Online Appendix: Crypto Assets and their Endogenous Convenience Yields

1 Model

There are an infinite number of islands that represent physically segmented marketplaces or online platforms. We assume that on each island there are a continuum of buyers and sellers for investment or consumption goods. To overcome the limited commitment problem between the buyers and sellers, trades are facilitated by an asset. Specifically, buyers pay sellers using either the asset or a security backed by the asset and receive investment or consumption goods in exchange. We assume that the asset does not pay any dividend and there are A_t units of it. We interpret the asset as either fiat money or cryptocurrency.

Time is infinite and discrete, indexed by $t = 1, 2, \ldots$. Each time period is divided into four subperiods. Buyers are long lived. They discount future utility with per-period discount factor $0 < \beta < 1$. (Hence no discounting between sub-periods.) Sellers, on the other hand, live for only one period. They enter the economy in the beginning of a period and leave at the end of the period. In subperiod 1, agents trade an investment good. In subperiod 3, they trade a consumption good. These markets are decentralized in the sense that each buyer is matched with at least two sellers who engage in Bertrand competition.

After a goods market closes, a frictionless asset market opens up where the agents have the opportunity to buy and sell the asset. In each period, therefore, asset markets take place in the second and fourth subperiods. We assume that in the asset market agents can produce a numeraire good that provides one util one-to-one using labor which has a cost of one util. This numeraire good is perishable and if produced must be consumed in that period. We denote the net utility from the numeraire good by n, i.e. for an agent this is the utility from numeraire good consumed minus the disutility from producing it. Besides being the numeraire, as will become clear, the ability to produce the numeraire good with labor allows buyers to balance their budgets in the asset markets.

Investment goods market In the first subperiod, a fraction γ_I of buyers receive an investment opportunity. In order to benefit from this opportunity, these buyers need to obtain an investment good from the sellers. Sellers can produce the investment good at unit marginal cost. A buyer with an investment opportunity obtains utility zi from receiving i units of the investment good. Buyers and sellers trade the investment good in exchange for securities backed by the asset. We refer to this market as Market I.

Consumption goods market In the third subperiod buyers purchase either a specialized consumption good, denoted by L, or a general consumption good, denoted by H, from the sellers. We assume that with probability λ all buyers in a given island need the specialized good and with probability $1-\lambda$ they all need the general good. In the former case we say the island is a type L island and in the latter case a type H island. We assume that islands' types are drawn independently, across islands and time. Buyers pay for the consumption good with the asset.[1] We refer to these markets as Market L and Market H.

Sellers produce either type of consumption good at unit marginal cost. A buyer's utility from consuming c units of the consumption good (specialized or general) is $u(c)$. It is harder to find a specialized good that suits a buyer's need than a general good. When a buyer values specialized goods (i.e. if the island is type L), the probability that the buyer meets a seller who offers specialized goods that she values is γ_L. When she values general goods (i.e. if the island is type H), the probability that she meets a seller who offers the general good that she values is γ_H. We assume $0 < \gamma_L < \gamma_H \leq 1$, that is, it is harder to find the specialized good than the general good. The utility function u is positive, twice differentiable, increasing and concave, i.e.

[1] As will become clear, there is no asymmetric information in the consumption goods market so agents do not need to use asset backed securities. An alternative interpretation is that buyers pay with a security that promises resale price of the asset.

$u(c) > 0$ for $c > 0$, $-\infty < u''(c) < 0 < u'(c)$ for $c > 0$. To guarantee equilibrium existence we further assume that utility is increasing sufficiently rapidly near zero consumption so that $u'(0^+) > 1 + \frac{1-\beta}{\beta(\lambda\gamma_L+(1-\lambda)\gamma_H)}$ and $\lim_{c\to\infty}[u'(c) + cu''(c)] = 0$.

Information environment Buyers on a given island learn their island's type privately in subperiod 1, and in particular, the island's type is not observed by the sellers of the investment good. Hence, there is asymmetric information between the buyers and sellers of the investment good in subperiod 1. After subperiod 1, each island's type in that period becomes common knowledge and information is symmetric across all agents.

