The Price Impact of Borrowing and Short-Sale Constraints

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Abstract

This study explores how trading constraints such as borrowing and short-sale constraints affect asset prices in the presence of asymmetric information. In a symmetric information environment, borrowing and short-sale constraints exacerbate downward and upward price movements, respectively. However, in the presence of information asymmetry, the price impact of each constraint is different. In an asymmetric information environment, prices play an important role in shaping uninformed investor expectation. Uninformed investors are uncertain whether trading constraints restrict informed investors from transmitting information to prices, and thus they demand an information-disadvantaged premium in holding stocks. This creates a large price decline. Hence, information asymmetry combined with short-sale constraints *dampens* the *upward* price movement while information asymmetry combined with borrowing constraints *intensifies* the *downward* price movement. The model also generates the following empirical predictions: 1) prices at extreme tails are less informative of the asset fundamental; 2) bad news creates greater return volatility than good news; 3) crashes are more likely than bubbles; and 4) the skewness in returns is more pronounced in stocks with severe information asymmetry and with a greater percentage of constrained informed investors.

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1 Introduction

In this paper, we analyze the asset pricing implications of two prevalent trading constraints in the financial market: borrowing and short-sale constraints. Borrowing and short-sale constraints are regarded in the existing literature as important market frictions that may contribute to market abnormalities such as sudden crashes or contagions (Kyle and Xiong (2001); Xiong (2001)), as well as bubbles (Harrison and Kreps (1978); Hong and Stein (2003); Scheinkman and Xiong (2003)). This paper extends the research by exploring the distinct effects of borrowing and short-sale constraints on asset prices when information is *asymmetric*.

Extensive literature studies the impact of each of these trading constraints separately. It has been shown both theoretically and empirically that borrowing constraints may lead to *underpricing* and result in fire sales while short-sale constraints may cause *overpricing*. For example, Shleifer and Vishny (1997) show there is a limit of arbitrage when leveraged investors face borrowing constraints. Xiong (2001), Xiong and Kyle (2001), Yuan (2005), Gromb and Vayanos (2002), and Liu and Longstaff (2004) study borrowing constraints that are endogeneous in wealth/asset prices.[1] Empirically, Coval and Stafford (2005) find evidence for asset fire sales, using market prices of mutual fund transactions caused by capital flows.

For the overpricing impact of short-sale constraints, the evidence is also extensive.[2] For instance, Allen, Morris, and Postlewaite (1993) show it is possible for short-sale constraints to generate finite bubbles. Other theoretical works include Miller (1977), Jarrow (1980), Diamond and Verrecchia (1981), Harrison and Kreps (1978), Hong and Stein (2003), Scheinkman and Xiong (2003), and Brunnermeier and Abreu (2003).

[1] Xiong (2001), Xiong and Kyle (2001), and Yuan (2005) show borrowing-constrained arbitragers can have a price destablizing effect and can induce correlations in asset prices. Gromb and Vayanos (2002) show that borrowing-constrained competitive arbitragers may take excess (or little) risk. Liu and Longstaff (2004) show it is optimal for risk averse arbitragers to underinvest in arbitrage due to collateral constraints. Grossman and Villa (1992) show that risk-neutral agents facing a fixed amount of borrowing alter the optimal portfolio strategy even when the constraint is not binding.

[2] A number of empirical studies have documented the overpricing impact of short-sale constraints, including Seneca (1967), Seneca and Stark (1993), Figlewski (1981), Figlewski and Webb (1993), D'Avolio (2002), Krishnamurthy (2002), Lamont and Stein (2004), Ofek and Richardson (2003), Geczy, Musto, and Reed (2002), and Asquith, Pathak, and Ritter (2005).

However, most of these studies do not explore constraint effects in an asymmetric information setting. Those that consider the asymmetric information setting include Diamond and Verrecchia (1981), Bai, Chang, and Wang (2006), and Marin and Olivier (2006) on the effect of short-sale constraints, and Yuan (2005) on the effect of borrowing constraints.[3] A central theme of these studies is that the asset price becomes less informative when investors are constrained by either borrowing or short-sale constraints. Most of these studies predict binding constraints when prices are low, which in turn suggest greater likelihood of a crash.

In this paper, we further the theoretical observations of these studies and show that the price impacts of short-sale and borrowing constraints are *different* in the presence of information asymmetry. This finding is counter to results in the existing literature on trading constraints. Specifically, we find that short-sale constraints are more likely to bind when prices are high rather than low. Intuitively, this corresponds to the empirical phenomenon that informed investors are short-sale constrained when the high asset price is caused by a high level of noise demand. Instead of resulting in a bubble (as predicted in the existing symmetric information literature) or causing a crash (as predicted in the existing asymmetric information literature), short-sale constraints when combined with information asymmetry *dampen* the *upward* price movement and thus make bubbles difficult to form. By contrast, borrowing constraints are more likely to bind when prices are low. Empirically. this captures the features of margin constraints: A drop in the asset price causes a decrease in the collateral value and consequently the amount of the margin loan investors can borrow. We find that the interaction of borrowing constraints with information asymmetry *exacerbates* the *downward* price movement, making crashes in asset prices more likely.

To study the price impacts of borrowing and short-sale constraints, we employ a standard noisy rational expectations equilibrium (REE) model of asset prices with both informed and uninformed investors. Informed investors receive a noisy signal about the asset payoff while uninformed investors observe only the price, from which they extract the informed investors' private signal. In this model, there is a noisy demand/supply shock so that prices do not fully reveal private information, similar to that used by Hellwig (1980)

[3]Additionally, the setting in Barlevy and Veronesi (2003) incorporates risk-neutral agents under both a short-sale constraint and a wealth constraint, with the shocks distributed exponentially. However, their emphasis is on the tail events on asset prices (such as crises) rather than on the impact of trading constraints.

and Grossman and Stiglitz (1980). In addition, some informed investors face borrowing and short-sale constraints. Consistent with the margin requirements and short-sale constraints observed in the financial market, we specify both constraints as restrictions on investors' demand and the borrowing constraint is price dependent.

We find that, when informed investors are not constrained, the asset price is informative since the unconstrained trading transmits their signal to the asset price. However, when a small *adverse* shock to the fundamentals lowers the price, informed investors may become borrowing constrained. In this case, their ability to trade on their private information is limited, resulting a noisy price. Uninformed investors now cannot separate liquidity selling (or noise asset selling) from informed investor information-based selling. Thus, they find it increasingly difficult to extract the informed signal from the falling price, and will bail out when the price falls. This behavior results in a demand that is price inelastic or possibly backward-bending, which exacerbates the downward price movement. The inelastic uninformed demand induces several feedback effects. For example, the falling asset price tightens informed investors' borrowing constraints, leading to greater volatility and possibly price multiplicity.

Conversely, we find that, when a small *positive* shock to the fundamentals increases the price, informed investors may be constrained out of the market due to short-sale restrictions. Again, in this scenario, informed investors' private information is not embedded in the market clearing price, resulting a noisy price. Uninformed investors are less willing to purchase the asset since they cannot distinguish noise demand from information-based buying. Due to adverse selection concerns, this additional uncertainty about informed investor constraints causes uninformed investors to demand an information-disadvantaged premium to hold the asset. Their demand becomes more elastic as the price increases, inducing a dampening effect. Hence, large upward price movements become less likely.

Additionally, in this economy, uninformed investors do not observe informed investors' constraint status and can only infer from the price the probabilities of informed investors being restricted by trading constraints. This introduces another source of uncertainty, since price informativeness changes with different informed investors' constrained status. In the presence of both trading constraints, the informed investors' constraint status varies more than the case when informed investors face only one constraint. It could range from unconstrained, to constrained by either or both trading constraints. Therefore, the perceived uncertainty to uninformed investors is greater. Consequently, the skewness in return and excess volatility are much more pronounced, demonstrating the importance of considering both market frictions in understanding properties of asset prices.

Overall, our analysis shows that the interaction between trading constraints and information asymmetry generates a different impact on asset prices depending on the type of constraints.[4] This finding is in contrast to the results in the existing literature showing both borrowing and short-sale constraints lead to crash in an asymmetric information environment. Furthermore, our findings generate several empirical implications: 1) prices at extreme tails are less informative of the fundamentals; 2) bad news creates greater return volatility than good news; 3) crashes are more likely than bubbles; and 4) the skewness in returns is more pronounced in stocks with severe information asymmetry and with a greater percentage of constrained informed investors

In addition to our findings regarding information asymmetry and constraint effects, our paper makes several technical contributions to the literature. First, we show that the non-linear REE solution presented in Yuan (2005)[5] can be generalized to any asymmetric information setting if the information structure is hierarchical. In particular, we generalize the non-linear REE solution method to a CARA-normal setting under both borrowing and short-sale constraints.

Second, this paper is among the first to study the asset pricing implications of short-sale constraints versus borrowing constraints in an asymmetric

[4]There are several strands of the literature that study the return volatility's asymmetric response to news. For example, Black (1976), Christie (1982), Glosten, Jagannathan, and Runkle (1993), Braun, Nelson, and Sunier (1995), among others, label this phenomenon as the "leverage" effect. Pindyck (1984), French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), Bekaert and Wu (2000), and Wu (2001) propose the volatility feedback effect as an alternative explanation. Detemple (1986), Feldman (1986), David (1997), and Veronesi (1999) argue for rational learning and stochastic uncertainty among investors as an explanation. Finally, Barberis, Shleifer, and Vishny (1998), Barberis, Huang, and Santos (2001), and McQueen and Vorknick (2004) offer explanations based on the perspective that investors are exposed to certain behavioral biases. The explanations in these models differ from ours in that information asymmetry plays a central role in our explanation. Further, predictions from our model are information asymmetry based and hence are different as well.

[5]Yuan (2005) studies a borrowing-constrained economy with mean-variance investors and normally distributed shocks

information setting. In this respect, we complement the findings in Yuan (2005), who studies the asset pricing implication of borrowing constraints, Bai, Chang, and Wang (2006) and Marin and Olivier (2006), who study the asset pricing implications of short-sale constraints.

Our study captures different market phenomenon from the latter two studies on short-sale constraints. The differences are due to choice of model setup. Instead of independent noise trading, these two studies introduce noise trading through informed investor hedging need on their non-tradable asset. This modeling difference causes several significant differences in results. For example, short-sale constraints are likely to bind when prices are *high* in our study, which captures the phenomenon that informed investors are short-sale constrained when the high asset price is caused by a high level of noise demand, a scenario similar to the "tech" bubble. A decrease in price informativeness in this case lowers the likelihood of bubbles but will *not* cause crashes. By contrast, in Bai, Chang, and Wang (2006) and Marin and Olivier (2006), short-sale constraints are likely to bind when asset prices are *low*. This is because informed investors are endowed with excess non-traded risky assets. To hedge this un-traded risk, they have to short-sell the traded asset that is positively correlated with the non-traded asset. Consequently, the sharp drop of price informativeness due to short-sale constraints causes a crash in the price of the traded asset. Therefore, they capture a different set of market conditions. Furthermore, the source of uncertainty in our study is also different from that identified in these two studies. In Bai, Chang, and Wang (2006) and Marin and Olivier (2006), at a given price, informed investors' demand can be inferred and so is their constraint status. By comparison, in our study, informed investors' constraint status cannot be inferred with certainty since the high price could be caused either by a high realization of private signals or by a high level of noise trading. This introduces an additional source of perceived uncertainty to uninformed investors and causes equilibrium price more skewed and more volatile.

