Local Risk Neutrality Puzzle and Decision Costs

Kathy Yuan*

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*University of Michigan. I thank Costis Skiadas, Emre Ozdenoren, and Annette Vissing-Jorgensen for helpful comments. All errors are mine.

Abstract

This study shows that introducing (fixed or proportional) decision cost helps to reconcile expected-utility agents' risk attitudes toward small and large gambles. As an example, we show that in the presence of decision costs, stock market non-participation can be explained with plausible risk aversion coefficients.

1 Introduction

Experimental evidence shows that across a range of wealth levels, people typically reject games with small to moderate stakes in which the potential gain is less than twice the potential loss. For example, the majority of subjects surveyed by Barberis, Huang, and Thaler (2003) turned down a 50:50 bet offering a \$550 gain against a \$500 loss. Such observed risk aversion to small-scale gambles, however, implies unrealistic risk aversion over large stakes within the existing theories of decision making under risk.[1] The intuition is that within the expected-utility framework, turning down a modest-stakes gamble means that the marginal utility of money must diminish very quickly for small changes in wealth. In fact, within all expected-utility models, only approximate risk neutrality over modest stakes would preclude such unrealistic implications (Rabin 2000). This local risk neutrality puzzle leads researchers to search for alternative models, such as loss aversion and narrow framing, to capture risk attitudes toward small and large-stake gambles.[2]

We argue that decision cost may help to explain the local risk neutrality puzzle. We introduce theorems that calibrate a relationship between decision cost and risk attitudes over small and large-stake gambles. Theorems show that even with a relatively small decision cost, the marginal utility of extra dollar diminishes far slower and attitude toward both small and reasonably large-scale gambles can be explained in an expected-utility framework. In our calibration exercise, we examine how large decision costs have to be in order to explain risk attitude toward reasonable large and small gambles. The small gamble we consider is a 50:50 bet offering a \$550 gain against a \$500 loss; and the large gamble is a 50:50 bet offering a \$20 million

[1]Rabin (2000) shows that turning down a 50:50 gamble that offers \$10 loss against \$10.10 gain implies turning down a 50:50 gamble that offers \$1000 loss against ∞ gain, within the expected utility models. Earlier, Kandel and Stambaugh (1991) have also pointed out that power utility functions cannot reconcile attitudes to both large and small-scale gambles simultaneously. Barberis, Huang, and Thaler (2003) find that this problem extends to the intertemporal setting as well.

[2]Rabin (2000) indicates loss aversion may explain risk attitudes toward both small and large-scale gambles. Barberis, Huang, and Thaler (2003) argue that narrow framing could explain the local neutrality puzzle, especially considering the cases where the gambles are not immediately played.

gain against a \$10,000 loss.[3] We find that a fixed cost of \$3.60 or a proportional cost of 0.69% are consistent with rejecting the small-stake gamble and accepting the large-stake gamble considered in this paper.

Psychological studies of decision making suggest at least three types of decision costs: the opportunity costs of the *time* it takes; the tendency to make *errors* under decision overload, and the *psychic* costs of anxiety and regret (Loewenstein 1999). Schwartz (2004) idenfities the following psychological factors underlying the psychic cost: People are more likely to regret their decisions; People are more likely to anticipate regretting their decisions adn this anticipated regret prevents people from actually deciding; When decisions have disappointing results, people tend to blame themselves because they feel that with so many options available, unsatisfying results must be their faults. Decision costs in this context can be thought of as either time and effort spent or, more important, psychological stress incurred in participating in the gamble. Presumably, information processing cost and propensity to error is minimal when a simple 50:50 bet with stakes as small as 550/500 is presented. Similarly, time and effort spent in investigating the fairness of gamble and participating the gamble may exist, but are not substantial. However, potential psychological stress may stop subjects from taking the small gamble altogether: A small gain will not make subjects feel much richer, but a small loss will definitely make subjects feel personally responsible and stupid. In the case of large-scale gambles, decision costs such as psychological stress are less important since the monetary stakes are very high.

Barberis, Huang, and Thaler (2003) argue that preferences with first-order risk aversion (such as loss aversion, disappointment aversion) can explain risk attitudes toward small and large-scale gambles only if the gambles are played out immediately. If the gambles are played with some delay, subjects are likely to face other sources of risks at the same time (such as labor income risk, house price risk); it is optimal for subjects, with first-order risk averse preferences, to accept the small gamble since it offers diversification benefits when merged with these other sources of risks. Following this reasoning, they argue that subjects must have preferences that

[3] The choice of gambles in our calibration exercise is based on Barberis, Huang, Thaler (2003).

depend on the outcome of the gamble over and above what the outcome implies for aggregate wealth risk, a feature called narrow framing. We show that with decision costs, subjects may not accept small gambles that are not immediately played either. By definition, subjects "frame" each gamble as a separation decision besides other "gambles" they are taking in their lives, and hence involve separate decision costs.