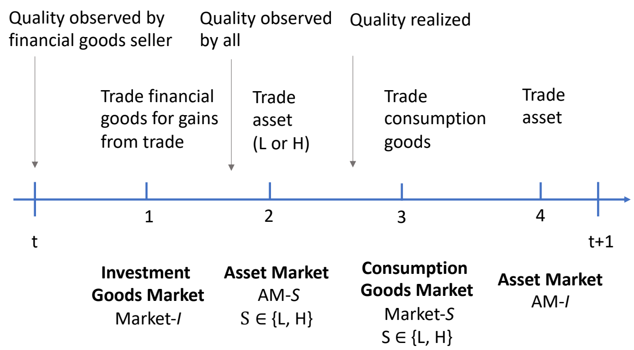
Asset markets In subperiod 2 agents trade the asset in a frictionless market. We denote this asset market by AM-S where $S \in \{L, H\}$ depending on the information available about the island's type. We denote the price of the asset in AM-S by ϕ^S. The price in this asset market depends on the island's type because, as we noted above, the island's type becomes commonly known after subperiod 1. In the fourth, and final, subperiod, agents, once again, trade the asset in a frictionless market which we denote by AM-I. We denote the price of the asset in AM-I by ϕ^I.

Timeline Figure 1.1 summarizes the event timeline of this economy within each time period.

2 Solving the Model

As usual we solve the model backwards beginning with the asset market in subperiod 4, moving back to consumption goods markets L and H in subperiod 3, asset markets L and H in subperiod 2 and finally investment goods market in subperiod one.

Figure 1.1: Timeline



2.1 AM-I

We refer to a buyer's continuation value from owning a units of the asset in the asset market AM-I by $W_t^I(a)$ and from entering Market I at time $t+1$ with a units of asset by $V_{t+1}^I(a)$.[2] Hence,

$$W_t^I(a) = \max_{n,\tilde{a}} -n + \beta V_{t+1}^I(\tilde{a})$$

$$s.t. \phi_t^I \tilde{a} \leq \phi_t^I a + n$$

[2]Recall that AM-I takes place in the fourth subperiod of period t and precedes Market I that takes place in the first subperiod of period $t+1$. By assumption agents discount the payoff that they obtain in time $t+1$Market I when they compute their continuation values in time t AM-I.

Substituting for n we get,

$$W_t^I(a) = \phi_t^I a + \left[\max_{\tilde{a}} -\phi_t^I \tilde{a} + \beta V_{t+1}^I(\tilde{a})\right]$$
$$= \phi_t^I a + W_t^I(0)$$

where

$$W_t^I(0) = \max_{\tilde{a}} -\phi_t^I \tilde{a} + \beta V_{t+1}^I(\tilde{a}).$$

Taking the first order condition and using market clearing ($\tilde{a} = A_t$) we obtain:

$$\phi_t^I = \beta \frac{\partial}{\partial a} V_{t+1}^I (A_t)$$

where A_t is the total supply of the asset.

2.2 Markets L and H

We denote buyers continuation values from owning a units of the asset in Market S where $S \in \{L, H\}$ by $V_t^S(a)$. Recall that in Market S a buyer is matched with a seller with probability γ_S. We assume that the buyer makes the seller a take it or leave it offer of $(a - \tilde{a})$ units of the asset in exchange for c units of the consumption good. Hence, if the seller accepts the offer, the buyer retains $\tilde{a}$ of the asset with which he enters AM-I. The seller, on the other hand, enters AM-I with $(a - \tilde{a})$ units of the asset, which she sells at price ϕ_t^I and obtains $\phi_t^I(a - \tilde{a})$ units of the numeraire good. If the seller refuses the offer, then the seller obtains reservation value of zero. Hence, we can write the buyer's value function as:

$$V_t^S(a) = \max_{c, \tilde{a} \geq 0} \gamma_S \left[u(c) + W_t^I(\tilde{a})\right] + (1 - \gamma_S) W_t^I(a)$$

subject to

$$c \leq \phi_t^I (a - \tilde{a})$$

Note that the constraint can be viewed as either the budget constraint for the buyer or the participation constraint of the seller. Since the constraint must be satisfied with equality we can substitute for c and write the buyer's value function as:

$$V_t^S(a) = \max_{\tilde{a}\geq 0} \gamma_S \left\{ u\left[\phi_t^I(a-\tilde{a})\right] + \phi_t^I \tilde{a} \right\} + (1-\gamma_S)\phi_t^I a + W_t^I(0).$$

First order condition for the buyer's optimization problem is:

$$-\phi_t^I u'\left[\phi_t^I(a-\tilde{a})\right] + \phi_t^I \leq 0$$

or

$$u'\left[\phi_t^I(a-\tilde{a})\right] \geq 1$$

with equality if $\tilde{a} > 0$. There are two cases to consider.