The remainder of the paper is structured as follows. In Section 2, rational expectation equilibrium (REE) models for an economy with asymmetric information and trading constraints are developed. In Section 3, we present the equilibrium solution and analyze the properties of equilibrium prices. Section 4 concludes. All proofs are presented in the Appendix.

2 The Model

2.1 An Economy with Information Asymmetry

The following model is an extension of the Grossman and Stiglitz model (1980) in one aspect: it includes borrowing and short-sale constrained investors. In this model, there are two dates, time 0 and time 1. At time 0, investors trade competitively in the market based on their private information. At time 1, payoffs from the assets are realized and consumption occurs.

The model assumes an underlying probability space, $(\Omega, \mathcal{F}, \mathbf{Q})$, on which all random variables are defined. A state of nature is denoted by $\omega \in \Omega$. It is also assumed that all random variables belong to a linear space, $\mathcal{N}$, of joint normally-distributed random variables on Ω.

In our model, there is one risk-free and one risky asset. The risk-free asset pays R units, while the risky asset pays v units of the single consumption good. Taking the risk-free asset to be the numeraire, we let P be the price vector for the risky assets. Investor k divides his initial wealth, $W_{0,k}$, between the risk-free and risky assets. We let D_k be the risky asset's holding by agent k. Thus, investor k's final wealth is given by:

$$W_{1,k} = W_{0,k}R + D_k(v - RP). \tag{1}$$

In this model, each investor maximizes the expected utility of consumption based on his or her own information set. We assume that, for agent k, the utility function exhibits constant absolute risk aversion, *i.e.*, $E_0\left[-e^{-w_{1,k}/\rho}\right]$, where E_0 is the expectation operator, conditional on investor information at time 0. Again, to simplify notation, we assume that all investors have the same risk aversion parameter, ρ. Generalization of this concept to heterogenous risk aversion parameters is shown in Admati (1985). We assume that investors are competitive and form a continuum with measure 1. Investors are of one of two classes: informed or uninformed.[6] Prior to trading, informed investors receive private information related to the payoff of the risky asset. The signal, s, is a noisy signal of the asset final payoff, v, given as follows:

$$s = v + \epsilon_s, \tag{2}$$

[6]We denote the measure of informed investors as w_i and the measure of uninformed investors as w_{ui}, where $w_i + w_{ui} = 1$. We denote the measure of unconstrained informed investors as w_i^{uc} and the measure of constrained informed investors as w_i^c, where $w_i^c + w_i^{uc} = w_i$.

where ϵ_s represents the noise of the signal and is independent of v. For informed investors, the information set consists of the equilibrium price vector and the realization of a private information signal, s, which is correlated with v. By contrast, the uninformed investor information set consists of only the equilibrium price.[7]

Another ingredient of the model is the existence of noise in the form of a random supply of the risky asset, m. Thus, the no-trade theorem does not apply (Milgrom and Stokey 1982).

In addition, we assume v, m, and ϵ_s are mutually independent and jointly normally-distributed with means of $0, \overline{m}, 0$ and variances of $\Sigma_v, \Sigma_m, \Sigma_s$, respectively. We assume that the risk-free asset is the numeraire asset and $R = 1$.

2.2 Short-Sale and Borrowing Constraints

A unique feature of our model is the introduction of a short-sale constraint and a price-dependent borrowing constraint on informed investor demand for the risky asset. These constraints are empirically relevant. The short-sale constraint is typically observed when the asset price is high relative to the fundamentals while the borrowing constraint arises when the stock price is low relative to the fundamental.[8] Inclusion of these constraints in the model is essential for an in-depth understanding of the properties of asset price distributions.

We incorporate the constraints into the model by assuming that only a fraction of informed investors (w_i^c) face short-sale and borrowing constraints.[9] The following definition provides a description of the short-sale constraint.

Definition 1 (Short-Sale Constraints) *Informed investors are short-sale constrained when their demand is bounded from below by d, a constant.*

Note that when $d = 0$, definition 1 is the short-sale constraint commonly

[7] We denote informed agents by i, uninformed agents by ui and generic agents by k.

[8] When the asset price is low, borrowing-constrained informed investors cannot justify a holding position on a beaten-down stock to outside lenders. Their borrowing capacity is tied to asset values. Hence, borrowing constraints can be modeled as a restriction on informed investor demand that depends on asset prices.

[9] The model assumes that only a fraction of informed investors are constrained, for the sake of generality. Constraints on uninformed investors in this type of problem are normally immaterial since they do not affect the inference problem of uninformed investors.

observed for retail investors. Next, following Yuan (2005), we capture the borrowing constraint by the following linear structure.

Definition 2 (Borrowing Constraints) *Informed investors are borrowing constrained if their stock demand is bounded from above by* $n(P) = aP + b$,[10] *where* $a > 0$ *and* $a < w_i^{uc}\rho(\tau_v + \tau_s)/w_i^c$.[11]

2.3 The Equilibrium Concept

This section defines the equilibrium concept for the above-specified constrained economy. It is based on the rational expectations model developed by Grossman (1976) and Hellwig (1980). The following is a standard equilibrium definition.

Definition 3 *A constrained REE in a constrained economy is a price vector,* P*, and allocation function,* D*, such that:*

- P *is* (s, m) *measurable.*
- *For an unconstrained agent* k, $D_k \in \arg\max_{D_k \in \mathcal{R}^n} E(U(W_k)|\mathcal{F}_k)$. $\mathcal{F}_k$ *is agent* k*'s information set.*
- *For a short-sale constrained agent* i, $D_i^c \in \arg\max_{D_i \geq d} E(U(W_i)|\mathcal{F}_i)$. $\mathcal{F}_i$ *is agent* i*'s information set.*
- *For a borrowing-constrained agent* i, $D_i^c \in \arg\max_{D_i \leq aP+b} E(U(W_i)|\mathcal{F}_i)$. $\mathcal{F}_i$ *is agent* i*'s information set.*

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[10] The financial constraint on informed investor demand is stylized but realistic. For example, investors often establish margin accounts with dealers. Let us assume the investor has a margin account for the risky asset and the margin requirement is 30%. At the trading date, an investor' wealth consists of a position (long or short) in the risky asset (Q shares) and a position (long or short with a value of A) in the riskfree asset ($W = QP + A$). He can leverage up using the margin account ($70\%W$). The upper bound of his position in the risky asset is $(1 + 70\%)Q + 70\%A/P$, which is endogenous in price. Thus, our definition can be considered as a linearized version of this constraint.

[11] We use τ to denote the precision of a random variable, that is, the inverse of the variance; later, we use 1 to denote indicator functions. The first restriction on a is to ensure that it is a borrowing constraint. The second is to ensure that the demand curve of constrained and unconstrained informed investors combined remains downward-sloping with respect to P so that the result of possible multiple equilibria is not trivial. We use bc and sc to denote borrowing and short-sale constraints, respectively, and $\overline{bc}$ and $\overline{sc}$ to denote the corresponding complements.

- *The market clearing condition is satisfied by:* $w_i^{uc} D_i + w_i^c D_i^c + w_{ui} D_{ui} = m$*, where* D_i *is unconstrained informed investor demand,* D_i^c *is constrained informed agent demand, and* D_{ui} *is uninformed agent demand.*

3 Asset Prices with Borrowing and Short-Sale Constraints

In this section, we first start with the equilibrium solution(s) for an economy with borrowing constraints and short-sale constraints, extending the results outlined in Yuan (2005). We next contrast the properties of equilibrium prices under the different constraint scenarios and investigate the corresponding price impacts.

3.1 Equilibrium under Borrowing and Short-Sale Constraints

In an economy with borrowing and short-sale constraints, if the price is low (or high) enough relative to the private signal held by informed investors, informed investors can be borrowing (or short-sale) constrained out of the market. As a result, the price informativeness could vary depending on the price level. Since the asset price is in the investor information set, the varying price informativeness makes it difficult to solve for the investor inference problem. This is especially true considering when informed investors are not constrained by either short-sale or borrowing constraints, the distribution of their private signals is doubly truncated (*i.e.*, to the left and to the right).

As a first step towards solving for the equilibrium, we examine informed investors' inference and optimization problem. Note that, in this setting, the information structure is hierarchical. Specifically, informed investors' private signal strictly dominates the price as a signal for the fundamentals. This means, for informed investors, the asset price is a redundant signal and can be ignored. Consequently, their inference problem can be worked out in closed-form. Further, for a given signal and a given price, we can solve their demand explicitly, as expressed in the following result.

Lemma 1 *Informed investor demand is represented by:*

$$D_i(s,P) = \begin{cases} d_s^{sc}s - d_p^{sc}P + d_0^{sc} & \textit{if } s < \kappa_1^{sc}P + \kappa_0^{sc}; P > (d-b)/a \\ d_s^{uc}s - d_p^{uc}P & \textit{if } \kappa_1^{sc}P + \kappa_0^{sc} < s < \kappa_1^{bc}P + \kappa_0^{bc}; P > (d-b)/a \\ d_s^{c}s - d_p^{c}P & \textit{if } P \le (d-b)/a \\ d_s^{bc}s - d_p^{bc}P + d_0^{bc} & \textit{if } s > \kappa_1^{bc}P + \kappa_0^{bc}; P > (d-b)/a. \end{cases} \tag{3}$$

Next, we consider a fictitious economy with only informed investors and an asset supply given by $\hat{m}(P) = m - w_{ui}D_{ui}(P)$. In this fictitious economy, we have

$$\begin{aligned} t_{sc} &= \frac{\hat{m}(P) - d_0^{sc} + d_p^{sc}P}{d_s^{sc}}, \quad t_{uc} = \frac{\hat{m}(P) + d_p^{uc}P}{d_s^{uc}}, \\ t_{bc} &= \frac{\hat{m}(P) - d_0^{bc} + d_p^{bc}P}{d_s^{bc}}, \quad t_c = \frac{\hat{m}(P) + d_p^{c}P}{d_s^{c}}, \end{aligned}$$

which are observable to uninformed investors since $D_{ui}(P)$ is in their information set. They can compute $\hat{m}(P)$ for a given $D_{ui}(P)$. These are the sufficient statistics for the information in P in the respective region where informed investors are either short-sale constrained, unconstrained, borrowing constrained, or totally constrained out of the market. Given the information conveyed in P, we can solve for uninformed investor optimal demand. Once we obtain a solution for $D_{ui}(P)$, the result in the following proposition provides a simple procedure to solve for equilibrium prices.