Similar to preferences with first-order risk aversion, introducing decision costs increases the magnitude of losses and raises an individual's risk aversion toward small-scale gambles relative to large-scale gambles. It differs from preferences with first-order risk aversion in three aspects: First, decision costs also decrease the magnitude of gains; second, decision costs do not introduce kinks in utility function as in the case of preferences of first-order risk aversion; third, the value of extra dollar depreciates due to the concavity feature of expected-utility models, even though the rate of depreciation of the marginal dollar is smaller than the expected-utility case without decision costs. Introducing decision costs to expected-utility models to study an individual's attitude toward risk may help us gauge to what extent expected utility incorporating various individual decision-making features can or cannot explain existing puzzles. How much decision costs help to explain the local risk neutrality puzzle depends on whether reasonable sizes of decision costs can deliver realistic rates of depreciation of marginal dollars observed in reality. The findings on the magnitude of decision costs needed to explain local risk neutrality puzzles in this paper are based on two data points taken from the experimental work in Barberis, Huang, and Thaler (2003), and contrasted with findings in empirical literature on transaction costs (Vissing-Jorgensen 2002). Our calibration exercise shows that in an expected utility framework with plausible risk aversion coefficients, a fixed decision cost in the magnitude of \$25 or a proportional decision cost of 0.48% will preclude investors from investing in the stock market all together.

The remainder of the paper is organized as follows. Section 2 develops two calibration theorems that incorporate fixed or proportional decision costs and presents numerical examples on the impact of different magnitudes of decision costs on attitudes toward risk. Section 3 analyzes the implication of decision costs in financial markets. More specifically, we consider the stock market participation puzzle. Section 4 concludes.

2 Some Calibrations of Risk-Taking Behavior with Decision Costs

Consider an expected utility maximizer over wealth, w, with Von Neuman-Morgenstern preferences $U(w)$. Assume that for all w, $U(w)$ is (strictly) increasing and (weakly) concave. Suppose further that, for some of initial wealth levels and for some $g > l > 0$, the individual rejects bets losing \$$l$ or gaining \$$g$, each with a 50% chance. For a certain size of fixed decision costs (c),[4] the theorem below places an upper bound on the rate at which utility increases above a given wealth level, and a lower bound on the rate at which utility decreases below that wealth level.

Theorem 1 (Fixed Decision Cost) *Suppose that for all w, $U(w)$ is strictly increasing and weakly concave. Suppose that there exists $\overline{w} > \underline{w}$, $g - c \geq l + c \geq 0$, such that for all $w \in [\underline{w}, \overline{w}]$, $0.5U(w - l - c) + 0.5U(w + g - c) < U(w)$. Then for all $w \in [\underline{w}, \overline{w}]$, for all $x > 0$,*

(i) if $g - c \leq 2l + 2c$, then

$$U(w) - U(w - x - c) \geq \begin{cases} 2\sum_{i=1}^{k^*(x)} (\frac{g-c}{l+c})^{i-1} r(w) & \text{if } w - \underline{w} \geq x + c \geq 2l + 2c \\ 2\sum_{i=1}^{k^*(w-\underline{w})} (\frac{g-c}{l+c})^{i-1} r(w) + A & \text{if } x + c \geq w - \underline{w} \end{cases}$$

(ii)

$$U(w + x - c) - U(w) \leq \begin{cases} 2\sum_{i=1}^{k^{**}(x)} (\frac{l+c}{g-c})^{i-1} r(w) & \text{if } x - c \leq \overline{w} - w \\ 2\sum_{i=1}^{k^{**}(\overline{w}-w)} (\frac{l+c}{g-c})^{i-1} r(w) + B & \text{if } x - c \geq \overline{w} - w \end{cases}$$

[4]The fixed cost is expressed in term of monetary units. If the investor takes the game, he/she will take either a gain of \$$g$ − \$$c$ or a loss of \$$l$ − \$$c$.

where, letting $k^* \equiv int((x+c)/(2l+2c))$, $k^{**} \equiv int((x-c)/(2g-2c))$, $r(w) = U(w) - U(w-l-c)$, $A = (x+c-(w-\underline{w}))(\frac{g-c}{l+c})^{k^*(w-\underline{w})}r(w)$, *and* $B = (x-c-(\overline{w}-w))(\frac{l+c}{g-c})^{k^{**}(\overline{w}-w)}r(w)$.

