of AAVE holders (stakers) to lock tokens into a Smart Contract-based component called the **Safety Module** (SM). The locked AAVE will be used as a mitigation tool in case of a Shortfall Event (i.e., when there is a deficit). In the instance of a Shortfall Event, part of the locked AAVE are auctioned on the market to be sold against the assets needed to mitigate the occurred deficit. To contribute to the safety of the protocol and receive incentives, AAVE holders will deposit their tokens into the SM. In return, they receive rewards (periodic issuance of AAVE known as Safety Incentives (SI)) and fees generated from the protocol (see reserve factor above).

B.6 Recovery Issuance

In case the SM is not able to cover all of the deficit incurred, an ad-hoc Recovery Issuance event is triggered where new AAVE is issued and sold in an open auction.

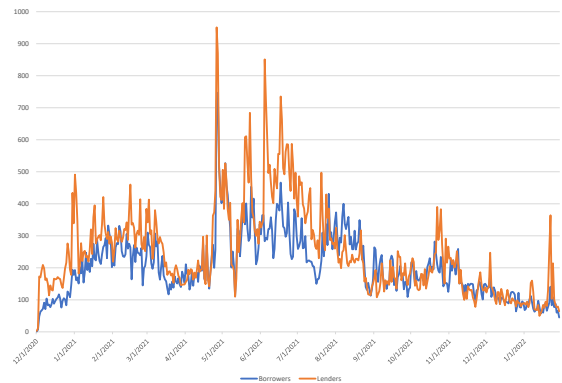
B.7 Some Basic Statistics

Figures 11-13 show some basic statistics describing the Aave lending protocol. In April 2022, Aave supports the lending of 31 tokens and the total market size is about 11 billion USD. As shown in Figure 11 (a), the total value locked in Aave has increased substantially from mid 2020 to mid 2021, and has gone through a few ups and downs since then. The numbers of active lenders and borrowers, reported in panel (b), have also fluctuated over time. Figure 12 shows the average compositions of deposits and borrows. Aave does not show explicitly which deposited crypto assets are used as collaterals. These graphs however suggest that stablecoins such as USDC and USDT are borrowed disproportionately relative to their deposits. Stablecoins account for over 75% of loans. At the same time, the frequencies of borrowing assets like ETH and BTC (WETH and WBTC in the figures) are lower than those of depositing them, suggesting that they are mostly used as collaterals. It is also observed that the leverage of these loans is relatively high since the distribution of the health factors is skewed towards the left in Figure 13 (a), with 40% with a health factor below 1.³⁶ Liquidations happen frequently as a result of the volatile collateral prices and high leverage. Panel (b) shows the time series of collateral liquidations.

³⁶In practice, a position with health factor below one may not be liquidated immediately due to the execution costs involved.

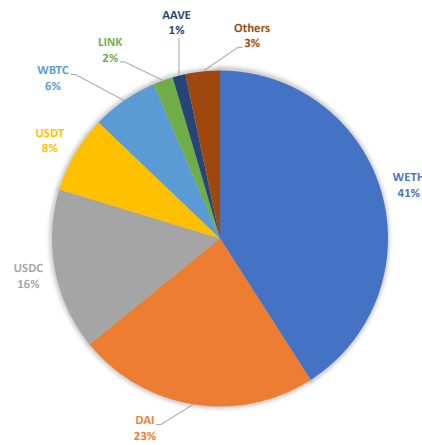


(a) Total Value (USD) Locked in Aave

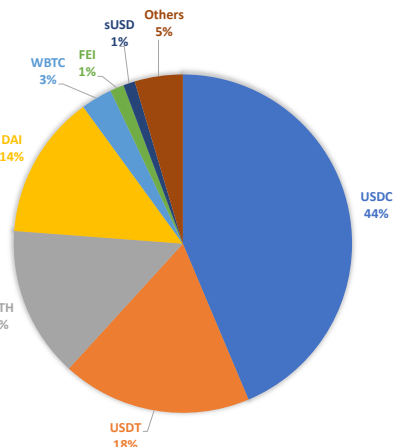


(b) Number of Unique Users per Day

Figure 11: Aave v2 TVL and Users Over time

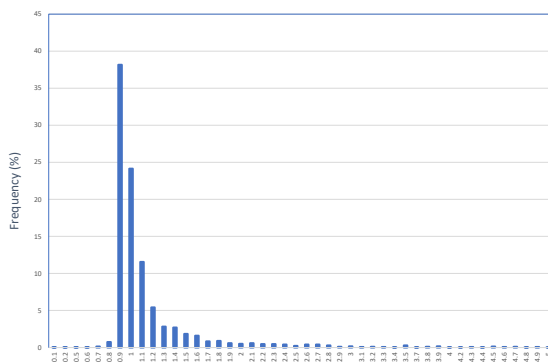


(a) Avg. Deposit Composition (Jan 2021-Jan 2022)