Proposition 1 *In this borrowing and short-sale constrained economy,*

- *informed investor aggregate demand, $D_i(s,P)$, is characterized by equation (3). Uninformed investor demand, $D_{ui}(P)$, is uniquely characterized by:*
$$D_{ui}(P) = \arg\max_{D_{ui} \in \mathcal{R}^n} E(U(W_{ui})|P, D_{ui}); \tag{4}$$
- *the equilibrium price $P(s,m)$ is an element of the set of P that satisfies:*

$$P = \begin{cases} \left(d_s^{sc}s - \hat{m}(P) + d_0^{sc}\right)/d_p^{sc} & \textit{if } s < \kappa_1^{sc}P + \kappa_0^{sc}; P > (d-b)/a \\ \left(d_s^{uc}s - \hat{m}(P)\right)/d_p^{uc} & \textit{if } \kappa_1^{sc}P + \kappa_0^{sc} < s < \kappa_1^{bc}P + \kappa_0^{bc}; P > (d-b)/a \\ \left(d_s^{c}s - \hat{m}(P)\right)/d_p^{c} & \textit{if } P \le (d-b)/a \\ \left(d_0^{bc} + d_s^{bc}s - \hat{m}(P)\right)/d_p^{sc} & \textit{if } s > \kappa_1^{bc}P + \kappa_0^{bc}; P > (d-b)/a. \end{cases} \tag{5}$$

This result outlines a procedure to solve for the equilibrium. Specifically, note that the right side of equation (4) depends implicitly on $D_{ui}(P)$. Therefore, uninformed investor optimal demand is a fixed point of equation (4), which, we show later, is unique. After obtaining $D_{ui}(P)$, we first compute $\hat{m}(P)$. Next, we consider the fictitious economy with only informed investors and an asset supply given by $\hat{m}(P)$. We then solve for informed investor aggregate demand. Finally, given $\hat{m}(P)$, we solve equation (5) to find the market clearing prices. This non-linear REE solution technique generalizes the results outlined in Yuan (2005). As long as the information structure is hierarchical, that is, informed investor inference problem is independent of that of uninformed investors, this solution technique can be applied to any asymmetric information setting.

The following result expresses the complicated algebraic equation, which $D_{ui}(P)$ is a solution of.

Corollary 1 *When $P \leq (d-b)/a$, the following equation expresses uninformed investor optimal demand:*

$$D_{ui}(P) = \frac{(\tau_p - \tau_s)\tau_v}{\tau_s + \tau_p w_{ui}/w_i^{uc}} \frac{P}{\rho} + \frac{\overline{m}\tau_p}{w_i^{uc}\tau_s + w_{ui}\tau_p}.$$

When $P \geq (d-b)/a$, uninformed investor demand for the risky asset is the unique fixed point of the following algebraic equation:

$$\begin{aligned}
&\frac{e^{t_1^{bc}D_{ui}+t_2^{bc}D_{ui}^2}Pr_{bc}}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)}\begin{pmatrix}\left(t_1^{bc}+2t_2^{bc}D_{ui}\right)\left(1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)\right)\\ -t_4^{bc}\phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)+\frac{1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)}t_5^{bc}\phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)\end{pmatrix}\\
&+\frac{e^{t_1^{sc}D_{ui}+t_2^{sc}D_{ui}^2}Pr_{sc}}{\Phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)}\begin{pmatrix}\left(t_1^{sc}+2t_2^{sc}D_{ui}\right)\Phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)\\ +t_4^{sc}\phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)-\frac{\Phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)}{\Phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)}t_5^{sc}\phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)\end{pmatrix}\\
&+\frac{e^{t_1^{uc}D_{ui}+t_2^{uc}D_{ui}^2}(1-Pr_{bc}-Pr_{sc})}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)}\\
&\begin{pmatrix}\left(t_1^{uc}+2t_2^{uc}D_{ui}\right)\left(\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\right)\\ +t_4^{uc}\phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-t_4^{uc}\phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\\ -\frac{\left(\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\right)t_6^{uc}\left(\phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)\right)}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)}\end{pmatrix}=0. \qquad (6)
\end{aligned}$$

All constants are defined in the appendix.

The following corollaries characterize the equilibrium for an economy with only borrowing or only short-sale constraints, respectively. Both are special cases of proposition 1.

Corollary 2 *In an economy when some informed investors face borrowing constraints,*

- *informed investor aggregate demand, $D_i(s,P)$, is characterized by:*

$$D_i(s,P) = \begin{cases} d_s^{uc}s - d_p^{uc}P & \text{if } s < \kappa_1^{bc}P + \kappa_0^{bc} \\ d_s^{bc}s - d_p^{bc}P + d_0^{bc} & \text{if } s \geq \kappa_1^{bc}P + \kappa_0^{bc}. \end{cases} \tag{7}$$

- *Uninformed investor demand for the risky asset is the unique fixed point of the following algebraic equation:*

$$\begin{aligned} 0 = & \frac{e^{t_1^{bc}D_{ui}+t_2^{bc}D_{ui}^2}Pr_{bc}}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)} \begin{pmatrix} \left(t_1^{bc}+2t_2^{bc}D_{ui}\right)\left(1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)\right) \\ -t_4^{bc}\phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right) \\ +\frac{1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)}t_5^{bc}\phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right) \end{pmatrix} \\ & + \frac{e^{t_1^{uc}D_{ui}+t_2^{uc}D_{ui}^2}(1-Pr_{bc})}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)} \begin{pmatrix} \left(t_1^{uc}+2t_2^{uc}D_{ui}\right)\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right) \\ +t_4^{uc}\phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right) \\ -\frac{\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)t_6^{uc}\phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)} \end{pmatrix}. \end{aligned} \tag{8}$$

- *the equilibrium price $P(s,m)$ is an element of the set of P that satisfies:*

$$P = \begin{cases} \left(d_s^{uc}s - \hat{m}(P)\right)/d_p^{uc} & \text{if } s < \kappa_1^{bc}P + \kappa_0^{bc} \\ \left(d_0^{bc} + d_s^{bc}s - \hat{m}(P)\right)/d_p^{bc} & \text{if } s \geq \kappa_1^{bc}P + \kappa_0^{bc}. \end{cases} \tag{9}$$

Corollary 3 *In an economy when some informed investors face short-sale constraints,*

- *informed investor aggregate demand, $D_i(s,P)$, is characterized by:*

$$D_i(s,P) = \begin{cases} d_s^{sc}s - d_p^{sc}P + d_0^{sc} & \text{if } s < \kappa_1^{sc}P + \kappa_0^{sc} \\ d_s^{uc}s - d_p^{uc}P & \text{if } s \geq \kappa_1^{sc}P + \kappa_0^{sc}. \end{cases} \tag{10}$$

- *Uninformed investor demand for the risky asset is the unique fixed point of the following algebraic equation:*

$$0 = \frac{e^{t_1^{sc} D_{ui} + t_2^{sc} D_{ui}^2} Pr_{sc}}{\Phi\left(t_3^{sc} + t_5^{sc} D_{ui}\right)} \left(\begin{array}{l} \left(t_1^{sc} + 2 t_2^{sc} D_{ui}\right) \Phi\left(t_3^{sc} + t_4^{sc} D_{ui}\right) \\ + t_4^{sc} \phi\left(t_3^{sc} + t_4^{sc} D_{ui}\right) - \frac{\Phi\left(t_3^{sc} + t_4^{sc} D_{ui}\right)}{\Phi\left(t_3^{sc} + t_5^{sc} D_{ui}\right)} t_5^{sc} \phi\left(t_3^{sc} + t_5^{sc} D_{ui}\right) \end{array} \right)$$
$$+ \frac{e^{t_1^{uc} D_{ui} + t_2^{uc} D_{ui}^2} (1 - Pr_{sc})}{1 - \Phi\left(t_5^{uc} + t_6^{uc} D_{ui}\right)} \left(\begin{array}{l} \left(t_1^{uc} + 2 t_2^{uc} D_{ui}\right)\left(1 - \Phi\left(t_5^{uc} + t_4^{uc} D_{ui}\right)\right) \\ - t_4^{uc} \phi\left(t_5^{uc} + t_4^{uc} D_{ui}\right) \\ + \frac{\left(1 - \Phi\left(t_5^{uc} + t_4^{uc} D_{ui}\right)\right) t_6^{uc} \phi\left(t_5^{uc} + t_6^{uc} D_{ui}\right)}{1 - \Phi\left(t_5^{uc} + t_6^{uc} D_{ui}\right)} \end{array} \right). \quad (11)$$

- *the equilibrium price $P(s, m)$ is an element of the set of P that satisfies:*

$$P = \begin{cases} \left(d_s^{sc} s - \hat{m}(P) + d_0^{sc}\right) / d_p^{sc} & \text{if } s < \kappa_1^{sc} P + \kappa_0^{sc} \\ \left(d_s^{uc} s - \hat{m}(P)\right) / d_p^{uc} & \text{if } s \geq \kappa_1^{sc} P + \kappa_0^{sc}. \end{cases} \quad (12)$$

The following corollary describes uninformed investor inferences in these constrained economies.

Corollary 4 *For a given P,*

$$\begin{array}{rcl} E[v|P = \frac{d_0^{bc} + d_s^{bc} s - \hat{m}(P)}{d_p^{bc}}, (s, m) \in \{bc\}] & \geq & E[v|P = \frac{d_s^{uc} s - \hat{m}(P)}{d_p^{uc}}, (s, m) \in \{uc\}], \\ Var[v|P = \frac{d_0^{bc} + d_s^{bc} s - \hat{m}(P)}{d_p^{bc}}, (s, m) \in \{bc\}] & \geq & Var[v|P = \frac{d_s^{uc} s - \hat{m}(P)}{d_p^{uc}}, (s, m) \in \{uc\}] \\ E[v|P = \frac{d_0^{sc} + d_s^{sc} s - \hat{m}(P)}{d_p^{sc}}, (s, m) \in \{sc\}] & \leq & E[v|P = \frac{d_s^{uc} s - \hat{m}(P)}{d_p^{uc}}, (s, m) \in \{uc\}] \\ Var[v|P = \frac{d_0^{sc} + d_s^{sc} s - \hat{m}(P)}{d_p^{sc}}, (s, m) \in \{sc\}] & \geq & Var[v|P = \frac{d_s^{uc} s - \hat{m}(P)}{d_p^{uc}}, (s, m) \in \{uc\}], \end{array}$$

where E and Var denote the conditional mean and variance, respectively.

This corollary suggests that, when some informed investors are borrowing or short-sale constrained out of the market, asset prices are less informative and hence, to uninformed investors, the perceived asset volatility is higher conditional on the asset price, which is a much noisier public signal.

However, when informed investors are borrowing constrained, uninformed investors suspect the asset price would be higher if informed investors were able to borrow. Thus, the perceived asset value is higher when informed investors are borrowing-constrained. The effect on the perceived volatility indicates that uninformed investors are less willing to purchase the risky asset as the price falls. The effect on conditional expectation indicates that uninformed investors are more willing to accommodate the distressed selling of informed investors as the price falls. These two countervailing effects may create a backward-bending region in uninformed investor demand, making the risky asset a Giffen good for uninformed investors.

Conversely, when informed investors are short-sale constrained, uninformed investors suspect the asset price would be lower if informed investors were able to short-sell. Thus, the perceived asset value is lower when informed investors are short-sale constrained. Both effects on the perceived volatility and the conditional expectation indicate that uninformed investors would reduce their demand drastically when informed investors are short-sale constrained. We examine these comparative static results in detail in the next section.