Suppose now that decision costs are proportional to the size of the stakes, that is, the investor will take either a gain of $(1-\delta)\$g$ or a loss of $(1+\delta)\$l$, the following theorem characterizes the upper (lower) bounds of the rate at which utility increases (decreases) at a given wealth level.

Theorem 2 (Proportional Decision Cost) *Suppose that for all* w, $U(w)$ *is strictly increasing and weakly concave. Suppose that there exists* $\overline{w} > \underline{w}$, $(1-\delta)g > (1+\delta)l > 0$, *such that for all* $w \in [\underline{w}, \overline{w}]$, $0.5U(w-(1+\delta)l) + 0.5U(w+(1-\delta)g) < U(w)$. *Then for all* $w \in [\underline{w}, \overline{w}]$, *for all* $x > 0$,

(i) if $g \leq 2(1+\delta)l$, *then*

$$U(w) - U(w-(1+\delta)x) \geq \begin{cases} 2\sum_{i=1}^{k^*(x)} \lambda^{i-1} r(w) & \text{if } w - \underline{w} \geq x(1+\delta) \geq 2(1+\delta)l \\ 2\sum_{i=1}^{k^*(w-\underline{w})} \lambda^{i-1} r(w) + A & \text{if } x(1+\delta) \geq w - \underline{w} \end{cases}$$

(ii)

$$U(w+(1+\delta)x) - U(w) \leq \begin{cases} 2\sum_{i=1}^{k^{**}(x)} (1/\lambda)^{i-1} r(w) & \text{if } x(1-\delta)g) \leq \overline{w} - w \\ 2\sum_{i=1}^{k^{**}(\overline{w}-w)} (1/\lambda)^{i-1} r(w) + B & \text{if } x(1-\delta)g) \geq \overline{w} - w \end{cases}$$

where, letting $\lambda = \frac{(1-\delta)g}{(1+\delta)l}$, $k^* \equiv int(\frac{x}{2l})$, $k^{**} \equiv int(\frac{x}{2g})$, $r(w) = U(w) - U(w-(1+\delta)l)$, $A = (x(1+\delta)-(w-\underline{w}))\lambda^{k^*(w-\underline{w})}r(w)$, *and* $B = (x-(w-\overline{w}))(1/\delta)^{k^{**}(\overline{w}-w)}r(w)$.

Table 1 illustrates the implications of these two theorems. It presents the smallest size of fixed and proportional costs needed for the following to be true: An individual who is known to reject, for all initial wealth levels, a 50:50 bet offering a \$550 gain against a \$500 loss, accepts a 50:50 bet offering a \$20 million gain against a \$10,000 loss. The interesting point of the table is how small these costs are.

Table 1: Risk Attitudes Toward Small and Large Gambles with Fixed and Proportional Decision Costs

Both gambles are 50:50 bets, with losses indicated in L column and gains in G column. Table displays an expected-utility individual's decisions to accept or reject the gambles for a given level of decision costs.

L	G	Fixed-Decision Cost		Proportional-Decision Cost	
		$c < 3.6$	$c > 3.6$	$c < 1.12\%$	$c > 1.12\%$
\$500	\$550	Reject	Reject	Reject	Reject
\$10,000	\$20,000,000	Reject	Accept	Reject	Accept

To compare the rates at which utility increases (decreases) above (below) certain wealth level, Table 2 considers an individual who is known to reject, for all initial wealth levels, 50:50, lose \$500 and gain g bets, for $g = \$550, \$555, \$560, \$570$. For each of these small gambles, it presents how large the gain has to be in order for the individual to accept the gamble for a certain loss L, where each L is a column in the table. The table presents results for the no-decision cost case, the fixed-decision cost case ($c = \$3.6$), and the proportional decision cost case ($c = 1.12\%$). Panel A of the table shows the results in Rabin (2000). Panels B and C contrast the findings in the cases of fixed and proportional decision costs: Utility increases at much slower diminishing rates with decision costs, even with small sizes of decision costs.