Case 1: $u'(\phi_t^I a) < 1$. This is the case when the agent has brought too much assets to the goods market. Denote c^* to be such that $u'(c^*) = 1$

$$\begin{aligned} V_t^S(a) &= \gamma_S \left\{ u(c^*) + \phi_t^I a - c^* \right\} + (1-\gamma_S)\phi_t^I a + W_t^I(0) \\ &= \phi_t^I a + \gamma_S(u(c^*) - c^*) + W_t^I(0) \end{aligned}$$

Case 2: $u'(\phi_t^I a) \geq 1$. This is the case when the agent spends all the assets that he has brought to purchase consumption goods.

$$V_t^S(a) = \gamma_S u(\phi_t^I a) + (1-\gamma_S)\phi_t^I a + W_t^I(0)$$

2.3 AM-L and AM-H

In the asset market AM S, $S \in \{L, H\}$, agents trade the asset with symmetric information. In particular, they know that in the next subperiod, buyers and sellers will trade in the consumption goods market of type

S. We refer to a buyer's continuation value from owning a units of the asset in AM-S by $W_t^S(a)$ and from entering consumption goods Market S with a units of asset by $V_t^S(a)$. Hence,

$$W_t^S(a) = \max_{n,\tilde{a}} -n + V_t^S(\tilde{a})$$

$$s.t. \phi_t^S \tilde{a} \leq \phi_t^S a + n$$

where n denotes the number of numeraire goods that agents have to give up by changing the asset holding from a to $\tilde{a}$. Recall that buyers can produce the numeraire good one-to-one from labor. If $\tilde{a} > a$, they pay for the difference with the numeraire good which is n in the budget constraint. The production of the numeraire creates disutility which is given by $-n$ in the payoff function. Similarly, if $\tilde{a} < a$, buyers receive numeraire (so $n < 0$) which gives them utility $-n$. Since the budget constraint must hold with equality, we substitute for n in the payoff function and obtain,

$$\begin{aligned} W_t^S(a) &= \phi_t^S a + \left[\max_{\tilde{a}} -\phi_t^S \tilde{a} + V_t^S(\tilde{a})\right] \\ &= \phi_t^S a + W_t^S(0) \end{aligned}$$

where

$$W_t^S(0) = \max_{\tilde{a}} -\phi_t^S \tilde{a} + V_t^S(\tilde{a}).$$

Taking the first order condition and using market clearing we obtain:

$$\phi_t^S = \frac{\partial}{\partial a} V_t^S(A_t)$$

where A_t is the total supply of the asset.

We can now use the expressions for $V_t^S(a_t)$ to solve for ϕ_t^S and $W_t^S(a)$. Recall that there are two cases.

Case 1: $u'(\phi_t^I a) < 1$. In this case,

$$\frac{\partial}{\partial a} V_t^S(a) = \phi_t^I$$

so we obtain

$$\phi_t^S = \phi_t^I,$$

and

$$W_t^S(a) = \phi_t^I a + W_t^S(0).$$

Case 2:$u'(\phi_t^I a) \geq 1$. In this case,

$$\frac{\partial}{\partial a} V_t^S(a) = \phi_t^I \left[\gamma_S u'(\phi_t^I a) + (1-\gamma_S)\right]$$

evaluating the right hand side at $a = A_t$, we obtain

$$\phi_t^S = \phi_t^I \left[\gamma_S u'(\phi_t^I A_t) + (1-\gamma_S)\right],$$

and

$$W_t^S(a) = \phi_t^I \left[\gamma_S u'(\phi_t^I A_t) + (1-\gamma_S)\right] a + W_t^S(0).$$

2.4 Market I

Now, we move to the first subperiod when Market I takes place. In this market there is asymmetric information. Buyers know their island's type but sellers do not know. This asymmetric information creates adverse selection in the market. If the island is type H, then buyers on the island know that the asset price will be ϕ_t^H next period. These buyers would buy the investment good, and give up a security claim on the asset, only if the seller gives enough of the investment good in exchange. However, sellers need to break even, and they need to take into account that the island may be type L in which case the asset price will be ϕ_t^L. Hence, there is a classic lemons problem in Market I.