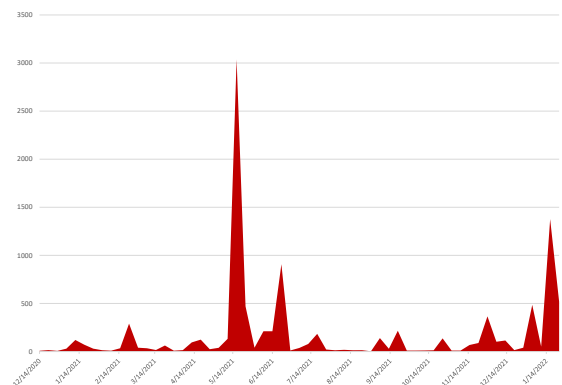


(b) Avg. Loan Composition (Jan 2021-Jan 2022)

Figure 12: Asset Compositions in Aave v2



(a) Health Factor (January 2022)



(b) Number of Liquidations per Week

Figure 13: Liquidation Risk in Aave v2

C Volatility of Collateral Value

See Table 3.

D Price Exploits

We discuss some evidence where borrowers pledged inflated collateral assets to obtain loans from lending protocols which later suffered big financial losses due to the bad debt.

As discussed in the Introduction, borrowers can have information advantage relative to the lending protocol when the smart contract relies on an inaccurate price feeds. For example, during the Terra collapse in May 2022, as a result of the extreme volatility in the price of LUNA tokens, the price feed used by DeFi smart contracts for the LUNA token was significantly higher than the actual market value of the token. Attackers exploited the price discrepancy to borrow loans collateralized on inflated LUNA from the Venus Protocol, the biggest lending platform on BSC, leading to a loss of about \$11.2 million to the protocol. The protocol later increased the haircut of LUNA from 45% to 100%. Similar exploits have depleted the entire lending pool of Avalanche lending protocol Blizz Finance, which has lost about \$8.28 million due to this incident.

Similar price exploits can also happen when price oracles are based on on-chain AMMs that are subject to liquidity problems or price manipulation. At times, token prices on DEX can deviate substantially from those on CEX. There are multiple incidents indicating that borrowers exploit lending protocols by borrowing against over-valued collateral assets. For instance, on May 18, 2021, the Venus Protocol faced a massive collateral liquidation. This incident occurred because a large sum of XVS was collateralized at a high price (possibly after price manipulation causing price to shoot up from \$80 to \$145 in three hours) to borrow 4,100 BTC and nearly 10,000 ETH from the lending protocol. When the price of XVS dropped four hours later, the loans became undercollateralized, resulting in \$200 million in liquidations and more than \$100 million in bad debts, with the borrowers profiting from this exploit. In this particular episode, borrowers were able to exploit their information advantage of the overpricing of XVS while lenders were unable to exclude XVS being used as a collateral. Similar exploits happened to Ethereum-based lending protocols Cheese Bank (with \$3.3 million loss in November 2020), Vesper Finance (with \$3 millions loss in November 2021), and Inverse Finance (with \$15.6 million loss in April 2022).

Table 3: The Volatility of Collateral Value (January 2021 - April 2022)