3 An Example

We have shown that a small decision cost can explain aversion to a small-size gamble without making counterintuitive predictions about attitudes to large-size gambles. We now show that this analysis also has useful implications for financial markets. We consider the stock market participation puzzle. The finance literature has documented the fact that many investors are reluctant to allocate any money to the stock market even though stocks have a high mean return (Mankiw and Zeldes (1991), Haliassos and Bertaut (1995)). The puzzle is deepened when considering that for most households, stock market risk has a correlation close to zero with other important risks, such as labor income risk, proprietary income risk, and house price risk (Heaton and Lucas 1996). Here with decision costs that reconcile investors' risk-averse attitude toward both small and large gambles, we calibrate investors' portfolio allocation problem using the commonly used stock market return process, and find whether the preference parameters required to explain the non-participation are reasonable or not. We incorporate a specification for pre-existing risk in our calibration, similar to Vissing-Jorgensen (2002), who invokes transaction costs of investing in the stock market to explain the non-participation puzzle.

We consider a setup specified in Barberis, Huang, and Thaler (2003), where an investor has a fixed fraction of her wealth (θ_n) in a non-financial asset (e.g., the labor income risk) with a gross return specified as

$$R^n_{t+1} = e^{g_n + \sigma_n \epsilon^n_{t+1}}.$$

Now she decides what fraction of her wealth (θ_s) to invest in the stock market. The stock market has a gross return of

$$R^s_{t+1} = e^{g_s + \sigma_s \epsilon^s_{t+1}},$$

where

$$\begin{pmatrix} \epsilon^c \\ \epsilon^n \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1, \varphi \\ \varphi, 1 \end{pmatrix} \right) \text{i.i.d. over time.}$$

The remaining fraction of her wealth, $1 - \theta_n - \theta_s$, is to be invested in a risk-free asset earning R_f, so that the overall return on wealth is

$$R^w_{t+1} = (1 - \theta_n - \theta_s) R_f + \theta_n R^n_{t+1} + \theta_s R^s_{t+1}.$$

The return process parameters are drawn from Barberis, Huang, and Thaler (2003) and are given in Table 3. In particular, g_s and σ_s are chosen to match historical annual data on aggregate stock returns; and the correlation between the stock market and investor's pre-existing risks, φ, is chosen to be 0.1 to reflect the low correlation between the pre-existing risk and stock market returns. We also assume a decision cost (a fixed cost of \$25 or a proportional cost of 0.48%) is incurred when investing in the stock market. We solve this portfolio problem for a recursive utility with the power utility form, where the consumption and portfolio problems are separable and the portfolio problem is given by,

$$max_{\theta_s} E((R^w_{t+1} - 1_{\{\theta_s \neq 0\}} c)^{1-\gamma}) \qquad \text{if a fixed cost, } c\text{, is incurred; or}$$
$$max_{\theta_s} E((R^w_{t+1} - 1_{\{\theta_s \neq 0\}} \delta R^w_{t+1})^{1-\gamma}) \qquad \text{if a proportional cost, } \delta\text{, is incurred.}$$

Our calibration analysis shows that for power utility without decision costs, $\gamma > 93$ is required to generate stock market non-participation; but with the magnitude of decision costs (fixed or proportional costs) specified here, $\gamma = 2$ is sufficient to generate a 0% allocation to stocks.

The result here is supported by the findings in the transaction cost and stock market participation literature. For example, Vissing-Jogensen (2002) estimates a model of the benefits of stock market particiaption and finds that a per period cost of \$50 is sufficient to explain the choices of half of stock market nonparticipants in the presence of pre-existing risks such as labor income. Although calibration exercises are similar, decision costs differ from transaction costs in concept. Transaction costs are normally considered as monetary costs to set up brokerage accounts, commission or the bid-ask spread investors have to pay to buy or sell stocks, or the price impact or illquidity cost of a trade. By decision cost, we emphasize that a decision to invest in stock market may involve more than stated monetary costs. The stress arised from having to pick stocks to invest and facing possible losses may deter investors from investing at all.

4 Conclusion

We show, through a calibration theorem, that introducing decision costs helps to address the local risk neutrality puzzle partially and has plausible implications in the financial market. The reason why it is partial is because that the marginal utility still depreciates, even though at a slower speed. We need experimental evidence to answer whether decision costs deliver a reasonable depreciation rate or not.