One way to solve the adverse selection problem is to issue a debt security claim with face value D where

$$D = (1-h)\phi^H.$$

Here h is the haircut on the security.

The actual payment of the security is

$$y_t^S = \min(D, \phi_t^S).$$

for $s = L, H$. Setting a high haircut (or a low face value) relaxes the participation constraint of type H borrowers because it lowers the amount that type H borrower gives up in exchange for the investment good. In this section, we take the haircut h as fixed and solve for the equilibrium price of the security, which we denote by p_t, and asset prices ϕ_t^L, ϕ_t^H and ϕ_t^I. We consider two cases. The first is a pooling equilibrium where both types of borrowers trade the security in exchange for the intermediate good. The second is a separating equilibrium where only the low type trades the security in exchange for the intermediate good.

The price of the security is determined by the break even condition for the sellers. In the pooling equilibrium, the price of the security is

$$p_t = \lambda \min(D, \phi_t^L) + (1 - \lambda) \min(D, \phi_t^H). \tag{2.1}$$

In the separating equilibrium, the price of the security is

$$p_t = \min(D, \phi_t^L). \tag{2.2}$$

Suppose a borrower enters Market I with a units of the asset. The borrower learns the type of the island and decides how many units of the asset backed security to sell given the island's type. Since the security is backed by the asset the borrower can sell at most a units of the security. We assume that the borrower makes the required payment y_t^S per unit of security and retains a units of the asset in AM-S next sub-period. This assumption is innocuous since the asset trades at price ϕ_t^S in AM-S (and there is no discounting between subperiods). Hence, the value of the borrower from holding a units of the asset in Market I is given by:

$$V_t^I(a) = \lambda \max_{0\le\tilde{a}_L\le a} \left[zp_t\tilde{a}_L - y_t^L\tilde{a}_L\right] + (1-\lambda) \max_{0\le\tilde{a}_H\le a} \left[zp_t\tilde{a}_H - y_t^H\tilde{a}_H\right]$$
$$+ \lambda W_t^L(a) + (1-\lambda)W_t^H(a)$$

Since $W_t^S(a) = \phi_t^S a + W_t^S(0)$ we can rewrite the value function as:

$$V_t^I(a) = \lambda \max_{0\le\tilde{a}_L\le a} \left[zp_t\tilde{a}_L - y_t^L\tilde{a}_L\right] + (1-\lambda) \max_{0\le\tilde{a}_H\le a} \left[zp_t\tilde{a}_H - y_t^H\tilde{a}_H\right]$$
$$+ \lambda\phi_t^L a + (1-\lambda)\,\phi_t^H a + \lambda W_t^L(0) + (1-\lambda)W_t^H(0) \tag{2.3}$$

2.5 Solving for Asset Prices

We focus on steady state and drop time subscripts for the remainder of the section. We are now ready to solve for the equilibrium steady state asset prices: ϕ^L, ϕ^H and ϕ^I.

Recall from previous discussion that there are two cases. We next show that case 1 ($u'(\phi_t^I a) < 1$) is inconsistent with a stationary equilibrium. To see this note that in case 1, $\phi^L = \phi^H = \phi^I$. As a result, $y^L = y^H = p = (1-h)\phi^I$. Substituting

$$V^I(a) = (z-1)\,(1-h)\phi^I a + \phi^I a + \lambda W^L(0) + (1-\lambda)W^H(0)$$

and

$$\phi^I = \beta\frac{\partial}{\partial a}V^I\,(A) = \beta\,[1 + (z-1)\,(1-h)]\,\phi^I$$

Hence, in steady state of stationary equilibrium, $\phi^L = \phi^H = \phi^I = 0$. But this is inconsistent since Case 1 requires $u'(\phi^I a) = u'(0) < 1$ but by assumption $u'(0) > 1$.