3.2 The Price Impact

In this section, we illustrate the equilibrium properties using numerical examples. The examples are chosen to reflect "reasonable" parameters, where the risky asset is a stock. We start with a numerical example where 15% of investors are informed. This percentage corresponds to the amount of total market capitalization held by institutional investors other than pension funds and insurance companies. We further assume that, a majority (in this case, 14%) of informed investors face possible borrowing and short-sale constraints. For simplicity, we normalize investor risk tolerance to 1.65.

A key parameter of our model is the quality of the information signal received by informed investors. In our example, we assume that informed investors receive a high-quality signal: the signal-to-noise ratio is 20. The parameters, a, b, and d are chosen so that there exist price regions where informed investors are borrowing or short-sale constrained or unconstrained, respectively. To perform a comparative static analysis on the price impact of trading constraints, we vary the percentage of informed investors who face trading constraints.

3.2.1 Symmetric Impact on Perceived Volatilities

When informed investors are constrained by either borrowing or short-sale constraints, they are unable to submit their optimal demand for the risky asset and, hence, the market price is less informative of their private signal. This decreased price informativeness creates greater perceived uncertainty for uninformed investors, who rely on the market price as a public signal for the fundamental value of the asset. This reliance is evident in Figure 1, where the conditional variance is significantly higher when prices are relatively high (*i.e.*, when short-sale constraints are possibly binding) or when prices are relatively low (*i.e.*, when borrowing constraints are possibly binding). This leads to our first observation.

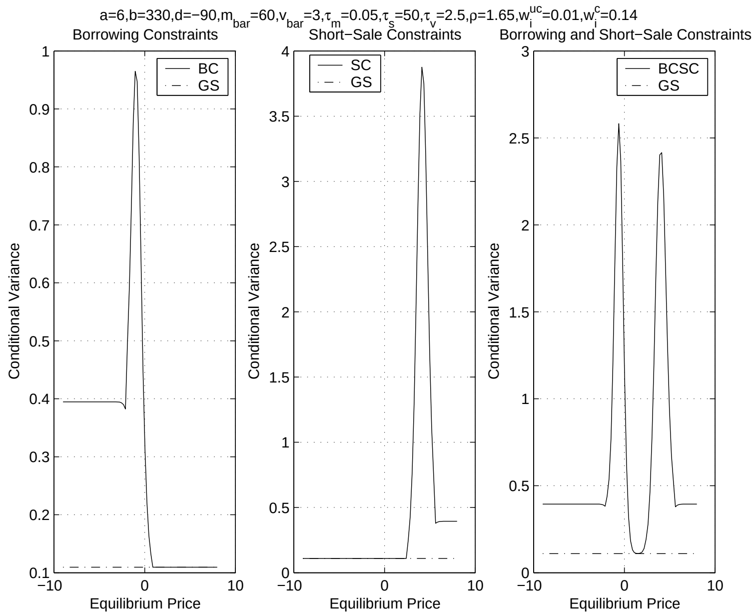

Figure 1: **Conditional Variance.** The dash-dotted lines in the graphs represent uninformed investor conditional variance of v, the fundamental value of the asset, in an economy without any trading constraints. The solid lines in the graphs represent conditional variance in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively.

Observation 1 *Borrowing and short-sale constraints have a symmetric impact on the perceived volatility of the risky asset for uninformed investors: When constraints are binding, the perceived volatility is higher.*

Furthermore, the graphs in Figure 1 show that uninformed investors perceive greatest asset volatility when they are uncertain whether informed investors are constrained or not. This indicates that the informed investor constraint status is another source of uncertainty in this economy. This finding leading to the following observation.

Observation 2 *The perceived volatility of the risky asset is higher because (1) informed investors are constrained from transmitting their private signals to prices, and (2) there is an additional source of uncertainty: the constraint status of informed investors.*

The graph in Figure 2 shows that the conditional variance is higher when a smaller percentage of informed investors face trading constraints. This result is summarized in the following observation.

Observation 3 *The decrease in price informativeness is smaller when fewer informed investors are subject to trading constraints.*

3.2.2 Asymmetric Impact on Conditional Expectation

Although the impact of trading constraints on conditional variance is symmetric, their impact on conditional expectation is not. When informed investors are borrowing constrained (that is, when prices are relatively low), uninformed investors rationally infer that informed investors hold a better signal than the price otherwise reveals and thus update their belief of the value of the risky asset upwards. By contrast, when informed investors possibly short-sale constrained (that is, when prices are relatively high), uninformed investors rationally infer that informed investors hold a worse signal than the price otherwise reflects and thus revise their belief of the asset value downwards. This result is shown in the graphs in Figure 3 and is stated in the following observation.

Observation 4 *The binding borrowing constraints causes uninformed investors to revise their expectation of the value of the risky asset upwards in the low price region, while the binding short-sale constraints causes them to revise their expectation downwards in the high price region.*

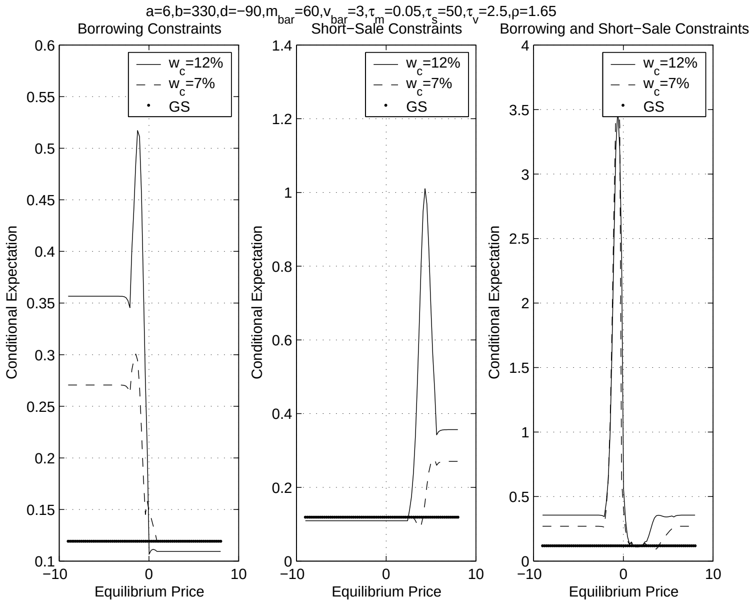

Figure 2: **Comparative Static on Conditional Variance.** The dotted lines in the graphs represent uninformed investor conditional variance of v, the fundamental value of the asset, in an economy without any trading constraints. The solid and the dashed lines in the graphs represent conditional variance in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively. The solid (dashed) lines correspond to the economies where 12% (7%) of investors are informed and constrained.

Furthermore, this revision of uninformed investor beliefs depends on the loss of price informativeness created by trading constraints. When the public signal or asset price, is less informative, uninformed investors rely on their prior beliefs to infer the value of the risky asset. The graphs in Figure 4 show that the expected mean is less elastic with respect to prices when a greater percentage of informed investors are constrained.

Observation 5 *When more informed investors face trading constraints, uninformed investors rely more on their prior belief rather than noisy prices to form their expectation of the value of the risky asset.*

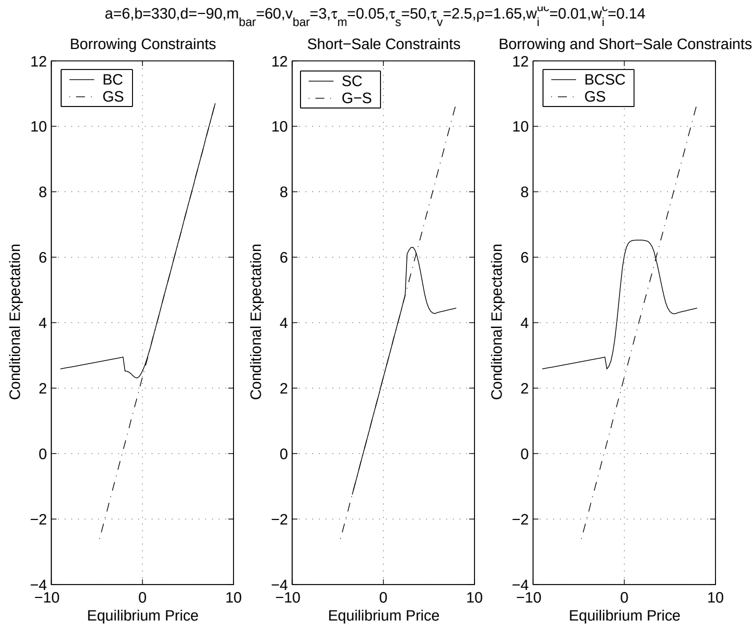

Figure 3: **Conditional Expectation.** The dash-dotted lines in the graphs represent uninformed investor conditional expectation of v, the fundamental value of the asset, in an economy without any trading constraints. The solid lines in the graphs represent their conditional expectation in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively.

3.2.3 Asymmetric Optimal Stock Holdings by Uninformed Investors

We have shown that borrowing and short-sale constraints have a significant impact on uninformed investor inferences about the fundamental value of the risky asset. Relative to the case without any trading constraints, uninformed investors in a constrained economy have higher perceived uncertainty, a lower expectation of the asset value when the price is relatively high, and a higher expectation when the price is relatively low. The impact of trading constraints on the uninformed investors' optimal stock holding is summarized below.

Observation 6 *Compared to the case without any trading constraints, unin-*

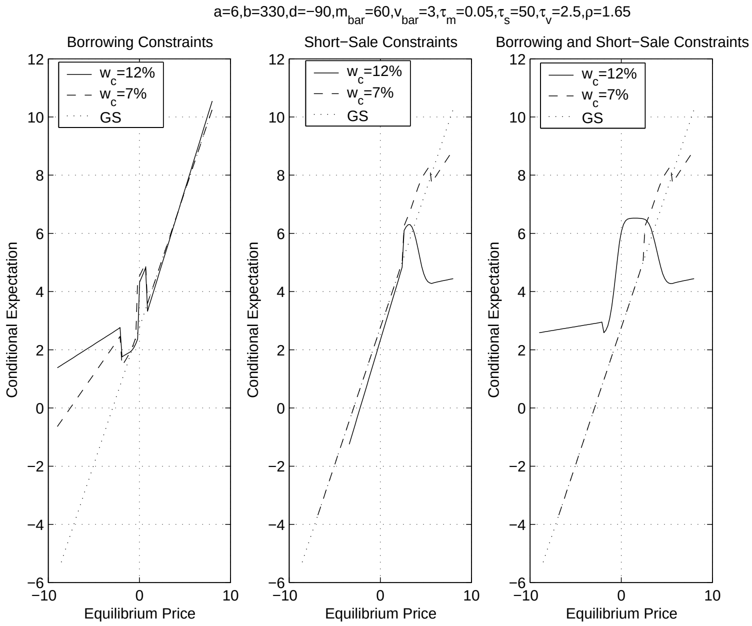

Figure 4: **Comparative Static on Conditional Expectation.** The dotted lines in the graphs represent uninformed investor conditional expectation of v, the fundamental value of the asset, in an economy without any trading constraints. The solid and the dashed lines in the graphs represent conditional expectation in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively. The solid (dashed) lines correspond to the economies where 12% (7%) of investors are informed and constrained.

formed investor demand in a constrained economy is smaller when the price is high (i.e., when informed investors are short-sale constrained), and may turn backwards when the price is low (i.e., when informed investors are borrowing constrained).