The insight from the analysis in this paper, however, is that to reconcile agents' risk attitudes toward small and large-scale gambles, large-scale gambles have to be totally different ball games from the small scale gambles to the agents. Decision costs is one way to separate large from small-scale gambles since decision costs may dominate the gains from small-scale gambles in magnitude. Another approach is by Barberis, Huang, and Thaler (2003), who argue first order risk aversion, combined with narrow framing, also address the local risk neutrality puzzle and have plausible financial applications. The first order risk aversion introduces a kink around the small-scale gamble. Narrow framing is introduced to reflect the fact that investors seem to treat stock market risk separately from other un-insurable pre-exisitng risks such as labor income risks. In our decision cost approach, investors effectively "frame" each stock market investment as a separate decision besides other risks they are taking in their lives. Hence these two approaches in this sense are similar.

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Appendix A: Proof of Theorem 1

Proof. For notational ease, let $r(w) = U(w) - U(w - l - c)$.
Part (i).

By the concavity of $U(\cdot) : U(\lambda w + (1-\lambda)(w - l - c)) \geq \lambda U(w) + (1-\lambda)U(w - l - c)$
$\Rightarrow U(w - (1-\lambda)(l + c)) - U(w - l - c) \geq \lambda \left(U(w) - U(w - l - c)\right).$

Choose $\lambda = \frac{g-l-2c}{l+c}$ so that

$$(1-\lambda)(l + c) = 2l - g + 3c.$$

Hence,

$$\begin{aligned}
& U(w - 2l + g - 3c) - U(w - l - c) \geq \frac{g - l - 2c}{l + c}(U(w) - U(w - l - c)) \\
\Rightarrow \quad & U(w - 2l + g - 3c) - U(w - 2c - 2l) \\
& \geq \frac{g - l - 2c}{l + c}(U(w) - U(w - l - c)) + U(w - l - c) - U(w - 2c - 2l) \\
\Rightarrow \quad & U(w - 2c - 2l + g - c) - U(w - 2c - 2l) \\
& \geq \frac{g - c}{l + c}(U(w) - U(w - l - c)) - (U(w) - U(w - l - c)) + (U(w - l - c) - U(w - 2l - 2c)) \\
\Rightarrow \quad & U(w - 2c - 2l + g - c) - U(w - 2c - 2l) \geq \frac{g - c}{l + c}(U(w) - U(w - l - c))
\end{aligned}$$

Hence, if $w - 2c - 2l \geq \underline{w}$, since by assumption $U(w - 2c - 2l + g - c) + U(w - 2c - 2l - l - c) \leq 2U(w - 2c - 2l)$, we know that the following is true:

$$U(w - 2c - 2l) - U(w - 2c - 3l - c) \geq \frac{g - c}{l + c}(U(w) - U(w - l - c)).$$

More generally,

$$
\begin{aligned}
& U(w-(2k-1)c-(2k-1)l-c)-U(w-2kc-2kl) \\
& \geq U(w-2(k-1)c-2(k-1)l)-U(w-(2k-1)c-(2k-1)l-c) \\
& \text{By concavity,} \\
\Rightarrow \quad & U(w-2kc-2kl+g-c)-U(w-(2k-1)c-(2k-1)l) \\
& \geq \frac{g-l-2c}{l+c}(U(w-2(k-1)c-2(k-1)l)-U(w-(2k-1)c-(2k-1)l)) \\
\Rightarrow \quad & U(w-2kc-2kl+g-c)-U(w-2kc-2kl) \\
& \geq \frac{g-c}{l+c}(U(w-2(k-1)c-2(k-1)l)-U(w-(2k-1)c-(2k-1)l)) \\
& \text{By rejecting the gamble,} \\
\Rightarrow \quad & U(w-2kc-2kl)-U(w-2kc-(2k+1)l-c) \\
& \geq \frac{g-c}{l+c}(U(w-2(k-1)c-2(k-1)l)-U(w-(2k-1)c-(2k-1)l))
\end{aligned}
$$

These lower bounds on marginal utilities yield the lower bound on total utilities $U(w)-U(w-x-c)$ in part (i) of the Theorem.