In case 2, we have two subcases. We refer to the subcase where the security is traded in a pooling as case 2.1 and in a separating equilibrium as case 2.2.

In case 2.1 substituting the pooling security price (2.1) into (2.3) and differentiating with respect to a we obtain:

$$\phi^I = \beta(z-1)\left[\lambda \min(D,\phi^L) + (1-\lambda)\min(D,\phi^H)\right] + \beta\left[\lambda\phi^L + (1-\lambda)\phi^H\right]$$

In case 2.2 substituting the separating security price (2.2) into (2.3) and differentiating with respect to a we obtain:

$$\phi^I = \beta\lambda(z-1)\min(D,\phi^L) + \beta\left[\lambda\phi^L + (1-\lambda)\phi^H\right].$$

In both cases we also have:

$$\phi^S = \phi^I\left[\gamma_S u'(\phi^I A) + (1-\gamma_S)\right].$$

Moreover, the asset is traded in the pooling equilibrium if and only if

$$zp = z\left\{\lambda \min(D,\phi^L) + (1-\lambda)\min(D,\phi^H)\right\} \geq \min(D,\phi^H)$$

That is if

$$\frac{\min(D,\phi^L)}{\min(D,\phi^H)} = \min(1, \frac{1}{1-h}\frac{\phi^L}{\phi^H}) \geq \zeta \equiv 1 - \frac{z-1}{\lambda z}. \tag{2.4}$$

The following proposition summarizes the discussion and shows that a stationary equilibrium exists.

Proposition 1. *There exists at least one stationary equilibrium. In a stationary equilibrium prices ϕ^L, ϕ^H and ϕ^I are given by:*

$$\phi^I = \begin{cases} \beta(z-1)\left[\lambda \min(D,\phi^L) + (1-\lambda)\min(D,\phi^H)\right] + \beta\left[\lambda\phi^L + (1-\lambda)\phi^H\right] & \text{if (2.4) holds} \\ \beta\lambda(z-1)\min(D,\phi^L) + \beta\left[\lambda\phi^L + (1-\lambda)\phi^H\right] & \text{otherwise} \end{cases}$$

and $\phi^S = \phi^I\left[\gamma_S u'(\phi^I A) + (1-\gamma_S)\right]$ for $S = L, H$.

Proof. Set $A = 1$. Let $\phi^S(x) = x\left[\gamma_S u'(x) + (1-\gamma_S)\right]$. Note that $\phi^S(0) = 0$ and

$$\frac{\partial \phi^S}{\partial x} = \gamma_S\left[u'(x) + xu''(x)\right] + (1-\gamma_S).$$

From our assumptions, $\frac{\partial \phi^S(0)}{\partial x} > 0$ and $\lim_{x\to\infty} \frac{\partial \phi^S(x)}{\partial x} \to 1-\gamma_S < 1$. Define

$$\phi^I_P(x) = \beta(z-1)\left[\lambda\min(D(x),\phi^L(x)) + (1-\lambda)\min\left(D(x),\phi^H(x)\right)\right] + \beta\left[\lambda\phi^L(x) + (1-\lambda)\phi^H(x)\right], \tag{2.5}$$

$$\phi^I_S(x) = \beta(z-1)\lambda\min(D(x),\phi^L(x) + \beta\left[\lambda\phi^L(x) + (1-\lambda)\phi^H(x)\right] \tag{2.6}$$

where $D(x) = (1-h)\phi^H(x)$. The functions $\phi^I_P(x)$ and $\phi^I_S(x)$ are continuous and differentiable almost everywhere, and in particular, are differentiable at at $x = 0$. Moreover,

$$\begin{aligned} \frac{\partial \phi^I_P(x)}{\partial x} \geq \frac{\partial \phi^I_S(x)}{\partial x} &\geq \beta\left[\lambda\frac{\partial \phi^L(x)}{\partial x} + (1-\lambda)\frac{\partial \phi^H(x)}{\partial x}\right] \\ &= \beta\left[\left(\lambda\gamma_L + (1-\lambda)\gamma_H\right)\left[u'(x) + cu''(x)\right] + 1 - (\lambda\gamma_L + (1-\lambda)\gamma_H)\right]. \end{aligned}$$