Figure 5 graphs examples of uninformed investor demand with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively. Note that the backward-bending region occurs when prices are low, or, when borrowing constraints are more likely to bind. The intuition for these results is as follows. In the high-price region, an *increase* in price reflects a higher short-sale constraint probability. The resultant decrease in price informativeness leads uninformed investors to revise downwards their belief of the asset value and upwards their perceived uncertainty. Hence, uninformed investors reduce their demand drastically as price increases, that is, their demand is downward-sloping but more price elastic. However, in the low-price region, a *falling* price also reduces price informativeness and causes uninformed investors to reduce their demand, resulting a price in-elastic demand or even a backward-bending demand curve.

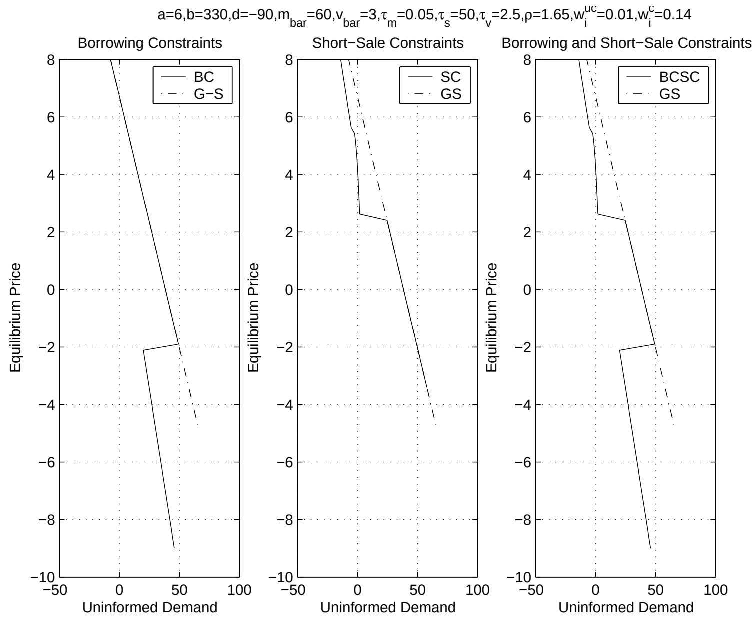

Figure 5: **Uninformed Investor Demand.** The dash-dotted lines in the graphs represent the demand for the risky asset by uninformed investors in an economy without any trading constraints. The solid lines in the graphs represent their demand in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively.

3.2.4 Asymmetric Price Impact: Bubbles and Crashes

In the previous section, we have showed that, with borrowing constraints, uninformed investor demand could turn backwards when the asset price is low. This possibility could result in a backward-bending market excess demand curve (total demand minus the market fixed supply of the risky asset).

Such an example is shown in Figure 6, indicating price multiplicity and, hence, higher volatility in the low price region. This leads to the following observation.

Observation 7 *With borrowing constraints, crashes from a small adverse shock are more likely and volatility is higher when prices are low.*

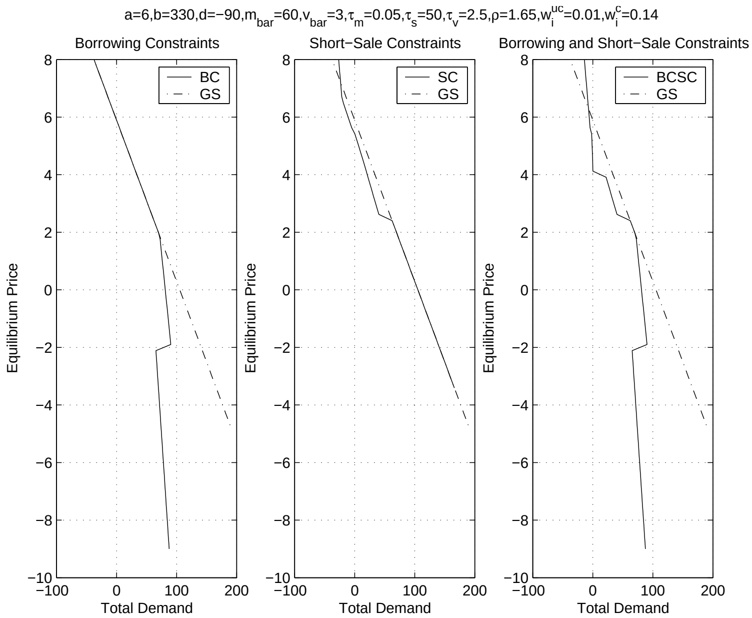

Figure 6: **Uninformed and Informed Investor Demand.** The dash-dotted lines in the graphs represent the demand for the risky asset by informed and uninformed investors in an economy without any trading constraints. The solid lines in the graphs represent their demand in an economy with borrowing constraints, short-sale constraints, and both borrowing and short-sale constraints, respectively. In all these examples, informed investors hold a private signal of 2.72.

Intuitively, a small adverse shock to the fundamental could result in a low asset price and consequently cause informed investors to be borrowing constrained. These investors may have to conduct noise selling to meet their margin requirements. This may put further downward pressure on the asset price. Furthermore, as shown in the previous results, uninformed investors would not purchase the distressed asset due to higher perceived uncertainty and adverse selection concerns. The information asymmetry between informed and uninformed investors *multiplies* the effect of borrowing constraints and *exacerbates* the *downward* price movement, causing higher price volatility or even price multiplicity.

By contrast, a small positive shock to the fundamental is less likely to build a bubble. First, a higher price may create short-sale constraints, which do not put upward pressure on the asset price. Second, as shown in Figure 5, uninformed investors demand less in the high price region where price informativeness is reduced. Uninformed investors, in this case, demand an information disadvantaged premium to hold the risky asset. Therefore, in the high price region, information asymmetry plays a *dampening* effect on the *upward* price movement. This discussion leads to our next observation.

Observation 8 *With short-sale constraints, bubbles are less likely as high prices are less informative and uninformed investors are likely to reduce their demand drastically as the price increases.*

Yuan (2005) has shown the equilibrium price has an asymmetric distribution when some informed investors face borrowing constraints. One may conjecture that symmetric trading constraints (*i.e.*, short-sale constraints bind when prices are high and borrowing constraints bind when prices are low) create a more symmetric price distribution. However, an examination of the likelihood of bubbles and crashes in this constrained economy yields the opposite finding. The degree of asymmetry in the price distribution is higher when both trading constraints are present. The comparative static analyses in the previous section lead to our last observation.

Observation 9 *In this borrowing and short-sale constrained economy, the asset price distribution is negatively skewed. The skewness is more pronounced with severe information asymmetry and with a greater percentage of constrained informed investors.*

4 Concluding Remarks

This study explores the asset pricing implications of borrowing and short-sale constraints in the presence of information asymmetry. Our analysis shows that price informativeness varies with the price level in a constrained economy. We find that borrowing constraints and short-sale constraints have different price impacts in an asymmetric information environment. When informed investors are constrained by either borrowing or short-sale constraints, asset prices become less informative. However, since less informative prices cause uninformed investors to demand an information-disadvantaged premium, the downward price movement is exacerbated while the upward price movement is lessened. This finding is contrary to the theoretical findings in the existing literature but is in-line with empirical observations. The result of our study indicate it is important to consider the impact of an asymmetric information environment as well as market frictions such as borrowing and short-sale constraints for an in-depth understanding of the properties of asset returns and volatilities.

A Proof of Lemmas 1

The constrained informed investor optimization problem is:

$$\max_{D_i^c} E_0[U(v, D_i^c|s, P] + \mu_{D_i^c} D_i^c - \lambda(D_i^c - aP - b). \tag{A1}$$

The unconstrained informed investor optimization problem is:

$$\max_{D_i^{uc}} E_0[U(v, D_i^{uc})|s, P]. \tag{A2}$$

Solving the above optimization, we find that the informed investor demand $f^c : \mathcal{R}^2 \to \mathcal{R}$ is:

$$D_i(s,P) = \begin{cases} d_s^{sc}s - d_p^{sc}P + d_0^{sc} & \text{if } s < \kappa_1^{sc}P + \kappa_0^{sc}; P > -b/a \\ d_s^{uc}s - d_p^{uc}P & \text{if } \kappa_1^{sc}P + \kappa_0^{sc} < s < \kappa_1^{bc}P + \kappa_0^{bc}; P > -b/a \\ d_s^{c}s - d_p^{c}P & \text{if } P \le -b/a \\ d_s^{bc}s - d_p^{bc}P + d_0^{bc} & \text{if } s > \kappa_1^{bc}P + \kappa_0^{bc}; P > -b/a. \end{cases}$$

We denote our inference constants as follows:

$$\begin{aligned} d_s^{sc} &= w_i^{uc}\rho\tau_s, & d_p^{sc} &= w_i^{uc}\rho(\tau_s + \tau_v), \\ d_s^{uc} &= (w_i^{uc} + w_i^c)\rho\tau_s, & d_p^{uc} &= (w_i^{uc} + w_i^c)\rho(\tau_s + \tau_v), \\ d_s^{bc} &= w_i^{uc}\rho\tau_s, & d_p^{bc} &= w_i^{uc}\rho(\tau_s + \tau_v) - w_i^c a, \\ d_s^{c} &= w_i^{uc}\rho\tau_s, & d_p^{c} &= w_i^{uc}\rho(\tau_s + \tau_v), \\ \kappa_1^{sc} &= (\tau_s + \tau_v)/\tau_s, & d_0^{bc} &= w_i^c b, d_0^{sc} = w_i^c d, \\ \kappa_1^{bc} &= (\tau_s + \tau_v + a/\rho)/\tau_s, & \kappa_0^{bc} &= b/(\rho\tau_s), \kappa_0^{sc} = d/(\rho\tau_s). \end{aligned}$$

B Proof of Proposition 1

Case 1: When informed investors are not constrained, the market clearing condition can be expressed as:

$$d_s^{uc}s - d_p^{uc}P = \hat{m}(P) \quad \text{and} \quad P = p_s^{uc}s - p_m^{uc}\hat{m},$$

where

$$p_s^{uc}s = d_s^{uc}/d_p^{uc} \quad \text{and} \quad p_m^{uc} = d_m^{uc}/d_p^{uc}.$$

Case 2: When informed investors are borrowing constrained, the market clearing condition can be expressed as:

$$d_s^{bc} s - d_p^{bc} P + d_0^{bc} = \hat{m}(P) \quad \text{and} \quad P = p_s^{bc} s - p_m^{bc}(\hat{m} - d_o^{bc}),$$

where

$$p_s^{bc} = d_s^{bc}/d_p^{bc} \quad \text{and} \quad p_m^{bc} = d_m^{bc}/d_p^{bc}.$$

Case 3: When informed investors are short-sale constrained, the market clearing condition can be expressed as:

$$d_0^{sc} + d_s^{sc} s - d_p^{sc} P = \hat{m}(P) \quad \text{and} \quad P = p_s^{sc} s - p_m^{sc}(\hat{m} - d_o^{sc}),$$

where

$$p_s^{sc} = d_s^{sc}/d_p^{sc} \quad \text{and} \quad p_m^{sc} = d_m^{sc}/d_p^{sc}.$$

Case 4: When informed investors are both short-sale and borrowing constrained, *i.e.*, $P \leq (d-b)/a$, the market clearing condition can be expressed as:

$$d_s^c s - d_p^c P = \hat{m}(P) \quad \text{and} \quad P = p_s^c s - p_m^c \hat{m},$$

where

$$p_s^c = d_s^c/d_p^c \quad \text{and} \quad p_m^c = d_m^c/d_p^c.$$

Therefore, we obtain the results shown in the proposition(s).