	Daily Volatility	Largest daily increase	Largest daily decrease
<i>Stable Coins</i>			
DAI	0.32%	1.26%	-1.33%
TUSD	0.39%	2.97%	-2.01%
USDC	0.34%	1.94%	-1.57%
<i>Other Coins</i>			
AAVE	7.15%	31.33%	-33.47%
BAT	7.48%	47.60%	-31.05%
BAL	6.62%	22.65%	-31.03%
CRV	8.89%	51.18%	-43.16%
ENJ	8.96%	56.46%	-35.61%
ETH	5.19%	24.53%	-26.30%
KNC	7.19%	30.57%	-31.98%
LINK	6.66%	30.38%	-35.65%
MANA	10.92%	151.66%	-29.79%
MKR	7.10%	51.31%	-24.24%
REN	8.05%	44.84%	-35.82%
SNX	7.36%	25.22%	-36.24%
UNI	7.14%	45.32%	-32.94%
WBTC	4.01%	19.04%	-13.75%
WETH	5.21%	25.96%	-26.12%
XSUSHI	7.65%	33.19%	-29.54%
YFI	6.82%	46.00%	-36.35%
ZRX	7.57%	56.02%	-36.31%
<i>Other Benchmarks</i>			
Stock Market (SPY ETF)	1.00%	2.68%	-3.70%
Treasury (BATS ETF)	0.35%	1.25%	-1.72%
AAA Bond (QLTA ETF)	0.41%	1.11%	-1.33%
Gold (GLD ETF)	0.89%	2.74%	-3.42%

Source: CoinGecko.

E Some Empirical evidence (for Online Appendix)

Here we report some evidence to support the case that our model can be useful for understanding the relationship between DeFi lending, crypto prices and market sentiment.

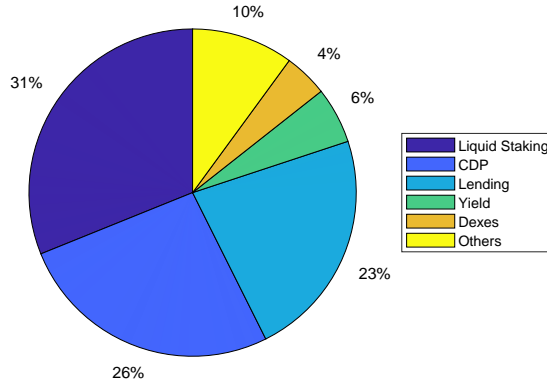
E.1 Effects of DeFi Lending on ETH Price

Our model predicts that DeFi lending should be positively correlated with crypto prices due to the price-liquidity feedback loop. Since the Ethereum blockchain is the main platform for DeFi, we use WETH TVL data from DeFiLlama to test this hypothesis. The sample is from 2018 January to 2022 March. Figure 14 shows that lending accounts for about 23% of DeFi TVL. We run an OLS

$$\log(ETHP) = \alpha_0 + \alpha_1 \log(LTCP) + \alpha_2 BURN + \alpha_3 DEFI + \alpha_4 LEND,$$

where $ETHP$ is the price of ETH, $LTCP$ is the price of Litecoin (LTC), $BURN$ is the amount of ETH burned since the London Fork as a percentage of ETH supply, $DEFI$ is the fraction of WETH locked into DeFi protocols, and $LEND$ is the fraction of WETH locked into DeFi lending. Since Litecoin has limited use in DeFi, we use its price to capture non-DeFi factors that can influence the price of ETH. As expected, results in Table 2 suggests that the prices of ETH and LTC are highly correlated. Also, unsurprisingly, by removing tokens from the circulating supply, BURN has a positive effect on the ETH price. Finally, after controlling for the general effects of DeFi on the price of ETH, TVL in DeFi lending is still positively correlated with the price of ETH, consistent with the prediction of our model.

Figure 14: Composition of WETH TVL in DeFi (March 2022)



Data Source: DeFiLlama.

Table 4: **DeFi Lending and Crypto Prices**

	Estimate	Std. Err.	T-Stat	p
Intercept	1.0845	0.07905	13.72	1.6765e-40
Log(LTCP)	1.0545	0.017673	59.665	0
BURN	0.42739	0.027956	15.288	3.1158e-49
DEFI	4.9181	0.92868	5.2957	1.3566e-07
LEND	36.438	2.5999	14.015	4.3029e-42
<hr/>				
No. obs. :	1546			
R^2	0.925	Adj. R^2	0.925	

E.2 Collateral Composition and Market Sentiment

Our model predicts that good market sentiment can help mitigate adverse selection, improving the quality of the collateral pool. We use the Aave platform data to examine the relationship between collateral composition and market sentiment. The market sentiment are measured by the “Crypto Fear & Greed Index” (FGI) for Bitcoin and other large cryptocurrencies.³⁷ The construction of the Index is based on measures of market volatility, market momentum/volume, social media, surveys, token dominance and Google Trends data. The Index is supposed to measure the emotions and sentiments from different sources, with a value of 0 indicating “Extreme Fear” while a value of 100 indicating “Extreme Greed”. Since Aave does not provide collateral data, we need to use outstanding deposits of collateralizable tokens as a proxy. Basing on their internal risk assessment, Aave assigns risk ratings to each token ranging from C+ to A+. We use these risk parameters to measure the quality of these assets. Figure 15 shows how the composition changes over time. Note that tokens have different USD prices. Hence, changing prices will affect their (nominal) shares in the pool. To remove the effects of token price changes on the composition, we fix their prices at the median level over the sample period (Jan 2021- April 2022). So the results derived below capture only variations in token quantities and not in their prices.