Hence at $x = 0$, both derivatives are greater than 1 since

$$u'(0) > 1 + \frac{1-\beta}{\beta\left(\lambda\gamma_L + (1-\lambda)\gamma_H\right)}.$$

Moreover, at every point of differentiability,

$$\frac{\partial \phi^I_S(x)}{\partial x} \leq \frac{\partial \phi^I_P(x)}{\partial x} \leq \beta(z-1)\max\left((1-h)\frac{\partial \phi^H(x)}{\partial x}, \frac{\partial \phi^L(x)}{\partial x}\right) + \beta\left[\lambda\frac{\partial \phi^L(x)}{\partial x} + (1-\lambda)\frac{\partial \phi^H(x)}{\partial x}\right]$$

By assumption the limit of the RHS as $x \to \infty$ is less than $\beta z(1-\gamma_L)$ which is less than 1. Hence, both functions have (potentially multiple) strictly positive fixed points. Let $\hat{\phi}^I_P > 0$ and $\hat{\phi}^I_S > 0$ be the smallest of these fixed points. Since $\phi_P^I(x) > \phi_S^I(x)$ for all $x \geq 0$, $\hat{\phi}_P^I > \hat{\phi}_S^I$. Since $\frac{\phi^L(x)}{\phi^H(x)}$ is increasing in x, we must have either $\frac{1}{1-h}\frac{\phi^L\left(\hat{\phi}_P^I\right)}{\phi^H\left(\hat{\phi}_P^I\right)} > \zeta$ or $\frac{1}{1-h}\frac{\phi^L\left(\hat{\phi}_S^I\right)}{\phi^H\left(\hat{\phi}_S^I\right)} < \zeta$ or both. Hence, either there exists a pooling equilibrium where $\phi^I = \hat{\phi}_P^I$ or a separating equilibrium where $\phi^I = \hat{\phi}_S^I$ or both types equilibria co-exist. □

2.6 Multiple Equilibria

We now describe the conditions under which the economy admits multiplicity in asset prices. As a first step we find $\phi^I(\zeta)$ such that

$$\frac{1}{1-h}\frac{\phi^L\left(\phi^I(\zeta)\right)}{\phi^H\left(\phi^I(\zeta)\right)} = \zeta$$

where $\phi^S\left(\phi^I\right) = \phi^I\left[\gamma_S u'(\phi^I A) + (1-\gamma_S)\right]$ for $S = L, H$. There are multiple equilibria iff $\hat{\phi}_S^I < \phi^I(\zeta) < \hat{\phi}_P^I$. Rearranging we obtain:

$$u'(\phi^I(\zeta)) = 1 + \chi \tag{2.7}$$

where $\chi = \frac{1-(1-h)\zeta}{(1-h)\zeta\gamma_H - \gamma_L}$. A necessary condition for multiple equilibria is $0 < \chi < \infty$, or equivalently $\frac{\gamma_L}{\gamma_H} < (1-h)\zeta$. To see why first consider the case $\frac{\gamma_L}{\gamma_H} > (1-h)\zeta$. In this case, (2.7) implies $u'(\phi^I(\zeta)) < 1$. If there is a pooling equilibrium, $u'(\hat{\phi}_P^I) > 1$ and $\phi^I(\zeta) < \hat{\phi}_P^I$ implying $u'(\phi^I(\zeta)) > 1$, leading to a contradiction. If $\frac{\gamma_L}{\gamma_H} = (1-h)\zeta$ than $u'(\phi^I(\zeta)) = \infty$. In this case, we must have $\phi^I(\zeta) \leq 0$ and there must be a unique pooling equilibrium.