C Proof of Corollary 1

We first define the following constants:

$$\begin{aligned}
\kappa^{sc} &= \kappa_1^{sc} P + \kappa_0^{sc}, \tau_{ss} = 1/(1/\tau_v + 1/\tau_s), & \kappa^{bc} &= \kappa_1^{bc} P + \kappa_0^{bc}, \\
\frac{1}{\tau_{P^{uc}}} &= \left(\frac{1}{\rho\left(w_i^{uc} + w_i^c\right)\tau_s}\right)^2 \frac{1}{\tau_m}, & \frac{1}{\tau_{P^{bc}}} &= \frac{1}{\tau_{P^{sc}}} = \left(\frac{1}{\rho w_i^{uc} \tau_s}\right)^2 \frac{1}{\tau_m}, \\
\theta^{sc} &= \frac{\tau_{P^{sc}}}{\tau_{ss} + \tau_{p^{sc}}} \kappa^{sc} / \frac{1}{\sqrt{(\tau_{ss} + \tau_{p^{sc}})}}, & \theta^{bc} &= \frac{\tau_{P^{bc}}}{\tau_{ss} + \tau_{p^{bc}}} \kappa^{bc} / \frac{1}{\sqrt{(\tau_{ss} + \tau_{p^{bc}})}}.
\end{aligned}$$

We also use $\Phi(\cdot)(\phi(\cdot))$ to denote the cumulative (probability) distribution function of a standard normal variable, *i.e.*, $\Phi(\cdot) \equiv NORMCDF(0,1)$ and $\phi(\cdot) \equiv NORMPDF(0,1)$. We then express the probabilities of informed investors being respectively short-sale constrained, borrowing constrained, and unconstrained in the following equations.

$$
\begin{aligned}
Pr_{sc} &= \int_{\{s \leq (\kappa_1^{sc} P + \kappa_0^{sc})\}} \phi\left(\frac{s}{1/\sqrt{\tau_{ss}}}\right) ds = \Phi\left(\frac{(\kappa_1^{sc} P + \kappa_0^{sc})}{1/\sqrt{\tau_{ss}}}\right), \\
Pr_{bc} &= 1 - \Phi\left(\frac{(\kappa_1^{bc} P + \kappa_0^{bc})}{1/\sqrt{\tau_{ss}}}\right), Pr_{uc} = 1 - (Pr_{bc} + Pr_{sc}).
\end{aligned}
$$

As specified in the text, uninformed investors divide their initial wealth, W_0, between risky and risk-free assets. We let D_{ui} represent the risky asset's holdings by uninformed investors. Thus, the uninformed investor's final wealth is given by:

$$W_{1,ui} = W_{0,ui} R + D_{ui}(v - RP).$$

The expected utility of an uninformed investor, conditional on informed investors being short-sale constrained, can be expressed in the following form:

$$
\begin{aligned}
& E_0[-e^{-w_1/\rho} | P, s \leq \kappa_1^{sc} P + \kappa_0^{sc}] = E_0[e^{-(W_0 R + D_{ui}(v - RP))/\rho} | P, s \leq \kappa_1^{sc} P + \kappa_0^{sc})] \\
= \; & e^{-(W_0 R - D_{ui} RP)/\rho} E_0[-e^{-D_{ui} v/\rho} | P, s \leq \kappa_1^{sc} P + \kappa_0^{sc})].
\end{aligned}
$$

Using iterative expectation, we can write this expression as:

$$E_0[-e^{-D_{ui} v/\rho} | P, s \leq \kappa_1^{sc} P + \kappa_0^{sc}] = E_0[E_0[-e^{-D_{ui} v/\rho} | s] | P, s \leq \kappa_1^{sc} P + \kappa_0^{sc}].$$

Since

$$f(y) \sim N(0, \frac{1}{\tau_0}), \quad x = y + \epsilon_x, \quad f(\epsilon_x) \sim N(0, \frac{1}{\tau_x}), \quad f(y|x) \sim N(\frac{\tau_x}{\tau_0 + \tau_x} x, \frac{1}{\tau_0 + \tau_x}),$$

we obtain

$$
\begin{aligned}
& E_0[-e^{-D_{ui} v/\rho} | s] = \int_{-\infty}^{\infty} -e^{-D_{ui} v/\rho} f\,(v | P, s)\, dv \\
= & \int_{-\infty}^{\infty} -e^{-D_{ui} v/\rho} \frac{1}{\sqrt{1/(\tau_v + \tau_s)}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{v - \frac{\tau_s}{\tau_v + \tau_s} s}{\sqrt{\frac{1}{\tau_v + \tau_s}}}\right)^2} dv = -e^{-(\frac{\tau_s}{\rho(\tau_v + \tau_s)} D_{ui} s - \frac{D_{ui}^2}{2\rho^2(\tau_v + \tau_s)})}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& E_0[-e^{-D_{ui}v/\rho}|P, s \le \kappa_1^{sc}P + \kappa_0^{sc}] = E_0[-e^{-(\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}s - \frac{D_{ui}^2}{2\rho^2(\tau_v+\tau_s)})}|P, s \le \kappa_1^{sc}P + \kappa_0^{sc}] \\
= \; & -e^{-(-\frac{D_{ui}^2}{2\rho^2(\tau_v+\tau_s)})} E_0[e^{-\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}s}|P, s \le \kappa_1^{sc}P + \kappa_0^{sc}] \\
= \; & -e^{-(-\frac{D_{ui}^2}{2\rho^2(\tau_v+\tau_s)})} \int_{s \le (\kappa_1^{sc}P + \kappa_0^{sc})} e^{-\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}s} \frac{f(s|P)}{\int_{s \le (\kappa_1^{sc}P+\kappa_0^{sc})} f(s|P)\, ds} ds \\
= \; & -e^{-(-\frac{D_{ui}^2}{2\rho^2(\tau_v+\tau_s)})} e^{-\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}\left(\frac{\tau_{Psc}}{\tau_{ss}+\tau_{Psc}}\frac{d_P^{sc}P+\bar{m}-(1-w_i)D_{ui}-d_0^{sc}}{d_s^{sc}} - \frac{1}{2(\tau_{ss}+\tau_{Psc})}\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}\right)} \\
& \Phi\left(\frac{(\kappa_1^{sc}P + \kappa_0^{sc}) - \left(\frac{\tau_{Psc}}{\tau_{ss}+\tau_{Psc}}\frac{d_P^{sc}P+\bar{m}-(1-w_i)D_{ui}-d_0^{sc}}{d_s^{sc}} - \frac{1}{(\tau_{ss}+\tau_{Psc})}\frac{\tau_s}{\rho(\tau_v+\tau_s)}D_{ui}\right)}{1/\sqrt{\tau_{Psc}+\tau_{ss}}}\right) / \\
& \Phi\left(\frac{(\kappa_1^{sc}P + \kappa_0^{sc}) - \frac{\tau_{Psc}}{\tau_{ss}+\tau_{Psc}}\frac{d_P^{sc}P+\bar{m}-(1-w_i)D_{ui}-d_0^{sc}}{d_s^{sc}}}{1/\sqrt{\tau_{Psc}+\tau_{ss}}}\right).
\end{aligned}
$$

We use g to denote $\frac{1}{(\tau_v+\tau_s)}$, h to denote $\frac{\tau_s}{(\tau_v+\tau_s)}$, g^{sc} to denote $\frac{1}{\tau_{ss}+\tau_{Psc}}$, and h^{sc} to denote $\frac{\tau_{Psc}}{\tau_{ss}+\tau_{Psc}}$. The expected utility of uninformed investors conditional on informed investors being short-sale constrained can then be simplified as:

$$
\begin{aligned}
& E_0[-e^{-w_1/\rho}|P, s \le \kappa_1^{sc}P + \kappa_0^{sc}] \\
= \; & -e^{\frac{D_{ui}^2}{2\rho^2}g} e^{-(W_0R - D_{ui}RP)/\rho} e^{-\frac{h}{\rho}D_{ui}\left(h^{sc}\left(\frac{d_P^{sc}}{d_s^{sc}}P + \frac{1}{d_s^{sc}}\bar{m} - \frac{1-w_i}{d_s^{sc}}D_{ui}\right) - \frac{1}{2\rho}g^{sc}hD_{ui}\right)} \\
& \Phi\left(\frac{(\kappa_1^{sc}P + \kappa_0^{sc}) - h^{sc}\frac{d_P^{sc}P+\bar{m}-(1-w_i)D_{ui}-d_0^{sc}}{d_s^{sc}} + hg^{sc}D_{ui}/\rho}{\sqrt{g^{sc}}}\right) / \\
& \Phi\left(\frac{(\kappa_1^{sc}P + \kappa_0^{sc}) - h^{sc}\frac{d_P^{sc}P+\bar{m}-(1-w_i)D_{ui}-d_0^{sc}}{d_s^{sc}}}{\sqrt{g^{sc}}}\right).
\end{aligned}
$$

The expected utility of uninformed investors condition on informed investors being borrowing constrained or unconstrained can be similarly expressed.

$$\begin{aligned}
&E_0[-e^{-w_1/\rho}|P, s \geq \kappa_1^{bc}P + \kappa_0^{bc}] \\
= \;& -e^{\frac{D_{ui}^2}{2\rho^2}g} e^{-(W_0R - D_{ui}RP)/\rho} e^{-\frac{h}{\rho}D_{ui}\left(h^{bc}\left(\frac{d_P^{bc}}{d_s^{bc}}P - \frac{d_0^{bc}}{d_s^{bc}} + \frac{1}{d_s^{bc}}\bar{m} - \frac{1-w_i}{d_s^{bc}}D_{ui}\right) - \frac{1}{2\rho}g^{bc}hD_{ui}\right)} \\
& \left(1 - \Phi\left(\frac{(\kappa_1^{bc}P + \kappa_0^{bc}) - h^{bc}\left(\frac{d_P^{bc}}{d_s^{bc}}P - \frac{d_0^{bc}}{d_s^{bc}} + \frac{1}{d_s^{bc}}\bar{m} - \frac{1-w_i}{d_s^{bc}}D_{ui}\right) + hg^{bc}D_{ui}/\rho}{\sqrt{g^{bc}}}\right)\right) / \\
& \left(1 - \Phi\left(\frac{(\kappa_1^{bc}P + \kappa_0^{bc}) - h^{bc}\left(\frac{d_P^{bc}}{d_s^{bc}}P - \frac{d_0^{bc}}{d_s^{bc}} + \frac{1}{d_s^{bc}}\bar{m} - \frac{1-w_i}{d_s^{bc}}D_{ui}\right)}{\sqrt{g^{bc}}}\right)\right),
\end{aligned}$$

where $g^{bc} = 1/(\tau_{ss} + \tau_p^{bc})$ and $h^{bc} = \tau_{P^{bc}}/(\tau_{ss} + \tau_{P^{bc}})$.