Part (ii).

$$
\begin{aligned}
\text{By the concavity of } U(\cdot) \quad & : \\
U(w+2g-2c)-U(w+g-l-2c) \quad & \leq \quad \frac{1}{\lambda}(U(w+2g-2c-(1-\lambda)(g+l))-U(w+g-l-2c)) \\
& = \quad \frac{g+l}{g-c}(U(w+2g-l-3c)-U(w+g-l-2c)) \\
& = \quad (\frac{l+c}{g-c}+1)(U(w+2g-l-3c)-U(w+g-l-2c))
\end{aligned}
$$

$$
\Rightarrow \quad U(w+2g-2c)-U(w+2g-2l-3c) \leq \frac{l+c}{g-c}(U(w+2g-l-c)-U(w+g-l-c))
$$

If $w+2g-2c \leq \overline{w}$, then $U(w+3g-3c)-U(w+2g-2c) \leq U(w+2g-2c)-U(w+2g-l-3c)$ by assumption. We know that the following is true:

$$
\begin{aligned}
U(w+3g-3c)-U(w+2g-2c) \quad & \leq \quad \frac{l+c}{g-c}(U(w+2g-l-2c)-U(w+g-l-c)) \\
& \leq \quad \frac{l+c}{g-c}(U(w+g-c)-U(w)) \\
& \leq \quad \frac{l+c}{g-c}(U(w)-U(w-l-c))
\end{aligned}
$$

More generally, by the concavity of $U(\cdot)$

$$\begin{aligned}
& U(w+2mg-2mc)-U(w+(2m-1)g-l-2mc) \\
\leq\ & \frac{1}{\lambda}(U(w+2mg-2mc-(1-\lambda)(g+l))-U(w+(2m-1)g-l-2mc)) \\
=\ & \frac{g+l}{g-c}(U(w+2mg-2mc-l-c)-U(w+(2m-1)g-l-2mc)) \\
=\ & (\frac{l+c}{g-c}+1)(U(w+2mg-2mc-l-c)-U(w+(2m-1)g-l-2mc))
\end{aligned}$$

$$\begin{aligned}
\Rightarrow\quad & U(w+2mg-2mc)-U(w+2mg-2mc-l-c) \\
\leq\ & \frac{l+c}{g-c}(U(w+2mg-2mc-l-c)-U(w+(2m-1)g-l-2mc))
\end{aligned}$$

If $w+2mg-2mc \leq \overline{w}$, then $U(w+(2m+1)g-(2m+1)c)-U(w+2mg-2mc) \leq U(w+2mg-2c)-U(w+2mg-2mc-l-c)$ by assumption. We know that the following is true:

$$\begin{aligned}
& U(w+(2m+1)g-(2m+1)c)-U(w+2mg-2mc) \\
\leq\ & \frac{l+c}{g-c}(U(w+2mg-2mc-l-c)-U(w+(2m-1)g-l-2mc)) \\
\leq\ & \frac{l+c}{g-c}(U(w+(2m-1)g-(2m-1)c)-U(w-2(m-1)g-2(m-1)c))
\end{aligned}$$

These upper bounds on marginal utilities yield the upper bound on utilities $U(w+x-c)-U(w)$ in part (ii) of the Theorem. ■

Appendix B: Proof of Theorem 2

Proof. For notational ease, let $r(w)=U(w)-U(w-(1+\delta)l)$.
Part (i).

By the concavity of $U(\cdot): U(\lambda w+(1-\lambda)(w-(1+\delta)l)) \geq \lambda U(w)+(1-\lambda)U(w-(1+\delta)l)$
$\Rightarrow U(w-(1-\lambda)(1+\delta)l)-U(w-(1+\delta)l) \geq \lambda\left(U(w)-U(w-(1+\delta)l)\right).$

Choose $\lambda=\frac{(1-\delta)g-(1+\delta)l}{(1+\delta)l}$ so that

$$(1-\lambda)(1+\delta)l=2(1+\delta)l-(1-\delta)g.$$

Hence,

$$
\begin{aligned}
& U(w-2(1+\delta)l+(1-\delta)g)-U(w-(1+\delta)l) \\
& \geq \frac{(1-\delta)g-(1+\delta)l}{(1+\delta)l}(U(w)-U(w-(1+\delta)l)) \\
\Rightarrow \quad & U(w-2(1+\delta)l+(1-\delta)g)-U(w-2(1+\delta)l) \\
& \geq \frac{(1-\delta)g-(1+\delta)l}{(1+\delta)l}(U(w)-U(w-(1+\delta)l))+U(w-(1+\delta)l)-U(w-2(1+\delta)l) \\
\Rightarrow \quad & U(w-2(1+\delta)l+(1-\delta)g)-U(w-2(1+\delta)l) \\
& \geq \frac{(1-\delta)g}{(1+\delta)l}(U(w)-U(w-(1+\delta)l))-(U(w)-U(w-(1+\delta)l)) \\
& +(U(w-(1+\delta)l)-U(w-2(1+\delta)l)) \\
\Rightarrow \quad & U(w-2(1+\delta)l+(1-\delta)g)-U(w-2(1+\delta)l) \geq \frac{(1-\delta)g}{(1+\delta)l}(U(w)-U(w-(1+\delta)l))
\end{aligned}
$$