Let

$$\phi^I(\zeta) = (u')^{-1}[1+\chi]$$

$$\phi^L(\phi^I(\zeta)) = [1+\chi\gamma_L](u')^{-1}[1+\chi]$$

$$\phi^H(\phi^I(\zeta)) = [1+\chi\gamma_H](u')^{-1}[1+\chi]$$

A sufficient condition for multiple equilibria is $\phi_P^I\left(\phi^I(\zeta)\right) > \phi^I(\zeta) > \phi_L^I\left(\phi^I(\zeta)\right)$. Using the definitions

$$\phi_P^I\left(\phi^I(\zeta)\right) = \beta(z-1)\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)(1-h)\phi^H(\phi^I(\zeta))\right] + \beta\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)\phi^H(\phi^I(\zeta))\right]. \tag{2.8}$$

$$\phi_S^I\left(\phi^I\left(\zeta\right)\right) = \beta(z-1)\lambda\phi^L(\phi^I(\zeta)) + \beta\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)\phi^H(\phi^I(\zeta))\right]. \tag{2.9}$$

Plugging in the conditions are:

$$\beta\lambda z\left[1+\chi\gamma_L\right] + \beta(1-\lambda)\left[\left(z-1\right)\left(1-h\right)+1\right]\left[1+\chi\gamma_H\right] > 1$$

$$\beta\lambda z\left[1+\chi\gamma_L\right] + \beta(1-\lambda)\left[1+\chi\gamma_H\right]) < 1$$

Or:

$$1 < \frac{1-\beta\lambda z\left[1+\chi\gamma_L\right]}{\beta(1-\lambda)\left[1+\chi\gamma_H\right])} < \left(z-1\right)\left(1-h\right)+1$$

Plugging in for χ:

$$\frac{1-\lambda\beta z\left[1+\chi\gamma_L\right]}{\beta\left(1-\lambda\right)\left[1+\chi\gamma_H\right]} = \frac{(1-h)\zeta\left(\gamma_H - \lambda\beta z\left(\gamma_H-\gamma_L\right)\right)-\gamma_L}{\beta\left(1-\lambda\right)\left(\gamma_H-\gamma_L\right)}$$

Hence multiplicity conditions can be written as:

$$1-\frac{1-(1-h)\zeta}{1-\beta\left(1-\lambda\right)-\beta\lambda(1-h)} < \frac{\gamma_L}{\gamma_H} < 1-\frac{1-(1-h)\zeta}{1-\beta\left(1-\lambda\right)-\beta\lambda(1-h)+\beta(1-h)\left(z-1\right)\left(1-\lambda\right)}.$$

The first inequality guarantees existence of a pooling and the second a separating equilibrium. Also note that when h is large enough there is only a pooling equilibrium.

3 Haircut as a limit to borrowing

We assume that lenders break even. In addition we assume:

$$p_t \leq (1-h)\phi^H$$

The idea is that haircut puts a limit on how much borrowers can borrow given the past price of the asset which had been determined in the asset market I which takes place before market I opens up. Pooling price is:

$$p^P = \left(\lambda \min(D, \phi^L) + (1-\lambda) \min(D, \phi^H)\right)$$

Separating price is:

$$p^S = \min(D, \phi^L)$$

Prices ϕ^L and ϕ^H are given by:

$$\phi^S = \phi^I \left[\gamma_S u'(\phi^I) + (1-\gamma_S)\right]$$

In a pooling equilibrium:

$$(1-h)\phi^H = \lambda \min(D, \phi^L) + (1-\lambda) \min(D, \phi^H)$$

Suppose the low type default and the high type doesn't. We can solve for D

$$(1-h)\phi^H = \lambda\phi^L + (1-\lambda)D$$

$$D^P = \frac{(1-h)\phi^H - \lambda\phi_L}{(1-\lambda)}$$

In a separating equilibrium:

$$(1-h)\phi^H = \min(D, \phi^L)$$

$$D^S = (1-h)\phi^H$$

The asset is traded in the pooling equilibrium if and only if

$$zp = z\left\{\lambda \min(D, \phi^L) + (1-\lambda) \min(D, \phi^H)\right\}$$

$$\geq \min(D, \phi^H)$$

That is if

$$\frac{\min(D,\phi^L)}{\min(D,\phi^H)} \geq \zeta \equiv 1 - \frac{z-1}{\lambda z}. \tag{3.1}$$

$$\frac{\min(D,\phi^L)}{\min(D,\phi^H)} = \frac{(1-\lambda)\phi^L}{(1-h)\phi^H - \lambda\phi_L} \geq \zeta \equiv 1 - \frac{z-1}{\lambda z}$$