$$\begin{aligned}
&E_0[-e^{-w_1/\rho}|P, \kappa_1^{sc}P + \kappa_0^{sc} < s < \kappa_1^{bc}P + \kappa_0^{bc}] \\
= \;& -e^{\frac{D_{ui}^2}{2\rho^2}g} e^{-(W_0R - D_{ui}RP)/\rho} e^{-\frac{h}{\rho}D_{ui}\left(h^{uc}\left(\frac{d_P^{uc}}{d_s^{uc}}P + \frac{1}{d_s^{uc}}\bar{m} - \frac{1-w_i}{d_s^{uc}}D_{ui}\right) - \frac{1}{2\rho}g^{uc}hD_{ui}\right)} \\
& \left(\begin{array}{c} \Phi\left(\frac{(\kappa_1^{bc}P + \kappa_0^{bc}) - h^{uc}\left(\frac{d_P^{uc}}{d_s^{uc}}P + \frac{1}{d_s^{uc}}\bar{m} - \frac{1-w_i}{d_s^{uc}}D_{ui}\right) + hg^{uc}D_{ui}/\rho}{\sqrt{g^{uc}}}\right) \\ -\Phi\left(\frac{(\kappa_1^{sc}P + \kappa_0^{sc}) - h^{uc}\left(\frac{d_P^{uc}}{d_s^{uc}}P + \frac{1}{d_s^{uc}}\bar{m} - \frac{1-w_i}{d_s^{uc}}D_{ui}\right) + hg^{uc}D_{ui}/\rho}{\sqrt{g^{uc}}}\right) \end{array}\right) / \\
& \left(\begin{array}{c} \Phi\left(\frac{(\kappa_1^{bc}P + \kappa_0^{bc}) - h^{uc}\left(\frac{d_P^{uc}P + \bar{m} - (1-w_i)D_{ui}}{d_s^{uc}}\right)}{\sqrt{g^{uc}}}\right) \\ -\Phi\left(\frac{\kappa_1^{sc}P + \kappa_0^{sc} - h^{uc}\left(\frac{d_P^{uc}P + \bar{m} - (1-w_i)D_{ui}}{d_s^{uc}}\right)}{\sqrt{g^{uc}}}\right) \end{array}\right),
\end{aligned}$$

where $g^{uc} = 1/(\tau_{ss} + \tau_{P^{uc}})$ and $h^{uc} = \tau_{P^{uc}}/(\tau_{ss} + \tau_{P^{uc}})$.

Since we assume the risk-free asset is the numeraire asset and $R = 1$, the first-order condition of the optimal demand problem for the risky asset can be expressed in the following equation:

$$\frac{e^{t_1^{bc}D_{ui}+t_2^{bc}D_{ui}^2}Pr_{bc}}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)}\left(\begin{array}{l}\left(t_1^{bc}+2t_2^{bc}D_{ui}\right)\left(1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)\right)\\ -t_4^{bc}\phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)+\frac{1-\Phi\left(t_3^{bc}+t_4^{bc}D_{ui}\right)}{1-\Phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)}t_5^{bc}\phi\left(t_3^{bc}+t_5^{bc}D_{ui}\right)\end{array}\right)$$
$$+\frac{e^{t_1^{sc}D_{ui}+t_2^{sc}D_{ui}^2}Pr_{sc}}{\Phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)}\left(\begin{array}{l}\left(t_1^{sc}+2t_2^{sc}D_{ui}\right)\Phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)\\ +t_4^{sc}\phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)-\frac{\Phi\left(t_3^{sc}+t_4^{sc}D_{ui}\right)}{\Phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)}t_5^{sc}\phi\left(t_3^{sc}+t_5^{sc}D_{ui}\right)\end{array}\right)$$
$$+\frac{e^{t_1^{uc}D_{ui}+t_2^{uc}D_{ui}^2}(1-Pr_{bc}-Pr_{sc})}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)} \tag{C1}$$
$$\left(\begin{array}{l}\left(t_1^{uc}+2t_2^{uc}D_{ui}\right)\left(\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\right)\\ +t_4^{uc}\phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-t_4^{uc}\phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\\ -\frac{\left(\Phi\left(t_3^{uc}+t_4^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_4^{uc}D_{ui}\right)\right)t_6^{uc}\left(\phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)\right)}{\Phi\left(t_3^{uc}+t_6^{uc}D_{ui}\right)-\Phi\left(t_5^{uc}+t_6^{uc}D_{ui}\right)}\end{array}\right)=0$$

where

$$\begin{aligned}
t_1^{bc} &= \left(\frac{P}{\rho}-\frac{h}{\rho}\frac{h^{bc}}{d_s^{bc}}(d_p^{bc}P+\overline{m}-d_0^{bc})\right), & t_2^{bc} &= \left(\frac{g}{2\rho^2}+\frac{h}{\rho}\left(\frac{h^{bc}}{d_s^{bc}}(1-w_i)+\frac{g^{bc}h}{2\rho}\right)\right),\\
t_3^{bc} &= \frac{(\kappa_1^{bc}P+\kappa_0^{bc})-h^{bc}(d_p^{bc}P+\overline{m}-d_0^{bc})/d_s^{bc}}{\sqrt{g^{bc}}}, & t_4^{bc} &= \frac{h^{bc}(1-w_i)/d_s^{bc}+hg^{bc}/\rho}{\sqrt{g^{bc}}},\\
t_5^{bc} &= \frac{h^{bc}(1-w_i)/d_s^{bc}}{\sqrt{g^{bc}}}, & t_5^{sc} &= \frac{h^{sc}(1-w_i)/d_s^{sc}}{\sqrt{g^{sc}}},\\
t_1^{sc} &= \left(\frac{P}{\rho}-\frac{h}{\rho}\frac{h^{sc}}{d_s^{sc}}(d_p^{sc}P+\overline{m})\right), & t_2^{sc} &= \left(\frac{g}{2\rho^2}+\frac{h}{\rho}\left(\frac{h^{sc}}{d_s^{sc}}(1-w_i)+\frac{g^{sc}h}{2\rho}\right)\right),\\
t_3^{sc} &= \frac{(\kappa_1^{sc}P+\kappa_0^{sc})-h^{sc}(d_p^{sc}P+\overline{m})/d_s^{sc}}{\sqrt{g^{sc}}}, & t_4^{sc} &= \frac{h^{sc}(1-w_i)/d_s^{sc}+hg^{sc}/\rho}{\sqrt{g^{sc}}},\\
t_1^{uc} &= \left(\frac{P}{\rho}-\frac{h}{\rho}\frac{h^{uc}}{d_s^{uc}}(d_p^{uc}P+\overline{m})\right), & t_2^{uc} &= \left(\frac{g}{2\rho^2}+\frac{h}{\rho}\left(\frac{h^{uc}}{d_s^{uc}}(1-w_i)+\frac{g^{uc}h}{2\rho}\right)\right),\\
t_3^{uc} &= \frac{(\kappa_1^{bc}P+\kappa_0^{bc})-h^{uc}(d_p^{uc}P+\overline{m})/d_s^{uc}}{\sqrt{g^{uc}}}, & t_4^{uc} &= \frac{h^{uc}(1-w_i)/d_s^{uc}+hg^{uc}/\rho}{\sqrt{g^{uc}}},\\
t_5^{uc} &= \frac{(\kappa_1^{sc}P+\kappa_0^{sc})-h^{uc}(d_p^{uc}P+\overline{m})/d_s^{uc}}{\sqrt{g^{uc}}}, & t_6^{uc} &= \frac{h^{uc}(1-w_i)/d_s^{uc}}{\sqrt{g^{uc}}}.
\end{aligned}$$

Since the left side is a continuous function of D_{ui}, a fixed point exists. To see that the fixed point is unique, we will show that the left side is decreasing in D_{ui}. To see this, note that an increase in D_{ui} results in a distribution over v that first order stochastically dominates the earlier one.

When $P \leq (d-b)/a$, informed investors facing trading constraints are constrained out of the market. In this case, the uninformed investor's inference and optimization problem is standard. The following equation expresses uninformed investor optimal demand:

$$D_{ui} = \frac{(\tau_p - \tau_s)\tau_v}{\tau_s + \tau_p w_{ui}/w_i^{uc}} \frac{P}{\rho} + \frac{\overline{m}\tau_p}{w_i^{uc}\tau_s + w_{ui}\tau_p}, \tag{C2}$$

where

$$\frac{1}{\tau_p} = \frac{1}{\tau_s} + \left(\frac{\rho}{w_i^{uc}\tau_s}\right)^2 \frac{1}{\tau_m}.$$

In this case, the equilibrium price is also standard.

$$P = \frac{\tau_s}{\tau_s + \tau_v} s - \frac{\rho}{w_i^{uc}(\tau_s + \tau_v)} (m - w_{ui}(\frac{(\tau_p - \tau_s)\tau_v}{\tau_s + \tau_p w_{ui}/w_i^{uc}} \frac{P}{\rho} + \frac{\overline{m}\tau_p}{w_i^{uc}\tau_s + w_{ui}\tau_p})). \tag{C3}$$

D Proofs of Corollary 4

We first denote the following:

$$\begin{aligned}
\theta_{uc}^{bc} &= \kappa^{bc}/\sqrt{1/\tau_v + 1/\tau_{p^{uc}}}, \quad \theta_c^{bc} = \kappa^{bc}/\sqrt{1/\tau_v + 1/\tau_{p^c}}, \quad \theta^{bc} = \kappa^{bc}/\sqrt{1/\tau_v + 1/\tau_s}, \\
\theta_{uc}^{sc} &= \kappa^{sc}/\sqrt{1/\tau_v + 1/\tau_{p^{uc}}}, \quad \theta_c^{sc} = \kappa^{sc}/\sqrt{1/\tau_v + 1/\tau_{p^c}}, \quad \theta^{sc} = \kappa^{bc}/\sqrt{1/\tau_v + 1/\tau_s}, \\
\frac{1}{\tau_{P^c}} &= \left(\frac{1}{\rho w_i^{uc}\tau_s}\right)^2 \frac{1}{\tau_m}, \quad \eta^{sc} = (d)\left(w_i^{uc} + w_i^c\right), \quad \eta^{bc} = (aP + b)\left(w_i^{uc} + w_i^c\right).
\end{aligned}$$

Since we have truncated normal distributions, we denote the following hazard function related terms.

$$\lambda^+ = \frac{\phi(\theta)}{1 - \Phi(\theta)}, \lambda^- = \frac{-\phi(\theta)}{\Phi(\theta)}, \delta^+ = \lambda^+(\lambda^+ - \theta), \delta^- = \lambda^-(\lambda^- - \theta).$$