We study how sentiment is related to the overall quality of the collateral pool proxied by the weighted average of the ratings of all outstanding collateralizable deposits.³⁸ We run an OLS regressing $\log(\text{Rating})$ on a dummy and $\log(\text{FGI})$ as follows

³⁷The Index is developed by the “Alternative.me” website since early 2018 (<https://alternative.me/crypto/fear-and-greed-index/>).

³⁸We convert ratings into numerical values as follows: Rating = 6 for “A”, = 5 for “A-”, ..., =1 for “C+”.

Figure 15: Composition of Collateralizable Asset Mix

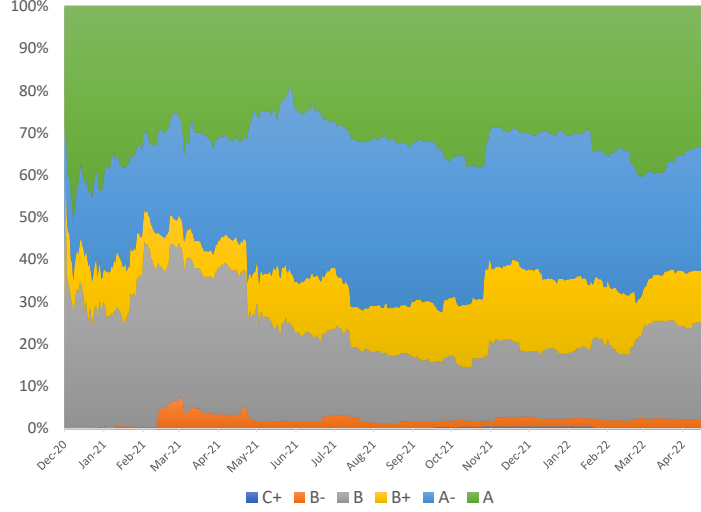


Figure Source: Dune Analytics

$$\text{Log}(\text{Rating}) = \alpha_0 + \alpha_1 \text{Dummy} + \alpha_2 \log(\text{FGI})$$

where Dummy=1 for days after April 26 (the date when Aave provided incentives to users who borrow/lend certain tokens). We report the result in Table 4. Both variables are significant, suggesting that the average rating of the collateral mix goes up when the sentiment captured by the FGI is high, as predicted by our model.

Table 5: **Sentiment and Collateral Rating**

	Estimate	Std. Err.	T-Stat	p
Intercept	1.4469	0.010123	142.93	0
Dummy	0.058287	0.0029707	19.62	4.2179e-64
Log(FGI)	0.01467	0.0022778	6.4405	2.7814e-10
No. obs. :	507			
R^2	0.464	Adj. R^2	0.461	

Figure 16: Effects of FG Index on Average Risk Rating

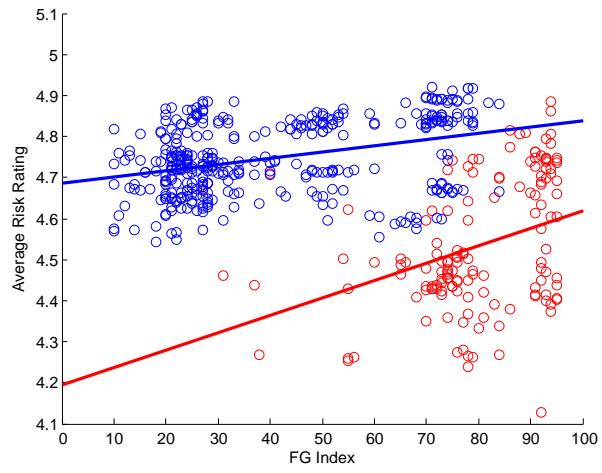


Figure Source: Dune Analytics

Blue (red) markers denote the sample period with (without) incentives