Hence, if $w-2(1+\delta)l \geq \underline{w}$, since by assumption, $U(w-2(1+\delta)l+(1-\delta)g)+U(w-2(1+\delta)l-(1+\delta)l) \leq 2U(w-2(1+\delta)l)$, we know that the following is true:

$$
U(w-2(1+\delta)l)-U(w-3(1+\delta)l) \geq \frac{(1-\delta)g}{(1+\delta)l}(U(w)-U(w-(1+\delta)l)).
$$

More generally,

$$
\begin{aligned}
& U(w-(2k-1)(1+\delta)l)-U(w-2k(1+\delta)l) \\
& \geq U(w-2(k-1)(1+\delta)l)-U(w-(2k-1)(1+\delta)l) \\
& \text{By concavity,} \\
\Rightarrow \quad & U(w-2k(1+\delta)l+(1-\delta)g)-U(w-(2k-1)(1+\delta)l) \\
& \geq \frac{(1-\delta)g-(1+\delta)l}{(1+\delta)l}(U(w-2(k-1)(1+\delta)l)-U(w-(2k-1)(1+\delta)l)) \\
\Rightarrow \quad & U(w-2k(1+\delta)l+(1-\delta)g)-U(w-2k(1+\delta)l) \\
& \geq \frac{(1-\delta)g}{(1+\delta)l}(U(w-2(k-1)(1+\delta)l)-U(w-(2k-1)(1+\delta))) \\
& \text{By rejecting the gamble,} \\
\Rightarrow \quad & U(w-2k(1+\delta)l)-U(w-(2k+1)(1+\delta)l) \\
& \geq \frac{(1-\delta)g}{(1+\delta)l}(U(w-2(k-1)(1+\delta)l)-U(w-(2k-1)(1+\delta)l))
\end{aligned}
$$

These lower bounds on marginal utilities yield the lower bound on total utilities $U(w)-U(w-(1+\delta)x)$ in part (i) of the Theorem.

Part (ii)

By the concavity of $U(\cdot)$:

$$\begin{aligned} & U(w+2(1-\delta)g)-U(w+(1-\delta)g-(1+\delta)l) \\ \leq\ & \frac{1}{\lambda}(U(w+2(1-\delta)g-(1-\lambda)((1-\delta)g+(1-\delta)l))-U(w+(1+\delta)g-(1-\delta)l)) \\ =\ & \frac{(1-\delta)g+(1-\delta)l}{(1+\delta)g-(1-\delta)c}(U(w+2(1-\delta)g-(1+\delta)l)-U(w+(1-\delta)g-(1+\delta)l)) \\ =\ & (\frac{(1+\delta)l}{(1-\delta)g}+1)(U(w+2(1-\delta)g-(1+\delta)l)-U(w+(1-\delta)g-(1+\delta)l)) \end{aligned}$$

$$\begin{aligned} \Rightarrow\quad & U(w+2(1-\delta)g)-U(w+2(1-\delta)g-2(1+\delta)l) \\ \leq\ & \frac{(1+\delta)l}{(1-\delta)g}(U(w+2(1-\delta)g-(1+\delta)l)-U(w+(1-\delta)g-(1+\delta)l)) \end{aligned}$$

If $w+2(1-\delta)g \leq \overline{w}$, then $U(w+3(1-\delta)g)-U(w+2(1-\delta)g) \leq U(w+2(1-\delta)g)-U(w+2(1-\delta)g-(1+\delta)l)$ by assumption. We know that the following is true:

$$\begin{aligned} & U(w+3(1-\delta)g)-U(w+2(1-\delta)g) \\ \leq\ & \frac{(1+\delta)l}{(1-\delta)g}(U(w+2(1-\delta)g-(1+\delta)l)-U(w+(1-\delta)g-(1+\delta)l)) \\ \leq\ & \frac{(1+\delta)l}{(1-\delta)g}(U(w+(1-\delta)g)-U(w)) \\ \leq\ & \frac{(1+\delta)l}{(1-\delta)g}(U(w)-U(w-(1+\delta)l)) \end{aligned}$$