We now express the conditional moments of the truncated normal variables in closed-form following Johnson and Kotz (1974). The following expressions represent uninformed investor inference on conditional moments of v when some informed investors face only borrowing constraints.

$$\begin{aligned} E^{bc} &= E[v| \left\{P = P^{bc}, D_{ui} = D^*_{ui}, (s,m) \in \{bc\}\right\}] \\ &= E\left[v| \left\{P = p^{bc}_s s - p^{bc}_m(\hat{m} - w^c_i b), D_{ui} = D^*_{ui}, s \geq \kappa^{bc}, \hat{m} \geq \eta^{bc}\right\}\right] \\ &= e^{bc}_P P - e^c_{D_{ui}} D^*_{ui} + e^c_\lambda \lambda^{bc+} + e^{bc}_0, \end{aligned} \tag{D1}$$

where

$$\begin{aligned} e^{bc}_P &= \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}} \left(\frac{\tau_s + \tau_v}{\tau_s} - \frac{w^c_i a}{w^{uc}_i \tau_s \rho}\right),\ e^c_{D_{ui}} = \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}} \left(\frac{1 - w_i}{w^{uc}_i \tau_s \rho}\right), \\ e^c_\lambda &= \frac{\tau_s}{\tau_s + \tau_v} \frac{1}{\sqrt{\tau_{p^c} + \tau_{ss}}},\ e^{bc}_0 = \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}} \left(\frac{\overline{m} - w^c_i b}{w^{uc}_i \tau_s \rho}\right), \quad \text{and} \end{aligned}$$

$$\begin{aligned} V^{bc} &= Var\left[v| \left\{P = P^{bc}, D_{ui} = D^*_{ui}, (s,m) \in \{bc\}\right\}\right] \\ &= Var\left[v| \left\{P = p^{bc}_s s - p^{bc}_m(\hat{m} - w^c_i b), D_{ui} = D^*_{ui}, s \geq \kappa^{bc}, \hat{m} \geq \eta^{bc}\right\}\right] \\ &= v_0 + v^c_\delta(1 - \delta^{bc+}), \end{aligned} \tag{D2}$$

where

$$v_0 = \frac{1}{\tau_v + \tau_s},\ v^c_\delta = \left(\frac{\tau_s}{\tau_v + \tau_s}\right)^2 \left(\frac{1}{\tau_{p^c} + \tau_{ss}}\right).$$

$$\begin{aligned} E^{\overline{\{bc\}}} &= E\left[v| \left\{P = P^{uc}, D_{ui} = D^*_{ui}, (s,m) \in \overline{\{bc\}}\right\}\right] \\ &= E\left[v| \left\{\widetilde{P} = p^{uc}_s \widetilde{s} - p^{uc}_m \hat{m}, D_{ui} = D^*_{ui}, s \leq \kappa^{bc}, \hat{m} \leq \eta^{bc}\right\}\right] \\ &= e^{uc}_P P - e^{uc}_{D_{ui}} D^*_{ui} + e^{uc}_\lambda \lambda^{bc-}_{uc} + e^{uc}_0, \end{aligned} \tag{D3}$$

$$\begin{aligned} e^{uc}_P &= \frac{\tau_{p^{uc}}}{\tau_{p^{uc}} + \tau_{ss}},\ e^{uc}_{D_{ui}} = \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^{uc}}}{\tau_{p^{uc}} + \tau_{ss}} \frac{(1 - w_i)}{w_i \rho \tau_s}, \\ e^{uc}_\lambda &= \frac{\tau_s}{\tau_s + \tau_v} \frac{1}{\sqrt{\tau_{p^{uc}} + \tau_{ss}}},\ e^{uc}_0 = \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^{uc}}}{\tau_{p^{uc}} + \tau_{ss}} \frac{\overline{m}}{w_i \rho \tau_s}, \quad \text{and} \end{aligned}$$

$$\begin{aligned} V^{\overline{\{bc\}}} &= Var\left[v| \left\{P = P^{uc}, D_{ui} = D^*_{ui}, (s,m) \in \overline{\{bc\}}\right\}\right] \\ &= Var\left[v| \left\{P = p^{uc}_s s - p^{uc}_m \hat{m}, D_{ui} = D^*_{ui}, s \leq \kappa^{bc}, \hat{m} \leq \eta^{bc}, \right\}\right] \\ &= v_0 + v^{uc}_\delta(1 - \delta^{bc-}_{uc}), \end{aligned} \tag{D4}$$

where

$$v_0 = \frac{1}{\tau_v + \tau_s}, \; v_\delta^{uc} = \left(\frac{\tau_s}{\tau_v + \tau_s}\right)^2 \left(\frac{1}{\tau_{p^{uc}} + \tau_{ss}}\right).$$

When some informed investors face only borrowing constraints, we can express uninformed investor inference on the conditional moments of the truncated normal variables in the following closed-form expressions.

$$\begin{aligned} E^{sc} &= E[v| \{P = P^{sc}, D_{ui} = D_{ui}^*, (s,m) \in \{sc\}\}] \\ &= E\left[v| \{P = p_s^{sc} s - p_m^{sc}(\hat{m} - w_i^c d), D_{ui} = D_{ui}^*, s \leq \kappa^{sc}, \hat{m} \geq \eta^{sc}\}\right] \\ &= e_P^{sc} P - e_{D_{ui}}^c D_{ui}^* + e_\lambda^c \lambda_c^{sc-} + e_0^{sc}, \end{aligned} \tag{D5}$$

where

$$e_P^{sc} = \frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}}, \; e_0^{sc} = \frac{\tau_s}{\tau_s + \tau_v} \frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}} \left(\frac{\overline{m} - w_i^c d}{w_i^{uc} \tau_s \rho}\right), \quad \text{and}$$

$$\begin{aligned} V^{sc} &= Var[v| \{P = P^{sc}, D_{ui} = D_{ui}^*, (s,m) \in \{sc\}\}] \\ &= Var\left[v| \{P = p_s^{sc} s - p_m^{sc}(\hat{m} - w_i^c d), D_{ui} = D_{ui}^*, s \leq \kappa^{sc}, \hat{m} \geq \eta^{sc}\}\right] \\ &= v_0 + v_\delta^c (1 - \delta_c^{sc-}). \end{aligned} \tag{D6}$$

$$\begin{aligned} E^{\overline{\{sc\}}} &= E\left[v| \left\{P = P^{uc}, D_{ui} = D_{ui}^*, (s,m) \in \overline{\{sc\}}, \right\}\right] \\ &= E\left[v| \{P = p_s^{uc} s - p_m^{uc} \hat{m}, D_{ui} = D_{ui}^*, s \geq \kappa^{sc}, \hat{m} \geq \eta^{sc}\}\right] \\ &= e_P^{uc} P - e_{D_{ui}}^{uc} D_{ui}^* + e_\lambda^{uc} \lambda_{uc}^{sc+} + e_0^{uc}, \text{ and} \end{aligned} \tag{D7}$$

$$\begin{aligned} V^{\overline{\{sc\}}} &= Var[v| \left\{P = P^{uc}, D_{ui} = D_{ui}^*, (s,m) \in \overline{\{sc\}}\right\}] \\ &= Var\left[v| \{P = p_s^{uc} s - p_m^{uc} \hat{m}, D_{ui} = D_{ui}^*, s \geq \kappa^{sc}, \hat{m} \geq \eta^{sc}, \}\right] \\ &= v_0 + v_\delta^{uc} (1 - \delta_{uc}^{sc+}). \end{aligned} \tag{D8}$$

When both borrowing and short-sale constraints exist but are not binding for informed investors, the distribution of their signals is doubly truncated normal. In this case, the conditional moments of the asset's fundamental have the following closed-form expressions..

$$
\begin{aligned}
E^{\overline{\{\{bc\}\bigcup\{sc\}\}}} &= E\left[v|\left\{P=P^{uc}, D_{ui}=D_{ui}^{*}, (s,m)\in\overline{\{\{bc\}\bigcup\{sc\}\}}\right\}\right] \\
&= E\left[v|\left\{\widetilde{P}=p_s^{uc}\widetilde{s}-p_m^{uc}\hat{m}, D_{ui}=D_{ui}^{*}, \kappa^{sc}\leq s\leq\kappa^{bc}, \eta^{sc}\leq\hat{m}\leq\eta^{bc}\right\}\right] \\
&= e_0^{uc}+e_\lambda^{uc}\frac{\phi(\theta_{uc}^{bc})-\phi(\theta_{uc}^{sc})}{\Phi(\theta_{uc}^{sc})-\Phi(\theta_{uc}^{bc})}, \text{ and} \qquad \text{(D9)}
\end{aligned}
$$

$$
\begin{aligned}
V^{\overline{\{\{bc\}\bigcup\{sc\}\}}} &= Var\left[v|\left\{P=P^{uc}, D_{ui}=D_{ui}^{*}, (s,m)\in\overline{\{\{bc\}\bigcup\{sc\}\}}\right\}\right] \\
&= Var\left[v|\left\{\widetilde{P}=p_s^{uc}\widetilde{s}-p_m^{uc}\hat{m}, D_{ui}=D_{ui}^{*}, \kappa^{sc}\leq s\leq\kappa^{bc}, \eta^{sc}\leq\hat{m}\leq\eta^{bc}\right\}\right] \\
&= v_0+v_\delta^{uc}\left(1+\frac{\theta_{uc}^{bc}\phi(\theta_{uc}^{bc})-\theta_{uc}^{sc}\phi(\theta_{uc}^{sc})}{\Phi(\theta_{uc}^{sc})-\Phi(\theta_{uc}^{bc})}-\left(\frac{\phi(\theta_{uc}^{bc})-\phi(\theta_{uc}^{sc})}{\Phi(\theta_{uc}^{sc})-\Phi(\theta_{uc}^{bc})}\right)^2\right). \qquad \text{(D10)}
\end{aligned}
$$

When informed investors do not face any short-sale or borrowing constraints, the inference problem is standard.

$$
\begin{aligned}
E^{uc} &= E\left[v|\left\{P=P^{uc}, D_{ui}=D_{ui}^{*},\right\}\right] \\
&= E\left[v|\left\{\widetilde{P}=p_s^{uc}\widetilde{s}-p_m^{uc}\hat{m}, D_{ui}=D_{ui}^{*}\right\}\right] \\
&= e_P^{uc}P-e_{D_{ui}}^{uc}D_{ui}^{*}+e_0^{uc}, \text{ and} \qquad \text{(D11)}
\end{aligned}
$$

$$
\begin{aligned}
V^{uc} &= Var\left[v|\left\{P=P^{uc}, D_{ui}=D_{ui}^{*},\right\}\right] \\
&= Var\left[v|\left\{\widetilde{P}=p_s^{uc}\widetilde{s}-p_m^{uc}\hat{m}, D_{ui}=D_{ui}^{*}\right\}\right] \\
&= v_0+v_\delta^{uc}. \qquad \text{(D12)}
\end{aligned}
$$

The results in the corollaries are immediate through a comparison of the above terms.

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