Solve for $\phi^I(\zeta)$

$$\phi^S = \phi^I \left[\gamma_S u'(\phi^I) + (1-\gamma_S)\right]$$

$$(1-h)\zeta\phi^H = (1-\lambda+\lambda\zeta)\,\phi^L$$

$$\frac{(1-h)\,\zeta}{1-\lambda+\lambda\zeta}\left[\gamma_H u'(\phi^I) + (1-\gamma_H)\right] = \left[\gamma_L u'(\phi^I) + (1-\gamma_L)\right]$$

$$u'(\phi^I(\zeta)) = 1 + \frac{1-\chi}{\chi\gamma_H - \gamma_L}$$

$$\phi^L\left(\phi^I(\zeta)\right) = \phi^I(\zeta)\left[1 + \frac{\gamma_L(1-\chi)}{\chi\gamma_H - \gamma_L}\right]$$

$$\phi^H\left(\phi^I(\zeta)\right) = \phi^I(\zeta)\left[1 + \frac{\gamma_H(1-\chi)}{\chi\gamma_H - \gamma_L}\right]$$

A sufficient condition for multiple equilibria is $\phi^I_P\left(\phi^I(\zeta)\right) > \phi^I(\zeta) > \phi^I_L\left(\phi^I(\zeta)\right)$. Using the definitions

$$p^P = \left(\lambda \min(D,\phi^L) + (1-\lambda)\min(D,\phi^H)\right)$$

$$\phi^I_P\left(\phi^I(\zeta)\right) = \beta(z-1)\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)(1-h)\phi^H(\phi^I(\zeta))\right] + \beta\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)\phi^H(\phi^I(\zeta))\right]. \tag{3.2}$$

$$\phi^I_S\left(\phi^I(\zeta)\right) = \beta(z-1)\lambda\phi^L(\phi^I(\zeta)) + \beta\left[\lambda\phi^L(\phi^I(\zeta)) + (1-\lambda)\phi^H(\phi^I(\zeta))\right]. \tag{3.3}$$

$$\beta\frac{(\gamma_H - \gamma_L)}{\chi\gamma_H - \gamma_L}\left[\lambda z\chi + (1-\lambda)(z - h(z-1))\right] \geq 1$$

$$\zeta = \frac{\lambda z - z + 1}{\lambda z}$$

4 Special case- Trading equity claims of the asset in the investment good market

Zero haircut (trading asset directly).

$$\phi^I = \begin{cases} \beta z\left[\lambda\phi^L + (1-\lambda)\phi^H\right] & \text{if pooling} \\ \beta\left[\lambda z\phi^L + (1-\lambda)\phi^H\right] & \text{if separating} \end{cases}$$

where $\phi^S = \beta\phi^I\left[\gamma_S u'(\phi^I A) + (1-\gamma_S)\right]$ for $S = L, H$

$$\phi^I = \begin{cases} \beta^2 z\phi^I\left[\lambda\left(\left[\gamma_L u'(\phi^I A) + (1-\gamma_L)\right]\right) + (1-\lambda)\left(\gamma_H u'(\phi^I A) + (1-\gamma_H)\right)\right] & \text{if pooling} \\ \beta^2\phi^I\left[\lambda z\left(\left[\gamma_L u'(\phi^I A) + (1-\gamma_L)\right]\right) + (1-\lambda)\left(\gamma_H u'(\phi^I A) + (1-\gamma_H)\right)\right] & \text{if separating} \end{cases}$$

In pooling equilibrium

$$1 = \beta^2 z\left[\bar{\gamma}u'(\phi^I A) + (1-\bar{\gamma})\right]$$

where

$$\bar{\gamma} = \lambda\gamma_L + (1-\lambda)\gamma_H$$

In separating equilibrium

$$\begin{aligned} 1 &= \beta^2\left[\left(\lambda z\gamma_L u'(\phi^I A) + \lambda z(1-\gamma_L)\right) + (1-\lambda)\gamma_H u'(\phi^I A) + (1-\lambda)(1-\gamma_H)\right] \\ &= \beta^2\left[\hat{\gamma}u'(\phi^I A) + \lambda z + 1 - \lambda - \hat{\gamma}\right] \end{aligned}$$

where

$$\hat{\gamma} = \lambda z\gamma_L + (1-\lambda)\gamma_H$$