More generally, by the concavity of $U(\cdot)$

$$\begin{aligned} & U(w+2m(1-\delta)g)-U(w+(2m-1)(1-\delta)g-(1+\delta)l) \\ \leq\ & \frac{1}{\lambda}(U(w+2m(1-\delta)g-(1-\lambda)((1-\delta)g+(1+\delta)l))-U(w+(2m-1)(1-\delta)g-(1+\delta)l)) \\ =\ & \frac{(1+\delta)g+(1-\delta)l}{(1-\delta)g}(U(w+2m(1-\delta)g-(1+\delta)l)-U(w+(2m-1)(1-\delta)g-(1+\delta)l)) \\ =\ & (\frac{(1+\delta)l}{(1-\delta)g}+1)(U(w+2m(1-\delta)g-(1+\delta)l)-U(w+(2m-1)(1-\delta)g-(1+\delta)l)) \end{aligned}$$

$$\begin{aligned} \Rightarrow\quad & U(w+2m(1-\delta)g)-U(w+2m(1-\delta)g-(1+\delta)l) \\ \leq\ & \frac{(1+\delta)l}{(1-\delta)g}(U(w+2m(1-\delta)g-(1+\delta)l)-U(w+(2m-1)(1-\delta)g-(1+\delta)l)) \end{aligned}$$

If $w + 2m(1-\delta)g \leq \overline{w}$, then $U(w + (2m+1)(1-\delta)g) - U(w + 2m(1-\delta)g) \leq U(w + 2m(1-\delta)g) - U(w + 2m(1-\delta)g - (1+\delta)l)$ by assumption. We know that the following is true:

$$
\begin{aligned}
& U(w + (2m+1)(1-\delta)g) - U(w + 2m(1-\delta)g) \\
\leq\; & \frac{(1+\delta)l}{(1-\delta)g}(U(w + 2m(1-\delta)g - (1+\delta)l) - U(w + (2m-1)(1-\delta)g - (1+\delta)l)) \\
\leq\; & \frac{(1+\delta)l}{(1-\delta)g}(U(w + (2m-1)(1-\delta)g) - U(w - 2(m-1)(1-\delta)g))
\end{aligned}
$$

These upper bounds on marginal utilities yield the upper bound on utilities $U(w + (1-\delta)x) - U(w)$ in part (ii) of the Theorem. ■

Table 2: Rates at Which Utility Increases

The table displays the lowest gain at which an expected-utility individual accepts a 50:50 bet for a loss of $2,000-$9,000, if she rejects a 50:50 bet with $500 loss and a gain of $550, $555, $560, $570, respectively. Panel A shows the standard expected-utility case without decision costs. Panel B shows the case with fixed-decision costs. Panel C shows the case with proportional decision costs.

Panel A: No Decision Costs								
	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$9,000
$550	$4,400	$5,500	$7,700	$11,000	$15,400	$24,200	∞	∞
$555	$4,440	$5,550	$8,880	$13,320	$21,090	∞	∞	∞
$560	$4,480	$6,720	$11,200	$23,520	∞	∞	∞	∞
$570	$4,560	$7,980	$15,960	∞	∞	∞	∞	∞
Panel B: Fixed Decision Costs (c=$3.6)								
	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$9,000
$550	$2,189	$3,282	$4,375	$6,560	$8,746	$12,024	$17,488	$26,231
$555	$2,209	$3,312	$4,415	$6,620	$9,929	$13,237	$20,957	$44,116
$560	$2,229	$3,342	$4,455	$6,680	$10,019	$15,583	$27,824	∞
$570	$2,269	$3,402	$5,668	$7,933	$12,464	$23,792	∞	∞
Panel C: Proportional Decision Costs (c=1.12%)								
	$2,000	$3,000	$4,000	$5,000	$6,000	$7,000	$8,000	$9,000
$550	$3,300	$4,400	$6,600	$8,800	$11,000	$15,400	$20,900	$30,800
$555	$3,330	$4,440	$6,660	$8,880	$12,210	$17,760	$26,640	∞
$560	$3,360	$4,480	$6,720	$10,080	$13,440	$21,280	$47,040	∞
$570	$3,420	$4,560	$7,980	$11,400	$19,380	∞	∞	∞

Table 3: Return Process Parameter Values

Parameters g_s and σ_s (g_n and σ_n) are the mean and standard deviation of log stock market returns (log returns on a non-financial asset); θ_n is the fixed fraction of wealth held in the non-financial asset; w is the correlation of log returns on the stock market and the non-financial asset; and R_f is the risk-free rate.

Parameter	g_s	σ_s	g_n	σ_n	θ_n	φ	R_f
Value	0.06%	0.20%	0.04%	0.03%	0.75	0.10	1.02%